Journal of Cosmology and
Astroparticle Physics

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Journal of Cosmology and Astroparticle Physics
An IOP and SISSA journal

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Received December 5, 2017
Revised March 26, 2018
Accepted May 17, 2018
Published May 30, 2018

Abstract. This paper aims to put constraints on the transition redshift $z_{\rm t}$, which determines the onset of cosmic acceleration, in cosmological-model independent frameworks. In order to perform our analyses, we consider a flat universe and assume a parametrization for the comoving distance $D_{\rm C}(z)$ up to third degree on z, a second degree parametrization for the Hubble parameter $H(z)$ and a linear parametrization for the deceleration parameter $q(z)$. For each case, we show that type Ia supernovae and $H(z)$ data complement each other on the parameter space and tighter constrains for the transition redshift are obtained. By combining the type Ia supernovae observations and Hubble parameter measurements it is possible to constrain the values of $z_{\rm t}$, for each approach, as 0.806 ± 0.094, 0.870 ± 0.063 and 0.973 ± 0.058 at 1σ c.l., respectively. Then, such approaches provide cosmological-model independent estimates for this parameter.

Keywords: redshift surveys, supernova type Ia - standard candles

ArXiv ePrint: 1712.01075

https://doi.org/10.1088/1475-7516/2018/05/073

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Contents

1 Introduction

The idea of a late time accelerating universe is indicated by type Ia supernovae (SNe Ia) observations [1–8] and confirmed by other independent observations such as Cosmic Microwave Background (CMB) radiation [9–11], Baryonic Acoustic Oscillations (BAO) [12–16] and Hubble parameter, $H(z)$, measurements [17–19]. The simplest theoretical model supporting such accelerating phase is based on a cosmological constant Λ term [20, 21] plus a Cold Dark Matter component [22–24], the so-called ΛCDM model. The cosmological parameters of such model have been constrained more and more accurately [11, 18, 25] as new observations are added. Beyond a constant Λ based model, several other models have been also suggested recently in order to explain the accelerated expansion. The most popular ones are based on a dark energy fluid [26, 27] endowed with a negative pressure filling the whole universe. The nature of such exotic fluid is unknown, sometimes attributed to a single scalar field or even to mass dimension one fermionic fields. There are also modified gravity theories that correctly describe an accelerated expansion of the Universe, such as: massive gravity theories [28], modifications of Newtonian theory (MOND) [29–31], $f(R)$ and $f(T)$ theories that generalize the general relativity [32–34], models based on extra dimensions, as brane world models [35–39], string [40] and Kaluza-Klein theories [41], among others. Having adopted a particular model, the cosmological parameters can be determined by using statistical analysis of observational data.

However, some works have tried to explore the history of the universe without to appeal to any specific cosmological model. Such approaches are sometimes called cosmography or cosmokinetic models [42–47], and we will refer to them simply as kinematic models. This nomenclature comes from the fact that the complete study of the expansion of the Universe (or its kinematics) is described just by the Hubble expansion rate $H = \dot{a}/a$, the deceleration parameter $q = -a\ddot{a}/\dot{a}^2$ and the jerk parameter $j = -\dddot{a}a^3/(a\dot{a}^3)$, where a is the scale factor in the Friedmann-Roberson-Walker (FRW) metric. The only assumption is that space-time is homogeneous and isotropic. In such parametrization, a simple dark matter dominated universe has $q = 1/2$ while the accelerating ΛCDM model has $j = -1$. The deceleration parameter allows to study the transition from a decelerated phase to an accelerated one,

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while the jerk parameter allows to study departures from the cosmic concordance model, without restricting to a specific model.

Concerning the deceleration parameter, several studies have attempted to estimate at which redshift z_t the universe undergoes a transition to accelerated phase [18, 19, 44, 48–50, 54, 55]. A model independent determination of the present deceleration parameter q_0 and deceleration-acceleration transition redshift z_t is of fundamental importance in modern cosmology. As a new cosmic parameter [54], it should be used to test several cosmological models.

In order to study the deceleration parameter in a cosmological-model independent framework, it is necessary to use some parametrization for it. This methodology has both advantages and disadvantages. One advantage is that it is independent of the matter and energy content of the universe. One disadvantage of this formulation is that it does not explain the cause of the accelerated expansion. Furthermore, the value of the present deceleration parameter may depend on the assumed form of $q(z)$. One of the first analyses and constraints on cosmological parameters of kinematic models was done by Elgarøy and Multamäki [46] by employing Bayesian marginal likelihood analysis. Since then, several authors have implemented the analysis by including new data sets and also different parametrizations for $q(z)$.

For a linear parametrization, $q(z) = q_0 + q_1 z$, the values for q_0 and z_t found by Cunha and Lima [48] were $q_0 \sim -0.7$, $z_t = 0.43^{+0.09}_{-0.05}$ from 182 SNe Ia of Riess et al. [4], $-1.17 \leq q_0 \leq -0.16$, $z_t = 0.61^{+3.68}_{-0.21}$ from SNLS data set [3] and $-1.0 \leq q_0 \leq -0.36$, $z_t = 0.60^{+0.28}_{-0.11}$ from Davis et al. data set [5]. Guimarães, Cunha and Lima [49] found $q_0 = -0.71 \pm 0.21$ and $z_t = 0.49^{+0.27}_{-0.09}$ using a sample of 307 SNe Ia from Union compilation [6]. Also, for the linear parametrization, Rani et al. [50] found $q_0 = -0.52 \pm 0.12$ and $z_t \approx 0.98$ using a joint analysis of age of galaxies, strong gravitational lensing and SNe Ia data. For a parametrization of type $q(z) = q_0 + q_1 z/(1+z)$, Xu, Li and Lu [51] used 307 SNe Ia together with BAO and $H(z)$ data and found $q_0 = -0.715 \pm 0.045$ and $z_t = 0.609^{+0.110}_{-0.070}$. For the same parametrization, Holanda, Alcaniz and Carvalho [52] found $q_0 = -0.85^{+1.35}_{-1.25}$ by using galaxy clusters of elliptical morphology based on their Sunyaev-Zeldovich effect (SZE) and X-ray observations. Such parametrization has also been studied in [46, 48].

As one may see, the determination of the deceleration parameter is a relevant subject in modern cosmology, as well as the determination of the Hubble parameter H_0. In seventies, Sandage [53] foretold that the determination of H_0 and q_0 would be the main role of cosmology for the forthcoming decades. The inclusion of the transition redshift z_t as a new cosmic discriminator has been advocated by some authors [54]. An alternative method to access the cosmological parameters in a model independent fashion is by means of the study of $H(z) = \dot{a}/a = -[1/(1+z)]dz/dt$. In the so called *cosmic chronometer* approach, the quantity dz is obtained from spectroscopic surveys and the only quantity to be measured is the differential age evolution of the universe (dt) in a given redshift interval (dz). By using the results from Baryon Oscillation Spectroscopic Survey (BOSS) [14–16], Moresco et al. [55] have obtained a cosmological-model independent determination of the transition redshift as $z_t = 0.4 \pm 0.1$ (see also [17–19]).

In the present work we study the transition redshift by means of a third order parametrization of the comoving distance, a second order parametrization of $H(z)$ and a linear parametrization of $q(z)$. By combining luminosity distances from SNe Ia [56] and $H(z)$ measurements, it is possible to determine z_t values in these cosmological-model independent frameworks. In such approach we obtain an interesting complementarity between the observational data and, consequently, tighter constraints on the parameter spaces.

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The paper is organized as follows. In section 2, we present the basic equations concerning the obtainment of z_t from luminosity distance, $H(z)$ and $q(z)$. Section 3 presents the data set used and the analyses are presented in section 4. Conclusions are left to section 5.

2 Basic equations

Let us discuss (from a more observational viewpoint) the possibility to enlarge Sandage's vision by including the transition redshift, z_t, as the third cosmological number. To begin with, consider the general expression for the deceleration parameter $q(z)$ as given by:

$$q(z) = -\frac{\ddot{a}}{aH^2} = \frac{1+z}{H}\frac{dH}{dz} - 1\,, \tag{2.1}$$

from which the transition redshift, z_t, can be defined as $q(z_t) = 0$, leading to:

$$z_t = \left[\frac{d\ln H(z)}{dz}\right]^{-1}_{|z=z_t} - 1. \tag{2.2}$$

Let us assume a flat Friedmann-Robertson-Walker cosmology. In such a framework, the luminosity distance, d_L (in Mpc), is given by:

$$d_L(z) = (1+z)d_C(z), \tag{2.3}$$

where d_C is the comoving distance:

$$d_C(z) = c\int_0^z \frac{dz'}{H(z')}, \tag{2.4}$$

with c being the speed of light in km/s and $H(z)$ the Hubble parameter in km/s/Mpc. For mathematical convenience, we choose to work with dimensionless quantities. Then, we define the dimensionless distances, $D_C \equiv \frac{d_C}{d_H}$, $D_L \equiv \frac{d_L}{d_H}$, $d_H \equiv c/H_0$ and the dimensionless Hubble parameter, $E(z) \equiv \frac{H(z)}{H_0}$. Thus, we have:

$$D_L(z) = (1+z)D_C(z), \tag{2.5}$$

and

$$D_C(z) = \int_0^z \frac{dz'}{E(z')}, \tag{2.6}$$

from which follows

$$E(z) = \left[\frac{dD_C(z)}{dz}\right]^{-1}. \tag{2.7}$$

From (2.2) we also have:

$$z_t = \left[\frac{d\ln E(z)}{dz}\right]^{-1}_{|z=z_t} - 1. \tag{2.8}$$

Therefore, from a formal point of view, we may access the value of z_t through a parametrization of both $q(z)$ and $H(z)$, at least around a redshift interval involving the transition redshift. As a third method we can also parametrize the co-moving distance, which is directly related to the luminosity distance, in order to study the transition redshift. In which follows we present the three different methods considered here.

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2.1 $z_{\rm t}$ from comoving distance, $D_{\rm C}(z)$

In order to put limits on $z_{\rm t}$ by considering the comoving distance, we can write $D_{\rm C}(z)$ by a third degree polynomial such as:

$$D_{\rm C} = z + d_2 z^2 + d_3 z^3. \tag{2.9}$$

where d_2 and d_3 are free parameters. Naturally, from eqs. (2.7) and (2.9), one obtains

$$E(z) = \frac{1}{1 + 2d_2 z + 3d_3 z^2}. \tag{2.10}$$

Solving eq. (2.8) with $E(z)$ given by (2.10), we find

$$z_{\rm t} = \frac{-2d_2 - 3d_3 \pm \sqrt{4d_2^2 - 6d_2 d_3 + 9d_3^2 - 9d_3}}{9d_3}, \tag{2.11}$$

where we may see there are two possible solutions to $z_{\rm t}$. From a statistical point of view, aiming to constrain $z_{\rm t}$, maybe it is better to write the coefficient d_3 in terms of $z_{\rm t}$ and d_2 via eq. (2.8) as

$$d_3(d_2, z_{\rm t}) = -\frac{1 + 2d_2 + 4d_2 z_{\rm t}}{3z_{\rm t}(2 + 3z_{\rm t})}. \tag{2.12}$$

Then

$$E(z) = \left[1 + 2d_2 z - \frac{1 + 2d_2 + 4d_2 z_{\rm t}}{z_{\rm t}(2 + 3z_{\rm t})} z^2\right]^{-1}. \tag{2.13}$$

Finally, from eqs. (2.5), (2.9) and (2.12) the dimensionless luminosity distance is

$$D_{\rm L}(z) = (1 + z)\left[z + d_2 z^2 - \frac{1 + 2d_2 + 4d_2 z_{\rm t}}{3z_{\rm t}(2 + 3z_{\rm t})} z^3\right]. \tag{2.14}$$

Equations (2.14) and (2.13) are to be compared with luminosity distances from SNe Ia and $H(z)$ measurements, respectively, in order to determine $z_{\rm t}$ and d_2.

2.2 $z_{\rm t}$ from $H(z)$

In order to assess $z_{\rm t}$ from eq. (2.2) by means of $H(z)$ we need an expression for $H(z)$. If one wants to avoid dynamical assumptions, one must to resort to kinematical methods which uses an expansion of $H(z)$ over the redshift.

The simplest expansion of $H(z)$ over the redshift, the linear expansion, gives no transition. To realize this, let us take

$$\frac{H(z)}{H_0} = E(z) = 1 + h_1 z. \tag{2.15}$$

From (2.8), we have

$$1 + z_{\rm t} = \left[\frac{d \ln E(z)}{dz}\right]^{-1}_{z=z_{\rm t}} = \frac{1}{h_1} + z_{\rm t} \Rightarrow \frac{1}{h_1} = 1. \tag{2.16}$$

Therefore, the transition redshift is undefined in this case.

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Let us now try the next simplest $H(z)$ expansion, namely, the quadratic expansion:

$$\frac{H(z)}{H_0} = E(z) = 1 + h_1 z + h_2 z^2. \tag{2.17}$$

In this case, inserting (2.10) into (2.16), we are left with:

$$1 + z_t = \frac{1 + h_1 z_t + h_2 z_t^2}{h_1 + 2h_2 z_t} \Rightarrow (1 + z_t)(h_1 + 2h_2 z_t) = 1 + h_1 z_t + h_2 z_t^2, \tag{2.18}$$

from which follows an equation for z_t:

$$h_2 z_t^2 + 2h_2 z_t + h_1 - 1 = 0, \tag{2.19}$$

whose solution is:

$$z_t = -1 \pm \sqrt{1 + \frac{1 - h_1}{h_2}}. \tag{2.20}$$

We may exclude the negative root, which would give $z_t < -1$ and this value is not possible (negative scale factor). Thus, if one obtains the h_1 and h_2 coefficients from a fit to $H(z)$ data, one may obtain a model independent estimate of transition redshift from

$$z_t = -1 + \sqrt{1 + \frac{1 - h_1}{h_2}}. \tag{2.21}$$

Equation (2.21) already is an interesting result, and shows the reliability of the quadratic model as a kinematic assessment of transition redshift. It is easy to see that taking $h_2 = 0$ into (2.19) does not furnish any information about the transition redshift.

In order to constrain the model with SNe Ia data, we obtain the luminosity distance from eqs. (2.5), (2.6) and (2.17). We have

$$D_C = \int_0^z \frac{dz'}{E(z')} = \int_0^z \frac{dz'}{1 + h_1 z' + h_2 z'^2}, \tag{2.22}$$

which gives three possible solutions, according to the sign of $\Delta \equiv h_1^2 - 4h_2$ such as

$$D_C = \begin{cases} \frac{2}{\sqrt{-\Delta}} \left[\arctan\left(\frac{2h_2 z + h_1}{\sqrt{-\Delta}}\right) - \arctan\frac{h_1}{\sqrt{-\Delta}}\right], & \Delta < 0 \\ \frac{2z}{h_1 z + 2}, & \Delta = 0 \\ \frac{1}{\sqrt{\Delta}} \ln\left|\left(\frac{\sqrt{\Delta} + h_1}{\sqrt{\Delta} - h_1}\right)\left(\frac{\sqrt{\Delta} - h_1 - 2h_2 z}{\sqrt{\Delta} + h_1 + 2h_2 z}\right)\right|, & \Delta > 0\,. \end{cases} \tag{2.23}$$

However, in order to obtain the likelihood for the transition redshift, we must reparametrize the eq. (2.17) to show its explicit dependency on this parameter. Notice also that from eq. (2.21) we may eliminate the parameter h_1:

$$h_1 = 1 + h_2[1 - (1 + z_t)^2], \tag{2.24}$$

thus we may write $D_C(z)$ from (2.23) just in terms of z_t and h_2, from which follows the luminosity distance $D_L = D_C(z)(1 + z)$.

2.3 z_t from $q(z)$

Now let us see how to assess z_t by means of a parametrization of $q(z)$. From (2.1) one may find $E(z)$

$$E(z) = \exp\left[\int_0^z \frac{1+q(z')}{1+z'}dz'\right]. \tag{2.25}$$

If we assume a linear z dependence in $q(z)$, as

$$q(z) = q_0 + q_1 z, \tag{2.26}$$

which is the simplest $q(z)$ parametrization that allows for a transition, one may find

$$E(z) = e^{q_1 z}(1+z)^{1+q_0-q_1}, \tag{2.27}$$

while the comoving distance $D_C(z)$ (2.6) is given by

$$D_C(z) = e^{q_1} q_1^{q_0-q_1}\left[\Gamma(q_1-q_0, q_1) - \Gamma(q_1-q_0, q_1(1+z))\right], \tag{2.28}$$

where $\Gamma(a,x)$ is the incomplete gamma function defined by [57] as $\Gamma(a,x) \equiv \int_x^\infty e^{-t}t^{a-1}dt$, with $a > 0$.

From (2.26) (or from (2.27) and (2.8)) it is easy to find:

$$z_t = -\frac{q_0}{q_1} \qquad \text{and} \qquad q_0 = -q_1 z_t, \tag{2.29}$$

from which follows $D_C(z)$ and $D_L(z)$ as a function of just z_t and q_1, which can be constrained from observational data.

3 Samples

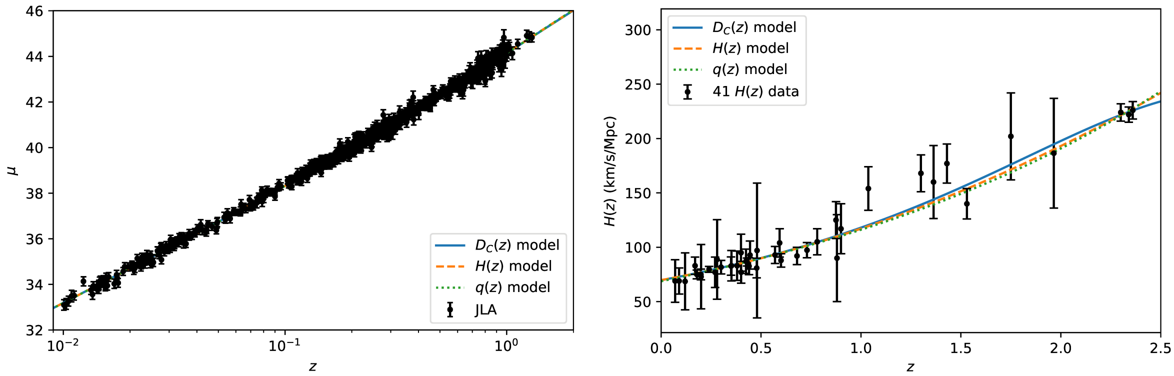
3.1 $H(z)$ data

Hubble parameter data in terms of redshift yields one of the most straightforward cosmological tests because it is inferred from astrophysical observations alone, not depending on any background cosmological models.

At the present time, the most important methods for obtaining $H(z)$ data are[1] (i) through "cosmic chronometers", for example, the differential age of galaxies (DAG), (ii) measurements of peaks of acoustic oscillations of baryons (BAO) and (iii) through correlation function of luminous red galaxies (LRG).

The data we work here are a combination of the compilations from Sharov and Vorontsova [58] and Moresco et al. [55] as described on Jesus et al. [59]. Sharov and Vorontsova [58] added 6 $H(z)$ data in comparison to Farooq and Ratra [18] compilation, which had 28 measurements. Moresco et al. [55], on their turn, have added 7 new $H(z)$ measurements in comparison to Sharov and Vorontsova [58]. By combining both datasets, Jesus et al. [59] have arrived at 41 $H(z)$ data, as can be seen on table 1 of [59] and figure 1b here.

[1]See [54] for a review.



(a) **(b)**

Figure 1. a) SNe Ia distance moduli from JLA. The data for μ were estimated from eq. (4.2), with SNe Ia parameters from the $D_\mathrm{C}(z)$ model (table 1) and the error bars comes from the diagonal of the covariance matrix. The lines represent the best fit from SNe+$H(z)$ data for each model. b) 41 $H(z)$ data compilation. The lines represent the best fit from SNe+$H(z)$ data for each model.

3.2 JLA SNe Ia compilation

The JLA compilation [56] consists of 740 SNe Ia from the SDSS-II [60] and SNLS [61] collaborations. Actually, this compilation produced recalibrated SNe Ia light-curves and associated distances for the SDSS-II and SNLS samples in order to improve the accuracy of cosmological constraints, limited by systematic measurement uncertainties, as, for instance, the uncertainty in the band-to-band and survey-to-survey relative flux calibration. The light curves have high quality and were obtained by using an improved SALT2 (Spectral Adaptive Light-curve Templates) method [56, 62–64]. The data set includes several low-redshift samples ($z < 0.1$), all three seasons from the SDSS-II ($0.05 \leq z \leq 0.4$) and three years from SNLS ($0.2 < z < 1.4$). See figure 1a and more details in next section.

4 Analyses and results

In our analyses, we have used flat priors over the parameters, so the posteriors are always proportional to the likelihoods. For $H(z)$ data, the likelihood distribution function is given by $\mathscr{L}_H \propto e^{-\frac{\chi^2_H}{2}}$, where

$$\chi^2_H = \sum_{i=1}^{41} \frac{\left[H_{\mathrm{obs},i} - H(z_i, H_0, z_\mathrm{t}, \theta_{\mathrm{mod},j})\right]^2}{\sigma^2_{H_i,\mathrm{obs}}}, \tag{4.1}$$

where $\theta_{\mathrm{mod},j}$ is the specific parameter for each model, namely d_2, h_2 or q_1, for $D_\mathrm{C}(z)$, $H(z)$, $q(z)$ parametrizations, respectively.

As explained on [56], we may assume that supernovae with identical color, shape and galactic environment have, on average, the same intrinsic luminosity for all redshifts. In this case, the distance modulus $\mu = 5\log_{10}(d_\mathrm{L}(\mathrm{pc})/10)$ may be given by

$$\mu = m^*_B - (M_B - \alpha \times X_1 + \beta \times C) \tag{4.2}$$

where X_1 describes the time stretching of the light-curve, C describes the supernova color at maximum brightness, m^*_B corresponds to the observed peak magnitude in the rest-frame

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B band, α, β and M_B are nuisance parameters. According to [61], M_B may depend on the host stellar mass ($M_{\rm stellar}$) as

$$M_B = \begin{cases} M_B^1 & \text{if } M_{\rm stellar} < 10^{10} M_\odot. \\ M_B^1 + \Delta_M & \text{otherwise.} \end{cases} \tag{4.3}$$

For SNe Ia from JLA, we have the likelihood $\mathscr{L}_{SN} \propto e^{-\frac{\chi^2_{SN}}{2}}$, where

$$\chi^2_{SN} = [\hat{\boldsymbol{\mu}}(\theta_{SN}) - \boldsymbol{\mu}(z, z_{\rm t}, \theta_{{\rm mod},j})]^T \boldsymbol{C}^{-1} [\hat{\boldsymbol{\mu}}(\theta_{SN}) - \boldsymbol{\mu}(z, z_{\rm t}, \theta_{{\rm mod},j})] \tag{4.4}$$

where $\theta_{SN} = (\alpha, \beta, M_B^1, \Delta_M)$, $\boldsymbol{C}$ is the covariance matrix of $\hat{\boldsymbol{\mu}}$ as described on [56], $\boldsymbol{\mu}(z, z_{\rm t}, \theta_{{\rm mod},j}) = 5\log_{10}(d_{\rm L}(z, z_{\rm t}, \theta_{{\rm mod},j})/10\,{\rm pc})$ computed for a fiducial value $H_0 = 70\,{\rm km/s/Mpc}$.

In order to obtain the constraints over the free parameters, we have sampled the likelihood $\mathscr{L} \propto e^{-\chi^2/2}$ through Monte Carlo Markov Chain (MCMC) analysis. A simple and powerful MCMC method is the so-called Affine Invariant MCMC Ensemble Sampler by [65], which was implemented in Python language with the emcee software by [66]. This MCMC method has advantage over the simple Metropolis-Hastings (MH) method, since it depends only on one scale parameter of the proposed distribution and also on the number of walkers, while MH method is based on the parameter covariance matrix, that is, it depends on $n(n+1)/2$ tuning parameters, where n is the dimension of parameter space. The main idea of the Goodman-Weare affine-invariant sampler is the so called "stretch move", where the position (parameter vector in parameter space) of a walker (chain) is determined by the position of the other walkers. Foreman-Mackey et al. modified this method, in order to make it suitable for parallelization, by splitting the walkers in two groups, then the position of a walker in one group is determined *only* by the position of walkers of the other group.[2]

We used the freely available software emcee to sample from our likelihood in n-dimensional parameter space. We have used flat priors over the parameters. In order to plot all the constraints on each model in the same figure, we have used the freely available software getdist,[3] in its Python version. The results of our statistical analyses can be seen on figures 2–8 and on table 1.

In figures 2–4, we have the combined results for each parametrization. As one may see, we have always chosen $z_{\rm t}$ as one of our fiducial parameter. The other parameters from each model is later obtained as derived parameters. As one may see in figures 2–4, the combination of JLA+$H(z)$ yields strong constraints over all parameters, especially $z_{\rm t}$. Also, we find negligible difference in the SNe Ia parameters for each model. We have also to emphasize that the constraints over H_0 comes just from $H(z)$ while for JLA H_0 is fixed. We choose not to include other constraints over H_0 due to the recent tension from different limits over the Hubble constant. At the end of this section, we compare our results with different constraints over H_0.

In figures 5–7, we show explicitly the independent constraints from JLA and $H(z)$ over the cosmological parameters. As one may see in figure 5, SNe Ia sets weaker constraints over $z_{\rm t}$ for $D_{\rm C}(z)$ parametrization. Almost all the $z_{\rm t}$ constraint comes just from $H(z)$. For d_2, $H(z)$ and JLA yields similar constraints. For d_3, $H(z)$ yields slightly better constraints. For figures 6 and 7, the constraints over $z_{\rm t}$ from SNe Ia are improved and one may see how SNe Ia and $H(z)$ complement each other in order to constrain the transition redshift. In figure 6,

[2]See [67] for a comparison among various MCMC sampling techniques.

[3]getdist is part of the great MCMC sampler and CMB power spectrum solver COSMOMC, by [68].

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Parameter	$D_C(z)$	$H(z)$	$q(z)$
$\boldsymbol{\alpha}$	$0.1412 \pm 0.0065 \pm 0.013$	$0.1412 \pm 0.0066 \pm 0.013$	$0.1408 \pm 0.0064 \pm 0.013$
$\boldsymbol{\beta}$	$3.105 \pm 0.080 \pm 0.16$	$3.101 \pm 0.082 \pm 0.16$	$3.094 \pm 0.080 \pm 0.16$
$\boldsymbol{M_B^1}$	$-19.073 \pm 0.044^{+0.089}_{-0.090}$	$-19.039 \pm 0.023^{+0.047}_{-0.045}$	$-19.033 \pm 0.023^{+0.045}_{-0.046}$
$\boldsymbol{\Delta_M}$	$-0.069 \pm 0.023 \pm 0.046$	$-0.071 \pm 0.023^{+0.045}_{-0.046}$	$-0.071 \pm 0.023^{+0.046}_{-0.047}$
$\boldsymbol{H_0}$	$69.1 \pm 1.5 \pm 3.0$	$68.8 \pm 1.6 \pm 3.2$	$68.6 \pm 1.6^{+3.3}_{-3.2}$
$\boldsymbol{z_t}$	$0.806 \pm 0.094^{+0.19}_{-0.18}$	$0.870 \pm 0.063^{+0.13}_{-0.12}$	$0.973 \pm 0.058^{+0.12}_{-0.11}$
$\boldsymbol{d_2}$	$-0.253 \pm 0.016^{+0.033}_{-0.031}$	—	—
d_3	$0.0299 \pm 0.0044^{+0.0085}_{-0.0090}$	—	—
h_1	—	$0.522 \pm 0.065 \pm 0.13$	—
$\boldsymbol{h_2}$	—	$0.192 \pm 0.026^{+0.051}_{-0.052}$	—
q_0	—	—	$-0.434 \pm 0.065 \pm 0.13$
$\boldsymbol{q_1}$	—	—	$0.446 \pm 0.062 \pm 0.12$

Table 1. Constraints from JLA+$H(z)$ for $D_C(z)$, $H(z)$ and $q(z)$ parametrizations. The parameters without bold faces were treated as derived parameters. The central values correspond to the mean and the 1 σ and 2 σ c.l. correspond to the minimal 68.3% and 95.4% confidence intervals.

one may see that the constraint over h_1 is better from JLA. The constraint over h_2 is better from $H(z)$. In figure 7, one may see that the constraint over q_0 is better from JLA. The constraint over q_1 is better from $H(z)$.

For all parametrizations, the best constraints over the transition redshift comes from $H(z)$ data, as first indicated by [54]. Moresco et al. [55] also found stringent constraints over z_t from $H(z)$ in their parametrization, however, they did not compare with SNe Ia constraints.

Figure 8 summarizes our combined constraints over z_t for each parametrization. As one may see, the $q(z)$ model yields the strongest constraints over z_t. The other parametrizations are important for us to realize how much z_t is still allowed to vary. All constraints are compatible at 1σ c.l.

Table 1 shows the full numerical results from our statistical analysis. As one may see the SNe Ia constraints vary little for each parametrization. In fact, we also have found constraints from the (faster) JLA binned data [56], however, when comparing with the (slower) full JLA constraints, we have found that the full JLA yields stronger constraints over the parameters, especially z_t. So we decided to deal only with the JLA full data.

By using 30 $H(z)$ data, plus H_0 from Riess et al. (2011) [69], Moresco et al. [55] found $z_t = 0.64^{+0.1}_{-0.06}$ for ΛCDM and $z_t = 0.4 \pm 0.1$ for their model independent approach. Only our $D_C(z)$ parametrization is compatible with their model-independent result. Their ΛCDM result is compatible with all our parametrizations, although it is marginally compatible with $q(z)$.

Another interesting result that can be seen in table 1 is the H_0 constraint. As one may see, the constraints over H_0 are consistent through the three different parametrizations,

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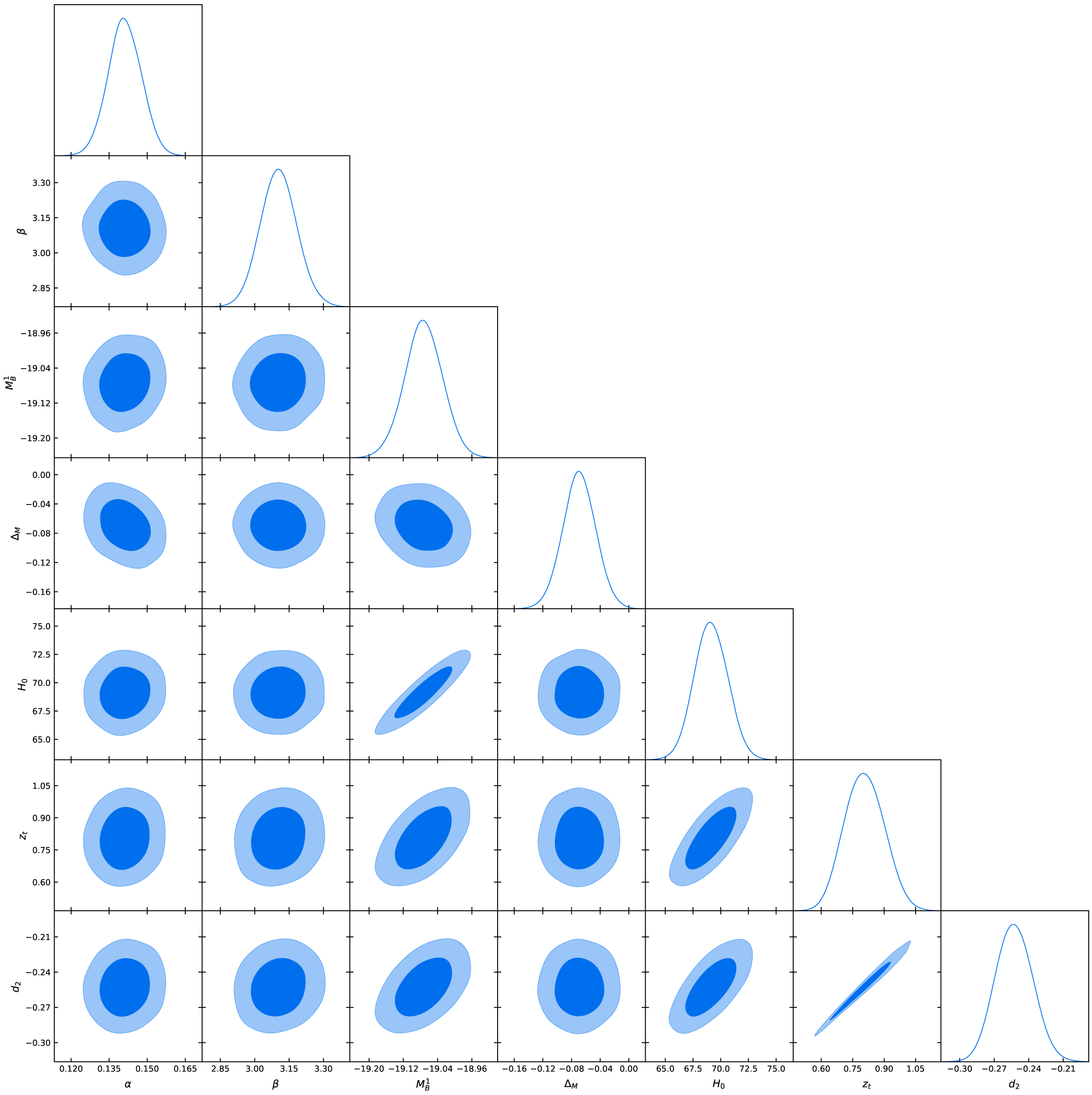

Figure 2. Combined constraints from JLA and $H(z)$ for $D_{\rm C}(z) = z + d_2 z^2 + d_3 z^3$.

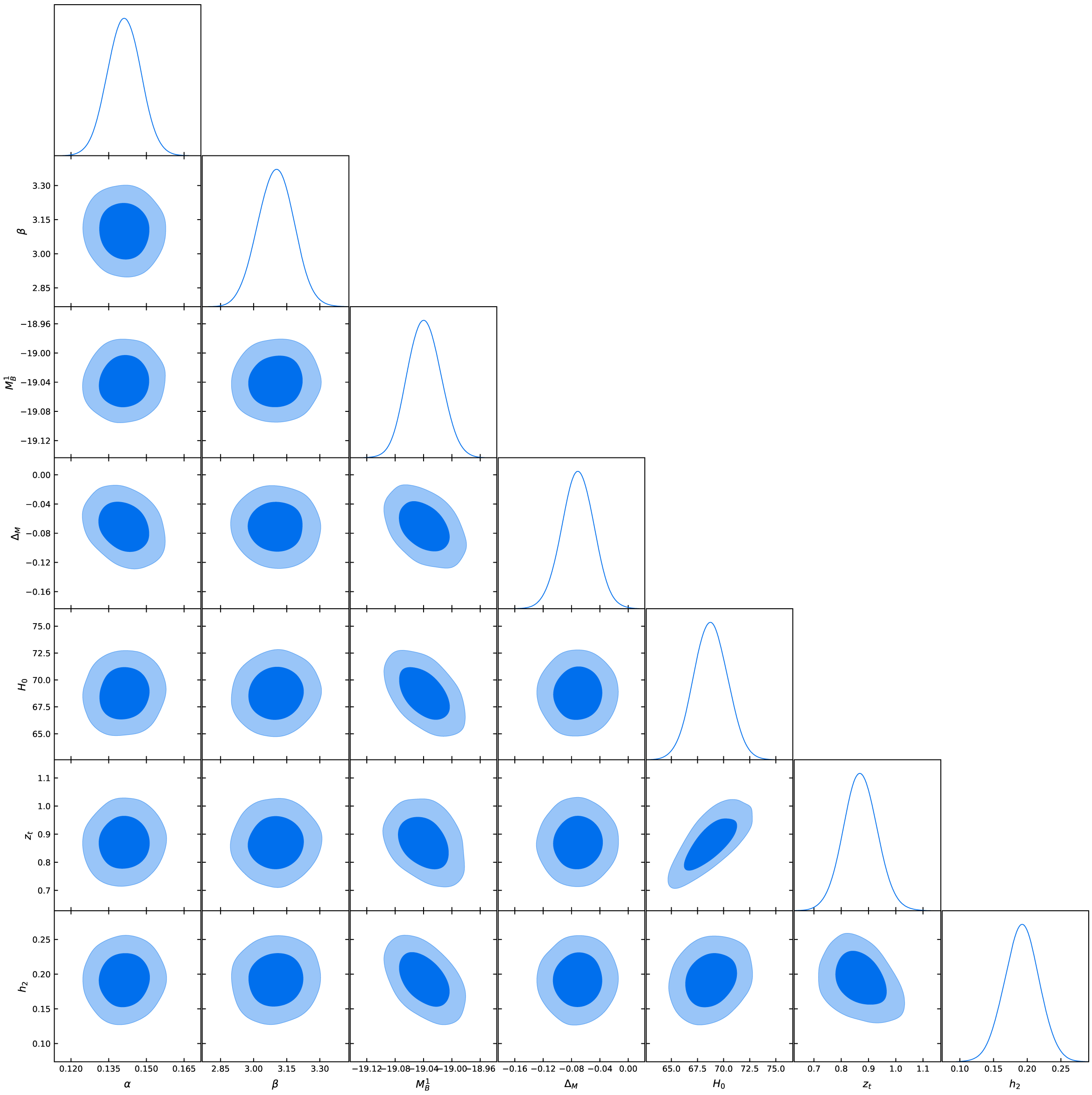

Figure 3. Combined constraints from JLA and $H(z)$ for $H(z) = H_0(1 + h_1 z + h_2 z^2)$.

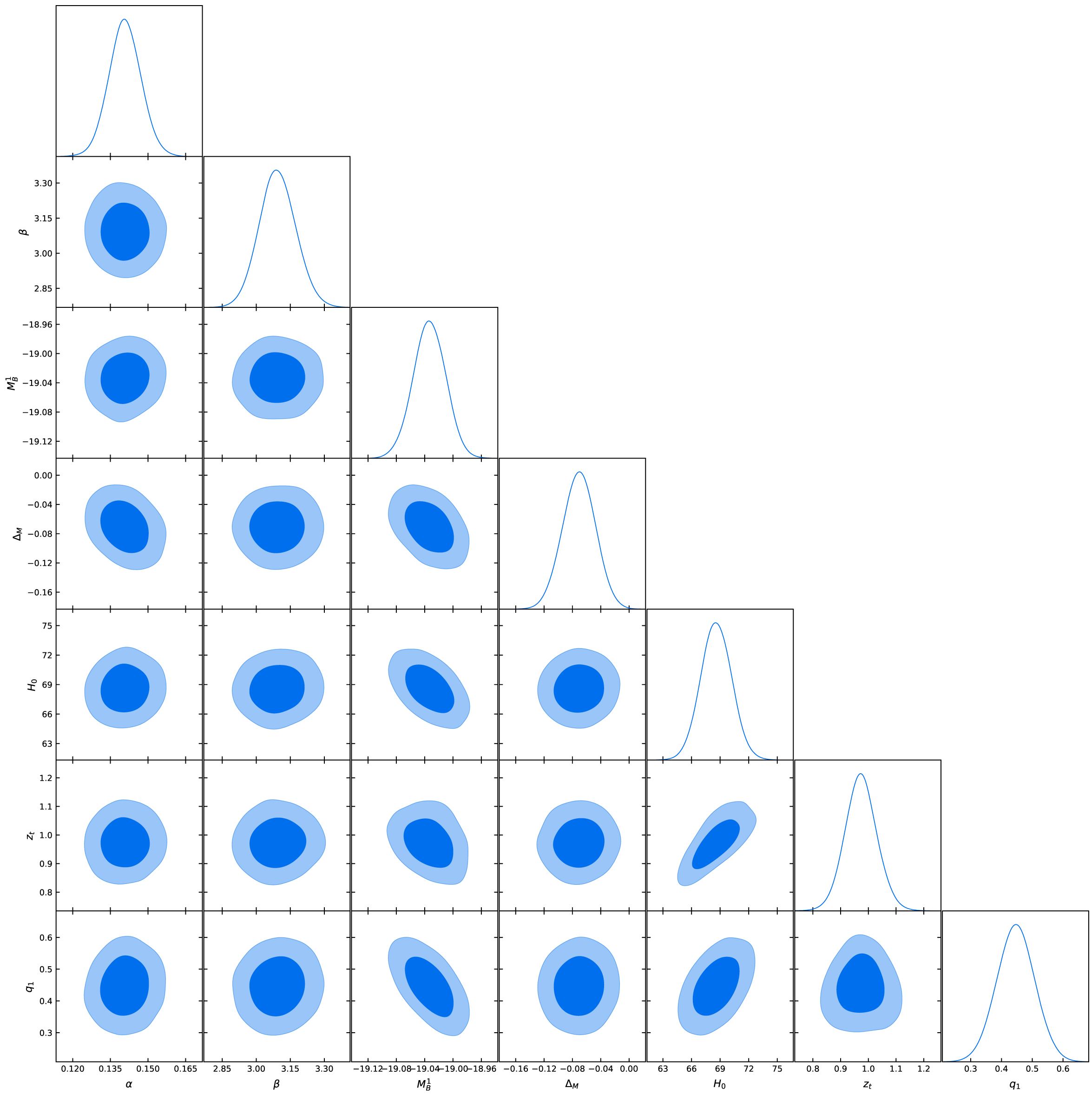

Figure 4. Combined constraints from JLA and $H(z)$ for $q(z) = q_0 + q_1 z$.

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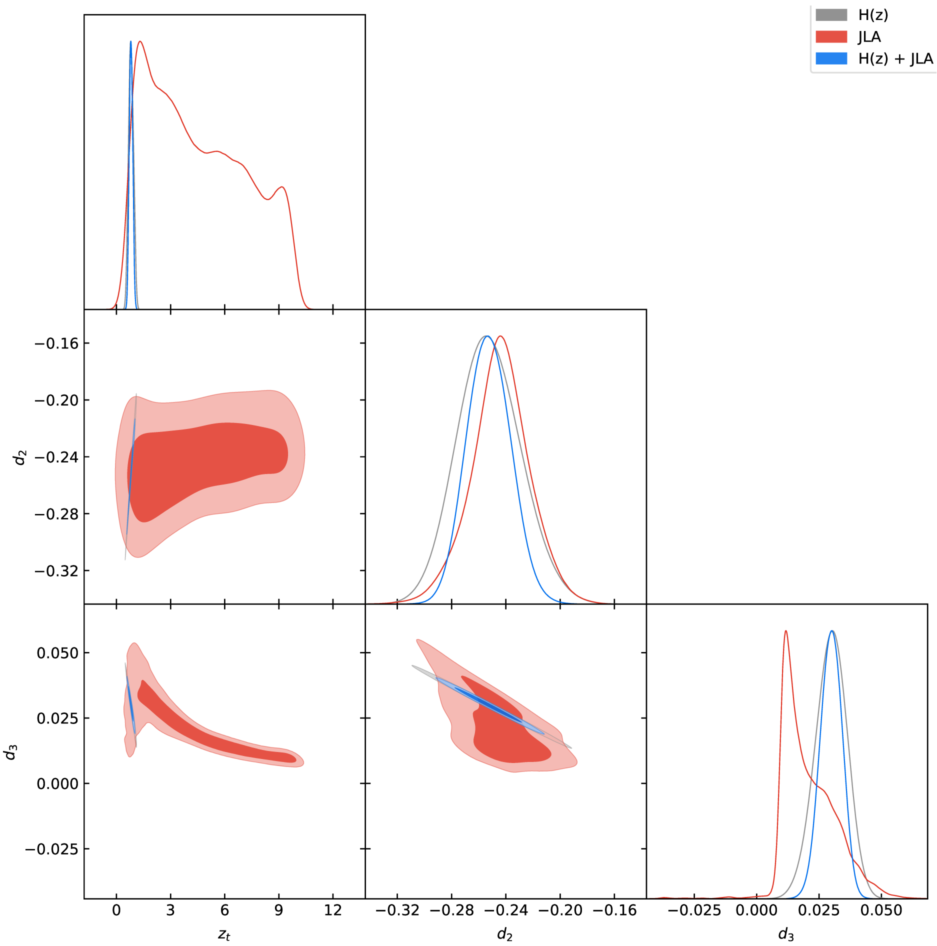

Figure 5. Constraints from JLA and $H(z)$ for $D_{\rm C}(z) = z + d_2 z^2 + d_3 z^3$.

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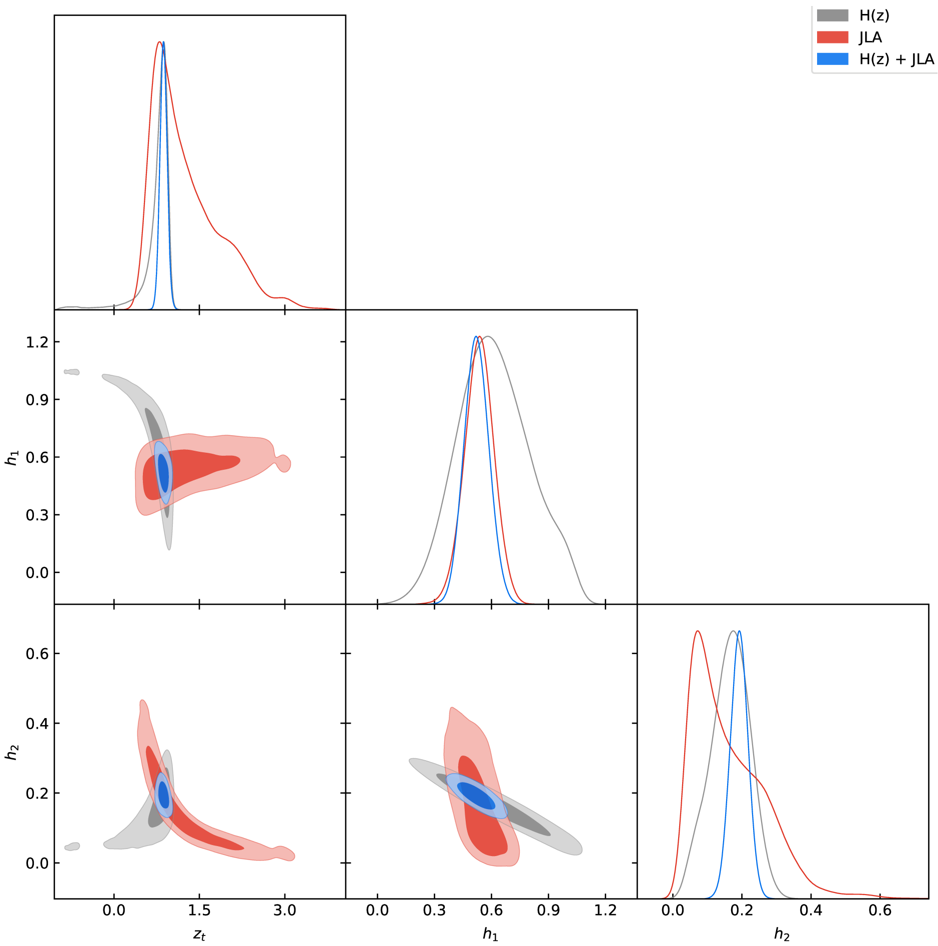

Figure 6. Constraints from JLA and $H(z)$ for $H(z) = H_0(1 + h_1 z + h_2 z^2)$.

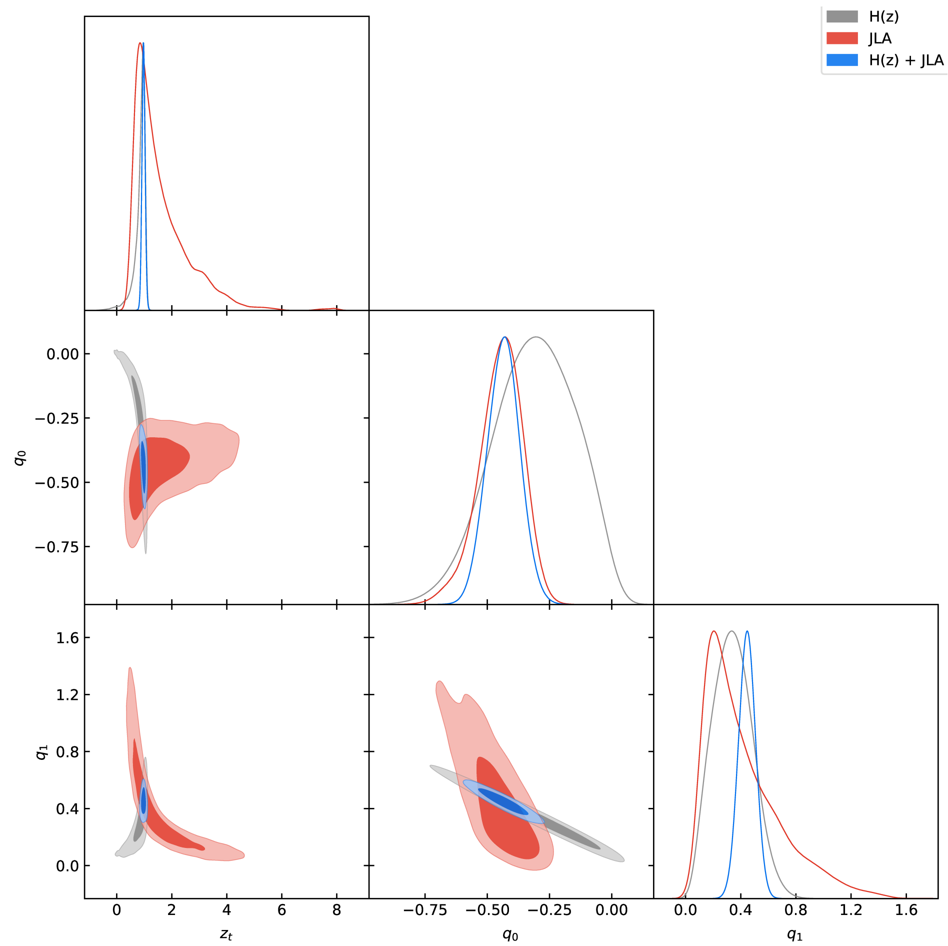

Figure 7. Constraints from JLA and $H(z)$ for $q(z) = q_0 + q_1 z$.

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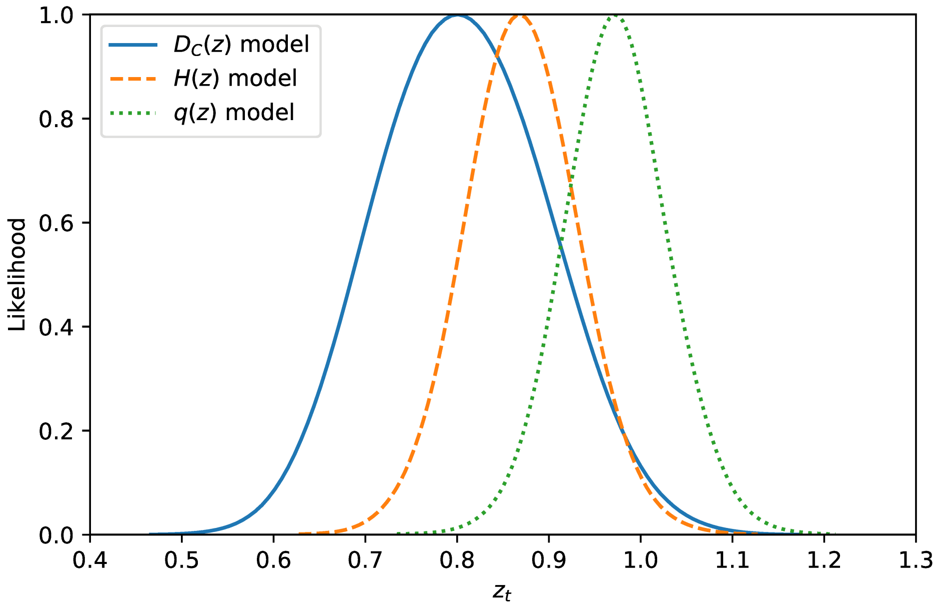

Figure 8. Likelihoods for transition redshift from JLA and $H(z)$ data combined. Blue solid line corresponds to $D_C(z)$ parametrization, orange long-dashed line corresponds to $H(z)$ parametrization and green short-dashed line corresponds to $q(z)$ parametrization.

with a little smaller uncertainty for $D_C(z)$, $H_0 = 69.1 \pm 1.5\,\mathrm{km/s/Mpc}$. The constraints over H_0 are quite stringent today from many observations [70, 71]. However, there is some tension among H_0 values estimated from Cepheids [70] and from CMB [71]. While Riess et al. advocate $H_0 = 73.24 \pm 1.74\,\mathrm{km/s/Mpc}$, the Planck collaboration analysis yields $H_0 = 66.93 \pm 0.62\,\mathrm{km/s/Mpc}$, a 3.4σ lower value. It is interesting to note, from our table 1 that, although we are working with model independent parametrizations and data at intermediate redshifts, our result is in better agreement with the high redshift result from Planck. In fact, all our results are compatible within 1σ with the Planck's result, while it is incompatible at 3σ with the Riess' result.

5 Conclusion

The accelerated expansion of Universe is confirmed by different sets of cosmological observations. Several models proposed in literature satisfactorily explain the transition from decelerated phase to the current accelerated phase. A more significant question is when the transition occurs from one phase to another, and the parameter that measures this transition is called the transition redshift, z_t. The determination of z_t is strongly dependent on the cosmological model adopted, thus the search for methods that allow the determination of such parameter in a model independent way are of fundamental importance, since it would serve as a test for several cosmological models.

In the present work, we wrote the comoving distance D_C, the Hubble parameter $H(z)$ and the deceleration parameter $q(z)$ as third, second and first degree polynomials on z, respectively (see equations (2.9), (2.17) and (2.26)), and obtained, for each case, the z_t value. Only a flat universe was assumed and the estimates for z_t were obtained, independent of a specific cosmological model. As observational data, we have used Supernovae type Ia and

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Hubble parameter measurements. Our results can be found in figures 2–7. As one may see from figures 5–7, the analyses by using SNe Ia (red color) and $H(z)$ data (blue color) are complementary to each other, providing tight limits in the parameter spaces. As a result, the values obtained for the transition redshift in each case were 0.806 ± 0.094, 0.870 ± 0.063 and 0.973 ± 0.058 at 1σ c.l., respectively (see figure 8).

Acknowledgments

JFJ is supported by Fundação de Amparo à Pesquisa do Estado de São Paulo — FAPESP (Process no. 2017/05859-0). RFLH acknowledges financial support from Conselho Nacional de Desenvolvimento Científico e Tecnológico (CNPq) and UEPB (No. 478524/2013-7, 303734/2014-0). SHP is grateful to CNPq, for financial support (No. 304297/2015-1, 400924/2016-1).

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