

# Constraints on the original ejection velocity fields of asteroid families

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Accepted 2016 January 5. Received 2016 January 5; in original form 2015 November 27

## ABSTRACT

Asteroid families form as a result of large-scale collisions among main belt asteroids. The orbital distribution of fragments after a family-forming impact could inform us about their ejection velocities. Unfortunately, however, orbits dynamically evolve by a number of effects, including the Yarkovsky drift, chaotic diffusion, and gravitational encounters with massive asteroids, such that it is difficult to infer the ejection velocities eons after each family's formation. Here, we analyse the *inclination* distribution of asteroid families, because proper inclination can remain constant over long time intervals, and could help us to understand the distribution of the component of the ejection velocity that is perpendicular to the orbital plane ( $v_W$ ). From modelling the initial break up, we find that the distribution of  $v_W$  of the fragments, which manage to escape the parent body's gravity, should be more peaked than a Gaussian distribution (i.e. be leptokurtic) even if the initial distribution was Gaussian. We surveyed known asteroid families for signs of a peaked distribution of  $v_W$  using a statistical measure of the distribution peakedness or flatness known as kurtosis. We identified eight families whose  $v_W$  distribution is significantly leptokurtic. These cases (e.g. the Koronis family) are located in dynamically quiet regions of the main belt, where, presumably, the initial distribution of  $v_W$  was not modified by subsequent orbital evolution. We suggest that, in these cases, the inclination distribution can be used to obtain interesting information about the original ejection velocity field.

**Key words:** celestial mechanics – Minor planets, asteroids: general.

## 1 INTRODUCTION

Asteroid families form as a result of collisions between asteroids. These events can either lead to a formation of a large crater on the parent body, from which fragments are ejected, or catastrophically disrupt it. More than 120 families are currently known in the main belt (Nesvorný, Brož & Carruba 2015) and the number of their members ranges from several thousands to just a few dozens, for the smaller and compact families. A lot of progress has been made in the last decades in developing sophisticated impact hydrocodes able to reproduce the main properties of families, mainly accounting for their size distribution, and, in a few cases (the Karin and Veritas clusters) the ejection velocities of their members (Michel et al. 2015). However, while the sizes of asteroids can be either measured directly through radar observations or occultations of stars, or inferred if the geometric albedo of the asteroid is known, correctly assessing ejection velocities is a more demanding task. The orbital element distribution of family members can, at least in principle, be converted into ejection velocities from Gauss' equations (Zappalà

et al. 1996), provided that both the true anomaly and the argument of perihelion of the family parent body are known (or assumed).

Orbital elements of family members, however, are not fixed in time, but can be changed by gravitational and non-gravitational effects, such as resonant dynamics (Morbidelli & Nesvorný 1999), close encounters with massive asteroids (Carruba et al. 2003), and Yarkovsky (Bottke et al. 2001) effects, etc. Separating which part of the current distribution in proper elements may be caused by the initial velocity field and which is the consequence of later evolution is a quite complex problem. Interested readers are referred to Vokrouhlický et al. (2006a,b,c) for a discussion of Monte Carlo methods applied to the distribution of asteroid families proper semimajor axis. Yet, insights into the distribution of the ejection velocities are valuable for better understanding of the physics of large-scale collisions (Nesvorný et al. 2006; Michel et al. 2015). They may help to calibrate impact hydrocodes, and improve models of the internal structure of asteroids.

Here, we analyse the inclination distribution of asteroid families. The proper inclination is the proper element least affected by dynamical evolution, and it could still bear signs of the original ejection velocity field. We find that a family formed in an event in which the ejection velocities were not much larger than the escape

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velocity from the parent body should be characterized by a peaked (leptokurtic) initial distribution (relative to a Gaussian), while families formed in hyper-velocity impacts, such as the case of the Eos family (Vokrouhlický et al. 2006a), should have either a normal or less peaked (platykurtic) distribution. The subsequent dynamical evolution should then act to bring this initial distribution to appear more Gaussian (or mesokurtic). The relative importance of the subsequent evolution depends on which specific proper element is considered, and on how active the local dynamics are. Using the proper inclination we attempt to identify cases where the local dynamics either did not have time or was not effective in erasing the initial, presumably leptokurtic, distributions. These cases can be used to better understand the conditions immediately after a parent body disruption.

This paper is divided as follows. In Section 2, we model the distribution of ejection velocities after a family-forming event. We explain how a peakedness of an expected distribution can be measured by the Pearson kurtosis. Section 3 shows how dynamics can modify the initial distribution by creating a new distribution that is more Gaussian in shape. In Section 4, we survey the known asteroid families, to understand in which cases the traces of the initially leptokurtic distribution can be found. Section 5 presents our conclusions.

## 2 MODEL FOR THE INITIAL $v_W$ DISTRIBUTION

Proper orbital elements can be related to the components of the velocity in infinity,  $v_{\text{inf}}$ , along the direction of the orbital motion ( $v_t$ ), in the radial direction ( $v_r$ ), and perpendicular to the orbital plane ( $v_W$ ) through the Gauss equations (Murray & Dermott 1999):

$$\frac{\delta a}{a} = \frac{2}{na(1-e^2)^{1/2}} [(1 + e \cos(f))\delta v_t + (e \sin(f))\delta v_r], \quad (1)$$

$$\delta e = \frac{(1-e^2)^{1/2}}{na} \left[ \frac{e + \cos(f) + e \cos^2(f)}{1 + e \cos(f)} \delta v_t + \sin(f)\delta v_r \right], \quad (2)$$

$$\delta i = \frac{(1-e^2)^{1/2}}{na} \frac{\cos(\omega + f)}{1 + e \cos(f)} \delta v_W, \quad (3)$$

where  $\delta a = a - a_{\text{ref}}$ ,  $\delta e = e - e_{\text{ref}}$ ,  $\delta i = i - i_{\text{ref}}$ , and  $a_{\text{ref}}$ ,  $e_{\text{ref}}$ ,  $i_{\text{ref}}$  define a reference orbit (usually the centre of an asteroid family, defined in our work as the centre of mass in a 3D proper element orbital space,<sup>1</sup> and  $f$  and  $\omega$  are the (generally unknown) true anomaly and perihelion argument of the disrupted body at the time of impact. From the initial distribution ( $a, e, i$ ), it should therefore be possible to estimate the three velocity components, assuming one knows the values of  $f$  and  $\omega$ . This can be accomplished by inverting equations (1), (2) and (3). With the exception of extremely young asteroid families (e.g. the Karin cluster, Nesvorný et al. 2002, 2006), this approach is not viable in most cases. Apart from the obvious limitation that we do not generally know the values of  $f$  and  $\omega$ , a more fundamental difficulty is that several gravitational (e.g. mean-motion and secular resonances, close encounters with massive asteroids) and

non-gravitational (e.g. Yarkovsky and YORP) effects act to change the proper elements on long time-scales. The Gauss equations thus cannot be directly applied in most cases to obtain information about the original ejection velocities.

In this work, we propose a new method to circumvent this difficulty. Of the three proper elements, the proper inclination is the one that is the least affected by dynamics. For example, unlike the proper semimajor axis, the proper inclination is not directly affected by the diurnal version of the Yarkovsky effect [it is affected by the seasonal version, but this is usually a much weaker effect, whose strength is of the order of 10 per cent of that of the diurnal version, for typical values of asteroid spin obliquities and rotation periods (Vokrouhlický & Farinella 1999)]. Also, unlike the proper eccentricity, which is affected by chaotic diffusion in mean-motion resonances, the proper inclination is more stable. In addition, the inclination is related to a single component of the ejection velocities,  $v_W$ , via equation (3). This equation can be, at least in principle, inverted to provide information about  $\delta v_W$ .

What kind of a probability distribution function (*pdf*) is to be expected for the original values of  $v_W$ ? Velocities at infinity  $V_{\text{inf}}$  are obtained from the ejection velocities  $V_{\text{ej}}$  through the relationship:

$$V_{\text{inf}} = \sqrt{V_{\text{ej}}^2 - V_{\text{esc}}^2}, \quad (4)$$

where  $V_{\text{esc}}$  is the escape velocity from the parent body. Following Vokrouhlický et al. (2006a,b,c), we assume that the ejection velocity field follows a Gaussian distribution of zero mean and standard deviation given by:

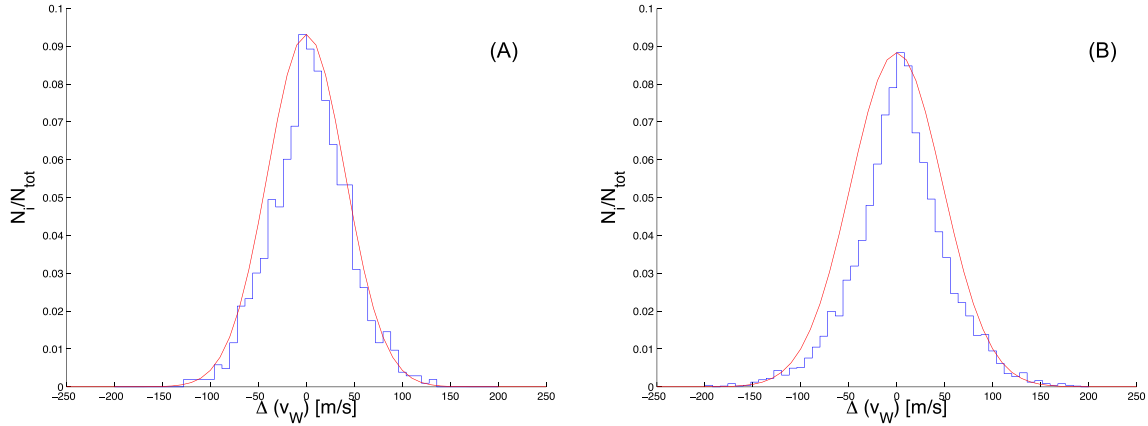
$$\sigma_{V_{\text{ej}}} = V_{\text{EJ}} \left( \frac{5km}{D} \right), \quad (5)$$

where  $D$  is the body diameter in km, and  $V_{\text{EJ}}$  is a parameter describing the width of the velocity field. Only objects with  $V_{\text{ej}} > V_{\text{esc}}$  succeed in escaping from the parent body. As a result, the initial distribution of  $V_{\text{inf}}$  should generally be more peaked than a Gaussian one. Fig. 1, panel A, displays the frequency distribution function  $fdf$  for 1031 objects with  $2.5 < D < 3.5$  km computed for a parent body with an escape velocity of  $130 \text{ m s}^{-1}$  and  $V_{\text{EJ}} = 0.5V_{\text{esc}}$ . We assumed that  $f = 30^\circ$  and  $(\omega + f) = 50^\circ$ . Since  $f$  and  $(\omega + f)$  appear only as multiplying factors in the expression for  $v_W$ , different choices of these parameters do not affect the shape of the distribution. One can notice how the  $v_W$  distribution is indeed more peaked than that of the Gaussian distribution with the same standard deviation (shown in red in Fig. 1).

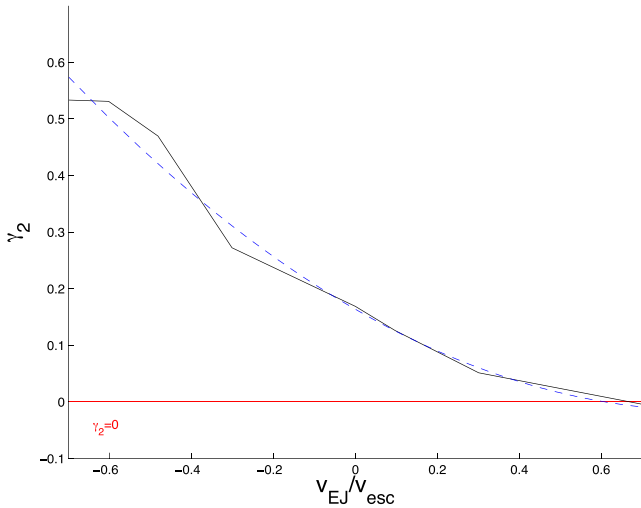
A useful parameter to understand if a given distribution is (or is not) normally distributed is the kurtosis (Carruba et al. 2013b). Pearson kurtosis (Pearson 1929), defined as  $\gamma_2 = \frac{\mu_4}{\sigma^4} - 3$ , where  $\mu_4$  is the fourth moment about the mean and  $\sigma$  is the standard deviation, gives a measure of the ‘peakedness’ of the distribution. The Gaussian distribution has  $\gamma_2 = 0$  and is the most commonly known example of a mesokurtic distribution. Larger values of  $\gamma_2$  are associated with leptokurtic distributions, which have longer tails and more acute central peaks. The opposite case, with  $\gamma_2 < 0$ , is known as platykurtic.

We generated values of  $v_W$  for various values of  $V_{\text{EJ}}$  for ten thousand 5 km bodies originating from a parent body with  $V_{\text{esc}} = 130 \text{ m s}^{-1}$  (results can be re-scaled for any other body size using equation (5), and a different value of  $D$ ). Fig. 2 shows how the kurtosis value of the  $v_W$  distribution changes as a function of  $V_{\text{EJ}}/V_{\text{esc}}$ . As expected, for smaller values of  $V_{\text{EJ}}/V_{\text{esc}}$  the  $v_W$  distribution is more peaked, and the kurtosis values are larger. For  $V_{\text{EJ}}/V_{\text{esc}} > 1$ , the kurtosis value approaches 0, because the influence of parent

<sup>1</sup> For families formed in cratering events, the centre of mass of the asteroid family usually coincides with the orbital position of the parent body. One notable exception to this rule is the Hygiea family, for reasons discussed in Carruba et al. (2014), such as possible dynamical evolution caused by close encounters with massive bodies with 10 Hygiea itself after the family formation.



**Figure 1.** The  $v_W$  distribution for a simulated family. Panel A shows the distribution for objects with diameters  $2.5 < D < 3.5$  km. Panel B shows the results for  $2.0 < D < 8.0$  km. The size distribution was set to be  $N dN = CD^{-\alpha} dN$  with  $\alpha = 3.6$ . We also set  $V_{EJ}/V_{esc} = 0.5$ . A Gaussian distribution with the same standard deviation of the  $v_W$  distribution is shown in both panels for reference.



**Figure 2.** Dependence of the kurtosis  $\gamma_2$  of the  $v_W$  distribution on  $V_{EJ}/V_{esc}$  (solid blue line). The scaling of the tick marks on the  $x$ -axis are proportional to the  $\log_{10} V_{EJ}/V_{esc}$ . The blue dashed line is the second order polynomial that best-fitted the data. The red line is  $\gamma_2 = 0$ , corresponding to a mesokurtic distribution.

body’s gravity diminishes. Most families for which an estimate of  $V_{EJ}$  is available (see Nesvorný et al. 2015, Section 5 and references within) have estimated values of  $V_{EJ}/V_{esc}$  in the range from 0.4 to 1.2, and should therefore have been significantly leptokurtic when they formed.

What typical values of  $\gamma_2$  one would expect for a real asteroid family, where different sizes of fragments are considered together? As the large objects typically have lower values of  $v_W$ , this implies that they would contribute to the peak of the distribution, and when considered with smaller fragments, the whole distribution should correspond to a larger value of  $\gamma_2$ . To illustrate this effect, we simulated a family using a size distribution  $N dN = CD^{-\alpha} dN$  with  $\alpha = 3.6$ , and  $V_{EJ}/V_{esc} = 0.5$ . The computed value of  $\gamma_2$  for bodies in the size range from 2 to 8 km was found to be 0.96 (see Fig. 1, panels B), while the one for a restricted range  $2.5 < D < 3.5$  km was 0.21 (Fig. 1, panel A). This shows that one must be careful when interpreting the real families, where the escape velocity and size distribution effects can combine together to produce larger

values of  $\gamma_2$ . To isolate the escape velocity effect, it is best to use a restricted range of sizes.

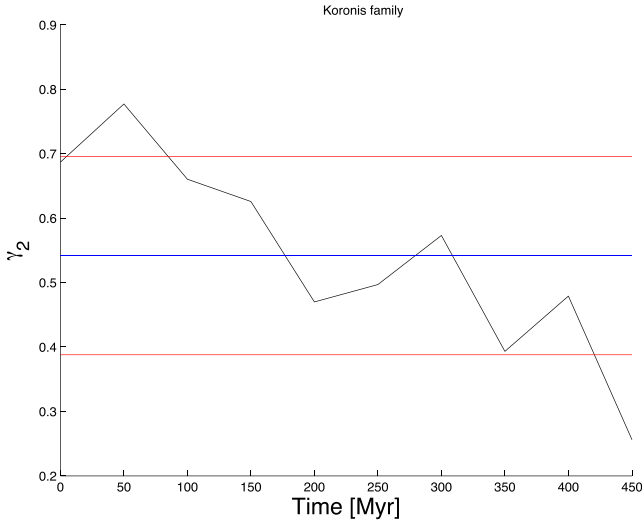
Finally, we compare our simple model for the  $v_W$  distribution with the results of impact simulations. We have taken these results from Nesvorný et al. (2006), where a Smooth Particle Hydrodynamic (SPH) code was used to model the formation of the Karin family. In this case, the parent body was assumed to have  $D = 33$  km, which gives  $V_{esc} \simeq 35 \text{ m s}^{-1}$ . Different impact conditions (impact speed, impact angle, projectile size, etc.) were studied in this work. The dynamical evolution of fragments and their re-accretion following the initial impact was followed by the PKDGRAV code.

Using these simulations, we extracted the values of  $v_W$  for the escaping fragments and estimated the value of  $V_{EJ}$  from equation (4). In a specific case that produced the best fit to the observed size distribution of the Karin family, we obtained  $V_{EJ}/V_{esc} = 0.2$  for fragments with  $1 < D < 5$  km. Finally, we found that the  $v_W$  distribution has  $\gamma_2 = 0.5$ , and is therefore significantly peaked. This is in very good agreement with results from our simple method to generate fragments (see above), which for the same size range and  $V_{EJ}/V_{esc}$  gives  $\gamma_2 = 0.53$ . This suggests, again, that the initial  $v_W$  distribution of asteroid families should be leptokurtic.

### 3 EFFECTS OF LONG-TERM DYNAMICS

Now that we have some expectations for the  $\gamma_2$  values just after a family formation, we turn our attention to  $\gamma_2$  for real asteroid families, where the inferred  $v_W$  values may have been affected by the long-term dynamics. We use the Koronis family to illustrate this effect. The Koronis family is located at low eccentricities and low inclinations, in a region that is relatively dynamically quiet (at least in what concerns the proper inclination; Bottke et al. 2001).

Bottke et al. (2001) simulated the long-term dynamical evolution of the Koronis family. Their simulation included the gravitational effects of four outer planets and the Yarkovsky effect. Different sizes of simulated family members were considered. We opted to illustrate here the case with  $D = 2$  km, for which Bottke et al. (2001) numerically integrated the orbits of 210 bodies over 450 Myr. The initial distribution of family members in Bottke et al. (2001) was obtained using the same method as described in Section 2. We computed the values of  $v_W$  by inverting equation (3). Since Pearson’s kurtosis is dependent on the presence of outliers, we eliminated from our distributions objects with values more than  $4\sigma$



**Figure 3.** The time evolution of the kurtosis for a simulated Koronis family (Bottke et al. 2001). As described in the main text, we computed  $\gamma_2$  of the  $v_W$  component and  $D = 2$  km bodies (black line). The blue line shows the mean value over the 450 Myr integration span, and the red lines are one standard deviation from the mean.

away from the mean. This allowed us to avoid possible distortions in the computed  $\gamma_2$  values caused by a few distant objects.

Fig. 3 shows that the initially leptokurtic distribution tends to become more mesokurtic with time. This can be understood as follows. In statistics, the central limit theorem states that the averages of random variables drawn from uncorrelated distributions are normally distributed. If the dynamical effects produce changes of  $i_p$  compatible with the central limit theorem, one would expect that, with time, the distributions of  $v_w$  should indeed become more and more mesokurtic, as the contribution from dynamics increases.

To illustrate things in the case of the (real) Koronis family, we computed the value of  $\gamma_2$  for all its 5118 members taken from Nesvorný et al. (2015), and for members with  $4.5 < D < 5.5$  km. We obtained  $\gamma_2 = 0.993$  and  $0.712$ , respectively. Since both these values are considerably leptokurtic, we believe that the dynamical evolution of the Koronis family did not had enough time to alter the initial distribution of inclinations. Therefore, in the specific case of the Koronis family, we may still be observing traces of the primordial distribution of  $v_w$ .

#### 4 KURTOSIS OF THE REAL ASTEROID FAMILIES

We selected the families identified in Nesvorný et al. (2015) that: (i) have at least 100 members with  $2 < D < 4$  km, and (ii) have a reasonably well defined age estimate.<sup>2</sup> The first selection criterion is required to have a reasonable statistics for the  $\gamma_2(v_w)$  computation. If we assume that the error is proportional to the square root of the number of objects, a sample of 100 objects would give us an uncertainty in  $\gamma_2(v_w)$  of 10 per cent. We use a restricted size range,

<sup>2</sup> Families were determined in Nesvorný et al. 2015 using the Hierarchical Clustering Method (HCM) in the  $(a, e, \sin(i))$  proper element domain. This method may not identify peripheral regions of some large families, the so-called *halo* of Brož & Morbidelli (2013), as belonging to the dynamical group. For families such as the Eos and Koronis groups, results from Nesvorný et al. (2015) should be considered as conservative estimates.

$2 < D < 4$  km, to avoid the influence of the size-dependent velocity effects on the kurtosis. The age estimate is useful to have some input for a physical interpretation of our results. Also, since  $\gamma_2(v_w)$  is sensitive to the presence of outliers in the distribution, we use the Jarque–Bera statistical test (jbstest hereafter) to confirm that the  $v_w$  distribution differs (or not) from a Gaussian one. We choose to work with this specific test, instead of others, because the jbstest is particularly sensitive to whether sample data have the skewness and kurtosis matching a normal distribution (Jarque & Bera 1987). The test, implemented on MATrix LABoratory (MATLAB), provides a  $p$  parameter stating the probability that a given  $v_w$  distribution follows (or not) the Gaussian *pdf*. The null hypothesis level is usually set at 5 per cent. Finally, we also checked that the  $v_w$  distribution was not too asymmetrical, and verified that its skewness, the parameter that measures a possible asymmetry, was in the range between  $-0.25$  and  $0.25$ . A skewness value outside this range would indicate an asymmetric ejection velocity field, or some dynamical effects that are beyond the scope of this study.

Of the 122 families listed in Nesvorný et al. (2015), 48 cases satisfied these requirements. At this stage of our analysis we do not eliminate interlopers and do not consider in detail the local dynamics. Also, families with haloes may have a further spread out distribution in inclination than what accounted for by HCM, and therefore larger values of  $\gamma_2(v_w)$ . For these families, our results should be considered as conservative. Values of  $\gamma_2(v_w)$  may be affected by these factors, but first we would like to see what families may be more interesting in terms of a simple first-order criteria. Table 1 reports the values of  $\gamma_2(v_w)$  for the whole family (3rd column), for  $2.0 < D < 4.0$  km ( $D_3$ ) members (4th column), and the  $p_{jbstest}$  coefficient of the jbstest for the  $D_3$  population. Note that the maximum value of  $p_{jbstest}$  is 50.0 per cent.

Following the notation of Nesvorný et al. (2015), the first two columns in Table 1 report the Family Identification Number (FIN) and the family name. Finally, the sixth column displays the family age estimate, with its error. The age estimate was obtained using equation (1) in Nesvorný et al. (2015), and values of the  $C_0$  parameter, its error [also estimated from equation (1) and the value of  $\delta C_0$  from Nesvorný et al. 2015<sup>3</sup>], and geometric albedos from table 2 of that paper. Densities were taken from Carry (2012) for the namesake body of each family, when available. If not, a standard value of  $1.2 \text{ g cm}^{-3}$  for C-complex families and  $2.5 \text{ g cm}^{-3}$  for the S-complex families was used. In a few cases, for which a better age estimate was available in the literature, we used these latter values. This includes the following cases (labeled by † in Table 1): Karin (Nesvorný et al. 2002), Eos (Vokrouhlický et al. 2006b), Hygiea (Carruba et al. 2013a), and Koronis, Themis, Meliboea, and Ursula (Carruba et al. 2015). Moreover, the families with values of skewness not in the range from  $-0.25$  to  $0.25$  are identified by a ‘(s)’ after family’s name in Table 1.

To select families whose  $v_w$  component may have not changed significantly with time, we adopted the following criteria:  $\gamma_2(v_w)$  for the  $D_3$  population has to be larger than  $0.25$ , the estimated error on values of  $\gamma_2(v_w)$ , skewness in the range from  $-0.25$  and  $0.25$ , and  $p_{jbstest}$  has to be lower than 5 per cent. After this pre-selection was carried out, we were left with a sample of nine candidates. This is much higher than the two families one would expect to randomly

<sup>3</sup> The error on family ages should be considered as nominal, since they do not account for the uncertainties on values of fundamental parameters such as the asteroids bulk densities and thermal conductivities (Masiero et al. 2012).



**Table 1.** Values of  $\gamma_2(v_W)$  of the whole family (3rd column), the  $2.0 < D < 4.0$  km ( $D_3$ ) members (4th column), the  $p$  coefficient of the jbstest (5th column), and estimated family age with its error (6th column) for the families in Nesvorný et al. (2015) that satisfy our selection criteria. Families with values of skewness not in the range from  $-0.25$  to  $0.25$  are identified by a '(s)' after the family's name. Daggers identify the cases for which a more refined age estimate was available in the literature.

FIN	Family name	$\gamma_2(v_W)$ All	$\gamma_2(v_W)$ $D_3$	$p_{\text{jbstest}}$ (per cent)	Age [Myr]
003	434 Hungaria (s)	0.42	0.21	0.5	$135 \pm 13$
401	4 Vesta	-0.42	-0.27	0.1	$1440 \pm 720$
402	8 Flora	-0.59	-0.60	0.1	$4360 \pm 2180$
403	298 Baptistina	-0.03	-0.16	0.1	$110 \pm 10$
404	20 Massalia	0.26	1.07	0.1	$320 \pm 20$
405	44 Nysa (s)	0.38	0.74	0.1	$1660 \pm 80$
406	163 Erigone	-0.32	-0.42	0.8	$190 \pm 25$
408	752 Sulamitis (s)	1.28	1.56	0.1	$320 \pm 30$
413	84 Klio (s)	0.12	0.19	2.5	$960 \pm 250$
701	25 Phocaea	-0.41	-0.40	1.4	$1460 \pm 1460$
501	3 Juno (s)	1.05	1.05	0.1	$740 \pm 150$
502	15 Eunomia	-0.42	-0.44	0.8	$3200 \pm 2240$
504	128 Nemesis	0.19	0.20	3.5	$440 \pm 20$
505	145 Adeona	0.20	0.05	19.2	$620 \pm 190$
506	170 Maria (s)	-0.17	-0.38	0.1	$1950 \pm 1950$
507	363 Padua	-0.28	-0.18	4.8	$410 \pm 80$
510	569 Misa	0.14	0.02	20.3	$700 \pm 140$
511	606 Brangane (s)	0.48	-0.48	3.2	$60 \pm 5$
512	668 Dora	0.16	0.28	1.0	$1190 \pm 600$
513	808 Merxia	0.24	0.34	9.9	$340 \pm 30$
514	847 Agnia (s)	0.47	0.24	0.3	$200 \pm 10$
515	1128 Astrid (s)	3.69	4.83	0.1	$140 \pm 10$
516	1272 Gefion	0.36	0.16	11.9	$980 \pm 290$
517	3815 Konig	0.94	0.48	27.0	$70 \pm 10$
518	1644 Rafita	0.63	0.72	0.3	$480 \pm 100$
519	1726 Hoffmeister	2.21	1.82	0.1	$270 \pm 10$
533	1668 Hanna	0.15	0.15	19.3	$240 \pm 20$
535	2732 Witt	-0.22	-0.21	50.0	$790 \pm 200$
536	2344 Xizang (s)	2.47	1.87	0.1	$220 \pm 20$
539	369 Aeria (s)	-0.56	-0.21	1.6	$180 \pm 20$
802	148 Gallia	2.02	3.39	0.1	$650 \pm 60$
803	480 Hansa	0.81	1.17	0.1	$2430 \pm 600$
804	686 Gersuind	-0.12	-0.64	5.3	$800 \pm 80$
805	945 Barcelona	1.48	1.32	0.5	$250 \pm 10$
807	4203 Brucato	0.29	0.01	50.0	$480 \pm 100$
601	10 Hygiea	0.58	0.29	0.1	$2420 \pm 580(\dagger)$
602	24 Themis	0.03	0.08	0.1	$1500 \pm 320(\dagger)$
605	158 Koronis	0.99	1.06	0.1	$2360 \pm 490(\dagger)$
606	221 Eos	-0.60	-0.60	0.1	$1300 \pm 200(\dagger)$
607	283 Emma (s)	0.17	-0.69	0.8	$400 \pm 40$
608	293 Brasilia	-0.14	-0.45	17.7	$160 \pm 10$
609	490 Veritas	-0.16	-0.32	5.4	$1820 \pm 180$
610	832 Karin (s)	-0.29	0.89	1.0	$5.8 \pm 0.2(\dagger)$
611	845 Naema	0.10	-0.65	19.2	$210 \pm 10$
612	1400 Tirela	-0.14	-0.35	21.9	$1980 \pm 500$
613	3556 Lixiaohua	1.30	-0.38	50.0	$430 \pm 20$
631	375 Ursula (s)	0.06	0.70	4.3	$1060 \pm 60(\dagger)$
901	31 Euphrosyne (s)	1.97	1.02	0.4	$1380 \pm 70$

fulfill this criteria, based on the number of families in our sample, and this suggests that a real effect might actually be observed in our data. Ordered by a decreasing value of  $\gamma_2(v_W(D_3))$ , these eight candidates are the Gallia, Hoffmeister, Barcelona, Hansa, Massalia, Koronis, Rafita, Hygiea and Dora families. We discarded the case of the Hoffmeister family, since this family is significantly

affected by the  $\nu_{1C} = g - g_C$  resonance with Ceres (Novaković et al. 2015).

Fig. 4 shows the distributions (blue line) of the  $v_W$  values for  $D_3$  members of the Gallia, Barcelona, Massalia, and Koronis families, four of our selected families with leptokurtic distributions of  $v_W$ . As can be observed from this figure, these families have  $v_W$  more peaked than a Gaussian, and this is particularly evident for the case of the Gallia family. Fig. 5 displays values of  $\gamma_2(v_W(D_3))$  versus the estimated ages of the eight families satisfying our selection criteria. One can notice that there is not a clear correlation between family age and  $\gamma_2(v_W(D_3))$ . While there are four families younger than 700 Myr, three families (Hansa, Hygiea, and Rafita) are estimated to be older than 2 Gyr. What these eight families have in common is that they are located in orbital regions not very affected by dynamical mechanisms capable of changing proper  $i$ . Gallia, Barcelona, and Hansa are families in stable islands at high inclinations, all in regions with not many mean-motion resonances (Carruba 2010). As previously discussed, Koronis is located in a region at low inclination relatively quiet in terms of dynamics, and the same can be said for Hygiea (Carruba et al. 2013a), and, possibly, for the Rafita and Massalia families (the latter also a relatively young family, Vokrouhlický et al. 2006b).

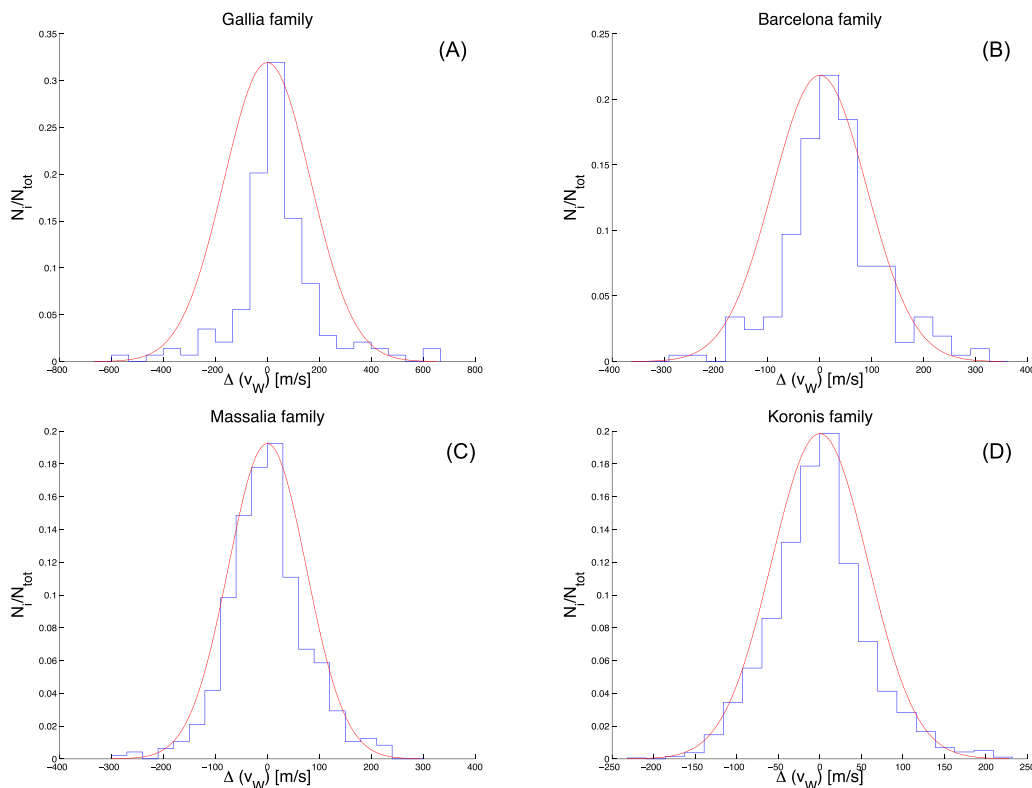
Finally, among the families with large values of both skewness and kurtosis, the cases of the Karin and Astrid families are of a particular interest to us. Karin is a very young family, already identified as such in Nesvorný et al. (2002, 2006), while Astrid is a family located in a dynamically quiet region. Nesvorný et al. (2006) showed that the velocity field of Karin was probably asymmetrical, so the relatively high value of the skewness that we found for the Karin cluster ( $-0.60$ ) should not be surprising. Astrid has the highest observed value of  $\gamma_2(v_W(D_3))$ . This family, however, also interacts with a secular resonance that produces a large dispersion of the inclination distribution. If the resonant population of objects is removed, the value of  $\gamma_2(v_W(D_3))$  drops to 0.3, with a skewness within the acceptable range. The still large value of  $\gamma_2(v_W(D_3))$  of this revised Astrid group makes it a very good candidate for a family with a partially pristine  $v_W$  velocity field.

## 5 CONCLUSIONS

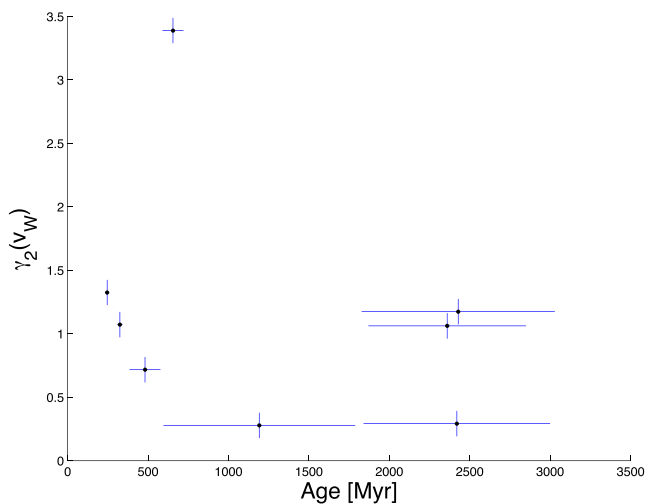
The main results of this work can be summarized as follows.

(i) We developed a model for how the original field of velocities at infinity should appear for newly created families. Families with a low  $v_{\text{EJ}}/v_{\text{esc}}$  ratio, where  $v_{\text{EJ}}$  is a parameter describing the standard deviation of ejection velocity field, assumed normal (equation 5), and  $v_{\text{esc}}$  is the escape velocity from the parent body, should have values of velocities at infinity following a peaked distribution. This distribution is characterized by positive values of kurtosis  $\gamma_2$  (leptokurtic distribution). This is confirmed by SPH simulations for the Karin cluster, that show a leptokurtic distribution of  $v_W$  values, the component of the velocity of infinity perpendicular to the orbital plane. Since the proper inclination is less affected by dynamical evolution than proper semimajor axis and eccentricity, and since  $v_W$  can be obtained from the distribution in proper  $i$  (equation 3), we decided to concentrate our analysis on the distribution of  $v_W$  values. We studied the influence that dynamics has on the distribution of  $v_W$  values. An initially peaked  $v_W$  distributions tend to become more normal with time.

(ii) We computed values of  $\gamma_2(v_W(D_3))$ , the kurtosis of the distribution of  $v_W$  values for objects with  $2 < D < 4$  ( $D_3$ ), for all families listed in Nesvorný et al. (2015) that (i) have an age estimate, (ii) have



**Figure 4.** The  $v_W$  distributions (blue line) of  $D_3$  members of the Gallia (panel A), Barcelona (panel B), Massalia (panel C), and Koronis families (panel D). The red line displays the Gaussian distribution with the same standard deviation of the  $v_W$  distribution and zero mean, normalized to the maximum value of the frequency distribution of each family.



**Figure 5.**  $\gamma_2(v_W(D_3))$  versus the estimated age of eight families satisfying our selection criteria. The horizontal blue lines display the estimated error on the ages, while the vertical ones show the nominal errors on  $\gamma_2(v_W(D_3))$ , assumed equal to 0.1.

a  $D_3$  population of at least 100 objects, and (iii) are not following a Gaussian distribution, according to the Jarque–Bera test. We also required the families to have values of skewness, the parameter identifying possible asymmetries in the  $v_W$  distribution, between  $-0.25$  and  $0.25$ . Eight families, the Gallia, Barcelona, Hansa, Massalia, Koronis, Rafita, Hygiea and Dora families, satisfied these criteria and have values of  $\gamma_2(v_W(D_3))$  larger than  $0.25$ , which suggests that they could still bear traces of the original values of  $v_W$ . Four of these

families are younger than 700 Myr. All of these groups appear to be located in regions not strongly affected by dynamical mechanisms able to modify proper inclination. Among asymmetrical families, the Karin and Astrid groups stand out as interesting cases.

Overall, while a more in depth study of the families selected in this work, accounting for a better analysis of local dynamics and of the role of possible interlopers, should be performed, our analysis already allowed us to select several asteroid families that could still bear traces of the original distribution of velocities at infinity. While to select families that could still bear traces of the original values of  $v_W$  we concentrated on the shape of the  $v_W$  distribution, obtaining better estimates of the *magnitude* of  $v_W$  for these selected families could much improve our knowledge of the physical mechanisms at work in the family-forming event, and serve as a useful constraints for simulations describing cratering and catastrophic destruction events, other than the observed Size Frequency Distribution (Durda et al. 2007).

## ACKNOWLEDGEMENTS

We thank an anonymous reviewer for comments and suggestions that significantly improved the quality of this paper. This paper was written while the first author was a visiting scientist at the Southwest Research Institute (SwRI) in Boulder, CO, USA. We would like to thank the São Paulo State Science Foundation (FAPESP) that supported this work via the grant 14/24071-7, the Brazilian National Research Council (CNPq, grant 305453/2011-4), and the National Science Foundation (NSF). The authors are grateful to W. F. Bottke for allowing them to use the results of the Koronis family simulations, to S. Aljbaae for revising earlier version of this manuscript,

and to D. Durda for comments and suggestions that improved the quality of this work.

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