

# A Strategy for Transmission Network Expansion Planning Considering Multiple Generation Scenarios

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## Abstract

Transmission network expansion planning (TNEP) is subject to several constraints that are driven by new market rules, such as generation uncertainty, demand growth, seasonal climate changes and technological advances. When considering multiple generation scenarios, due to different operational conditions, it is necessary to find an expansion plan that allows the expanded system to operate properly for each scenario. This paper presents a strategy that provides several expansion plans that operate efficiently in all the previously defined generation scenarios and that keep reasonable values for the expansion costs. Thus, the developed mathematical model allows us to find adequate expansion plans with expansion costs that can be controlled, with small infeasibilities in the operation of the system that may not be significant in the long-term planning. The proposed approach is applied to the 24-bus system and the Colombian 93-bus system. The results indicate that the plans obtained using the proposed strategy present much lower expansion costs compared to the conventional TNEP model with multiple generation scenarios, with very low infeasibilities in the operation of the system, which can be corrected in the short-term expansion planning.

**Keywords:** Generation scenarios; Mixed-integer linear programming; Power systems optimisation; Transmission network expansion planning.

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## Nomenclature

The notation used throughout this paper is reproduced below for quick reference.

### *Indices:*

$i$	Index for buses
$ij, ji$	Index for corridors
$s$	Index for scenarios
$y$	Index for new lines

### *Sets:*

$B$	Set of buses
$C$	Set of corridors
$S$	Set of scenarios
$Y$	Set of candidate lines

### *Functions:*

$v$	Investment to build new lines
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*Constants:*

$\alpha$	Penalty factor for the load shedding
$\beta$	Penalty factor for generation displacements
$x_{ij}$	Reactance of a line in corridor $ij$
$\delta$	Limit for the percentage of total load shedding in the system
$\bar{\theta}$	Maximum voltage angle
$\theta_s^{ref}$	Reference angle in scenario $s$
$\sigma_{ij}$	Overload factor for corridor $ij$
$c_{ij}$	Cost of a new line in corridor $ij$
$d_i$	Load demand at bus $i$
$\bar{f}_{ij}$	Maximum power flow allowed for a line in corridor $ij$
$\bar{f}_{ij}^{new}$	Maximum power flow allowed in the lines of corridor $ij$ with overload
$\underline{g}_{i,s}$	Minimum generation at bus $i$ , scenario $s$
$\bar{g}_{i,s}$	Maximum generation at bus $i$ , scenario $s$
$g_{i,s}^*$	Ideal generation at bus $i$ , scenario $s$
$\bar{n}_{ij}$	Maximum number of lines that can be built in corridor $ij$
$n_{ij}^o$	Number of existing lines in corridor $ij$
$\bar{r}_i$	Maximum load shedding at bus $i$

*Variables:*

$\theta_{i,s}$	Voltage angle at bus $i$ , scenario $s$
$f_{ij,s}^o$	Power flow in the existing lines of corridor $ij$ , scenario $s$
$f_{ij,y,s}$	Power flow in line $y$ of corridor $ij$ , scenario $s$
$g_{i,s}$	Generation at bus $i$ , scenario $s$
$r_{i,s}$	Fictitious generation corresponding to the load shedding at bus $i$ , scenario $s$
$w_{ij,y}$	Binary variable that decides if line $y$ will be built in corridor $ij$

**1. Introduction**

Long-term transmission network expansion planning (TNEP) is a classic problem of power systems. The objective is to find the optimal expansion plan that identifies the transmission lines that must be installed in the electrical system to allow a proper operation within a predefined planning horizon with the lowest investment cost. The data of this problem are the current topology of the network, the candidate lines for addition, the generation and demand for a planning horizon and the investment restrictions. The optimal expansion plan should define where, how many and when the new lines should be installed.

Several mathematical models and optimisation techniques have been used to solve the TNEP problem. The direct current (DC) model is the formulation most often used to represent the electric network and the electric quantities. However, there are more relaxed models, such as the transportation model, and more accurate models, like the alternating current (AC) model. In this work, we use the DC model [1,2] to formulate the problem.

The traditional modelling for this problem considers only one generation scenario. The generation scenario choice is arbitrary; however, it is common to use the most probable or the worst-case scenario. Thus, the generation in each generation bus is fixed at a previously specified value. Nevertheless, when it is intended to consider climatic problems, renewable generation sources (dispatchable or not), competitive market operation requirements or any other type of uncertainty, the expansion plan should allow the expanded system to operate in a suitable way for different generation scenarios. Especially when considering generation and demand uncertainties, the most used way to represent these uncertainties is to build a set of representative generation and demand scenarios and to expand the electric system taking them into account [3,4]. The process of constructing future scenarios for generation and demand is based on historical and forecasted data, while strategies must be used to define a reduced set of representative scenarios [5]. This work addresses the TNEP problem considering multiple generation scenarios and proposes a strategy to generate expansion plans with nonprohibitive expansion costs.

In the TNEP problem, for each mathematical model and depending on the complexity of the system to be solved, there is a most suitable optimisation technique to be used. These techniques can be the exact methods [6], heuristics [7], or metaheuristics [8]. In [6], the authors propose a mixed-integer linear disjunctive formulation for the TNEP problem to be solved by a branch-and-bound algorithm together with a greedy randomized adaptive search procedure. Reference [7] presents a constructive heuristic algorithm for the TNEP problem, that adds one line to the network at each iteration, based on the result of a linear programming problem. The authors of [8] present a strategy based on a constructive heuristic algorithm to reduce the search space of the TNEP and a particle swarm algorithm to solve the problem. In this work, we use the linear disjunctive model that is an exact optimisation formulation, which considers the DC model modified to have an equivalent optimal solution, with the advantage that it can be solved by a mixed-integer linear programming optimisation solver.

Regarding the planning horizon, the TNEP problem can be classified as static or multistage. With static planning, there is only a planning horizon, and with multistage planning, the planning horizon is divided into several stages and new transmission lines are installed at each stage. In this work, only static planning is considered. Reliability criteria and uncertainties in the generation and demand can also be considered in the TNEP problem.

One of the pioneering works in analysing the TNEP problem considering multiple generation scenarios was presented by Fang and Hill [9]. The described expansion planning considers competitive market operation requirements. However, in the proposal, independent plans are processed for each generation scenario, and only one of them is determined to be the most appropriate, considering an approach based on the minimisation of the maximum regret. Thus, the

authors do not present a single expansion plan that simultaneously meets all the presented generation scenario requirements, and because of that, when the solution obtained by the method is considered to operate in a different scenario, for which it was not originally planned, the load shedding in the system is very high.

Other optimisation proposals considering a set of generation scenarios can be found in recent works such as [10–15], that use stochastic programming to represent uncertainties. Reference [10] presents a stochastic programming formulation for the TNEP problem that considers the uncertainties related to wind generation and the load, and uses a Benders decomposition algorithm to solve the problem. The authors of [11] present a method based on risk/investment to solve the TNEP problem considering multiple generation and load future scenarios. In [12], the authors use a non-dominated sorting genetic algorithm (NSGA-II) to solve the multi-stage TNEP problem considering three objective functions: the total social cost, maximum regret and the maximum adjustment cost. The uncertainties of the problem are considered using scenarios. Reference [13] presents a mixed-integer nonlinear programming model for the generation and transmission expansion planning using a stochastic formulation with probabilistic constraints to represent the uncertainties related to the demands, availability of the power plants and the transmission capacity factor of the lines. In [14], the authors present a Benders decomposition algorithm to solve the TNEP problem considering a stochastic formulation to represent the high penetration of renewable generation in transmission systems considering network contingencies. The authors of [15] present a stochastic adaptive robust optimization approach for the generation and transmission expansion planning problem considering uncertainties in the demand and generation costs. However, these studies address the TNEP problem with multiple generation scenarios integrated into much more complex optimisation strategies. Also, the tests show systems that need to add few transmission lines; therefore, it is not possible to verify the high expansion costs due the need to meet several generation scenarios.

Reference [16] presents a robust optimisation approach based on a Benders decomposition algorithm to solve the TNEP problem considering uncertain generation and load. The tests include systems with renewable generation, and it is possible to verify that the investment costs are high when compared to the conventional TNEP problem. Reference [17] also presents a robust approach to the TNEP problem, that considers both uncertain future demand growth and generation availability. The solution method is based on a constraint-and-column generation method. In [18], an approach for the robust TNEP problem with improved performance is presented. The results of the proposed algorithm are compared with [17], demonstrating the performance improvement in terms of computational times. In [19], a method for robust multistage TNEP is presented. The results for the robust multistage planning present lower costs than the robust static model.

Reference [20] presents a hybrid NSGA-II/Chu-Beasley algorithm for solving the multi-objective TNEP problem considering multiple generation scenarios. The objective function minimises both the investment in new lines and the total load shedding in the system. Two test systems are considered, the Garver 6-bus and the 24-bus systems, and it is possible to verify that the investment cost is drastically increased when the multiple generation scenarios are considered in the problem—for the 24-bus system, the investment cost is more than six times higher than the investment cost for a single generation scenario.

Reference [21] presents an imperialist competitive algorithm to solve the TNEP problem considering the uncertainty in wind generation and the costs of investments, repair, maintenance and losses, together with the AC operation of the network. In [22], the authors use the NSGA-II algorithm to solve the TNEP problem considering load correlation as a multi-objective problem with the following objective functions: investment cost, congestion cost and risk cost. Reference, [23] proposes an approach for solving the TNEP problem in large-scale systems based on a network reduction strategy.

Heuristics and meta-heuristics for solving the TNEP problem considering deregulated power systems can be found in [24–28]. In [24], a combined genetic algorithm and linear programming method is used to solve the TNEP problem in a deregulated market environment. In [25], an improved harmony search algorithm is used, and  $N-1$  security constraints are considered. Reference [26] presents a particle swarm optimisation algorithm for the multistage TNEP problem in electricity markets considering  $N-1$  security constraints. Reference, [27] presents a heuristic approach for the TNEP problem in electricity markets. Finally, [28] presents a method based on the NSGA-II algorithm and a fuzzy decision-making strategy for TNEP in a deregulated environment, considering the AC operation of the system. In all these works, it can be verified that the investment costs are much higher than the solution for the traditional TNEP problem that considers a monopoly electricity market.

In this paper, we discuss the TNEP problem with multiple generation scenarios, proposing to find a single expansion plan that operates properly in each one of the previously specified generation scenarios. It can be seen that an expansion plan considering multiple generation scenarios presents a high investment cost. Therefore, the proposal of this paper also presents strategies to find good-quality expansion plans with lower investment costs.

In summary, this paper presents a mathematical model for the TNEP problem that considers multiple generation scenarios and expands the mathematical model so that it is possible to generate expansion plans with a significant reduction in the expansion costs and with small infeasibilities in the operation of some generation scenarios, which can be corrected in the short-term expansion planning.

It must be emphasised that the solutions obtained for the long-term TNEP must be analysed and reinforced with the use of other tools, such as power flow analysis, reactive power planning, short circuit, transient and stability analysis. Therefore, even solutions that are feasible to the DC model, with very high investment costs, may be infeasible for the complete AC model. The advantage of applying the proposed approach is that it can produce a set of high-quality solutions with low investment costs, and the network planner can choose a solution that is adequate for other requirements.

The proposed model was implemented in the mathematical modelling language AMPL [29] and solved with the commercial solver CPLEX [30]. The IEEE 24-bus system is used for the tests, for which a set of results with much lower expansion costs (compared to the results for the traditional TNEP model with multiple generation scenarios) and with very small infeasibilities in the operation of the system were obtained.

The rest of the paper is organised as follows: Section 2 presents the mathematical formulation proposed to solve the TNEP problem with multiple generation scenarios and other alternative formulations; in Section 3, we present the tests and results, as well as an analysis of these results; the main conclusions are presented in Section 4.

## 2. Mathematical Formulation

The most used mathematical modelling for the TNEP problem is the DC model. This model considers only the active power balance in the system and neglects the losses. Even so, it is a mixed-integer nonlinear programming problem that can be highly complex. Thus, when an optimisation solver is to be used, the DC model is replaced by the linear disjunctive model, which is a mixed-integer linear programming problem (MILP) that has the same optimal solution of the DC model [6,31].

### 2.1 Linear Disjunctive Model with Multiple Generation Scenarios

The linear disjunctive model [6] can be adapted for the TNEP problem with multiple generation scenarios. In this case, the network must be expanded to properly operate in the planning horizon for each one of the different generation scenarios previously defined. This model assumes the form (1)–(11).

$$\min v = \sum_{ij \in C} \sum_{y \in Y} c_{ij} w_{ij,y} \quad (1)$$

subject to

$$\sum_{ji \in C} \left( f_{ji,s}^o + \sum_{y \in Y} f_{ji,y,s} \right) - \sum_{ij \in C} \left( f_{ij,s}^o + \sum_{y \in Y} f_{ij,y,s} \right) + g_{i,s}^* = d_i \quad \forall i \in B, s \in S \quad (2)$$

$$f_{ij,s}^o = n_{ij}^o \frac{(\theta_{i,s} - \theta_{j,s})}{x_{ij}} \quad \forall ij \in C, s \in S \quad (3)$$

$$|f_{ij,s}^o| \leq n_{ij}^o \bar{f}_{ij} \quad \forall ij \in C, s \in S \quad (4)$$

$$|x_{ij} f_{ij,y,s} - (\theta_{i,s} - \theta_{j,s})| \leq 2\bar{\theta}(1 - w_{ij,y}) \quad \forall ij \in C, y \in Y, s \in S \quad (5)$$

$$|f_{ij,y,s}| \leq \bar{f}_{ij} w_{ij,y} \quad \forall ij \in C, y \in Y, s \in S \quad (6)$$

$$|\theta_{i,s}| \leq \bar{\theta} \quad \forall i \in B, s \in S \quad (7)$$

$$\sum_{y \in Y} w_{ij,y} \leq \bar{n}_{ij} \quad \forall ij \in C \quad (8)$$

$$w_{ij,y} \leq w_{ij,y-1} \quad \forall ij \in C, y \in Y | y > 1 \quad (9)$$

$$\theta_s^{ref} = 0 \quad \forall s \in S \quad (10)$$

$$w_{ij,y} \in \{0,1\} \quad \forall ij \in C, y \in Y \quad (11)$$

The objective function (1) represents the investment for the construction of new transmission lines. Equation (2) represents Kirchhoff's current law for each bus of the system in each generation scenario. Constraint (3) represents Kirchhoff's voltage law for each fundamental loop formed by the existing lines in each generation scenario. Constraints (4) and (6) determine the transmission capacity limits for the existing lines and each new line, respectively, in each generation scenario. In these constraints, the use of the absolute value is necessary because the flows are bidirectional. Constraint (5) represents Kirchhoff's voltage law for each fundamental loop generated by a new line in each generation scenario. Note that the addition of the  $y^{th}$  line means that  $w_{ij,y} = 1$ , and in this context, the power flow equation (5) for a new line is equivalent to the power flow equation (3) of an existing line. If  $w_{ij,y} = 0$ , then the  $y^{th}$  line is not built in corridor  $ij$  and the parameter  $2\bar{\theta}$ , must be large enough in order to not impose a limit on the angular difference  $(\theta_{i,s} - \theta_{j,s})$  between buses  $i$  and  $j$  in equation (5). Constraint (7) represents the angular limit at each bus in each generation scenario. Constraint (8) represents the limit for the number of new lines that can be installed in each corridor. Constraint (9) represents a surrogate constraint that forms the sequential installation of new lines in corridor  $ij$  and avoids the generation of equal solutions with different values of  $w_{ij,y}$ . Equation (10) imposes the angular reference to the system in each generation scenario. Finally, (11) indicates that the variable  $w_{ij,y}$  must be binary, representing the biggest source of complexity in the problem.

The objective function and the constraints in this model are practically the same as in the model with a single generation scenario, with the addition of the index  $s$  in  $g_{i,s}^*$ , to represent the generation scenarios, and in the operation variables  $f_{ij,s}^o$ ,  $f_{ij,y,s}$  and  $\theta_{i,s}$ , to represent the operation

of the system in each scenario. Also, it should be noted that for  $|S|$  generation scenarios, the number of operation variables increases  $|S|$  times, as does the number of constraints (2)–(7) and (10). However, the number of binary variables that represent the addition of transmission lines, which is the main source of complexity of the problem, remains unchanged.

Considering multiple generation scenarios reduces the feasible region of the problem, therefore, the mathematical model (1)–(11) provides expansion plans with very high costs when compared to the expansion plans considering a single generation scenario. This fact is verified in the results of this paper. Thus, the following sections present several alternative mathematical models that allow us to generate expansion plans for the TNEP problem with multiple generation scenarios with a significant reduction of the expansion costs (and small infeasibilities for some generation scenarios).

## 2.2 Linear Disjunctive Model with Multiple Generation Scenarios and Small Generation Displacements

The mathematical modelling presented in the previous section can be modified so that it is possible to displace the generations in a small range. Thus, with a small variation of the ideal generation points, an expansion plan can be generated with a reduction in the expansion costs. The new model only changes the objective function to (12) and the power balance constraint (2) to (13), and it adds constraint (14) to the model (1)–(11). The resulting model is shown in (12)–(14).

$$\min v = \sum_{ij \in C} \sum_{y \in Y} c_{ij} w_{ij,y} + \beta \sum_{i \in B} \sum_{s \in S} |g_{i,s} - g_{i,s}^*| \quad (12)$$

subject to (3)–(11)

$$\sum_{ji \in C} \left( f_{ji,s}^o + \sum_{y \in Y} f_{ji,y,s} \right) - \sum_{ij \in C} \left( f_{ij,s}^o + \sum_{y \in Y} f_{ij,y,s} \right) + g_{i,s} = d_i \quad \forall i \in B, s \in S \quad (13)$$

$$\underline{g}_{i,s} \leq g_{i,s} \leq \bar{g}_{i,s} \quad \forall i \in B, s \in S \quad (14)$$

It should be noted that, in the original model, the generation  $g_{i,s}^*$  is a known parameter, and in the modified model, the generation  $g_{i,s}$  is a variable that can assume any value in a small range, from  $\underline{g}_{i,s}$  to  $\bar{g}_{i,s}$ , previously specified, but that is encouraged to assume the predefined ideal value. Thus, the objective function is still linear, because the absolute value of its second term can be easily linearised. Parameter  $\beta$ , given in MUS\$/MW, only maintains the generations at the ideal values if there is no possibility of reducing the expansion costs.



### 2.3 Linear Disjunctive Model with Multiple Generation Scenarios and Small Line Overloads

In this modelling strategy, the linear disjunctive model of Section 2.1 is not changed. However, the data used will be modified to allow a small overload on the transmission lines, allowing expansion plans with lower investment costs.

This strategy consists of allowing the overload of the transmission lines in acceptable percentages, i.e., the values for the overloads should be small. Therefore, an increase of up to 5% in the maximum transmission capacity of a line is established. The only change occurs in  $\bar{f}_{ij}^{new} = \sigma_{ij} \bar{f}_{ij}$  for each transmission line in the database. In the tests, values of  $\sigma_{ij}$ , ranging from 1.00 to 1.05 (maximum permissible overload of 5%), were used.

### 2.4 Linear Disjunctive Model with Multiple Generation Scenarios and Small Load Shedding

The linear disjunctive model for the TNEP problem with multiple generation scenarios can also be modified so that the expansion plans can be obtained at lower costs by allowing small load shedding in some generation scenarios. In modern systems, considering the smart grid paradigm, load shedding can represent the load flexibility (demand response). With the long-term TNEP, it may be acceptable to allow a small load shedding to achieve expansion plans with a significant reduction in expansion costs. In this context, the mathematical formulation is shown in (15)–(19).

$$\min v = \sum_{ij \in C} \sum_{y \in Y} c_{ij} w_{ij,y} + \alpha \sum_{i \in B} \sum_{s \in S} r_{i,s} \quad (15)$$

subject to (3)–(11)

$$\sum_{ji \in C} \left( f_{ji,s}^o + \sum_{y \in Y} f_{ji,y,s} \right) - \sum_{ij \in C} \left( f_{ij,s}^o + \sum_{y \in Y} f_{ij,y,s} \right) + g_{i,s} + r_{i,s} = d_i \quad \forall i \in B, s \in S \quad (16)$$

$$\sum_{i \in B} \sum_{s \in S} r_{i,s} \leq (1 - \delta) \sum_{i \in B} d_i \quad (17)$$

$$0 \leq r_{i,s} \leq \bar{r}_i \quad \forall i \in B, s \in S \quad (18)$$

$$0 \leq g_{i,s} \leq g_{i,s}^* \quad \forall i \in B, s \in S \quad (19)$$

Equation (15) represents the objective function that minimises the costs of investment in transmission lines and the costs of load shedding in all the generation scenarios. Equation (16) replaces equation (2) for the active power balance in each bus. In fact, the load shedding is represented as an artificial generation  $r_{i,s}$  in each demand bus in each generation scenario. Constraint (17) imposes a limit for the total load shedding in the system. Note that when  $\delta = 0$ , the total load shedding in the system is unconstrained, while when  $\delta = 1$ , the total load shedding

in the system must be zero. Constraint (18) restricts the artificial generation (the load shedding) to a maximum value,  $\bar{r}_i \leq d_i$ , at each bus of the system. Constraint (19) limits the generation at each generation bus to the ideal value. Note that, since in (15)–(18) the demand will decrease with the load shedding, (19) is different from (14), allowing only a decrease in the ideal generation values, so that the power balance (16) is ensured.

The parameter  $\alpha$  (MUS\$/MW) represents the cost related to load shedding. Its value should be adjusted so that a small load shedding, under certain conditions, is preferable than building an additional transmission line. Thus, good-quality expansion plans can be found.

### 2.5 *Linear Disjunctive Model with Multiple Generation Scenarios, Small Line Overloads, Small Load Shedding and Small Generation Displacements*

The three modelling strategies previously shown independently can be integrated into a single, more generic mathematical model. This mathematical model can be used for each type of strategy, to incorporate either two strategies simultaneously or the three strategies at the same time. Therefore, in a more general way, a single expansion plan can be found with small displacements of the ideal generations, small load shedding and small overloads in the transmission lines. The formulation of this mathematical model changes the objective function and adds some constraints, as shown in (20)–(24).

$$\min v = \sum_{ij \in C} \sum_{y \in Y} c_{ij} w_{ij,y} + \alpha \sum_{i \in B} \sum_{s \in S} r_{i,s} + \beta \sum_{i \in B} \sum_{s \in S} |g_{i,s} - g_{i,s}^*| \quad (20)$$

subject to (3)–(11)

$$\sum_{ji \in C} \left( f_{ji,s}^o + \sum_{y \in Y} f_{ji,y,s} \right) - \sum_{ij \in C} \left( f_{ij,s}^o + \sum_{y \in Y} f_{ij,y,s} \right) + g_{i,s} + r_{i,s} = d_i \quad \forall i \in B, s \in S \quad (21)$$

$$\sum_{i \in B} \sum_{s \in S} r_{i,s} \leq (1 - \delta) \sum_{i \in B} d_i \quad (22)$$

$$0 \leq r_{i,s} \leq \bar{r}_i \quad \forall i \in B, s \in S \quad (23)$$

$$\underline{g}_{i,s} \leq g_{i,s} \leq \bar{g}_{i,s} \quad \forall i \in B, s \in S \quad (24)$$

The objective function represented by equation (20) replaces objective function (1), constraint (21) replaces constraint (2) and constraints (22), (23) and (24) must be added to the basic model represented in (3)–(11). In addition, the database must be modified to allow small increments of the transmission capacity of the lines, using the relationship shown in Section 2.3.

The linear disjunctive model is particularly important because it is used as the base for the development of modified mathematical models, such as those presented in this section. The expansion plans found by the mathematical models proposed in this paper are compared and analysed in the following section.

### 3. Tests and Results

In order to evaluate the performance of the presented models, the IEEE 24-bus system (with 41 corridors) and the Colombian 93-bus system (with 155 corridors) were used, considering the TNEP problem with four different generation scenarios (G1, G2, G3, G4), that represent each season of the year, with equal durations. The mathematical models were programmed in AMPL and solved using the solver CPLEX version 12.8 with default parameters. In the tests, a computer with a 3.2 GHz Intel® Core™ i7-8700 processor with 16 GB of RAM was used.

#### 3.1 24-Bus System

The data of the four-generation scenarios of the IEEE 24-bus system presented in the appendix were designed by Fang and Hill [9], in which the IEEE 24-bus system data is also found. The total demand in the system is 8550.00 MW. The maximum addition of three transmission lines in each corridor will be allowed in all the test cases and it is assumed in this case that the maximum load shedding at a bus is equal to the demand of the bus,  $\bar{r}_i = d_i$ , and the maximum total load shedding in the system unconstrained.

##### 3.1.1 Expansion Plan Using the Linear Disjunctive Model with a Single Generation Scenario

Table 1 presents the results when the system is expanded, considering each generation scenario individually, as proposed in [9].

**Table 1** Expansion plan for each individual scenario as presented by [9]

Scenario	Number of New Lines	Cost (MUS\$)
G1	$n_{1-5} = 1, n_{3-24} = 1, n_{6-10} = 1, n_{7-8} = 3, n_{14-16} = 1, n_{15-21} = 1, n_{15-24} = 1, n_{16-17} = 2, n_{16-19} = 1, n_{17-18} = 1$	454
G2	$n_{3-24} = 1, n_{6-10} = 1, n_{7-8} = 1, n_{10-12} = 1, n_{14-16} = 1, n_{15-21} = 1, n_{15-24} = 1, n_{16-17} = 2, n_{17-18} = 1, n_{2-8} = 1$	451
G3	$n_{1-5} = 1, n_{3-24} = 1, n_{6-10} = 1, n_{7-8} = 2, n_{10-12} = 1, n_{16-17} = 1, n_{14-23} = 1$	292
G4	$n_{3-24} = 1, n_{6-10} = 1, n_{7-8} = 2, n_{10-12} = 1, n_{12-13} = 1, n_{14-16} = 1, n_{15-24} = 1, n_{16-17} = 1$	376

Table 1 shows four expansion plans, one plan for each individual generation scenario and the respective investment cost. These plans are only shown to give an overview of the costs of independent expansion plans, such as those made in [9]; they do not represent the expansion strategy proposed in this work. Also, these expansion costs can be used for comparative analysis with those found with the strategies proposed in this work. Obviously, the expansion plans found are feasible only for each corresponding generation scenario, and they do not operate properly for the other scenarios. Since [9] did not use an exact method to solve the TNEP problem, optimality

could not be ensured. In fact, the expansion plans obtained in [9] are not optimal. Table 2 shows the optimal solution for each generation scenario G1–G4.

**Table 2** Optimal expansion plan for each individual scenario

Scenario	Number of New Lines	Cost (MUS\$)
G1	$n_{1-5} = 1, n_{3-24} = 1, n_{6-10} = 1, n_{7-8} = 2, n_{14-16} = 1, n_{15-24} = 1, n_{16-17} = 2, n_{16-19} = 1, n_{17-18} = 2$	390
G2	$n_{1-5} = 1, n_{3-24} = 1, n_{6-10} = 1, n_{7-8} = 1, n_{10-12} = 1, n_{14-16} = 1, n_{15-24} = 1, n_{16-17} = 2, n_{17-18} = 2$	392
G3	$n_{6-10} = 1, n_{7-8} = 2, n_{10-12} = 1, n_{14-16} = 1, n_{16-17} = 1, n_{20-23} = 1$	218
G4	$n_{3-24} = 1, n_{6-10} = 1, n_{7-8} = 2, n_{9-11} = 1, n_{10-12} = 1, n_{14-16} = 2, n_{16-17} = 1$	342

Table 3 present the load shedding values for the expansion plans of Tables 1 and 2, in which it can be verified that the load shedding is zero only for the generation scenarios for which the corresponding plan was obtained.

**Table 3** Load shedding for the expansion plans presented in [9] and for the optimal solutions for each scenario

Plan	Cost (MUS\$)	G1 (MW)	G2 (MW)	G3 (MW)	G4 (MW)	Total Load Shedding (MW)
G1 [9]	454	0.00	118.58	395.94	164.35	678.87
G2 [9]	451	380.65	0.00	352.74	94.04	827.43
G3 [9]	292	660.00	660.00	0.00	120.10	1440.10
G4 [9]	376	359.39	357.76	240.58	0.00	957.73
G1 optimal	390	0.00	124.98	387.26	167.46	679.70
G2 optimal	392	372.05	0.00	352.32	95.98	820.35
G3 optimal	218	479.97	386.07	0.00	132.73	998.77
G4 optimal	342	361.65	357.76	276.18	0.00	995.59

The proposal presented in [9] chooses the plans for G1 and G3 as solutions for the problem. It can be verified, in Table 3, that both plans present high values of load shedding.

### 3.1.2 Expansion Plan Using the Linear Disjunctive Model with Multiple Generation Scenarios

Table 4 shows the optimal expansion plan considering the four generation scenarios simultaneously and using the mathematical modelling shown in Section 2. Thus, this expansion plan allows the expanded system to operate properly for each one of the four different generation scenarios. The expansion cost is 532 MUS\$, with 15 new transmission lines. Obviously, this optimal expansion plan allows the system to operate adequately for the four generation scenarios, but the expansion cost increases significantly when compared to the isolated plans shown in Tables 1 and 2.

**Table 4** Optimal expansion plan for the four generation scenarios considered simultaneously

Corridor	Number of New Lines	Cost (MUS\$)
1–5	1	22
3–24	1	50
6–10	1	16
7–8	2	32
10–12	1	50
14–16	1	54
15–24	1	72
16–17	2	72
16–19	1	32
17–18	2	40
20–23	1	30
13–14	1	62
Total	15	532

### 3.1.3 Expansion Plans Found with the Modified Models for the IEEE 24-Bus System

Tests were performed with the modified models, and many alternative expansion plans were found. In the tests, the following parameters were used:  $\beta = 0.01$  MUS\$/MW, a small value only to avoid inadequate displacements of the generations in relation to the ideal value, overloads in the transmission lines varying between 1% and 5% and parameter  $\alpha$  ranging from  $\alpha = 0.3$  MUS\$/MW to 0.6 MUS\$/MW. The data for this system is presented in the appendix.

The best results are shown in Table 5. The first column of Table 5 indicates the number of the plan; the second column shows the three control parameters used to obtain the corresponding plan; the third column shows the expansion cost; the fourth column shows the maximum line overload obtained in the solution, considering all the scenarios; the fifth column shows the total load shedding for the solution, i.e., the sum of  $r_{i,s}$  in all buses and in all scenarios; and the sixth column shows the maximum generation displacement percentage in a bus in the obtained solution.

The dashes in Table 5 indicate that the corresponding strategy was not used. Thus, plan number 5 was obtained when only a 4% overload was allowed on the transmission lines and, in this context, a plan of 472 MUS\$ was obtained. In this plan, the largest power flow capacity violation appears in scenario G1 in line 12–13, which carries 519.55 MW, violating the limit of 500 MW by 3.91%, but it represents an expansion plan with an investment reduction of 60 MUS\$ compared to the 532 MUS\$ plan without violations.

Plan number 7 was found when only load shedding was allowed, and in this context, a plan of 470 MUS\$ was found. In this plan, there is a load shedding of 45.26 MW at bus 10 in scenario G1 and 13.37 MW at bus 10 in scenario G4, totalling a load shedding of 58.63 MW (note that, although the plan found in [9] for G1, shown in Table 1, has an expansion cost of 454.00 MUS\$, its total load shedding is 678.87 MW). This proposal represents a reduction of 62 MUS\$ when compared to the plan without violations. Plan number 4 was found when only small displacements of the generations around the ideal values were allowed; in this context, a plan of 500 MUS\$ was

found. In this plan, there is a maximum displacement of 7.69% for the generation of bus 15 in scenario G3 (the ideal generation of 325 MW is increased to 350 MW). However, this proposal represents a cost reduction of 32 MUS\$ when compared to the plan without violations.

**Table 5** Expansion plans for the IEEE 24-bus system

Plan	Control Parameters (MUS\$/MW)			Cost (MUS\$)	Maximum Line Overload (%)	Total Load Shedding (MW)	Maximum Generation Displacement (%)
	$\sigma$	$\alpha$	$\beta$				
1	—	—	—	532	—	—	—
2	1.02	—	—	516	1.91	—	—
3	1.03	—	—	512	2.08	—	—
4	—	—	0.01	500	—	—	7.69
5	1.04	—	—	472	3.91	—	—
6	1.02	—	0.01	472	2.00	—	10.59
7	—	0.60	—	470	—	58.63	—
8	1.05	—	—	450	4.11	—	—
9	—	0.40	—	450	—	92.29	—
10	—	0.45	0.01	450	—	30.58	7.56
11	1.03	—	0.01	450	3.00	—	6.17
12	1.01	0.35	—	450	1.00	54.95	—
13	1.02	0.35	—	450	2.00	26.25	—
14	1.03	0.40	—	450	3.00	12.01	—
15	1.04	0.40	—	450	4.00	1.21	—
16	1.01	0.40	0.01	450	1.00	11.98	11.11
17	1.02	0.40	0.01	450	2.00	1.18	11.11
18	1.03	0.45	0.01	450	3.00	0.00	6.17
19	—	0.30	0.01	428	—	77.48	7.99
20	1.01	0.30	0.01	428	1.00	62.42	11.11
21	1.02	0.35	0.01	428	2.00	54.90	11.11
22	1.03	0.40	0.01	428	3.00	47.37	11.11
23	1.04	0.45	0.01	428	4.00	39.85	11.11
24	1.05	0.50	0.01	428	5.00	32.32	11.11
25	1.02	0.30	0.01	356	2.00	264.31	11.11
26	1.03	0.35	0.01	356	3.00	235.66	11.11
27	1.04	0.40	0.01	356	4.00	206.74	11.11
28	1.05	0.40	0.01	356	5.00	179.27	11.11
29	1.05	0.35	0.01	306	5.00	306.44	11.11
30	1.05	0.30	0.01	276	5.00	390.93	11.11

Among the plans found using more than one strategy, plan number 17 is interesting, with an investment of 450 MUS\$ (a reduction of 82 MUS\$ compared to the plan without violations). This plan produces an overload of 2% in line 12–13 in scenario G1 and in line 15–21 in scenario G2 (510 MW, for a capacity of 500 MW), 0.13% in line 15–21 in scenario G1 (1001.32 MW, for a capacity of 1000 MW) and 1.20% in line 12–13 in scenario G4 (506.00 MW, for a capacity of 500 MW), a maximum displacement of the generation of 11.11% (from 315 MW to 350 MW) in bus 23 and scenario G1 and a load shedding of 1.18 MW in bus 10 and scenario G1.

It should be noted that even expansion plans with very low levels of investment, such as the plan number 27, with an investment of 356 MUS\$ (a reduction of 176 MUS\$ in relation to the

plan without violations), operate in a better condition than any of the plans found for each generation scenario individually, like those presented previously in Tables 1 and 2.

As an example, we show the lines added in expansion plans 4 and 7:

- Plan 4, with investment of 500 MUS\$:  $n_{1-5} = 1$ ,  $n_{3-24} = 1$ ,  $n_{6-10} = 1$ ,  $n_{7-8} = 2$ ,  $n_{9-11} = 1$ ,  $n_{10-12} = 1$ ,  $n_{14-16} = 1$ ,  $n_{15-24} = 1$ ,  $n_{16-17} = 2$ ,  $n_{16-19} = 1$ ,  $n_{17-18} = 1$ ,  $n_{20-23} = 1$ .
- Plan 7, with investment of 470 MUS\$:  $n_{1-5} = 1$ ,  $n_{3-24} = 1$ ,  $n_{6-10} = 1$ ,  $n_{7-8} = 2$ ,  $n_{10-12} = 1$ ,  $n_{14-16} = 1$ ,  $n_{15-24} = 1$ ,  $n_{16-17} = 2$ ;  $n_{16-19} = 1$ ,  $n_{17-18} = 2$ ,  $n_{20-23} = 1$ .

Finally, regarding the computational effort, the computational times were low in all cases, being less than five seconds in each one.

#### 3.1.4 Sensitivity Analysis

In order to evaluate the behaviour of the proposed models when some control parameters change, a sensitivity analysis was carried out. Four tests were performed, with the objective of showing how the investment costs and load shedding values are dependent on the control parameters.

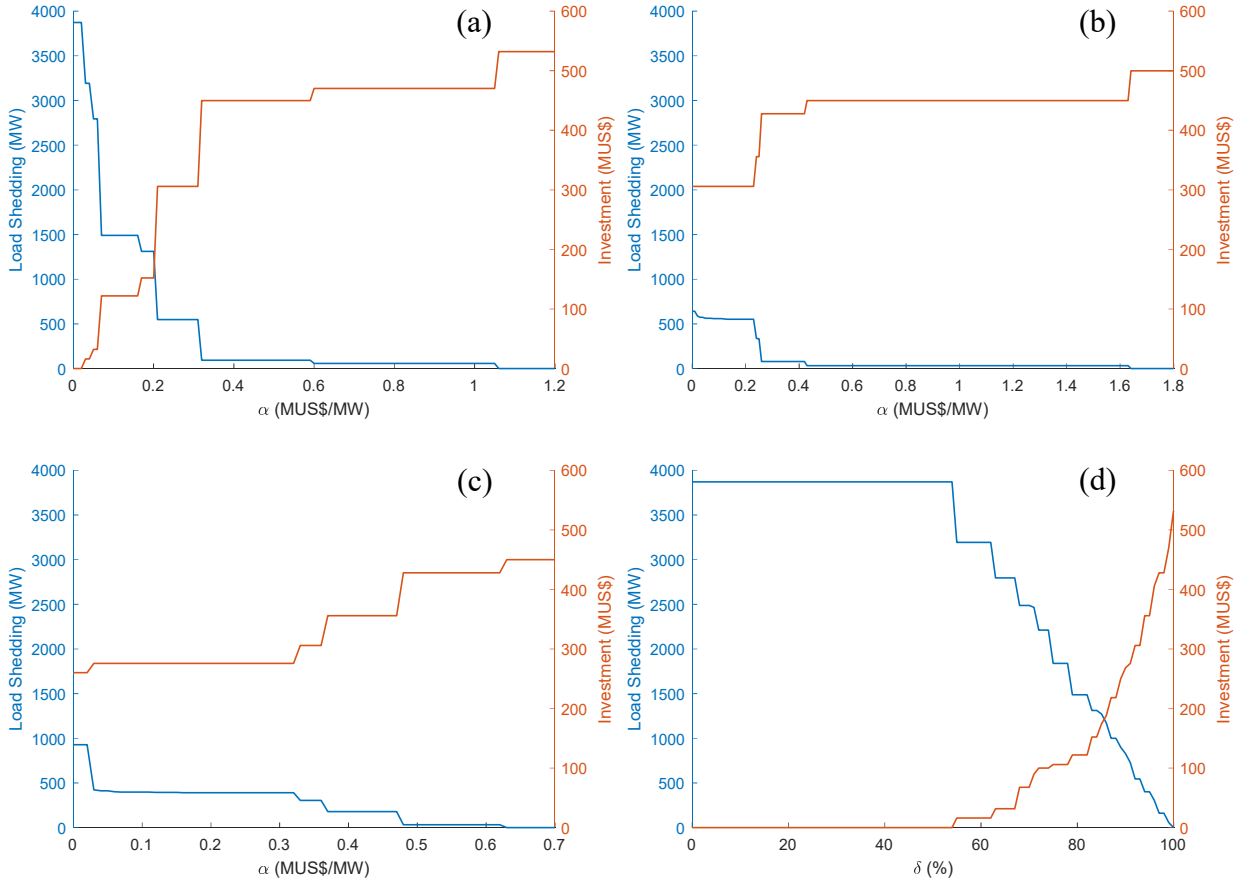
For the test shown in Figure 1(a),  $\delta = 0$  and  $\sigma = 1.00$ , i.e., the total load shedding in the system is unconstrained and no overload is allowed in the lines. Besides that, model (15)–(19) is considered, and therefore, the generations can vary from zero until the ideal values, as shown in (19). In this case, for  $\alpha$  varying from 0 MUS\$/MW until 1.2 MUS\$/MW, the objective function goes from 0 MUS\$ until 532 MUS\$, while the load shedding varies from 3871.89 MW until 0 MW. The values of objective function and load shedding for plans 7 and 9 from Table 5 can be identified in Figure 1(a).

Figure 1(b) shows that the investment cost varies from 306 MUS\$ until 500 MUS\$ and the load shedding varies from 642.71 MW until 0 MW when the model (20)–(24) is solved with  $\delta = 0$ ,  $\sigma = 1.00$ ,  $\beta = 0.01$  MUS\$/MW and  $\alpha \in [0, 1.8]$  MUS\$/MW. Note that in this case, even for  $\alpha = 0$ , i.e., no cost for load shedding, there is an investment of 306 MUS\$. This happens because constraint (24) limits the minimum generation in the system, while the power balance equation (21) must be satisfied, indirectly imposing a limit for the load shedding in the system. Plans 4, 10 and 19 from Table 5 can be identified in Figure 1(b).

Figure 1(c) shows the results for the same case shown in Figure 1(b), but with  $\sigma = 1.05$ , for  $\alpha \in [0, 0.7]$  MUS\$/MW. Note that, in this case, the investment cost varies from 260 MUS\$ until 450 MUS\$ (lower values than the previous case, since an overload is allowed in the lines) and the

load shedding varies from 928.17 MW until 0 MW. Plans 24, 28, 29 and 30 from Table 5 can be identified in Figure 1(c).

Finally, Figure 1(d) shows the results for the model (15)–(19) with  $\alpha = 0.01$  MUS\$/MW (a small value for the load shedding),  $\sigma = 1.00$  (no overload in the lines) and  $\delta \in [0,1]$ . In this case, when  $\delta = 0$ , the total load shedding in the system is unconstrained in (17) and Figure 1(d) shows a solution with an investment cost of 0 MUS\$ and a total load shedding of 3871.89 MW. When no load shedding is allowed in the system, a solution with an investment cost of 532 MUS\$ and a total load shedding of 0 MW is obtained.



**Fig. 1.** Sensitivities of the objective function and total load shedding in the system considering: (a)  $\delta = 0$ ,  $\alpha \in [0, 1.2]$  MUS\$/MW,  $\sigma = 1.00$ , in model (15)–(19), (b)  $\delta = 0$ ,  $\alpha \in [0, 1.8]$  MUS\$/MW,  $\sigma = 1.00$  and  $\beta = 0.01$  MUS\$/MW, (c)  $\delta = 0$ ,  $\alpha \in [0, 0.7]$  MUS\$/MW,  $\sigma = 1.05$  and  $\beta = 0.01$  MUS\$/MW, (d)  $\delta \in [0, 1]$ ,  $\alpha = 0.01$  MUS\$/MW,  $\sigma = 1.00$ , in the model (15)–(19)

### 3.2 Colombian 93-Bus System

Complete data of the Colombian 93-Bus system is available in [32]. The total demand in the system is 14559.00 MW. The maximum addition of four transmission lines in each corridor will be allowed in all the test cases and it is assumed in this case that the maximum load shedding at a bus is equal to the demand of the bus,  $\bar{r}_i = d_i$ , and the maximum total load shedding in the system unconstrained.



### 3.2.1 Expansion Plans Found with the Proposed Models for the 93-Bus System

Table 6 presents the results of the proposed method for ten tests using different control parameters.

**Table 6** Expansion plans for the Colombian 93-bus system

Plan	Control Parameters (MUS\$/MW)			Cost (MUS\$)	Maximum Line Overload (%)	Total Load Shedding (MW)	Maximum Generation Displacement (%)	Computational Time (min)
	$\sigma$	$\alpha$	$\beta$					
1	—	—	—	779.31	—	—	—	26.7
2	1.05	—	—	721.38	4.82	—	—	119.5
3	—	0.40	—	775.29	—	4.67	—	74.13
4	—	0.30	—	712.64	—	201.11	—	170.33
5	—	—	0.01	587.29	—	—	10.00	47.88
6	1.05	0.40	—	699.21	5.00	27.84	—	121.67
7	1.05	0.30	—	511.26	5.00	639.11	—	305.70
8	1.05	—	0.01	530.38	5.00	—	10.00	23.87
9	1.05	0.40	0.01	522.62	5.00	5.05	10.00	15.94
10	1.05	0.30	0.01	513.14	5.00	25.35	10.00	18.52

Plan number 1, shown in Table 6, operates without any constraint violation in all the generation scenarios of the model (1)–(11). However, its investment cost is 779.31 MUS\$. By applying the proposed strategies, it was possible to obtain good-quality expansion plans with objective functions down to 511.26 MUS\$ (plan 7), what represents a reduction of 34.39% in the investment cost when compared to plan number 1.

Plans number 5 and number 10 are interesting. In plan number 5, by only allowing a generation displacement of up to 10%, it was possible to obtain a solution with an investment cost of 587.29 MUS\$, what represents a reduction of 24.64% in the investment cost when compared to plan number 1. Plan number 10 presents an investment cost of 513.14 MUS\$, what represents a reduction of 34.15% in the investment cost when compared to plan number 1, and it was obtained allowing a maximum overload of 5% in the system's lines, a load shedding of 25.35 MW (0.17% of the total load) and generation displacements of up to 10%.

The numbers of lines constructed in these plans are as follows:

- Plan 1, with investment of 779.31 MUS\$:  $n_{43-88} = 2$ ,  $n_{57-81} = 2$ ,  $n_{15-18} = 1$ ,  $n_{30-64} = 1$ ,  $n_{30-65} = 1$ ,  $n_{57-84} = 2$ ,  $n_{55-84} = 2$ ,  $n_{59-67} = 1$ ,  $n_{55-62} = 2$ ,  $n_{27-29} = 1$ ,  $n_{29-64} = 1$ ,  $n_{48-63} = 1$ ,  $n_{62-73} = 1$ ,  $n_{45-81} = 2$ ,  $n_{72-73} = 2$ ,  $n_{19-82} = 2$ ,  $n_{82-85} = 1$ ,  $n_{68-86} = 2$ .
- Plan 5, with investment of 587.29 MUS\$:  $n_{43-88} = 2$ ,  $n_{45-54} = 2$ ,  $n_{30-65} = 1$ ,  $n_{55-57} = 1$ ,  $n_{55-84} = 1$ ,  $n_{56-57} = 1$ ,  $n_{55-62} = 1$ ,  $n_{16-21} = 1$ ,  $n_{27-29} = 1$ ,  $n_{29-64} = 1$ ,  $n_{48-63} = 1$ ,  $n_{62-73} = 1$ ,  $n_{54-56} = 1$ ,  $n_{72-73} = 2$ ,  $n_{19-82} = 2$ ,  $n_{82-85} = 1$ ,  $n_{68-86} = 1$ .

- Plan 10, with investment of 513.14 MU\$:  $n_{43-88} = 2$ ,  $n_{45-54} = 1$ ,  $n_{57-84} = 1$ ,  $n_{55-84} = 1$ ,  $n_{56-57} = 1$ ,  $n_{55-62} = 1$ ,  $n_{27-64} = 1$ ,  $n_{19-66} = 1$ ,  $n_{48-63} = 1$ ,  $n_{62-73} = 1$ ,  $n_{54-56} = 1$ ,  $n_{72-73} = 1$ ,  $n_{19-82} = 1$ ,  $n_{82-85} = 1$ ,  $n_{68-86} = 1$ .

Finally, regarding the computational effort, since the TNEP problem is solved offline, the computational times shown in Table 6, between 15.94 minutes and 305.70 minutes are acceptable.

## 4. Conclusion

This paper proposes a method to solve the transmission network expansion planning problem with multiple generation scenarios. The objective is to find a set of expansion plans that operate adequately for the different generation scenarios considered. Strategies were presented to reduce the high investment cost of the expansion plan that is obtained when the traditional problem is solved.

Tests were conducted using the IEEE 24-bus system and the Colombian 93-bus system. The results indicate that the modifications incorporated in the proposed mathematical model lead to solutions with substantial reductions in the investment costs, with very small infeasibilities in the operation compared to other approach presented in the literature. Thus, many alternative expansion plans can be found by simply changing the penalty and overload factors. A sensitivity analysis was conducted to evaluate the influence of the values of the penalty and overload factors in the investment costs. By allowing lower penalty factors it was possible to obtain lower investment costs. The expansion plans found by the presented mathematical models can be considered of great practical interest, and the best alternatives can be identified when the requirements of the planning entity are taken into account.

## 5. Acknowledgment

This work was supported by the Coordination for the Improvement of Higher Education Personnel (CAPES) – Finance Code 001, the Brazilian National Council for Scientific and Technological Development (CNPq), under grant 305852/2017-5, and the São Paulo Research Foundation (FAPESP), under grants 2014/23741-9 and 2015/21972-6.

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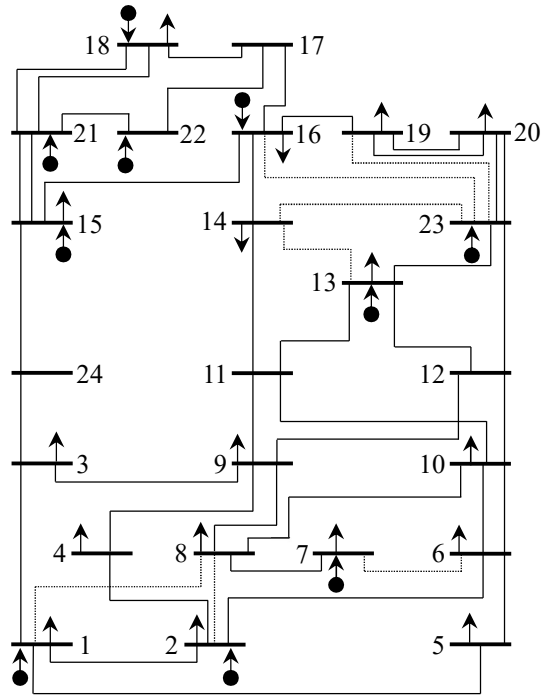
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## Appendix

Table 7 presents the data of minimum and maximum generation (a small range variation) for each generation bus in each scenario of the 24-bus system. The ideal generation data for the four generation scenarios of the IEEE 24-bus system is presented in Table 8, as are the demand data [9]. The data for the corridors and lines of the IEEE 24-bus system are shown in Table 9. Finally, Figure 2 shows the IEEE 24-bus system with 41 corridors used in the tests.

**Table 7** Generation data for the IEEE 24-bus system—multiple scenarios within a generation range

Bus	G1 (MW)		G2 (MW)		G3 (MW)		G4 (MW)	
	Min	Max	Min	Max	Min	Max	Min	Max
1	540	576	430	470	530	576	480	560
2	540	576	540	576	530	576	480	560
7	860	900	690	750	860	900	770	830
13	1720	1773	1380	1450	1400	1500	1540	1640
15	620	645	620	645	290	350	550	620
16	420	465	420	465	250	310	400	450
18	1160	1200	1160	1200	550	640	680	750
21	1150	1200	1150	1200	900	990	1000	1110
22	860	900	860	900	860	900	860	900
23	290	350	900	990	1900	1980	1350	1450



**Fig. 2.** IEEE 24-bus system

**Table 8** Generation and demand data for the IEEE 24-bus system

Bus	Load (MW)	G1 (MW)	G2 (MW)	G3 (MW)	G4 (MW)
1	324	576	465	576	520
2	291	576	576	576	520
3	540	0	0	0	0
4	222	0	0	0	0
5	213	0	0	0	0
6	408	0	0	0	0
7	375	900	722	900	812
8	513	0	0	0	0
9	525	0	0	0	0
10	585	0	0	0	0
11	0	0	0	0	0
12	0	0	0	0	0
13	795	1773	1424	1457	1599
14	582	0	0	0	0
15	951	645	645	325	581
16	300	465	465	282	419
17	0	0	0	0	0
18	999	1200	1200	603	718
19	543	0	0	0	0
20	384	0	0	0	0
21	0	1200	1200	951	1077
22	0	900	900	900	900
23	0	315	953	1980	1404
24	0	0	0	0	0

**Table 9** Data for the IEEE 24-bus system's corridors

Corridor	Power Flow Capacity (MW)	Reactance (p.u.)	Line Cost (MU\$)	Number of Existing Lines
1-2	175	0.0139	3	1
1-3	175	0.2112	55	1
1-5	175	0.0845	22	1
2-4	175	0.1267	33	1
2-6	175	0.1920	50	1
3-9	175	0.1190	31	1
3-24	400	0.0839	50	1
4-9	175	0.1037	27	1
5-10	175	0.0883	23	1
6-10	175	0.0605	16	1
7-8	175	0.0614	16	1
8-9	175	0.1651	43	1
8-10	175	0.1651	43	1
9-11	400	0.0839	50	1
9-12	400	0.0839	50	1
10-11	400	0.0839	50	1
10-12	400	0.0839	50	1
11-13	500	0.0476	66	1
11-14	500	0.0418	58	1
12-13	500	0.0476	66	1
12-23	500	0.0966	134	1
13-23	500	0.0865	120	1
14-16	500	0.0389	54	1
15-16	500	0.0173	24	1
15-21	500	0.0490	68	2
15-24	500	0.0519	72	1
16-17	500	0.0259	36	1
16-19	500	0.0231	32	1
17-18	500	0.0144	20	1
17-22	500	0.1053	146	1
18-21	500	0.0259	36	2
19-20	500	0.0396	55	2
20-23	500	0.0216	30	2
21-22	500	0.0678	94	1
1-8	500	0.1344	35	0
2-8	500	0.1267	33	0
6-7	500	0.1920	50	0
13-14	500	0.0447	62	0
14-23	500	0.0620	86	0
16-23	500	0.0822	114	0
19-23	500	0.0606	84	0