arXiv:1504.04038v2 [hep-th] 3 Jun 2015

D-Oscillons in the Standard Model-Extension

R. A. C. Correa*
CCNH, Universidade Federal do ABC, 09210-580, Santo André, SP, Brazil

Roldão da Rocha†
CMCC, Universidade Federal do ABC, 09210-580, Santo André, SP, Brazil and International School for Advanced Studies (SISSA), Via Bonomea 265, 34136 Trieste, Italy

A. de Souza Dutra‡
Universidade Estadual Paulista (UNESP), Campus de Guaratinguetá, 12516-410, SP, Brazil

In this work we investigate the consequences of the Lorentz symmetry violation on extremely long-living, time-dependent, and spatially localized field configurations, named oscillons. This is accomplished for two interacting scalar field theories in $(D+1)$ dimensions in the context of the so-called Standard Model-Extension. We show that D-dimensional scalar field lumps can present a typical size $R_{\rm min} \ll R_{KK}$, where R_{KK} is the extent of extra dimensions in Kaluza-Klein theories. The size $R_{\rm min}$ is shown to strongly depend upon the terms that control the Lorentz violation of the theory. This implies either contraction or dilation of the average radius $R_{\rm min}$, and a new rule for its composition, likewise. Moreover, we show that the spatial dimensions for existence of oscillating lumps have an upper limit, opening new possibilities to probe the existence of D-dimensional oscillons at TeV energy scale. In addition, in a cosmological scenario with Lorentz symmetry breaking, we show that in the early Universe with an extremely high energy density and a strong Lorentz violation, the typical size $R_{\rm min}$ was highly dilated. As the Universe had expanded and cooled down, it then passed through a phase transition towards a Lorentz symmetry, wherein $R_{\rm min}$ tends to be compact.

PACS numbers: 11.25.-w, 11.27.+d, 89.70.Cf
Keywords: Oscillons, Lorentz violation, nonlinear models, Kaluza-Klein models

I. INTRODUCTION

The Lorentz invariance represents the essential symmetry in the Standard Model of elementary particles. Notwithstanding, Lorentz symmetry may be violated at high energies [1], constituting thus a fundamental tool in several fields [2–7]. For instance, by using a scalar-vector-tensor theory with Lorentz violation, the exact Lorentz violation inflationary solutions can be found without an inflaton potential [8]. Topological defects in a Lorentz symmetry violation (LSV) framework have been recently addressed [10–12], providing in particular the LSV as an asymmetry between defects and anti-defects [9]. Motivated by these results, travelling solitons in Lorentz and CPT breaking systems were studied [12], where the solutions present a critical behavior controlled by the choice of a scalar. Other prominent interests regarding LSV further arise in various contexts, encompassing, e. g., gravity, monopoles and vortices [13–15].

Topologically stable configurations play a prominent role on non-linear models. Among non-linear field configurations, a distinguished class of time-dependent stable solutions are exemplified by the breathers, in sine-Gordon like models. Another time-dependent field configuration whose stability is granted by charge conservation are the so-called Q-balls, as named by Coleman [16], or alternatively non-topological solitons [17]. However, considering the fact that many physical systems interestingly may present a metastable behavior, a further class of non-linear systems may present a very long-living configuration, usually known as oscillon. This class of solutions was discovered by Bogolyubsky and Makhankov [18], and then rediscovered posteriorly by Gleiser [19]. These solutions appeared in the study of the dynamics of first-order phase transitions and bubble nucleation. Since then, an increasing amount of works has been dedicated to the study of these objects [19–45].

Oscillons are quite general configurations found in various contexts, as the Abelian-Higgs $U(1)$ models [36], the standard model $SU(2)\times U(1)$ [26, 28], inflationary cosmological models [29, 30], axion models [31], expanding Universe scenarios [27, 33, 41] and systems involving phase transitions as well [20]. In a recent work by Gleiser *et al.* [38] the problem of the hybrid inflation characterized by two real scalar fields interacting quadratically was analyzed, where a new class of oscillons arises both in excited and ground states as well.

The usual oscillon aspect is typically that of a bell shape which oscillates sinusoidally. They are long lived time-dependent D-dimensional scalar field lumps. Moreover, their lifetimes are long enough to yield noteworthy effects. In fact, their collapse happens in very short time scales, thus they might be evinced in large com-

*Electronic address: rafael.couceiro@ufabc.edu.br
†Electronic address: roldao.rocha@ufabc.edu.br
‡Electronic address: dutra@feg.unesp.br

pact extra dimensions frameworks as an abrupt burst of particles from a small region [25]. Recently, Amin and Shirokoff have shown that depending upon the intensity of the self-interacting scalar field coupling constant, it is possible to observe oscillons with a kind of plateau at its top [41]. Indeed, these new oscillons were shown to be more robust with respect to collapse instabilities in three spatial dimensions. In a recent work, the impact of the Lorentz and CPT breaking symmetries was discussed in the context of the so-called flat-top oscillons [42].

At this point it is worth to remark that Segur and Kruskal [44] have shown that the asymptotic expansion does not represent in general an exact solution for the scalar field. Indeed, it simply represents a first order asymptotic expansion. They also showed in one spatial dimension that the solutions radiate as well [44]. Besides, the computation of the emitted radiation of the oscillons was extended for two and three spatial dimensions [39]. Another important result was put forward by Hertzberg [40], computing the decaying rate of quantized oscillons and showing moreover that the quantum rate decay is very distinct from the classical one.

Our main aim here is to show that in a LSV framework oscillons present a typical size $R_{\rm min}$ that strongly depends upon the LSV parameters. In addition we shall prove that in the early Universe, with an extremely high energy density and strong Lorentz violation, the typical size $R_{\rm min}$ is characterized by a high dilation, in a cosmological scenario with LSV. We show that as the Universe expanded and cooled down, occurring a phase transition towards a Lorentz symmetry framework, the oscillon size $R_{\rm min}$ is compacted.

This work is organized as follows: in Sect. II we introduce a Lagrangian regarding the Standard Model-Extension. Hence, a two scalar fields theory, in $(D+1)$-dimensional space-time, is taken into account in a LSV framework. In Sect. III oscillons configurations are thus modelled with respect to a Gaussian formulation. Thus in Sect. IV the stability of the oscillons in the Standard Model-Extension is studied and some kinds of potentials, including the quadratic, cubic and quartic ones, are analyzed, providing a minimal size for the oscillon. Moreover, we show that the dimension for the existence of oscillating lumps presents an upper limit. Sect. VI provides an application of the studied framework, concerning a double-well potential.

II. STANDARD MODEL-EXTENSION LAGRANGIAN IN D DIMENSIONS

In this section the Lagrangian in the Standard Model-Extension (SME) context is introduced, describing a theory with two scalar fields in $(D+1)$-dimensional space-time in a Lorentz symmetry breaking framework. Our aim in working with a two scalar fields theory with Lorentz violation (LV) comes from the fact that such theories can provide observable effects. Therefore, the approach is much more favorable from an experimental point of view. Here, we will consider a local field theory in which the Lagrangian L can be written as a volume integral over a density function $\mathcal{L}$

$$L = \int d^D x \, \mathcal{L}(\phi, \chi, \dot{\phi}, \dot{\chi}, \nabla\phi, \nabla\chi), \tag{1}$$

where the Lagrangian density may depend on the field functions $\phi(\mathbf{x}, t)$ and $\chi(\mathbf{x}, t)$, on its time derivatives $\dot{\phi}(\mathbf{x}, t)$ and $\dot{\chi}(\mathbf{x}, t)$, and also on the gradients $\nabla\phi(\mathbf{x}, t)$ and $\nabla\chi(\mathbf{x}, t)$. It is important to remark that the restriction to local Lagrange density is sufficiently general for the framework of all recent field theories.

Furthermore, since any deformation away from spherical symmetry leads to more energetic configurations [16], we will deal with spherically symmetric field configurations. In this case, we can write the D-dimensional spherical coordinate system as

$$\begin{aligned}
x_1 &= r \sin\theta_1 \cdots \sin\theta_{D-1}, \\
x_2 &= r \sin\theta_1 \cdots \sin\theta_{D-2} \cos\theta_{D-1}, \\
&\;\;\vdots \\
x_k &= r \sin\theta_1 \cdots \sin\theta_{D-k} \cos\theta_{D-k+1}, \quad 2 \le k \le D-1, \\
&\;\;\vdots \\
x_D &= r \cos\theta_1 .
\end{aligned} \tag{2}$$

Here, it is important to highlight that this system is the generalization of spherical coordinates in three dimensions. Thus, using the above spherical coordinates system yields the volume element $d^D x = r^{D-1} dr d\Omega_D$, where $d\Omega_D$ is the so-called element of the D-dimensional solid angle, given by

$$d\Omega_D = \sin^{D-2}\theta_1 \sin^{D-3}\theta_2 \cdots \sin\theta_{D-2} d\theta_1 \cdots d\theta_{D-1}, \tag{3}$$

Now, the integral

$$\int_0^\pi d\theta \sin^n\theta = \frac{\sqrt{\pi}\,\Gamma[(n+1)/2]}{\Gamma[(n+2)/2]}, \tag{4}$$

yields the total solid angle in D dimensions $\Omega_D = \frac{2\pi^{D/2}}{\Gamma(D/2)}$.

Thus, we can rewrite the Lagrangian (1), in D-dimensional spherical coordinates, in the form

$$L = \frac{2\pi^{D/2}}{\Gamma(D/2)} \int dr r^{D-1} \mathcal{L}(\phi, \chi, \dot{\phi}, \dot{\chi}, \nabla\phi, \nabla\chi), \tag{5}$$

where the fields ϕ and χ now are functions of $r = \sqrt{x_1^2 + \cdots + x_D^2}$, and also of t.

However, in order to work with a LSV theory, we assume that the Lagrangian density L has the form

$$\mathcal{L} = \frac{1}{2}\partial_a\phi\partial^a\phi + \frac{1}{2}\partial_a\chi\partial^a\chi + K^{ab}\partial_a\phi\partial_b\chi - V(\phi, \chi), \tag{6}$$

where $a,b = 0,1$ and $V(\phi,\chi)$ denotes the scalar field potential. Also, we are using the following definition

$$\partial_a := (\partial/\partial t, \partial/\partial r),\ \partial^a := (\partial/\partial t, -\partial/\partial r). \tag{7}$$

Moreover, in the above Lagrangian (6)

$$K^{ab} = \begin{pmatrix} K^{00} & K^{01} \\ K^{10} & K^{11} \end{pmatrix}, \tag{8}$$

is a dimensionless rank-2 symmetric tensor that encompasses the Lorentz symmetry breaking. It is worth to emphasize that in general K^{ab} has arbitrary parameters, but if this matrix is real, symmetric, and traceless, the CPT symmetry is conserved [51–54]. Moreover, under CPT operation, namely $\partial_a \mapsto -\partial_a$, the term $K^{ab}\partial_a\phi\partial_b\chi$ goes as $K^{ab}\partial_a\phi\partial_b\chi \mapsto +K^{ab}\partial_a\phi\partial_b\chi$. Thus, K^{ab} is CPT-even [42, 54]. Furthermore, the tensor K^{ab} should be symmetric in order to avoid a vanishing contribution. The LSV parameters are denoted by $K^{00} \sim K^{11} = \alpha$ and $K^{01} \sim K^{10} = \beta$.

It is appropriate to stress that in (6) the coefficients for LV cannot be removed from the Lagrangian by using either variable or field redefinitions. In fact, coordinate choices and field redefinitions solely make the Lorentz symmetry violation to go to another sector of the theory.

The aim here is to analyze the behavior of oscillons in a class of potentials that are as general as possible. We then consider systems that can be decoupled by applying a field redefinition

$$\phi = \frac{\psi+\sigma}{\sqrt{2}}, \qquad \chi = \frac{\psi-\sigma}{\sqrt{2}}. \tag{9}$$

Hence, the preceding rotation shows that it is possible to put the Lagrangian (5) in the form

$$L = \frac{2\pi^{D/2}}{\Gamma(D/2)}\int dr r^{D-1}\left[\frac{1}{2}(1+\alpha)(\partial_t\psi)^2 + \frac{1}{2}(1-\alpha)(\partial_t\sigma)^2 -\frac{1}{2}(1-\alpha)(\partial_r\psi)^2 - \frac{1}{2}(1+\alpha)(\partial_r\sigma)^2 + \beta\partial_t\psi\,\partial_r\psi -\beta\partial_t\sigma\partial_r\sigma - V(\psi,\sigma)\right]. \tag{10}$$

Let us now consider a class of potentials $V(\phi,\chi)$ such that the rotation enables to write

$$V(\psi,\sigma) := U_1(\psi) + U_2(\sigma), \tag{11}$$

where U_1 and U_2 are arbitrary. In this case, we can find the original potential described in terms of the fields ϕ and χ, performing the rotation (9) back.

Note that under the condition (11) the Lagrangian reads a sum of two independent Lagrangians

$$L = \sum_{j=1}^{2} L_j,$$

with

$$L_j = \frac{2\pi^{D/2}}{\Gamma(D/2)}\int dr r^{D-1}\left[A_j(\partial_t\Phi_j)^2 - B_j(\partial_r\Phi_j)^2 C_j(\partial_t\Phi_j)(\partial_r\Phi_j) - U_j(\Phi_j)\right], \tag{12}$$

where the following notation

$$\Phi_1 = \psi, \qquad \Phi_2 = \sigma,$$
$$A_j = [1+(-1)^{j+1}\alpha]/2, \tag{13}$$
$$B_j = [1+(-1)^{j}\alpha]/2, \tag{14}$$
$$C_j = (-1)^{j+1}\beta, \tag{15}$$

is used.

It is important to remark that the Lagrangian (12) is represented by a sum of two independent Lagrangians, where the fields ϕ and χ are connected through a inverse rotation with respect to Eq. (9). In the next section, we show the analytic approach to find oscillons.

III. MODELLING OSCILLONS CONFIGURATIONS

The analytic approach to investigate the dynamics of time-dependent oscillating scalar field configurations was introduced by Gleiser [25]. Motivated by numerical investigations [19, 23], Gleiser showed that oscillons are well approximated by a Gaussian curve. Thus, in the present work, we also assume that oscillons solutions can be modelled as

$$\Phi_j(r,t) = G_j(t)\exp\left(\frac{-q_j r^2}{R_j^2}\right) + \Phi_j^v. \tag{16}$$

where $q_j > 0$ and Φ_j^v is the asymptotic value when $r \to \infty$, which is determined by the form of the potentials $U_j(\Phi_j)$. At this point, it is important to remark that any ansatz is an assumption, where boundary conditions can be taken into account. After an ansatz has been established, the equations are solved for the general function. Hence we can verify the validity of the assumption provided by the ansatz. Eq. (16) is the natural choice. In fact, as shown in Refs. [19, 20], oscillons can be found by profiles either Gaussian or hyperbolic tangent type. Here we use the Gaussian ansatz in Eq. (16), where $G_j(t)$ is an amplitude that can be identified with $\Phi_j(0,t) - \Phi_j^v$, where R_j denotes the core radius. Similarly to the case of Lorentz symmetry, our case regarding LSV violation makes the evolution of the configuration to split into different stages. The lifetime of the oscillon configuration is sensitive to the choices of R_j and $\Phi_j(0,t)$. The lifetime of the oscillon configuration is thus related to perturbations arising by the different choices of initial parameters, what increases the amount of radiation being emitted. As shown in Ref. [19], due to the tiny, however steady, oscillon radiation, at some point the maximum amplitude decreases below the inflection point of the potential and $\Phi_j(0,t) \to \Phi_j^v$ exponentially fast. During the nonlinear evolution of these configurations a regime of dynamical stability was shown to be achieved, where the energy is conserved within a localized region [19–22]. Such a Gaussian ansatz is further acquired by numerical analysis in an overlapping context. The analytical results obtained were verified numerically to great accuracy, confirming to be adequate for the goals investigated in, e .g., [19–22].

Here we are interested in the particular case of polynomial potentials, which are written as

$$U_j(\Phi_j) = \sum_{n=0}^{N} \frac{g_{j,n}}{n!}\Phi_j^n - U_j(\Phi_j^v), \tag{17}$$

where N denotes the maximal power of the potential, $g_{j,n}$ are scalars, and the vacuum energy $U_j(\Phi_j^v)$ is taken away from the potential to prevent spurious divergences upon spatial integration. An immediate consequence of the definition of $U_j(\Phi_j)$ in Eq. (17) is that the potential $V(\Phi_1,\Phi_2)$ now takes the form

$$V(\Phi_1,\Phi_2) = \sum_{n=0}^{N} \frac{g_{1,n}}{n!}\Phi_1^n + \sum_{n=0}^{N} \frac{g_{2,n}}{n!}\Phi_2^n - U(\Phi_1^v,\Phi_2^v), \tag{18}$$

where $U(\Phi_1^v,\Phi_2^v) = U_1(\Phi_1^v) + U_2(\Phi_2^v)$ is the vacuum energy.

Hence, using Eqs. (16) and (17) into Eq. (12), and integrating over space coordinates, it yields

$$\begin{aligned} L_j = \pi^{D/2}\Bigg\{ & \varepsilon_j^{-D/2} A_j \dot{G}_j^2 - \frac{2q_j^2}{R_j^4} D\varepsilon_j^{-(D+2)/2} B_j G_j^2 \\ & - \frac{q_j}{R_j^2}\,\varepsilon_j^{-(D+1)/2}\frac{\Gamma[(D+1)/2]}{\Gamma(D/2)} C_j G_j \dot{G}_j \\ & - \sum_{n=2}^{N} \frac{G_j^n}{n!}\varepsilon_{j,n}^{-D/2} U_j^{(n)}(\Phi_j^v)\Bigg\}, \end{aligned} \tag{19}$$

where $\varepsilon_j := 2q_j/R_j^2$, $\varepsilon_{j,n} := nq_j/R_j^2$, $U_j^{(n)}(\Phi_j^v) := dU_j^n(\Phi_j^v)/d\Phi^n$, and the dot stands for the derivative with respect to time.

IV. STABILITY OF THE OSCILLONS IN THE STANDARD MODEL-EXTENSION

In the preceding sections we introduced the Lagrangian in the SME context. In this section, we are going to deduce the equations of motion for the functions $G_j(t)$ in (16) and introduce the so-called effective frequency ω_j as well, which is necessary to examine the stability of the oscillon. Hence, from the Lagrangian (19) the corresponding equations of motion read

$$\begin{aligned} & \ddot{G}_j + \frac{2Dq_j^2}{R_j^4}\frac{B_j}{A_j\varepsilon_j}G_j \\ & + \frac{\varepsilon_j^{D/2}}{2A_j}\sum_{n=2}^{N}\frac{G^{n-1}}{(n-1)!}\varepsilon_{j,n}^{-D/2}U_j^{(n)}(\Phi_j^v) = 0. \end{aligned} \tag{20}$$

In order to analyze the stability of the system, let us expand the amplitude as

$$G_j(t) = G_{j,0}(t) + \delta G_j(t), \tag{21}$$

where $G_{j,0}(t)$ is the solution of Eq. (20), and $\delta G_j(t)$ describes a small perturbation.

Therefore, by applying Eq. (21) into Eq. (20) and subsequently linearizing the result, it yields

$$\begin{aligned} \delta\ddot{G}_j = & -\frac{DB_j}{2A_j\varepsilon_j}\left(\frac{2q_j}{R_j^2}\right)^2 \\ & + \frac{\varepsilon_j^{D/2}}{2A_j}\left[\sum_{n=2}^{N}\frac{G_{j,0}^{n-2}}{(n-2)!}\varepsilon_{j,n}^{-D/2}U_j^{(n)}(\Phi_j^v)\right]\delta G_j. \end{aligned} \tag{22}$$

In the light of the analysis accomplished in this section, the following effective frequency is introduced

$$\begin{aligned} \omega_j^2(R_j,A_j,B_j,G_{j,0}) := & \frac{DB_j}{2A_j\varepsilon_j}\left(\frac{2q_j}{R_j^2}\right)^2 \\ & + \frac{\varepsilon_j^{D/2}}{2A_j}\left[\sum_{n=2}^{N}\frac{G_{j,0}^{n-2}}{(n-2)!}\varepsilon_{j,n}^{-D/2}U_j^{(n)}(\Phi_j^v)\right]. \end{aligned} \tag{23}$$

We can see, from the above effective frequency, that this quantity leads to a distinct definition of frequency in a scenario with LSV. Indeed, here there is an explicit dependence of the parameters responsible by the Lorentz violation.

For simplicity, let us use the above definition to rewrite Eq.(22) in the form

$$\delta\ddot{G}_j = -\omega_j^2(R_j,A_j,B_j,G_{j,0})G_j. \tag{24}$$

The above equation enables us to understand the stability of the motion. In fact, if $\omega_j^2 < 0$, instabilities occur. On the other hand, when $\omega_j^2 > 0$, the system is stable.

As a straightforward example, consider the case where $U_j(\Phi_j) = 0$. Consequently the effective frequency becomes

$$\omega_j^2 = \frac{D}{R_j^2}\left(\frac{q_jB_j}{A_j}\right). \tag{25}$$

The above established result prominently enables us to show that the frequency is a function of the parameters responsible by the effects of the Lorentz violation.

A. Quadratic potentials

In this section we will consider the case of quadratic potentials, where $U_j(\Phi_j)$ is given by

$$U_j(\Phi_j) = g_{j,1}\Phi_j + \frac{g_{j,2}\Phi_j^2}{2} - U(\Phi_j^v). \tag{26}$$

In this case, we have

$$\omega_j^2 = \frac{D}{R_j^2}\left(\frac{q_j B_j}{A_j}\right) + \frac{U^{(2)}(\Phi_j^v)}{2A_j}. \tag{27}$$

Therefore, imposing that A_j, B_j, and $U^{(2)}(\Phi_j^v)$ are positive it implies that $\omega_j^2 > 0$, precluding thus any kind of instability. On the other hand, if $A_j > 0$, $B_j > 0$, and $U^{(2)}(\Phi_j^v) < 0$, instabilities are possible when

$$\frac{D}{R_j^2} < \frac{\left|U^{(2)}(\Phi_j^v)\right|}{2q_j B_j}, \tag{28}$$

which provides a minimal values for the oscillon typical size:

$$R_j \geq \sqrt{2q_j B_j}\left[\frac{D}{\left|U^{(2)}(\Phi_j^v)\right|}\right]^{1/2}. \tag{29}$$

Nevertheless, instabilities are allowed when $A_j > 0$, $B_j < 0$ and $U^{(2)}(\Phi_j^v) < 0$, such that

$$R_j \geq \sqrt{2q_j\left|B_j\right|}\left[\frac{D}{U^{(2)}(\Phi_j^v)}\right]^{1/2}. \tag{30}$$

At this point, we observe that once one recover the expression of the original fields ϕ and χ by using the results obtained for Φ_1 and Φ_2, the resulting minimal size of the ϕ and χ oscillons is approximately the one of the biggest of the decoupled fields Φ_1 and Φ_2. This can seen in the case plotted in the Figure 1. In fact, this is a general feature of these configurations and will equally appear in the next examples.

B. Cubic potentials

Now, in this section we will analyze the case of the cubic potential. Therefore, we consider the following potential

$$U_j(\Phi_j) = g_{j,1}\Phi_j + \frac{g_{j,2}\Phi_j^2}{2!} + \frac{g_{j,3}\Phi_j^3}{3!} - U(\Phi_j^v). \tag{31}$$

At this point, it is important to remark that in the cubic potential the parity is broken. Consequently, the potential has an inflection point which is determined by the relation $U_j^{(2)}(\Phi_j) = 0$. As a consequence, the inflection point is provided by

$$\Phi_j^{\rm inf} = -g_{j,2}/g_{j,3}. \tag{32}$$

For simplicity, however without loss of generality, we take $g_{j,1} = 0$. Thus, the vacuum state corresponds to the values

$$\Phi_j^v = \begin{cases} 0, & \text{for } g_{j,2} > 0, \\ -2g_{j,2}/g_{j,3}, & \text{for } g_{j,2} < 0. \end{cases} \tag{33}$$

Now, by using the potential (31) we can show that the effective frequency can be written as

$$\omega_j^2 = \frac{D}{R_j^2}\left(\frac{q_j B_j}{A_j}\right) + \frac{U^{(2)}(\Phi_j^v)}{2A_j} + \left(\frac{2}{3}\right)^{D/2}\frac{G_{j,0}U^{(3)}(\Phi_j^v)}{2A_j},$$

which reads

$$\omega_j^2 = \frac{D}{R_j^2}\left(\frac{q_j B_j}{A_j}\right) + \frac{g_{j,3}}{2A_j}\left[\frac{g_{j,2}}{g_{j,3}} + \Phi_j^v + \left(\frac{2}{3}\right)^{D/2} G_{j,0}\right].$$

To satisfy the condition $\omega_j^2 < 0$, which is necessary to guarantee the existence of oscillons, the sign of $G_{j,0}$ must be opposite to that of $g_{j,3}$. Indeed, long-lived oscillons can exist if the oscillations above the vacuum of $U_j^{(2)}(\Phi_j) < 0$ for a sustained period of time. Therefore, the relations of existence, to be analyzed in the following sub-subsections, hold. In addition, in the following cases the minimal radius explicitly depends of the LSV parameter α, through the B_j in Eq.(14).

1. *For $g_{j,2} > 0$ and $g_{j,3} \gtrless 0$*

Here, we find that

$$R_j \geq \sqrt{\frac{2Dq_j B_j}{\left|g_{j,3}\right|\left[-\Phi_j^{\rm inf} + \left(\frac{2}{3}\right)^{D/2} G_{j,0}\right]}}. \tag{34}$$

2. *For $g_{j,2} < 0$ and $g_{j,3} \gtrless 0$*

In this case, it follows that the oscillon radius obeys the constraint

$$R_j \geq \sqrt{\frac{2Dq_j B_j}{\left|g_{j,3}\right|\left[\Phi_j^{\rm inf} + \left(\frac{2}{3}\right)^{D/2}\left|G_{j,0}\right|\right]}}. \tag{35}$$

Moreover, since that R_j^2 must be a positive number, the amplitudes $G_{j,0}$ must obey the condition

$$\left|G_{j,0}\right| \lesseqqgtr \left(\frac{3}{2}\right)^{D/2}\left|\Phi_j^{\rm inf}\right|, \quad \text{for } B_j \lessgtr 0\,.$$

Again, as advertised in the previous section, the minimal radius of the ϕ and χ fields will be the bigger one between R_1 and R_2.

C. Quartic potentials

We are now going to study the case of quadratic potentials, represented by

$$U_j(\Phi_j) = g_{j,1}\Phi_j + \frac{g_{j,2}\Phi_j^2}{2!} + \frac{g_{j,3}\Phi_j^3}{3!} + \frac{g_{j,4}\Phi_j^4}{4!} - U(\Phi_j^v). \quad (36)$$

Thus, by using (23) it forthwith yields that

$$\omega_j^2 = \frac{D}{R_j^2}\frac{q_j B_j}{A_j} + \frac{1}{2A_j}\left[U^{(2)}(\Phi_j^v) + \left(\frac{2}{3}\right)^{D/2} G_{j,0}U^{(3)}(\Phi_j^v) + \frac{1}{2^{D/2+1}}G_{j,0}^2 U^{(4)}(\Phi_j^v)\right]. \quad (37)$$

Again, the condition for the existence of oscillating lumps is described by $\omega_j^2 < 0$. However, the results depend upon the sign of $U^{(4)}(\Phi_j^v) = g_{j,4}$, such that two conditions $g_{j,4} \gtrless 0$ should be analyzed. Firstly, let us analyze the case where $g_{j,4} > 0$. In addition let us assume $A_j > 0$ and $B_j > 0$ and that

$$\omega_j^2 := \Omega(G_{0,j}), \quad (38)$$

which is a parabola with positive concavity with a minimum localized at

$$G_{0,j}^{\min} = -\left(\frac{4}{3}\right)^{D/2}\frac{U^{(3)}(\Phi_j^v)}{U^{(4)}(\Phi_j^v)}. \quad (39)$$

Consequently, it yields

$$\Omega(G_{0,j}^{\min}) = \frac{D}{R_j^2}\left(\frac{q_j B_j}{A_j}\right) + \frac{1}{2A_j}\left\{U^{(2)}(\Phi_j^v) - 2^{(D-2)/2}\left(\frac{2}{3}\right)^{D}\frac{\left[U^{(3)}(\Phi_j^v)\right]^2}{U^{(4)}(\Phi_j^v)}\right\}. \quad (40)$$

From the inequality $\omega_j^2 < 0$, we conclude that

$$R_j \geq \sqrt{\frac{2Dq_jB_j}{\frac{1}{2}\left(\frac{2^{3/2}}{3}\right)^D \frac{\left[U^{(3)}(\Phi_j^v)\right]^2}{U^{(4)}(\Phi_j^v)} - U^{(2)}(\Phi_j^v)}}. \quad (41)$$

In the above expression the denominator must be positive. As a consequence, we immediately obtain

$$D \leq \frac{\ln\left\{\frac{2U^{(2)}(\Phi_j^v)U^{(4)}(\Phi_j^v)}{\left[U^{(3)}(\Phi_j^v)\right]^2}\right\}}{\ln(2^{3/2}/3)}. \quad (42)$$

From the condition (42), the dimension for the existence of oscillating lumps is realized to have an upper limit. On the other hand, the condition $2^{3/2}/3 < 1$, implies that the potential must obey the constraint

$$\frac{2U^{(2)}(\Phi_j^v)U^{(4)}(\Phi_j^v)}{\left[U^{(3)}(\Phi_j^v)\right]^2} < 1 \quad (43)$$

as well.

V. AN APPLICATION

In this section, in order to apply the approach presented in the previous sections for a realistic case, let us consider the symmetric double-well potential, which is the most important model to find both topological defects and a wide class of problems involving phase transitions. Here, we choose the potential to have the form

$$U_j(\Phi_j) = \frac{\lambda_j}{4}\left[\Phi_j^2 - \left(\Phi_j^v\right)^2\right]^2, \quad (44)$$

where in Eq. (17) $N = 4$, $g_{j,1} = g_{j,3} = 0$, $g_{j,2} = -\lambda_j(\Phi_j^v)^2$ and $g_{j,4} = 6\lambda_j$. The potential (44) straightforwardly implies that

$$U_j^{(1)}(\Phi_j^v) = 0, \qquad U_j^{(2)}(\Phi_j) = 2\lambda_j\left(\Phi_j^v\right)^2, \quad (45)$$
$$U_j^{(3)}(\Phi_j) = 6\lambda_j\Phi_j^v, \qquad U_j^{(4)}(\Phi_j) = 6\lambda_j.$$

It is worth to emphasize that the condition provided by Eq. (43) reduces to the value 2/3. Moreover, for the sake of simplicity, we will apply the scale $R_j = \tilde{R}_j/\sqrt{\lambda_j}\Phi_j$. Thus, Eq. (41) reads

$$R_j \geq \sqrt{\frac{2Dq_jB_j}{3\left(\frac{2^{3/2}}{3}\right)^D - 2}}. \quad (46)$$

By assuming $q_j = 1$ it implies for $D = 2$ that $R_j \geq \sqrt{6B_j}$. On the other hand, choosing $D = 3$ we obtain $R_j \gtrsim 2.42\sqrt{2B_j}$. Furthermore, Eq.(42) imply that $D \leq 6$.

In what follows we depict the behavior of the oscillon as a function of the coordinates r and t as well. It is remarkable the appearance of a kind of double oscillon profile during some time along the evolution of the fields configurations obtained in this work, as it can be observed in the χ profile plotted in the Figure 1. Moreover, since ϕ and χ come from combinations of Φ_1 and Φ_2 with different time dependency, a clear beating behavior shows up in their profiles, as one can see in the Figure 2. As a consequence, these objects would appear in periodic bursts.

VI. CONCLUSIONS

In this work we investigate the consequences of the Lorentz symmetry violation on extremely long-living, time-dependent, and spatially localized field configurations which are called oscillons. This is accomplished in $(D+1)$ dimensions for two interacting scalar field theories in the so-called Standard Model-Extension context. We show that D-dimensional scalar field lumps can be found in typical size $R_{\min} \ll R_{KK}$, where R_{KK} is the associated length scale of the extra dimensions in Kaluza-Klein

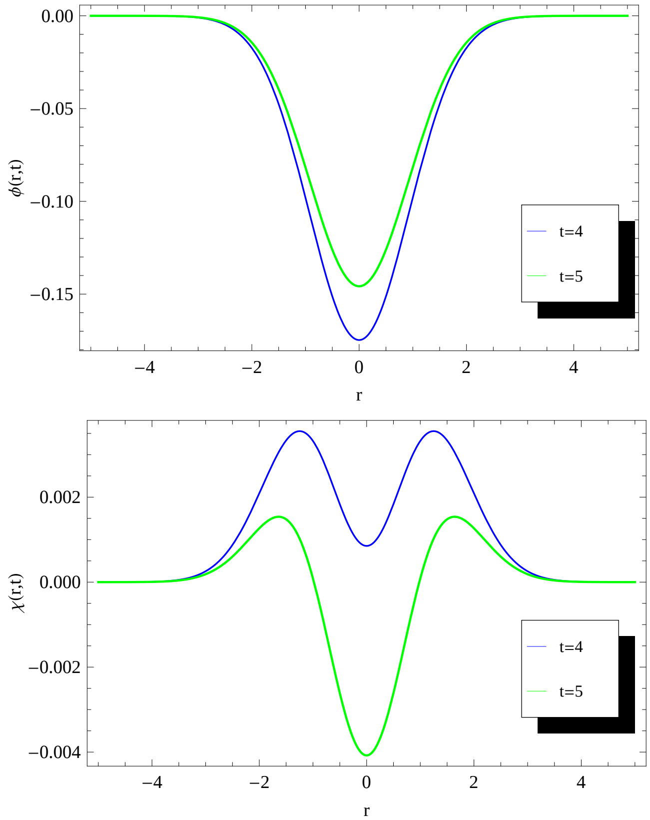

FIG. 1: Profile of the oscillon with $D = 2$, $\alpha = 0.1$ and $\lambda_1 = \lambda_2 = 1$.

D	α	$R^{(1)}_{min} = \sqrt{\frac{2DB_1}{3\left(\frac{2^{3/2}}{3}\right)^D - 2}}$	$R^{(2)}_{min} = \sqrt{\frac{2DB_2}{3\left(\frac{2^{3/2}}{3}\right)^D - 2}}$
2	0	1.73205	1.73205
2	0.01	1.72337	1.74069
2	0.03	1.70587	1.75784
3	0	2.41553	2.41553
3	0.01	2.40342	2.42758
3	0.05	2.34194	2.48694

Table I: Typical size $R_{\rm min}$ for the symmetric double-well potential.

theories. In fact, if the fundamental gravity scale is denoted by M, the length scale of the extra dimensions is $R_{KK} \sim M^{-1}(M_{\rm Pl}/M)^{2/(D-3)}$, and $D-3 \geq 1$ is the number of extra dimensions. Thus, if $M \approx 1$ TeV then $R_{KK} \approx 10^{32/(d-3)} \times 10^{-19} m$ (see, e. g., [50] for a comprehensive review). Here $R_{\rm min}$ is shown to strongly depend on the terms that regulate the Lorentz violation in the theory, implying either contraction or dilation of $R_{\rm min}$, accordingly. Oscillons in a LSV framework present thus a set of dimensionally-dependent properties. Moreover, the minimum radius that allows the initial configurations

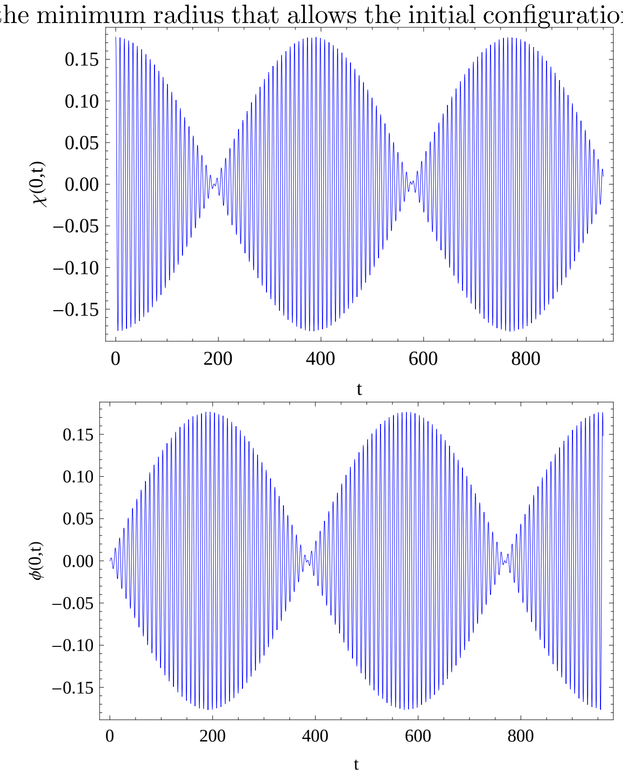

FIG. 2: Configurations ϕ and χ in $r = 0$ for $D = 2$, $\alpha = 0.1$ and $\lambda_1 = \lambda_2 = 1$.

to be led to oscillons is also ruled by the dimensionality of space. If such configurations were to be probed by observations, their sizes and energies would uniquely provide the space dimensions. Alternatively, it can also probe the existence of D-dimensional oscillons at the TeV energy scale. In a cosmological scenario with Lorentz symmetry breaking, we argue that in the early Universe with an extremely high energy density and a strong Lorentz violation, the typical size $R_{\rm min}$ was found highly dilated. With the Universe expansion and cooling, a phase transition towards a Lorentz symmetry had occurred, and the size $R_{\rm min}$ tended to shrink.

Acknowledgments

RACC thanks to UFABC and CAPES for financial support. RdR thanks to CNPq grants No. 303027/2012-6 and No. 473326/2013-2 for partial financial support. ASD was supported in part by the CNPq.

[1] V. A. Kostelecky and S. Samuel, Phys. Rev. D **39**, 683 (1989).
[2] J. Diaz and V. A. Kostelecky, Phys. Lett. B **700**, 25 (2011).
[3] A. Gomes, V. A. Kostelecky, and A. Vargas, Phys. Rev. D **90**, 076009 (2014).
[4] A. E. Bernardini and O. Bertolami, Phys. Rev. D **77**, 085032 (2008).
[5] A. E. Bernardini and R. da Rocha, Phys. Rev. D **75**, 065014 (2007).
[6] A. E. Bernardini and R. da Rocha, Europhys. Lett. **81**, 40010 (2008).
[7] M. Chaichian, A. D. Dolgov, V. A. Novikov, and A. Tureanu, Phys. Lett. B **699**, 177 (2011).
[8] S. Kanno and J. Soda, Phys. Rev. D **74**, 063505 (2006).
[9] M. N. Barreto, D. Bazeia, and R. Menezes, Phys. Rev. D **73**, 065015 (2006).
[10] A. de Souza Dutra, M. Hott, and F. A. Barone, Phys. Rev. D **74**, 085030 (2006).
[11] D. Bazeia, M. M. Ferreira Jr., A. R. Gomes, and R. Menezes, Physica D **239**, 942 (2010).
[12] A. de Souza Dutra and R. A. C. Correa, Phys. Rev. D **83**, 105007 (2011).
[13] R. Casana, M. M. Ferreira, Jr., E. da Hora, and A. B. F. Neves, Eur. Phys. J. C **74**, 3064 (2014).
[14] C. H. C. Villalobos, J. M. Hoff da Silva, M. B. Hott, and H. Belich, Eur. Phys. J. C **74**, 3, 2799 (2014).
[15] A. F. Ferrari, M. Gomes, J. R. Nascimento, E. Passos, A. Petrov, A. J. da Silva, Phys. Lett. B **652**, 174 (2007).
[16] S. Coleman, Nucl. Phys. B **262**, 263 (1985).
[17] T. D. Lee and Y. Pang, Phys. Rep. **221,** 251 (1992).
[18] I. L. Bogolyubsky and V. G. Makhankov, Pis'ma Zh. Eksp. Teor. Fiz. **24**, 15 (1976).
[19] M. Gleiser, Phys. Rev. D **49**, 2978 (1994).
[20] E. J. Copeland, M. Gleiser, and H. -R. Müller, Phys. Rev. D **52**, 1920 (1995).
[21] M. Gleiser and R. C. Howell, Phys. Rev. E **68**, 065203 (2003).
[22] M. Gleiser and A. Sornborger, Phys. Rev. E **62**, 1368 (2000).
[23] A. B. Adib, M. Gleiser and C. A. S. Almeida, Phys. Rev. D **66**, 085011 (2002).
[24] E. P. Honda and M. W. Choptuik, Phys. Rev. D **65**, 084037 (2002).
[25] M. Gleiser, Phys. Lett. B **600**, 126 (2004).
[26] E. Farhi, N. Graham, V. Khmeani, R. Markov, and R. Rosales, Phys. Rev. D **72**, 101701 (2005).
[27] N. Graham and N. Stamatopoulos, Phys. Lett. B **639**, 541 (2006).
[28] N. Graham, Phys. Rev. Lett. **98**, 101801 (2007).
[29] A. D. Linde, Phys. Rev. D **49**, 748 (1994).
[30] M. Gleiser, Int. J. Mod. Phys. D **16**, 219 (2007).
[31] E. W. Kolb and I. I. Tkachev, Phys. Rev. D **49**, 5040 (1994).
[32] P. M. Saffin and A. Tranberg, JHEP **01**, 30 (2007).
[33] E. Farhi, N. Graham, A. H. Guth, N. Iqbal, R. R. Rosales, N. Stamatopoulos, Phys. Rev. D **77**, 085019 (2008).
[34] H. Arodz, P. Klimas, and T. Tyranowski, Phys. Rev. D **77**, 047701 (2008).
[35] G. Fodor, P. Forgács, Z. Horváth, and A. Lukács, Phys. Rev. D **78**, 025003 (2008).
[36] M. Gleiser, J. Thorarinson, Phys. Rev. D **79**, 025016 (2009).
[37] M. Gleiser, D. Sicilia, Phys. Rev. D **80**, 125037 (2009).
[38] M. Gleiser, N. Graham, and N. Stamatopoulos, Phys. Rev. D **83**, 096010 (2011).
[39] G. Fodor, P. Forgács, Z. Horváth, and M. Mezei, Phys. Lett. B **674**, 319 (2009).
[40] M. P. Hertzberg, Phys. Rev. D **82**, 045022 (2010).
[41] M. A. Amin, D. Shirokoff, Phys. Rev. D **81**, 085045 (2010).
[42] R. A. C. Correa and A. de Souza Dutra, Adv. High Energy Phys. **2015**, 673716 (2015).
[43] M. Gleiser, N. Graham and N. Stamatopoulos, Phys. Rev D **82** 043517 (2010).
[44] H. Segur, M. D. Kruskal, Phys. Rev. Lett. **58**, 747 (1987).
[45] T. Romanczukiewicz and Ya. Shnir, Phys. Rev. Lett. **105**, 081601 (2010).
[46] M. Gleiser and N. Stamatopoulos, Phys. Rev. D **86**, 045004 (2012).
[47] M. A. Amin, P. Zukin, and E. Bertschinger, Phys. Rev. D **85**, 103510 (2012).
[48] M. A. Amin, R. Easther, H. Finkel, R. Flauger, and M. P. Hertzberg, Phys. Rev. Lett. **108**, 241302 (2012).
[49] M. A. Amin, JCAP **12**, 001 (2010).
[50] R. Maartens and K. Koyama, Living Rev. Rel. **13**, 5 (2010).
[51] D. Colladay and V. A. Kostelecky, Phys. Rev. D **55**, 6760 (1997).
[52] V. A. Kostelecky and R. Lehnert, Phys. Rev. D **63**, 065008 (2001).
[53] M. A. Anacleto, F. A. Brito, and E. Passos, Phys. Lett. B **694**, 149 (2010).
[54] R. A. C. Correa, R. da Rocha, and A. de Souza Dutra, Ann. Phys. **359**, 198 (2015).