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Vibration Control on a Tensegrity Structure Using Shape
Memory Alloys

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Vibration Control on a Tensegrity Structure Using Shape
Memory Alloys

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Gonçalves

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Vibration Control on a Tensegrity Structure Using Shape Memory Alloys

Daniel Martins

Keywords: Tensegrity structure, SMA, Vibration control

Abstract - A tensegrity system is composed by two types of elements, tensile and compressive parts, which promotes the structural stability of the system, in this case the tensile parts are cables (steel cables), and the compressive parts are bars, in this work made of steel. An attractive characteristic of a tensegrity structure is the capability to be deployable, for this reason, is interesting for space applications due to the small volume that it occupies in a possible transport to a station outside the Earth. In general, these structures can be large with many repetitive cells. In this work, a tensegrity boom structure consisting of ten periodic cells made of bars and cables is studied. The mathematical model, obtained by finite element method is validated experimentally considering the case of one structural cell. In order to make the system adaptable to external excitation, one of the steel cables in the elementary cell is replaced by a SMA (shape memory alloy) wire (Nitinol) allowing the dynamics characteristics of the system to be changed according to an electrical current applied to the SMA, to heat the alloy through Joule effect. Different configurations for placing the SMA cable are studied with the objective of reducing the vibration amplitudes for harmonic force excitation.

Utilização de estruturas do tipo tensegrity como isolador de vibração

Daniel Martins

Palavras chave: Estrutura tensegrity, SMA, Controle de vibração

Resumo - Um sistema de tensegrity é composto por dois tipos de elementos, elementos de tração e compressão, os quais promovem a estabilidade estrutural do sistema, neste trabalho as partes de tração são cabos (cabos de aço), e as partes compressivas são barras de aço. Uma característica atrativa de uma estrutura de tensegridade é a capacidade de ser retrátil, por esta razão, é interessante para aplicações espaciais devido ao pequeno volume que ocupa um possível transporte para uma estação espacial. Em geral, essas estruturas podem ser grandes com muitas células repetitivas. Neste trabalho, estuda-se uma estrutura de tensegrity composta por dez células periódicas de barras e cabos. O modelo matemático, obtido pelo método dos elementos finitos, é validado experimentalmente considerando o caso de uma célula estrutural. Para tornar o sistema adaptável à excitação externa, um dos cabos de aço na célula elementar é substituído por um fio de liga de memória de forma (SMA) que permite que as características dinâmicas do sistema sejam alteradas de acordo com uma corrente elétrica aplicada no SMA, para que através do efeito Joule a liga seja aquecida. Diferentes configurações para a colocação do cabo SMA são estudadas com o objetivo de reduzir as amplitudes de vibração para uma excitação de força harmônica.

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List of Symbols

Designation	Explanation
Latin symbols	
A	Element Cross-section area [m ²]
$\mathbf{B}$	Boolean matrix
C	Direction cosines
$\mathbf{C}$	Damping matrix [N·s/m]
E	Young's modulus [GPa]
$\mathbf{f}$	External load vector [N]
$()^H$	Superscript, Hermetian operator
J	Performance function
$\bar{J}_n$	n^{th} performance function arithmetic mean
$\mathbf{K}$	Global stiffness matrix [N/m]
$\bar{\mathbf{K}}$	Element stiffness matrix [N/m]
$\bar{\mathbf{K}}_{\mathbf{E}}$	Linear stiffness matrix [N/m]
$\bar{\mathbf{K}}_{\mathbf{G}}$	Geometrical stiffness matrix [N/m]
L	Length of the element [m]
$\mathbf{M}$	Global mass matrix [kg]
n_n	Finite element node number
p	Coordinate position
$()^T$	Super-script, Transpose operator

T	Cable tension [N]
$\mathbf{T}_i$	i^{th} coordinate transformation matrix
$\mathbf{u}$	Vector of nodal displacement
$\mathbf{U}$	Vector of complex displacement amplitudes
v	Velocity [m/s]
Greek symbols	
α	Element rotation angle
β	Damping constant
ρ	Material density [kg/m ³]
ϕ_j	n^{th} Mode shape
ω	Angular frequency [rad/s]
ω_n	Natural frequency [rad/s]
Abbreviations	
3D	Three dimensional
DAQ	Data acquisition
DOF	Degree of freedom
FEM	Finite element method
FRF	Frequency response function
NASA	National Aeronautics and Space Administration
PSD	Power spectral density
RT	Room temperature
SMA	Shape memory alloy

Chapter 1

Introduction

Tensegrity is a class of spatial structural systems, composed of tensile and compressive elements (Skelton and de Oliveira, 2009). These elements could be simple elements such as cables and bars, assembled in a self-balanced fashion, as Kenneth Snelson's *X-Piece*, shown in figure 1.1.

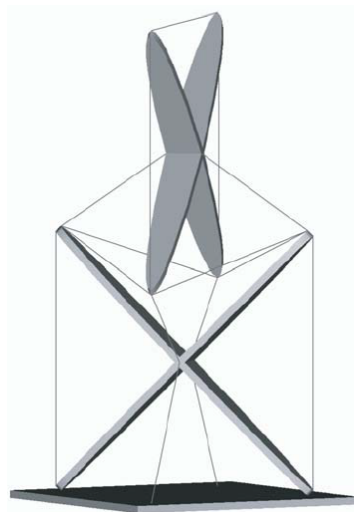


Figure 1.1: Illustration of early tensegrity structure by Kenneth Snelson's *X-Piece*. Source: Tibert (2002).

When compared with traditional structures tensegrity structures are lighter and occupy much less space and volume (Cefalo and Mirats-Tur, 2011). As a result of these characteristics, this type of structure received attention in aerospace applications as

Furuya (1992) highlights the packaging efficiency and the lower number of joints that have complicated mechanisms. As Sultan (2003) explains, classical deployable structures have several deficiencies, as the existence of complicated rigid to rigid joints and the use of telescopic struts for deployment.

Deployable structures and mechanisms have been extensively employed in space and earth applications. Several works and experimental tests on these structures have been conducted over the years since the former Soviet Union joined the “Space Race” launching its first satellite *Sputnik 1* on October 4, 1957 presented by figure 1.2 (Tibert, 2002). These studies and experiments covered broad applications of deployable antennas, solar arrays, booms, and other instruments, to analyze and understand their dynamic properties in working conditions (Liu et al., 2016; Gonçalves, 2007; Tibert, 2002; Pellegrino, 2014; Puig et al., 2010).

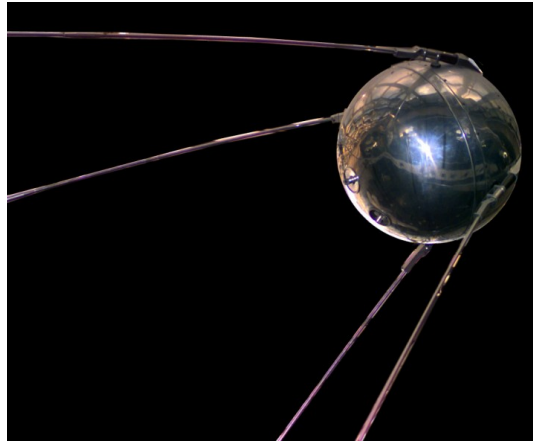


Figure 1.2: Sputnik 1 satellite. Source: NASA (2003).

Probably one of the first applications considering the use of tensegrity systems was proposed by Buckminster (1962) and by Snelson (1965), as structural frameworks with tensile and compressive (rigid bodies) parts in a lattice form, being separated from each other. Several studies with this type of structure were conducted over the years (Sultan, 2009) in different science areas, such as cell biology (Stamenović et al., 1996; Ingber, 1997), nature (Volkh et al., 2002; Skelton and de Oliveira, 2009; Simmons et al., 1996), architecture (Cimmino et al., 2017), demonstrating diverse applications and studies

on the mechanical behavior of these structures (Motro et al., 1986), and also showing that the behavior of self-stressed reticulated spatial systems is nonlinear due to their flexibility (Kebiche et al., 1999).

Through these studies and experiments, the dynamic behavior of some tensegrity structures was examined and showed that the system can function as a counterpart to conventional trusses, even though some of those systems were ineffective at eliminating vibrations (Oppenheim and Williams, 2001a,b). To use this type of structure and to model its dynamics, Motro et al. (1986) shows that linear models can be used to describe the approximate dynamics of tensegrity structures and Sultan et al. (2002) shows that the modal dynamic range commonly increases with pretension. This pretension comes from different tensile elements such as tendons, membranes and cables (Skelton and de Oliveira, 2009; Yang and Sultan, 2017a,b), and can be assembled in several different ways resulting in different structures shapes, and also an ample set of applications. These structures have a great advantage in the model accuracy of the system statics and dynamics since the rigid bodies are not compliant, there are no elastic bodies in bending, only elastic axially loaded strings, being that the material bending is not required in this model (Skelton and de Oliveira, 2009).

Motivated by these applications, the necessity to control the vibration conditions also became a need in order to achieve optimized results as vibration isolation. Several types of control can be used to accomplish the desirable results as Raja and Narayanan (2015) shows. Natural frequencies can be shifted when the self-stress level in the tensegrity structure is modified, using active control (Ali and Smith, 2010). Chan et al. (2004) presented a small-scale active three bar structure, with local integral force feedback and acceleration feedback control, achieving effective damping for the next 2 resonant bending modes. Robust active control algorithm (H_∞) in vibration control can be used to provide an efficient performance on the attenuation of the modes, in one degree of freedom (Amouri et al., 2013). The Linear-quadratic regulator (LQR) can also be integrated into tensegrity system control and offer different actuator placement schemes

as Feng et al. (2018) demonstrates.

Following this idea, the structure considered in this work comprises of a tensegrity structure composed of 40 compressive (bars) and 102 tensile parts (cables) and has 53 nodes localized at the joints, as presented in section 1.2, and it is made by ten repetitive (elementary) cells. In each cell one of the side cables is actually a shape memory alloy (SMA) wire, acting as a semi active control similar to what dos Santos et al. (2015) proposed, however not utilizing the whole structure, but the elementary cells acting as control cells.

A physical structure was built and subjected to experimental tests with the objective of validating the mathematical model proposed in this work. The experimental apparatus is a single cell part of the structure. The SMA wire on the cell was subjected to the two conditions proposed in this work, the first when the SMA is heated (presenting the shape memory effect), and the second when is at RT (room temperature) named respectively as control cell and passive cell. The control cell SMA wire was heated by an electrical current during the experimental process to reach the transformation temperature. The SMA wire was connected to the edge of each bar in the same way as all the other cables are due to its malleability, and it was trained (or programmed) to be straight upon heating (transformation to austenite) as Jani et al. (2014) presents. The SMA wire was trained by Wayne Metals.

In order to control the proposed structure by utilizing this semi-active control, two different control strategies were studied in this work. The first analyzed specific ranges (7 - 15 Hz and 200 - 300 Hz) utilizing control cells in different positions. The second strategy, aims to control specific structural modes by calculating the elements strain energies as Preumont (1997) does for conventional truss-like structures utilizing active control. Based on this method, now applying for the semi-active control, the control cell was placed on the unit cell with higher strain energy value, and like actuators. In this context, this work analyzes the dynamic response of the tensegrity structure with control cells in comparison with a passive structure to verify its effectiveness.

1.1 Literature review

1.1.1 Tensegrity structures

Tensegrity structures are a class of prestressable truss-like structures (Masic, 2004). As Pugh (1976) and Skelton and de Oliveira (2009) defines, the configuration of tensegrity is established when a set of discontinuous compression components interacts with a set of continuous tensile components to define a stable volume in space. In these structures there are elements of compression and tension, and they are connected to each other in a net of continuous tension. Skelton and de Oliveira (2009) define the compressive parts as rigid bodies, and tensile parts as strings, and by the string connectivity with the rigid bodies, the system is defined spatially and stabilize this configuration in the lack of external forces.

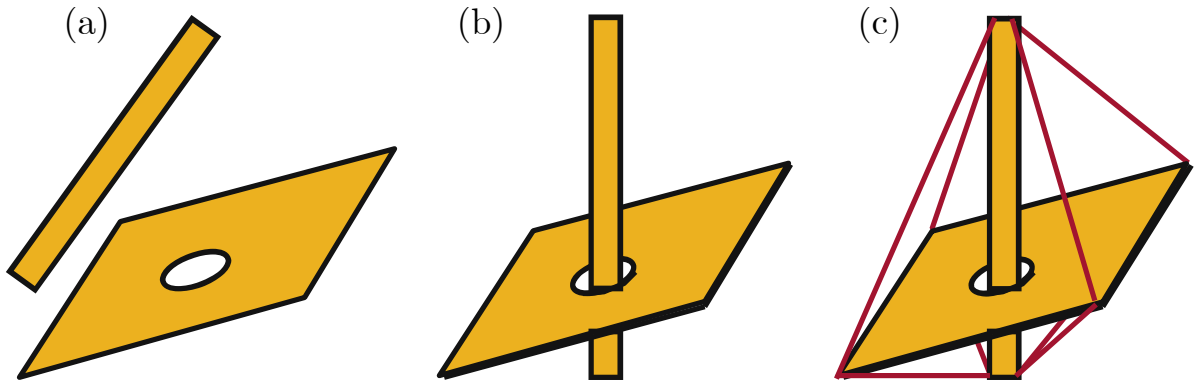


Figure 1.3: Illustration of two-body configuration to differentiate: a) not a tensegrity configuration, b) what could be a tensegrity configuration, and c) a tensegrity configuration. Adapted from: Skelton and de Oliveira (2009).

The classification that what could be a tensegrity structure is explained by Skelton and de Oliveira (2009) utilizing three different examples presented in figure 1.3. The figure 1.3(a) shows a configuration that the rigid bodies could not be structurally stabilized (self-sustained) by any set of strings, as a difference from figure 1.3(b), in which a set of tensile members could stabilize the structure, characterizing a tensegrity configuration. Figure 1.3(c) shows a structurally stable configuration, hence a tensegrity

system. Skelton and de Oliveira (2009) also classify the system in different classes based on the number of the rigid bodies that is part of the structure as represented by figure 1.4.

Figure 1.4(a) shows a class 2 tensegrity and figure 1.4(b) shows a class 3 tensegrity. This definition is helpful to classify the elementary cells in a structure made by several elementary cells.

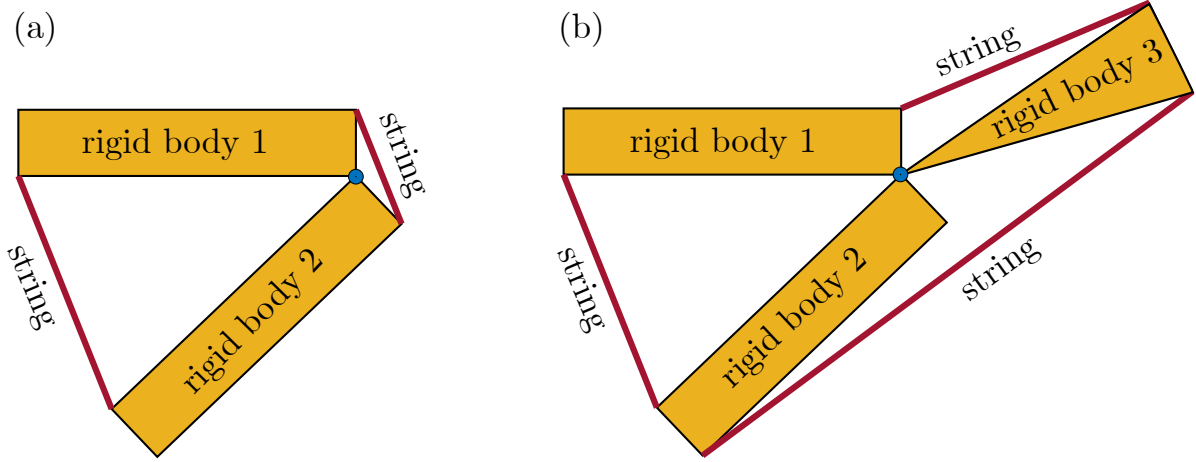


Figure 1.4: Illustration of a class 2 and class 3 tensegrity system, with rigid bodies (yellow) connected by strings (dark red). Adapted from: Skelton and de Oliveira (2009).

Several types of tensegrity structures have been constructed over the years using different methods, which can be divided into the following three classes (Motro, 2003). One class is named node-on-node methods, which simply connect elementary cells by joining the nodes of both cells together. These methods often produces bar-bar connections which if following Pugh's definition do not fall with the well-accepted, but if considered the whole structure those connected bars will result in a larger bar element and then agreeing with the definition. Wang (1998) utilized this method to construct a reciprocal prism and crystalcell pyramid tensegrity structures. Motro (2003) showed a double-layer grid by assembling quadruplex cylindrical tensegrity modules. Kebiche et al. (1999) connected five class 4 elementary cells, and applied the total Lagrangian formulation for the structure taking into account the geometrical non-linearities to study the behavior

of self-stressed and under external force structure for different tension levels.

The second class is referred to as node-on-cable methods, which need to break strings in one cell in order to form joint nodes of other cells (Li et al., 2009). Pugh's definition for these methods is fulfilled but so far there is a very limited type of tensegrity structure shapes that have been achieved as, Hanaor and Liao (1991) presented a double-layer dome assembly with class 3 cylindrical elementary cells.

The third class is the cable-on-cable, which combine two cables of elementary cells by adding a new node. With this, the structures which are constructed utilizing this method in most cases are not self-equilibrated due the lack of bars or struts, and need additional supports to maintain equilibrium (Li et al., 2009), by this method is applied only to few structures.

Elementary cells

Following Pugh's definition this configuration does not allow connections between two bars in an elementary cell. There are several types of elementary cell shapes as Snelson (2012) presents, and can be seen in figure 1.5.

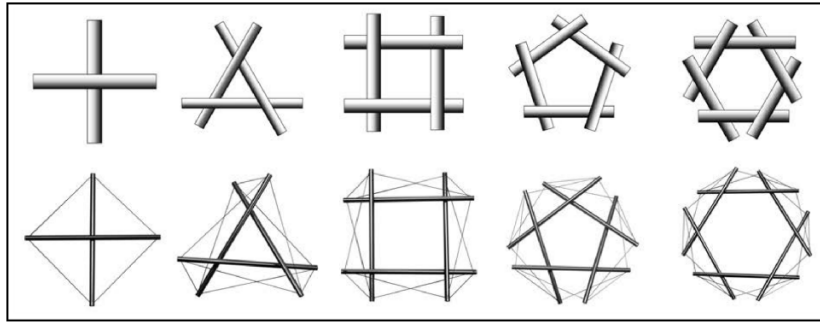


Figure 1.5: Illustration of basic two, three, four, five and six rigid bodies elementary cells . Source: Snelson (2012).

These cells as Motro (2003) explains, can be arranged in planes, and the structure can be assembled in different ways forming a larger tensegrity structure. Utilizing the different ways of construction, different structures can be assembled, and the mechanical behaviour changes with the construction mode. Figure 1.6 presents a double layer grid

(dome) constructed utilizing the node-on-cable principle.

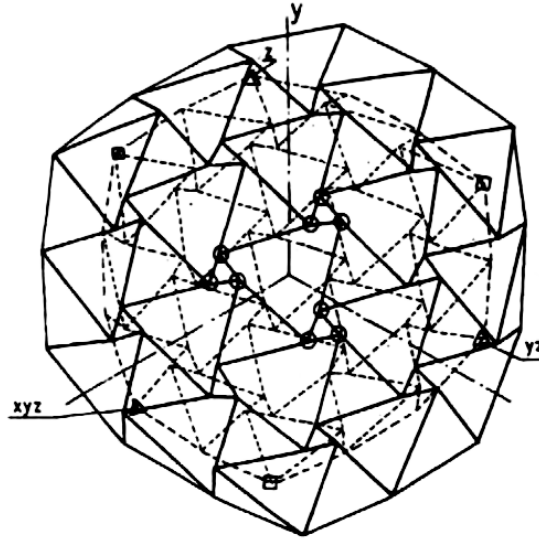
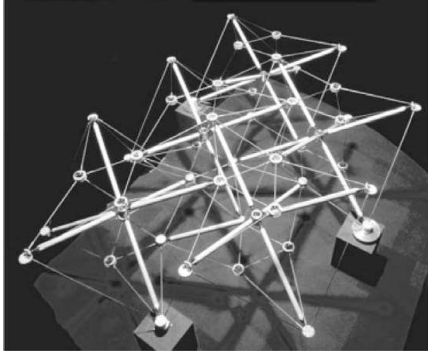


Figure 1.6: Illustration of double layer grid forming a tensegrity dome, possibly originated from Hanaor and Liao (1991) work. Source: Motro (2003).

1.1.2 Vibration control and active tensegrity structures

As mentioned before, there are several works with active tensegrity structures, and there has been an increasing interest due to the growing demands in engineering challenges (Korkmaz, 2011). Ali and Smith (2010) presented a 5 module (5 elementary cells) structure that covers a surface area of 15 m^2 based on Adam and Smith (2007) work. This structure is shown in figure 1.8 (a). Ali shows that the deformation modes frequencies (flexural and torsion modes) does not change significantly with the self-stress level of the tensegrity structure, unlike infinitesimal modes. Ali presents by showing numerical simulations and experimental measurements, that resonance peaks can be shifted by changing the self-stress level in tensegrity structures. Figure 1.8 (b) shows the vibration amplitude for different excitation frequencies and stress levels.

(a)



(b)

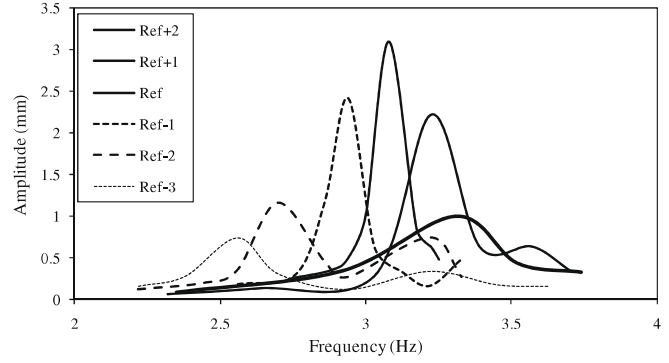


Figure 1.7: (a) Tensegrity structure with five elementary cells, 30 struts and 120 tendons. (b) Vibration amplitude for different excitation frequencies and stress levels. Source: Ali and Smith (2010).

Feng et al. (2018) focus on a control strategy to vibration mitigation for a tensegrity beam composing of five class 4 elementary cells. Feng makes use of different actuator placement scenarios and uses a linear-quadratic regulator (LQR) applied on these scenarios, changing the actuator positions in different structure regions. The simulated results showed that the LQR algorithm is capable of mitigation vibration of the tensegrity system, and by changing the positions of the actuators is possible to change the dynamic response of the system. Figure 1.8 shows five different scenarios tested by Feng et al. (2018) in comparison with an uncontrolled structure, in which $G_{y_k y_k}(f)$ is the estimated amplitude of power spectral density (PSD) with frequency (f) by system outputs y_k .

Moussa et al. (2001) presented the direct relation between the first natural frequency of tensegrity systems and the levels of self-stress in the elements. Moussa showed that the first natural frequency of single or multi-module tensegrity systems is a function of the pre-stress forces in the elements of these structures, and the first natural frequency can serve as a base for an estimate of the tension (pre-stress).

Kan et al. (2018) presented a nonlinear dynamic analysis of clustered tensegrity structures utilizing FEM, and also make a deployment analysis, showing the motion characteristics of the two proposed tensegrity structures. The structures utilized by

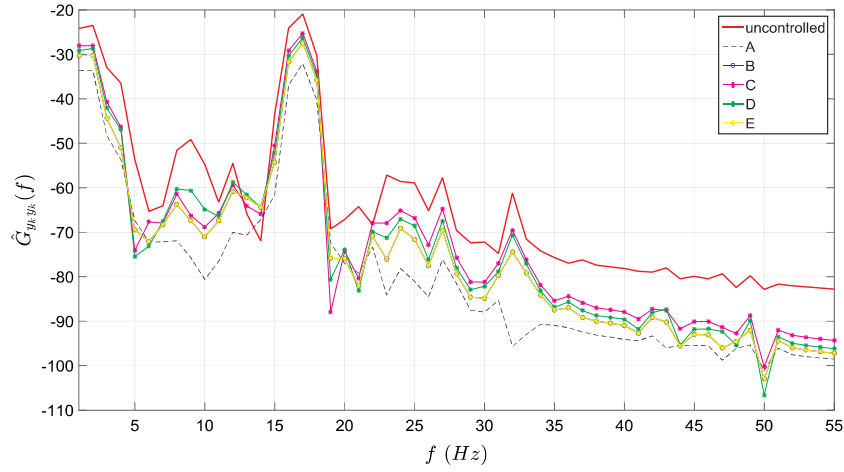


Figure 1.8: Comparison between five different scenarios of actuator placements, and a passive structure in PSD of the tensegrity structure. Source: Feng et al. (2018).

Kan et al. (2018) have a class 4 elementary cell and are presented by figure 1.9, similar to Feng et al. (2018) structure.

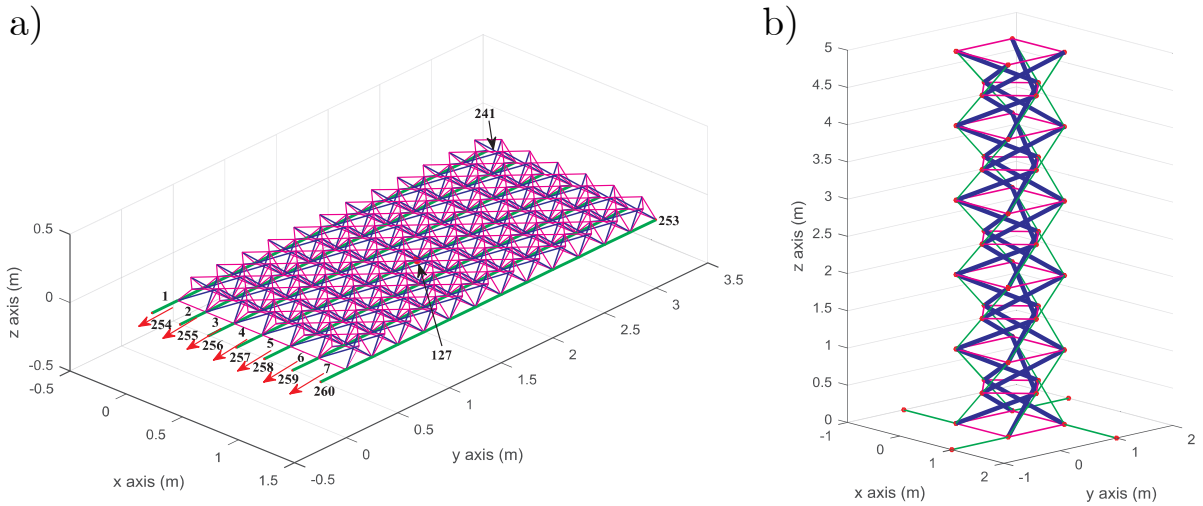


Figure 1.9: Clustered tensegrity grid and tower constructed utilizing a class 4 elementary cell. Source: Kan et al. (2018) .

Kan et al. (2018) presented the dynamic model utilizing FEM, demonstrating multiple examples of the clustered tensegrity, and showed with the modal analysis that the clustered tensegrity inherits the property of the classical tensegrity where natural frequencies rises with the increasing pre-stress amplitudes. The elementary cell used by Kan's work is similar to this work elementary cell, similar FEM analysis, and also the

connection method utilized in the construction of the tensegrity grid and tower.

dos Santos et al. (2015) presented an adaptive shape-morphing tensegrity structure, with frequency self-tuning capabilities, and the work addresses the dynamic response of the structure. The control method is based on the use of a shape memory alloy (SMA). The SMA wire is connected to all elementary cells of the structure and by changing the temperature of the wire, the shape of the structure will also change. The illustration of this structure is presented by figure 1.10.

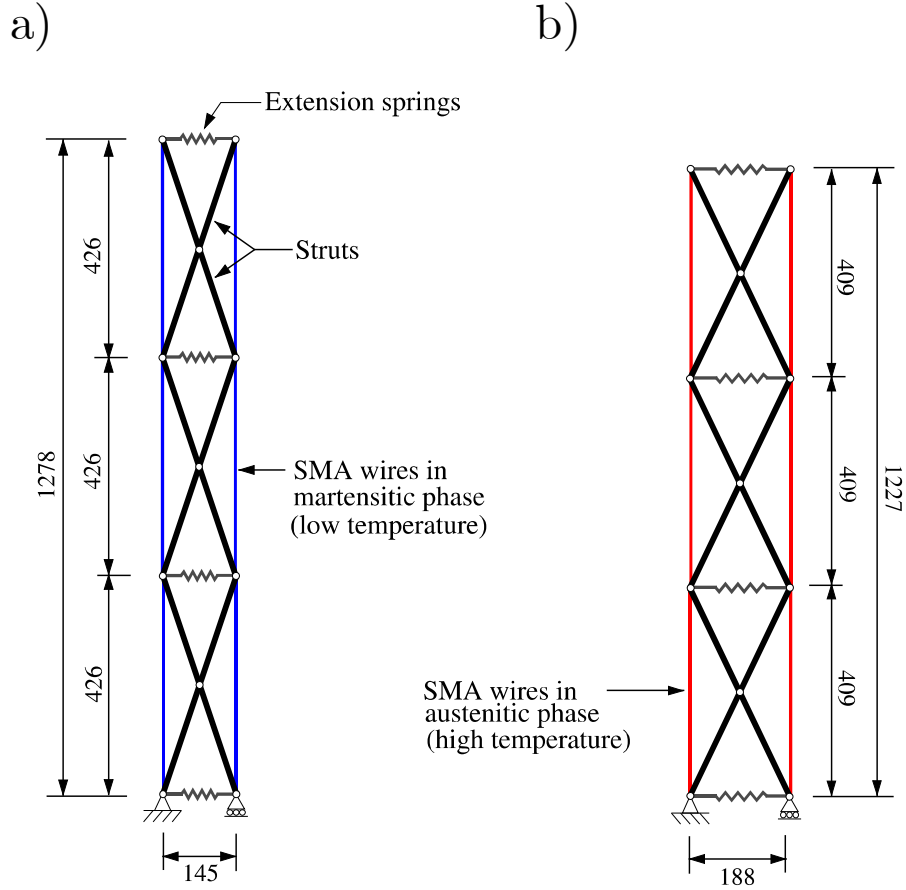


Figure 1.10: Illustration of shape morphing tensegrity structure utilizing SMA. (a) Low temperature SMA tower (SMA wire in martensitic phase), (b) high temperature SMA tower (SMA wire in austenitic phase). Source: dos Santos et al. (2015) .

By analyzing the dynamic response, it was possible to see a frequency shift of the structure associated with physical and geometric effects of the shape-morphing. The physical effect was due the change on the elastic modulus of the SMA caused by the temperature and the shape memory alloy effect, of the martensitic transformation.

With both effects, the control was able to reduce the rms and the absolute accelerations of the structure up to 80%, showing the effectiveness of this type of control utilizing SMA.

Shape memory alloy effect

The shape memory effect, as Otsuka and Wayman (1999) describes, is a special property of particular alloys exhibiting martensitic transformations. This effect is shown in figure 1.11.

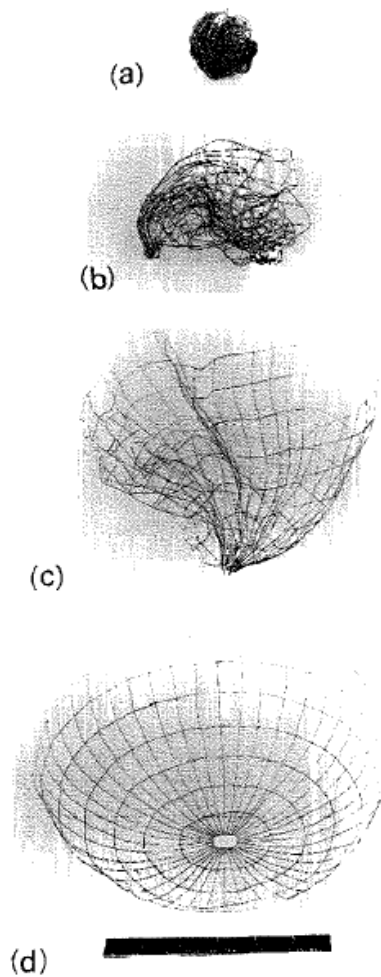


Figure 1.11: Shape memory effect demonstration by a space antenna. Transformation from martensitic to austenitic state. Source: Otsuka and Wayman (1999).

As figure 1.11 shows, the antenna deformed in the martensitic state (fig. 1.11(a)) regress to the original “programmed” shape (fig. 1.11(b-d)) upon heating. As Duerig et al. (2013) explains this effect is based on materials that can easily transform to and from martensite, and this martensitic transformation is described by Otsuka and Wayman (1999) as a diffusionless phase transformation in solids, in which atoms move cooperatively, and often by a shear-like mechanism. Based on the martensitic transformation, a relevant characteristic that distinguishes the different types of SMA is the start and finish temperature transformation (Otsuka and Wayman, 1999).

Several works utilizing this type of alloy have been researched over the years, showing diverse applications in medicine (Watanabe et al., 2006; Fattorini et al., 2016) and engineering, as Barbarino et al. (2014) shows for applications to morphing aircraft, and like-wise Sales et al. (2014) studying transient aerodynamic features of these morphing airfoils.

The shape memory alloy used in this work is a Nitinol (Nickel-Titanium) made by Wayne Metals, and for this type of alloy the manufacturer makes use of this transformation temperature to distinguish different grades of Nitinol, however taking austenite for a parameter. The austenite transformation temperature of this work SMA is from 35 to 85 °C, by this in temperatures above 85 °C the alloy is at the austenitic phase.

1.2 Proposed tensegrity structure

The tensegrity structure used in this work is based on a variation of Karl Ioganson and Kenneth Snelson’s structures. They presented a three-body configuration, with 9 cables, and both are presented by figure 1.12.

A similar structure was used by Kebiche et al. (1999), but in his work five elementary cells were used as figure 1.13 exemplifies the structure elementary cell and the connections with the other cells. This type of elementary cell has been used and analyzed in several works over the years (Kebiche et al., 1999; Sultan, 2003; Kan et al., 2018).

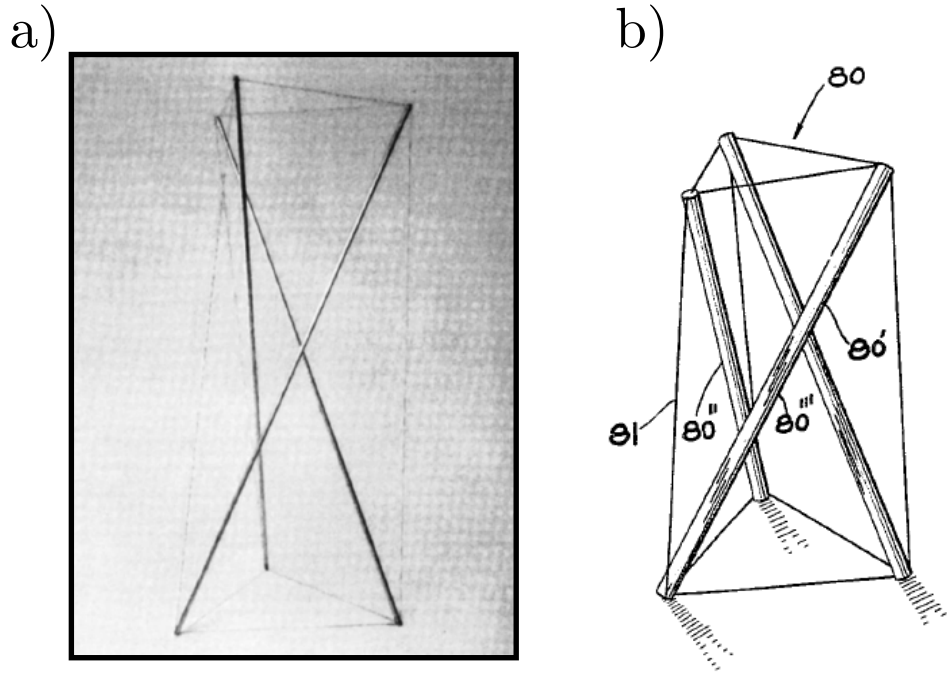


Figure 1.12: Three body tensegrity structure.(a) Karl loganson art piece dated from 1920. Source: Gough (1998). (b) Illustration of Kenneth Snelson's structure. Source: Snelson (1965)

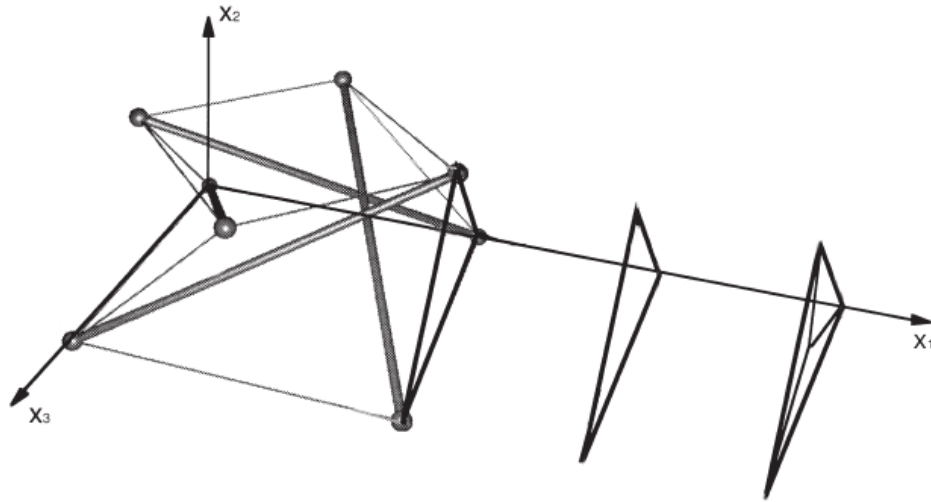


Figure 1.13: Class 4 tensegrity structure assembly constructed by node-on-node method. Adapted from: Kebiche et al. (1999).

This work structure is composed of ten elementary cells assembled in a similar way as Kebiche et al. (1999) presented. The structure is presented by figure 1.14.

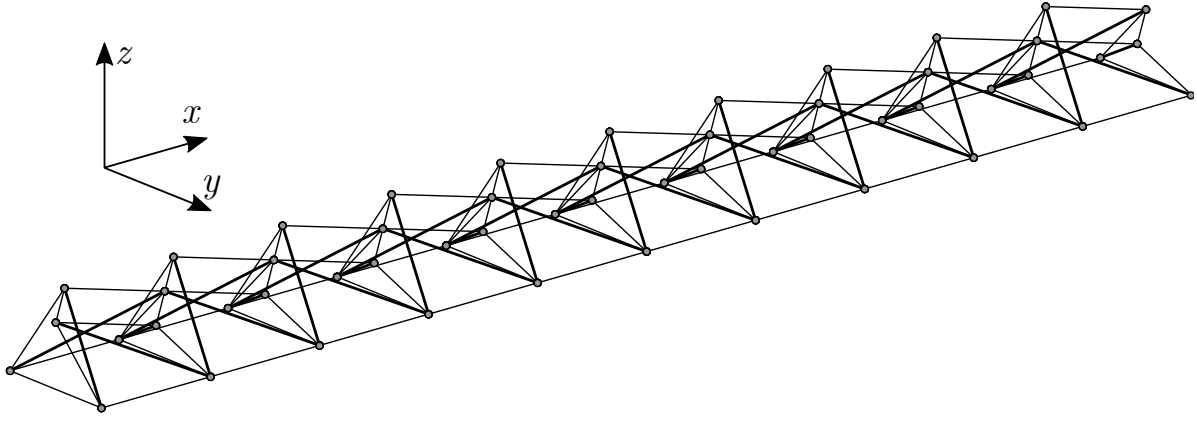


Figure 1.14: Tensegrity structure with ten class 4 elementary cells constructed utilizing node-on-node method.

1.3 Objectives

The dissertation objectives are listed as follows:

- Model a tensegrity elementary cell structure and validate it with an experimental apparatus through an experiment;
- To investigate the influence of a shape memory alloy cable on the dynamics of the tensegrity structure.
- To investigate strategies of semi-active vibration control using SMA cables on a 10 periodic cells tensegrity structure.

1.4 Summary of the dissertation

The dissertation is divided in 4 chapters, and is represented in the following way. The structure modeling and analysis are presented in Chapter 2, consisting in a introduction to the modeling of a tensegrity structure, and aims to present the governing equations of its dynamics and to model the structure utilizing FEM.

The experimental apparatus and procedure are presented in Chapter 3. The experimental structure construction is described. The experimental procedure is detailed,

along with the comparison results of both experimental and mathematical model.

Chapter 4 presents the tensegrity structure, and a dynamic analysis for this structure. Is presented a performance function to simplify the results, and an attenuation expression to compare three different control strategies. This chapter shows the results analysis for different strategies. The first strategy utilizes elementary cells with a heated SMA (control cell) in different locations of the structure. The second strategy is to replace the cell with higher strain energy with a control cell. The analysis is obtained by utilizing FEM. Finally, Chapter 5 is a summary of this dissertation and includes the final conclusions, contributions and recommendations for further work.

Chapter 2

Structure modeling and analysis

The objective of this chapter is to present the method for modeling the tensegrity elementary cell and the tensegrity numerical modeling utilizing a traditional finite element method. The dynamic equations for the elementary cell, the relation to the tensegrity structure, its natural frequencies and modal shapes are presented.

2.1 Definition of an elementary cell

The tensegrity elementary cell used in this work is a class 4 tensegrity structure and is composed of 4 bars and 12 cables, and has 8 nodes localized at the joints, one of the side cables is a SMA as shown in figure 2.1.

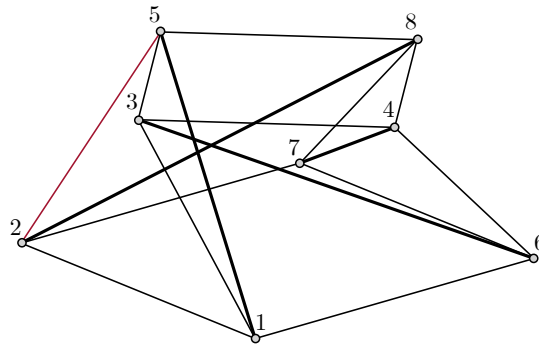


Figure 2.1: Illustration of the tensegrity elementary cell, representing the bars in thick black line, cables in a thinner black line and the SMA in a dark red line.

In figure 2.1, the thick black lines represent rigid bars, and cables are represented in thinner lines. The SMA is represented by the dark red thinner line and the nodes are localized at the joints. This type of structure elements are one-dimensional, with the properties defined by a single orthogonal axis of the cross-section of the bar or cable.

The lower cables represented in figure 2.1 by connections 1-2, 2-7, 7-6 and 6-1 have 168 mm. The side and upper cables represented by connections 1-3, 6-4, 7-8, 3-4, 4-8, 8-5 and 5-3 have 119 mm. The SMA is a side cable represented by connection 3-5, and has the length of 119 mm. The bars represented by connections 1-5, 2-8, 7-4 and 6-3 have 205 mm. Figures 2.2 and 2.3 presents the proposed structure in a rendered 3D (three-dimensional) model.

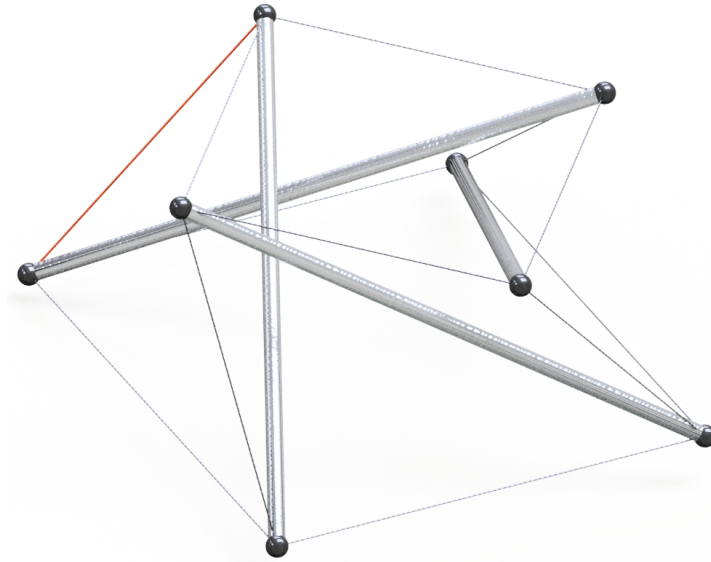


Figure 2.2: Rendered 3D model of the tensegrity structure in perspective. Following the same pattern as the illustration (fig. 2.1), the black thin lines are cables and the SMA is presented by a dark red line.

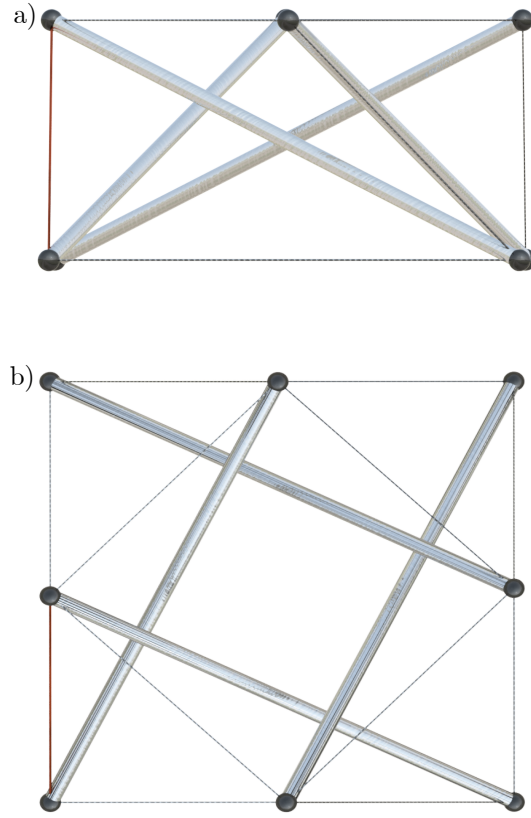


Figure 2.3: Rendered 3D model of the tensegrity structure 2 principal views showing the cell symmetry. a) Lateral view, b) Superior view.

2.2 Finite element of a tensegrity structure

The FEM as described by Bang (2000) is a numerical solution technique to solve the mathematical expressions that describes the system behavior, in which usually consists of differential equations and boundary conditions. This modeling method utilizes joints or nodes on determined structure parts, and each of these joints or nodes, represent the system movement for finite DOF. Thus, for this work system dynamic analysis, the motion equations of the elements are considered in the frequency domain, so it is not time dependent. By this, the structure dynamic response is analyzed with the frequency response function (FRF). This work also considers that bars and cables do not bend, and there is no momentum transmission at the joints of the structure, that is, there is no joint rotation, only translation in the three directions (X, Y, Z).

Since Motro et al. (1986) showed that a linearized dynamic model is a good approximation of the nonlinear behavior of tensegrity systems around equilibrium configurations even for active structures (Ali and Smith, 2010), several studies on the dynamics of tensegrity structures were conducted over the years based on this assumption (Sultan, 2009). So a linearized equation of motion to describe a dynamic model around an equilibrium position is used to describe the tensegrity structure dynamic behavior, as follows:

$$\mathbf{M}\ddot{\mathbf{u}} + \mathbf{C}\dot{\mathbf{u}} + \mathbf{K}\mathbf{u} = \mathbf{f} \quad (2.1)$$

in which $\mathbf{M}$ and $\mathbf{K}$ are mass and stiffness global matrices. The term $(\mathbf{f})$ is the external load vector, and $\ddot{\mathbf{u}}, \dot{\mathbf{u}}$ and $\mathbf{u}$ are the vectors of nodal acceleration, velocity and displacement respectively. It is considered that, the structure has a structural damping, treated as equivalent Rayleigh damping (Rayleigh, 1945), or proportional to the stiffness matrix as:

$$\mathbf{C} = \beta\mathbf{K} \quad (2.2)$$

in which β is a damping constant. The element mass matrix is given by equation 2.3, in which the overbar represents variables in the local coordinate system.

$$\overline{\mathbf{M}} = \frac{\rho AL}{6} \begin{bmatrix} 2\mathbf{I}_3 & -\mathbf{I}_3 \\ -\mathbf{I}_3 & 2\mathbf{I}_3 \end{bmatrix} \quad (2.3)$$

in which, ρ , A and L are the elements material densities, cross-section areas, and lengths. Similarly to what is shown by Ali and Smith (2010) and (Guest, 2006), the local stiffness matrix $\overline{\mathbf{K}}$ can be decomposed into two other matrices, $\overline{\mathbf{K}}_E$ and $\overline{\mathbf{K}}_G$, linear stiffness matrix and geometrical stiffness matrix correspondingly, $\overline{\mathbf{K}} = \overline{\mathbf{K}}_E + \overline{\mathbf{K}}_G$ in which:

$$\bar{\mathbf{K}}_E = \frac{EA}{L} \begin{bmatrix} \mathbf{I}_0 & -\mathbf{I}_0 \\ -\mathbf{I}_0 & \mathbf{I}_0 \end{bmatrix}; \quad \bar{\mathbf{K}}_G = \frac{T}{L} \begin{bmatrix} \mathbf{I}_3 & -\mathbf{I}_3 \\ -\mathbf{I}_3 & \mathbf{I}_3 \end{bmatrix}; \quad (2.4)$$

in which,

$$\mathbf{I}_0 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}; \quad \mathbf{I}_3 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad (2.5)$$

The terms defining equation 2.3 and equation 2.4 are described in table 2.1, in which the subindices $_b$, $_c$ and $_s$ are the bar, cable and SMA parameters respectively. The bars and the cables are made of stainless steel, and the SMA is made of Nitinol. Based on this equations, the relevant properties for this study are the cross-section area (A), the material density (ρ), the material Young's modulus (E), and tension of the cable (T). The material and geometric properties of the structure are shown in table 2.1. The SMA is treated as a cable with two Young's modulus one when the cable is at RT and other when is heated.

Table 2.1: Material and Geometric Properties of the Tensegrity Structure.

Property	Value
Bar Young's modulus (E_b)	200 GPa
Bar Density (ρ_b)	7850 kg/m ³
Bar Cross Section Area (A_b)	2.8274×10^{-5} m ²
Cable Young's modulus (E_c)	200 GPa
Cable Density (ρ_c)	7850 kg/m ³
Cable Cross Section Area (A_c)	3.3183×10^{-7} m ²
SMA Young's modulus (E_s)	40 GPa
SMA Young's modulus (E_{sh}) (Heated)	70 GPa
SMA Density (ρ_s)	6450 kg/m ³
SMA Cross Section Area (A_s)	7.854×10^{-7} m ²

Coordinate transformation matrix

The elementary matrices given in the last section describe mass, stiffness and damping of each element given in local coordinates (element coordinates). To obtain the system model, these matrices are assembled using finite element procedures considering the conditions of force equilibrium and compatibility. The condition of compatibility, requires that all the elements are given in the same coordinate system, for this, a coordinate transformation operation is performed. Similitar to what Gonçalves (2007) shows for a beam in space, the element local coordinate system can be defined by the location of the edges 0 and 1 in the global coordinate system XYZ shown in figure 2.4 and α is the rotation angle in the x axis.. The length L of the i -th element is defined by equation 2.6 in which p_X^1 is the coordinate in the X axis for the edge 1:

$$L_i = \left[(p_X^1 - p_X^0)^2 + (p_Y^1 - p_Y^0)^2 + (p_Z^1 - p_Z^0)^2 \right]^{1/2} \quad (2.6)$$

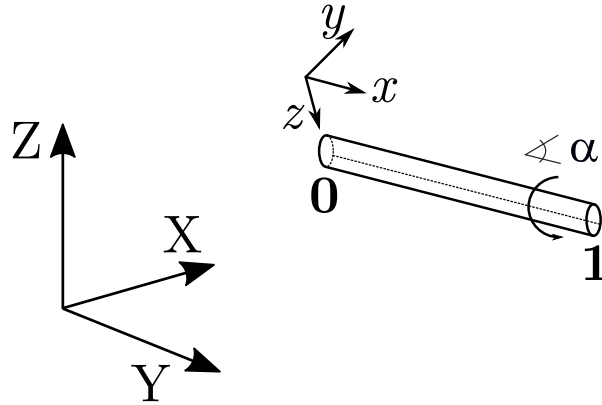


Figure 2.4: Element in space showing the relation of Global (XYZ) and Local (xyz) coordinate systems.

The transformation matrix can be calculated from the cosines of the Euler angles of a vector, also named, the direction cosines C , defined as:

$$C_X = \frac{(p_X^1 - p_X^0)}{L_i}, C_Y = \frac{(p_Y^1 - p_Y^0)}{L_i}, C_Z = \frac{(p_Z^1 - p_Z^0)}{L_i} \quad (2.7)$$

As Azevedo (2003) and Gonçalves (2007) show, that the coordinate transformation matrix $\mathbf{T}$ of the i – th beam is defined by:

$$\mathbf{T}_i = \begin{bmatrix} C_X & C_Y & C_Z \\ -\frac{C_X C_Y \cos \alpha + C_Z \sin \alpha}{C_{XZ}} & C_{XZ} \cos \alpha & -\frac{C_Y C_Z \cos \alpha + C_X \sin \alpha}{C_{XZ}} \\ -\frac{C_X C_Y \sin \alpha + C_Z \cos \alpha}{C_{XZ}} & C_{XZ} \sin \alpha & \frac{C_Y C_Z \sin \alpha + C_X \cos \alpha}{C_{XZ}} \end{bmatrix} \quad (2.8)$$

in which $C_{XZ} = (C_X^2 + C_Z^2)^{1/2}$. This relation is not valid when the beam is oriented vertically or $C_X = C_Z = 0$. In this case the matrix that can be used is presented as follows:

$$\mathbf{T}_i = \begin{bmatrix} 0 & C_Y & 0 \\ -C_Y \cos \alpha & 0 & \sin \alpha \\ C_Y \sin \alpha & 0 & \cos \alpha \end{bmatrix} \quad (2.9)$$

The transformation matrix is inserted in a identity matrix resulting in a 6×6 matrix as follows:

$$\mathbf{T} \mathbf{e}_i = \begin{bmatrix} \mathbf{T}_i & \mathbf{I}_2 \\ \mathbf{I}_2 & \mathbf{T}_i \end{bmatrix} \quad \mathbf{I}_2 = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \quad (2.10)$$

To obtain the resulting global stiffness matrix $\mathbf{K}_i$ and global mass matrix $\mathbf{M}_i$ for the element i , the equation 2.11 was applied, utilizing the transformation matrix inserted

in a identity matrix ($\mathbf{T}\mathbf{e}_i$) and the transposed transformation matrix ($\mathbf{T}\mathbf{e}_i^T$):

$$\begin{aligned}\mathbf{K}_i &= (\mathbf{T}\mathbf{e}_i)^T \overline{\mathbf{K}} \mathbf{T}\mathbf{e}_i \\ \mathbf{M}_i &= (\mathbf{T}\mathbf{e}_i)^T \overline{\mathbf{M}} \mathbf{T}\mathbf{e}_i\end{aligned}\tag{2.11}$$

To arrange the stiffness and mass matrices of the element ($\mathbf{K}_i$, $\mathbf{M}_i$) in the global matrices, one of the methods is to use a Boolean matrix $\mathbf{B}$ (Gonçalves, 2007). The Boolean matrix must have as many rows as the degrees of freedom of the element, and as many columns as the global degrees of freedom. Whenever the local degree of freedom line is equal to the column of the overall degree of freedom, there will be a value of 1, otherwise the value will be 0. To obtain the global matrices of stiffness and mass, the following expression can be used:

$$\begin{aligned}\mathbf{K} &= \mathbf{B}_i^T \mathbf{K}_i \mathbf{B}_i \\ \mathbf{M} &= \mathbf{B}_i^T \mathbf{M}_i \mathbf{B}_i\end{aligned}\tag{2.12}$$

2.3 Chapter remarks

This chapter presented the elementary information of a tensegrity unit, called here cell. The chapter described the finite element modeling procedure to be used in the following chapters. In the next chapter the model of a tensegrity cell is validated by experimental tests.

Chapter 3

Experimental procedure

The objective of this chapter is to describe the experimental procedure used to validate the mathematical model obtained by finite element method described in the previous chapter. Moreover, the cable tensions are obtained by adjusting the first natural frequencies of the model to be coincident to those observed in the experimental results for the cases where the shape memory alloy is at normal room temperature and heated.

3.1 Experimental apparatus

An experimental apparatus was built to validate the mathematical model. The figure 3.1(a) shows the experimental model based on the tensegrity elementary cell. As described in 2, this work structure bars and cables are made of stainless steel. The shape memory alloy used in this work is a Nitinol (Nickel-Titanium) made by Wayne Metals.

Figure 3.1(b) presents the bars edge, and how the cables are connected. The edge of each bar has a screw hole. This screw hole is useful to the experimental process to connect the accelerometer and also an equivalent mass on the bars. In this experimental apparatus all the cables are held by a screw inserted at the edge of each bar.

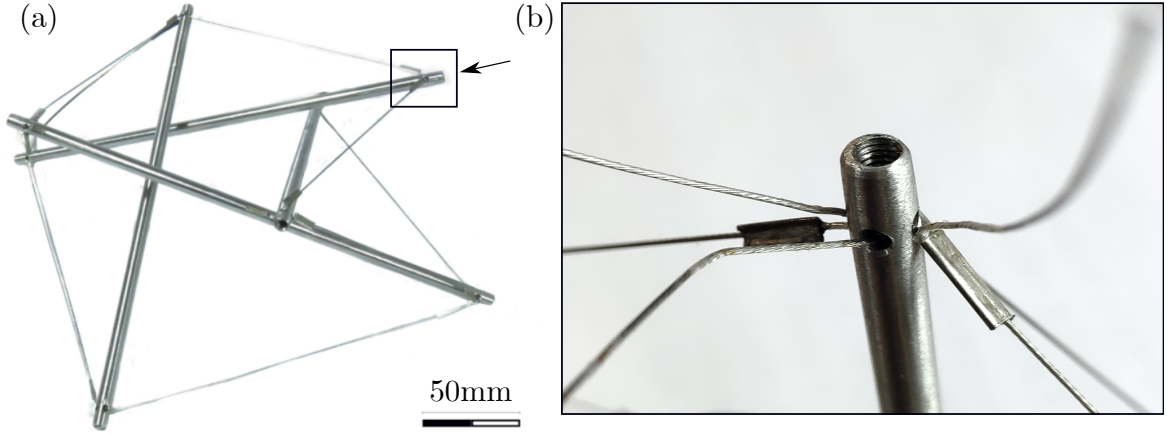


Figure 3.1: Experimental apparatus built to compare with the mathematical model of a tensegrity elementary cell - (a) Experimental apparatus of the tensegrity elementary cell, (b) Detail of the bars edge with a screw hole and the cable connections.

3.2 Experimental setup and procedure

The experimental procedure is the transmissibility between the base and top displacements of the tensegrity elementary cell. In this experiment, nodes (3, 4, 5, 8) were considered free, while nodes (1, 2, 6, 7) were clamped with no rotation allowed, as figure 3.2 shows.

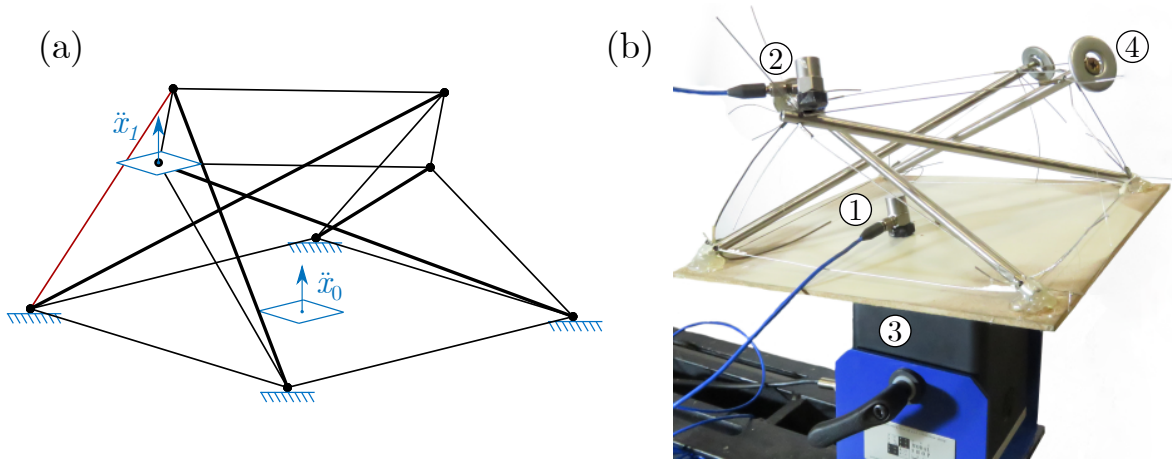


Figure 3.2: Experimental scheme and setup, showing the transmissibility ratio ($\ddot{x}_1/\ddot{x}_0$) (a) Experimental scheme with the elementary cell boundary conditions, (b) Experimental setup with part of the data acquisition system numbered.

In the experimental setup shown in figure 3.2 (b), it is shown two piezoelectric ac-

celerometers (PCB model ICP 352C33), a shaker (Model K2007E01 by The Modal Shop INC) which was used to excite the system. The DAQ device is a *National Instruments* module (Model NI USB-4431, 24-Bit 102.4 KS/s) connected to a computer with the software *LabVIEW 2013 Professional Development System* with the routine in block diagram represented by figure 3.3. The excitation signal was generated by the software *Audacity* in a computer using the sound board output connected to the shaker. Instead of white noise, commonly used for excitation, this procedure used a brown noise signal which has higher energy at lower frequencies, inversely proportional to the frequency squared. This signal was used to equalize the excitation levels in a frequency range, because both the computer sound board and the shaker attenuate the signals at lower frequencies.

The signals from the accelerometers were acquired with a sample frequency of 1 KHz, in a frequency range up to 100 Hz. After the acquisition it was possible to convert the time signals to the frequency domain using the power spectral density (PSD) of each signal. This was obtained using the Labview Frequency Response block. For the signal processing procedure, Hanning window was used to reduce the leakage before converting from time to frequency domain and 10 RMS averages were performed to obtain the final frequency responses.

The two accelerometers and the shaker are numbered as 1, 2 and 3 in figure 3.2 (b).

The experimental model was connected to the shaker, with one accelerometer in the base of the structure, and the other one coupled in an upper node. Three additional washers (4) equivalent to the accelerometer mass were included in the experimental and mathematical model to have the same mass at each node.

The procedure was performed twice, in the first the SMA was at room temperature (RT) and for the second, the temperature of the SMA wire was elevated until the transformation from martensite to austenite. To heat the SMA an electrical current was applied, and the transformation from the martensic phase to the austenite phase occurred after the transition temperature, and the SMA effect happened by the wire

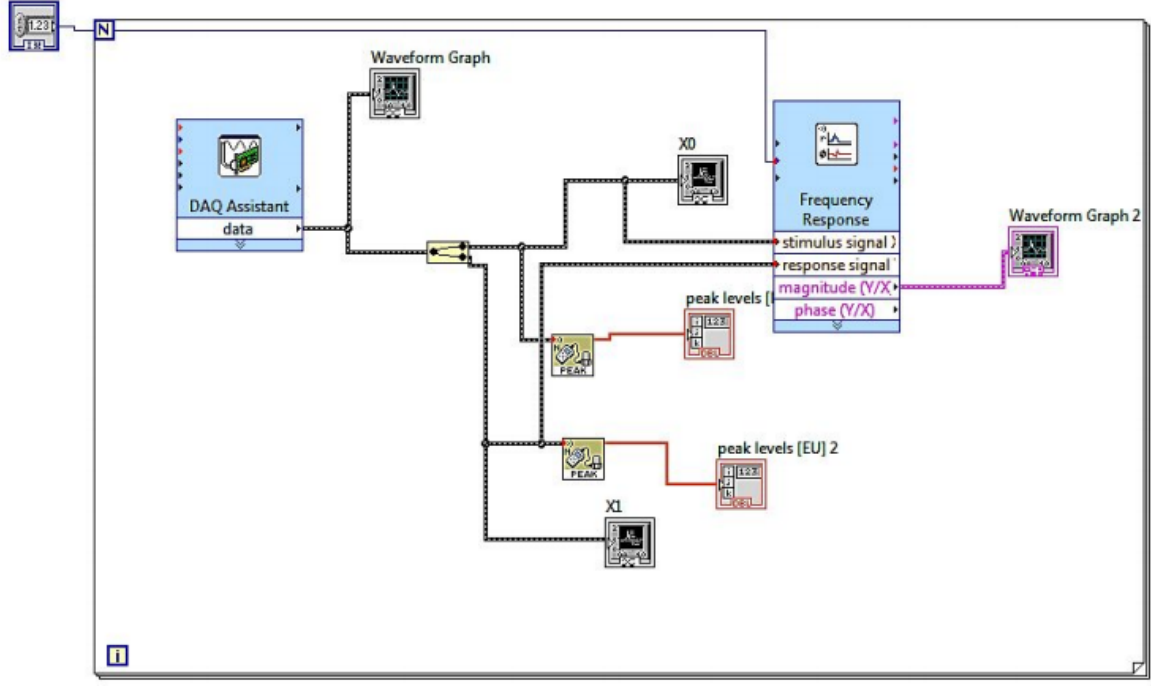


Figure 3.3: *LabVIEW 2013* block diagram routine used in the experimental procedure.

shape-morphing to the original programmed state. The austenite transformation temperature of this work SMA is from 35 to 85 °C. In both cases the structure was subjected to random excitation and after calculating the power spectral density (PSD) from the accelerometer data, it was possible to obtain the ratio of accelerations (top/base) ($\ddot{x}_1/\ddot{x}_0$) as figure 3.2 (a) shows, indicating an acceleration transmissibility in the frequency domain.

3.3 Analysis and Results

Figure 3.4 shows that the first resonance frequency is reduced even though the heated SMA Young's modulus (E_{sh}) increases.

This occurs because the SMA used in the experiment was straight annealed. When heated, it tends to expand, causing a reduction on the tension on the other cables of the structure consequently reducing the stiffness as equation 2.4 shows, and then resulting on an overall reduction of the first natural frequency of the system. This tendency to

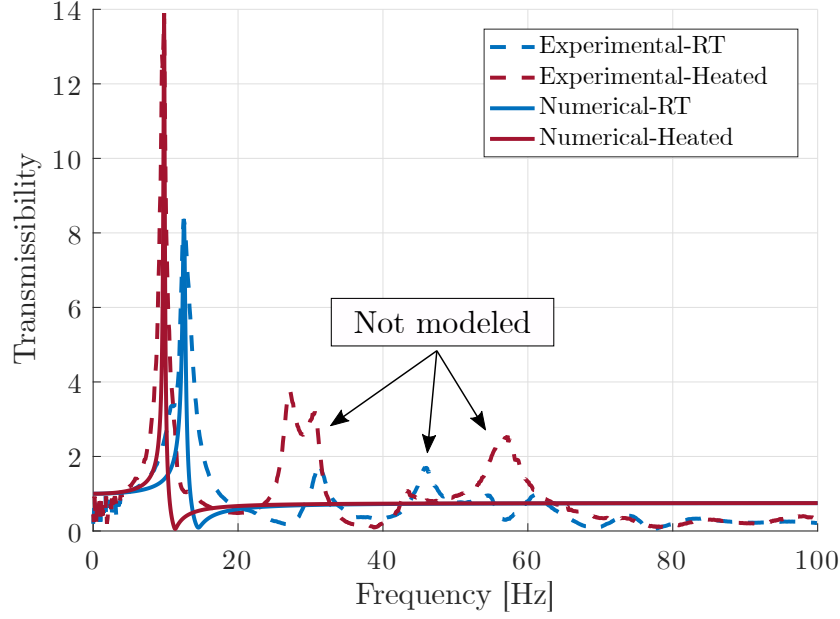


Figure 3.4: Comparison of experimental and numerical transmissibility of the tensegrity elementary cell with the SMA at RT and heated.

expand is shown by figure 3.5 by the red arrows. Once the SMA is fixed at the edge of the bar in the same way as the other cables as shown by figure 3.1, it tends to move and with this expansion the tension of the other cables is lowered.

As described by Kebiche et al. (1999), the tension on the cables could be determined according to their lengths, by this, the relation between the length of the cables is determined. Utilizing the same method as Moussa et al. (2001), based on the first natural frequency, the tension at RT for the upper, side and lower cables were initially 3.8 N, 3.8 N and 2.69 N, respectively and when heated were 2.3 N, 2.3 N and 1.63 N. The tensegrity elementary cell resonance frequency using the SMA at RT and heated are 12.5 Hz and 9.8 Hz, respectively.

The tension and the structural damping were adjusted based on the experimental results, and applied on the mathematical model presented in Chapter 2. The adjustment was made utilizing a parametric variation of Tension (T) and damping constant (β), varying first the tension to find the approximated value of the natural frequency, then varying β to find the amplitude. This sequence was important because the damping

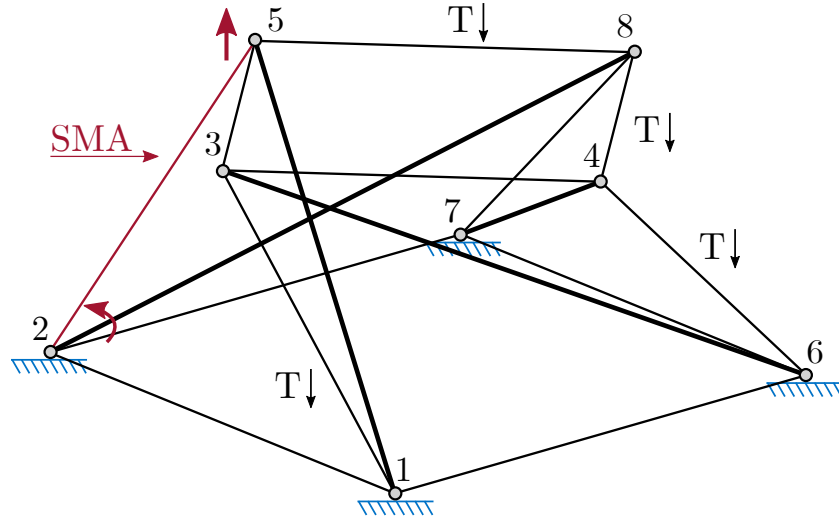


Figure 3.5: Heated SMA effect on the elementary cell affecting the cables tension. Black arrows indicating that tension (T) is lower after the SMA is heated, and the dark red arrows, indicating the SMA tendency to expand.

value is dependent of both Tension and damping. The damping constant is $\beta = 3.45 \times 10^{-5}$. This adjustment was based on the first resonance peak because it represents the highest energy on the studied range. With the adjustments on the model and the validation, this element cell properties were replicated to the tensegrity structure.

3.4 Chapter Remarks

This chapter presented a discussion about the experimental apparatus and experimental procedure used to validate a tensegrity cell model discussed in the last chapter. After calculating the PSD from the accelerometer data, the results of the experimental apparatus and mathematical model were compared. The results presented a shift to the left on the first resonance peak, which was the focus of the analysis, this represents the reduction of tension (and consequently stiffness) of the elementary cell. This reduction was caused by the SMA wire which tends to expand when heated.

The adjusted elementary cell model was replicated to the tensegrity structure. In the following chapters the elementary cell with the heated SMA is named active cell, and the cell with SMA at RT is named inactive cell.

Chapter 4

Analysis and Results

The analysis in this chapter extrapolates the concepts observed in chapters 2 and 3 for a single cell to apply to a larger tensegrity structure. The tensegrity structure and its boundary conditions are presented. A performance function to simplify the results and an attenuation expression are presented to better simplify the results and compare the proposed control strategies. Two dynamic analyses considering a free motion and a harmonic motion are presented, and for the harmonic motion two control strategies and results are presented and discussed .

4.1 Tensegrity structure

The tensegrity structure considered in this work is composed of ten elementary cells shown in figure 1.14. The elementary cell is presented at the beginning of this chapter by figures 2.1, 2.2 and 2.3. In these figures, the thin lines represent the SMA and cables, and the thick lines represent bars with axial stiffness. The numbering scheme of the structure is shown in figure 4.1. The elementary cells are connected by the nodes and share the same from the previous unit cell. The cables are independent of each other, and the SMA is one of the side cables on each unit cell (dark red line in figure 4.1).

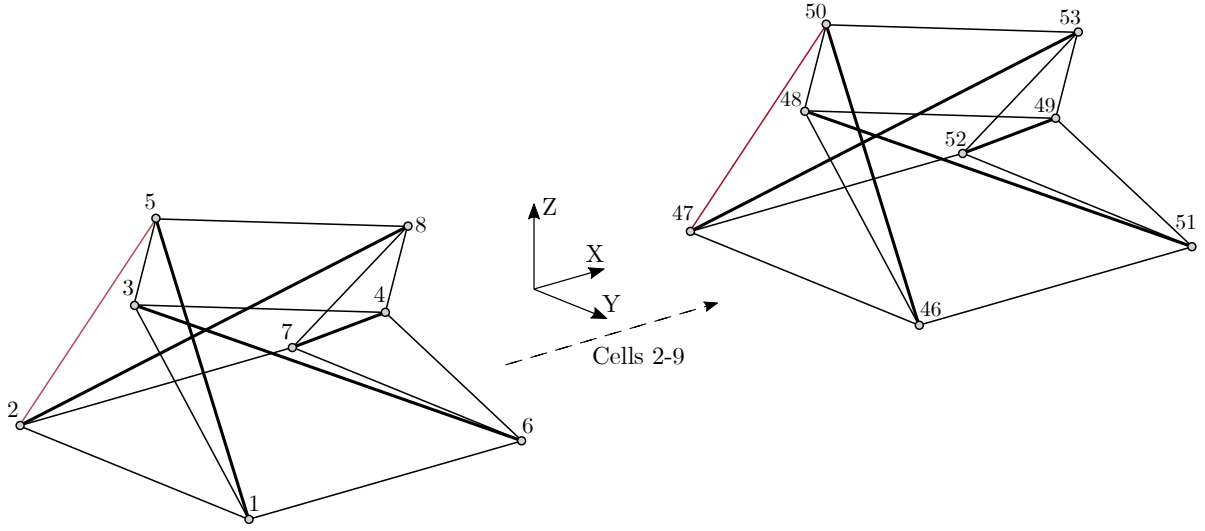


Figure 4.1: Tensegrity structure node numbering scheme from 1 to 53, starting from the left to the right.

4.2 Dynamic Response

In this section two analysis are presented, a free motion and a harmonic motion analysis. The method presented in chapter 2 is considered to obtain the parameters used in this section, considering a tensegrity structure with 10 elementary cells. The equation of motion of the structure 4.1 is represented by equation 4.1.

$$\mathbf{M}\ddot{\mathbf{u}} + \mathbf{C}\dot{\mathbf{u}} + \mathbf{K}\mathbf{u} = \mathbf{f} \quad (4.1)$$

in which the overdots denotes differentiation with respect to time (t). Considering harmonic excitation the vector of nodal displacement and force vector can be written in the form:

$$\begin{aligned} \mathbf{u}(t) &= \mathbf{U}e^{j\omega t} \\ \mathbf{f}(t) &= \mathbf{F}e^{j\omega t} \end{aligned} \quad (4.2)$$

Applying equation 4.2 in 4.1, the equation of motion in frequency domain can be

written as:

$$[-\omega^2 \mathbf{M} + j\omega \mathbf{C} + \mathbf{K}] \mathbf{U} = \mathbf{F} \quad (4.3)$$

$\mathbf{U}$ is a vector of complex displacement amplitudes for each degree of freedom in the structure, and can be calculated by:

$$\mathbf{U} = \mathbf{D}^{-1} \mathbf{F} \quad (4.4)$$

in which,

$$\mathbf{D} = [-\omega^2 \mathbf{M} + j\omega \mathbf{C} + \mathbf{K}] \quad (4.5)$$

By this, the vector of complex velocities $\mathbf{V}$ is given by:

$$\mathbf{V} = j\omega \mathbf{U} \quad (4.6)$$

For the tensegrity structure (10 cell structure presented in section 4.1), there are 150 degrees of freedom (DOF), consequently, the size of the matrices $\mathbf{M}$, $\mathbf{C}$ and $\mathbf{K}$ are 150 by 150. The displacement vector has dimension 150 by 1. The harmonic force vector $\mathbf{F}$ is 150 by 1 and at z direction of node 8 (matrix position 24), has a -1 N harmonic force applied. Two types of analysis are considered, free motion, and harmonic forced response. These are discussed in the following sections.

4.2.1 Free motion analysis

Using the equation 4.1 and following the method to obtain the mass and stiffness matrices from Chapter 2, it was possible to analyze the natural frequencies of the structure by the eigenvalue problem. In this analysis the external force vector ($\mathbf{f}$) and the damping matrix $\mathbf{C}$ are neglected, and then the eigenvalue problem is given by the following

equation:

$$(\mathbf{K} - \omega_i^2 \mathbf{M})\phi_i = 0 \quad (4.7)$$

in which ω_i is the natural frequency and ϕ_i is the corresponding vector that contains all the degrees of freedom of the system. The modal shapes and natural frequencies are related to the eigenvectors and eigenvalues, respectively (Craig and Kurdila, 2006). Some of the modal shapes can be seen in figures 4.2 and 4.3, presenting modes that the structure behave similar to those of an uniform beam, and modes that the elementary cells move independently.

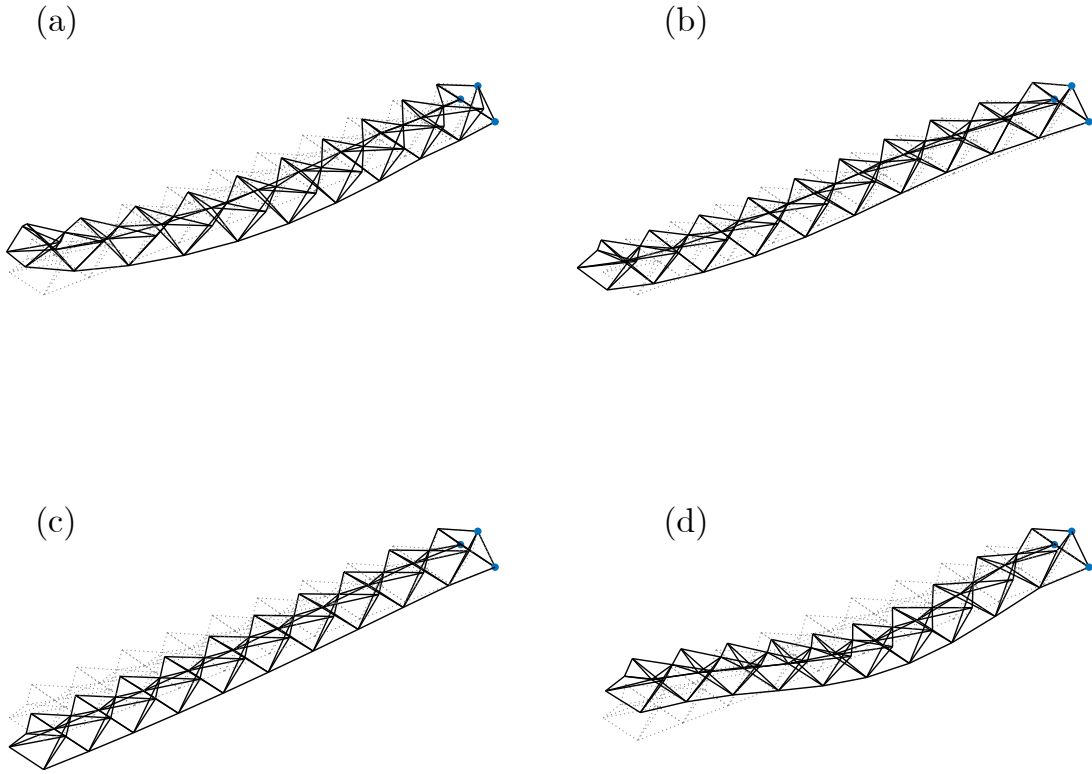


Figure 4.2: Tensegrity structure modal shapes with behavior as uniform beam - (a) 1st mode shape (4 Hz), (b) 2nd mode shape (6 Hz), (c) 22th mode shape (53 Hz), (d) 23th mode shape (98 Hz).

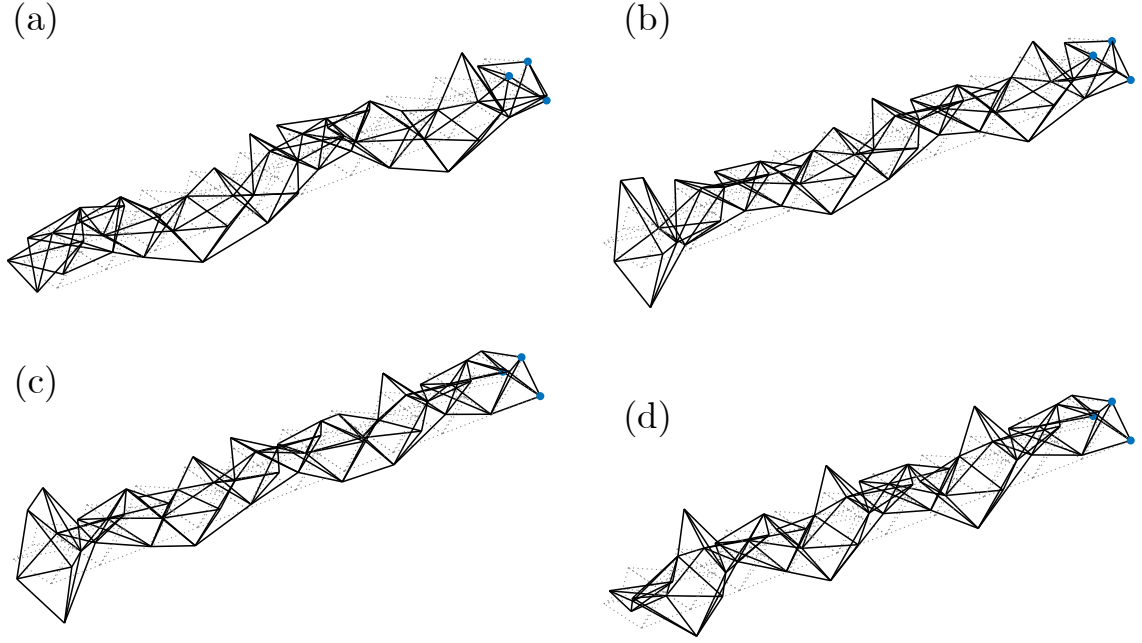


Figure 4.3: Tensegrity structure modal shapes with elementary cells behaving independently - (a) 26th mode shape (206 Hz), (b) 27th mode shape (230 Hz), (c) 28th mode shape (243 Hz), (d) 29th mode shape (255 Hz).

4.2.2 Harmonic force excitation

In this subsection a harmonic force ($\mathbf{F}$) is considered. This force is applied at node 8 as figure 4.4 shows. With the objective of understanding the dynamic behavior of such system, a Performance Function J is defined as the sum of the linear squared velocities of the nodes $n_1 = 1$, $n_2 = 2$, and $n_3 = 3$, as shown by figure 4.4. The performance function simplify the results from nine FRFs to only one FRF, so instead of analyzing and optimize for nine responses the focus is only in one.

This performance function is proportional to the kinetic energy at these nodes and it is calculated by:

$$J(\omega) = \mathbf{V}_m^H(\omega) \mathbf{V}_m(\omega) \quad (4.8)$$

in which, ω is the angular frequency. $\mathbf{V}_m^H(\omega)$ is the Hermitian form of the vector $\mathbf{V}_m(\omega)$.

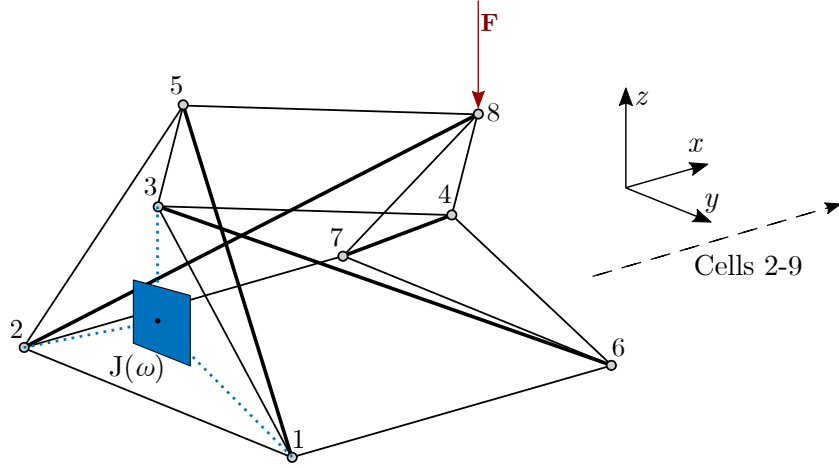


Figure 4.4: Illustration of the performance function ($J(\omega)$) reducing the three DOF in each node (1, 2 and 3) into only one response. The harmonic force ($\mathbf{F}$) represented by the dark red arrow, is applied at node 8 for the analysis on section 4.3.

Abolishing the symbol ω for simplicity, the vector $\mathbf{V}_m$ is defined by:

$$\mathbf{V}_m = [v_x^{n_1} \ v_y^{n_1} \ v_z^{n_1} \ v_x^{n_2} \ v_y^{n_2} \ v_z^{n_2} \ v_x^{n_3} \ v_y^{n_3} \ v_z^{n_3}]^T \quad (4.9)$$

in which for an example $v_y^{n_1}$ is the nodal velocity n_1 in the y direction for an ω , and T denotes the transpose of the vector.

4.3 Proposed control strategies

Different control strategies are presented in this work, the first strategy utilizes cells with a heated SMA wire, named control cells, to control the system dynamic response in two different ranges. This strategy is divided into placing single control cells and multiple control cells in the structure at different locations. The second strategy is to control specific modes by calculating the strain energy of the elementary cells for the respective mode, and replacing the cell with the highest strain energy with a cell with a heated SMA wire. These strategies use different setups presented at the beginning of each subsection.

The control strategies are compared with a passive structure. To compare the dif-

ferent setups is proposed an attenuation expression in the performance function given by :

$$a(n)[\text{dB}] = 20 \log_{10} \frac{\bar{J}_n}{\bar{J}_0} \quad (4.10)$$

in which, n is the setup number and $\bar{J}_0$ is the performance function arithmetic mean for the setup with no control cell. The overline represents the mean, and uses the same principle as $\mathcal{H}_2$ presented by Zhou et al. (1996).

During the structure analysis, the nodes 51, 52 and 53 were considered to be fully clamped and a harmonic force is applied at node 8, as shown in the figure 4.5. The same boundary condition is considered for all analysis in this Subsection. The cell numbering is from the left to the right, thus, the clamped cell is the 10th cell.

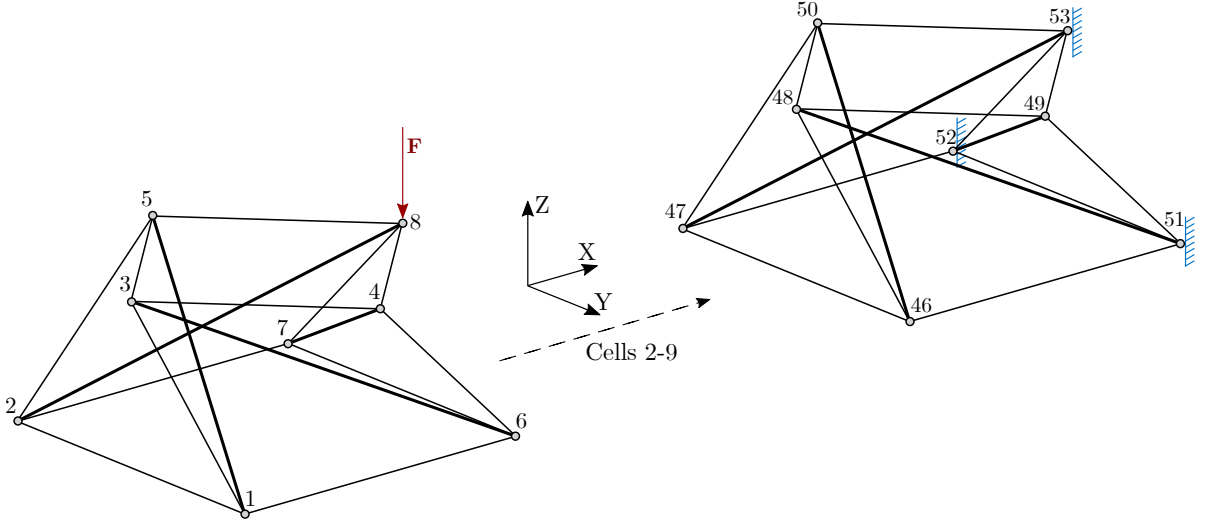


Figure 4.5: Tensegrity structure node numbering scheme, and boundary conditions, including the the harmonic force ($\mathbf{F}$) represented by the dark red arrow, and the fixed nodes (51, 52 and 53).

Each FRF presented was obtained with the model presented in Chapter 2, and the adjustments presented in Chapter 3. The overall analyzed frequency band is from 7 to 300 Hz, and covers 29 natural frequencies (ω_n) of the structure. The ω_n values are presented in appendix A. Figure 4.6 presents a FRF of the passive structure from 0 to 500 Hz, and shows that there is a gap between two ranges, so the analyzed frequency

band is divided into two ranges as, Range 1 from 7 Hz to 15 Hz and Range 2 from 200 to 300 Hz. The ranges were considered based on the concentration of energy.

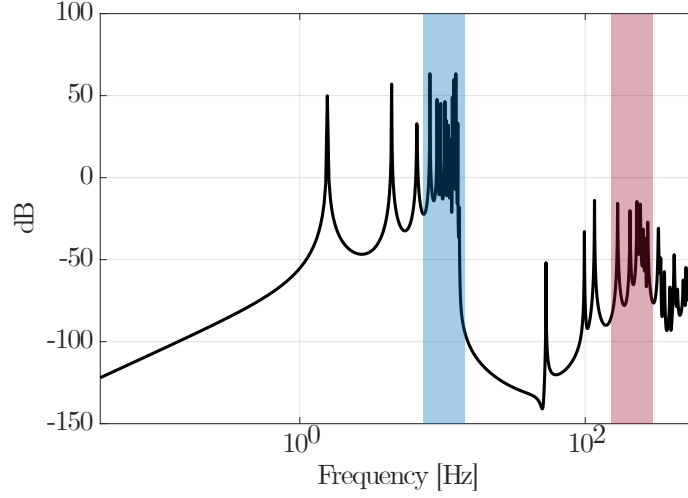


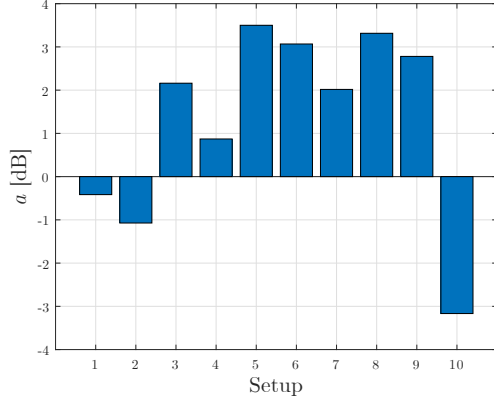
Figure 4.6: Frequency response function of the structure with no control from 0 to 500 Hz, and the highlight of the two studied ranges in blue and red colors. dB ref. 1 Ns/m

4.3.1 Single control cell allocation

In this section a single control cell is placed at the structure. Ten different setups are analyzed, and these setups are based on the position of control cells over the structure. For this structure with ten elementary cells, there are ten different setups with one control cell in each of them. They are named based on the position number of the control cells, from cells 1 to 10. These setups are proposed in order to verify the behavior of the tensegrity structure with the change of stiffness on diverse locations.

The relation between the passive structure and the setups using Eq. 4.10 for the lower frequency is shown by figure 4.7 (a). It shows that setups 3 to 9 have lowered the J function amplitude, and setups 1, 2 and 10, increased the mean FRF amplitude. In figure 4.7 (b) is represented the results for the higher frequency, which is possible to see that all setups lowered the mean FRF amplitude. Notably, for range 1 and 2, setups 5 and 3 lowered the mean amplitude by 3.4 and 6.2 dB respectively. Based on the attenuation equation 4.10 it is clear that the mean amplitude for range 2 with setup

(a)



(b)

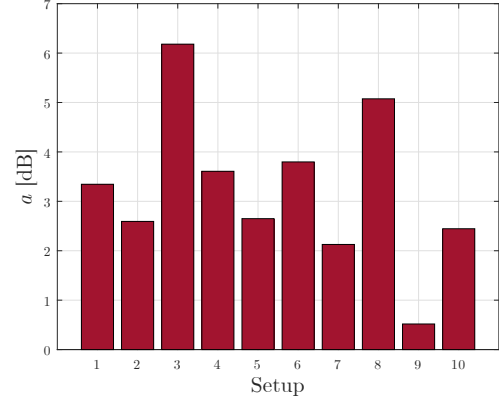


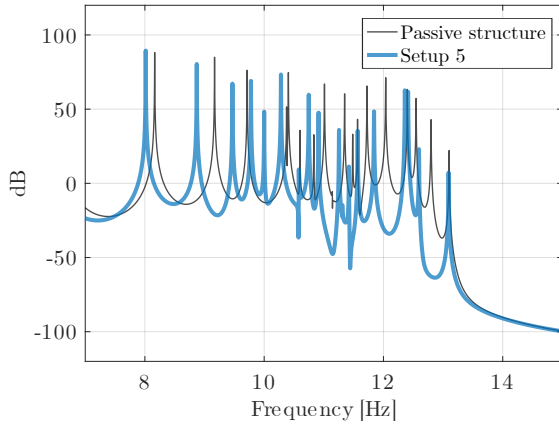
Figure 4.7: Attenuation comparing setups with an control cell and the passive structure. (a) First range amplitude attenuation for single control cell setups, (b) Second range amplitude attenuation for single control cell setups.

3, is half of the inactive structure.

First Range response

The two best results for the first range, setups 5 and 8, are presented by figure 4.8.

(a)



(b)

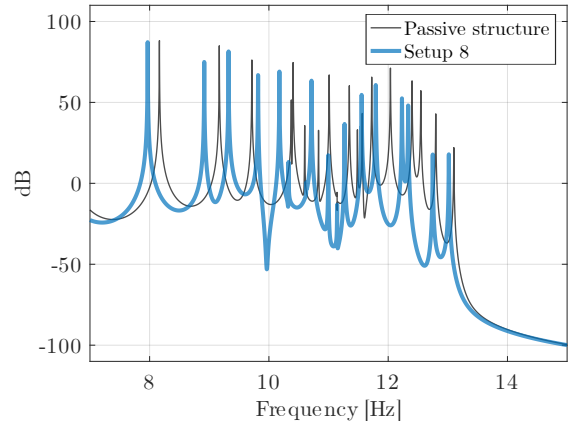


Figure 4.8: FRF comparison between setups with the higher attenuation value and the passive structure for the first range. (a) 5th control cell, (b) 8th control cell - FRF Range 1. dB ref. 1 Ns/m

For both examples mostly resonance peaks were not just shifted but also lowered, caused by the reduction of the damping from the reduction of the system stiffness as equation 2.2 shows. Lower amplitude between peaks also were created with the control

cell leading to a lower performance function. Overall, in the two best results the mean reduction was around 3.2 dB.

Second Range response

The two best results, setups 3 and 8, for the second range are presented by figure 4.9. For this range some peaks disappeared and a gap is made between 250 and 270 Hz in both responses. It is shown that the control cells like for the first range lowered the amplitude between peaks, and in most cases lowered the peaks. Overall, in the two best results the attenuation was 5.6 dB.

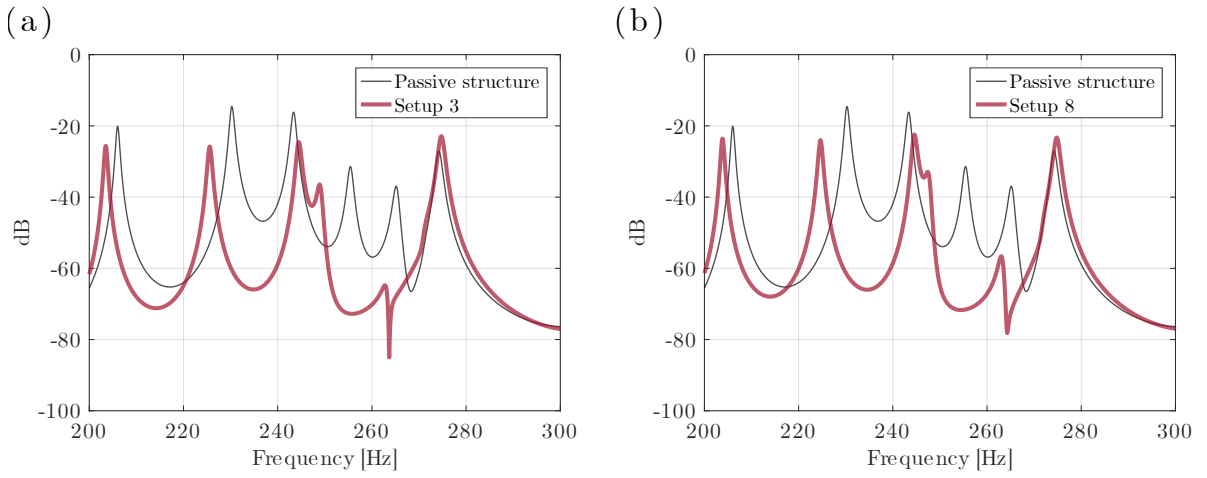


Figure 4.9: FRF comparison between setups with the higher attenuation value and the passive structure for the second range. (a) 3rd control cell, (b) 8th control cell - FRF Range 2. dB ref. 1 Ns/m

For both ranges some setups presented a decline on the amplitude of mobility FRF. The stiffness reduction caused by the control cells did not just shift the peaks overall, but also created bigger gaps between peaks in both ranges. With this, it is evident that the position of control cells at the boom structure have a major impact on the response of an harmonic force applied on the structure, changing the natural frequencies and the dynamic response of the whole structure.

4.3.2 Multiple control cells

The second analysis is based on the use of multiple control cells at the same time in different positions through the structure. The figure 4.10, presents the configurations of the four setups tested in this section, with control cells represented by red color. The proposed setups aim different aspects of the structure, testing the stiffness reduction on the extremities and the central region, and a periodic positioning of control cells.

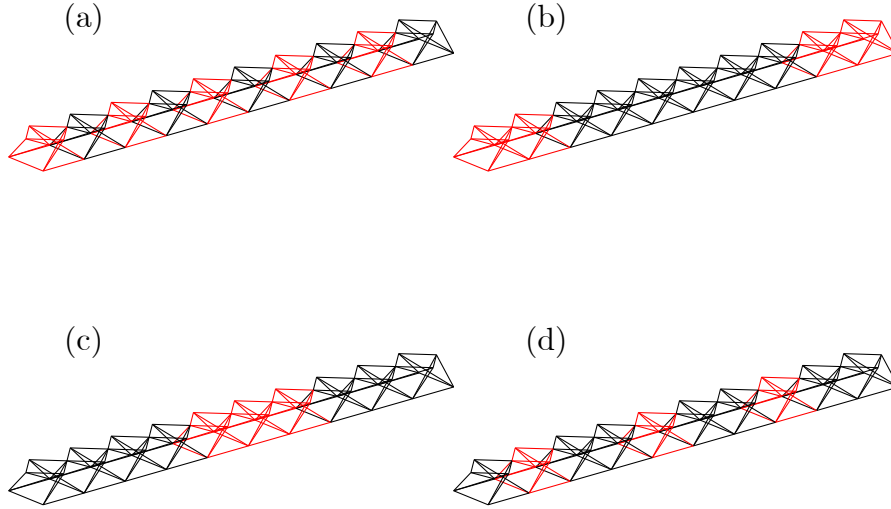


Figure 4.10: Illustration of multiple control cells setups. (a) Structure with interleaved control cells - Setup M1, (b) Structure with control cells at the extremities - Setup M2, (c) Structure with 3 control cells at the central region - Setup M3, (d) Structure with 3 interleaved cells - Setup M4

The setups have at least 3 and 5 maximum control cells distributed at the tensegrity boom structure as, structure with interleaved control cells, structure with control cells at the extremities, structure with 3 control cells at the central region and Structure with 3 interleaved control cells, named as Setup M1, Setup M2, Setup M3 and Setup M4, respectively. The attenuation for the two ranges is shown by figure 4.11, where for the first range, both Setups M1 and M2 presents a increased amplitude in response of the harmonic force applied for the analysis, when compared to the passive structure, in around 5 dB. The setup 3 have a mitigation about 8.5 dB and is the best of the four setups for the range. For the second range the first setup have the best results with

attenuation of 7.9 dB. In all setups for this case the amplitude diminishes, different from the first range with just 2 better results.

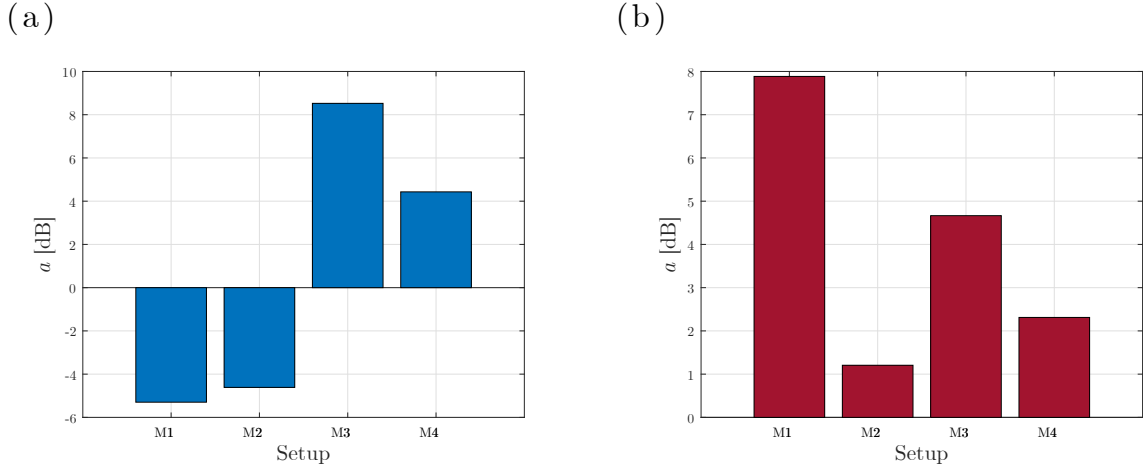


Figure 4.11: Attenuation comparing the structure with multiple control cells and the passive structure. (a) First range amplitude attenuation for multiple control cells setups, (b) Second range amplitude attenuation for multiple control cells setups.

First Frequency Range

The first range Setups M3 and M4 presents the best results, and the mobility FRF is represented by figure 4.12.

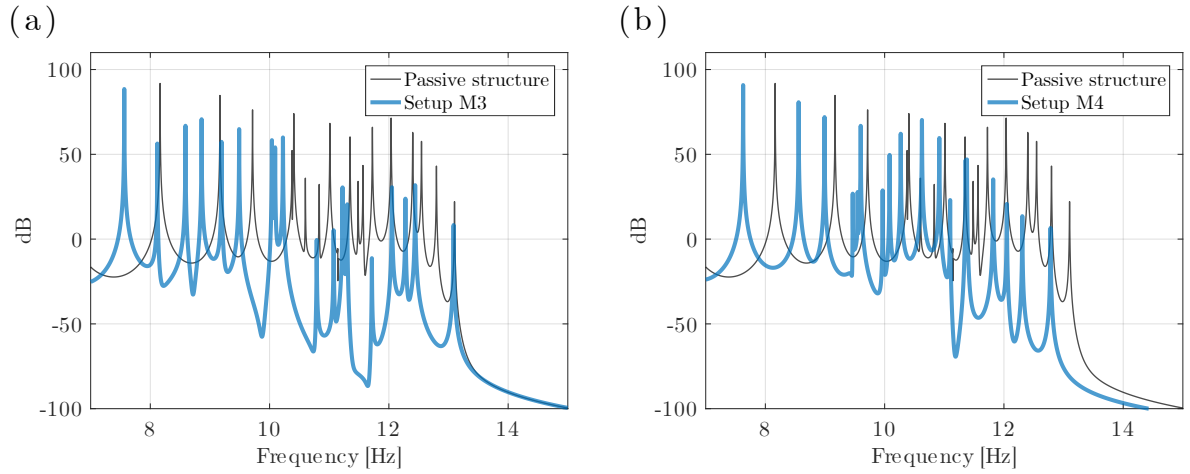


Figure 4.12: FRF comparison between setups with the higher attenuation value and the passive structure for the first range. (a) Setup M3 FRF, (b) Setup M4 FRF - Range 1. dB ref. 1 Ns/m

Similar to the single control cell setups, the Setup M3 created deeper valleys and gaps in the response, notably from 10 to 13 Hz, and also did not just shift the peaks, shown by figure 4.12(a) in the last resonance peak. For the Setup M4, is visible that was a shift to the left, and like all the setups tested, shows an amplitude reduction. Overall, in the two best results the mean reduction was around 6.2 dB.

Second Frequency Range

The best results for the second range are presented by figure 4.13. It is visible that the results are similar to the single control cell as well, and reducing the amplitude for the range, with a shift for specific natural frequencies. Overall, in the two best results the mean reduction was around 6.3 dB.

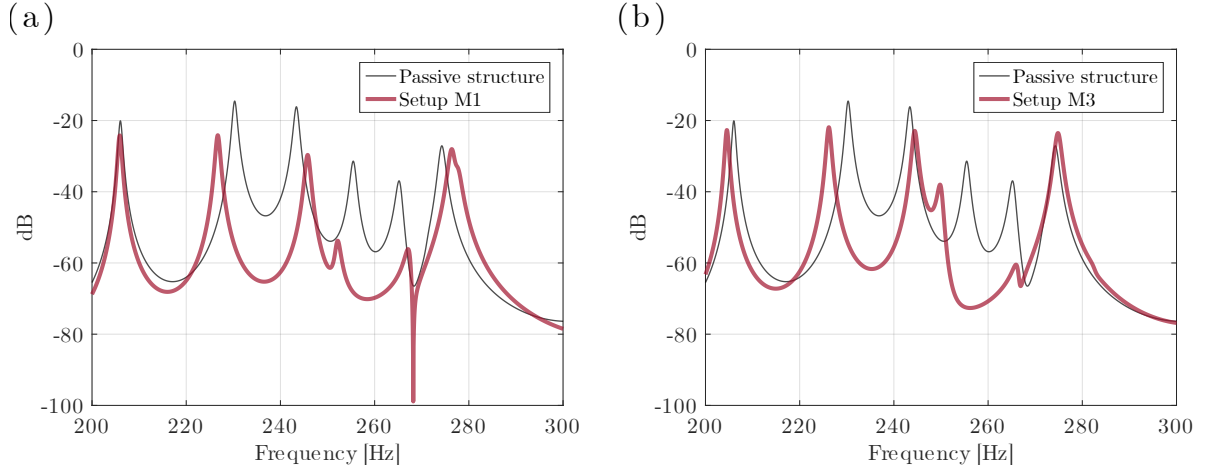


Figure 4.13: FRF comparison between setups with the higher attenuation value and the passive structure for the second range. (a) Setup M1 FRF, (b) Setup M3 FRF - Range 2. dB ref. 1 Ns/m

The results for both ranges shows a reduction on the amplitude of mobility FRF. Two of the eight cases tested presented a increase in the amplitude response, making it visible that, the control cell location even with more than one, have a role on the response of an harmonic force applied on the structure changing the modal energy distribution. The mean attenuation presented by the best setups in both ranges was about 8 dB, presenting a reduction over half of the amplitude value.

4.3.3 Vibration control of specific modes and frequencies

To control specific modes and frequencies was used a similar approach as Preumont (1997) presents, replacing the element with higher strain energy with a control device, in his work actuators were used as this control device. Analogously to this approach, the strain energy was calculated using Equation 4.11, for all the elements of the structure, but the considered strain energy to replace with a control, was an elementary cell not just an element, so with this, was calculated the cell strain energy. The cell strain energy was calculated by a simple sum of the strain energy of all elements that compose an elementary cell in the tensegrity structure, so for a modal analysis, each mode will have the response for the whole cell, not just an element.

$$v_i = \frac{\phi_i^T \mathbf{B}_i^T \mathbf{T}_i^T \mathbf{K}_i \mathbf{T}_i \mathbf{B}_i \phi_i}{\phi_i^T \mathbf{K} \phi_i} \quad (4.11)$$

in which, ϕ_i is the collection matrix containing the i^{th} mode shape components associated with the degree of freedom of the element. $\mathbf{B}_i$, is the Boolean matrix associated with the element i . $\mathbf{T}_i$ is the coordinate transformation matrix for the element i , $\mathbf{K}_i$ is the stiffness matrix of the element i and $\mathbf{K}$ is the global stiffness matrix.

Aiming the first 32 modes that comprehends from 0 to 300 Hz, in which the two first studied ranges are part of, the figure 4.14 shows the individual normalized energy value for each mode in each elementary cell.

It is possible to see that the structural modal strain energy, is not concentrated in one cell for some modes, and in contrast, some modes have one or two cells with almost all the modal strain energy. For this analysis three different vibrational modes were chosen to control using the method presented in this subsection. As mentioned before, the cell with the higher modal strain energy was activated. The structures with control cells for the chosen specific modes are presented by figure 4.15.

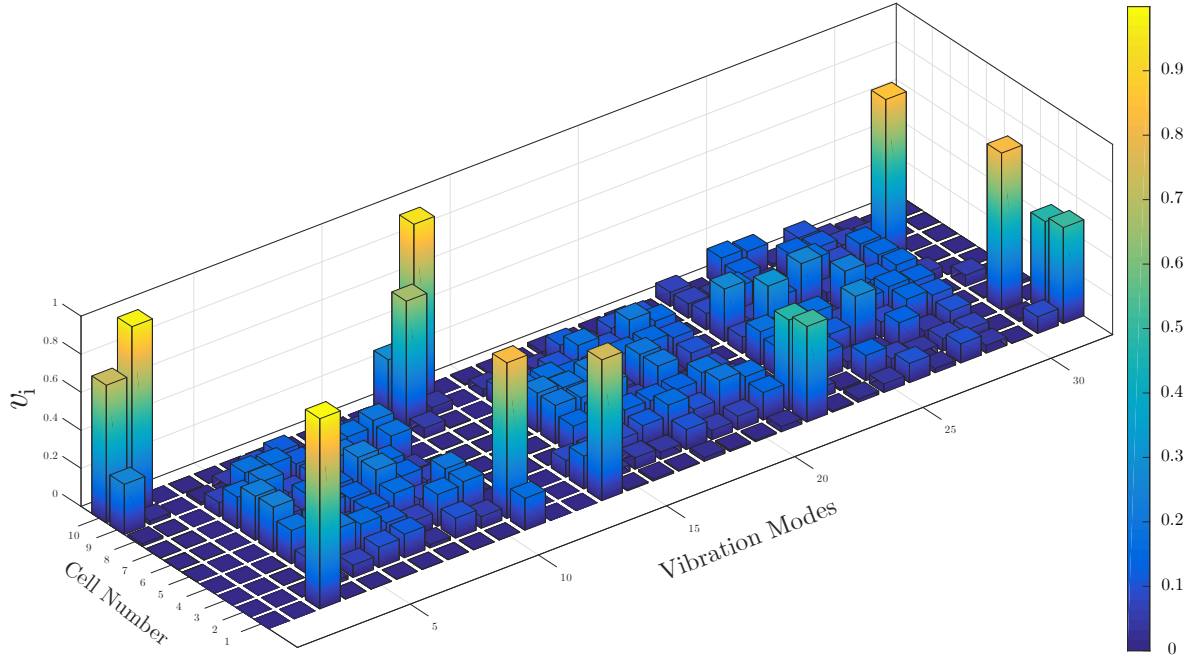


Figure 4.14: Individual cell normalized strain energy in the tensegrity structure for corresponding vibration modes.

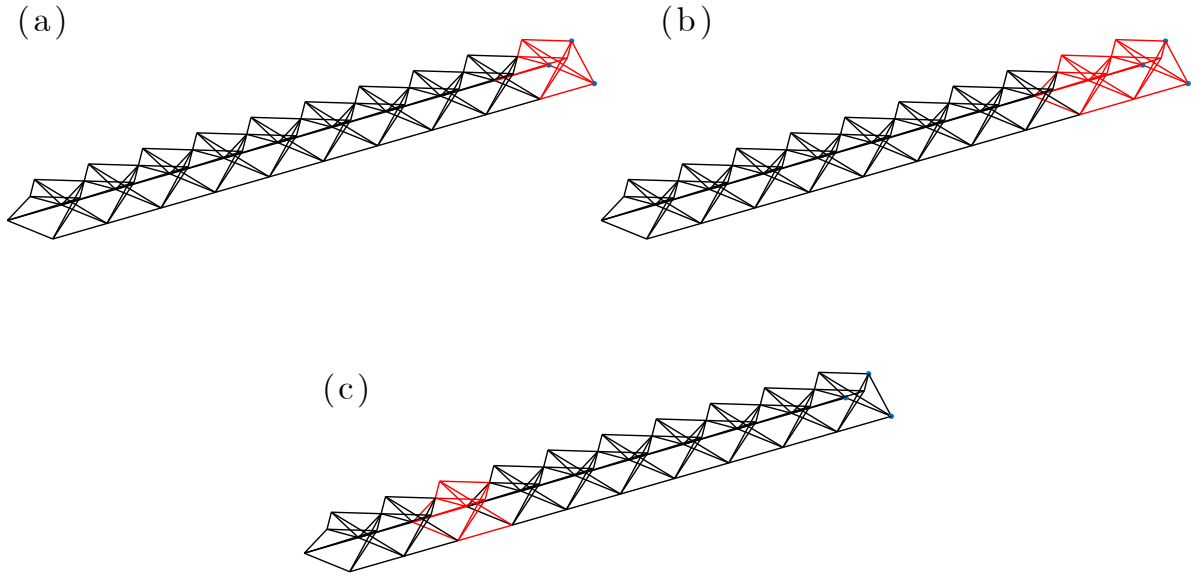


Figure 4.15: Setups with control cells represented in red to control specific modes (a) Setup S1 for 3rd mode, (b) Setup S2 for 11th mode, (c) Setup S3 for 30th mode

As presented by figure 4.15 the chosen modes are the 3rd, 11th and 30th modes and the associated setups for these modes are, Setup S1, Setup S2 and Setup S3 respectively.

Figure 4.16 presents the response for the different setups. The black thin line represent the inactive structure response, and the blue thick line represent the active setup. The blue and red dashed lines represents, the vibrational mode on the active and inactive structure respectively.

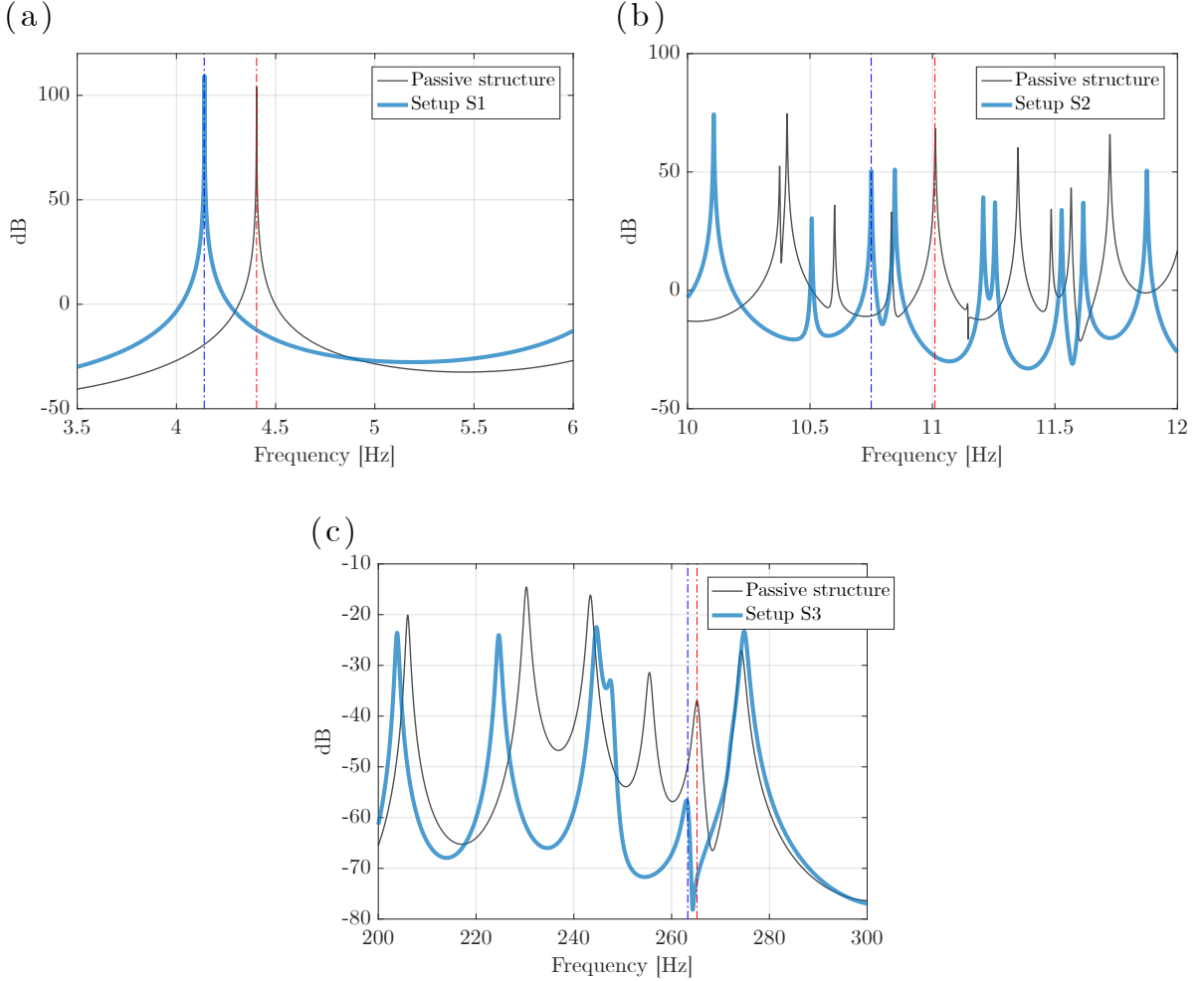


Figure 4.16: FRF comparison between the setups with control cells and the passive structure. dB ref. 1 Ns/m - (a) Setup S1 to control the 3rd Mode, (b) Setup S2 to control the 11th Mode, (c) Setup S3 to control the 30th Mode.

All setups response presented a change in the natural frequency. The Setup S1 shifted the vibrational mode to the left as a result of the lower stiffness. Setup S2 and S3 response, had a change by shifting the resonance peak and also by lowering the vibrational mode response amplitude. A deeper valley was created in both cases, notably the change caused by Setup S3 has a amplitude difference at the original natural

frequency over 40 dB.

The strain energy was compared before and after the control. The change in the v_i values presented in figure 4.14 is less than 0.05, meaning that there is a small energy difference between the control and passive cell.

4.4 Chapter remarks

This chapter presented the tensegrity structure and a dynamic analysis, along with the analysis and results for two proposed control strategies using the control cells. The first consisted in a control of two different ranges utilizing single and multiple control cells in the structure, and the second was the control of specific modes based on the strain energy of each cell, replacing the cell with higher strain energy with a control cell. The results showed that is possible to control the vibration and change the dynamic of the structure using control cells, without changing the mass. This change in the response is mainly the shift of resonance peaks, and in some cases gaps and the mitigation of those peaks. Figure 4.17 shows the comparison between passive structure, best single control cell structure, best multiple control cells structure. For the first range (fig. 4.17 (a)), Setup M3 presents bigger gaps than Setup 5. For the second range is possible to see similar gaps between both setups but, with shifted resonance frequencies.

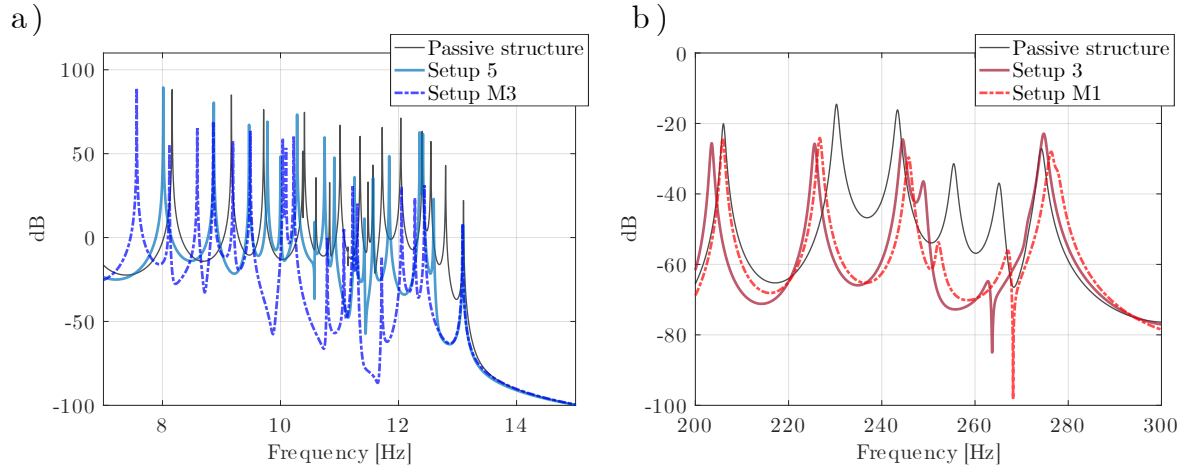


Figure 4.17: FRF comparison between passive structure, best single control cell structure, best multiple control cells structure for both ranges. (a) First range, (b) second range. dB ref. 1 Ns/m

Chapter 5

Conclusions and recommendations for further work

5.1 Conclusions

Deployable structures have been intensively researched for its advantages over non-deployable structures as, small volume they occupy during storage, characteristic that has a major advantage for space applications. To design more advanced structures that could be adapted for diverse applications, control methods are applied. This work was aimed to analyze the capability of controlling a tensegrity structure using shape memory alloy at the individual elementary cells, acting as a semi-active control. The structure considered here has a class 4 tensegrity elementary cell, and the structure is composed of ten of these cells.

When modelling the tensegrity structure, it is assumed that a linearized equation of motion around an equilibrium configuration can describe the structural dynamics. After obtaining a finite element model of a tensegrity elementary cell, an experiment was performed and with the experimental results it was possible to adjust the cable tensions and damping parameters of the model. This adjustment was performed considering the first resonance frequency and amplitude.

This procedure was done twice, with the SMA cable at room temperature, with the SMA cable at room temperature (bellow the transition temperature) and with a electrical current applied to heat the SMA and cause the transition to the austenitic phase. Because the SMA cable was straight annealed when heated, it tends to expand, causing a reduction on the tension on the other cables of the structure consequently reducing the stiffness.

Different control strategies have been analyzed. Single and multiple cell activation, to control two different frequency ranges, and the activation of cells for specific modes control. For the first strategy, the first frequency range (Range 1) is considered from 7 to 15 Hz, and the second (Range 2) from 200 to 300 Hz. The free motion analysis showed that for the first modes the structure predominantly behave similar to those of an uniform beam, and at higher frequency modes the elementary cells moves independently. The analysis with a harmonic force excitation showed that, the control cells mainly shifted the resonance peaks, and created gaps in the FRF. For this strategy an attenuation comparison is made, presenting the overall results for all cells in the first and second strategy setups, the attenuation reached 6.2 dB with a single control cell, and 8 dB with multiple control cells.

The second strategy is based on the control specific modes, by calculating the sum of all elements strain energy in the elementary cells, and replacing a passive cell with a cell with a heated SMA. The analysis of this strategy showed to be effective to control specific modes, by mainly shifting the peaks as a result of the stiffness reduction caused by the control cell, and two out of three studied setups, presented an amplitude reduction.

The general conclusion of the dissertation is that the use of SMA on the tensegrity elementary cells as a control strategy is effective. Vibration levels can be reduced by using this type of control, changing the dynamic response of the structure without changing the mass. The control for specific modes however, would have low effectiveness when compared to the attenuation on the studied ranges as described in this dissertation, caused by the low distribution of the strain energy from the control cell to the other

cells, for the specific vibrational modes.

5.2 Contributions

Parts of this work have been published and presented at VIth Symposium on Intelligent Materials and Control, as *Investigation of the dynamic characteristics of a smart tensegrity structure*, and it was presented at the XIIIth International Conference on Recent Advances in Structural Dynamics, as *On the dynamics of a smart tensegrity structure using shape memory alloy*, and by that, was accepted to be published at Journal of Physics: Conference series.

5.3 Recommendations for further work

The work developed in this dissertation has been based on the use of a linearized equation of motion to describe the system dynamics, and supported by references and with an experimental test. The simulations showed that the control cells can create gaps in the FRF, and a band gap investigation is a recommended direction for future work. A possible approach for further work could consider bending in bars and momentum transmission at the joints of the structure. Due to the periodicity of the structure, the spectral elements method should be considered to further work as a viable method for larger periodic structure.

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Appendix A

Natural Frequencies

In this Appendix, the natural frequencies for this work structure in a free-fixed condition are presented by table A.1. The natural frequencies are obtained by the eigenvalue problem presented by equation 4.7.

Table A.1: Boom structure natural frequencies.

Natural frequency number	Frequency [Hz]
4	8,164
5	9,168
6	9,714
7	10,376
8	10,406
9	10,600
10	10,832
11	11,011
12	11,144
13	11,349
14	11,484

Continued on next page

Table A.1 – *Continued from previous page*

Natural frequency number	Frequency [Hz]
15	11,566
16	11,724
17	12,038
18	12,400
19	12,547
20	12,798
21	13,101
22	53,140
23	98,678
24	116,191
25	168,755
26	206,016
27	230,269
28	243,395
29	255,465
30	265,268
31	271,025