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MARIANA CRISTINA DE OLIVEIRA

THE np_x CHART: when the parameters are known and unknown

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MARIANA CRISTINA DE OLIVEIRA

THE np_x CHART: when the parameters are known and unknown

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Advisor: Prof. Dr Marcela Aparecida Guerreiro Machado

Co-advisor: Prof. Dr. Subhabrata Chakraborti

Co-advisor: Prof. Dr. Felipe Schoemer Jardim

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Coorientadores: Prof. Dr. Subhabrata Chakraborti

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POTENTIAL IMPACT OF THIS RESEARCH

In the thesis, statistical tools were suggested for monitoring processes in real-world scenarios where parameters like mean and variance are estimated. These tools were found to be more cost-effective, simpler, and, depending on the sample size, more efficient than existing methods.

IMPACTO POTENCIAL DESTA PESQUISA

Na tese, foram sugeridas ferramentas estatísticas para monitorar processos em cenários do mundo real, onde parâmetros como média e variância são estimados. Essas ferramentas mostraram-se mais econômicas, simples e, dependendo do tamanho da amostra, mais eficientes do que os métodos existentes.

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Profª. Drª. PALOMA MARIA SILVA ROCHA RIZOL
UNESP

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ABSTRACT

Control charts are powerful tools used by many industries to monitor some quality characteristics and detect special causes of variation. If it is impractical to numerically represent the quality characteristic, attribute control charts, such as the np chart, are often used to monitor the number of nonconforming units. Attribute control charts are considered simpler, faster, and cheaper than variable control charts (such as the $\overline{X}$ chart) because they do not require the measurement of some quality characteristic; instead, professionals only need to count the number of nonconforming units in a sample. For this reason, the np control chart is also used to monitor not only the number of nonconforming units, but also the process average. The np_x control chart was introduced by Wu et al. (2009)(WU et al., 2009) to monitor the mean of a continuous variable using attribute inspection. The authors evaluated the performance of the np_x graph without considering the practical case where the parameters (such as p), necessary to calculate the control limits, must be estimated. The present work focuses on evaluating the performance of the np_x control chart when the parameters are estimated. Despite the initial fascination of the np_x chart for its perceived advantages over variable control charts – such as simplicity, speed and cost-effectiveness – inconsistencies in the results presented by Wu et al. (2009)(WU et al., 2009) are revealed. By addressing these inconsistencies, we offer alternative insights into the effectiveness of the np_x graph. Through an extensive investigation, involving derivations and numerical analysis, the scenarios in which the bilateral and unilateral np_x graphs outperform the $\overline{X}$ graph were determined. With this background, now with the corrected np_x graph, the impacts on parameter estimation were analyzed in detail. It was found that performance is affected mainly when the standard deviation of the process is estimated (cases KU and UU), presenting itself significantly different from what is nominally expected if the estimation of the parameters is not taken into account in the design of the graph. The number of reference samples needed to mitigate this effect makes the application of the np_x graph unfeasible. Therefore, adjustments to the UCL and k_w parameters are necessary. Such adjustments were investigated and proved to be efficient, enabling the np_x graph to offer equal or better performance to the $\overline{X}$ graph using numbers of usual reference samples. This work contributes significantly to a more nuanced understanding of the comparative advantages of different control chart methodologies, for variable and attribute data.

KEYWORDS: Attribute control charts; Parameter Estimation; Average Run Length; False Alarm Rate.

RESUMO

As cartas de controle são ferramentas poderosas usadas por muitas indústrias para monitorar algumas características de qualidade e detectar causas especiais de variação. Se for impraticável representar numericamente a característica de qualidade, gráficos de controle de atributos, como o gráfico np, são frequentemente usados para monitorar o número de unidades não conformes. As cartas de controle de atributos são consideradas mais simples, rápidas e baratas do que as cartas de controle variáveis (como o gráfico $\overline{X}$) porque não exigem a medição de alguma característica de qualidade; em vez disso, os profissionais só precisam contar o número de unidades não conformes em uma amostra. Por esse motivo, o gráfico de controle np também é utilizado para monitorar não apenas o número de unidades não conformes, mas também a média do processo. O gráfico de controle np_x foi introduzido por Wu et al. (2009)(WU et al., 2009) para monitorar a média de uma variável contínua usando inspeção de atributos. Os autores avaliaram o desempenho do gráfico np_x sem considerar o caso prático onde os parâmetros (como p), necessários para calcular os limites de controle, devem ser estimados. O presente trabalho tem como foco a avaliação de desempenho do gráfico de controle np_x quando os parâmetros são estimados. Apesar do fascínio inicial do gráfico np_x pelas suas vantagens percebidas sobre os gráficos de controle variável – como simplicidade, velocidade e custo-benefício – inconsistências nos resultados apresentados por Wu et al. (2009)(WU et al., 2009) são reveladas. Ao abordar essas inconsistências, oferecemos insights alternativos sobre a eficácia do gráfico np_x. Através de uma extensa investigação, envolvendo derivações e análise numérica, foram determinados os cenários em que os gráficos np_x bilaterais e unilaterais superam o gráfico $\overline{X}$. Com esse pano de fundo, agora com o gráfico np_x corrigido, foram detalhadamente analisados os impactos na estimação dos parâmetros. Verificou-se que o desempenho é afetado principalmente quando o desvio padrão do processo é estimado (casos KU e UU) apresentando-se significativamente diferente do que é nominalmente esperado se a estimação dos parâmetros não for levada em consideração na concepção do gráfico. O número de amostras de referência necessárias para atenuar esse efeito torna a aplicação do gráfico np_x inviável. Dessa forma, ajustes no parâmetros UCL e k_w se fazem necessários, tais ajustes foram investigados e mostraram-se eficientes, possibilitando que o gráfico np_x ofereça desempenho igual ou melhor ao gráfico $\overline{X}$ utilizando números de amostras de referência usuais. Esta trabalho contribui significativamente para uma compreensão mais matizada das vantagens comparativas de diferentes metodologias de gráficos de controle, para dados de variáveis e atributos.

PALAVRAS CHAVE: Gráficos de controle por atributo; Estimação de parâmetros; Número médio de amostras; Taxa de alarme falso.

LIST OF FIGURES

Figure 1 Thesis Structure. 25

Figure 2 Control chart's general structure. 27

Figure 3 Publications related to variable control charts in the last 10 years in database Scopus and Web of science. 29

Figure 4 Publications related to attribute control charts in the last 10 years in database Scopusand Web of science. 31

Figure 5 Publications related to estimation of parameters and control charts in the last 10 years in database Scopus and Web of science. 33

Figure 6 Go/No Go Gauge 39

Figure 7 Comparison of ATS values of the np_x and $\overline{X}$ control charts 54

LIST OF TABLES

Table 1 – Charting parameters and incorrect ATS and EQL values of the control charts ($\tau = 370$) provided by Wu *et al.*(WU et al., 2009). . . . 41
Table 2 – Corrected results using the charting parameters used by Wu *et al.* (2009) and performance measured ATS and EQL calculated for the np_x chart and the $\overline{X}$chart ($\tau = 370$). . . . 42
Table 3 – UCL_{npx} and k_w values of the np_x control chart, charting parameters: $\tau = 370$, $n_{npx} = 6$, $h_{npx} = 1$. . . . 43
Table 4 – UCL_{npx} and k_w values of the np_x control chart, charting parameters: $\tau = 370$, $n_{npx} = 12$, $h_{npx} = 1$. . . . 43
Table 5 – UCL_{npx} and k_w values of the np_x control chart, charting parameters: $\tau = 370$, $n_{npx} = 6$, $h_{npx} = 0.5$. . . . 44
Table 6 – Charting parameters and ATS values of the control charts ($\tau = 370$). . . . 44
Table 7 – Optimum charting parameters and performance measured ATS and EQL of the control charts ($\tau = 370$). . . . 45
Table 8 – Charting parameters and ATS values of the control charts ($\tau = 370$). . . . 48
Table 9 – The minimum sample size for the two-sided np_x chart n_{npx} which guarantee a $EQL_{npx} \leq EQL_{\overline{X}}$, $ATS_0 = 370$ varying the $n_{\overline{X}}$ from 2 to 15 for a sampling interval of $h = 1$ for both charts. . . . 49
Table 10 – Charting parameters and performance measured ATS and EQL calculated for the np_x and $\overline{X}$ charts one-sided ($\tau = 370$). . . . 50
Table 11 – The minimum sample size for the one-sided np_x chart n_{npx} which guarantee a $EQL_{npx} \leq EQL_{\overline{X}}$, $ATS_0 = 370$ varying the $n_{\overline{X}}$ from 2 to 15 for a sampling interval of $h = 1$ for both charts. . . . 51
Table 12 – Charting parameters and performance measured ATS and EQL calculated for the np_x chart and the $\overline{X}$ chart ($\tau = 370$). . . . 54
Table 13 – Simulation of $100,000$ $CATS_0$ considering the mean is estimated and the standard deviation is known ($n = 38$, $UCL_{np_x} = 9$ and $k_w = 1.6599319$). . . . 58
Table 14 – Simulation of $100,000$ $CEQL$ considering the mean is estimated and the standard deviation is known ($n = 38$, $UCL_{np_x} = 9$ and $k_w = 1.6599319$). . . . 58
Table 15 – Comparison of ATS of the $\overline{X}$ chart and the two-sided np_x chart when parameters are known and when the mean is estimated ($\tau = 370$). . . . 59
Table 16 – Simulation of $CATS_0$ when the standard deviation is estimated ($n = 38$, $UCL_{np_x} = 9$ and $k_w = 1.6599319$). . . . 63
Table 17 – Simulation of $CEQL$ when the standard deviation is estimated ($n = 38$, $UCL_{np_x} = 9$ and $k_w = 1.6599$). . . . 63
Table 18 – Comparison of ATS of the $\overline{X}$ chart and the two-sided np_x chart when parameters are known and when the standard deviation is estimated ($\tau = 370$). . . . 64

Table 19 – Simulation of $CATS_0$ when the mean and the standard deviation are estimated ($n = 38, UCL_{np_x} = 9$ and $k_w = 1.6599319$). 68
Table 20 – Simulation of $CEQL$ when the mean and the standard deviation are estimated ($n = 38, UCL_{np_x} = 9$ and $k_w = 1.6599$). 68
Table 21 – Comparison of ATS of the $\overline{X}$ chart and the two-sided np_x chart when parameters are known and when the mean and the standard deviation is estimated ($\tau = 370$). . 69
Table 22 – k_w and UCL which satisfy the constraint $ATS_0 = \tau = 370$ for the np_x chart when the mean is estimated and the standard deviation is known for different number of samples m. 72
Table 23 – Comparison of ATS of the $\overline{X}$ chart and the two-sided np_x chart when parameters are known and when the mean is estimated ($\tau = 370$). 73
Table 24 – k_w and UCL which satisfy the constraint $ATS_0 = \tau = 370$ for the np_x chart when the mean is estimated and the standard deviation is known for different number of samples m. 75
Table 25 – Comparison of ATS of the $\overline{X}$ chart and the two-sided np_x chart when parameters are known and when the standard deviation is estimated ($\tau = 370$). 76
Table 26 – k_w and UCL which satisfy the constraint $ATS_0 = \tau = 370$ for the np_x chart when the mean is estimated and the standard deviation is known for different number of samples m. 77
Table 27 – Comparison of ATS of the $\overline{X}$ chart and the two-sided np_x chart when parameters are known and when both the mean and the standard deviation are estimated ($\tau = 370$). 78
Table 28 – k_w and UCL which satisfy the constraint $ATS_0 = \tau = 370$ for the np_x chart when the mean is estimated and the standard deviation is known for $n = 5, h = 1$ and different number of samples m. 91
Table 29 – EQL for all combinations of k_w and UCL which satisfy the constraint $ATS_0 = \tau = 370$ for the np_x chart when the mean is estimated and the standard deviation is known for $n = 5$ and $h = 1$. 91
Table 30 – k_w and UCL which satisfy the constraint $ATS_0 = \tau = 370$ for the np_x chart when the mean is estimated and the standard deviation is known for $n = 10, h = 1$ and different number of samples m. 91
Table 31 – EQL for all combinations of k_w and UCL which satisfy the constraint $ATS_0 = \tau = 370$ for the np_x chart when the mean is estimated and the standard deviation is known for $n = 10$ and $h = 1$. 92
Table 32 – k_w and UCL which satisfy the constraint $ATS_0 = \tau = 370$ for the np_x chart when the mean is estimated and the standard deviation is known for $n = 15, h = 1$ and different number of samples m. 92
Table 33 – EQL for all combinations of k_w and UCL which satisfy the constraint $ATS_0 = \tau = 370$ for the np_x chart when the mean is estimated and the standard deviation is known for $n = 15$ and $h = 1$. 93

Table 34 – k_w and UCL which satisfy the constraint $ATS_0 = \tau = 370$ for the np_x chart when the mean is estimated and the standard deviation is known for $n = 20, h = 1$ and different number of samples m. 93
Table 35 – EQL for all combinations of k_w and UCL which satisfy the constraint $ATS_0 = \tau = 370$ for the np_x chart when the mean is estimated and the standard deviation is known for $n = 20$ and $h = 1$. 94
Table 36 – k_w and UCL which satisfy the constraint $ATS_0 = \tau = 370$ for the np_x chart when the mean is estimated and the standard deviation is known for $n = 25, h = 1$ and different number of samples m. 95
Table 37 – EQL for all combinations of k_w and UCL which satisfy the constraint $ATS_0 = \tau = 370$ for the np_x chart when the mean is estimated and the standard deviation is known for $n = 25$ and $h = 1$. 96
Table 38 – k_w and UCL which satisfy the constraint $ATS_0 = \tau = 370$ for the np_x chart when the mean is estimated and the standard deviation is known for $n = 38, h = 1$ and different number of samples m. 97
Table 39 – EQL for all combinations of k_w and UCL which satisfy the constraint $ATS_0 = \tau = 370$ for the np_x chart when the mean is estimated and the standard deviation is known for $n = 38$ and $h = 1$. 98
Table 40 – k_w and UCL which satisfy the constraint $ATS_0 = \tau = 370$ for the np_x chart when the mean is estimated and the standard deviation is known for $n = 5, h = 0.5$ and different number of samples m. 99
Table 41 – EQL for all combinations of k_w and UCL which satisfy the constraint $ATS_0 = \tau = 370$ for the np_x chart when the mean is estimated and the standard deviation is known for $n = 5$ and $h = 0.5$. 99
Table 42 – k_w and UCL which satisfy the constraint $ATS_0 = \tau = 370$ for the np_x chart when the mean is estimated and the standard deviation is known for $n = 10, h = 0.5$ and different number of samples m. 99
Table 43 – EQL for all combinations of k_w and UCL which satisfy the constraint $ATS_0 = \tau = 370$ for the np_x chart when the mean is estimated and the standard deviation is known for $n = 10$ and $h = 0.5$. 100
Table 44 – k_w and UCL which satisfy the constraint $ATS_0 = \tau = 370$ for the np_x chart when the mean is estimated and the standard deviation is known for $n = 15, h = 0.5$ and different number of samples m. 100
Table 45 – EQL for all combinations of k_w and UCL which satisfy the constraint $ATS_0 = \tau = 370$ for the np_x chart when the mean is estimated and the standard deviation is known for $n = 15$ and $h = 0.5$. 101
Table 46 – k_w and UCL which satisfy the constraint $ATS_0 = \tau = 370$ for the np_x chart when the mean is estimated and the standard deviation is known for $n = 20, h = 0.5$ and different number of samples m. 101

Table 47 – EQL for all combinations of k_w and UCL which satisfy the constraint $ATS_0 = \tau = 370$ for the np_x chart when the mean is estimated and the standard deviation is known for $n = 20$ and $h = 0.5$. 102
Table 48 – k_w and UCL which satisfy the constraint $ATS_0 = \tau = 370$ for the np_x chart when the mean is estimated and the standard deviation is known for $n = 25$, $h = 0.5$ and different number of samples m. 103
Table 49 – EQL for all combinations of k_w and UCL which satisfy the constraint $ATS_0 = \tau = 370$ for the np_x chart when the mean is estimated and the standard deviation is known for $n = 25$ and $h = 0.5$. 104
Table 50 – k_w and UCL which satisfy the constraint $ATS_0 = \tau = 370$ for the np_x chart when the mean is known and the standard deviation is estimated for $n = 5$, $h = 1$ and different number of samples m. 109
Table 51 – EQL for all combinations of k_w and UCL which satisfy the constraint $ATS_0 = \tau = 370$ for the np_x chart when the mean is known and the standard deviation is estimated for $n = 5$ and $h = 1$. 109
Table 52 – k_w and UCL which satisfy the constraint $ATS_0 = \tau = 370$ for the np_x chart when the mean is known and the standard deviation is estimated for $n = 10$, $h = 1$ and different number of samples m. 109
Table 53 – EQL for all combinations of k_w and UCL which satisfy the constraint $ATS_0 = \tau = 370$ for the np_x chart when the mean is known and the standard deviation is estimated for $n = 10$ and $h = 1$. 110
Table 54 – k_w and UCL which satisfy the constraint $ATS_0 = \tau = 370$ for the np_x chart when the mean is known and the standard deviation is estimated for $n = 15$, $h = 1$ and different number of samples m. 110
Table 55 – EQL for all combinations of k_w and UCL which satisfy the constraint $ATS_0 = \tau = 370$ for the np_x chart when the mean is known and the standard deviation is estimated for $n = 15$ and $h = 1$. 111
Table 56 – k_w and UCL which satisfy the constraint $ATS_0 = \tau = 370$ for the np_x chart when the mean is known and the standard deviation is estimated for $n = 20$, $h = 1$ and different number of samples m. 111
Table 57 – EQL for all combinations of k_w and UCL which satisfy the constraint $ATS_0 = \tau = 370$ for the np_x chart when the mean is known and the standard deviation is estimated for $n = 20$ and $h = 1$. 112
Table 58 – k_w and UCL which satisfy the constraint $ATS_0 = \tau = 370$ for the np_x chart when the mean is known and the standard deviation is estimated for $n = 25$, $h = 1$ and different number of samples m. 113
Table 59 – EQL for all combinations of k_w and UCL which satisfy the constraint $ATS_0 = \tau = 370$ for the np_x chart when the mean is known and the standard deviation is estimated for $n = 25$ and $h = 1$. 114

Table 60 – k_w and UCL which satisfy the constraint $ATS_0 = \tau = 370$ for the np_x chart when the mean is known and the standard deviation is estimated for $n = 38, h = 1$ and different number of samples m. 115
Table 61 – EQL for all combinations of k_w and UCL which satisfy the constraint $ATS_0 = \tau = 370$ for the np_x chart when the mean is known and the standard deviation is estimated for $n = 38$ and $h = 1$. 116
Table 62 – k_w and UCL which satisfy the constraint $ATS_0 = \tau = 370$ for the np_x chart when the mean is known and the standard deviation is estimated for $n = 5, h = 0.5$ and different number of samples m. 117
Table 63 – EQL for all combinations of k_w and UCL which satisfy the constraint $ATS_0 = \tau = 370$ for the np_x chart when the mean is known and the standard deviation is estimated for $n = 5$ and $h = 0.5$. 117
Table 64 – k_w and UCL which satisfy the constraint $ATS_0 = \tau = 370$ for the np_x chart when the mean is known and the standard deviation is estimated for $n = 10, h = 0.5$ and different number of samples m. 117
Table 65 – EQL for all combinations of k_w and UCL which satisfy the constraint $ATS_0 = \tau = 370$ for the np_x chart when the mean is known and the standard deviation is estimated for $n = 10$ and $h = 0.5$. 118
Table 66 – k_w and UCL which satisfy the constraint $ATS_0 = \tau = 370$ for the np_x chart when the mean is known and the standard deviation is estimated for $n = 15, h = 0.5$ and different number of samples m. 118
Table 67 – EQL for all combinations of k_w and UCL which satisfy the constraint $ATS_0 = \tau = 370$ for the np_x chart when the mean is known and the standard deviation is estimated for $n = 15$ and $h = 0.5$. 119
Table 68 – k_w and UCL which satisfy the constraint $ATS_0 = \tau = 370$ for the np_x chart when the mean is known and the standard deviation is estimated for $n = 20, h = 0.5$ and different number of samples m. 119
Table 69 – EQL for all combinations of k_w and UCL which satisfy the constraint $ATS_0 = \tau = 370$ for the np_x chart when the mean is known and the standard deviation is estimated for $n = 20$ and $h = 0.5$. 120
Table 70 – k_w and UCL which satisfy the constraint $ATS_0 = \tau = 370$ for the np_x chart when the mean is known and the standard deviation is estimated for $n = 25, h = 0.5$ and different number of samples m. 121
Table 71 – EQL for all combinations of k_w and UCL which satisfy the constraint $ATS_0 = \tau = 370$ for the np_x chart when the mean is known and the standard deviation is estimated for $n = 25$ and $h = 0.5$. 122
Table 72 – k_w and UCL which satisfy the constraint $ATS_0 = \tau = 370$ for the np_x chart when the mean and the standard deviation are estimated for $n = 5, h = 1$ and different number of samples m. 126

Table 73 – EQL for all combinations of k_w and UCL which satisfy the constraint $ATS_0 = \tau = 370$ for the np_x chart when the mean and the standard deviation are estimated for $n = 5$ and $h = 1$. . . . 127
Table 74 – k_w and UCL which satisfy the constraint $ATS_0 = \tau = 370$ for the np_x chart when the mean and the standard deviation are estimated for $n = 10, h = 1$ and different number of samples m. . . . 127
Table 75 – EQL for all combinations of k_w and UCL which satisfy the constraint $ATS_0 = \tau = 370$ for the np_x chart when the mean and the standard deviation are estimated for $n = 10$ and $h = 1$. . . . 127
Table 76 – k_w and UCL which satisfy the constraint $ATS_0 = \tau = 370$ for the np_x chart when the mean and the standard deviation are estimated for $n = 15, h = 1$ and different number of samples m. . . . 128
Table 77 – EQL for all combinations of k_w and UCL which satisfy the constraint $ATS_0 = \tau = 370$ for the np_x chart when the mean and the standard deviation are estimated for $n = 15$ and $h = 1$. . . . 128
Table 78 – k_w and UCL which satisfy the constraint $ATS_0 = \tau = 370$ for the np_x chart when the mean and the standard deviation are estimated for $n = 20, h = 1$ and different number of samples m. . . . 129
Table 79 – EQL for all combinations of k_w and UCL which satisfy the constraint $ATS_0 = \tau = 370$ for the np_x chart when the mean and the standard deviation are estimated for $n = 20$ and $h = 1$. . . . 130
Table 80 – k_w and UCL which satisfy the constraint $ATS_0 = \tau = 370$ for the np_x chart when the mean and the standard deviation are estimated for $n = 25, h = 1$ and different number of samples m. . . . 131
Table 81 – EQL for all combinations of k_w and UCL which satisfy the constraint $ATS_0 = \tau = 370$ for the np_x chart when the mean and the standard deviation are estimated for $n = 25$ and $h = 1$. . . . 132
Table 82 – k_w and UCL which satisfy the constraint $ATS_0 = \tau = 370$ for the np_x chart when the mean and the standard deviation are estimated for $n = 38, h = 1$ and different number of samples m. . . . 133
Table 83 – EQL for all combinations of k_w and UCL which satisfy the constraint $ATS_0 = \tau = 370$ for the np_x chart when the mean and the standard deviation are estimated for $n = 38$ and $h = 1$. . . . 134
Table 84 – k_w and UCL which satisfy the constraint $ATS_0 = \tau = 370$ for the np_x chart when the mean and the standard deviation are estimated for $n = 5, h = 0.5$ and different number of samples m. . . . 135
Table 85 – EQL for all combinations of k_w and UCL which satisfy the constraint $ATS_0 = \tau = 370$ for the np_x chart when the mean and the standard deviation are estimated for $n = 5$ and $h = 0.5$. . . . 135

Table 86 – k_w and UCL which satisfy the constraint $ATS_0 = \tau = 370$ for the np_x chart when the mean and the standard deviation are estimated for $n = 10$, $h = 0.5$ and different number of samples m. . . . 135
Table 87 – EQL for all combinations of k_w and UCL which satisfy the constraint $ATS_0 = \tau = 370$ for the np_x chart when the mean and the standard deviation are estimated for $n = 10$ and $h = 0.5$. . . . 136
Table 88 – k_w and UCL which satisfy the constraint $ATS_0 = \tau = 370$ for the np_x chart when the mean and the standard deviation are estimated for $n = 15$, $h = 0.5$ and different number of samples m. . . . 136
Table 89 – EQL for all combinations of k_w and UCL which satisfy the constraint $ATS_0 = \tau = 370$ for the np_x chart when the mean and the standard deviation are estimated for $n = 15$ and $h = 0.5$. . . . 137
Table 90 – k_w and UCL which satisfy the constraint $ATS_0 = \tau = 370$ for the np_x chart when the mean and the standard deviation are estimated for $n = 20$, $h = 0.5$ and different number of samples m. . . . 137
Table 91 – EQL for all combinations of k_w and UCL which satisfy the constraint $ATS_0 = \tau = 370$ for the np_x chart when the mean and the standard deviation are estimated for $n = 20$ and $h = 0.5$. . . . 138
Table 92 – k_w and UCL which satisfy the constraint $ATS_0 = \tau = 370$ for the np_x chart when the mean and the standard deviation are estimated for $n = 25$, $h = 0.5$ and different number of samples m. . . . 139
Table 93 – EQL for all combinations of k_w and UCL which satisfy the constraint $ATS_0 = \tau = 370$ for the np_x chart when the mean and the standard deviation are estimated for $n = 25$ and $h = 0.5$. . . . 140

LIST OF ABBREVIATIONS AND ACRONYMS

ATS	Unconditional Average Time to a Signal: ATS = E(CATS)
CATS	Conditional Average Time to a Signal
CFAR	Conditional False Alarm Rate
EQL	Extra Quadratic Loss
CEQL	Conditional Extra Quadratic Loss
Case KK	Case where both the mean and standard deviation are known
Case KU	Case where the mean is known, and the standard deviation is unknown
Case UK	Case where the mean is unknown, and the standard deviation is known
Case UU	Case where both the mean and standard deviation are unknown
SPC	Statistical Process Control
UCL	Upper Control Limit
IC	In Control

LIST OF SYMBOLS

$\overline{X}$	Sample mean
$\overline{\overline{X}}$	The grand mean of the m Phase I samples estimator
k_w	Coeficient
μ	Process mean
μ	In-control process mean
σ	Process standard deviation
σ_0	In-Control process standard deviation
Φ	Cumulative distribution function of a Standard Normal Random Variable
δ	Scaled shift of the mean
τ	Letra Grega Tau
m	Number of Phase I samples
n	Sample size
Y	Chi-square random variable
Z	Standard Normal Random Variable
μ	In-control process mean

CONTENTS

1 INTRODUCTION . . . **21**
1.1 Contextualization . . . 21
1.2 Research Objectives . . . 23
1.3 Methodology . . . 23
1.4 Thesis structure . . . 24

2 CONTROL CHART . . . **26**
2.1 Control charts with estimated parameters . . . 31

3 THE np_x CONTROL CHART WITH KNOWN PARAMETERS . . . **38**
3.1 np_x Control Chart . . . 38
3.2 Different Results . . . 42
3.3 The optimal design for the np_x chart for the two-sided design . . . 46
3.4 Implementation and design of the np_x charts for the one-sided case . . . 50
3.5 Practical Example . . . 52

4 THE np_x CONTROL CHART WITH ESTIMATED PARAMETERS . . . **55**
4.1 Mean standard unknown but variance specified (UK) . . . 55
4.2 Mean standard specified but variance unknown (KU) . . . 60
4.3 Mean and variance both unknown (UU) . . . 65

5 THE np_x CONTROL CHART OPTIMIZATION . . . **71**
5.1 Adjustment in Case UK . . . 71
5.2 Adjustment in Case KU . . . 74
5.3 Adjustment in Case UU . . . 74

6 CONCLUSION . . . **79**
6.1 SUGGESTIONS FOR FUTURE WORK . . . 80

BIBLIOGRAPHY . . . **81**

APPENDIX A – OPTIMIZATION OF THE np_x CONTROL CHART . . . **85**
A.1 Find the optimal values of UCL and k_w . . . 85
A.2 The np_x chart performance using optimal values of UCL and k_w . . . 86

APPENDIX B – np_x CHART: UK CASE . . . **87**
B.1 Conditional approach . . . 87
B.2 Unconditional approach - derivations . . . 88
B.3 Adjustment - Codes in R . . . 89

B.4 Adjusted parameters UCL and k_w 90

APPENDIX C – np_x CHART: KU CASE 105

C.1 conditional approach 105

C.2 UNCONDITIONAL approach - DERIVATIONS 106

C.3 Adjustment - Codes in R 107

C.4 Adjusted parameters UCL and k_w 108

APPENDIX D – np_x CHART: UU CASE 123

D.1 conditional approach 123

D.2 UNCONDITIONAL approach - DERIVATIONS 124

D.3 Adjustment - Codes in R 125

D.4 Adjusted parameters UCL and k_w 126

ANNEX A – PAPER UNDER REVIEW IN THE JOURNAL QUALITY AND RELIABILITY ENGINEERING INTERNATIONAL 141

1 INTRODUCTION

This chapter provides context for the research by outlining its motivation, research questions, objectives, and the structure of the document.

1.1 CONTEXTUALIZATION

The quality of processes is undoubtedly of paramount importance in the field of business, so controlling and improving quality is an excellent strategy used by companies. The statistical process control area (SPC) provides a series of tools used to improve the process stability and reduce the variability. Control charts are one of the oldest tools in the area, used to monitor and detect changes in the process in order to keep it stable (AYKROYD; LEIVA; RUGGERI, 2019).

Processes based on qualitative characteristics, that is, by attribute, cannot be monitored with traditional control charts, as the popular $\overline{X}$, for example. In this cases, it is necessary to apply an attribute control chart: p, np and c are the most used to evaluate processes where the analyzed characteristics are difficult to be measured in a numeral scale (CASTAGLIOLA et al., 2014).

Monitoring processes through attributes offers advantages in terms of time and cost efficiency. Typically, the equipment required for attribute monitoring is less expensive than that used for variable monitoring. However, according to Montgomery, attribute monitoring is generally considered less efficient for tracking specific characteristics compared to variable monitoring, (MONTGOMERY, 2012). Therefore, it is essential to develop methods that enhance the reliability of attribute control charts for monitoring variable processes.

In 2009, Wu *et al.* developed an attribute control chart called np_x which employs an attribute inspection to monitor the process mean using a gage to classify the items as conforming or nonconforming (WU et al., 2009).

Due to the simplicity of the np_x control chart, many researchers have focused on its improvement. For example, Ho and Costa (HO; COSTA, 2011) examined the effects of wandering process means on the performance of the np_x chart. Sampaio, Ho, and Medeiros (SAMPAIO; HO; MEDEIROSC, 2014) proposed a combined np_x-$\overline{X}$ chart to monitor the process mean in a two-stage sampling situation, where the sample is divided into two sub-samples: n_1 and n_2. First, the sub-sample of size n_1 is evaluated by attribute inspection. If it is out of control, then the sub-sample n_2 is evaluated, and the mean is calculated. The process is considered out of control only if the mean exceeds the limit. For monitoring process variability based on the np_x chart, see Ho and Quinino (HO; QUININO, 2013), and da Silva et al. (SILVA; HO; QUININO, 2022b). Quinino et al. (QUININO; CRUZ; QUININO, 2021) proposed a control chart that combines attribute and variable inspections to monitor the process mean. In their method, the first sample undergoes variable inspection using the $\overline{X}$ chart, followed by k samples subjected to attribute inspection using the np_x chart. If neither chart signals a shift in the process mean, the cycle restarts with one sample for variable inspection and k samples for attribute inspection, continuing until a shift is detected by either chart. Additionally, Zhou et al. (ZHOU; YE; ZHENG, 2022) introduced the np_{CEV} chart to monitor the process mean using the conditional expected value (CEV) weighting method. Da Silva et al. (SILVA; HO; QUININO, 2022a) proposed a control chart for monitoring the process mean based on attribute inspection with sequential sample sizes.

The concept of using an attribute chart to monitor a continuous variable, as proposed by Wu *et al.* (WU et al., 2009), has made a notable impact on process monitoring literature. This drew attention to the fact that all work carried out to date considered the process parameters as known. Typically, these parameters are unknown and must be estimated from m historical samples, each of size n, collected when the process is presumably in control. This is known as Phase I analysis in Statistical Process Control (SPC). For an overview of Phase I, see Chakraborti *et al.* (CHAKRABORTI; HUMAN; GRAHAM, 2008) and Jones-Farmer *et al.* (JONES-FARMER et al., 2014).

Subsequently, the chart's control limits are established using these estimates for prospective process monitoring (Phase II), where samples (also of size n) are collected at regular intervals h. These samples are classified as conforming or nonconforming using a gauge, for example, and the number of nonconforming units in each sample is compared with the control limits. If the plotting statistic falls outside the control limits, it indicates a high probability that the process mean has changed, requiring the manager to identify and correct the potential problem.

Most research on the development and performance evaluation of Phase II control charts assumes that the in-control process parameters are known. This assumption simplifies both the design and performance evaluation of the chart, as the run length (the number of samples until an alarm) follows the well-known geometric distribution.

In practice, however, these parameters are usually unknown and must be estimated during Phase I. When estimates are used instead of known parameters, their variability can leads to chart performance that deviates from that of charts designed with known parameters. For example, the run length no longer follows a geometric distribution, and the actual false alarm rate (FAR) may exceed the desired nominal rate. Frequent false alarms result to unnecessary costs, such as wasted time for managers investigating non-existent issues, which disrupts operations and reduces productivity. Therefore, when designing a control chart, it is crucial for managers to consider the impact of parameter estimation on chart performance to avoid unnecessary costs.

When designing control charts with estimated parameters, four cases are possible:

1. Both the in-control process mean (μ_0) and standard deviation (σ_0) are unknown. This situation, known as "Case UU" (Unknown mean and Unknown standard deviation), is the most common in practice and considered the standard unknown case.

2. The process mean is known, but the standard deviation is unknown. This is "Case KU" (Known mean, Unknown standard deviation), where the standard deviation must be estimated during Phase I.

3. The in-control process mean is unknown and needs to be estimated during Phase I, while the standard deviation is known. This is "Case UK" (Unknown mean, Known standard deviation).

4. Both the in-control process mean and standard deviation are known. This is "Case KK" (Known mean and Known standard deviation), the simplest and most desirable case. Here, the perfor-

mance of the control chart can be easily studied and measured since the run length distribution is known.

The np_x chart is commonly used in quality control to monitor processes over time. Using estimated parameters instead of true parameters can significantly impact the performance of the np_x chart. Estimating parameters introduces uncertainty, which can affect the chart's sensitivity to detecting process shifts and its overall effectiveness.

Building upon this background, the current study will explore the impact on the average time to signal (for both in-control and out-of-control conditions) and extra quadratic loss considering a shift range in the process mean for the UU, KU, and UK scenarios of the np_x chart. This research aims to elucidate the compromises inherent in these scenarios and offer valuable insights for practitioners in quality control, emphasizing practical considerations for real-world applications.

1.2 RESEARCH OBJECTIVES

Clearly defined research objectives provide a roadmap for the entire research process, guiding decisions on study design, data collection methods, and analysis techniques (CRESWELL, 2008).

The general objective of this thesis is to investigate the effects of estimation of parameters (μ and σ) on an attribute control chart used to monitor the mean of a continuous variable.

In order to achieve the general objective of this research, this thesis has the following specific objectives

1. Review the available literature on attribute control charts used to monitor the mean of a continuous variable to highlight gaps in the literature;
2. Replicate the results of the control chart by attributes np_x to monitor the mean of a continuous variable.
3. Obtain the properties of the np_x control chart in the condition in which the parameters (μ and σ) are estimated.
4. Evaluate the performance of the np_x chart under the effect of parameters estimation;
5. Evaluate optimizing the np_x control chart considering the estimated parameters;
6. Propose future studies for the continuation of the study and contribution to the research field.

1.3 METHODOLOGY

The methodology employed in this research plays a pivotal role in ensuring the credibility and validity of the findings. As Creswell (CRESWELL, 2008) aptly stated, the methodology serves as the "blueprint" for the study, guiding the researcher in the systematic investigation of the research questions or objectives. By meticulously designing and implementing a robust methodology, this study aims to uphold the principles of rigor and transparency in academic research.

The initial stage involved conducting a preliminary examination of control chart literature to identify gaps requiring further study. Specifically focusing on control charts with estimated parameters, relevant papers were examined to understand the current state of research, including best practices, findings, and limitations from previous studies. Notably, it was found that no prior research had considered the estimation of parameters for np_x control chart.

Based on the initial step, certain decisions were made. It was presumed that the designed control chart would be utilized for normally distributed processes. The unconditional approach was chosen to estimate the process parameters (μ and σ), given its wide applicability and ability to represent a variety of users. Recognizing the estimated parameters as random variables following a normal distribution, the model requires adjustments to accommodate this randomness and the associated variability.

The effect of estimating limits on the np_x control chart's performance is examined separately for cases where either one or both of the process parameters, the mean and the variance are unknown. Precise formulas are obtained for the distribution of run lengths, the Average Run Length (ARL), and the extra quadratic loss (EQL). We assess these exact $EQLs$ and $ARLs$ across various scenarios. These findings offer practical understanding of how the np_x chart performs when parameters are estimated. Additionally, these derivations can serve as illustrative instances of probability and distribution calculations through conditioning.

The optimization of the np_X chart is also proposed. After the optimization study, an illustrative simulation study is presented to demonstrate the use of optimized parameters.

To perform all the calculations mentioned above, programs were written in the R language and are provided in the appendix section.

1.4 THESIS STRUCTURE

This thesis is structured into six chapters. Figure 1 shows the proposed thesis' structure and the alignment of the chapters with the specific objectives of the Thesis.

Chapter 1 presents the contextualization and research question, research motivation, research objectives, and the methodological release.

Chapter 2 presents the relevant theoretical background. The fundamentals of control charts and the types of control charts are presented. An overview of the literature on control charts with estimated parameters is also presented.

Chapter 3 presents the design of the np_x chart considering the parameters are known. The divergences on results and necessaries corrections are also presented.

Chapter 4 presents the precise formulas for the distribution of run lengths, the Average Run Length (ARL), and the extra quadratic loss (EQL) for each cases where one or both parameters are estimated.

Chapter 5 presents the proposal for the np_x control chart optimization and an illustrative simulation example.

Finally, Chapter 6 summarize this thesis's main contributions and provide suggestions for further research involving related topics.

Figure 1 – Thesis Structure.

Chapter 1 – Introduction	Contextualization
Chapter 2 – Control Chart	Specific Objective 1
Chapter 3 – The np_x control chart with known parameters	Specific Objective 2
Chapter 4 – The np_x control chart with estimated parameters	Specific Objectives 3 and 4
Chapter 5 – The np_x control chart optimization	Specific Objective 5
Chapter 6 – Conclusions	Specific Objective 6

source: Elaborated by the author.

2 CONTROL CHART

The quality of a product or service is determined by its ability to satisfy users' needs. In large-scale industries, production is intricate and spans multiple stages, each managed by different operators who generally lack direct interaction with customers. Thus, it is imperative to translate customer needs into detailed product specifications to ensure that the final product adheres to these specifications and meets customer expectations. To ensure compliance with these specifications, a reliable production process control system is vital for maintaining product quality. Quality control can be executed systematically, by inspecting all parts, or statistically, by collecting and analyzing samples. Given that quality control is an expensive procedure that does not add direct value to the product, it is beneficial to reduce the number of parts inspected without compromising process reliability. Statistical control facilitates the monitoring of all product characteristics through samples, thereby ensuring overall quality while enhancing efficiency (GODINA et al., 2018).

Statistical Process Control (SPC) encompasses a set of statistical tools designed to monitor and enhance the quality of production processes. By taking samples at various stages and analyzing the data statistically, SPC enables informed decisions to ensure the process remains in control and the final product meets desired specifications. The application of SPC involves monitoring processes, identifying problem areas, optimizing processes, testing part reliability, and conducting various analytical operations. Both basic and advanced statistical quality control methods are utilized in SPC, including control charts, Pareto charts, and experimental design, among others. Consequently, SPC is extensively employed by professionals in academic research and engineering applications (GODINA et al., 2018).

The control chart is one of the oldest tools in the field, it is considered a powerful tool for monitoring and detecting changes in processes to maintain stability (HUSSAIN et al., 2020). The study of control charts began in 1920 in the United States of America, marking the formal inception of statistical quality control. Control charts continue to be extensively utilized in various industries, retaining their status as a robust tool for process monitoring and quality assurance.(WEESE et al., 2016).

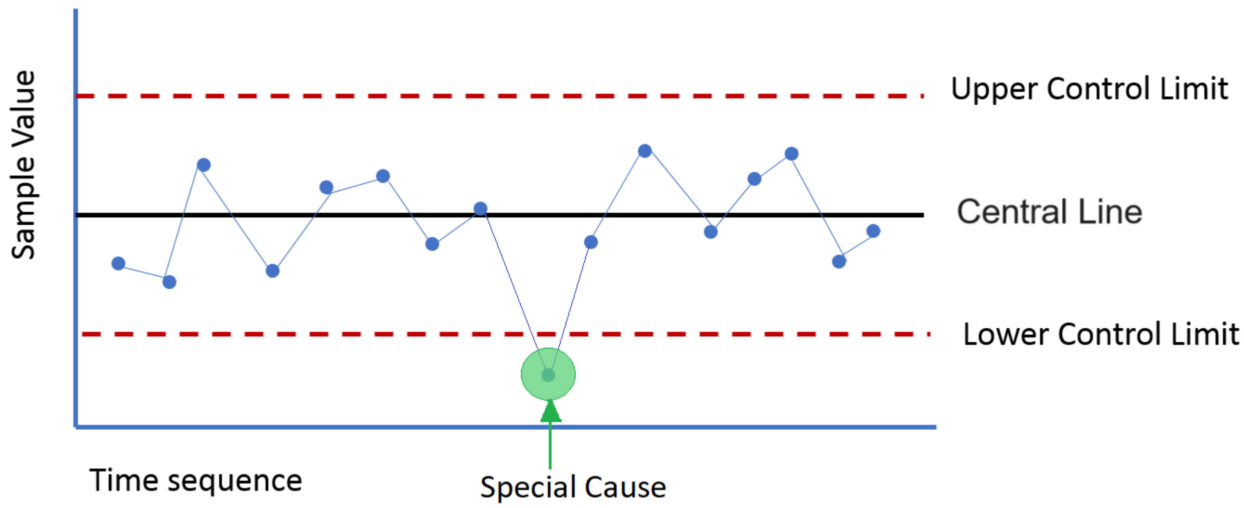
When collecting data at various stages of a process, it is common to observe disparities, meaning that a portion of variability will always be observed in a process. This variability can often be due to common causes, that is, inherent to the process itself. But they can also occur due to atypical factors, that is, due to special causes, representing sporadic deviations in their behavior. A process is deemed in control when it operates solely under the influence of common causes, reflecting natural variability. Conversely, the presence of a special cause signifies an out-of-control process. Process variability frequently is the main source of quality issues in industries and control charts are used for the purpose of detecting these assignable causes that affect process stability (CHAKRABORTI; GRAHAM, 2019).

There are four crucial parameters associated with control charts: the sample size (n), the number of samples (m), the sampling interval (h) and the control limit coefficient (k) (KEYVANDARIAN et al., 2017). Larger samples are beneficial for detecting smaller shifts but are associated with higher costs and time requirements, whereas smaller samples may suffice if significant shifts are expected.

In practice, variable charts often utilize approximately 30 samples with a size of 5. Additionally, the frequency of data collection is influenced by factors such as time constraints, costs, and anticipated shifts (CHAKRABORTI; GRAHAM, 2019).

Fundamentally, a control chart monitors the process, through the distribution presented by a variable that has an interesting quality characteristic, called the variable of interest.Control charts conventionally consist of three primary components: the Central Line (CL), the Upper Control Limit (UCL), and the Lower Control Limit (LCL) that are defined according to the specifications of the product. The central line denotes the target value, while the control limits establish the boundaries that, if surpassed, indicate an out-of-control event. Samples of size n are taken at regular intervals of time h. For each sample, the value of an appropriate statistic is determined, for example: the sample mean $\overline{X}$. Then these values are plotted on the control chart and if the variable presents a behavior that is not expected, the control chart will sign an alarm, indicating that the process is out of control The basic structure of a control chart is shown in Figure 2 (VACHáLEK et al., 2021).

Figure 2 – Control chart's general structure.



source: Elaborated by the author.

The upper and lower limits are not necessarily symmetrical to the central limit. The spacing in relation to the center line, CL, is established based on the natural variability of the X measurements of the product quality characteristic estimated with the process in control. A control chart's performance is evaluated by how well it can discriminate between process shifts due to random causes and those due to special causes. It involves its ability to detect rapidly process deviations and the ability to not signal when they are not present (MALEKI; AMIRI; CASTAGLIOLA, 2017).

In order to evaluate the control chart performance, it is defined two types of errors:

- The error type I occurs when the control chart signalizes special causes, but the process is in an in-control condition. The probability of Type I Error occur is denominated α.

- The error type II occurs when the control chart does not signalize a special cause, however the process is out of control. The probability of Type II Error occur is denominated β.

It is important to consider the distance between the upper and lower control limits from the central line. A smaller distance increases the probability of a Type I error (α), where a process is incorrectly identified as out-of-control. Conversely, larger distances between the central line and the control limits raise the probability of a Type II error (β), where an actual out-of-control process goes undetected. Thus, there is a trade-off: as the probability of a Type I error (α) increases, the probability of a Type II error (β) decreases, and vice versa (HUSSAIN et al., 2020). When considering practical implications, if the production of a significant number of nonconforming units incurs substantial costs, it is imperative to minimize Type II errors. In contrast, if halting the process for investigation entails greater expenses, minimizing Type I errors becomes paramount.

The earlier the control chart detects signals, the fewer nonconforming items are likely to be manufactured. Therefore, it is crucial to assess the effectiveness of control charts in promptly identifying these signals. It is possible and common to measure the performance of control charts by the average number of samples until an alarm (ARL), which represents the mean of a random variable known as run length (RL). The run length denotes the number of samples taken until an alarm signals an out-of-control process. RL follows a geometric distribution since samples are repeatedly taken until the first alarm occurs (the first success on a geometrical distribution) (CHAKRABORTI, 2007).

When the process is in control, $\delta = 0$ and the performance measurement is $ARL_0 = \frac{1}{\alpha}$. When the process is out of control, $ARL = \frac{1}{1-\beta}$. When a process is under control, it is desirable that the average run length (ARL_0) is large, in order to guarantee few false alarms. When a process is out of control, it is desirable that the average run length (ARL) is small, in order to ensure rapid detection of the special cause.

In order to increase the efficiency of control charts, some statistical specifications about the population under analysis are assumed. For example: independence or not of the observations and type of distribution presented. Therefore, it is essential to analyze the characteristics of the population to assume reliable statistical qualities and thus obtain a reliable result when applying the appropriate control chart. Data are often transformed to obtain a distribution that allows the use of traditional methods, however, this practice can reduce the detection power of the control chart. Preferably, real data should be used so that process monitoring is as close to reality as possible. The same must be considered regarding the independence of observations, because, if the data is self-correlated, there are suitable methods for this case that make it possible to work with real data instead of changing the frequency in order to reduce the relationship between them (AYKROYD; LEIVA; RUGGERI, 2019).

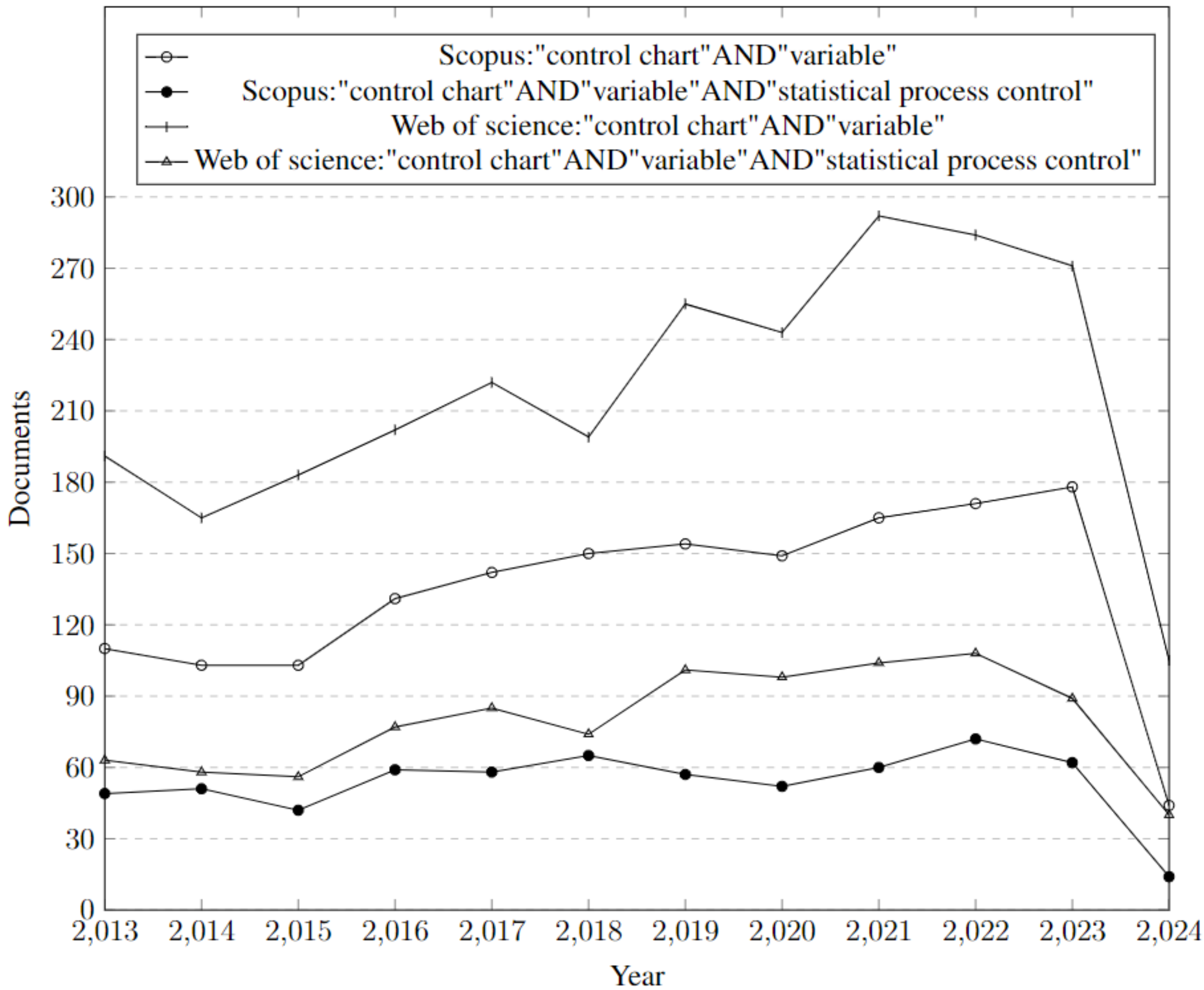
Control charts are primarily classified into two main categories based on the type of quality characteristic: variable control charts and attribute control charts. Variable control charts monitor quality characteristics that are measured on a continuous scale, such as length, temperature, and weight. In contrast, attribute control charts are used for discrete or countable quality characteristics, such as the number of nonconforming products. Additionally, depending on the number of quality characteristics being monitored, various univariate and multivariate control charts have been developed to differentiate between assignable and common causes of process variation (MALEKI; AMIRI; CASTAGLIOLA, 2018).

Variable control charts monitor continuous quality characteristics, such as the diameter of a hole or the length of a shaft. Traditional charts used for this purpose include $\overline{X}$, S, and R charts (COSTA; EPPRECHT; CARPINETTI, 2005).

The $\overline{X}$ chart is commonly employed to monitor the stability of the process mean due its simplicity and satisfactory performance. The main disadvantages are time consuming and cost to monitor a process using variable inspection (MONTGOMERY, 2012).

Several studies are carried out to increase the efficiency of control charts in detecting small special causes in processes. Combinations of traditional statistical techniques are used with more advanced approaches, such as: cumulative sums (CUSUM), exponentially weighted moving average (EWMA) and synthetic graphs. In practice, they are still rarely used because they are complex. Figure 3 presents publications in the last 10 years in the Scopus database containing the words "control charts", "variables" and "statistical process control".

Figure 3 – Publications related to variable control charts in the last 10 years in database Scopus and Web of science.



source: Adapted from Scopus and Web of science.

As previously discussed, control chart represents one of the most important tool in statistical process control. This assertion is underscored by the extensive research conducted in this domain, with over 100 publications per year recorded in the Scopus database over the past decade. This substantial body of literature highlights the significant relevance and importance of control charts as a fundamental tool in quality management and process control.

Initially, control charts were predominantly used in industrial production. However, their application has significantly expanded to fields such as business, education, and healthcare. In some instances, the characteristic of interest is difficult to measure on a numerical scale due to the complexity of measurement, the lack of appropriate equipment, or the destructive nature of the measurement process, among other factors. Under these circumstances, the characteristic of interest is categorical, meaning items are classified as either conforming or nonconforming based on specific criteria. This is common in visual inspections, dimensional inspections with gauges, or functional tests where outcomes are simply categorized as good or bad. Therefore, selecting the correct variable of interest is essential, and attribute data are frequently employed in these cases (AYKROYD; LEIVA; RUGGERI, 2019; CHUKHROVA; JOHANNSSEN, 2019).

Processes based on qualitative characteristics, that is, by attribute, cannot be monitored with traditional control charts ($\overline{X}$, S or R, for example). In these cases, the control charts called p, np and c are the most used to monitor the quality of the process (CASTAGLIOLA et al., 2014).

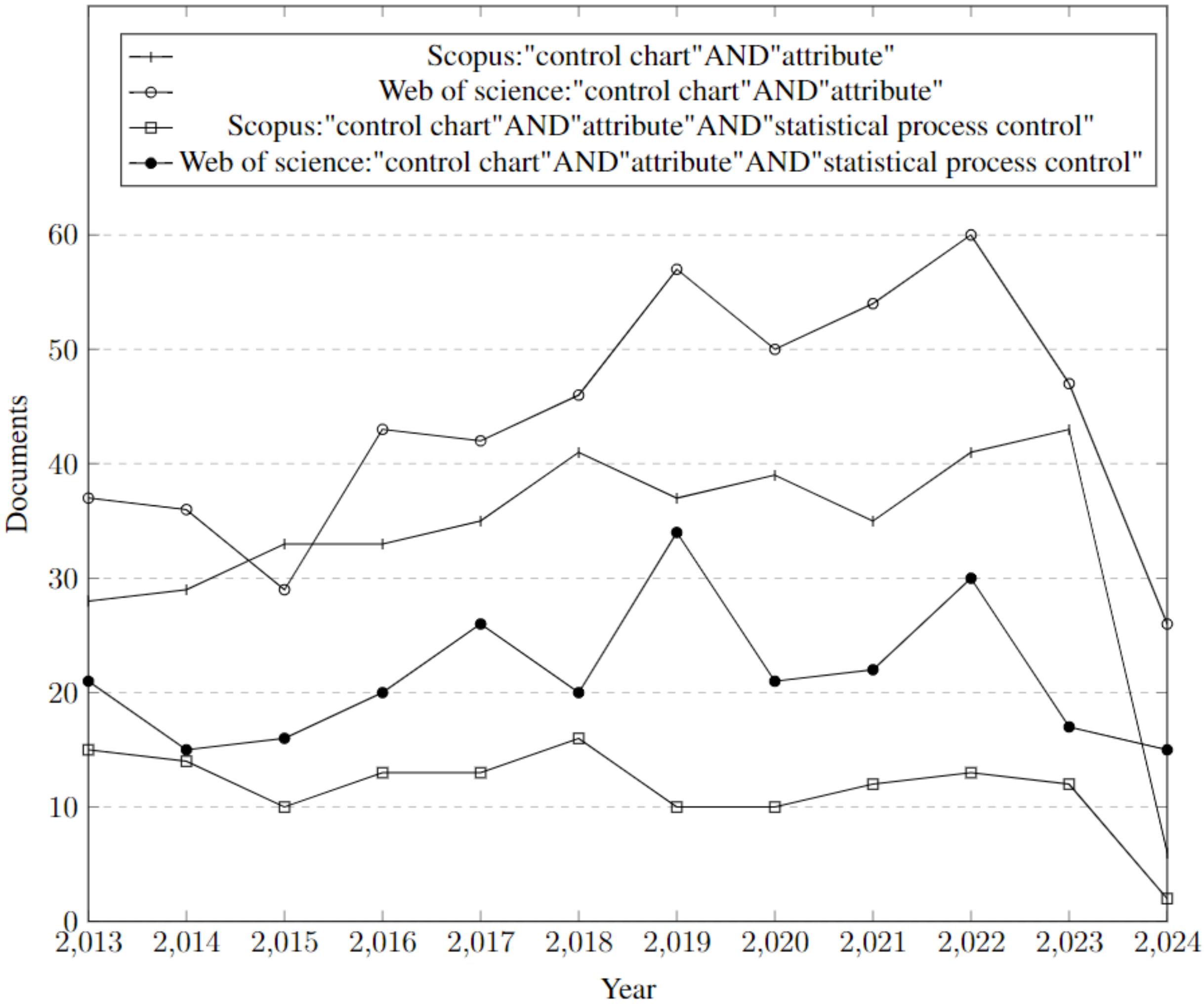
The discrete random variable X takes a value of 1 for nonconforming items and a value of 0 for conforming items. The quality of the process is measured based on how the sequence $X_1, X_2, X_3 \ldots X_t$ is distributed on a control chart. There is a wide variety of attribute control charts, but all are based on a few statistical models, as follows:

- Binomial Model (p, np, binomial CUSUM, etc.) – used to monitor nonconforming products;
- Bernoulli Model (Bernoulli CUSUM) – used to monitor nonconforming products when sampling is done individually;
- Poisson Model (c, D, u, etc.) – used to monitor nonconformities;
- Multinomial Model (multinomial CUSUM, multinomial EWMA, etc.) – used to monitor more than two attributes in a product, for example: conforming, nonconforming, acceptable, etc. (SHEPHERD; CHAMP; RIGDON, 2016; TIPLICA, 2015).

Attribute-based control charts offer several advantages over variable-based charts, including ease of design and implementation, lower costs, and reduced operational time (HO; QUININO, 2016). Consequently, research has been conducted to demonstrate the efficiency of using attribute control charts to monitor continuous variables. Notable studies in this area include *"Attribute control charts for monitoring the covariance matrix of bivariate processes"* (MACHADO; HO; COSTA, 2018), *"A synthetic control chart for monitoring the small shifts in a process mean based on an attribute inspection"* (ZHOU; LIU; ZHENG, 2020) and *"The Trinomial ATTRIVAR control chart"* (SIMõES; COSTA; MACHADO, 2020).

Figure 4 illustrates the progression of research concerning attribute control charts based on Scopus data base in the last ten years.

Figure 4 – Publications related to attribute control charts in the last 10 years in database Scopusand Web of science.



source: Adapted from Scopus and Web of science.

It is evident that the analysis and advancement of control charts designed to monitor processes using attributes have been growing. This may have been driven by the aforementioned advantages. However, when compared with research using variable control charts (Figure 2) we see that there is a significant gap, which means that there is much to be explored about the performance of control charts by attributes, especially when they are applied to monitor continuous variables.

2.1 CONTROL CHARTS WITH ESTIMATED PARAMETERS

Typically, population parameters (mean, variance, proportion, etc.) are assumed to be known for the development of studies in the field. However, in practice, these parameters are rarely known and must therefore be estimated based on a dataset provided by samples taken from the controlled process. This presents a challenge for research in studying the performance of control charts when parameters are estimated, aiming to obtain more reliable results through the progressive approximation of the reality of processes (WU; CASTAGLIOLA; KHOO, 2016).

The parameter estimation is the result of an iterative process, known as Phase I. The process begins by defining control limits based on a large amount of data (observations or subgroups); then, the sample points are plotted on the control chart. Sample points located outside the control limits are considered out of control and removed from the analysis. The process restarts by estimating new control limits from the remaining data, and the iteration continues until no sample points are located outside the control limits. In this phase, the remaining data is considered in control, and the final control limits can be constructed from this data (CHAKRABORTI; HUMAN; GRAHAM, 2008).

The performance of control charts when parameters are assumed to be known differs from when parameters are estimated, primarily due to the variability attributed to the estimators during Phase I, as previously described. It is known that estimating both quantitative and qualitative variables significantly affects the operational properties of control charts, in both in-control and out-of-control cases. The occurrence of false alarms typically increases when parameters are estimated (ZHANG et al., 2011).

When obtaining samples for Phase I is not feasible, there exists the option of employing the chart proposed by Quesenberry (referred to as Q-charts). The Q-chart operates under the assumption that the process is initially stable, and therefore it is not necessary to collect samples until achieving an in-control process. Although the use of this graph is attractive, it has some limitations: it is not realistic, data points are plotted outside the original measurement scale, and its capability for detecting changes is diminished owing to data transformation (CASTAGLIOLA et al., 2014).

In 2006, a study was conducted on the effects of parameter estimation on the performance of control charts. This research made significant contributions to the field of statistical process control by not only analyzing these effects but also proposing thirteen key challenges that warrant further exploration. Among these challenges, the first underscores the critical need to investigate the implications of parameter estimation when employing attribute control charts (JENSEN et al., 2006).

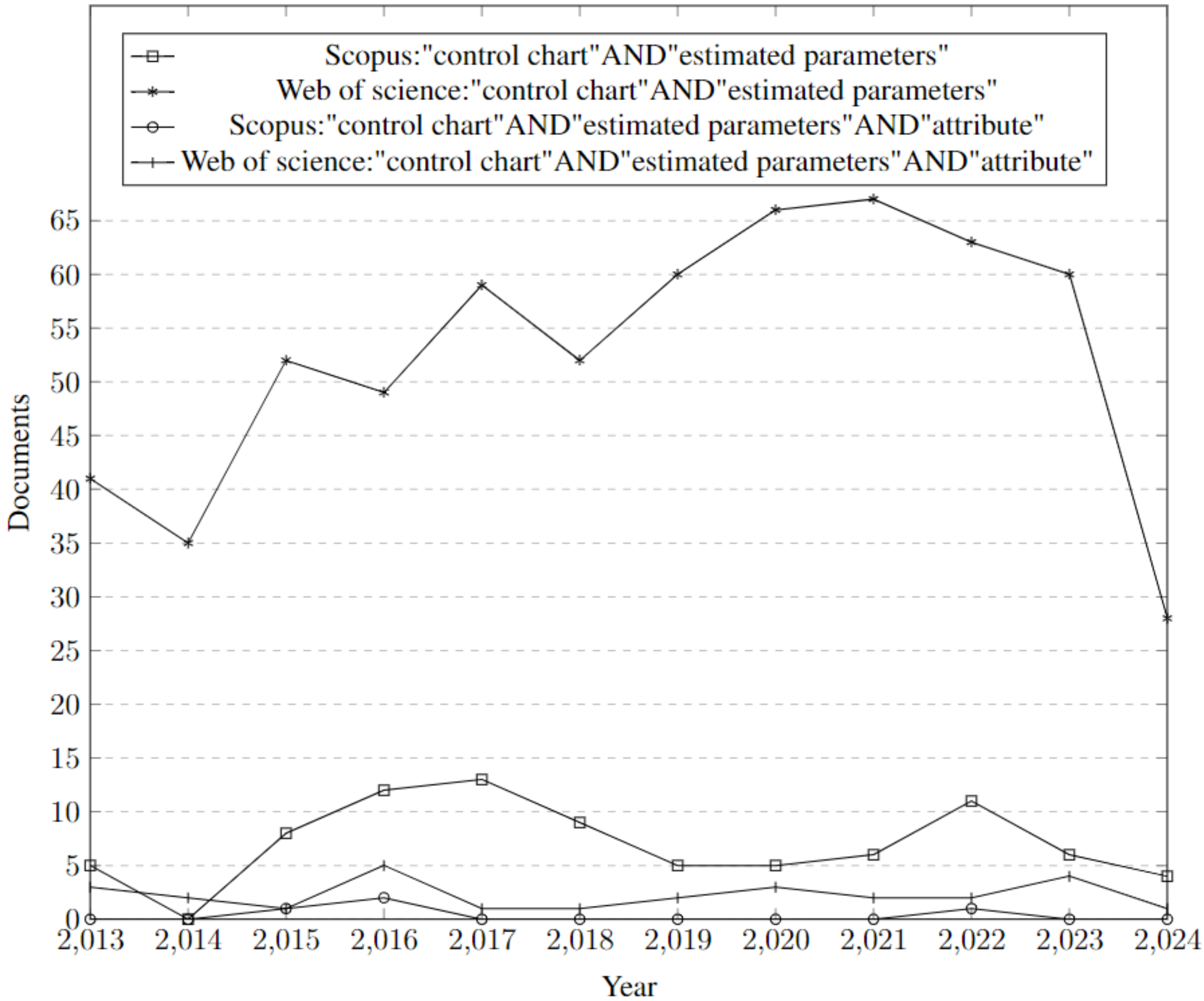
Studies have been conducted to analyze the impact of parameter estimation, primarily focusing on control charts for continuous variables. For instance, Mehmood et al. (MEHMOOD; QAZI; RIAZ, 2018), Hu et al. (HU et al., 2018), and Jardim et al. (JARDIM; CHAKRABORTI; EPPRECHT, 2019; JARDIM; CHAKRABORTI; EPPRECHT, 2020) have contributed to this area.

Although the importance of studying the relationship between estimated parameters and the efficiency of attribute control charts is noticiable, there is still a paucity of studies that substantially contribute to the field. Most authors neglect the importance of conducting research on attribute charts (NEZHAD; JAFARIAN-NAMIN; FARAZ, 2019). See Figure 5 for a depiction of publications in the Scopus database over the last decade concerning parameter estimation and control charts.

The data show that there is much to be explored in this area, particularly regarding attribute inspection. So far, no work has been conducted analyzing attribute control charts for monitoring a continuous variable with estimated parameters.

Braun (BRAUN, 1999) was the first to conduct a study to understand the impact of parameter estimation on attribute control charts. His intention was to extend the work done by Chen (1998) with estimated parameters for variable charts, using the same methodology for attribute charts. To achieve this, the author analyzed the c chart with a Poisson distribution, random variables, and with estimated average c ($\widehat{c}$), and the p chart with a Binomial distribution, random variables, and with estimated p

Figure 5 – Publications related to estimation of parameters and control charts in the last 10 years in database Scopus and Web of science.



source: Adapted from Scopus and Web of science.

$(\widehat{p})$. Braun compared the results of the average run length (ARL) and the standard deviation of the run length $(SDRL)$ obtained for estimated parameters with the results for known parameters (ideal situation). For the c chart, it was found that for out-of-control situations, the ARL values are higher when the parameters are estimated, which is consistent with the results obtained by Chen for variable charts and indicates a possible additional cost due to failure to detect out-of-control behavior when estimating control limits with 25 samples. However, for in-control cases, the results differ from those obtained by Chen; the ARL values are not always lower for estimated parameters, probably because the width of the control limits follows the function of $\widehat{c}$. For the p chart, adopting 25 samples, the ARL values are higher when the parameters are estimated, in both cases: in and out of control for small values of np. However, for large values of n and p, lower ARL values are obtained for estimated parameters. Braun concludes, therefore, that just like for variables, the effect of parameter estimation for attributes exists, can be detected, and can be quite concerning. To obtain behaviors close to ideal, many samples of large size are required.

Chakraborti and Human analyzed the performance of the p control chart when attributes are estimated. The p chart is straightforward to apply; however, the nonconformity fraction (p) is almost always unknown. The authors used binary data because the approximation of the binomial distribution to the normal distribution is quite weak. An analysis was conducted for the case of cardboard boxes for storing frozen orange juice concentrate; in the first case, the parameter p was given. In the second case, p was estimated, starting Phase I with 30 samples (m) of 50 boxes each (n), and the iteration process continued until achieving a controlled process and estimating the final control limits. These limits were used to monitor the process in Phase II. The authors found that a very large number of reference data is required when estimating the parameter p for the false alarm rate to be close to nominal. Another important observation is that the number of data points in each sample (n) should be considerably larger than the number of samples (m) to achieve a false alarm rate close to the expected. Carefully defining the number and size of samples is also important in determining the average value of the distance between one special cause and another, as in some cases, this can be zero, which is unrealistic. The authors recommend using 10 samples with 20 observations each. Additionally, they suggest a general rule to be used: $n/m \geq 0.5$ (CHAKRABORTI; HUMAN, 2006).

Testik conducted a study on the effect of parameter estimation on the performance of Poisson CUSUM charts. This type of chart excels at detecting small shifts in the mean of the characteristic of interest. The author varied the intensity of changes (δ), the sample sizes (n), and the estimated mean values. The performance of the chart was evaluated using the average run length (ARL) between special causes, the standard deviation of the run length ($SDRL$), and the percentiles (RL) of the distribution. The results showed that parameter estimation can significantly reduce the detection power of the chart. To mitigate this issue, a sample should have at least 200 data points for estimation during Phase I (TESTIK, 2007).

Chakraborti and Human conducted a similar study on the c chart. They found that the false alarm rate (FAR) and the average run length (ARL) between alarms vary significantly when parameters are estimated. During Phase I, they determined that a large number of samples is necessary to ensure that the difference between results for estimated parameters and known parameters is not significant. It is important to note that even with a sufficiently large volume of data, the results will not be as expected due to the discrete nature of the distribution. To achieve satisfactory chart performance, there is also the option of using a different chart parameter (k) than usual: $k = 3$. In this case, control limits will be established based on $k - sigma$, where $k > 0$ (CHAKRABORTI; HUMAN; GRAHAM, 2008).

In 2012, a study on c and np control charts when parameters are estimated was conducted. The study aimed to analyze the number of samples (m) needed in Phase I to achieve an average run length (ARL) similar to the case of known parameters and thereby propose control chart parameters dedicated to the number of samples used in Phase I. For the c chart, it was found that as m increases, the difference between ARL for estimated and known parameters decreases. However, unlike the $\overline{X}$ chart, this does not occur without variation. At times, the ARL values are higher for estimated parameters, and at other times, they are lower. When investigating the practical number of samples required, it was observed that in some cases, more than $10,000$ samples would be needed to achieve a difference of less than 5% in the distribution properties of the charts. Since this number is often impractical, the authors suggest using alternative control chart parameters (k) rather than the classic $k = 3$. Similar results were obtained when investigating the np chart, allowing for the same considerations to be extended.

The authors mention that the new chart parameters (k') help reduce the standard deviation of the run length ($SDRL$) for c and np charts with estimated parameters (CASTAGLIOLA; WU, 2012).

Subsequently, a studied on the behavior of synthetic c and np charts with estimated parameters and compared the results with those obtained from known parameters. They focused on in-control conditions due to the greater dispersion in results, noting that the values become closer as the number of out-of-control conditions increases. Both cases exhibited significantly different average run length (ARL) values for in-control situations between known and estimated parameters. The ARL values varied depending on the values of c for the $c - chart$ and n and p for the np chart, but there was no consistent pattern: sometimes ARL values were higher for known parameters, and other times they were higher for estimated parameters. The study determined the minimum number of subgroups (m) needed in Phase I to approximate the results between the known and estimated parameter cases, finding very high values ($> 10,000$) depending on c or n and p. Such a high number of samples is impractical due to execution time, difficulty in obtaining samples, and the associated high costs. To mitigate this problem, the authors suggest calculating alternative chart parameters (h', c', and k') that take into account the value of m, allowing ARL values to be as close as possible between known and estimated parameter cases (CASTAGLIOLA et al., 2014).

It can be observed that the studies presented thus far have focused on examining the effect of parameter estimation for in-control conditions. Consequently, the effects on out-of-control conditions remain under-investigated. To address this, Tiplica aimed to compute and analyze the ARL values for out-of-control conditions (ARL_1) using the c chart with estimated parameters. He fixed the ARL value for in-control conditions (ARL_0) and varied the number of samples, the average number of nonconformities, and the chart's constant k. As expected, the lower the ARL_0 value, the quicker the chart detects small changes in the average number of nonconformities, though this also increases the probability of false alarms. The chart's performance in detecting changes, especially small ones, improves with an increase in the number of samples (m). It was also found that the number of samples has a significant impact on the standard deviation of the run length ($SDRL$); the higher the m value, the lower the $SDRL$ (TIPLICA, 2015).

In 2017, a study was conducted on the effect of estimating the mean number of defects in a c chart with variable parameters ($c - VP$). The performance of the chart was compared to the results obtained with known parameters. It was found that results can vary significantly when the samples are small, taking much longer to detect changes when the parameters are estimated. As the sample size (n) increases, the chart's behavior becomes closer to that observed when the parameters are assumed to be known. It was also noted that it is possible to improve the performance of the charts with large samples by using a small number of samples (m), or with small samples by using a large number of samples (KEYVANDARIAN et al., 2017).

Several studies have been conducted to address the need for large samples for parameter estimation. The performance of the adapted c chart using the bootstrap method showed improvement under in-control conditions; however, the sensitivity to detect changes in the process was negatively affected (VAKILIAN; AMIRI; FARAZ, 2018). Another study utilized the Cornish-Fisher expansion to enhance the performance of the np chart for in-control conditions, and the bootstrap method was employed to adjust the control limits to reduce the variability of ARL under control (NEZHAD; JAFARIAN-NAMIN; FARAZ, 2019).

The performance of control charts with estimated parameters is generally measured using the average run length (ARL). For in-control conditions, ARL_0 is the expected number of samples before a false alarm; thus, the higher the ARL_0 value, the better the chart's performance. For out-of-control conditions, ARL_1 is the number of samples required to detect changes in the process; therefore, the lower the ARL_1 value, the quicker the detection of the change, which is the desired condition (NEZHAD; JAFARIAN-NAMIN; FARAZ, 2019).

Other parameters, such as the average ($AARL$), median ($MARL$), and variability ($SDARL$) of the ARL, can also be used to evaluate the performance of control charts. Jones and Steiner highlighted the importance of using ARL variability to determine the number of samples needed in Phase I of parameter estimation. The lower the $SDARL$ value, the closer ARL_0 is to the desired value. Since there is no guarantee that the chart's performance under in-control conditions is known, focusing on $SDARL$ variability becomes important (JONES; STEINER, 2012; NEZHAD; JAFARIAN-NAMIN; FARAZ, 2019).

Various studies emphasize the importance of studying the effect of parameter estimation on the performance of control charts in monitoring the process and quickly signaling changes, even if they are small. Some studies indicate challenges to be explored by researchers, such as:

- Tiplica highlights the importance of extending his study to other attribute charts with estimated parameters, evaluating the chart's performance for out-of-control conditions through ARL (TIPLICA, 2015).
- Exploring new combinations of the $\overline{X}^{att}$ chart with CUSUM or EWMA charts is a suggestion, as well as evaluating attribute means using multivariate charts (QUININO; BESSEGATO; CRUZ, 2017).
- The importance of applying the bootstrap method to other types of charts (beyond the c chart) under different adaptations (VAKILIAN; AMIRI; FARAZ, 2018).
- Nezhad, Jafarian-Namin, and Faraz suggest that their methodology be applied to other types of charts and also used to evaluate both in-control and out-of-control conditions (NEZHAD; JAFARIAN-NAMIN; FARAZ, 2019).
- Explore control charts based on attribute inspection to monitor the process mean and variance simultaneously, considering the estimation of the mean (μ_0) and standard deviation (δ_0) for in-control conditions (QUININO; CRUZ; HO, 2020).

There are two perspectives related to parameter estimation: the conditional and unconditional perspectives. The run length distribution for a given set of estimators is called the conditional distribution. Conditional run length properties vary based on the values of parameter estimators for a reference sample. When parameters are estimated, the conditional false alarm rate ($CFAR$) and conditional average duration ($CARL$) are random variables because they are conditional on the estimated parameters, which are also random variables. Therefore, these measurements alone cannot be used when parameters are estimated because they vary. The unconditional perspective is based on the average $CARL$ for a given set of estimators over the distribution of those estimators. The distribution RL_0 is obtained by averaging the distribution of parameter estimators, so a performance given by ARL_0 or $SDRL_0$ is an "average" performance over an infinite number of possible control charts, each constructed with a possible value from a set infinite number of estimated parameters, rather than a specific control chart performance (CHAKRABORTI; GRAHAM, 2019).

Analyzing the efficiency of control charts in relation to estimated parameters involves understanding how accurately these parameters represent the process. Inaccuracy or variability in parameter estimation can affect the chart's ability to reliably detect process changes. Several factors can influence the efficiency of control charts in practical applications:

1. Estimation Methods: Different estimation methods can produce varying levels of accuracy and precision in parameter estimation.

2. Distribution of Process Data: The efficiency of parameter estimation can be impacted by the data distribution. Many control charts proposed in the literature assume data normality due to their well-established theoretical foundation. However, in practice, the data distribution is not always known, and deviations from these assumptions can affect the accuracy of the estimates.

3. Measurement Error: Significant measurement errors in the process data can distort parameter estimates and reduce the efficiency of control charts.

4. Sample Size: Larger sample sizes generally lead to more accurate parameter estimates, potentially increasing the efficiency of control charts. However, there is a trade-off between sample size and the cost or time required to collect data.

By investigating these factors and their interactions, insights can be gained into how estimated parameters affect control chart efficiency. This can help identify strategies to improve performance, such as using robust estimation techniques or optimizing sample sizes.

3 THE np_x CONTROL CHART WITH KNOWN PARAMETERS

3.1 np_x CONTROL CHART

Wu $et\ al.$ (2009) proposed a new np control chat, called np_x chart, which employs an attribute inspection to monitor the mean value of a continuous variable X classifying each unit of a sample as conforming or nonconforming using a go/no go gauge. The attribute inspection is easier, faster, and cheaper to implement and monitor a process (MONTGOMERY, 2012). However, the attribute inspection can require a bigger sample size to have the same performance of a variable inspection. The np_x chart is compared to the $\overline{X}$ chart, and both control charts are required to have the same performance when the process is in-control, then the average time to signal for the in-control condition should be equal to 370 $(ATS_0 = \tau = 370)$. For the out-of-control condition, the performance of both control charts was measured according to the extra quadratic loss, EQL.

Therefore, an identical inspection cost per unit time (W) would be appropriated to evaluate the comparison between the performances of the np_x chart and the $\overline{X}$ chart, see equation 1:

$$W_{\overline{X}} = W_{npx} \ \rightarrow \ \frac{c_{\overline{X}} n_{\overline{X}}}{h_{\overline{X}}} = \frac{c_{npx} n_{npx}}{h_{npx}} \tag{1}$$

where $c_{\overline{X}}$ and c_{npx} are the inspection cost per unit, $n_{\overline{X}}$ and n_{npx} are the sample size, $h_{\overline{X}}$ and h_{npx} are the sampling interval, respectively for the $\overline{X}$ chart and np_x chart.

According to the identical inspection cost per unit time applied, if the EQL of both charts is equal, but one costs less than the other to monitor the process. Then the cheaper chart is better to be used because the process is monitored with the same efficient, but it costs less. Another possibility is to have the same inspection cost for both charts, but the power of detection of one is superior to the other. Then the chart which presents better performance (the smaller EQL) should be used.

The np_x chart uses statistical warning limits, equations 2 and 3, instead of specification limits.

$$w_L = \mu_0 - k_w \sigma_0 \tag{2}$$

$$w_U = \mu_0 + k_w \sigma_0 \tag{3}$$

where μ_0 and σ_0 are respectively the mean and standard deviation for the in-control process condition. And k_w is the warning limit coefficient.

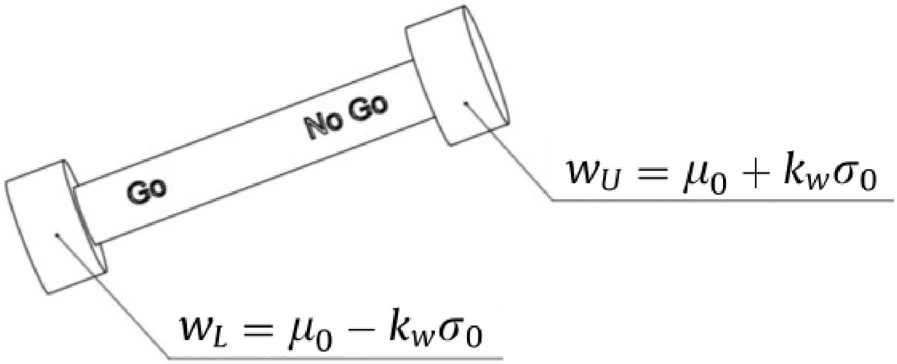
The gauge is designed according to the warning limits, see Figure 6. It is used to classify the produced units as conforming or nonconforming. The advantage of use warning limits is that they can be set at an optimal level to improve the detection power of the chart.

When a process shift occurs, the mean value will change, as shown on equation 4:

$$\mu = \mu_0 + \delta \sigma_0 \tag{4}$$

where δ is the magnitude of a mean shift in terms of σ_0.

Figure 6 – Go/No Go Gauge



source: Elaborated by the author.

The number of nonconforming units (D) on the chart are compared to the upper control limit of np_x chart (UCL_{npx}). The UCL_{npx} is the maximum number of nonconforming units accepted in a sample, thus, if D is larger than UCL_{npx} the process is signaled as out of control. Note that for the np_x chart, only an upper control limit (UCL_{npx}) is required, as there is no lower control limit.

The fraction of nonconforming items in a sample (p), classified by the go/no go gauge, follows a normal distribution, see equation 5.

$$p = Pr(X < w_L \; or \; X > w_U) = 1 - \Phi(k_w - \delta) + \Phi(-k_w - \delta) \tag{5}$$

where $\Phi()$ is the cumulative probability function of the standard normal distribution.

The number of nonconforming units follow a binomial distribution, so the probability of the number of nonconforming units (D) in a sample being larger than UCL_{npx} for a given fraction nonconforming p is demonstrated on equation 6:

$$P(p) = P(D > UCL|p) = 1 - \sum_{i=0}^{UCL_{np}} C_i^{n_{npx}} p^i (1-p)^{n_{npx}-i} \tag{6}$$

When the process is considered in control, $\delta = 0$, thus $\alpha = P(p_0)$. The performance can be measured about the average time to a signal when the process is in-control (ATS_0), equation 7.

$$ATS_0 = \frac{h}{\alpha} = \frac{h}{P(p_0)} \tag{7}$$

The power of detecting a mean shift is:

$$\beta = 1 - P(p) \tag{8}$$

So, when the process is out-of-control, the performance is given by $ATS(\delta)$, equation 9.

$$ATS(\delta) = (ARL - 0.5)h_{npx} = \left(\frac{1}{1-\beta} - 0.5\right)h_{npx} = \left(\frac{1}{P(p)} - 0.5\right)h_{npx} =$$
$$1 - \left[\left(\sum_{i=0}^{UCL_{npx}} C_i^{n_{npx}}(1-\phi(k_w-\delta)+\phi(-k_w-\delta))^i(\phi(k_w-\delta)-\phi(-k_w-\delta))^{n_{npx}-i}\right)^{-1} - 0.5\right]h_{npx} \tag{9}$$

Commonly, the average time to a signal is used as the objective function to be minimized at a specified process shift level. Although, the objective of the design is to make the control chart efficient at signaling a wide range of mean shifts. Researchers usually compare the performance of two charts by examining the corresponding out-of-control $ATS(\delta)$ values of the two charts at some discrete points of process shift across a range. For most of the cases, one chart is unable to be better than the other at all the points. Because of this, some overall out-of-control performance measure, which consider a range of possible values for δ simultaneously, are proposed in the literature (REYNOLDS; STOUMBOS, 2004).

Wu $et\ al.$ (WU et al., 2009) considered the EQL function (equation 10) as the overall out-of-control measure and assumed that a shift range are equally important, and a uniform distribution for δ is implied.

$$EQL = \frac{1}{\delta_{max} - \delta_{min}} \int_{\delta_{min}}^{\delta_{max}} \delta^2 ATS(\delta) d\delta = \int_{\delta_{min}}^{\delta_{max}} (\delta_{max} - \delta_{min})^{-1} * \delta^2 * 1-$$
$$\left[\left(\sum_{i=0}^{UCL_{npx}} C_i^{n_{npx}}(1-\phi(k_w-\delta)+\phi(-k_w-\delta))^i(\phi(k_w-\delta)-\phi(-k_w-\delta))^{n_{npx}-i}\right)^{-1} - 0.5\right]h_{npx} d\delta \tag{10}$$

The optimization design for the np_x chart is based on defining the two charting parameters: the upper control limit (UCL_{npx}) and the warning limit coefficient (k_w), which guarantee that the ATS_0 is equal to $\tau = 370$ and minimize the EQL function. The other two charting parameters: sample size (n_{npx}) and sampling interval (h_{npx}), are specifications.

Wu $et\ al.$ (WU et al., 2009) compared the effectiveness of the np_x chart and the $\overline{X}$ chart fixing the $\mu_0 = 0$ and $\sigma_0 = 1$ respectively. The out-of-control condition was analyzed varying the shift range from 0 to 3.5. Their results are presented in Table 1

When the sample size and sampling interval for the np_x chart and the $\overline{X}$ chart are the same, check Table 1 columns 2 and 3, the $\overline{X}$ chart presents a superior performance, since the ratio $EQL_{npx}/EQL_{\overline{X}}$ is larger than 1. representing that the $\overline{X}$ chart is approximately 31% more effective according than the np_x chart. If the sample size is doubled for the np_x chart (column 4) then the np_x chart surpasses the $\overline{X}$ chart in 15%. The same occurs if the sampling interval is halved (column 5) the np_x chart is approximately 12% more effective than the $\overline{X}$ chart.

So, the authors (WU et al., 2009) concluded that, recalling the identical inspection cost per unit of

Table 1 – Charting parameters and incorrect ATS and EQL values of the control charts ($\tau = 370$) provided by Wu $et\ al.$(WU et al., 2009).

1	2	3	4	5
δ	ATS			
	$\overline{X}$ $chart$	np_x $chart$	np_x $chart$	np_x $chart$
	$n_{\overline{X}} = 6$, $h_{\overline{X}} = 1$	$n_{npx} = 6$, $h_{npx} = 1$	$n_{npx} = 12$, $h_{npx} = 1$	$n_{npx} = 6$, $h_{npx} = 0.5$
	$UCL_{\overline{X}} = 1.2246$	$UCL_{npx} = 3$	$UCL_{npx} = 7$	$UCL_{npx} = 4$
		$w_u = k_w = 1.2724$	$w_u = k_w = 0.7544$	$w_u = k_w = 0.9651$
0.00	370.03	370.18	370.10	370.04
0.25	117.34	156.16	90.10	137.91
0.50	25.85	41.37	17.05	33.17
0.75	7.66	13.58	4.75	10.12
1.00	2.94	5.41	1.83	3.82
1.25	1.41	2.56	0.94	1.73
1.50	0.83	1.41	0.63	0.92
1.75	0.61	0.90	0.53	0.56
2.00	0.53	0.67	0.51	0.39
2.25	0.51	0.56	0.50	0.31
2.50	0.50	0.52	0.50	0.28
2.75	0.50	0.51	0.50	0.26
3.00	0.50	0.50	0.50	0.25
3.25	0.50	0.50	0.50	0.25
3.50	0.50	0.50	0.50	0.25
EQL	3.891	5.115	3.305	3.411
$EQL_{npx}/EQL_{\overline{X}}$	1.000	1.314	0.849	0.877

source: (WU et al., 2009).

time (equation 1), if the inspection cost per unit for the np_x chart is 50% lower than the $\overline{X}$ chart, the attribute inspection would be advantageous since we would have a better performance for the same inspection cost per unit of time.

Based on Wu's et al. results, a lot of studies have been done about the np_x chart approach, see for example: Ho and Costa ((HO; COSTA, 2011)) studied the effect of the wandering behavior of the process mean on the performance of the np_x chart. Sampaio *et al.* (SAMPAIO; HO; MEDEIROSC, 2014) proposed a combined np_x-$\overline{X}$ chart to monitor the process mean in a two-stage sampling, where the sample is divided in two sub-samples: n_1 and n_2. First the sub-sample of size n_1 is evaluated by attribute inspection, if its out of control than the sub-sample n_2 is evaluated and the mean is calculated and only is the mean is upper to the limit the process is considered to be out of control. The use of an attribute control chart to monitor the process variability based on the np_x chart was also studied (HO; QUININO, 2013; SILVA; HO; QUININO, 2022b). Quinino *et al.* (QUININO; CRUZ; QUININO, 2021) proposed a control chart based on attribute and variable inspection to monitor the process mean, where the first sample is submitted to variable inspection using the $\overline{X}$ chart followed by the remaining k samples submitted to an attribute inspection using the np_x chart. After those k samples, if the charts do not signal any shift in the process mean, then the inspection procedure starts over again, with one sample being submitted to variable inspection and the next k sample by attribute, until either the attribute np_x chart or the variable $\overline{X}$ chart signals an increase or decrease in the process mean. Zhou *et al.* (ZHOU; YE; ZHENG, 2022) proposed the np_{CEV} chart to monitor the process mean adopting the conditional expected value (CEV) weight method. And a control chart to monitor the process mean based on attribute inspection applying sequential sample size was proposed by Silva *et al.* (SILVA; HO; QUININO, 2022a).

The relevance of the np_x chart was the motivation to study the attribute inspection to monitor the process mean. The same concepts, techniques and equations were used to calculate the ATS_0 and EQL, however, the results found were different from the ones found by Wu $et\ al.$ (WU et al., 2009).

3.2 DIFFERENT RESULTS

Call to mind that the authors compared the effectiveness of the np_x chart and the $\overline{X}$ chart. For this, μ_0 and σ_0 were fixed as zero and one respectively. The produced ATS_0 must be equal to 370 for both charts. The out-of-control condition was analyzed varying the shift range from 0 to 3.5.

Using the same concepts, techniques and equations to calculate the ATS_0 and EQL, I got results completely different from the ones found by Wu $et\ al.$ (WU et al., 2009), see Table 2.

Table 2 – Corrected results using the charting parameters used by Wu $et\ al.$ (2009) and performance measured ATS and EQL calculated for the np_x chart and the $\overline{X}$chart ($\tau = 370$).

1	2	3	4	5
δ	ATS			
	$\overline{X}$ $chart$	np_x $chart$	np_x $chart$	np_x $chart$
	$n_{\overline{X}} = 6$, $h_{\overline{X}} = 1$	$n_{npx} = 6$, $h_{npx} = 1$	$n_{npx} = 12$, $h_{npx} = 1$	$n_{npx} = 6$, $h_{npx} = 0.5$
	$UCL_{\overline{X}} = 1.2246$	$UCL_{npx} = 3$	$UCL_{npx} = 7$	$UCL_{npx} = 4$
		$w_u = k_w = 1.2724$	$w_u = k_w = 0.7544$	$w_u = k_w = 0.9651$
0.00	370.37	55.64	8.88	27.59
0.25	115.36	43.26	7.04	22.32
0.50	25.86	23.29	4.44	13.06
0.75	7.67	10.66	2.43	6.44
1.00	2.94	4.90	1.34	3.09
1.25	1.43	2.46	0.82	1.57
1.50	0.83	1.39	0.61	0.88
1.75	0.61	0.90	0.53	0.55
2.00	0.53	0.68	0.50	0.39
2.25	0.50	0.56	0.50	0.31
2.50	0.50	0.52	0.50	0.28
2.75	0.50	0.51	0.50	0.26
3.00	0.50	0.50	0.50	0.25
3.25	0.50	0.50	0.50	0.25
3.50	0.50	0.50	0.50	0.25
EQL	3.891	3.895	2.339	2.196
$EQL_{npx}/EQL_{\overline{X}}$	1.000	1.058	0.635	0.598

source: Elaborated by the author.

Initially, the ATS_0 generated by the np_x chart diverges significantly from $\tau = 370$ across all cases, as observed in columns 3, 4, and 5 of Table 2. For instance, when $n = 6$ and $h = 1$, ATS_0 is recorded as 370.18 in Table 1 but drastically decreases to 55.64 in Table 2 (an 85% reduction). This discrepancy poses a problem as it leads to a higher occurrence of false alarms compared to expectations based on Wu et al.'s findings (WU et al., 2009), and notably more false alarms compared to the $\overline{X}$ chart, which consistently maintains $ATS_0 = 370$ across both tables. Consequently, this deviation indicates a failure to meet the design condition.

While the performance of the np_x chart appears to exhibit superior detection power when the process is out-of-control, as evidenced by the EQL values presented in Table 2 (e.g., 3.895 for $n = 6$ and $h = 1$) compared to the incorrect values provided in Table 1 (e.g., 5.115 for $n = 6$ and $h = 1$), this perception is deceptive. The apparent enhancement in performance is merely a consequence of the substantial disparity between the correct and incorrect ATS_0 values.

Taking into account the constraint that ATS_0 must be equal to $\tau = 370$, it was identified the combined values of k_w and UCL_{npx} which satisfies this condition given the sample size (n) and sampling interval (h) for the np_x chart. Then, the correspondent EQL_{npx} was calculated for each pair of parameters k_w and UCL_{npx}, according to the equation 10.

The results are presented in Table 3 for $n_{npx} = 6$ and $h_{npx} = 1$, Table 4 for $n_{npx} = 12$ and $h_{npx} = 1$ and Table 5 for $n_{npx} = 6$ and $h_{npx} = 0.5$.

Table 3 – UCL_{npx} and k_w values of the np_x control chart, charting parameters: $\tau = 370$, $n_{npx} = 6$, $h_{npx} = 1$.

UCL_{npx}	k_w	EQL_{npx}
0	3.5083183	14.475099
1	2.4658079	9.364985
2	**1.9312636**	**8.652093**
3	1.5466371	9.091237
4	1.2181876	10.565809
5	0.8904558	14.530189

source: Elaborated by the author.

Table 4 – UCL_{npx} and k_w values of the np_x control chart, charting parameters: $\tau = 370$, $n_{npx} = 12$, $h_{npx} = 1$.

UCL_{npx}	k_w	EQL_{npx}
0	3.6885716	12.277000
1	2.7193652	7.365772
2	2.2510745	6.302212
3	1.9377482	5.982668
4	**1.6961228**	**5.952972**
5	1.4941776	6.095324
6	1.3162326	6.385766
7	1.1528777	6.838506
8	0.9976724	7.521456
9	0.8447705	8.585813
10	0.6868793	10.435308
11	0.5087378	14.698688

source: Elaborated by the author.

Table 5 – UCL_{npx} and k_w values of the np_x control chart, charting parameters: $\tau = 370$, $n_{npx} = 6$, $h_{npx} = 0.5$.

UCL_{npx}	k_w	EQL_{npx}
0	3.6887431	11.768940
1	2.5893784	7.235402
2	**2.0329908**	**6.551387**
3	1.6369585	6.847554
4	1.3014878	7.961268
5	0.9690862	10.914016

source: Elaborated by the author.

Once again, the effectiveness of both the $\overline{X}$ chart and the np_x chart was compared using the charting parameters consistent with those employed by Wu et al. (WU et al., 2009): n_{npx}, h_{npx}, and UCL_{npx}. However, the warning limit coefficient (k_w) was determined based on Tables 3, 4, and 5 above. The results of this comparison are presented in Table 6.

Table 6 – Charting parameters and ATS values of the control charts ($\tau = 370$).

1	2	3	4	5
δ	ATS			
	$\overline{X}$ *chart*	np_x *chart*	np_x *chart*	np_x *chart*
	$n_{\overline{X}} = 6$, $h_{\overline{X}} = 1$	$n_{npx} = 6$, $h_{npx} = 1$	$n_{npx} = 12$, $h_{npx} = 1$	$n_{npx} = 6$, $h_{npx} = 0.5$
	$UCL_{\overline{X}} = 1.2246$	$UCL_{npx} = 3$	$UCL_{npx} = 7$	$UCL_{npx} = 4$
		$w_u = k_w = 1.5466$	$w_u = k_w = 1.1529$	$w_u = k_w = 1.3015$
0.00	370.37	369.93	369.97	369.99
0.25	115.36	261.87	249.67	265.28
0.50	25.86	111.13	94.86	113.37
0.75	7.67	38.95	26.41	38.55
1.00	2.94	14.06	7.94	13.09
1.25	1.43	5.74	2.87	4.97
1.50	0.83	2.71	1.31	2.19
1.75	0.61	1.48	0.76	1.11
2.00	0.53	0.94	0.57	0.65
2.25	0.50	0.68	0.52	0.44
2.50	0.50	0.57	0.50	0.34
2.75	0.50	0.52	0.50	0.29
3.00	0.50	0.51	0.50	0.26
3.25	0.50	0.50	0.50	0.25
3.50	0.50	0.50	0.50	0.25
EQL	3.680	9.091	6.839	7.961
$EQL_{npx}/EQL_{\overline{X}}$	1.000	2.470	1.858	2.163

source: Elaborated by the author.

Analyzing both charts at the same charting parameters, Table 6 columns 2 and 3, it is possible to observe the smaller is the mean shift, bigger is the difference between the $\overline{X}$ chart and the np_x power of detecting a mean shift. The maximum difference between charts is 127% and occurs for a mean shift of $\delta = 0.25$. The ratio between the EQL of both charts $(EQL_{npx}/EQL_{\overline{X}})$ shows that the $\overline{X}$chart is, on average, 147% more effective than the np_x chart across the entire mean shift range.

As verified by Wu $et\ al.$ (WU et al., 2009) if the sampling interval is decreased or the sample size is increased, the performance of the np_x chart improves. See Table 6, column 4, where the sample size is doubled, the ratio $(EQL_{npx}/EQL_{\overline{X}})$ shows that the $\overline{X}$ chart is, on average, 85.8% more effective than the np_x chart across the entire mean shift range. The same improvement on np_x chart performance can be observed when the sampling interval is halved (Table 6, column 5) when the $\overline{X}$ chart is approximately 116% superior to the np_x chart.

The accurate findings indicate that, even with the sample size doubled or the sampling interval halved, the np_x chart performs less effectively than the $\overline{X}$ chart.

Wu $et\ al.$ (WU et al., 2009) designed the np_x chart under two conditions aforementioned: $ATS_0 = \tau$ and EQL minimized. Thus, the optimum values of UCL_{npx} and k_w which satisfy the constraint $ATS_0 = \tau = 370$ and produce the minimum EQL are highlighted in **bold** on Tables 3, 4 and 5. These optimum values were used to compare the performance of the np_x chart and the $\overline{X}$ chart, see Table 7.

Table 7 – Optimum charting parameters and performance measured ATS and EQL of the control charts $(\tau = 370)$.

1	2	3	4	5
δ	ATS			
	$\overline{X}$ $chart$	np_x $chart$	np_x $chart$	np_x $chart$
	$n_{\overline{X}} = 6$, $h_{\overline{X}} = 1$	$n_{npx} = 6$, $h_{npx} = 1$	$n_{npx} = 12$, $h_{npx} = 1$	$n_{npx} = 6$, $h_{npx} = 0.5$
	$UCL_{\overline{X}} = 1.2246$	$UCL_{npx} = 2$	$UCL_{npx} = 4$	$UCL_{npx} = 2$
		$w_u = k_w = 1.9313$	$w_u = k_w = 1.6961$	$w_u = k_w = 2.0330$
0.00	370.37	369.93	369.97	369.99
0.25	115.36	261.87	249.67	265.28
0.50	25.86	111.13	94.86	113.37
0.75	7.67	38.95	26.4	38.551
1.00	2.94	14.06	7.94	13.09
1.25	1.43	5.74	2.87	4.97
1.50	0.83	2.71	1.31	2.19
1.75	0.61	1.48	0.76	1.11
2.00	0.53	0.94	0.57	0.65
2.25	0.50	0.68	0.52	0.44
2.50	0.50	0.57	0.50	0.34
2.75	0.50	0.52	0.50	0.29
3.00	0.50	0.51	0.50	0.26
3.25	0.50	0.50	0.50	0.25
3.50	0.50	0.50	0.50	0.25
EQL	3.680	8.652	5.953	6.551
$EQL_{npx}/EQL_{\overline{X}}$	1.000	2.351	1.618	1.780

source: Elaborated by the author.

At the same charting parameters$n_{npx} = n_{\overline{X}}$ and $h_{npx} = h_{\overline{X}}$, Table 7, columns 2 and 3, it is possible to observe that the $\overline{X}$ chart is, on average, 135% more effective than the np_x chart across the entire mean shift range. If the sample size is doubled (n_{npx} from 6 to 12), the difference between the power of detecting a mean shift for both charts decreases, but even so the $\overline{X}$ chart is, on average, 61.8% more effective than the np_x (Table 7, column 4).

Comparing the values on tables 6 and 7 it is clearly shown that the performance of the np_x chart improves when the optimal values are used. Analyzing both charts at the same charting parameters, when UCL_{npx} and k_w is choose only considering ATS_0 equal to 370, the $\overline{X}$ chart is 147% more effective than the np_x chart (Table 6, columns 2 and 3). However, this difference decreases to 135% (Table 7, columns 2 and 3) when the optimal values, $ATS_0 = 370$ and the minimum EQL_{npx}, are used. The same behavior can be observed when the charting parameters are different.

Contradicting the statements made by Wu *et al.* (WU et al., 2009) doubling the sample size or halving the sampling interval is not enough to the np_x chart outperforms the $\overline{X}$ chart. In fact, using their chart parameters, we got an ATS_0 value that is 85% smaller than what is shown in their paper, in section 3.1, page 145, where $ATS_0 = 370$. Also, for the two-sided chart, their claim that the sample size was needed to be increased over "twice" for the np_x chart to outperform the $\overline{X}$ control chart, is found to be incorrect. In fact, if the sample size is doubled, the $\overline{X}$ chart performance is found to be still approximately 62% superior to the np_x chart. We also found discrepancies when we tried to replicate their results in section 3.2 of their paper, where the authors compared the performance of the np_x chart and the $\overline{X}$ control for different sample sizes.

The observations and findings are significant enough to be brought to the attention of the readers and researchers in the field. Consequently, we reached out to the authors of the Wu *et al.* (WU et al., 2009) article, and received a response from one of the authors. They clarified that the results presented in Table 1 of their paper were indeed calculated for the np_x chart in the one-sided case (one warning limit), with $ATS_0 = \tau = 740$. However, this explanation is problematic for several reasons. Firstly, Section 3.1 of their paper explicitly states that they considered $ATS_0 = \tau = 370$ for both the np_x and $\overline{X}$ control charts. Secondly, throughout the main text, including prior to Table 1, there is no indication that a one-sided design was being used. Therefore, the results in Table 1 and the discussions in Section 3.1 are inconsistent with the specified ATS_0 value and design parameters of the np_x chart as described in their paper. Furthermore, the control limit value provided in their Table 1 for the $\overline{X}$ chart clearly corresponds to a two-sided design with $ATS_0 = \tau = 370$, as it employs the standard 3-Sigma limits. Additionally, the implementation equations and design procedure outlined in their article in Section 2 are for a two-sided design, with no formulas provided for a one-sided design.

3.3 THE OPTIMAL DESIGN FOR THE np_x CHART FOR THE TWO-SIDED DESIGN

In the interest of highlighting the implications of using the two-sided np_x chart instead of the two-sided $\overline{X}$ chart to monitor the process, it was investigated the minimum sample size necessary to guarantee at least the same performance for both control charts in terms of minimum EQL and satisfying the constraint that ATS_0 must be equal to $\tau = 370$.

The optimization procedure of the proposed chart is described as follows:

1. The sampling interval and the sample size are defined.

2. The UCL_{npx} is the maximum of nonconforming units accept in a sample, so it can vary from zero to $n - 1$ ($0 \leq UCL_{npxi} < n$,where $i = 0, 1, \ldots, n - 1$). For each value of UCL_{npx} there is

a k_w (k_{wi},where $i = 0, 1, \ldots, n-1$) that satisfy the condition of $ATS_0 = \tau = 370$. Then, we will have n combinations of the parameters UCL_{npx} and k_w that can be used.

3. To define which UCL_{npx} and k_w combination should be used, it is necessary to calculate the EQL_{npx} for each one (EQL_{npxi}, where $i = 0, 1, \ldots, n-1$). As the objective is to minimize the EQL_{npx}, the best combination will be the one which results in the minimum value of EQL_{npx}.

4. Recalling that the performance of the np_x chart should be at least equal to the $\overline{X}$ chart performance. The minimum EQL_{npx} founded on step 3 is compared to the $\overline{X}$ chart EQL, if it is equal or smaller, than the minimum sample size for a defined h_{npx} is founded. If the minimum EQL_{npx} founded on step 3 is larger than the $\overline{X}$ chart EQL, the sample size should be increased ($n = n + 1$) or the sampling interval reduced and restart the optimization procedure (steps 1 – 4).

Note that when the two design constraints are satisfied, the minimum $EQL_{npx} = EQL_{npxi}$, where $i = 0, 1, 2, \ldots, n-1$ will give us the optimum parameters UCL_{npxi} and k_{wi} that should be used. Thus, the optimization procedure will result in the smallest sample size given a sampling interval, which guarantee the average time to a signal equal to 370 for the in-control condition and the minimum extra quadratic loss within a shift range considering the correspondents upper control limit and warning limit coefficient.

Table 8 shows the combinations of values of n_{npx}, UCL_{npx} and k_w (for $h_{npx} = 1$ in column 3 and for $h_{npx} = 0.5$ in column 4) that generates $ATS_0 = 370$ and the largest EQL_{npx} smaller than the $EQL_{\overline{X}}$ for the same $\overline{X}$ chart parameters showed in Tables 1 and 2.

For the $\overline{X}$ chart, it was adopted $n_{\overline{X}} = 6, h_{\overline{X}} = 1$ and $UCL_{\overline{X}} = 1.2246$ (check column 2 of Table 8) to enable the comparison between the optimum parameters (Table 8) and Wu *et al.*'s(WU et al., 2009) parameters (Table 2). Also, as done by Wu *et al.*(WU et al., 2009), the performance was investigated based on the average time to a signal and the extra quadratic loss for a shift ranging from 0 to 3.5.

First of all, the same sampling interval was considered for for the np_x chart ($h_{npx} = 1$), (check columns 2 and 3 of Table 8), the optimal procedure described above was realized. To observe at least a corresponding performance in terms of ATS_0 and minimum EQL for the np_x chart and $\overline{X}$ chart, the smallest sample size is 38 ($\cong 3.17$ times larger than the indicated by Wu *et al.*(WU et al., 2009) - check Table 1) which provides a $EQL = 3.671$ using the charting parameters $UCL = 9$ and $k_w = 1.6599$. The ATS value of the np_x chart is larger than the ones of the $\overline{X}$ chart only for $\delta \leq 0.50$. When the shift becomes larger the ATS value of the np_x chart becomes smaller when compared to the $\overline{X}$ chart. The ratio $(EQL_{np}/EQL_{\overline{X}})$ shows that the np_x chart is on average 0.3% more effective than the $\overline{X}$ chart across the shift range 0 to 3.5.

It's worth mentioning that considering the same sampling interval ($h_{\overline{X}} = h_{npx} = 1$), to compensate this large sample size of 38 (compared to $n_{\overline{X}} = 6$ of the $\overline{X}$ chart). The cost per unit for the $\overline{X}$ chart should be significantly larger than for the np_x chart. Considering the identical inspection cost per unit of time in equation 1 and a sampling interval (h) of 1 for both charts, the cost per unit of the $\overline{X}$ chart, must be 6.33 times larger than the per unit of the np_x chart, since $c_{\overline{X}} = \frac{n_{npx}}{n_{\overline{X}}} c_{npx} = \frac{38}{6} c_{npx} = 6.33 c_{npx}$.

Similarly, when the sampling interval for the np_x chart is halved $h_{npx} = 0.5$ (Table 8, column 4), it is necessary a sample size of at least 17 units to satisfy the constraint about the ATS_0 and produce

Table 8 – Charting parameters and ATS values of the control charts ($\tau = 370$).

1	2	3	4
δ	ATS		
	$\overline{X}$ *chart*	np_x *chart*	np_x *chart*
	$n_{\overline{X}} = 6$, $h_{\overline{X}} = 1$	$n_{np} = 38$, $h_{np} = 1$	$n_{np} = 17$, $h_{np} = 0.5$
	$UCL_{\overline{X}} = 1.2246$	$UCL_{np} = 9$	$UCL_{np} = 7$
		$w_u = 1.6599$	$w_u = 1.7597$
0.00	370.37	370.00	370.08
0.25	115.36	176.42	206.69
0.50	25.86	32.22	51.85
0.75	7.67	5.43	10.85
1.00	2.94	1.37	2.73
1.25	1.43	0.63	0.92
1.50	0.83	0.51	0.43
1.75	0.61	0.50	0.29
2.00	0.53	0.50	0.26
2.25	0.50	0.50	0.25
2.50	0.50	0.50	0.25
2.75	0.50	0.50	0.25
3.00	0.50	0.50	0.25
3.25	0.50	0.50	0.25
3.50	0.50	0.50	0.25
EQL	3.680	3.671	3.583
$EQL_{npx}/EQL_{\overline{X}}$	1.000	0.997	0.974

source: Elaborated by the author.

the minimum $EQL = 3.583$. The ATS value of the np_x chart is smaller than the ones of the $\overline{X}$ chart for $\delta \geq 1.00$. On average, the np_x chart is 3.6% more effective than the $\overline{X}$ chart across the shift range according to the ratio between EQL (($EQL_{np}/EQL_{\overline{X}}$)).

It is called to mind that the inspection is based on equal cost per unit time (equation 1), which allows the np_x chart to use a larger sample size and/or smaller sampling interval. In this case, considering the same sampling interval for the np_x chart and the $\overline{X}$ chart, the ratio $C_{\overline{X}}/C_{np}$ is at least 6.33 and for $h_{np} = 0.5 * h_{\overline{X}}$, the ratio is at least 5.67.

The analysis for the two-sided np_x control chart reveled that the sample size needs to be increased much more than twice for the np_x chart to present an equivalent performance or outperform the $\overline{X}$ control chart, as recommended in Wu's *et al.* paper.

To conduct a comprehensive analysis of the minimum sample size conditions favoring the two-side np_x chart over the $\overline{X}$ chart, an investigation was undertaken across varying sample sizes $n_{\overline{X}}$ ranging from 2 to 15. Table 9 presents the findings. This ensures a performance at least as reliable as that of the $\overline{X}$ chart (i.e., $EQL_{npx} \leq EQL_{\overline{X}}$) for an average time to signal (ATS_0) of 370, with a sampling interval of (h) of 1. Table 9 also provides corresponding values of $UCL_{\overline{X}}$, $EQL_{\overline{X}}$, k_w, UCL_{npx}, and EQL_{npx}.

It's evident from Table 9 that, across all $n_{\overline{X}}$ values, the sample size required for the np_x chart is more than double that of the $\overline{X}$ chart (see columns 1 and 4). Notably, as the $n_{\overline{X}}$ sample size increases,

Table 9 – The minimum sample size for the two-sided np_x chart n_{npx} which guarantee a $EQL_{npx} \leq EQL_{\overline{X}}$, $ATS_0 = 370$ varying the $n_{\overline{X}}$ from 2 to 15 for a sampling interval of $h = 1$ for both charts.

$\overline{X}$ *chart*			np_x *chart*				
$n_{\overline{X}}$	$UCL_{\overline{X}}$	$EQL_{\overline{X}}$	n_{npx}	k_w	UCL_{npx}	EQL_{npx}	$n_{npx}/n_{\overline{X}}$
2	2.12	10.528	5	1.832	2	9.701	2.50
3	1.73	6.675	10	1.846	3	6.526	3.33
4	1.50	5.052	17	1.697	5	5.038	4.25
5	1.34	4.196	27	1.673	7	4.151	5.40
6	1.22	3.680	38	1.660	9	3.672	6.33
7	1.13	3.342	52	1.612	12	3.329	7.43
8	1.06	3.106	67	1.586	15	3.106	8.38
9	1.00	2.934	85	1.584	18	2.931	9.44
10	0.95	2.803	105	1.590	21	2.801	10.50
11	0.90	2.702	127	1.573	25	2.700	11.55
12	0.87	2.621	151	1.566	29	2.620	12.58
13	0.83	2.555	177	1.565	33	2.555	13.62
14	0.80	2.501	206	1.571	37	2.500	14.71
15	0.77	2.456	236	1.547	43	2.455	15.73

source: Elaborated by the author.

the proportionate increase in n_{npx} needed to satisfy the $EQL_{npx} \leq EQL_{\overline{X}}$ relation also rises.

This observation suggests that the attractiveness of using the np_x chart over $\overline{X}$ the chart increases with decreasing n_{npx}. For instance, if the inspection cost per unit of time to measure 3 units using the $\overline{X}$ chart is equivalent to the inspection cost per unit of time to inspect 10 units using a go/no-go gauge, then adopting the np_x chart over the $\overline{X}$ chart proves advantageous. This decision leads to approximately a 2% improvement in process performance $\left(\frac{EQL_{npx}}{EQL_{\overline{X}}} \cong 0.98\right)$.

The relation $n_{npx}/n_{\overline{X}}$ (check column 8 of Table 9) indicates how many times the sample size for the np_x chart should be increased relative to that of the $\overline{X}$ chart ($n_{npx} = \frac{n_{npx}}{n_{\overline{X}}} n_{\overline{X}}$). Since in practice n_{npx} is an integer. The ceiling function is used to obtain an integer value for n_{npx}, ensuring better performance for the np_x chart, Specifically $n_{npx} = \left\lceil \left(\frac{n_{npx}}{n_{\overline{X}}}\right) \right\rceil n_{\overline{X}}$.

From Table 9, it's noticeable that for $2 \leq n_{\overline{X}} \leq 15$, the largest integer smaller than $\frac{n_{npx}}{n_{\overline{X}}}$, denoted as $\left\lfloor \frac{n_{npx}}{n_{\overline{X}}} \right\rfloor$, precisely equals the sample size value of the $\overline{X}$ chart ($n_{\overline{X}}$), as indicated in column 1 of Table 9. This leads to $\left\lceil \frac{n_{npx}}{n_{\overline{X}}} \right\rceil = n_{\overline{X}} + 1$ resulting in the following relationship between the number of samples for the np_x and the $\overline{X}$ chart:

$$n_{npx} = n_{\overline{X}}(n_{\overline{X}} + 1) \tag{11}$$

.

So, for a given value of $n_{\overline{X}}$, one can quickly determine the value of n_{npx} that guarantees a superior performance for np_x chart (that is close to the optimal performance), using Equation 11.

In summary, in a two-sided design, the np_x appear less appealing due to its requirement for a notably larger sample size to achieve equivalent performance compared to the two-sided $\overline{X}$ chart, as indicated in Table 9. However, this scenario shifts when considering the one-sided np_x chart, where the np_x design becomes considerably more enticing. The subsequent section (Section 3.4) outlines the performance of the one-sided np_x chart and compares it with that of the one-sided $\overline{X}$ chart.

3.4 IMPLEMENTATION AND DESIGN OF THE np_x CHARTS FOR THE ONE-SIDED CASE

For the one-sided design, a slight adjustment is required in the fraction of nonconforming items in a sample (np_x), as the focus is now solely on detecting either oversizing or undersizing. This adjustment is presented in Equation 12.

$$p = Pr(X < w_L) = P(X > w_U) = 1 - \Phi(k_w - \delta) \tag{12}$$

The same optimization procedure detailed in Section 3.3 was employed for the one-sided design of the np_x chart. The resulting outcomes are summarized in Table 10. To facilitate a comparative analysis between the one-sided design (Table 10) and the two-sided design (Table 8), identical sample sizes and sampling intervals were utilized for the $\overline{X}$ ($n_{\overline{X}} = 6$ and $h_{\overline{X}} = 1$). Subsequently, the minimum sample size for the np_x chart was determined, ensuring at least equivalent performance to that of the $\overline{X}$ chart.

Table 10 – Charting parameters and performance measured ATS and EQL calculated for the np_x and $\overline{X}$ charts one-sided ($\tau = 370$).

1	2	3	4	5
δ	ATS			
	$\overline{X}$ *chart*	np_x *chart*	np_x *chart*	np_x *chart*
	$n_{\overline{X}} = 6$, $h_{\overline{X}} = 1$	$n_{npx} = 6$, $h_{npx} = 1$	$n_{npx} = 9$, $h_{npx} = 1$	$n_{npx} = 5$, $h_{npx} = 0.5$
	$UCL_{\overline{X}} = 1.1358$	$UCL_{npx} = 3$	$UCL_{npx} = 5$	$UCL_{npx} = 3$
		$w_u = k_w = 1.1653$	$w_u = k_w = 0.8583$	$w_u = k_w = 1.1177$
0.00	370.37	370.00	370.00	370.00
0.25	66.11	86.05	64.91	85.37
0.50	16.25	25.04	15.82	24.30
0.75	5.30	8.95	5.16	8.41
1.00	2.20	3.85	2.16	3.47
1.25	1.14	1.95	1.13	1.67
1.50	0.73	1.14	0.73	0.92
1.75	0.57	0.78	0.57	0.58
2.00	0.52	0.61	0.52	0.41
2.25	0.50	0.54	0.50	0.32
2.50	0.50	0.51	0.50	0.28
2.75	0.50	0.50	0.50	0.26
3.00	0.50	0.50	0.50	0.25
3.25	0.50	0.50	0.50	0.25
3.50	0.50	0.50	0.50	0.25
EQL	3.079	3.838	3.057	2,840
$EQL_{npx}/EQL_{\overline{X}}$	1.000	1.247	0.993	0.922

source: Elaborated by the author.

Upon examination of the results presented in Table 10, it is evident that the constraint of $ATS_0 =$

$\tau = 370$ is met across all cases. Utilizing identical sample sizes ($n_{\overline{X}} = n_{npx} = 6$) and sampling intervals ($h_{\overline{X}} = h_{npx} = 1$) for both the np_x and the $\overline{X}$ chart, as indicated in Table 10, columns 2 and 3, the ratio $EQL_{npx}/EQL_{\overline{X}}$ reveals that the $\overline{X}$ chart holds only a 24.7% superiority over the np_x chart. Remarkably, by simply increasing the sample size of the np_x chart to nine units (from $n_{npx} = 6$ to $n_{npx} = 9$) while maintaining the same sampling interval for both charts, as shown in Table 10, column 4, the np_x chart surpasses the $\overline{X}$ chart by 0.7%.

Given that the np_x chart operates as an attribute control chart, the cost per unit for inspection may be lower compared to that of the $\overline{X}$ chart (see Equation 1). Consequently, it becomes feasible to reduce the sampling interval for the np_x chart. With the sampling interval halved for the np_x chart, as illustrated in Table 10, column 5, only 5 units are required for the np_x chart to surpass the $\overline{X}$ chart by 7.8% based on the $EQL_{npx}/EQL_{\overline{X}}$ ratio.

Notably, the results shown on Table 10 one-sided design offers a more compelling option than the two-sided design as it does not require a doubling of the sample size for the np_x chart to outperform the $\overline{X}$ chart.

For a comprehensive assessment of the minimum sample size required for the one-sided np_x chart to surpass the $\overline{X}$ chart, Table 11 presents the minimum sample size needed for the np_x chart while varying the $n_{\overline{X}}$ values from 2 to 15. This ensures that the performance reliability equals or exceeds that of the $\overline{X}$ chart (i.e., where $EQL_{npx} \leq EQL_{\overline{X}}$) for an ATS of 370, with a sampling interval of $h = 1$. Table 11 also includes corresponding values of $UCL_{\overline{X}}$, $EQL_{\overline{X}}$, k_w, UCL_{npx}, and EQL_{npx}.

Table 11 – The minimum sample size for the one-sided np_x chart n_{npx} which guarantee a $EQL_{npx} \leq EQL_{\overline{X}}$, $ATS_0 = 370$ varying the $n_{\overline{X}}$ from 2 to 15 for a sampling interval of $h = 1$ for both charts.

$\overline{X}$ *chart*			np_x *chart*				
$n_{\overline{X}}$	$UCL_{\overline{X}}$	$EQL_{\overline{X}}$	n_{npx}	k_w	UCL_{npx}	EQL_{npx}	$n_{npx}/n_{\overline{X}}$
2	1.97	7.418	3	1.083	2	6.696	1.50
3	1.61	4.976	5	1.004	3	4.340	1.67
4	1.39	3.948	6	1.165	3	3.838	1.50
5	1.24	3.406	8	1.044	4	3.252	1.60
6	1.14	3.079	9	0.858	5	3.057	1.50
7	1.05	2.865	11	0.804	6	2.806	1.57
8	0.98	2.716	13	0.759	7	2.644	1.63
9	0.93	2.607	14	0.651	8	2.583	1.56
10	0.88	2.524	16	0.626	9	2.488	1.60
11	0.84	2.460	17	0.688	9	2.451	1.55
12	0.80	2.409	19	0.528	11	2.390	1.58
13	0.77	2.367	20	0.584	11	2.364	1.54
14	0.74	2.333	22	0.566	12	2.322	1.57
15	0.72	2.304	24	0.549	13	2.289	1.60

source: Elaborated by the author.

Table 11 demonstrates that doubling the sample size for the np_x chart (as indicated in column 8) is unnecessary across all values of $n_{\overline{X}}$. For example, consider the scenario where the inspection cost per unit of time to measure 3 units using the $\overline{X}$ chart is equivalent to the inspection cost per unit of time to inspect 5 units using a go/no-go gauge. In this case, adopting the np_x chart for process monitoring presents clear advantages. The performance improvement is substantial, with approximately 14% enhancement ($\frac{EQL_{npx}}{EQL_{\overline{X}}} \cong 0.86$), compared to utilizing the $\overline{X}$ chart.

This finding underscores the cost-effectiveness and efficiency of the np_x chart in certain scenarios, emphasizing its potential for enhancing quality control processes.

In contrast to the two-sided case (refer to Section 11), the relationship $n_{npx}/n_{\overline{X}}$ (see column 8 of Table 11)—indicating how many times the sample size for the np_x chart should be increased relative to the $\overline{X}$ chart ($n_{npx} = \frac{n_{npx}}{n_{\overline{X}}} n_{\overline{X}}$)—does not exhibit a consistent trend of proportionality with $n_{\overline{X}}$. Observing across all values of $n_{\overline{X}}$, the ratio $\frac{n_{npx}}{n_{\overline{X}}}$ fluctuates approximately between 1.5 and 1.6. Therefore, to encompass all cases that ensure better performance for the np_x chart, we can approximate the ratio as $\frac{n_{npx}}{n_{\overline{X}}} = 1.6$. In practical application, since n_{npx} must be an integer, it's necessary to consider the smallest integer larger than $\frac{n_{npx}}{n_{\overline{X}}}$, denoted as $\left\lceil \frac{n_{npx}}{n_{\overline{X}}} \right\rceil$. This approximation strategy ensures practical feasibility while maximizing the performance potential of the np_x chart relative to the $\overline{X}$ chart across various scenarios.

To succinctly express the relationship between the number of samples for the np_x chart (n_{npx}) and the $\overline{X}$ chart ($n_{\overline{X}}$), considering the approximation discussed, you can use the following expression:

$$n_{npx} = \lceil 1.6 n_{\overline{X}} \rceil \tag{13}$$

.

This formula captures the idea that the sample size for the np_x chart should be adjusted to approximately 1.6 times the sample size of the $\overline{X}$ chart in order to ensure better performance. The use of the ceiling function ensures that the sample size for the np_x chart is an integer.

To efficiently determine the values of n_{npx} that lead to superior performance for the np_x chart, approaching optimal levels, Equation 13 can be utilized for any given $n_{\overline{X}}$ value. In other words, the cost per unit of time to measure 3 units using the $\overline{X}$ chart is equivalent to the inspection cost per unit of time to inspect 5 units using a gauge.

The analysis reinforced that the one-sided np_x chart is more attractive, since it is not necessary to even double the sample size to achieve the same or better performance for the np_x chart compared to the $\overline{X}$ chart.

Adjusting the sample size and sampling interval emerges as a potent strategy to enhance performance, especially concerning the inspection cost per unit of time for both control charts. It's important to note that the inspection cost per unit of time doesn't encompass equipment costs, which would further favor the np_x chart. This is because a go/no-go gauge, typically used with the np_x chart, tends to be more cost-effective than precision measuring equipment like a micrometer used with the $\overline{X}$ chart.

3.5 PRACTICAL EXAMPLE

In this section a practical example is provided to highlight the advantages of employing the np_x chart to monitor the mean of a continuous variable. To this end, an example from Wu et al. (2009) was adapted, focusing on the diameter X of a shaft, a critical dimension in manufacturing contexts. In this example, the probability distribution of X can be approximated by a normal distribution and there is no evidence to reject the assumption of independence among X readings. The in-control parameters are: $\mu_0 = 8.00$ and $\sigma_0 = 12.00$ mm.

It's worth noting that both oversizing and undersizing must be promptly detected, necessitating the use of two-sided control charts for our analysis.

For the 3-sigma $\overline{X}$ chart, the in-control average time to signal (ATS) is set at $ATS_0 = 370$ hours. The sample size and sampling interval are defined as $n_{\overline{X}} = 4$ and $h_{\overline{X}} = 1$, respectively.

To estimate the inspection costs per unit, a real field experiment was conducted involving four operators inspecting the diameters X of 10 specimens each. Initially, they used a ring gauge, followed by measurements with a micrometer. Let t represent the average time spent on inspecting an individual specimen.

$$\frac{n_{np}}{n_{\overline{X}}} = \frac{c_{\overline{X}}}{c_{np}} = \frac{t_{\overline{X}}}{t_{np}} = \frac{9.525}{2.125} = 4.482 \tag{14}$$

where $n_{\overline{X}}$ and n_{npx} are the sample size, $c_{\overline{X}}$ and $c_{np}x$ are the inspection cost per unit, $t_{\overline{X}}$ and t_{npx} are the average time spent on an individual specimen, respectively for the $\overline{X}$ chart and np_x chart.

Based on the relationship between the average time spent on both types of charts, the sample size and sampling interval for the np_x chart are determined as $n_{np} = 17$ and $h_{np} = 1$, respectively.

The np_x chart was optimized as described on section 3.3 and the optimal charting parameters were defined as $UCL_{npx} = 5$ and $k_w = 1.6967520$.

Both control charts were evaluated over a shift range from 0 to 3.5, see Table 12.

Results show that the np_x chart is superior to the $\overline{X}$ chart in average because the EQL_{npx} (5.035) is smaller than the $EQL_{\overline{X}}$ (5.052) as shown in Table 12.

Figure 7 illustrates a comparative analysis of the ATS values for the np_x and $\overline{X}$ control charts across multiple shifts in the mean. Notably, the $\overline{X}$ chart outperforms the np_x chart only for the shifts 0.25 and 0.50 due to the $ATS_{npx}/ATS_{\overline{X}}$ ratio exceeding one. However, for all other shifts, the np_x chart either matches or surpasses the performance of the $\overline{X}$ chart.

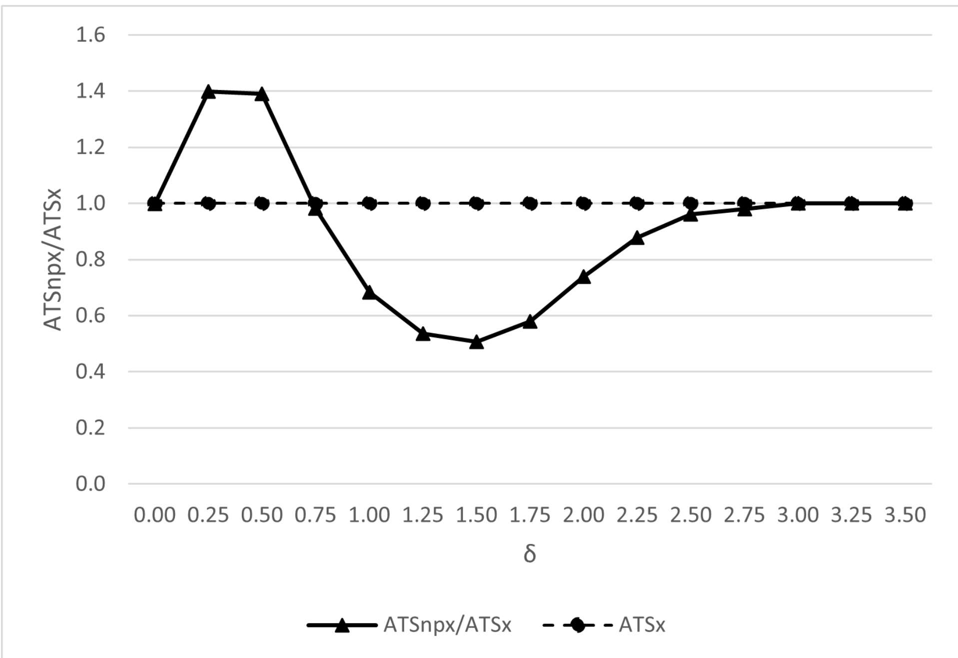
This is clearly a condition where using the np_x control chart instead of the $\overline{X}$ control chart to monitor the process would be advantageous.

Table 12 – Charting parameters and performance measured ATS and EQL calculated for the np_x chart and the $\overline{X}$ chart ($\tau = 370$).

1	2	3
δ	ATS	
	$\overline{X} chart$	$np_x chart$
	$n_{\overline{X}} = 4, h_{\overline{X}} = 1$	$n_{np} = 17, h_{np} = 1$
	$UCL_{\overline{X}} = 8.0180$	$UCL_{np} = 5$
		$k_w = 1.6967520$
0.00	370.37	369.97
0.25	154.71	216.28
0.50	43.39	60.33
0.75	14.46	14.19
1.00	5.80	3.96
1.25	2.74	1.47
1.50	1.50	0.76
1.75	0.95	0.55
2.00	0.69	0.51
2.25	0.57	0.50
2.50	0.52	0.50
2.75	0.51	0.50
3.00	0.50	0.50
3.25	0.50	0.50
3.50	0.50	0.50
EQL	5.052	5.035
$EQL_{npx}/EQL_{\overline{X}}$	1.000	0.997

source: Elaborated by the author.

Figure 7 – Comparison of ATS values of the np_x and $\overline{X}$ control charts



source: Elaborated by the author.

4 THE np_x CONTROL CHART WITH ESTIMATED PARAMETERS

Accurate parameter estimation is fundamental in control chart analysis, as it directly impacts the control chart's ability to detect deviations from expected process behavior. The np_x chart serves as a valuable tool for monitoring the mean of a continuous variable, particularly in situations where data can be categorized as conforming or nonconforming based on a predefined limits.

This study examines the influence of sample size on parameter estimation and its subsequent effect on the performance of the np_x control chart. Specifically, it investigates scenarios where one or both of the process parameters—mean and variance—are unknown.

Following the methodology outlined by Chakraborti (CHAKRABORTI, 2000), for each case, it is demonstrated that the lower bound for the Average Run Length (ARL) is determined by the reciprocal of the signal probability. Exact expressions are derived for the run length distribution, ARL, and false alarm rate. These expressions are then evaluated across various scenarios, offering practical insights into the performance of the np_x chart when parameters are estimated.

Additionally, the derivations presented serve as illustrative examples of probability and distribution calculations through conditioning, adding pedagogical value to the study.

4.1 MEAN STANDARD UNKNOWN BUT VARIANCE SPECIFIED (UK)

Variance is known: $\sigma^2 = \sigma_0^2$ and the mean μ is estimated by $\overline{\overline{X}}$ (mean of the m subgroups mean). Now the warning limits are based on the estimated parameters of the mean :

$$\widehat{w}_L = \overline{\overline{X}} - k_w \sigma_0 \tag{15}$$

$$\widehat{w}_U = \overline{\overline{X}} + k_w \sigma_0 \tag{16}$$

The probability of X be classified as nonconforming $\widehat{p}$ is now a random variable:

$$\begin{aligned}\widehat{p} = P(X < \widehat{w}_L \; ou \; X > \widehat{w}_U) = 1 - P(\widehat{w}_L \leq X \leq \widehat{w}_U) \\ = 1 - \Phi\left(Z \leq \frac{\overline{\overline{X}} - \mu_0}{\sigma_0} - \delta + k_W\right) + \Phi\left(Z \leq \frac{\overline{\overline{X}} - \mu_0}{\sigma_0} - \delta - k_W\right)\end{aligned}$$

where μ_0 is the in-control process mean $\delta = \left(\frac{\mu - \mu_0}{\sigma_0}\right)$ is the magnitude of a mean shift in terms of σ_0 and Φ is the cumulative standard normal distribution.

It is possible to demonstrate that $\frac{\overline{\overline{X}} - \mu_0}{\sigma_0}\sqrt{mn} = Z_1$ follows a normal distribution:

$$\frac{\overline{\overline{X}} - \mu_0}{\sigma_0} = \frac{Z_1}{\sqrt{mn}} \tag{17}$$

So we can replace $\frac{\overline{\overline{X}}-\mu_0}{\sigma_0}$ as shown in equation 18:

$$\widehat{p} = 1 - \Phi\left(\frac{Z_1}{\sqrt{mn}} - \delta + k_W\right) + \Phi\left(\frac{Z_1}{\sqrt{mn}} - \delta - k_W\right) \tag{18}$$

The run length (RL) of the np_x is used to measure the performance of the chart, it is a random variable. Commonly it is used the mean of the run length (ARL). When the parameters are estimated, we have two possibles approaches about the RL: the conditional and the unconditional one.

The conditional run length (CRL) consider a specific estimation, in other words, it changes from practitioner to practitioner. Thus, it follows a geometric distribution like when the parameters are considered to be known.

The probability of an alarm for the conditional approach can be expressed as shown below:

$$\begin{aligned} P(\widehat{p}) = P(D > UCL|p) &= 1 - \sum_{i=0}^{UCL_{np}} C_i^{n_{np}} \widehat{p}^i (1-\widehat{p})^{n_{np}-i} \\ &= 1 - \sum_{i=0}^{UCL_{np}} C_i^{n_{np}} \left(1 - \Phi\left(\frac{Z_1}{\sqrt{mn}} - \delta + k_W\right) + \Phi\left(\frac{Z_1}{\sqrt{mn}} - \delta - k_W\right)\right)^i \\ &\qquad \left(\Phi\left(\frac{Z_1}{\sqrt{mn}} - \delta + k_W\right) - \Phi\left(\frac{Z_1}{\sqrt{mn}} - \delta - k_W\right)\right)^{n_{np}-i} \end{aligned} \tag{19}$$

The power of detecting a mean shift is $\beta = 1 - P\left(\widehat{p}\right)$, the conditional average time to a signal when the process is out-of-control is:

$$\begin{aligned} CATS &= \left(E\left(RL|\overline{\overline{X}}\right) - 0.5\right)h = \left(\frac{1}{1-\beta} - 0.5\right)h = \left(\frac{1}{P\left(\widehat{p}\right)} - 0.5\right)h \\ &= \left(\left(1 - \sum_{i=0}^{UCL_{np}} C_i^{n_{np}} \left(1 - \Phi\left(\frac{Z_1}{\sqrt{mn}} - \delta + k_W\right) + \Phi\left(\frac{Z_1}{\sqrt{mn}} - \delta - k_W\right)\right)^i \right.\right. \\ &\qquad \left.\left. \left(\Phi\left(\frac{Z_1}{\sqrt{mn}} - \delta + k_W\right) - \Phi\left(\frac{Z_1}{\sqrt{mn}} - \delta - k_W\right)\right)^{n_{np}-i}\right)^{-1} - 0.5\right)h \end{aligned} \tag{20}$$

When the process is considered in control, $\delta = 0$, therefore $\alpha = P(\widehat{p}_0) = CFAR$ (Conditional False Alarm Rate), shown in equation 21.

$$\begin{aligned} CFAR = &\left(1 - \sum_{i=0}^{UCL_{np}} C_i^{n_{np}} \left(1 - \Phi\left(\frac{Z_1}{\sqrt{mn}} + k_W\right) + \Phi\left(\frac{Z_1}{\sqrt{mn}} - k_W\right)\right)^i \right. \\ &\qquad \left. \left(\Phi\left(\frac{Z_1}{\sqrt{mn}} + k_W\right) - \Phi\left(\frac{Z_1}{\sqrt{mn}} - k_W\right)\right)^{n_{np}-i}\right)h \end{aligned} \tag{21}$$

The conditional average time to a signal when the process is in-control $(CATS_0)$ is inversely proportional to $CFAR$:

$$CATS_0 = \frac{1}{CFAR} \tag{22}$$

Note that $CFAR$, $CATS_0$ and $CATS$ are random variables and they change from practitioner to practitioner. Thus, the unconditional ATS is:

$$\begin{aligned} ATS &= (E(RL) - 0.5)h = (E_{\overline{\overline{X}}} E(RL|\overline{\overline{X}}) - 0.5)h \\ &= \int_{-\infty}^{\infty} \left(\left(\left(\left(1 - \sum_{i=0}^{UCL_{np}} C_i^{n_{np}} \left(1 - \Phi\left(\frac{Z_1}{\sqrt{mn}} - \delta + k_W\right) + \Phi\left(\frac{Z_1}{\sqrt{mn}} - \delta - k_W\right)\right)^i \right.\right.\right.\right. \\ &\qquad \left.\left.\left.\left. \left(\Phi\left(\frac{Z_1}{\sqrt{mn}} - \delta + k_W\right) - \Phi\left(\frac{Z_1}{\sqrt{mn}} - \delta - k_W\right)\right)^{n_{np}-i} \right)^{-1} - 0.5 \right) h \right) \phi z_1 dz_1 \end{aligned} \tag{23}$$

To compare the performance of two charts, the Extra Quadratic Loss (EQL) could be used. For a given mean shift, the incurred quadratic loss is simply equal to δ^2. And the loss in quality is proportional to $ATS(\delta)$. The Extra Quadratic Loss (EQL) also becomes a random variable changing from practitioner to practitioner, it will be called $CEQL$, equation 24.

$$\begin{aligned} CEQL &= \frac{1}{\delta_{max} - \delta_{min}} \int_{\delta_{min}}^{\delta_{max}} \delta^2 CATS(\delta) d\delta = \frac{1}{\delta_{max} - \delta_{min}} \int_{\delta_{min}}^{\delta_{max}} \delta^2 \\ & \left[\left(\left(1 - \sum_{i=0}^{UCL_{np}} C_i^{n_{np}} \left(1 - \Phi\left(\frac{Z_1}{\sqrt{mn}} - \delta + k_W\right) + \Phi\left(\frac{Z_1}{\sqrt{mn}} - \delta - k_W\right)\right)^i \right.\right.\right. \\ &\qquad \left.\left.\left. \left(\Phi\left(\frac{Z_1}{\sqrt{mn}} - \delta + k_W\right) - \Phi\left(\frac{Z_1}{\sqrt{mn}} - \delta - k_W\right)\right)^{n_{np}-i} \right)^{-1} - 0.5 \right) * h \right] d\delta \end{aligned} \tag{24}$$

The overall EQL can be calculated as equation 25:

$$\begin{aligned} EQL &= \frac{1}{\delta_{max} - \delta_{min}} \int_{\delta_{min}}^{\delta_{max}} \delta^2 ATS(\delta) d\delta = \frac{1}{\delta_{max} - \delta_{min}} \int_{\delta_{min}}^{\delta_{max}} \int_{-\infty}^{\infty} \delta^2 \\ & \left[\left(\left(1 - \sum_{i=0}^{UCL_{np}} C_i^{n_{np}} \left(1 - \Phi\left(\frac{Z_1}{\sqrt{mn}} - \delta + k_W\right) + \Phi\left(\frac{Z_1}{\sqrt{mn}} - \delta - k_W\right)\right)^i \right.\right.\right. \\ &\qquad \left.\left.\left. \left(\Phi\left(\frac{Z_1}{\sqrt{mn}} - \delta + k_W\right) - \Phi\left(\frac{Z_1}{\sqrt{mn}} - \delta - k_W\right)\right)^{n_{np}-i} \right)^{-1} - 0.5 \right) * h \right] \phi z_1 dz_1 d\delta \end{aligned} \tag{25}$$

In order to compare the performance between the np_x chart considering that the parameters are known and the estimated parameters, it was simulated $100,000$ values of $CATS_0$ and $CEQL$ considering different numbers of samples m $(25, 50, 100, 500$ and $1000)$ each one of sample size $n = 38$, sampling interval $h = 1$, $UCL_{npx} = 9$ and $k_w = 1.6599319$, presented on Table 8, column 3.

The results are displayed on Tables 13 and 14.

Table 13 – Simulation of $100,000$ $CATS_0$ considering the mean is estimated and the standard deviation is known ($n = 38$, $UCL_{np_x} = 9$ and $k_w = 1.6599319$).

m	$CATS_0$ Mean	$CATS_0$ Standard Deviation	$CATS_0$ Median	$CATS_0$ 0.25 Quantile	$CATS_0$ 0.75 Quantile
25	364.70	6.60	367.25	363.07	369.00
50	367.07	3.41	368.39	366.26	369.26
100	368.27	1.73	368.94	367.86	369.38
500	369.25	0.35	369.39	369.18	369.48
1000	369.38	0.17	369.45	369.34	369.49

source: Elaborated by the author.

Table 14 – Simulation of $100,000$ $CEQL$ considering the mean is estimated and the standard deviation is known ($n = 38$, $UCL_{np_x} = 9$ and $k_w = 1.6599319$).

m	$CEQL$ Mean	$CEQL$ Standard Deviation	$CEQL$ Median	$CEQL$ 0.25 Quantile	$CEQL$ 0.75 Quantile
25	3.702	0.344	3.672	3.456	3.915
50	3.685	0.242	3.670	3.516	3.837
100	3.678	0.170	3.670	3.560	3.788
500	3.672	0.076	3.671	3.620	3.722
1000	3.672	0.053	3.671	3.635	3.707

source: Elaborated by the author.

As the sample size (m) increases, it becomes apparent that the mean of $CATS_0$ approaches the desired value of 370, Table 13 columns 1 and 2. Simultaneously, the standard deviation decreases with the increasing number of samples, check Table 13, column 3. For example, with 25 samples used to estimate μ_0, the mean of $CATS_0$ is 364.70, accompanied by a standard deviation of 6.60. In contrast, when the sample size increases to 500, the mean of $CATS_0$ becomes $369,25$, with a standard deviation of 0.35. Therefore, employing a larger sample size for estimating $CATS_0$ leads to results that closely align with the desired value. It is also possible to observe that estimating the mean of the process presents a small effect on the average time to a signal even when a small number of samples is used on the estimation, since the $CATS_0$ is 1.5% smaller than $\tau = 370.4$

The same trend is evident with $CEQL$: as the sample size increases, the mean of $CEQL$ decreases. For example, in Table 14, columns 1 and 2, when $m = 25$, the mean $CEQL$ is 3.702. However, with an increased sample size of 500, the mean $CEQL$ decreases to 3.672. Given that the objective is to minimize EQL, employing a larger sample size leads to more precise results. Furthermore, the effect of estimating μ_0 is minimal, as evidenced by the fact that the mean of $CEQL$ for $m = 25$ is only 0.8% greater than the EQL when the parameters are known (3.671, check Table 8 column 2).

The unconditional approach for the np_x chart was analyzed under the same conditions: varying the number of samples: $m = 25, 50, 100, 500$ and 1000, the sample size $n = 38$, sampling interval $h = 1$, $UCL_{npx} = 9$ and $k_w = 1.6599319$. The ATS and EQL values considering that μ_0 is estimated were compared to the ATS and EQL values when the parameters are known for the np_x chart and the $\overline{X}$ chart (Table 8).

Table 15 – Comparison of ATS of the $\overline{X}$ chart and the two-sided np_x chart when parameters are known and when the mean is estimated ($\tau = 370$).

1	2	3	4	5	6	7	8
δ	ATS						
	$\overline{X} chart$	np_x $chart$			np_x $chart$		
	$n_{\overline{X}} = 6$	$n_{npx} = 38$			$n_{npx} = 38$		
	$UCL_{\overline{X}} = 1.2246$	$UCL_{npx} = 9$			$UCL_{npx} = 9$		
		$w_u = k_w = 1.6599$			$w_u = k_w = 1.6599$		
		known parameters			estimated parameters		
			$m = 25$	$m = 50$	$m = 100$	$m = 500$	$m = 1000$
0.00	369.87	370.00	364.67	367.06	368.27	369.25	369.38
0.25	115.36	176.42	177.70	177.07	176.75	176.48	176.45
0.50	25.86	32.22	33.16	32.69	32.45	32.27	32.24
0.75	7.67	5.43	5.57	5.50	5.46	5.43	5.43
1.00	2.94	1.37	1.39	1.38	1.37	1.37	1.37
1.25	1.41	0.63	0.63	0.63	0.63	0.63	0.63
1.50	0.83	0.51	0.51	0.51	0.51	0.51	0.51
1.75	0.61	0.50	0.50	0.50	0.50	0.50	0.50
2.00	0.53	0.50	0.50	0.50	0.50	0.50	0.50
2.25	0.51	0.50	0.50	0.50	0.50	0.50	0.50
2.50	0.50	0.50	0.50	0.50	0.50	0.50	0.50
2.75	0.50	0.50	0.50	0.50	0.50	0.50	0.50
3.00	0.50	0.50	0.50	0.50	0.50	0.50	0.50
3.25	0.50	0.50	0.50	0.50	0.50	0.50	0.50
3.50	0.50	0.50	0.50	0.50	0.50	0.50	0.50
EQL	3.680	3.671	3.700	3.686	3.678	3.672	3.672
$EQL_{npx}/EQL_{\overline{X}}$	1.000	0.997	1.006	1.001	0.999	0.998	0.998

source: Elaborated by the author.

Table 15 presents the results considering the estimated and known parameters.

It can be observed that the difference between the unconditional values (Table 15) and the simulated values for ATS and EQL (Tables 13 and 14) is minimal, demonstrating that the simulation yielded satisfactory results.

Despite ATS_0 values not meeting the requirement of being exactly 370 when fewer than 500 samples are used, they closely approximate the desired value, as depicted in Table 15, columns 4, 5, and 6.

It is also possible to verify that the np_x chart with estimated parameters becomes advantageous compared to the $\overline{X}$ chart when at least 100 samples are used to estimate the process mean, as the ratio $EQL_{npx}/EQL_{\overline{X}}$ becomes less than 1 only in these cases (Table 15, columns 6, 7, and 8). However, when 25 samples are used to estimate the process mean, the $\overline{X}$ chart proves to be superior to the np_x chart by only 0.6%.Thus, the impact of estimating the process mean appears negligible under the examined conditions.

4.2 MEAN STANDARD SPECIFIED BUT VARIANCE UNKNOWN (KU)

Mean is known: $\mu = \mu_0$ and the variance σ^2 is estimated by S_p^2 (variance of the m subgroups variance).

Now the warning limits are based on the estimated parameters of the standard deviation (S_p)

$$\widehat{w}_U = \mu_0 + k_w S_p \tag{26}$$

$$\widehat{w}_L = \mu_0 - k_w S_p \tag{27}$$

The probability of X be classified as nonconforming $\widehat{p}$ is now a random variable:

$$\widehat{p} = P\left(X < \widehat{w}_L \; ou \; X > \widehat{w}_U\right) = 1 - P\left(\widehat{w}_L \leq X \leq \widehat{w}_U\right) = 1 - \Phi\left(k_w \frac{S_p}{\sigma} - \delta\right) + \Phi\left(-k_w \frac{S_p}{\sigma} - \delta\right)$$

where μ_0 is the in-control process mean $\delta = \left(\frac{\mu - \mu_0}{\sigma_0}\right)$ is the magnitude of a mean shift in terms of σ_0 and Φ is the cumulative standard normal distribution.

It is possible to demonstrate that $\frac{S_p^2}{\sigma^2} m(n-1) = Y$ follows a chi-squared distribution with $m(n-1)$ degrees of freedom. As we have $\frac{S_p}{\sigma}$ in the $\widehat{p}$ equation, it can be replaced as shown below:

$$\frac{S_p}{\sigma} = \sqrt{\frac{Y}{m(n-1)}} \tag{28}$$

Thus the $\widehat{p}$ equation now is:

$$\widehat{p} = 1 - \Phi\left(k_w \sqrt{\frac{Y}{m(n-1)}} - \delta\right) + \Phi\left(-k_w \sqrt{\frac{Y}{m(n-1)}} - \delta\right) \tag{29}$$

The probability of an alarm for the conditional approach can be expressed as shown below:

$$
\begin{aligned}
P(\widehat{p}) = P(D > UCL|p) = 1 - & \sum_{i=0}^{UCL_{np}} C_i^{n_{np}} \widehat{p}^i (1-\widehat{p})^{n_{np}-i} \\
= 1 - \sum_{i=0}^{UCL_{np}} C_i^{n_{np}} & \left(1 - \Phi\left(k_w\sqrt{\frac{Y}{m(n-1)}} - \delta\right) + \Phi\left(-k_w\sqrt{\frac{Y}{m(n-1)}} - \delta\right)\right)^i \\
& \left(\Phi\left(k_w\sqrt{\frac{Y}{m(n-1)}} - \delta\right) - \Phi\left(-k_w\sqrt{\frac{Y}{m(n-1)}} - \delta\right)\right)^{n_{np}-i}
\end{aligned} \tag{30}
$$

The power of detecting a mean shift is $\beta = 1 - P\left(\widehat{p}\right)$, the conditional average run length when the process is out-of-control is:

$$
\begin{aligned}
CATS = & \left(E\left(RL|\overline{\overline{X}}\right) - 0.5\right)h = \left(\frac{1}{1-\beta} - 0.5\right)h = \left(\frac{1}{P(\widehat{p})} - 0.5\right)h \\
= & \Bigg(\Bigg(1 - \sum_{i=0}^{UCL_{np}} C_i^{n_{np}} \left(1 - \Phi\left(k_w\sqrt{\frac{Y}{m(n-1)}} - \delta\right) + \Phi\left(-k_w\sqrt{\frac{Y}{m(n-1)}} - \delta\right)\right)^i \\
& \left(\Phi\left(k_w\sqrt{\frac{Y}{m(n-1)}} - \delta\right) - \Phi\left(-k_w\sqrt{\frac{Y}{m(n-1)}} - \delta\right)\right)^{n_{np}-i}\Bigg)^{-1} - 0.5\Bigg)h
\end{aligned} \tag{31}
$$

When the process is considered in control, $\delta = 0$, therefore $\alpha = P(\widehat{p}_0) = CFAR$ (Conditional False Alarm Rate).

$$
\begin{aligned}
CFAR = & \Bigg(1 - \sum_{i=0}^{UCL_{np}} C_i^{n_{np}} \left(1 - \Phi\left(k_w\sqrt{\frac{Y}{m(n-1)}}\right) + \Phi\left(-k_w\sqrt{\frac{Y}{m(n-1)}}\right)\right)^i \\
& \left(\Phi\left(k_w\sqrt{\frac{Y}{m(n-1)}}\right) - \Phi\left(-k_w\sqrt{\frac{Y}{m(n-1)}}\right)\right)^{n_{np}-i}\Bigg)h
\end{aligned} \tag{32}
$$

The conditional average run length when the process is in-control $(CATS_0)$ is inversely proportional to $CFAR$:

$$
CATS_0 = \frac{1}{CFAR} \tag{33}
$$

Note that $CFAR$, $CATS_0$ and $CATS$ are random variables and they change from practitioner to practitioner. Thus, the unconditional ATS is:

$$\begin{aligned}
ATS &= (E(RL) - 0.5)h = (E_{\overline{\overline{X}}} E(RL|\overline{\overline{X}}) - 0.5)h \\
&= \int_0^\infty \left(\left(\left(1 - \sum_{i=0}^{UCL_{np}} C_i^{n_{np}} \left(1 - \Phi\left(k_w \sqrt{\frac{Y}{m(n-1)}} - \delta \right) + \Phi\left(-k_w \sqrt{\frac{Y}{m(n-1)}} - \delta \right) \right) \right)^i \right.\right. \\
&\quad \left.\left. \left(\Phi\left(k_w \sqrt{\frac{Y}{m(n-1)}} - \delta \right) - \Phi\left(-k_w \sqrt{\frac{Y}{m(n-1)}} - \delta \right) \right)^{n_{np}-i} \right)^{-1} - 0.5 \right) h \, f_{x_v^2} dy
\end{aligned} \tag{34}$$

The Extra Quadratic Loss (EQL) also becomes a random variable changing from practitioner to practitioner, it will be called $CEQL$, equation 35.

$$\begin{aligned}
CEQL &= \frac{1}{\delta_{max} - \delta_{min}} \int_{\delta_{min}}^{\delta_{max}} \delta^2 CATS(\delta) d\delta = \frac{1}{\delta_{max} - \delta_{min}} \int_{\delta_{min}}^{\delta_{max}} \delta^2 \\
&\left[\left(\left(\left(1 - \sum_{i=0}^{UCL_{np}} C_i^{n_{np}} \left(1 - \Phi\left(k_w \sqrt{\frac{Y}{m(n-1)}} - \delta \right) + \Phi\left(-k_w \sqrt{\frac{Y}{m(n-1)}} - \delta \right) \right) \right)^i \right.\right.\right. \\
&\quad \left.\left.\left. \left(\Phi\left(k_w \sqrt{\frac{Y}{m(n-1)}} - \delta \right) - \Phi\left(-k_w \sqrt{\frac{Y}{m(n-1)}} - \delta \right) \right)^{n_{np}-i} \right)^{-1} - 0.5 \right) h \right] d\delta
\end{aligned} \tag{35}$$

The overall EQL can be calculated as equation 36:

$$\begin{aligned}
EQL &= \frac{1}{\delta_{max} - \delta_{min}} \int_{\delta_{min}}^{\delta_{max}} \delta^2 ATS(\delta) d\delta = \frac{1}{\delta_{max} - \delta_{min}} \int_{\delta_{min}}^{\delta_{max}} \int_0^\infty \delta^2 \\
&\left[\left(\left(\left(1 - \sum_{i=0}^{UCL_{np}} C_i^{n_{np}} \left(1 - \Phi\left(k_w \sqrt{\frac{Y}{m(n-1)}} - \delta \right) + \Phi\left(-k_w \sqrt{\frac{Y}{m(n-1)}} - \delta \right) \right) \right)^i \right.\right.\right. \\
&\quad \left.\left.\left. \left(\Phi\left(k_w \sqrt{\frac{Y}{m(n-1)}} - \delta \right) - \Phi\left(-k_w \sqrt{\frac{Y}{m(n-1)}} - \delta \right) \right)^{n_{np}-i} \right)^{-1} - 0.5 \right) h \right] f_{x_v^2} dy d\delta
\end{aligned} \tag{36}$$

To compare the performance of the np_x chart using known parameters against that using estimated parameters, $100,000$ values of $CATS_0$ and $CEQL$ were simulated. This involved varying sample sizes ($m = 25, 50, 100, 500$, and 1000), each with a sample size of $n = 38$ and a sampling interval of $h = 1$. The control limits were set to $UCL_{npx} = 9$ and $k_w = 1.6599319$, as shown in Table 8, column 3. The results of these simulations are presented in Tables 16 and 17.

The mean values of $CATS_0$ (Table 16, column 2) and $CEQL$ (Table 17, column 2) demonstrate sensitivity to the estimation of the process standard deviation. When 25 samples are used to estimate the process standard deviation, the mean of $CATS_0 = 445.04$, whereas the desirable value would be 370, and the mean of $CEQL = 3.879$, whereas for known parameters, $EQL = 3.671$ (Table 8, column 3). When the number of samples used in the estimation increases to 1000, the mean of $CATS_0 = 371.01$ and the mean of $CEQL = 3.675$, results much closer to the expected values.

Table 16 – Simulation of $CATS_0$ when the standard deviation is estimated ($n = 38$, $UCL_{np_x} = 9$ and $k_w = 1.6599319$).

m	$CATS_0$ Mean	$CATS_0$ Standard Deviation	$CATS_0$ Median	$CATS_0$ 0.25 Quantile	$CATS_0$ 0.75 Quantile
25	445.04	302.43	366.48	247.22	548.93
50	404.77	181.27	367.77	278.67	488.13
100	386.91	118.07	369.03	302.44	451.25
500	372.85	49.74	369.14	337.84	404.10
1000	371.01	34.86	369.11	346.71	393.43

source: Elaborated by the author.

Table 17 – Simulation of $CEQL$ when the standard deviation is estimated ($n = 38$, $UCL_{np_x} = 9$ and $k_w = 1.6599$).

m	$CEQL$ Mean	$CEQL$ Standard Deviation	$CEQL$ Median	$CEQL$ 0.25 Quantile	$CEQL$ 0.75 Quantile
25	3.879	0.952	3.660	3.230	4.270
50	3.769	0.600	3.665	3.347	4.072
100	3.720	0.400	3.669	3.433	3.949
500	3.680	0.172	3.669	3.560	3.790
1000	3.675	0.121	3.669	3.591	3.753

source: Elaborated by the author.

Additionally, as the number of samples used for standard deviation estimation increases, there is a notable reduction in variability among the simulated values, as evident in the third columns of Tables 16 and 17.

The unconditional approach for the np_x chart was examined under identical conditions, with variations in the number of samples ($m = 25, 50, 100, 500$, and 1000), a sample size of $n = 38$, sampling interval $h = 1$, $UCL_{npx} = 9$, and $k_w = 1.6599319$. The ATS and EQL values were compared under two scenarios: when the standard deviation is estimated for the np_x chart and when parameters are known, for both the np_x chart and the $\overline{X}$ chart (refer to Table 8). The results, presented in Table 18, depict the outcomes considering both estimated and known parameters.

The estimation of the standard deviation significantly impacts the performance of the np_x chart, particularly for slight changes in the process mean, as highlighted in Table 18, column 4. The ATS values for the in-control process ($\delta = 0$) and for changes in the process mean less than 1 ($\delta \leq 1$) are further from those when parameters are known (Table 18, column 3). As the number of samples (m) increases, this difference in ATS between known and estimated parameters diminishes. For example, considering a process mean change of magnitude $\delta = 0.25$, it is expect to get $ATS = 176.42$. However, for $m = 25$, the ATS value is 206.48. Yet, increasing $m = 100$ yields $ATS = 177.09$.

Similar observations hold when comparing the $\overline{X}$ chart with the np_x chart using estimated parameters. The ratio $EQL_{np}/EQL_{\overline{X}}$ reveals that with 25 samples used for standard deviation estimation, the $\overline{X}$ chart outperforms the np_x chart by 5.4%. However, if 500 samples are employed for estimation, the $\overline{X}$ and np_x charts are equivalent in terms of EQL.

Table 18 – Comparison of ATS of the $\overline{X}$ chart and the two-sided np_x chart when parameters are known and when the standard deviation is estimated $(\tau = 370)$.

1	2	3	4	5	6	7	8
δ	ATS						
	$\overline{X} chart$	$np_x\ chart$			$np_x\ chart$		
	$n_{\overline{X}} = 6$	$n_{npx} = 38$			$n_{npx} = 38$		
	$UCL_{\overline{X}} = 1.2246$	$UCL_{npx} = 9$			$UCL_{npx} = 9$		
		$w_u = k_w = 1.6599$			$w_u = k_w = 1.6599$		
		known parameters			estimated parameters		
			$m = 25$	$m = 50$	$m = 100$	$m = 500$	$m = 1000$
0.00	369.87	370.00	445.49	405.14	386.77	372.87	371.18
0.25	115.36	176.42	206.48	190.63	183.34	177.77	177.09
0.50	25.86	32.22	35.56	33.83	33.01	32.38	32.30
0.75	7.67	5.43	5.71	5.57	5.50	5.44	5.43
1.00	2.94	1.37	1.40	1.38	1.37	1.37	1.37
1.25	1.41	0.63	0.63	0.63	0.63	0.63	0.63
1.50	0.83	0.51	0.51	0.51	0.51	0.51	0.51
1.75	0.61	0.50	0.50	0.50	0.50	0.50	0.50
2.00	0.53	0.50	0.50	0.50	0.50	0.50	0.50
2.25	0.51	0.50	0.50	0.50	0.50	0.50	0.50
2.50	0.50	0.50	0.50	0.50	0.50	0.50	0.50
2.75	0.50	0.50	0.50	0.50	0.50	0.50	0.50
3.00	0.50	0.50	0.50	0.50	0.50	0.50	0.50
3.25	0.50	0.50	0.50	0.50	0.50	0.50	0.50
3.50	0.50	0.50	0.50	0.50	0.50	0.50	0.50
EQL	3.680	3.671	3.878	3.770	3.719	3.680	3.676
$EQL_{npx}/EQL_{\overline{X}}$	1.000	0.997	1.054	1.024	1.011	1.000	0.999

source: Elaborated by the author.

4.3 MEAN AND VARIANCE BOTH UNKNOWN (UU)

The variance σ^2 and the mean μ are estimated by S_p^2 and $\overline{\overline{X}}$ respectively for the m subgroups.

Now the warning limits are based on the estimated parameters of the mean and the standard deviation (S_p):

$$\widehat{w}_U = \overline{\overline{X}} + k_w S_p \tag{37}$$

$$\widehat{w}_L = \overline{\overline{X}} - k_w S_p \tag{38}$$

The probability of X be classified as nonconforming when process is in-control $\widehat{p}_0$ is now a random variable:

$$\begin{aligned}
\widehat{p} = P\left(X < \widehat{w}_L \; ou \; X > \widehat{w}_U\right) = 1 - P\left(\widehat{w}_L \leq X \leq \widehat{w}_U\right) = 1 - P\left(\overline{\overline{X}} - k_w S_p \leq X \leq \overline{\overline{X}} + k_w S_p\right) \\
= 1 - P\left(\frac{\overline{\overline{X}} - k_w S_p - \mu - \mu_0 + \mu_0}{\sigma} \leq \frac{X-\mu}{\sigma} \leq \frac{\overline{\overline{X}} + k_w S_p - \mu - \mu_0 + \mu_0}{\sigma}\right) \\
= 1 - P\left(\frac{\overline{\overline{X}} - \mu_0}{\sigma} - k_w \frac{S_p}{\sigma} - \frac{\mu - \mu_0}{\sigma} \leq Z \leq \frac{\overline{\overline{X}} - \mu_0}{\sigma} + k_w \frac{S_p}{\sigma} - \frac{\mu - \mu_0}{\sigma}\right) \\
= 1 - P\left(\frac{\overline{\overline{X}} - \mu_0}{\sigma} - k_w \frac{S_p}{\sigma} - \delta \leq Z \leq \frac{\overline{\overline{X}} - \mu_0}{\sigma} + k_w \frac{S_p}{\sigma} - \delta\right) \\
= 1 - \left[P\left(Z \leq \frac{\overline{\overline{X}} - \mu_0}{\sigma} - k_w \frac{S_p}{\sigma} - \delta\right) - P\left(Z \leq \frac{\overline{\overline{X}} - \mu_0}{\sigma} + k_w \frac{S_p}{\sigma} - \delta\right)\right] \\
= 1 - \Phi\left(\frac{\overline{\overline{X}} - \mu_0}{\sigma} - k_w \frac{S_p}{\sigma} - \delta\right) + \Phi\left(\frac{\overline{\overline{X}} - \mu_0}{\sigma} + k_w \frac{S_p}{\sigma} - \delta\right)
\end{aligned}$$

where μ_0 is the in-control process mean. μ is the process mean in Phase II. When $\mu = \mu_0$, $\delta = 0$, the process is in control. σ is the process standard deviation in Phase II. When $\mu = \mu_1 \neq \mu_0$, $\delta \neq 0$, the process is out of control. $\delta = \frac{(\mu - \mu_0)}{\sigma}$ = is the magnitude of a mean shift in terms of σ and ϕ is the cumulative standard normal distribution.

It is possible to demonstrate that $\frac{\overline{\overline{X}} - \mu_0}{\sigma}\sqrt{mn} = Z_1$ follows a normal distribution. So we can replace $\frac{\overline{\overline{X}} - \mu_0}{\sigma}$ as shown below:

$$\frac{\overline{\overline{X}} - \mu_0}{\sigma} = \frac{Z_1}{\sqrt{mn}} \tag{39}$$

It is also possible to demonstrate that $\frac{S_p^2}{\sigma^2} m(n-1) = Y$ follows a chi-squared distribution with $m(n-1)$ degrees of freedom. As we have $\frac{S_p}{\sigma}$ in the $\widehat{p}$ equation, it can be replaced as shown below:

$$\frac{S_p}{\sigma} = \sqrt{\frac{Y}{m(n-1)}} \tag{40}$$

Thus, the $\widehat{p}$ equation now is:

$$\widehat{p} = 1 - \Phi\left(\frac{Z_1}{\sqrt{mn}} + k_w\sqrt{\frac{Y}{m(n-1)}} - \delta\right) + \Phi\left(\frac{Z_1}{\sqrt{mn}} - k_w\sqrt{\frac{Y}{m(n-1)}} - \delta\right) \tag{41}$$

The probability of an alarm for the conditional approach can be expressed as shown below:

$$\begin{aligned}
P(\widehat{p}) = P(D > UCL|p) = 1 - \sum_{i=0}^{UCL_{np}} C_i^{n_{np}} \widehat{p}^i (1-\widehat{p})^{n_{np}-i} \\
= 1 - \sum_{i=0}^{UCL_{np}} C_i^{n_{np}} \left(1 - \Phi\left(\frac{Z_1}{\sqrt{mn}} + k_w\sqrt{\frac{Y}{m(n-1)}} - \delta\right) + \Phi\left(\frac{Z_1}{\sqrt{mn}} - k_w\sqrt{\frac{Y}{m(n-1)}} - \delta\right)\right)^i \\
\left(\Phi\left(\frac{Z_1}{\sqrt{mn}} + k_w\sqrt{\frac{Y}{m(n-1)}} - \delta\right) - \Phi\left(\frac{Z_1}{\sqrt{mn}} - k_w\sqrt{\frac{Y}{m(n-1)}} - \delta\right)\right)^{n_{np}-i}
\end{aligned} \tag{42}$$

The power of detecting a mean shift is $\beta = 1 - P(\widehat{p})$, the conditional average run length when the process is out-of-control is:

$$\begin{aligned}
CATS = \left(E\left(RL|\overline{\overline{X}}\right) - 0.5\right) h = \left(\frac{1}{1-\beta} - 0.5\right) h = \left(\frac{1}{P(\widehat{p})} - 0.5\right) h \\
= \left(\left(1 - \sum_{i=0}^{UCL_{np}} C_i^{n_{np}} \left(1 - \Phi\left(\frac{Z_1}{\sqrt{mn}} + k_w\sqrt{\frac{Y}{m(n-1)}} - \delta\right) + \Phi\left(\frac{Z_1}{\sqrt{mn}} - k_w\sqrt{\frac{Y}{m(n-1)}} - \delta\right)\right)^i \right.\right. \\
\left.\left. \left(\Phi\left(\frac{Z_1}{\sqrt{mn}} + k_w\sqrt{\frac{Y}{m(n-1)}} - \delta\right) - \Phi\left(\frac{Z_1}{\sqrt{mn}} - k_w\sqrt{\frac{Y}{m(n-1)}} - \delta\right)\right)^{n_{np}-i}\right)^{-1} - 0.5\right) h
\end{aligned} \tag{43}$$

When the process is considered in control, $\delta = 0$, therefore $\alpha = P(\widehat{p}_0) = CFAR$ (Conditional False Alarm Rate).

$$\begin{aligned}
CFAR = \left(1 - \sum_{i=0}^{UCL_{np}} C_i^{n_{np}} \left(1 - \Phi\left(\frac{Z_1}{\sqrt{mn}} + k_w\sqrt{\frac{Y}{m(n-1)}} - \delta\right)\right.\right. \\
\left. + \Phi\left(\frac{Z_1}{\sqrt{mn}} - k_w\sqrt{\frac{Y}{m(n-1)}} - \delta\right)\right)^i \\
\left.\left(\Phi\left(\frac{Z_1}{\sqrt{mn}} + k_w\sqrt{\frac{Y}{m(n-1)}} - \delta\right) - \Phi\left(\frac{Z_1}{\sqrt{mn}} - k_w\sqrt{\frac{Y}{m(n-1)}} - \delta\right)\right)^{n_{np}-i}\right) h
\end{aligned} \tag{44}$$

The conditional average run length when the process is in-control $(CATS_0)$ is inversely proportional to $CFAR$:

$$CATS_0 = \frac{1}{CFAR} \tag{45}$$

Note that $CFAR$, $CATS_0$ and $CATS$ are random variables and they change from practitioner to practitioner. Thus, the unconditional ATS is:

$$\begin{aligned} ATS &= (E(RL)-0.5)h = (E_{\overline{\overline{X}},S_p}E(RL|\overline{\overline{X}},S_p)-0.5)h = (E_{Z_1,Y}E(RL|Z_1,Y)-0.5)h \\ &= (E(CATS)-0.5)h = \int_{-\infty}^{\infty}\int_{0}^{\infty}\left(\left(1-\sum_{i=0}^{UCL_{np}}C_i^{n_{np}}\left(1-\Phi\left(\frac{Z_1}{\sqrt{mn}}+k_w\sqrt{\frac{Y}{m(n-1)}}-\delta\right)\right.\right.\right. \\ &\quad \left.\left.+\Phi\left(\frac{Z_1}{\sqrt{mn}}-k_w\sqrt{\frac{Y}{m(n-1)}}-\delta\right)\right)^i\left(\Phi\left(\frac{Z_1}{\sqrt{mn}}+k_w\sqrt{\frac{Y}{m(n-1)}}-\delta\right)\right. \\ &\quad \left.\left.\left.-\Phi\left(\frac{Z_1}{\sqrt{mn}}-k_w\sqrt{\frac{Y}{m(n-1)}}-\delta\right)\right)^{n_{np}-i}\right)^{-1}-0.5\right)h\ f_{x_v^2}dy\ \phi z_1 dz_1 \end{aligned} \tag{46}$$

The Extra Quadradic Loss (EQL) also becomes a random variable changing from practitioner to practitioner, it will be called $CEQL$, equation 47.

$$\begin{aligned} CEQL &= \frac{1}{\delta_{max}-\delta_{min}}\int_{\delta_{min}}^{\delta_{max}}\delta^2 CATS(\delta)d\delta = \frac{1}{\delta_{max}-\delta_{min}}\int_{\delta_{min}}^{\delta_{max}}\delta^2\left(\left(1-\sum_{i=0}^{UCL_{np}}C_i^{n_{np}}\right.\right. \\ &\left(1-\Phi\left(\frac{Z_1}{\sqrt{mn}}+k_w\sqrt{\frac{Y}{m(n-1)}}-\delta\right)+\Phi\left(\frac{Z_1}{\sqrt{mn}}-k_w\sqrt{\frac{Y}{m(n-1)}}-\delta\right)\right)^i \\ &\left.\left.\left(\Phi\left(\frac{Z_1}{\sqrt{mn}}+k_w\sqrt{\frac{Y}{m(n-1)}}-\delta\right)-\Phi\left(\frac{Z_1}{\sqrt{mn}}-k_w\sqrt{\frac{Y}{m(n-1)}}-\delta\right)\right)^{n_{np}-i}\right)^{-1}-0.5\right)h\ d\delta \end{aligned} \tag{47}$$

To compare the performance of two charts, the Extra Quadratic Loss (EQL) could be used. For a given mean shift, the incurred quadratic loss is simply equal to δ^2. And the loss in quality is proportional to $ATS(\delta)$. The overall EQL can be calculated as equation 48:

$$\begin{aligned} EQL &= \frac{1}{\delta_{max}-\delta_{min}}\int_{\delta_{min}}^{\delta_{max}}\delta^2 ATS(\delta)d\delta = \frac{1}{\delta_{max}-\delta_{min}}\int_{\delta_{min}}^{\delta_{max}}\int_{-\infty}^{\infty}\int_{0}^{\infty}\delta^2\left(\left(1-\sum_{i=0}^{UCL_{np}}C_i^{n_{np}}\right.\right. \\ &\left(1-\Phi\left(\frac{Z_1}{\sqrt{mn}}+k_w\sqrt{\frac{Y}{m(n-1)}}-\delta\right)+\Phi\left(\frac{Z_1}{\sqrt{mn}}-k_w\sqrt{\frac{Y}{m(n-1)}}-\delta\right)\right)^i \\ &\left.\left.\left(\Phi\left(\frac{Z_1}{\sqrt{mn}}+k_w\sqrt{\frac{Y}{m(n-1)}}-\delta\right)-\Phi\left(\frac{Z_1}{\sqrt{mn}}-k_w\sqrt{\frac{Y}{m(n-1)}}-\delta\right)\right)^{n_{np}-i}\right)^{-1}-0.5\right)h \\ &\qquad f_{x_v^2}dy\ \phi z_1 dz_1\ d\delta \end{aligned} \tag{48}$$

To compare the efficacy of the np_x chart with known parameters against that with estimated parameters, $100,000$ simulations were conducted for $CATS_0$ and $CEQL$ values. These simulations encompassed a range of sample sizes ($m = 25, 50, 100, 500$, and 1000), each consisting of $n = 38$ samples with a sampling interval of $h = 1$. Control limits were set to $UCL_{npx} = 9$ and $k_w = 1.6599319$, as specified in Table 8, column 3. The results of these simulations are summarized in Tables 19 and 20.

Table 19 – Simulation of $CATS_0$ when the mean and the standard deviation are estimated ($n = 38$, $UCL_{np_x} = 9$ and $k_w = 1.6599319$).

m	$CATS_0$ Mean	$CATS_0$ Standard Deviation	$CATS_0$ Median	$CATS_0$ 0.25 Quantile	$CATS_0$ 0.75 Quantile
25	439.14	300.30	360.05	243.53	541.69
50	402.34	180.31	365.43	276.13	486.14
100	385.78	118.47	367.48	301.80	449.64
500	372.75	49.58	369.18	337.91	403.97
1000	371.05	34.85	369.32	346.55	393.34

source: Elaborated by the author.

Table 20 – Simulation of $CEQL$ when the mean and the standard deviation are estimated ($n = 38$, $UCL_{np_x} = 9$ and $k_w = 1.6599$).

m	$CEQL$ Mean	$CEQL$ Standard Deviation	$CEQL$ Median	$CEQL$ 0.25 Quantile	$CEQL$ 0.75 Quantile
25	3.913	1.083	3.649	3.187	4.326
50	3.787	0.669	3.663	3.316	4.113
100	3.728	0.444	3.666	3.412	3.977
500	3.682	0.188	3.671	3.551	3.801
1000	3.676	0.132	3.670	3.584	3.761

source: Elaborated by the author.

In the case of both parameters been estimated: the mean and the standard deviation of the process, the mean values of $CATS_0$ and EQL are distant of the expected values: $ATS_0 = 370.00$ and $EQL = 3.671$ (Table 8, column 3). Also the $CATS_0$ presents a high standard deviation. For example: for the UU case using 25 samples in estimation of parameters, the mean of $CARL_0$ is 439.14 and the standard deviation is 300.30, check Table 19 columns 2 and 3. And the mean of $CEQL$ is 3.913 (Table 20 column 2), approximately 6.6% higher than the expected value.

Similar to the previous cases, as the number of samples increases, the mean of $CARL_0$ approaches the desired value of 370.00, and the standard deviation decreases. Additionally, the mean of $CEQL$ moves closer to 3.671. This underscores that using more samples for parameter estimation leads to results that are closer to the expected values.

The same analyzes done for the unconditional approach of the np_x chart for the UK and KU cases were done for the UU case. The number of samples were assorted as shown: $m = 25, 50, 100, 500$ and 1000. Each sample size of $n = 38$, sampling interval $h = 1$, $UCL_{npx} = 9$ and $k_w = 1.6599319$. The mean values of $CATS$ and $CEQL$ considering the estimation of the parameters (μ and σ) were compared to the values of ATS and EQL of the np_x chart and $\overline{X}$ chart with known parameters (see Table 8). The results are presented in Table 21.

According to the results shown in Table 21, columns 4 to 8, the performance of the np_x chart is negatively influenced by the estimation of both: the process mean and standard deviation. Notably,

Table 21 – Comparison of ATS of the $\overline{X}$ chart and the two-sided np_x chart when parameters are known and when the mean and the standard deviation is estimated ($\tau = 370$).

1	2	3	4	5	6	7	8
δ	ATS						
	$\overline{X} chart$	$np_x\ chart$			$np_x\ chart$		
	$n_{\overline{X}} = 6$	$n_{npx} = 38$			$n_{npx} = 38$		
	$UCL_{\overline{X}} = 1.2246$	$UCL_{npx} = 9$			$UCL_{npx} = 9$		
		$w_u = k_w = 1.6599$			$w_u = k_w = 1.6599$		
		known parameters			estimated parameters		
			$m = 25$	$m = 50$	$m = 100$	$m = 500$	$m = 1000$
0.00	369.87	370.00	438.28	402.01	385.34	372.60	371.04
0.25	115.36	176.42	207.71	191.22	183.62	177.83	177.12
0.50	25.86	32.22	36.62	34.31	33.24	32.42	32.32
0.75	7.67	5.43	5.86	5.64	5.53	5.45	5.44
1.00	2.94	1.37	1.42	1.39	1.38	1.37	1.37
1.25	1.41	0.63	0.64	0.63	0.63	0.63	0.63
1.50	0.83	0.51	0.51	0.51	0.51	0.51	0.51
1.75	0.61	0.50	0.50	0.50	0.50	0.50	0.50
2.00	0.53	0.50	0.50	0.50	0.50	0.50	0.50
2.25	0.51	0.50	0.50	0.50	0.50	0.50	0.50
2.50	0.50	0.50	0.50	0.50	0.50	0.50	0.50
2.75	0.50	0.50	0.50	0.50	0.50	0.50	0.50
3.00	0.50	0.50	0.50	0.50	0.50	0.50	0.50
3.25	0.50	0.50	0.50	0.50	0.50	0.50	0.50
3.50	0.50	0.50	0.50	0.50	0.50	0.50	0.50
EQL	3.680	3.671	3.915	3.787	3.727	3.682	3.676
$EQL_{np}/EQL_{\overline{X}}$	1.000	0.997	1.064	1.029	1.013	1.000	0.999

source: Elaborated by the author.

for $m = 25$, the ATS_0 exceeds the expected value by approximately 18%, thus the first constraint of the np_x chart $ATS_0 = \tau = 370$ is not satisfied. Only for $m \geq 100$ does the difference between the obtained value and the expected value for ATS_0 fall below 5%. The same can be observed for the EQL.

It's evident that in all cases (UK, KU, and UU), increasing the number of samples, m, used for parameter estimation brings the results closer to the expected ones. Particularly when the process standard deviation is estimated, the np_x chart's performance becomes more sensitive to parameter estimation. Consequently, the obtained results may deviate from the restriction of having ATS_0 equal to 370. Moreover, there is a substantial increase in EQL, which compromises the chart's performance within a range of 0 to 3.5 changes in the process mean.

The UU case is the most practical scenario, as in industrial settings, both the mean and standard deviation are unknown and must be estimated. However, having 1000 samples with 38 units each for this estimation is impractical. This would require $38,000$ items to estimate the mean and standard deviation and obtain results close to those where the parameters are assumed to be known.

One approach to address this impasse is to adjust the control limits: (UCL_{npx}) and k_w of the np_x chart with estimated parameters. This adjustment aims to achieve performance comparable to that obtained when parameters are assumed to be known, even with a limited number of samples used in the estimation process. In the next chapter, this approach will be thoroughly explored.

5 THE np_x CONTROL CHART OPTIMIZATION

In the previous chapter, it was demonstrated that the minimum number of reference samples required to estimate parameters for ensuring conditional performance in control, at least comparable to situations where parameters are known, can be very large. This is especially true when estimating the process's standard deviation, often rendering it impractical in many real-world scenarios. Parameter estimation can introduce uncertainties that may compromise the performance of the np_x chart. Adjusting the control limits to account for these uncertainties is a common approach to mitigate this issue. Therefore, this chapter presents the exact control limits adjusted for the np_x chart across all UK, KU, and UU cases, considering sample sizes of $m = 25, 50, 100, 500$, and 1000.

The performance of the np_x chart is evaluated based on the requirements that, under the in-control condition, ATS_0 equals 370, and under the out-of-control scenario, the Extra Quadratic Loss (EQL) is minimized. The gauge is constructed according to the warning limits (Equations 2 and 3), which depend on the k_w coefficient and the probability that the fraction of non-conforming items in a sample is dependent on the maximum number of non-conforming items accepted (UCL). Therefore, we must adjust the UCL and k_w parameters to meet these requirements.

Since the UCL (Upper Control Limit) represents the maximum number of non-conforming units accepted in a sample, it can range from 0, where no non-conforming units are acceptable, to $n - 1$, because if UCL equals n, a sample could contain all non-conforming items, preventing the chart from ever signaling. Therefore, for each value of UCL, we can find the corresponding k_w value that satisfies the constraint $ATS_0 = \tau = 370$. However, which of these n pairs of UCL and k_w should be chosen to guarantee the best performance given the sample size (n), sampling interval (h), and number of reference samples (m)? To answer this question, we must consider the objective function of the np_x chart, which is to minimize the EQL. Hence, we will select the pair of parameters that produces the lowest EQL under these conditions.

Adjustments for cases UK, KU and UU are respectively in sections 5.1, 5.2 and 5.3. In all cases the previously described procedure was employed to adjust the parameters UCL and k_w with the aim of optimizing the performance of the np_x control chart.

Consistent with the methodology utilized in chapter 4 the same number of samples were analyzed for optimization in this scenario ($m = 25, 50, 100, 500$, and 1000).

5.1 ADJUSTMENT IN CASE UK

Initially, all feasible combinations of UCL and k_w pairs satisfying the constraint of $ATS_0 = 370$ were generated, varying the sample size ($n = 5, 10, 15, 20$ and 25) and the sampling interval ($h = 1.0$ and 0.5). The next step involved determining the Extra Quadratic Loss (EQL) generated by each pair of UCL and k_w, considering a change range of $0 \leq \delta \leq 3.5$. Results are shown in Appendix B.4.

By analyzing the results obtained for EQL, it is possible to determine which parameters UCL and k_w should be used to guarantee the minimum EQL based on the number of reference samples (m), sampling interval (h) and sample size (n) available. See Table 22.

Table 22 – k_w and UCL which satisfy the constraint $ATS_0 = \tau = 370$ for the np_x chart when the mean is estimated and the standard deviation is known for different number of samples m.

1	2	3	4	5	6	7
				$h = 1$		
		$m = 25$	$m = 50$	$m = 100$	$m = 500$	$m = 1000$
n=5	UCL	3	3	3	3	3
	k_w	1.41854	1.41592	1.41456	1.41344	1.41330
	EQL	10.328	10.017	9.862	9.739	9.723
n=10	UCL	3	3	3	3	3
	k_w	1.85014	1.84839	1.84750	1.84677	1.84667
	EQL	6.787	6.659	6.595	6.543	6.537
n=15	UCL	4	4	4	4	4
	k_w	1.81746	1.81630	1.81571	1.81523	1.81517
	EQL	5.499	5.422	5.384	5.353	5.349
n=20	UCL	5	5	5	5	5
	k_w	1.78387	1.78301	1.78257	1.78222	1.78217
	EQL	4.805	4.751	4.725	4.704	4.701
n=25	UCL	7	7	7	7	7
	k_w	1.63268	1.63205	1.63173	1.63147	1.63143
	EQL	4.363	4.323	4.303	4.287	4.285
				$h = 0.5$		
		$m = 25$	$m = 50$	$m = 100$	$m = 500$	$m = 1000$
n=5	UCL	3	3	3	3	3
	k_w	1.51281	1.51004	1.50860	1.50741	1.50726
	EQL	7.957	7.663	7.518	7.401	7.387
n=10	UCL	3	3	3	3	3
	k_w	1.93248	1.93067	1.92973	1.92897	1.92887
	EQL	5.068	4.945	4.885	4.836	4.830
n=15	UCL	4	4	4	4	4
	k_w	1.88711	1.88591	1.88530	1.88480	1.88474
	EQL	3.991	3.918	3.881	3.852	3.849
n=20	UCL	6	6	6	6	6
	k_w	1.69340	1.69259	1.69217	1.69184	1.69179
	EQL	3.400	3.350	3.324	3.304	3.302
n=25	UCL	7	7	7	7	7
	k_w	1.68470	1.68405	1.68371	1.68345	1.68341
	EQL	3.027	2.989	2.969	2.954	2.952

source: Elaborated by the author.

For example, when $m = 25$, $h = 1$ and $n = 10$, the minimum EQL (6.787) is provided using the charting parameters $UCL = 3$ and $k_w = 1.85014$, check (Table 22, column 3).

In order to compare the optimized performance achieved through parameter estimation with the performance obtained when parameters are known, the values of $n = 38$ and $h = 1$ were utilized.

Table 23 shows the performance of the np_x chart in terms of ATS and EQL considering the adjusted parameters UCL and k_w for each m. The results are compared to the np_x chart and $\overline{X}$ chart when parameters are know.

Note that for any number of samples (m) the constraint of $ATS_0 = 370$ is satisfied, differently

Table 23 – Comparison of ATS of the $\overline{X}$ chart and the two-sided np_x chart when parameters are known and when the mean is estimated ($\tau = 370$).

1	2	3	4	5	6	7	8
δ	ATS						
	$\overline{X} chart$	$np_x\ chart$			$np_x\ chart$		
	$n_{\overline{X}} = 6$	$n_{npx} = 38$			$n_{npx} = 38$		
	$UCL_{\overline{X}} = 1.2246$	$UCL_{npx} = 9$			$UCL_{npx} = 9$		
		$k_w = 1.6599$	$k_w = 1.6610$	$k_w = 1.6605$	$k_w = 1.6603$	$k_w = 1.6601$	$k_w = 1.6601$
		known parameters			estimated parameters		
			$m = 25$	$m = 50$	$m = 100$	$m = 500$	$m = 1000$
0.00	369.87	370.00	370.40	370.40	370.40	370.40	370.40
0.25	115.36	176.42	180.26	178.55	177.68	176.99	176.90
0.50	25.86	32.22	33.53	32.90	32.59	32.34	32.31
0.75	7.67	5.43	5.61	5.52	5.48	5.44	5.44
1.00	2.94	1.37	1.39	1.38	1.37	1.37	1.37
1.25	1.41	0.63	0.63	0.63	0.63	0.63	0.63
1.50	0.83	0.51	0.51	0.51	0.51	0.51	0.51
1.75	0.61	0.50	0.50	0.50	0.50	0.50	0.50
2.00	0.53	0.50	0.50	0.50	0.50	0.50	0.50
2.25	0.51	0.50	0.50	0.50	0.50	0.50	0.50
2.50	0.50	0.50	0.50	0.50	0.50	0.50	0.50
2.75	0.50	0.50	0.50	0.50	0.50	0.50	0.50
3.00	0.50	0.50	0.50	0.50	0.50	0.50	0.50
3.25	0.50	0.50	0.50	0.50	0.50	0.50	0.50
3.50	0.50	0.50	0.50	0.50	0.50	0.50	0.50
EQL	3.680	3.671	3.721	3.697	3.686	3.676	3.675
$EQL_{npx}/EQL_{\overline{X}}$	1.000	0.997	1.011	1.005	1.002	0.999	0.999

source: Elaborated by the author.

from what was found before adjusting UCL and k_w. See Table 15 column 3, for example, for $m = 25$ the $ATS_0 = 364.67$. Although the $EQL_{npx}/EQL_{\overline{X}} > 1$, the $\overline{X}$ chart outperforms the np_x chart only 1%, it is definitely not significant.

5.2 ADJUSTMENT IN CASE KU

For the UK case, all feasible combinations of UCL and k_w pairs generated satisfying the constraint of $ATS_0 = 370$ varying the sample size ($n = 5, 10, 15, 20$ and 25) and the sampling interval ($h = 1.0$ and 0.5) and the Extra Quadratic Loss (EQL) associated with each UCL and k_w pair are detailed in Appendix C.4.

By examining the EQL results, we can identify the optimal UCL and k_w parameters that ensure the minimal EQL for given numbers of reference samples (m), sampling intervals (h), and sample sizes (n). Detailed findings are provided in Table 24. For example, with $m = 25$, $h = 1$, and $n = 10$, the minimum EQL (6.332) is achieved using $UCL = 3$ and $k_w = 1.81566$, as shown in Table 24, column 3.

Table 25 presents the performance of the np_x chart in terms of ATS and EQL, considering the adjusted parameters UCL and k_w for $n = 38$, $h = 1$ and each m. The results are compared side by side with those of the np_x chart and $\overline{X}$ chart when parameters are known.

In Chapter 4, it became evident that the estimated standard deviation significantly influences the observed values. Consequently, the adjustment of the UCL and k_w parameters becomes even more crucial. Notably, without adjusting the parameters and using only 25 reference samples, we observed in Table 18 an ATS_0 value of 445.49. The performance of the $\overline{X}$ chart was 5% superior to the np_x chart according to the ratio $EQL_{npx}/EQL_{\overline{X}}$ making it unfeasible to use the np_x chart under these conditions. However, upon adjusting the parameters to meet the restriction $ATS_0 = 370$, and considering the $EQL_{npx}/EQL_{\overline{X}}$ ratio, the np_x chart outperformed the $\overline{X}$ chart by 1.4% (Table 25, column 4).

It is noteworthy that for $n = 38$ and $h = 1$, the EQL obtained for the np_x chart with estimated parameters and adjusted UCL and k_w ($EQL_npx = 3,630$) is smaller than the EQL obtained when the parameters are known ($EQL_npx = 3,671$) (see Table 25, columns 3 and 4). This result underscores the importance of adjusting UCL and k_w when dealing with estimated parameters, enabling better performance even with a limited number of reference samples.

5.3 ADJUSTMENT IN CASE UU

Following the same procedure for the case UU, all combinations of UCL and k_w pairs which satisfies the constraint of $ATS_0 = 370$ were generated. This was done by varying the sample size ($n = 5, 10, 15, 20$ and 25) and the sampling interval ($h = 1.0$ and 0.5). The subsequent step involved calculating the Extra Quadratic Loss (EQL) for each UCL and k_w pair, considering a range of changes $0 \leq \delta \leq 3.5$. The results are presented in Appendix D.4.

Note that there are cases that the extra quadratic loss could not be determined for some values due to Triple integral has no resolution.

Table 24 – k_w and UCL which satisfy the constraint $ATS_0 = \tau = 370$ for the np_x chart when the mean is estimated and the standard deviation is known for different number of samples m.

1	2	3	4	5	6	7
				$h = 1$		
		$m = 25$	$m = 50$	$m = 100$	$m = 500$	$m = 1000$
n=5	UCL	2	2	2	2	2
	k_w	1.78051	1.80627	1.81936	1.82999	1.83132
	EQL	9.213	9.461	9.584	9.683	9.695
n=10	UCL	3	3	3	3	3
	k_w	1.81566	1.83101	1.83877	1.84502	1.84580
	EQL	6.332	6.431	6.481	6.521	6.526
n=15	UCL	4	4	4	4	4
	k_w	1.79209	1.80354	1.80931	1.81395	1.81453
	EQL	5.223	5.284	5.315	5.339	5.342
n=20	UCL	5	5	5	5	5
	k_w	1.76338	1.77272	1.77742	1.78119	1.78166
	EQL	4.610	4.654	4.676	4.694	4.696
n=25	UCL	7	7	7	7	7
	k_w	1.61665	1.62401	1.62770	1.63066	1.63103
	EQL	4.215	4.249	4.266	4.280	4.281
				$h = 0.5$		
		$m = 25$	$m = 50$	$m = 100$	$m = 500$	$m = 1000$
n=5	UCL	2	2	2	2	2
	k_w	1.876489247	1.906558223	1.92192595	1.934384095	1.935951729
	EQL	6.897703074	7.13524878	7.253715964	7.348543445	7.360439104
n=10	UCL	3	3	3	3	3
	k_w	1.89354	1.91102	1.91986	1.92699	1.92788
	EQL	4.635	4.730	4.777	4.815	4.819
n=15	UCL	4	4	4	4	4
	k_w	1.85879	1.87166	1.87815	1.88337	1.88402
	EQL	3.729	3.787	3.816	3.839	3.842
n=20	UCL	6	6	6	6	6
	k_w	1.67279	1.68220	1.68694	1.69080	1.69128
	EQL	3.217	3.258	3.278	3.295	3.297
n=25	UCL	7	7	7	7	7
	k_w	1.66698	1.67515	1.67926	1.68255	1.68297
	EQL	2.887	2.919	2.935	2.947	2.949

source: Elaborated by the author.

By examining the EQL outcomes, it becomes clear which UCL and k_w parameters should be selected to minimize EQL, based on the reference sample number (m), sampling interval (h), and sample size (n). This information is summarized in Table 26. For example, when $m = 25$, $h = 1$, and $n = 10$, the minimum EQL of 6.610 is achieved with charting parameters $UCL = 3$ and $k_w = 1.81967$ (see Table 26, column 3).

Table 27 illustrates the performance of the np_x chart in terms of ATS and EQL, considering the adjusted parameters UCL and k_w for each m. The results are compared with those of the np_x chart and $\overline{X}$ chart when parameters are known.

Table 25 – Comparison of ATS of the $\overline{X}$ chart and the two-sided np_x chart when parameters are known and when the standard deviation is estimated ($\tau = 370$).

1	2	3	4	5	6	7	8
δ	ATS						
	$\overline{X} chart$	$np_x\ chart$			$np_x\ chart$		
	$n_{\overline{X}} = 6$	$n_{npx} = 38$			$n_{npx} = 38$		
	$UCL_{\overline{X}} = 1.2246$	$UCL_{npx} = 9$			$UCL_{npx} = 9$		
		$k_w = 1.6599$	$k_w = 1.6483$	$k_w = 1.6542$	$k_w = 1.6571$	$k_w = 1.6595$	$k_w = 1.6598$
		known parameters			estimated parameters		
			$m = 25$	$m = 50$	$m = 100$	$m = 500$	$m = 1000$
0.00	369.87	370.00	370.40	370.40	370.40	370.40	370.40
0.25	115.36	176.42	174.48	175.65	176.23	176.70	176.75
0.50	25.86	32.22	31.20	31.74	32.01	32.22	32.25
0.75	7.67	5.43	5.21	5.32	5.38	5.42	5.43
1.00	2.94	1.37	1.32	1.34	1.36	1.36	1.37
1.25	1.41	0.63	0.62	0.62	0.62	0.63	0.63
1.50	0.83	0.51	0.51	0.51	0.51	0.51	0.51
1.75	0.61	0.50	0.50	0.50	0.50	0.50	0.50
2.00	0.53	0.50	0.50	0.50	0.50	0.50	0.50
2.25	0.51	0.50	0.50	0.50	0.50	0.50	0.50
2.50	0.50	0.50	0.50	0.50	0.50	0.50	0.50
2.75	0.50	0.50	0.50	0.50	0.50	0.50	0.50
3.00	0.50	0.50	0.50	0.50	0.50	0.50	0.50
3.25	0.50	0.50	0.50	0.50	0.50	0.50	0.50
3.50	0.50	0.50	0.50	0.50	0.50	0.50	0.50
EQL	3.680	3.671	3.630	3.652	3.663	3.672	3.673
$EQL_{npx}/EQL_{\overline{X}}$	1.000	0.997	0.986	0.992	0.995	0.998	0.998

source: Elaborated by the author.

Table 26 – k_w and UCL which satisfy the constraint $ATS_0 = \tau = 370$ for the np_x chart when the mean is estimated and the standard deviation is known for different number of samples m.

1	2	3	4	5	6	7
				$h = 1$		
		$m = 25$	$m = 50$	$m = 100$	$m = 500$	$m = 1000$
n=5	UCL	2	3	2	2	2
	k_w	1.38481	1.39878	1.82129	1.83037	1.83151
	EQL	9.881	10.979	9.746	9.715	9.712
n=10	UCL	3	3	3	3	3
	k_w	1.81967	1.83299	1.83976	1.84521	1.84590
	EQL	6.610	6.567	6.548	6.534	6.532
n=15	UCL	4	4	4	4	4
	k_w	1.79476	1.80486	1.80997	1.81408	1.81459
	EQL	5.389	5.366	5.355	5.347	5.346
n=20	UCL	5	5	5	5	5
	k_w	1.76537	1.77370	1.77790	1.78128	1.78171
	EQL	4.726	4.711	4.705	4.700	4.699
n=25	UCL	7	7	7	7	7
	k_w	1.61812	1.62473	1.62806	1.63073	1.63107
	EQL	4.302	4.292	4.288	4.284	4.284
				$h = 0.5$		
		$m = 25$	$m = 50$	$m = 100$	$m = 500$	$m = 1000$
n=5	UCL	2	2	2	2	2
	k_w	1.88478	1.91071	1.92399	1.93479	1.93616
	EQL	7.542	7.448	7.408	7.379	7.376
n=10	UCL	3	3	3	3	3
	k_w	1.89774	1.91311	1.92090	1.92719	1.92799
	EQL	4.901	4.860	4.841	4.827	4.826
n=15	UCL	4	4	4	4	4
	k_w	1.86156	1.87304	1.87883	1.88350	1.88409
	EQL	3.889	3.866	3.855	3.847	3.846
n=20	UCL	6	6	6	6	6
	k_w	1.67471	1.68314	1.68740	1.69089	1.69132
	EQL	3.328	3.312	3.305	3.301	3.300
n=25	UCL	7	7	7	7	7
	k_w	1.66852	1.67591	1.67963	1.68263	1.68300
	EQL	2.971	2.960	2.955	2.951	2.951

source: Elaborated by the author.

As observed in Chapter 4, the performance for the case UU is notably affected by the estimation of parameters. When the same UCL and k_w values for the case KK are employed for the case UU, it is observed that for $m = 25$, the $ATS_0 = 438.28$, and the ratio $EQL_{npx}/EQL_{\overline{X}}$ indicates that the $\overline{X}$ chart is 6.4% superior to the np_x chart, see Table 21, column 4. Considering the adjusted parameters UCL and k_w (Table 27, column 4) the constraint $ATS_0 = 370$ is satisfied and according to the ratio $EQL_{npx}/EQL_{\overline{X}}$, $\overline{X}$ chart outperforms the np_x chart by only 1.8%. This underscores the necessity of accurately adjusting the UCL and k_w parameters.

Table 27 – Comparison of ATS of the $\overline{X}$ chart and the two-sided np_x chart when parameters are known and when both the mean and the standard deviation are estimated $(\tau = 370)$.

1	2	3	4	5	6	7	8
δ	ATS						
	$\overline{X} chart$	$np_x\ chart$			$np_x\ chart$		
	$n_{\overline{X}} = 6$	$n_{npx} = 38$			$n_{npx} = 38$		
	$UCL_{\overline{X}} = 1.2246$	$UCL_{npx} = 9$	$UCL_{npx} = 13$	$UCL_{npx} = 8$		$UCL_{npx} = 9$	
		$k_w = 1.6599$	$k_w = 1.3641$	$k_w = 1.7418$	$k_w = 1.6574$	$k_w = 1.6595$	$k_w = 1.6598$
		known parameters	estimated parameters				
			$m = 25$	$m = 50$	$m = 100$	$m = 500$	$m = 1000$
0.00	369.87	370.00	370.30	370.41	370.40	370.40	370.40
0.25	115.36	176.42	183.38	176.98	177.10	176.87	176.84
0.50	25.86	32.22	34.39	32.46	32.32	32.28	32.28
0.75	7.67	5.43	5.60	5.51	5.42	5.43	5.43
1.00	2.94	1.37	1.35	1.40	1.36	1.37	1.37
1.25	1.41	0.63	0.62	0.64	0.63	0.63	0.63
1.50	0.83	0.51	0.51	0.51	0.51	0.51	0.51
1.75	0.61	0.50	0.50	0.50	0.50	0.50	0.50
2.00	0.53	0.50	0.50	0.50	0.50	0.50	0.50
2.25	0.51	0.50	0.50	0.50	0.50	0.50	0.50
2.50	0.50	0.50	0.50	0.50	0.50	0.50	0.50
2.75	0.50	0.50	0.50	0.50	0.50	0.50	0.50
3.00	0.50	0.50	0.50	0.50	0.50	0.50	0.50
3.25	0.50	0.50	0.50	0.50	0.50	0.50	0.50
3.50	0.50	0.50	0.50	0.50	0.50	0.50	0.50
EQL	3.680	3.671	3.747	3.686	3.675	3.674	3.674
$EQL_{npx}/EQL_{\overline{X}}$	1.000	0.997	1.018	1.001	0.999	0.998	0.998

source: Elaborated by the author.

6 CONCLUSION

For the np_x chart when parameters are known, the results founded reveal inconsistencies with Wu *et al.*'s findings (WU et al., 2009). The np_x chart requires a larger sample size than recommended by Wu *et al.* to match the performance of the $\overline{X}$ chart. Despite this, the np_x chart can be advantageous due to its potential for cheaper, faster, and easier attribute inspections in certain applications. Both one-sided and two-sided np_x chart designs can outperform the $\overline{X}$ chart, particularly with small sample sizes.

The np_x chart can monitor the mean more efficiently than the $\overline{X}$ chart, depending on inspection costs. Adjusting the sample size or sampling interval can enhance performance or reduce costs. Although the np_x chart's performance may not be as high as initially claimed by Wu *et al.*, it remains beneficial for monitoring a continuous variable's mean in some cases.

Parameter estimation proved to be detrimental to the performance of the np_x chart. The results revealed that the constraint of $ATS_0 = 370$ is not satisfied, and the EQL_{npx} increases, indicating that the chart's performance has worsened. This effect diminishes as the number of reference samples increases; the more samples used to estimate the mean and standard deviation, the closer the performance will be to that achieved when the parameters are known. This makes the practical use of the np_x chart with estimated parameters unfeasible, as satisfactory results were only obtained when at least 500 reference samples were used.

The impact on the performance of the np_x chart is more pronounced for the KU and UU cases. The results showed that the mean estimate (UK case) has little effect on the variation of ATS_0 and EQL_{npx}. In contrast, the process standard deviation estimate substantially affects both ATS_0 and EQL_{npx}. Since in practice both parameters are typically unknown (UU case), it is necessary to adjust the control chart limits to optimize performance.

The optimization of the np_x chart involves adjusting its UCL and k_w parameters. The results have proven valuable, as adjusting these parameters allows for achieving equal or superior performance for the np_x chart when parameters are estimated, compared to when parameters are known, considering 25 reference samples — a commonly used value in real-world scenarios. Therefore, parameter adjustment renders the use of the np_x chart feasible.

In summary, our study sheds light on the performance and practical considerations of the np_x chart in comparison to the $\overline{X}$ chart. Our findings challenge previous research by revealing inconsistencies in performance expectations, particularly concerning sample size requirements. Despite requiring larger sample sizes, the np_x chart offers advantages in cost and efficiency, especially for attribute inspections. However, parameter estimation significantly impacts the np_x chart's performance, rendering its practical use unfeasible in many cases. Nonetheless, by adjusting parameters such as UCL and k_w, it's possible to optimize the np_x chart's performance, making it a viable option even when parameters are estimated. This highlights the importance of careful parameter adjustment in implementing the np_x chart effectively in real-world scenarios.

6.1 SUGGESTIONS FOR FUTURE WORK

This study was limited to observing the performance of the np_x control chart under the conditions studied by Wu *et al.* (WU et al., 2009). It is suggested that future research explore the practical application of optimization for real-world cases of monitoring process averages using attribute inspection.

The approach used in this study was limited to unconditional analysis, considering all users. Future research could investigate the conditioned performance of the np_x chart with estimated parameters.

As highlighted in the literature review, further investigation into the effects of parameter estimation and optimization of attribute control charts is warranted. Therefore, expanding the study to include other types of attribute control charts is recommended.

In this study, measurement errors were not considered. Given the inherent imprecision of measurements, future research should delve deeper into the effects of measurement errors on the performance of the np_x chart when parameters are estimated.

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APPENDIX A – OPTIMIZATION OF THE np_x CONTROL CHART

This appendix presents the R codes for optimizing the control chart np_x for known parameters (case KK).

A.1 FIND THE OPTIMAL VALUES OF UCL AND k_w

These codes were employed to determine the minimum number of samples required to achieve performance equal to or better than the $\overline{X}$ chart. To accomplish this, all pairs of k_w and UCL that satisfy $ATS_0 = 370$ for a given sample size n are identified, and the resulting minimum EQL is compared to the EQL of the $\overline{X}$ chart. If this minimum EQL is equal to or less than the $EQL_{\overline{X}}$, the sample size n is deemed sufficient. If the minimum EQL_{np_x} exceeds the $EQL_{\overline{X}}$, the sample size n is increased by one, and the optimization process is repeated.

```
#This code is to optimize the npx chart considering the mean and the variance known
dmin <- 0 #the minimum shift
dmax <- 3.5 #the maximum shift

#OPTIMIZATION OF PARAMETERS
n <- 1 #sample size
h <- 1.0 #sampling interval
Tal <- 370.4 #desirable ATS0
L <- rep(NA, n)

#function uniroot to find the kw which the ARL0 will be Tal for the UCL=0
ATS0 <- function(kw){
((h/(1 - pbinom(0, n, (1-pnorm(kw)+pnorm(-kw)))))-Tal)
}
L[1] <- uniroot(ATS0, lower = 0,upper = 5)$root

#function uniroot to find the kw which the ARL0 will be Tal varing the UCL from 1 to n-1
for (UCL in 1:(n-1)){
ATS0 <- function(kw){
((h/(1 - pbinom(UCL, n, (1-pnorm(kw)+pnorm(-kw)))))-Tal)
}
L[(UCL+1)] <-uniroot(ATS0,lower = 0,upper = L[UCL])$root
}
L #print all the kw which guarantee ATS0=370.4 for UCL 0 to n-1

#Calculate the EQL for combinations of UCL and kw which guarantee ARL0=370
```

```
EQL <- rep(NA, n)
for (UCL in 0:(n-1)){
int <- function(delta) (1/(dmax-dmin))*delta^2*(((1/(1-pbinom(UCL,n,(1-pnorm(L[(UCL+1)]-delta)+pnorm(-L[(UCL+1)]-delta)))))-0.5)*h)
EQL[(UCL+1)] <- integrate(int,lower=dmin,upper=dmax)$value
}
EQL #print all the EQL values
min(EQL) #print the minimum EQL
```

A.2 THE np_x CHART PERFORMANCE USING OPTIMAL VALUES OF UCL AND k_w

After determining the sample size n and the parameters UCL and k_w, the performance of the np_x chart is evaluated for both in-control (ATS_0) and out-of-control conditions, including ATS and EQL_{npx}.

```
#CALCULATION WITH THE OPTIMUM PARAMETERS
#for the npx chart with knonm parameters in terms of ATS
UCL <- which.min(EQL)-1 #the UCL which produces the minimum EQL and ATS0=370.4
kw <- L[which.min(EQL)] #the kw which produces the minimum EQL and ATS0=370.4
UCL
kw

#calculate the ATS0, the ATS when the process is in-control
ATS0 <- h/(1-pbinom(UCL, n, (1-pnorm(kw)+pnorm(-kw))))
ATS0 # print the ATS0

#calculate the ATS for different values of delta varying from 0 to 3.5
delta <-0.25
while (delta <= 3.5) {
ATS <- ((1/(1-pbinom(UCL, n, (1-pnorm(kw-delta)+pnorm(-kw-delta)))))-0.5)*h
print(ATS)
delta <- delta+0.25
}

#calculate the EQL, the overall of ATS in a range of shifts(0 to 3.5)
int <- function(delta) (1/(dmax-dmin))*delta^2*(((1/(1-pbinom(UCL,n,(1-pnorm(kw-delta)+pnorm(-kw-delta)))))-0.5)*h)
EQL <- integrate(int,lower=dmin,upper=dmax)
EQL
```

APPENDIX B – np_x CHART: UK CASE

B.1 CONDITIONAL APPROACH

```
#This code is to simulated 100,000 values considering the mean is unknown and the variance is known
library(pracma) #it is necessary to use the function integral2, which solves a two dimensional numeric integral
N <- 100000 #number of CARL0 to be simulated
kw <- 1.6599319 #Limits 3-Sigma
n <- 38 #Sample size on phase I
m <- c(25, 50, 100, 500, 1000) #Number of samples on phase I
UCL <- 9 #Upper control limit
shift <- 0 #Shift on the process mean

#Creating the vectors to store the values simulated for the performance in terms of CARL and CEQL
meanCARL <- rep(NA, 5)
sdCARL <- rep(NA, 5)
medianCARL <- rep(NA, 5)
Q25CARL <- rep(NA, 5)
Q75CARL <- rep(NA, 5)
meanCEQL <- rep(NA, 5)
sdCEQL <- rep(NA, 5)
medianCEQL <- rep(NA, 5)
Q25CEQL <- rep(NA, 5)
Q75CEQL <- rep(NA, 5)

for (j in 1:5) { #to simulate the CARL and CEQL for all the numbers of samples considered "m"
CARL <- rep(NA, N)
CEQL <- rep(NA, N)
for (i in 1:N) { #to simulate 100000 CARL and CEQL
Z1 <- rnorm(1)
p <- 1 - pnorm(Z1/(sqrt(m[j]*n))-shift+kw) + pnorm(Z1/sqrt((m[j]*n))-shift-kw)
CARL[i] <- (1/(1-pbinom(UCL, n, p)))
dmin <- 0
dmax <- 3.5
int <- function(shift) (1/(dmax-dmin))*shift^2 * (1/(1 - pbinom(UCL, n,
(1 - pnorm(Z1/(sqrt(m[j]*n))-shift+kw) + pnorm(Z1/sqrt((m[j]*n))-shift-kw)))))
CEQL[i] <- integrate(int,0,3.5)$value
```

```
}
meanCARL[j] <- mean(CARL)
sdCARL [j] <- sd(CARL)
medianCARL [j] <- median(CARL)
Q25CARL [j] <- quantile(CARL,0.25)
Q75CARL [j] <- quantile(CARL,0.75)
meanCEQL[j] <- mean(CEQL)
sdCEQL [j] <- sd(CEQL)
medianCEQL [j] <- median(CEQL)
Q25CEQL [j] <- quantile(CEQL,0.25)
Q75CEQL [j] <- quantile(CEQL,0.75)
}
matriz_simulation <- rbind(m, meanCARL, sdCARL, medianCARL, Q25CARL, Q75CARL,
meanCEQL, sdCEQL, medianCEQL, Q25CEQL, Q75CEQL)
matriz_simulation #to create a matrix to store and show the simulated values for different number
of samples
```

B.2 UNCONDITIONAL APPROACH - DERIVATIONS

```
#This code is to calculate the unconditional ARL (considering all users)
ARL <- rep(NA, 75)
k <- 1

for (j in 1:5) {
shift <- 0
ARLm <- rep(NA, 15)
for (i in 1:15) {
f <- function(u) (1/(1-pbinom(UCL, n, (1 - pnorm(qnorm(u)/(sqrt(m[j]*n))+kw-shift) +
pnorm(qnorm(u)/(sqrt(m[j]*n))-kw-shift)))))
ARLm[i] <- integrate(f,0,1)$value
shift=shift+0.25
ARL[k] <- c(ARLm[i])
k=k+1
}
}
ARLtot <- matrix(ARL,15,5)
ARLtot
#This code is to calculate the unconditional EQL, the overall of ARL in a range of shifts(0 to 3.5)
EQL <- rep(NA,5)
```

```
for (i in 1:5) {
dmin <- 0
dmax <- 3.5
f <- function(u,shift) (1/(dmax-dmin))*shift^2 * (1/(1 - pbinom(UCL, n,
(1 - pnorm(qnorm(u)/(sqrt(m[i]*n))+kw-shift) + pnorm(qnorm(u)/(sqrt(m[i]*n))-kw-shift)))))
EQL[i] <- integral2(f,0,1,0,3.5)
}
print(EQL)
```

B.3 ADJUSTMENT - CODES IN R

```
library(pracma) #it is necessary to use the function integral2 and integral3, which solves a two and
three dimensional numeric integral respectively

#This code is to optimize the kw considering the mean unknown and the variance known
n <- 38
h <- 1
m <- c(25, 50, 100, 500, 1000)
shift <- 0
Tal <- 370.4
L <- rep(NA, 5*n)

#function uniroot to find the kw which the ATS0 will be Tal for the UCL=0
j <- 0
for (i in 1:5) {
j <- j+1
ARL0 <- function(kw) {
f<-function(u) ((1/(1-pbinom(0, n, (1 - pnorm(qnorm(u)/(sqrt(m[i]*n))+kw-shift)
+ pnorm(qnorm(u)/(sqrt(m[i]*n))-kw-shift)))))-0.5)*h
((integrate(f,0,1)$value)-Tal)
}
L[j] <- uniroot(ARL0, lower = 0,upper = 5)$root

#function uniroot to find the kw which the ARL0 will be Tal varing the UCL from 1 to n-1
for (UCL in 1:(n-1)){
ARL0 <- function(kw){
f<-function(u) ((1/(1-pbinom(UCL, n, (1 - pnorm(qnorm(u)/(sqrt(m[i]*n))+kw-shift)
+ pnorm(qnorm(u)/(sqrt(m[i]*n))-kw-shift)))))-0.5)*h ((integrate(f,0,1)$value)-Tal)
}
L[j+1] <- uniroot(ARL0, lower = 0,upper = L[j])$root
j <- j+1
```

```
}
}
Ltot <- matrix(L,n,5)
Ltot

#Calculate the EQL for combinations of UCL and kw which guarantee ARL0=370.4
dmin <- 0
dmax <- 3.5
EQL <- rep(NA, 5*n)
EQLmin <- rep(NA, 5)
UCL2 <- rep(NA, 5)
UCLopt <- rep(NA, 5)
kw_opt <- rep(NA, 5)
m_opt <- rep(NA, 5)
j<-1

for (i in 1:5) {
for (UCL in 0:(n-1)){
f <- function(u,shift) (1/(dmax-dmin))*shift2̂*(((1/(1-pbinom(UCL, n, (1 - pnorm(qnorm(u)/
(sqrt(m[i]*n))+L[j]-shift) + pnorm(qnorm(u)/(sqrt(m[i]*n))-L[j]-shift)))))-0.5)*h)
EQL[j] <- integral2(f,0,1,0,3.5)$Q
j<-j+1
}
EQLmin[i] <- EQL[which.min(EQL)]
UCL2[i] <- which.min(EQL)-1
kw_opt[i] <- L[which.min(EQL)]
m_opt[i] <- m[i]
}
EQLtot<-matrix(EQL,n,5)
EQLtot
```

B.4 ADJUSTED PARAMETERS UCL AND k_w

This section shows the k_w and UCL tables generated for different sample sizes during the np_x chart optimization process. Also the EQL value produced by each pair of k_w and UCL are presented.

Table 28 – k_w and UCL which satisfy the constraint $ATS_0 = \tau = 370$ for the np_x chart when the mean is estimated and the standard deviation is known for $n = 5, h = 1$ and different number of samples m.

1	2	3	4	5	6
UCL			k_w		
	$m = 25$	$m = 50$	$m = 100$	$m = 500$	$m = 1000$
0	3.47283	3.46679	3.46358	3.46090	3.46055
1	2.40222	2.39793	2.39567	2.39380	2.39356
2	1.83955	1.83621	1.83446	1.83303	1.83284
3	1.41854	1.41592	1.41456	1.41344	1.41330
4	1.02695	1.02502	1.02403	1.02322	1.02312

source: Elaborated by the author.

Table 29 – EQL for all combinations of k_w and UCL which satisfy the constraint $ATS_0 = \tau = 370$ for the np_x chart when the mean is estimated and the standard deviation is known for $n = 5$ and $h = 1$.

1	2	3	4	5	6
UCL			EQL		
	$m = 25$	$m = 50$	$m = 100$	$m = 500$	$m = 1000$
0	16.10319	15.65857	15.43338	15.25136	15.22853
1	10.79167	10.46040	10.29507	10.16251	10.14597
2	10.32774	10.01667	9.86205	9.73876	9.72340
3	11.49879	11.18233	11.02509	10.89980	10.88422
4	15.36190	15.01684	14.84532	14.70865	14.69162

source: Elaborated by the author.

Table 30 – k_w and UCL which satisfy the constraint $ATS_0 = \tau = 370$ for the np_x chart when the mean is estimated and the standard deviation is known for $n = 10, h = 1$ and different number of samples m.

1	2	3	4	5	6
UCL			k_w		
	$m = 25$	$m = 50$	$m = 100$	$m = 500$	$m = 1000$
0	3.64945	3.64608	3.64432	3.64295	3.64277
1	2.66117	2.65869	2.65741	2.65636	2.65623
2	2.17778	2.17574	2.17469	2.17383	2.17372
3	1.85014	1.84839	1.84750	1.84677	1.84667
4	1.59367	1.59217	1.59139	1.59076	1.59068
5	1.37537	1.37406	1.37339	1.37285	1.37278
6	1.17821	1.17708	1.17651	1.17604	1.17598
7	0.99088	0.98992	0.98944	0.98905	0.98900
8	0.80243	0.80165	0.80126	0.80094	0.80090
9	0.59377	0.59319	0.59290	0.59266	0.59263

source: Elaborated by the author.

Table 31 – EQL for all combinations of k_w and UCL which satisfy the constraint $ATS_0 = \tau = 370$ for the np_x chart when the mean is estimated and the standard deviation is known for $n = 10$ and $h = 1$.

1	2	3	4	5	6
UCL			EQL		
	$m = 25$	$m = 50$	$m = 100$	$m = 500$	$m = 1000$
0	13.23734	13.02497	12.91801	12.83408	12.82330
1	8.10411	7.95470	7.88002	7.82032	7.81287
2	7.03786	6.90392	6.83719	6.78396	6.77731
3	6.78738	6.65857	6.59450	6.54341	6.53704
4	6.88535	6.75738	6.69374	6.64303	6.63671
5	7.23162	7.10193	7.03743	6.98605	6.97965
6	7.84761	7.71385	7.64744	7.59453	7.58793
7	8.86795	8.72810	8.65853	8.60308	8.59619
8	10.68942	10.53955	10.46499	10.40551	10.39810
9	14.94475	14.77650	14.69271	14.62591	14.61759

source: Elaborated by the author.

Table 32 – k_w and UCL which satisfy the constraint $ATS_0 = \tau = 370$ for the np_x chart when the mean is estimated and the standard deviation is known for $n = 15, h = 1$ and different number of samples m.

1	2	3	4	5	6
UCL			k_w		
	$m = 25$	$m = 50$	$m = 100$	$m = 500$	$m = 1000$
0	3.75040	3.74804	3.74682	3.74583	3.74571
1	2.79908	2.79731	2.79640	2.79567	2.79557
2	2.34523	2.34374	2.34298	2.34236	2.34228
3	2.04548	2.04418	2.04352	2.04298	2.04291
4	1.81746	1.81630	1.81571	1.81523	1.81517
5	1.62989	1.62885	1.62832	1.62789	1.62783
6	1.46762	1.46668	1.46620	1.46582	1.46577
7	1.32213	1.32128	1.32085	1.32050	1.32046
8	1.18803	1.18727	1.18688	1.18656	1.18652
9	1.06148	1.06079	1.06045	1.06017	1.06013
10	0.93945	0.93885	0.93854	0.93829	0.93826
11	0.81907	0.81854	0.81827	0.81806	0.81803
12	0.69711	0.69666	0.69643	0.69624	0.69622
13	0.56849	0.56812	0.56793	0.56778	0.56776
14	0.42110	0.42082	0.42068	0.42057	0.42056

source: Elaborated by the author.

Table 33 – EQL for all combinations of k_w and UCL which satisfy the constraint $ATS_0 = \tau = 370$ for the np_x chart when the mean is estimated and the standard deviation is known for $n = 15$ and $h = 1$.

1	2	3	4	5	6
UCL			EQL		
	$m = 25$	$m = 50$	$m = 100$	$m = 500$	$m = 1000$
0	11.99788	11.86043	11.79125	11.73572	11.72876
1	7.10954	7.01492	6.96765	6.92985	6.92512
2	6.00772	5.92441	5.88287	5.84970	5.84557
3	5.62283	5.54400	5.50474	5.47343	5.46952
4	5.49885	5.42191	5.38362	5.35308	5.34927
5	5.51221	5.43582	5.39781	5.36750	5.36373
6	5.61981	5.54313	5.50497	5.47456	5.47077
7	5.80927	5.73166	5.69304	5.66226	5.65843
8	6.08534	6.00623	5.96685	5.93548	5.93156
9	6.46589	6.38469	6.34428	6.31207	6.30806
10	6.99048	6.90636	6.86456	6.83124	6.82709
11	7.73171	7.64413	7.60053	7.56578	7.56145
12	8.85252	8.76002	8.71394	8.67720	8.67263
13	10.76629	10.66661	10.61691	10.57726	10.57233
14	15.11547	15.00320	14.94726	14.90257	14.89701

source: Elaborated by the author.

Table 34 – k_w and UCL which satisfy the constraint $ATS_0 = \tau = 370$ for the np_x chart when the mean is estimated and the standard deviation is known for $n = 20, h = 1$ and different number of samples m.

1	2	3	4	5	6
UCL			k_w		
	$m = 25$	$m = 50$	$m = 100$	$m = 500$	$m = 1000$
0	3.82089	3.81907	3.81813	3.81738	3.81728
1	2.89262	2.89123	2.89052	2.88995	2.88988
2	2.45586	2.45468	2.45408	2.45359	2.45353
3	2.17109	2.17004	2.16951	2.16908	2.16903
4	1.95734	1.95639	1.95592	1.95553	1.95548
5	1.78387	1.78301	1.78257	1.78222	1.78217
6	1.63618	1.63539	1.63499	1.63467	1.63462
7	1.50598	1.50525	1.50488	1.50458	1.50454
8	1.38836	1.38769	1.38735	1.38708	1.38704
9	1.28004	1.27942	1.27910	1.27885	1.27882
10	1.17857	1.17800	1.17771	1.17748	1.17745
11	1.08232	1.08180	1.08153	1.08131	1.08129
12	0.98984	0.98936	0.98911	0.98891	0.98889
13	0.89988	0.89944	0.89922	0.89904	0.89901
14	0.81137	0.81098	0.81078	0.81061	0.81059
15	0.72313	0.72277	0.72260	0.72245	0.72243
16	0.63375	0.63344	0.63328	0.63315	0.63314
17	0.54118	0.54091	0.54078	0.54067	0.54066
18	0.44195	0.44173	0.44162	0.44154	0.44152
19	0.32696	0.32680	0.32672	0.32666	0.32665

source: Elaborated by the author.

Table 35 – EQL for all combinations of k_w and UCL which satisfy the constraint $ATS_0 = \tau = 370$ for the np_x chart when the mean is estimated and the standard deviation is known for $n = 20$ and $h = 1$.

1	2	3	4	5	6
UCL			EQL		
	$m = 25$	$m = 50$	$m = 100$	$m = 500$	$m = 1000$
0	11.25811	11.15714	11.10641	11.06576	11.06066
1	6.55858	6.48996	6.45566	6.42823	6.42482
2	5.47289	5.41307	5.38322	5.35940	5.35643
3	5.05605	4.99993	4.97198	4.94969	4.94691
4	4.87447	4.82019	4.79315	4.77160	4.76891
5	4.80473	4.75135	4.72478	4.70360	4.70096
6	4.80521	4.75212	4.72571	4.70465	4.70203
7	4.85236	4.79924	4.77280	4.75173	4.74910
8	4.94013	4.88668	4.86007	4.83887	4.83623
9	5.06621	5.01215	4.98524	4.96380	4.96113
10	5.22957	5.17472	5.14743	5.12567	5.12296
11	5.43871	5.38282	5.35500	5.33283	5.33007
12	5.70202	5.64487	5.61641	5.59374	5.59091
13	6.03314	5.97444	5.94522	5.92194	5.91904
14	6.45980	6.39926	6.36911	6.34507	6.34207
15	7.02371	6.96088	6.92957	6.90460	6.90150
16	7.80394	7.73817	7.70540	7.67928	7.67603
17	8.95801	8.88845	8.85376	8.82610	8.82266
18	10.90644	10.83144	10.79402	10.76415	10.76044
19	15.30370	15.21917	15.17699	15.14338	15.13917

source: Elaborated by the author.

Table 36 – k_w and UCL which satisfy the constraint $ATS_0 = \tau = 370$ for the np_x chart when the mean is estimated and the standard deviation is known for $n = 25, h = 1$ and different number of samples m.

1	2	3	4	5	6
UCL			k_w		
	$m = 25$	$m = 50$	$m = 100$	$m = 500$	$m = 1000$
0	3.87493	3.87344	3.87268	3.87206	3.87199
1	2.96305	2.96190	2.96132	2.96085	2.96079
2	2.53795	2.53697	2.53647	2.53607	2.53602
3	2.26304	2.26216	2.26172	2.26136	2.26132
4	2.05829	2.05749	2.05709	2.05676	2.05672
5	1.89348	1.89275	1.89237	1.89207	1.89204
6	1.75428	1.75361	1.75326	1.75298	1.75295
7	1.63268	1.63205	1.63173	1.63147	1.63143
8	1.52384	1.52325	1.52295	1.52270	1.52267
9	1.42460	1.42405	1.42377	1.42354	1.42352
10	1.33285	1.33233	1.33207	1.33186	1.33183
11	1.24686	1.24637	1.24613	1.24593	1.24590
12	1.16548	1.16503	1.16480	1.16461	1.16459
13	1.08771	1.08729	1.08707	1.08690	1.08688
14	1.01289	1.01249	1.01229	1.01213	1.01211
15	0.94030	0.93993	0.93975	0.93960	0.93958
16	0.86934	0.86900	0.86882	0.86868	0.86867
17	0.79948	0.79916	0.79901	0.79888	0.79886
18	0.73013	0.72984	0.72970	0.72958	0.72957
19	0.66072	0.66046	0.66033	0.66023	0.66021
20	0.59052	0.59028	0.59017	0.59007	0.59006
21	0.51856	0.51836	0.51825	0.51817	0.51816
22	0.44333	0.44315	0.44306	0.44299	0.44298
23	0.36208	0.36194	0.36186	0.36181	0.36180
24	0.26746	0.26735	0.26730	0.26726	0.26725

source: Elaborated by the author.

Table 37 – EQL for all combinations of k_w and UCL which satisfy the constraint $ATS_0 = \tau = 370$ for the np_x chart when the mean is estimated and the standard deviation is known for $n = 25$ and $h = 1$.

1	2	3	4	5	6
UCL			EQL		
	$m = 25$	$m = 50$	$m = 100$	$m = 500$	$m = 1000$
0	10.74941	10.66981	10.62990	10.59786	10.59386
1	6.19611	6.14256	6.11577	6.09438	6.09172
2	5.13426	5.08786	5.06472	5.04625	5.04394
3	4.71049	4.66726	4.64569	4.62847	4.62632
4	4.50711	4.46551	4.44478	4.42823	4.42616
5	4.40642	4.36574	4.34547	4.32929	4.32727
6	4.36568	4.32546	4.30543	4.28943	4.28743
7	4.36299	4.32296	4.30302	4.28711	4.28512
8	4.38844	4.34842	4.32848	4.31256	4.31058
9	4.43728	4.39710	4.37708	4.36107	4.35909
10	4.50905	4.46855	4.44830	4.43224	4.43024
11	4.59979	4.55879	4.53844	4.52221	4.52019
12	4.71203	4.67062	4.65003	4.63359	4.63155
13	4.84575	4.80377	4.78287	4.76621	4.76414
14	5.00829	4.96559	4.94433	4.92739	4.92528
15	5.20312	5.15958	5.13790	5.12062	5.11847
16	5.43664	5.39213	5.36997	5.35229	5.35010
17	5.72166	5.67602	5.65328	5.63514	5.63289
18	6.07281	6.02586	6.00246	5.98382	5.98150
19	6.51764	6.46911	6.44493	6.42568	6.42324
20	7.09889	7.04847	7.02333	7.00329	7.00079
21	7.89637	7.84355	7.81723	7.79624	7.79363
22	9.06924	9.01335	8.98547	8.96323	8.96047
23	11.03894	10.97864	10.94856	10.92455	10.92156
24	15.46080	15.39295	15.35906	15.33203	15.32865

source: Elaborated by the author.

Table 38 – k_w and UCL which satisfy the constraint $ATS_0 = \tau = 370$ for the np_x chart when the mean is estimated and the standard deviation is known for $n = 38, h = 1$ and different number of samples m.

1	2	3	4	5	6
UCL			k_w		
	$m = 25$	$m = 50$	$m = 100$	$m = 500$	$m = 1000$
0	3.9748	3.9738	3.9733	3.9729	3.9728
1	3.0909	3.0901	3.0897	3.0894	3.0894
2	2.6849	2.6842	2.6839	2.6836	2.6835
3	2.4255	2.4249	2.4246	2.4243	2.4243
4	2.2345	2.2339	2.2336	2.2334	2.2334
5	2.0824	2.0819	2.0816	2.0814	2.0814
6	1.9554	1.9549	1.9546	1.9544	1.9544
7	1.8456	1.8451	1.8449	1.8447	1.8447
8	1.7485	1.7480	1.7478	1.7476	1.7476
9	1.6610	1.6605	1.6603	1.6601	1.6601
10	1.5810	1.5806	1.5804	1.5802	1.5802
11	1.5071	1.5067	1.5065	1.5063	1.5063
12	1.4381	1.4378	1.4376	1.4374	1.4374
13	1.3733	1.3729	1.3727	1.3726	1.3726
14	1.3118	1.3115	1.3113	1.3112	1.3112
15	1.2534	1.2530	1.2529	1.2527	1.2527
16	1.1974	1.1970	1.1969	1.1968	1.1968
17	1.1435	1.1432	1.1431	1.1430	1.1429
18	1.0915	1.0912	1.0911	1.0910	1.0910
19	1.0411	1.0408	1.0407	1.0406	1.0406
20	0.9921	0.9918	0.9917	0.9916	0.9916
21	0.9442	0.9440	0.9438	0.9437	0.9437
22	0.8973	0.8971	0.8970	0.8969	0.8969
23	0.8513	0.8511	0.8510	0.8509	0.8509
24	0.8059	0.8057	0.8056	0.8055	0.8055
25	0.7611	0.7609	0.7608	0.7607	0.7607
26	0.7167	0.7165	0.7164	0.7163	0.7163
27	0.6725	0.6723	0.6722	0.6722	0.6722
28	0.6284	0.6282	0.6282	0.6281	0.6281
29	0.5842	0.5841	0.5840	0.5840	0.5839
30	0.5398	0.5397	0.5396	0.5395	0.5395
31	0.4949	0.4948	0.4947	0.4947	0.4947
32	0.4492	0.4491	0.4490	0.4490	0.4490
33	0.4024	0.4023	0.4022	0.4022	0.4022
34	0.3538	0.3537	0.3537	0.3537	0.3536
35	0.3026	0.3025	0.3025	0.3025	0.3025
36	0.2469	0.2468	0.2468	0.2467	0.2467
37	0.1818	0.1817	0.1817	0.1817	0.1817

source: Elaborated by the author.

Table 39 – EQL for all combinations of k_w and UCL which satisfy the constraint $ATS_0 = \tau = 370$ for the np_x chart when the mean is estimated and the standard deviation is known for $n = 38$ and $h = 1$.

1	2	3	4	5	6
UCL			EQL		
	$m = 25$	$m = 50$	$m = 100$	$m = 500$	$m = 1000$
0	9.9195	9.8686	9.8431	9.8226	9.8201
1	5.6352	5.6014	5.5845	5.5711	5.5694
2	4.6302	4.6013	4.5868	4.5753	4.5739
3	4.2135	4.1868	4.1735	4.1628	4.1615
4	3.9971	3.9717	3.9590	3.9488	3.9475
5	3.8722	3.8475	3.8351	3.8253	3.8241
6	3.7978	3.7737	3.7615	3.7519	3.7506
7	3.7532	3.7294	3.7174	3.7078	3.7067
8	3.7296	3.7059	3.6941	3.6846	3.6834
9	3.7211	3.6975	3.6857	3.6763	3.6751
10	3.7241	3.7006	3.6888	3.6794	3.6782
11	3.7370	3.7135	3.7016	3.6922	3.6911
12	3.7578	3.7341	3.7223	3.7129	3.7117
13	3.7857	3.7619	3.7500	3.7406	3.7394
14	3.8201	3.7962	3.7843	3.7748	3.7736
15	3.8610	3.8370	3.8250	3.8154	3.8142
16	3.9085	3.8843	3.8722	3.8626	3.8614
17	3.9631	3.9387	3.9265	3.9168	3.9156
18	4.0249	4.0002	3.9879	3.9781	3.9769
19	4.0943	4.0694	4.0570	4.0470	4.0458
20	4.1722	4.1470	4.1344	4.1244	4.1232
21	4.2596	4.2340	4.2213	4.2111	4.2099
22	4.3574	4.3316	4.3187	4.3084	4.3071
23	4.4676	4.4413	4.4283	4.4178	4.4165
24	4.5919	4.5652	4.5519	4.5413	4.5400
25	4.7329	4.7058	4.6923	4.6815	4.6802
26	4.8934	4.8658	4.8521	4.8411	4.8397
27	5.0789	5.0507	5.0367	5.0255	5.0241
28	5.2946	5.2658	5.2515	5.2400	5.2386
29	5.5483	5.5188	5.5041	5.4924	5.4910
30	5.8506	5.8204	5.8052	5.7932	5.7917
31	6.2223	6.1911	6.1755	6.1631	6.1616
32	6.6843	6.6521	6.6360	6.6232	6.6216
33	7.2855	7.2520	7.2353	7.2219	7.2203
34	8.1027	8.0676	8.0501	8.0362	8.0344
35	9.3007	9.2635	9.2450	9.2302	9.2284
36	11.2885	11.2484	11.2285	11.2125	11.2105
37	15.7405	15.6955	15.6731	15.6551	15.6529

source: Elaborated by the author.

Table 40 – k_w and UCL which satisfy the constraint $ATS_0 = \tau = 370$ for the np_x chart when the mean is estimated and the standard deviation is known for $n = 5, h = 0.5$ and different number of samples m.

1	2	3	4	5	6
UCL			k_w		
	$m = 25$	$m = 50$	$m = 100$	$m = 500$	$m = 1000$
0	3.65574	3.64945	3.64608	3.64331	3.64295
1	2.52883	2.52436	2.52199	2.52002	2.51977
2	1.94477	1.94126	1.93942	1.93791	1.93772
3	1.51281	1.51004	1.50860	1.50741	1.50726
4	1.11499	1.11290	1.11183	1.11095	1.11084

source: Elaborated by the author.

Table 41 – EQL for all combinations of k_w and UCL which satisfy the constraint $ATS_0 = \tau = 370$ for the np_x chart when the mean is estimated and the standard deviation is known for $n = 5$ and $h = 0.5$.

1	2	3	4	5	6
UCL			EQL		
	$m = 25$	$m = 50$	$m = 100$	$m = 500$	$m = 1000$
0	13.18180	12.76010	12.54615	12.37549	12.35381
1	8.45857	8.14495	7.98843	7.86334	7.84770
2	7.95671	7.66331	7.51755	7.40134	7.38687
3	8.80437	8.50650	8.35877	8.24114	8.22650
4	11.66844	11.34581	11.18558	11.05800	11.04209

source: Elaborated by the author.

Table 42 – k_w and UCL which satisfy the constraint $ATS_0 = \tau = 370$ for the np_x chart when the mean is estimated and the standard deviation is known for $n = 10, h = 0.5$ and different number of samples m.

1	2	3	4	5	6
UCL			k_w		
	$m = 25$	$m = 50$	$m = 100$	$m = 500$	$m = 1000$
0	3.82439	3.82089	3.81907	3.81757	3.81738
1	2.77812	2.77554	2.77421	2.77311	2.77298
2	2.27217	2.27005	2.26895	2.26806	2.26794
3	1.93248	1.93067	1.92973	1.92897	1.92887
4	1.66839	1.66682	1.66601	1.66535	1.66527
5	1.44476	1.44339	1.44269	1.44212	1.44205
6	1.24356	1.24238	1.24177	1.24128	1.24122
7	1.05296	1.05195	1.05143	1.05102	1.05096
8	0.86165	0.86082	0.86039	0.86005	0.86001
9	0.65016	0.64953	0.64921	0.64895	0.64892

source: Elaborated by the author.

Table 43 – EQL for all combinations of k_w and UCL which satisfy the constraint $ATS_0 = \tau = 370$ for the np_x chart when the mean is estimated and the standard deviation is known for $n = 10$ and $h = 0.5$.

1	2	3	4	5	6
UCL			EQL		
	$m = 25$	$m = 50$	$m = 100$	$m = 500$	$m = 1000$
0	10.86415	10.66209	10.56017	10.47818	10.46790
1	6.27951	6.13715	6.06604	6.00919	6.00210
2	5.31138	5.18403	5.12063	5.07005	5.06374
3	5.06769	4.94535	4.88454	4.83607	4.83003
4	5.12583	5.00448	4.94415	4.89611	4.89013
5	5.39301	5.27017	5.20911	5.16049	5.15445
6	5.87833	5.75204	5.68927	5.63927	5.63304
7	6.68308	6.55003	6.48441	6.43215	6.42564
8	8.09856	7.95753	7.88739	7.83150	7.82456
9	11.30358	11.14629	11.06802	11.00556	10.99778

source: Elaborated by the author.

Table 44 – k_w and UCL which satisfy the constraint $ATS_0 = \tau = 370$ for the np_x chart when the mean is estimated and the standard deviation is known for $n = 15, h = 0.5$ and different number of samples m.

1	2	3	4	5	6
UCL			k_w		
	$m = 25$	$m = 50$	$m = 100$	$m = 500$	$m = 1000$
0	3.92112	3.91867	3.91740	3.91637	3.91624
1	2.91142	2.90958	2.90864	2.90787	2.90778
2	2.43480	2.43326	2.43247	2.43183	2.43175
3	2.12283	2.12149	2.12080	2.12024	2.12017
4	1.88711	1.88591	1.88530	1.88480	1.88474
5	1.69414	1.69306	1.69251	1.69206	1.69200
6	1.52783	1.52685	1.52635	1.52595	1.52590
7	1.37913	1.37825	1.37780	1.37744	1.37739
8	1.24242	1.24163	1.24122	1.24089	1.24085
9	1.11364	1.11293	1.11256	1.11227	1.11223
10	0.98957	0.98893	0.98860	0.98834	0.98831
11	0.86729	0.86673	0.86645	0.86622	0.86619
12	0.74350	0.74302	0.74277	0.74258	0.74255
13	0.61288	0.61248	0.61228	0.61212	0.61210
14	0.46319	0.46288	0.46273	0.46261	0.46259

source: Elaborated by the author.

Table 45 – EQL for all combinations of k_w and UCL which satisfy the constraint $ATS_0 = \tau = 370$ for the np_x chart when the mean is estimated and the standard deviation is known for $n = 15$ and $h = 0.5$.

1	2	3	4	5	6
UCL			EQL		
	$m = 25$	$m = 50$	$m = 100$	$m = 500$	$m = 1000$
0	9.84879	9.71729	9.65112	9.59801	9.59136
1	5.46034	5.36990	5.32471	5.28859	5.28408
2	4.46376	4.38426	4.34463	4.31300	4.30906
3	4.11093	4.03579	3.99838	3.96852	3.96479
4	3.99120	3.91793	3.88147	3.85233	3.84872
5	3.99167	3.91898	3.88273	3.85393	3.85035
6	4.07362	4.00070	3.96445	3.93556	3.93196
7	4.22438	4.15064	4.11398	4.08476	4.08112
8	4.44801	4.37288	4.33552	4.30576	4.30205
9	4.75710	4.68005	4.64172	4.61119	4.60739
10	5.17917	5.09963	5.06004	5.02850	5.02458
11	5.77380	5.69081	5.64959	5.61673	5.61268
12	6.66581	6.57848	6.53501	6.50036	6.49604
13	8.15969	8.06592	8.01918	7.98192	7.97727
14	11.45266	11.34759	11.29526	11.25346	11.24826

source: Elaborated by the author.

Table 46 – k_w and UCL which satisfy the constraint $ATS_0 = \tau = 370$ for the np_x chart when the mean is estimated and the standard deviation is known for $n = 20, h = 0.5$ and different number of samples m.

1	2	3	4	5	6
UCL			k_w		
	$m = 25$	$m = 50$	$m = 100$	$m = 500$	$m = 1000$
0	3.98881	3.98691	3.98594	3.98515	3.98505
1	3.00198	3.00055	2.99981	2.99922	2.99914
2	2.54240	2.54119	2.54056	2.54006	2.54000
3	2.24542	2.24434	2.24379	2.24335	2.24329
4	2.02393	2.02296	2.02246	2.02206	2.02201
5	1.84512	1.84423	1.84378	1.84342	1.84337
6	1.69340	1.69259	1.69217	1.69184	1.69179
7	1.56003	1.55928	1.55890	1.55859	1.55855
8	1.43985	1.43916	1.43880	1.43852	1.43848
9	1.32934	1.32869	1.32837	1.32810	1.32807
10	1.22605	1.22546	1.22515	1.22491	1.22488
11	1.12810	1.12756	1.12728	1.12705	1.12703
12	1.03412	1.03362	1.03336	1.03316	1.03313
13	0.94280	0.94234	0.94211	0.94192	0.94190
14	0.85294	0.85252	0.85231	0.85214	0.85212
15	0.76338	0.76300	0.76281	0.76266	0.76264
16	0.67264	0.67231	0.67215	0.67201	0.67200
17	0.57865	0.57836	0.57822	0.57810	0.57809
18	0.47780	0.47757	0.47745	0.47736	0.47734
19	0.36066	0.36048	0.36039	0.36032	0.36031

source: Elaborated by the author.

Table 47 – EQL for all combinations of k_w and UCL which satisfy the constraint $ATS_0 = \tau = 370$ for the np_x chart when the mean is estimated and the standard deviation is known for $n = 20$ and $h = 0.5$.

1	2	3	4	5	6
UCL			EQL		
	$m = 25$	$m = 50$	$m = 100$	$m = 500$	$m = 1000$
0	9.23827	9.14136	9.09268	9.05361	9.04872
1	5.00202	4.93628	4.90344	4.87717	4.87389
2	4.01858	3.96137	3.93283	3.90999	3.90716
3	3.63990	3.58632	3.55962	3.53831	3.53565
4	3.47226	3.42047	3.39468	3.37410	3.37153
5	3.40575	3.35483	3.32949	3.30926	3.30674
6	3.40023	3.34962	3.32443	3.30434	3.30183
7	3.43527	3.38465	3.35945	3.33931	3.33682
8	3.50547	3.45453	3.42910	3.40892	3.40640
9	3.60747	3.55590	3.53030	3.50990	3.50736
10	3.74237	3.69017	3.66420	3.64351	3.64093
11	3.91287	3.85973	3.83330	3.81223	3.80961
12	4.12917	4.07485	4.04782	4.02629	4.02361
13	4.40196	4.34619	4.31846	4.29634	4.29359
14	4.74931	4.69187	4.66327	4.64046	4.63765
15	5.20729	5.14772	5.11807	5.09443	5.09148
16	5.83604	5.77381	5.74281	5.71810	5.71502
17	6.75726	6.69156	6.65882	6.63273	6.62948
18	8.28907	8.21836	8.18311	8.15499	8.15149
19	11.61857	11.53946	11.49999	11.46848	11.46456

source: Elaborated by the author.

Table 48 – k_w and UCL which satisfy the constraint $ATS_0 = \tau = 370$ for the np_x chart when the mean is estimated and the standard deviation is known for $n = 25, h = 0.5$ and different number of samples m.

1	2	3	4	5	6
UCL			k_w		
	$m = 25$	$m = 50$	$m = 100$	$m = 500$	$m = 1000$
0	4.04077	4.03922	4.03843	4.03779	4.03771
1	3.07029	3.06911	3.06850	3.06802	3.06796
2	2.62238	2.62137	2.62085	2.62044	2.62039
3	2.33525	2.33435	2.33389	2.33352	2.33347
4	2.12281	2.12199	2.12157	2.12123	2.12119
5	1.95266	1.95190	1.95152	1.95121	1.95117
6	1.80943	1.80873	1.80837	1.80808	1.80805
7	1.68470	1.68405	1.68371	1.68345	1.68341
8	1.57332	1.57272	1.57241	1.57216	1.57212
9	1.47197	1.47140	1.47111	1.47087	1.47084
10	1.37840	1.37786	1.37759	1.37737	1.37735
11	1.29082	1.29032	1.29007	1.28986	1.28984
12	1.20803	1.20756	1.20733	1.20713	1.20711
13	1.12904	1.12860	1.12837	1.12820	1.12817
14	1.05304	1.05263	1.05243	1.05226	1.05224
15	0.97930	0.97891	0.97872	0.97856	0.97855
16	0.90731	0.90695	0.90677	0.90663	0.90661
17	0.83644	0.83611	0.83594	0.83581	0.83579
18	0.76611	0.76581	0.76566	0.76554	0.76552
19	0.69568	0.69541	0.69527	0.69516	0.69514
20	0.62443	0.62418	0.62406	0.62396	0.62395
21	0.55135	0.55114	0.55103	0.55094	0.55093
22	0.47488	0.47469	0.47459	0.47452	0.47451
23	0.39219	0.39203	0.39195	0.39189	0.39188
24	0.29560	0.29548	0.29542	0.29538	0.29537

source: Elaborated by the author.

Table 49 – EQL for all combinations of k_w and UCL which satisfy the constraint $ATS_0 = \tau = 370$ for the np_x chart when the mean is estimated and the standard deviation is known for $n = 25$ and $h = 0.5$.

1	2	3	4	5	6
UCL			EQL		
	$m = 25$	$m = 50$	$m = 100$	$m = 500$	$m = 1000$
0	8.81625	8.73978	8.70139	8.67059	8.66673
1	4.69924	4.64786	4.62216	4.60164	4.59909
2	3.73536	3.69086	3.66871	3.65103	3.64882
3	3.35043	3.30908	3.28847	3.27201	3.26995
4	3.16528	3.12553	3.10572	3.08991	3.08794
5	3.07282	3.03395	3.01459	2.99914	2.99721
6	3.03256	2.99416	2.97504	2.95978	2.95788
7	3.02671	2.98851	2.96948	2.95430	2.95241
8	3.04561	3.00741	2.98839	2.97322	2.97132
9	3.08450	3.04617	3.02708	3.01186	3.00996
10	3.14324	3.10462	3.08539	3.07005	3.06813
11	3.21803	3.17903	3.15962	3.14409	3.14217
12	3.31083	3.27136	3.25164	3.23600	3.23405
13	3.42304	3.38293	3.36301	3.34714	3.34516
14	3.55795	3.51727	3.49701	3.48087	3.47886
15	3.71709	3.67566	3.65502	3.63860	3.63655
16	3.91104	3.86870	3.84762	3.83082	3.82873
17	4.14621	4.10280	4.08120	4.06397	4.06183
18	4.43591	4.39130	4.36907	4.35135	4.34915
19	4.79941	4.75334	4.73040	4.71209	4.70981
20	5.27280	5.22499	5.20116	5.18221	5.17980
21	5.91701	5.86702	5.84210	5.82223	5.81975
22	6.85454	6.80173	6.77541	6.75443	6.75181
23	8.40527	8.34844	8.32010	8.29749	8.29467
24	11.75894	11.69539	11.66367	11.63835	11.63520

source: Elaborated by the author.

APPENDIX C – np_x CHART: KU CASE

C.1 CONDITIONAL APPROACH

#This code is to simulated 100,000 values considering the mean is known and the variance is unknown
library(pracma) #it is necessary to use the function integral2, which solves a two dimensional numeric integral
N <- 100000 #numeros de CARL0 a serem simulados/num de usuarios
kw <- 1.6599319 #Limtes 3-Sigma
m <- c(25, 50, 100, 500, 1000) #Number of samples on phase I
n <- 38 #Tamanho de cada amostra da Fase I
UCL <- 9 #Upper control limit
shift <- 0 #Shift on the process mean

#Creating the vectors to store the values simulated for the performance in terms of CARL and CEQL
meanCARL <- rep(NA, 5)
sdCARL <- rep(NA, 5)
medianCARL <- rep(NA, 5)
Q25CARL <- rep(NA, 5)
Q75CARL <- rep(NA, 5)
meanCEQL <- rep(NA, 5)
sdCEQL <- rep(NA, 5)
medianCEQL <- rep(NA, 5)
Q25CEQL <- rep(NA, 5)
Q75CEQL <- rep(NA, 5)

for (j in 1:5) { #to simulate the CARL and CEQL for all the numbers of samples considered "m"
CARL <- rep(NA, N)
CEQL <- rep(NA, N)
for (i in 1:N) #to simulate 100000 CARL and CEQL
Y <- rchisq(1, (m[j]*(n-1)))
p <- 1 - pnorm(kw*sqrt((Y)/(m[j]*(n-1)))-shift) + pnorm(-kw*sqrt((Y)/(m[j]*(n-1)))-shift)
CARL[i] <- (1/(1-pbinom(UCL, n, p)))
dmin <- 0
dmax <- 3.5
int <- function(shift) (1/(dmax-dmin))*shift$^2 * (1/(1 - pbinom(UCL, n,$
(1 - pnorm(kw*sqrt((Y)/(m[j]*(n-1)))-shift) + pnorm(-kw*sqrt((Y)/(m[j]*(n-1)))-shift)))))
CEQL[i] <- integrate(int,0,3.5)$value

```
}
meanCARL[j] <- mean(CARL)
sdCARL [j] <- sd(CARL)
medianCARL [j] <- median(CARL)
Q25CARL [j] <- quantile(CARL,0.25)
Q75CARL [j] <- quantile(CARL,0.75)
meanCEQL[j] <- mean(CEQL)
sdCEQL [j] <- sd(CEQL)
medianCEQL [j] <- median(CEQL)
Q25CEQL [j] <- quantile(CEQL,0.25)
Q75CEQL [j] <- quantile(CEQL,0.75)
}
matriz_simulation <- rbind(m, meanCARL, sdCARL, medianCARL, Q25CARL, Q75CARL,
meanCEQL, sdCEQL, medianCEQL, Q25CEQL, Q75CEQL)
matriz_simulation #to create a matrix to store and show the simulated values for different number
of samples
```

C.2 UNCONDITIONAL APPROACH - DERIVATIONS

```
#Calculate the unconditional ARL
ARL <- rep(NA, 75)
k <- 1

for (j in 1:5) {
shift <- 0
ARLm <- rep(NA, 15)
for (i in 1:15) {
f <- function(w) (1/(1-pbinom(UCL, n, (1 - pnorm(kw*sqrt((qchisq(w,(m[j]*(n-1))))/(m[j]*(n-1)))
-shift) + pnorm(-kw*sqrt((qchisq(w,(m[j]*(n-1))))/(m[j]*(n-1)))-shift)))))
ARLm[i] <- integrate(f,0,1)$value
shift=shift+0.25
ARL[k] <- c(ARLm[i])
k=k+1
}
}
ARLtot <- matrix(ARL,15,5)
ARLtot
#calculate the EQL, the overall of ARL in a range of shifts(0 to 3.5)
EQL <- rep(NA,5)

for (j in 1:5) {
```

```
dmin <- 0
dmax <- 3.5
f <- function(w,shift) (1/(dmax-dmin))*shift^2 * (1/(1 - pbinom(UCL, n, (1-
pnorm(kw*sqrt((qchisq(w,(m[j]*(n-1))))/(m[j]*(n-1)))-shift) +
pnorm(-kw*sqrt((qchisq(w,(m[j]*(n-1))))/(m[j]*(n-1)))-shift)))))
EQL[j] <- integral2(f,0,1,0,3.5)
}
print(EQL)
```

C.3 ADJUSTMENT - CODES IN R

```
library(pracma) #it is necessary to use the function integral2 and integral3, which solves a two and
three dimensional numeric integral respectively

#This code is to optimize the kw considering the mean known and the variance unknown
n <- 38
h <- 1
m <- c(25, 50, 100, 500, 1000)
shift <- 0
Tal <- 370.4
L <- rep(NA, 5*n)

function uniroot to find the kw which the ARL0 will be Tal for the UCL=0
j <- 0
for (i in 1:5) {
j <- j+1
ARL0 <- function(kw){
f <- function(w) ((1/(1-pbinom(0, n, (1 - pnorm(kw*sqrt((qchisq(w,(m[i]*(n-1))))/
(m[i]*(n-1)))-shift) + pnorm(-kw*sqrt((qchisq(w,(m[i]*(n-1))))/(m[i]*(n-1)))-shift)))))-0.5)*h
((integrate(f,0,1)$value)-Tal)
}
L[j] <- uniroot(ARL0, lower = 0,upper = 5)$root

#function uniroot to find the kw which the ARL0 will be Tal varing the UCL from 1 to n-1
for (UCL in 1:(n-1)){
ARL0 <- function(kw){
f <- function(w) ((1/(1-pbinom(UCL, n, (1 - pnorm(kw*sqrt((qchisq(w,(m[i]*(n-1))))/
(m[i]*(n-1)))-shift) + pnorm(-kw*sqrt((qchisq(w,(m[i]*(n-1))))/(m[i]*(n-1)))-shift)))))-0.5)*h
((integrate(f,0,1)$value)-Tal)
L[j+1] <- uniroot(ARL0, lower = 0,upper = L[j])$root
j <- j+1
```

```
}
}
Ltot <- matrix(L,n,5)
Ltot

#Calculate the EQL for combinations of UCL and kw which guarantee ARL0=370.4
dmin <- 0
dmax <- 3.5
EQL <- rep(NA, 5*n)
EQLmin <- rep(NA, 5)
UCL2 <- rep(NA, 5)
UCLopt <- rep(NA, 5)
kw_opt <- rep(NA, 5)
m_opt <- rep(NA, 5)
j<-1

for (i in 1:5) {
for (UCL in 0:(n-1))
f <- function(w,shift) (1/(dmax-dmin))*shift2̂*(((1/(1-pbinom(UCL, n, (1 - pnorm
(L[j]*sqrt((qchisq(w,(m[i]*(n-1))))/(m[i]*(n-1)))-shift) + pnorm(-L[j]*sqrt
((qchisq(w,(m[i]*(n-1))))/(m[i]*(n-1)))-shift)))))-0.5)*h)
EQL[j] <- integral2(f,0,1,0,3.5)$Q
j<-j+1
}
EQLmin[i] <- EQL[which.min(EQL)]
UCL2[i] <- which.min(EQL)-1
kw_opt[i] <- L[which.min(EQL)]
m_opt[i] <- m[i]
}
EQLtot<-matrix(EQL,n,5)
EQLtot
```

C.4 ADJUSTED PARAMETERS UCL AND k_w

This section shows the k_w and UCL tables generated for different sample sizes during the np_x chart optimization process. Also the EQL value produced by each pair of k_w and UCL are presented.

Table 50 – k_w and UCL which satisfy the constraint $ATS_0 = \tau = 370$ for the np_x chart when the mean is known and the standard deviation is estimated for $n = 5, h = 1$ and different number of samples m.

1	2	3	4	5	6
UCL			k_w		
	$m = 25$	$m = 50$	$m = 100$	$m = 500$	$m = 1000$
0	3.3531	3.4059	3.4328	3.4547	3.4575
1	2.3193	2.3558	2.3744	2.3895	2.3914
2	1.7805	1.8063	1.8194	1.8300	1.8313
3	1.3790	1.3959	1.4045	1.4114	1.4123
4	1.0044	1.0136	1.0183	1.0221	1.0225

source: Elaborated by the author.

Table 51 – EQL for all combinations of k_w and UCL which satisfy the constraint $ATS_0 = \tau = 370$ for the np_x chart when the mean is known and the standard deviation is estimated for $n = 5$ and $h = 1$.

1	2	3	4	5	6
UCL			EQL		
	$m = 25$	$m = 50$	$m = 100$	$m = 500$	$m = 1000$
0	14.2332	14.7193	14.9620	15.1571	15.1813
1	9.5289	9.8290	9.9792	10.0992	10.1143
2	9.2128	9.4608	9.5835	9.6830	9.6955
3	10.4369	10.6511	10.7595	10.8466	10.8576
4	14.2910	14.4814	14.5774	14.6550	14.6648

source: Elaborated by the author.

Table 52 – k_w and UCL which satisfy the constraint $ATS_0 = \tau = 370$ for the np_x chart when the mean is known and the standard deviation is estimated for $n = 10, h = 1$ and different number of samples m.

1	2	3	4	5	6
UCL			k_w		
	$m = 25$	$m = 50$	$m = 100$	$m = 500$	$m = 1000$
0	3.5865	3.6143	3.6284	3.6397	3.6412
1	2.6113	2.6335	2.6448	2.6538	2.6550
2	2.1363	2.1548	2.1642	2.1717	2.1727
3	1.8157	1.8310	1.8388	1.8450	1.8458
4	1.5654	1.5779	1.5842	1.5893	1.5900
5	1.3526	1.3626	1.3676	1.3717	1.3722
6	1.1603	1.1681	1.1720	1.1751	1.1755
7	0.9775	0.9832	0.9861	0.9884	0.9887
8	0.7931	0.7970	0.7989	0.8005	0.8007
9	0.5883	0.5905	0.5915	0.5924	0.5925

source: Elaborated by the author.

Table 53 – EQL for all combinations of k_w and UCL which satisfy the constraint $ATS_0 = \tau = 370$ for the np_x chart when the mean is known and the standard deviation is estimated for $n = 10$ and $h = 1$.

1	2	3	4	5	6
UCL			EQL		
	$m = 25$	$m = 50$	$m = 100$	$m = 500$	$m = 1000$
0	12.4120	12.6119	12.7118	12.7922	12.8024
1	7.5514	7.6785	7.7418	7.7926	7.7990
2	6.5525	6.6618	6.7162	6.7597	6.7652
3	6.3317	6.4314	6.4810	6.5207	6.5257
4	6.4445	6.5375	6.5839	6.6210	6.6257
5	6.7986	6.8860	6.9295	6.9644	6.9688
6	7.4166	7.4989	7.5400	7.5730	7.5772
7	8.4338	8.5113	8.5502	8.5814	8.5853
8	10.2448	10.3177	10.3542	10.3833	10.3870
9	14.4704	14.5395	14.5743	14.6022	14.6057

source: Elaborated by the author.

Table 54 – k_w and UCL which satisfy the constraint $ATS_0 = \tau = 370$ for the np_x chart when the mean is known and the standard deviation is estimated for $n = 15, h = 1$ and different number of samples m.

1	2	3	4	5	6
UCL			k_w		
	$m = 25$	$m = 50$	$m = 100$	$m = 500$	$m = 1000$
0	3.7063	3.7259	3.7357	3.7436	3.7446
1	2.7620	2.7787	2.7870	2.7938	2.7946
2	2.3126	2.3273	2.3347	2.3407	2.3415
3	2.0166	2.0297	2.0362	2.0415	2.0422
4	1.7921	1.8035	1.8093	1.8139	1.8145
5	1.6077	1.6177	1.6227	1.6268	1.6273
6	1.4484	1.4570	1.4614	1.4648	1.4653
7	1.3056	1.3130	1.3167	1.3197	1.3200
8	1.1740	1.1802	1.1833	1.1859	1.1862
9	1.0498	1.0549	1.0575	1.0596	1.0598
10	0.9299	0.9340	0.9361	0.9378	0.9380
11	0.8115	0.8147	0.8164	0.8177	0.8178
12	0.6913	0.6938	0.6950	0.6960	0.6961
13	0.5644	0.5661	0.5669	0.5676	0.5677
14	0.4187	0.4196	0.4201	0.4205	0.4205

source: Elaborated by the author.

Table 55 – EQL for all combinations of k_w and UCL which satisfy the constraint $ATS_0 = \tau = 370$ for the np_x chart when the mean is known and the standard deviation is estimated for $n = 15$ and $h = 1$.

1	2	3	4	5	6
UCL			EQL		
	$m = 25$	$m = 50$	$m = 100$	$m = 500$	$m = 1000$
0	11.4767	11.5992	11.6606	11.7091	11.7155
1	6.7620	6.8414	6.8809	6.9125	6.9164
2	5.7030	5.7724	5.8069	5.8345	5.8380
3	5.3362	5.4011	5.4334	5.4591	5.4624
4	5.2226	5.2842	5.3149	5.3393	5.3424
5	5.2428	5.3015	5.3307	5.3541	5.3570
6	5.3545	5.4108	5.4389	5.4613	5.4641
7	5.5462	5.6004	5.6275	5.6491	5.6518
8	5.8229	5.8751	5.9012	5.9223	5.9249
9	6.2040	6.2537	6.2788	6.2990	6.3015
10	6.7261	6.7744	6.7986	6.8180	6.8205
11	7.4639	7.5104	7.5337	7.5524	7.5548
12	8.5787	8.6233	8.6456	8.6635	8.6658
13	10.4822	10.5246	10.5459	10.5630	10.5652
14	14.8089	14.8500	14.8706	14.8872	14.8893

source: Elaborated by the author.

Table 56 – k_w and UCL which satisfy the constraint $ATS_0 = \tau = 370$ for the np_x chart when the mean is known and the standard deviation is estimated for $n = 20, h = 1$ and different number of samples m.

1	2	3	4	5	6
UCL			k_w		
	$m = 25$	$m = 50$	$m = 100$	$m = 500$	$m = 1000$
0	3.7865	3.8018	3.8095	3.8156	3.8164
1	2.8626	2.8762	2.8830	2.8884	2.8891
2	2.4285	2.4409	2.4472	2.4522	2.4528
3	2.1462	2.1575	2.1632	2.1678	2.1684
4	1.9347	1.9450	1.9502	1.9544	1.9549
5	1.7634	1.7727	1.7774	1.7812	1.7817
6	1.6177	1.6261	1.6303	1.6337	1.6342
7	1.4894	1.4969	1.5007	1.5037	1.5041
8	1.3735	1.3802	1.3836	1.3863	1.3867
9	1.2668	1.2728	1.2758	1.2782	1.2785
10	1.1669	1.1722	1.1748	1.1769	1.1772
11	1.0721	1.0767	1.0790	1.0808	1.0810
12	0.9810	0.9849	0.9869	0.9885	0.9887
13	0.8923	0.8956	0.8973	0.8987	0.8988
14	0.8050	0.8078	0.8092	0.8103	0.8104
15	0.7179	0.7201	0.7213	0.7222	0.7223
16	0.6296	0.6313	0.6322	0.6329	0.6330
17	0.5380	0.5393	0.5400	0.5405	0.5406
18	0.4397	0.4406	0.4411	0.4414	0.4415
19	0.3256	0.3261	0.3264	0.3266	0.3266

source: Elaborated by the author.

Table 57 – EQL for all combinations of k_w and UCL which satisfy the constraint $ATS_0 = \tau = 370$ for the np_x chart when the mean is known and the standard deviation is estimated for $n = 20$ and $h = 1$.

1	2	3	4	5	6
UCL			EQL		
	$m = 25$	$m = 50$	$m = 100$	$m = 500$	$m = 1000$
0	10.8796	10.9672	11.0116	11.0467	11.0511
1	6.3068	6.3642	6.3928	6.4156	6.4185
2	5.2520	5.3028	5.3281	5.3484	5.3509
3	4.8485	4.8965	4.9203	4.9393	4.9417
4	4.6745	4.7204	4.7432	4.7616	4.7639
5	4.6098	4.6542	4.6763	4.6939	4.6961
6	4.6135	4.6566	4.6780	4.6951	4.6973
7	4.6630	4.7049	4.7257	4.7423	4.7444
8	4.7524	4.7931	4.8133	4.8295	4.8315
9	4.8791	4.9188	4.9386	4.9544	4.9564
10	5.0432	5.0818	5.1010	5.1164	5.1183
11	5.2526	5.2900	5.3086	5.3235	5.3254
12	5.5159	5.5519	5.5700	5.5844	5.5862
13	5.8454	5.8808	5.8984	5.9126	5.9144
14	6.2707	6.3050	6.3220	6.3356	6.3373
15	6.8321	6.8652	6.8817	6.8950	6.8967
16	7.6085	7.6405	7.6566	7.6695	7.6711
17	8.7572	8.7881	8.8036	8.8161	8.8176
18	10.6963	10.7264	10.7415	10.7536	10.7551
19	15.0767	15.1057	15.1203	15.1320	15.1335

source: Elaborated by the author.

Table 58 – k_w and UCL which satisfy the constraint $ATS_0 = \tau = 370$ for the np_x chart when the mean is known and the standard deviation is estimated for $n = 25, h = 1$ and different number of samples m.

1	2	3	4	5	6
UCL			k_w		
	$m = 25$	$m = 50$	$m = 100$	$m = 500$	$m = 1000$
0	3.8466	3.8592	3.8656	3.8706	3.8713
1	2.9376	2.9491	2.9549	2.9596	2.9601
2	2.5143	2.5251	2.5305	2.5349	2.5354
3	2.2410	2.2511	2.2562	2.2603	2.2608
4	2.0378	2.0472	2.0520	2.0557	2.0562
5	1.8746	1.8833	1.8876	1.8911	1.8916
6	1.7369	1.7449	1.7489	1.7521	1.7525
7	1.6167	1.6240	1.6277	1.6307	1.6310
8	1.5091	1.5159	1.5193	1.5220	1.5223
9	1.4112	1.4173	1.4204	1.4229	1.4232
10	1.3206	1.3262	1.3290	1.3312	1.3315
11	1.2357	1.2408	1.2433	1.2454	1.2456
12	1.1554	1.1600	1.1623	1.1641	1.1643
13	1.0787	1.0828	1.0848	1.0864	1.0867
14	1.0048	1.0084	1.0103	1.0117	1.0119
15	0.9331	0.9363	0.9379	0.9392	0.9394
16	0.8630	0.8658	0.8672	0.8684	0.8685
17	0.7940	0.7964	0.7976	0.7986	0.7987
18	0.7254	0.7275	0.7285	0.7293	0.7295
19	0.6567	0.6585	0.6593	0.6600	0.6601
20	0.5872	0.5886	0.5893	0.5899	0.5900
21	0.5159	0.5170	0.5176	0.5180	0.5181
22	0.4413	0.4421	0.4426	0.4429	0.4429
23	0.3607	0.3612	0.3615	0.3617	0.3618
24	0.2666	0.2669	0.2671	0.2672	0.2672

source: Elaborated by the author.

Table 59 – EQL for all combinations of k_w and UCL which satisfy the constraint $ATS_0 = \tau = 370$ for the np_x chart when the mean is known and the standard deviation is estimated for $n = 25$ and $h = 1$.

1	2	3	4	5	6
UCL			EQL		
	$m = 25$	$m = 50$	$m = 100$	$m = 500$	$m = 1000$
0	10.4535	10.5216	10.5559	10.5830	10.5864
1	5.9998	6.0445	6.0668	6.0845	6.0868
2	4.9618	5.0018	5.0217	5.0376	5.0396
3	4.5483	4.5863	4.6052	4.6203	4.6222
4	4.3508	4.3876	4.4058	4.4204	4.4222
5	4.2541	4.2898	4.3075	4.3217	4.3234
6	4.2159	4.2508	4.2681	4.2819	4.2837
7	4.2152	4.2493	4.2662	4.2797	4.2814
8	4.2422	4.2755	4.2920	4.3052	4.3069
9	4.2922	4.3247	4.3409	4.3538	4.3555
10	4.3647	4.3965	4.4124	4.4250	4.4266
11	4.4560	4.4871	4.5026	4.5150	4.5166
12	4.5685	4.5991	4.6143	4.6264	4.6279
13	4.7024	4.7323	4.7472	4.7591	4.7606
14	4.8648	4.8940	4.9086	4.9202	4.9217
15	5.0593	5.0878	5.1020	5.1134	5.1149
16	5.2928	5.3204	5.3341	5.3451	5.3465
17	5.5764	5.6035	5.6170	5.6279	5.6293
18	5.9261	5.9528	5.9658	5.9765	5.9778
19	6.3691	6.3951	6.4079	6.4182	6.4195
20	6.9481	6.9731	6.9857	6.9957	6.9970
21	7.7422	7.7665	7.7787	7.7885	7.7897
22	8.9104	8.9339	8.9458	8.9553	8.9565
23	10.8724	10.8954	10.9069	10.9162	10.9174
24	15.2804	15.3027	15.3139	15.3230	15.3241

source: Elaborated by the author.

Table 60 – k_w and UCL which satisfy the constraint $ATS_0 = \tau = 370$ for the np_x chart when the mean is known and the standard deviation is estimated for $n = 38, h = 1$ and different number of samples m.

1	2	3	4	5	6
UCL			k_w		
	$m = 25$	$m = 50$	$m = 100$	$m = 500$	$m = 1000$
0	3.9550	3.9639	3.9683	3.9719	3.9723
1	3.0724	3.0808	3.0851	3.0885	3.0889
2	2.6670	2.6752	2.6794	2.6827	2.6831
3	2.4083	2.4162	2.4202	2.4234	2.4238
4	2.2180	2.2257	2.2295	2.2326	2.2329
5	2.0667	2.0740	2.0777	2.0806	2.0810
6	1.9405	1.9474	1.9509	1.9537	1.9540
7	1.8314	1.8380	1.8413	1.8440	1.8443
8	1.7351	1.7413	1.7444	1.7469	1.7472
9	1.6483	1.6542	1.6571	1.6595	1.6598
10	1.5690	1.5746	1.5774	1.5796	1.5799
11	1.4958	1.5010	1.5037	1.5058	1.5060
12	1.4275	1.4324	1.4348	1.4368	1.4371
13	1.3632	1.3679	1.3702	1.3720	1.3723
14	1.3024	1.3068	1.3090	1.3107	1.3110
15	1.2446	1.2486	1.2507	1.2523	1.2525
16	1.1891	1.1929	1.1948	1.1964	1.1965
17	1.1358	1.1394	1.1411	1.1426	1.1427
18	1.0843	1.0876	1.0893	1.0906	1.0908
19	1.0344	1.0375	1.0390	1.0403	1.0404
20	0.9859	0.9887	0.9901	0.9913	0.9914
21	0.9385	0.9411	0.9424	0.9435	0.9436
22	0.8921	0.8945	0.8957	0.8966	0.8967
23	0.8464	0.8486	0.8497	0.8506	0.8507
24	0.8015	0.8035	0.8045	0.8053	0.8054
25	0.7571	0.7589	0.7598	0.7605	0.7606
26	0.7130	0.7147	0.7155	0.7161	0.7162
27	0.6692	0.6707	0.6714	0.6720	0.6721
28	0.6255	0.6268	0.6274	0.6279	0.6280
29	0.5816	0.5828	0.5834	0.5838	0.5839
30	0.5375	0.5385	0.5390	0.5394	0.5395
31	0.4929	0.4938	0.4942	0.4946	0.4946
32	0.4476	0.4483	0.4486	0.4489	0.4490
33	0.4010	0.4016	0.4019	0.4021	0.4021
34	0.3527	0.3532	0.3534	0.3536	0.3536
35	0.3018	0.3021	0.3023	0.3024	0.3024
36	0.2463	0.2465	0.2466	0.2467	0.2467
37	0.1814	0.1815	0.1816	0.1817	0.1817

source: Elaborated by the author.

Table 61 – EQL for all combinations of k_w and UCL which satisfy the constraint $ATS_0 = \tau = 370$ for the np_x chart when the mean is known and the standard deviation is estimated for $n = 38$ and $h = 1$.

1	2	3	4	5	6
UCL			EQL		
	$m = 25$	$m = 50$	$m = 100$	$m = 500$	$m = 1000$
0	9.7324	9.7749	9.7961	9.8132	9.8154
1	5.5113	5.5395	5.5536	5.5649	5.5663
2	4.5213	4.5469	4.5597	4.5699	4.5712
3	4.1110	4.1356	4.1479	4.1577	4.1589
4	3.8982	3.9223	3.9343	3.9438	3.9450
5	3.7756	3.7993	3.8113	3.8205	3.8217
6	3.7040	3.7268	3.7381	3.7472	3.7483
7	3.6607	3.6831	3.6943	3.7032	3.7043
8	3.6381	3.6602	3.6712	3.6800	3.6811
9	3.6303	3.6522	3.6630	3.6717	3.6728
10	3.6343	3.6556	3.6663	3.6749	3.6759
11	3.6465	3.6676	3.6794	3.6878	3.6888
12	3.6676	3.6885	3.6988	3.7071	3.7095
13	3.6958	3.7163	3.7266	3.7348	3.7372
14	3.7305	3.7508	3.7624	3.7704	3.7714
15	3.7723	3.7928	3.8031	3.8110	3.8120
16	3.8198	3.8400	3.8500	3.8581	3.8591
17	3.8742	3.8944	3.9043	3.9123	3.9133
18	3.9361	3.9559	3.9658	3.9736	3.9746
19	4.0056	4.0251	4.0348	4.0426	4.0436
20	4.0835	4.1027	4.1123	4.1200	4.1209
21	4.1707	4.1897	4.1991	4.2067	4.2076
22	4.2685	4.2872	4.2965	4.3040	4.3049
23	4.3785	4.3969	4.4060	4.4134	4.4143
24	4.5026	4.5206	4.5296	4.5369	4.5378
25	4.6433	4.6610	4.6699	4.6770	4.6779
26	4.8034	4.8208	4.8296	4.8366	4.8375
27	4.9882	5.0054	5.0140	5.0209	5.0218
28	5.2026	5.2202	5.2287	5.2354	5.2363
29	5.4564	5.4729	5.4812	5.4878	5.4886
30	5.7575	5.7739	5.7820	5.7886	5.7894
31	6.1278	6.1437	6.1518	6.1583	6.1592
32	6.5892	6.6045	6.6122	6.6184	6.6192
33	7.1881	7.2033	7.2109	7.2170	7.2178
34	8.0029	8.0177	8.0252	8.0312	8.0319
35	9.1973	9.2118	9.2191	9.2250	9.2257
36	11.1809	11.1947	11.2016	11.2071	11.2078
37	15.6222	15.6364	15.6435	15.6492	15.6499

source: Elaborated by the author.

Table 62 – k_w and UCL which satisfy the constraint $ATS_0 = \tau = 370$ for the np_x chart when the mean is known and the standard deviation is estimated for $n = 5, h = 0.5$ and different number of samples m.

1	2	3	4	5	6
UCL			k_w		
	$m = 25$	$m = 50$	$m = 100$	$m = 500$	$m = 1000$
0	3.5186	3.5796	3.6108	3.6362	3.6394
1	2.4338	2.4760	2.4976	2.5151	2.5173
2	1.8765	1.9066	1.9219	1.9344	1.9360
3	1.4662	1.4864	1.4967	1.5050	1.5061
4	1.0876	1.0990	1.1048	1.1095	1.1101

source: Elaborated by the author.

Table 63 – EQL for all combinations of k_w and UCL which satisfy the constraint $ATS_0 = \tau = 370$ for the np_x chart when the mean is known and the standard deviation is estimated for $n = 5$ and $h = 0.5$.

1	2	3	4	5	6
UCL			EQL		
	$m = 25$	$m = 50$	$m = 100$	$m = 500$	$m = 1000$
0	11.3664	11.8604	12.0914	12.2832	12.3076
1	7.2534	7.5436	7.6879	7.8031	7.8176
2	6.8977	7.1352	7.2537	7.3485	7.3604
3	7.7959	8.0034	8.1074	8.1908	8.2013
4	10.6544	10.8392	10.9323	11.0073	11.0167

source: Elaborated by the author.

Table 64 – k_w and UCL which satisfy the constraint $ATS_0 = \tau = 370$ for the np_x chart when the mean is known and the standard deviation is estimated for $n = 10, h = 0.5$ and different number of samples m.

1	2	3	4	5	6
UCL			k_w		
	$m = 25$	$m = 50$	$m = 100$	$m = 500$	$m = 1000$
0	3.7530	3.7849	3.8009	3.8139	3.8156
1	2.7221	2.7472	2.7600	2.7703	2.7715
2	2.2255	2.2465	2.2571	2.2657	2.2668
3	1.8935	1.9110	1.9199	1.9270	1.9279
4	1.6363	1.6506	1.6579	1.6637	1.6645
5	1.4187	1.4303	1.4361	1.4408	1.4414
6	1.2230	1.2320	1.2366	1.2402	1.2407
7	1.0373	1.0441	1.0475	1.0502	1.0506
8	0.8506	0.8553	0.8576	0.8595	0.8597
9	0.6435	0.6462	0.6475	0.6486	0.6488

source: Elaborated by the author.

Table 65 – EQL for all combinations of k_w and UCL which satisfy the constraint $ATS_0 = \tau = 370$ for the np_x chart when the mean is known and the standard deviation is estimated for $n = 10$ and $h = 0.5$.

1	2	3	4	5	6
UCL			EQL		
	$m = 25$	$m = 50$	$m = 100$	$m = 500$	$m = 1000$
0	10.0569	10.2581	10.3575	10.4376	10.4476
1	5.7514	5.8736	5.9344	5.9828	5.9889
2	4.8502	4.9543	5.0059	5.0471	5.0522
3	4.6347	4.7297	4.7769	4.8145	4.8192
4	4.7075	4.7961	4.8401	4.8753	4.8797
5	4.9821	5.0654	5.1068	5.1400	5.1442
6	5.4696	5.5482	5.5875	5.6189	5.6228
7	6.2704	6.3446	6.3818	6.4116	6.4154
8	7.6768	7.7470	7.7822	7.8105	7.8140
9	10.8533	10.9241	10.9569	10.9833	10.9867

source: Elaborated by the author.

Table 66 – k_w and UCL which satisfy the constraint $ATS_0 = \tau = 370$ for the np_x chart when the mean is known and the standard deviation is estimated for $n = 15, h = 0.5$ and different number of samples m.

1	2	3	4	5	6
UCL			k_w		
	$m = 25$	$m = 50$	$m = 100$	$m = 500$	$m = 1000$
0	3.8714	3.8937	3.9049	3.9139	3.9150
1	2.8701	2.8888	2.8982	2.9058	2.9067
2	2.3985	2.4150	2.4233	2.4300	2.4308
3	2.0907	2.1053	2.1127	2.1186	2.1194
4	1.8588	1.8717	1.8782	1.8834	1.8840
5	1.6693	1.6806	1.6862	1.6908	1.6914
6	1.5062	1.5160	1.5209	1.5249	1.5254
7	1.3605	1.3689	1.3731	1.3765	1.3769
8	1.2265	1.2336	1.2372	1.2401	1.2404
9	1.1002	1.1062	1.1092	1.1116	1.1119
10	0.9786	0.9834	0.9858	0.9878	0.9880
11	0.8585	0.8624	0.8643	0.8658	0.8660
12	0.7368	0.7396	0.7411	0.7422	0.7424
13	0.6081	0.6101	0.6111	0.6119	0.6120
14	0.4603	0.4614	0.4620	0.4625	0.4625

source: Elaborated by the author.

Table 67 – EQL for all combinations of k_w and UCL which satisfy the constraint $ATS_0 = \tau = 370$ for the np_x chart when the mean is known and the standard deviation is estimated for $n = 15$ and $h = 0.5$.

1	2	3	4	5	6
UCL			EQL		
	$m = 25$	$m = 50$	$m = 100$	$m = 500$	$m = 1000$
0	9.3369	9.4612	9.5232	9.5723	9.5785
1	5.1279	5.2040	5.2418	5.2720	5.2758
2	4.1737	4.2397	4.2731	4.2986	4.3018
3	3.8389	3.9003	3.9307	3.9549	3.9580
4	3.7293	3.7875	3.8163	3.8393	3.8422
5	3.7355	3.7914	3.8191	3.8412	3.8440
6	3.8214	3.8751	3.9018	3.9230	3.9257
7	3.9751	4.0267	4.0521	4.0724	4.0749
8	4.1993	4.2489	4.2736	4.2934	4.2958
9	4.5068	4.5547	4.5791	4.5987	4.6011
10	4.9284	4.9745	4.9975	5.0160	5.0183
11	5.5225	5.5670	5.5893	5.6040	5.6063
12	6.4063	6.4489	6.4702	6.4874	6.4895
13	7.8905	7.9314	7.9520	7.9685	7.9705
14	11.1654	11.2040	11.2235	11.2391	11.2411

source: Elaborated by the author.

Table 68 – k_w and UCL which satisfy the constraint $ATS_0 = \tau = 370$ for the np_x chart when the mean is known and the standard deviation is estimated for $n = 20, h = 0.5$ and different number of samples m.

1	2	3	4	5	6
UCL			k_w		
	$m = 25$	$m = 50$	$m = 100$	$m = 500$	$m = 1000$
0	3.9502	3.9675	3.9762	3.9832	3.9841
1	2.9687	2.9838	2.9914	2.9975	2.9983
2	2.5122	2.5260	2.5330	2.5385	2.5392
3	2.2179	2.2305	2.2369	2.2420	2.2426
4	1.9989	2.0104	2.0162	2.0208	2.0214
5	1.8224	1.8328	1.8381	1.8423	1.8428
6	1.6728	1.6822	1.6869	1.6908	1.6913
7	1.5415	1.5500	1.5542	1.5577	1.5581
8	1.4233	1.4308	1.4346	1.4377	1.4381
9	1.3146	1.3213	1.3246	1.3274	1.3277
10	1.2130	1.2189	1.2219	1.2243	1.2245
11	1.1166	1.1218	1.1244	1.1265	1.1267
12	1.0241	1.0286	1.0308	1.0327	1.0329
13	0.9342	0.9380	0.9399	0.9415	0.9417
14	0.8457	0.8489	0.8505	0.8518	0.8519
15	0.7573	0.7600	0.7613	0.7624	0.7625
16	0.6678	0.6699	0.6709	0.6718	0.6719
17	0.5749	0.5765	0.5773	0.5779	0.5780
18	0.4751	0.4762	0.4768	0.4772	0.4773
19	0.3590	0.3597	0.3600	0.3602	0.3603

source: Elaborated by the author.

Table 69 – EQL for all combinations of k_w and UCL which satisfy the constraint $ATS_0 = \tau = 370$ for the np_x chart when the mean is known and the standard deviation is estimated for $n = 20$ and $h = 0.5$.

1	2	3	4	5	6
UCL			EQL		
	$m = 25$	$m = 50$	$m = 100$	$m = 500$	$m = 1000$
0	8.8665	8.9553	8.9994	9.0349	9.0394
1	4.7608	4.8159	4.8432	4.8651	4.8679
2	3.8085	3.8566	3.8805	3.8995	3.9019
3	3.4430	3.4882	3.5106	3.5285	3.5307
4	3.2826	3.3260	3.3475	3.3646	3.3668
5	3.2208	3.2628	3.2835	3.3000	3.3021
6	3.2169	3.2577	3.2778	3.2952	3.2973
7	3.2555	3.2951	3.3147	3.3304	3.3323
8	3.3271	3.3657	3.3848	3.4000	3.4020
9	3.4301	3.4676	3.4862	3.5011	3.5029
10	3.5653	3.6019	3.6201	3.6347	3.6365
11	3.7376	3.7714	3.7892	3.8034	3.8052
12	3.9519	3.9864	4.0037	4.0174	4.0192
13	4.2242	4.2575	4.2741	4.2875	4.2891
14	4.5695	4.6021	4.6184	4.6315	4.6331
15	5.0253	5.0569	5.0727	5.0853	5.0869
16	5.6506	5.6812	5.6965	5.7088	5.7104
17	6.5669	6.5965	6.6113	6.6232	6.6247
18	8.0906	8.1192	8.1335	8.1450	8.1465
19	11.4061	11.4333	11.4469	11.4579	11.4592

source: Elaborated by the author.

Table 70 – k_w and UCL which satisfy the constraint $ATS_0 = \tau = 370$ for the np_x chart when the mean is known and the standard deviation is estimated for $n = 25, h = 0.5$ and different number of samples m.

1	2	3	4	5	6
UCL			k_w		
	$m = 25$	$m = 50$	$m = 100$	$m = 500$	$m = 1000$
0	4.0090	4.0233	4.0304	4.0362	4.0369
1	3.0422	3.0550	3.0614	3.0666	3.0672
2	2.5963	2.6083	2.6143	2.6191	2.6197
3	2.3110	2.3222	2.3278	2.3323	2.3329
4	2.1003	2.1107	2.1159	2.1201	2.1206
5	1.9318	1.9414	1.9463	1.9502	1.9506
6	1.7902	1.7991	1.8035	1.8071	1.8076
7	1.6670	1.6752	1.6793	1.6826	1.6830
8	1.5570	1.5645	1.5683	1.5713	1.5717
9	1.4571	1.4639	1.4674	1.4701	1.4705
10	1.3648	1.3710	1.3742	1.3767	1.3770
11	1.2784	1.2841	1.2870	1.2892	1.2895
12	1.1968	1.2019	1.2045	1.2066	1.2068
13	1.1189	1.1235	1.1258	1.1277	1.1279
14	1.0440	1.0481	1.0501	1.0518	1.0520
15	0.9712	0.9749	0.9767	0.9782	0.9783
16	0.9002	0.9034	0.9050	0.9063	0.9064
17	0.8302	0.8330	0.8344	0.8355	0.8356
18	0.7607	0.7631	0.7643	0.7653	0.7654
19	0.6911	0.6931	0.6941	0.6949	0.6950
20	0.6206	0.6223	0.6231	0.6238	0.6239
21	0.5483	0.5496	0.5503	0.5508	0.5509
22	0.4725	0.4735	0.4740	0.4744	0.4744
23	0.3905	0.3912	0.3915	0.3918	0.3918
24	0.2946	0.2950	0.2952	0.2953	0.2953

source: Elaborated by the author.

Table 71 – EQL for all combinations of k_w and UCL which satisfy the constraint $ATS_0 = \tau = 370$ for the np_x chart when the mean is known and the standard deviation is estimated for $n = 25$ and $h = 0.5$.

1	2	3	4	5	6
UCL			EQL		
	$m = 25$	$m = 50$	$m = 100$	$m = 500$	$m = 1000$
0	8.5249	8.5939	8.6284	8.6559	8.6594
1	4.5110	4.5539	4.5752	4.5922	4.5944
2	3.5714	3.6092	3.6279	3.6428	3.6447
3	3.1972	3.2325	3.2502	3.2643	3.2661
4	3.0172	3.0518	3.0689	3.0825	3.0842
5	2.9284	2.9620	2.9787	2.9919	2.9936
6	2.8906	2.9235	2.9397	2.9527	2.9543
7	2.8866	2.9187	2.9346	2.9473	2.9489
8	2.9069	2.9383	2.9539	2.9663	2.9678
9	2.9467	2.9776	2.9928	3.0050	3.0065
10	3.0061	3.0363	3.0513	3.0632	3.0647
11	3.0814	3.1109	3.1256	3.1373	3.1388
12	3.1745	3.2034	3.2177	3.2292	3.2306
13	3.2870	3.3152	3.3291	3.3403	3.3417
14	3.4215	3.4492	3.4630	3.4741	3.4754
15	3.5804	3.6075	3.6210	3.6318	3.6331
16	3.7736	3.8001	3.8134	3.8240	3.8253
17	4.0081	4.0339	4.0467	4.0571	4.0584
18	4.2968	4.3218	4.3343	4.3444	4.3457
19	4.6583	4.6829	4.6952	4.7050	4.7063
20	5.1303	5.1535	5.1654	5.1750	5.1762
21	5.7707	5.7939	5.8055	5.8149	5.8161
22	6.7039	6.7265	6.7378	6.7469	6.7480
23	8.2479	8.2698	8.2808	8.2896	8.2907
24	11.5896	11.6104	11.6212	11.6298	11.6309

source: Elaborated by the author.

APPENDIX D – np_x CHART: UU CASE

D.1 CONDITIONAL APPROACH

```
#This code is to simulated values considering the mean and the variance unknown
library(pracma) #it is necessary to use the function integral2 and integral3, which solves a two and three dimensional numeric integral respectively
N <- 100000 #number of CARL0 to be simulated (number of users)
kw <- 1.6599319 #Warning limit coefficient
m <- c(25,50,100,500,1000) #Number of samples on phase I
n <- 38 #Sample size on Phase I
UCL <- 9 #Upper control limit
shift <- 0
#Calculate the conditional Average Run Length and Conditional Extra Quadratic Loss
meanCARL <- rep(NA, 5)
sdCARL <- rep(NA, 5)
medianCARL <- rep(NA, 5)
Q25CARL <- rep(NA, 5)
Q75CARL <- rep(NA, 5)
meanCEQL <- rep(NA, 5)
sdCEQL <- rep(NA, 5)
medianCEQL <- rep(NA, 5)
Q25CEQL <- rep(NA, 5)
Q75CEQL <- rep(NA, 5)

for(j in 1:5){
CARL <- rep(NA, N)
CEQL <- rep(NA, N)
for (i in 1:N){
Z1 <- rnorm(1)
Y <- rchisq(1, (m[j]*(n-1)))
p <- 1 - pnorm(Z1/(sqrt(m[j]*n))+kw*sqrt((Y)/(m[j]*(n-1)))-shift) +
pnorm(Z1/(sqrt(m[j]*n))-kw*sqrt((Y)/(m[j]*(n-1)))-shift)
CARL[i] <- (1/(1-pbinom(UCL, n, p)))
dmin <- 0
dmax <- 3.5
int <- function(shift) (1/(dmax-dmin))*shift^2 * (1/(1 - pbinom(UCL, n,
(1 - pnorm(Z1/(sqrt(m[j]*n))+kw*sqrt((Y)/(m[j]*(n-1)))-shift) +
pnorm(Z1/(sqrt(m[j]*n))-kw*sqrt((Y)/(m[j]*(n-1)))-shift)))))
CEQL[i] <- integrate(int,0,3.5)$value
```

```
}
meanCARL[j] <- mean(CARL)
sdCARL[j] <- sd(CARL)
medianCARL[j] <- median(CARL)
Q25CARL[j] <- quantile(CARL,0.25)
Q75CARL[j] <- quantile(CARL,0.75)
meanCEQL[j] <- mean(CEQL)
sdCEQL[j] <- sd(CEQL)
medianCEQL[j] <- median(CEQL)
Q25CEQL[j] <- quantile(CEQL,0.25)
Q75CEQL[j] <- quantile(CEQL,0.75)
}
matriz_simulation <- rbind(m, meanCARL, sdCARL, medianCARL, Q25CARL, Q75CARL,
meanCEQL, sdCEQL, medianCEQL, Q25CEQL, Q75CEQL)
matriz_simulation
```

D.2 UNCONDITIONAL APPROACH - DERIVATIONS

```
#Calculate the unconditional ARL
ARL <- rep(NA, 75)
k <- 1

for (j in 1:5) {
shift<-0
ARLm <- rep(NA, 15)
for (i in 1:15) {
f <- function(w,u) (1/(1-pbinom(UCL, n, (1 - pnorm(qnorm(u)/(sqrt(m[j]*n))+
kw*sqrt((qchisq(w,(m[j]*(n-1))))/(m[j]*(n-1)))-shift) + pnorm(qnorm(u)/(sqrt(m[j]*n))-
kw*sqrt((qchisq(w,(m[j]*(n-1))))/(m[j]*(n-1)))-shift)))))
ARLm[i] <- integral2(f,0,1,0,1)
shift=shift+0.25
ARL[k] <- c(ARLm[i])
k <- k+1
}
}
ARLtot <- matrix(ARL,15,5)
ARLtot
#calculate the EQL, the overall of ARL in a range of shifts(0 to 3.5)
EQL <- rep(NA,5)

for (j in 1:5) {
```

```
dmin <- 0
dmax <- 3.5
int <- function(w,u,shift) (1/(dmax-dmin))*shift^2 * (1/(1 - pbinom(UCL, n,
(1 - pnorm(qnorm(u)/(sqrt(m[j]*n))+kw*sqrt((qchisq(w,(m[j]*(n-1))))/(m[j]*(n-1)))-shift) +
pnorm(qnorm(u)/(sqrt(m[j]*n))-kw*sqrt((qchisq(w,(m[j]*(n-1))))/(m[j]*(n-1)))-shift)))))
EQL[j] <- integral3(int,0,1,0,1,0,3.5)
}
EQL
```

D.3 ADJUSTMENT - CODES IN R

```
library(pracma) #it is necessary to use the function integral2 and integral3, which solves a two and
three dimensional numeric integral respectively

#This code is to optimize the kw considering the mean known and the variance unknown
n <- 38 #sample size
h <- 1 #sampling interval
m <- c(25, 50, 100, 500, 1000) #number of samples used to estimate the parameters k_w and UCL
shift <- 0 #shift in the mean of X
Tal <- 370.4 #the objective of ATS_0
L <- rep(NA, 5*n) #vector of k_w that guarantee ATS_0=Tal

#function uniroot to find the kw which the ATS0 will be Tal for the UCL=0
j <- 0
for (i in 1:5) {
j <- j+1
ATS0 <- function(kw){
f <- function(w,u) ((1/(1-pbinom(0, n, (1 - pnorm(qnorm(u)/(sqrt(m[i]*n)) + kw*sqrt((qchisq(w,
(m[i]*(n-1))))/(m[i]*(n-1)))-shift) + pnorm(qnorm(u)/(sqrt(m[i]*n))-kw*sqrt((qchisq(w,
(m[i]*(n-1))))/(m[i]*(n-1)))-shift)))))-0.5)*h
((integral2(f,0,1,0,1)$Q)-Tal)
}
L[j] <- uniroot(ATS0, lower = 0,upper = 5)$root

#function uniroot to find the kw which the ARL0 will be Tal varing the UCL from 1 to n-1
for (UCL in 1:(n-1)){
ATS0 <- function(kw){
f <- function(w,u) ((1/(1-pbinom(UCL, n, (1 - pnorm(qnorm(u)/(sqrt(m[i]*n))+kw*sqrt((qchisq(w,
(m[i]*(n-1))))/(m[i]*(n-1)))-shift) + pnorm(qnorm(u)/(sqrt(m[i]*n))-kw*sqrt((qchisq(w,
(m[i]*(n-1))))/(m[i]*(n-1)))-shift)))))-0.5)*h
((integral2(f,0,1,0,1)$Q)-Tal)
```

```
}
L[j+1] <- uniroot(ATS0, lower = 0,upper = L[j])$root
j <- j+1 }
}
Ltot <- matrix(L,n,5)
Ltot

#Calculate the EQL for combinations of UCL and kw which guarantee ARL0=370.4
dmin <- 0
dmax <- 3.5
EQL <- rep(NA, 5*n)
j<-5
i<-1
UCL<-0
int <- function(w,u,shift) (1/(dmax-dmin))*shift2̂*(((1/(1-pbinom(UCL, n, (1 - pnorm(qnorm(u)/
(sqrt(m[j]*n))+L[i]*sqrt((qchisq(w,(m[j]*(n-1))))/(m[j]*(n-1)))-shift) + pnorm(qnorm(u)/(sqrt(m[j]*n))
-L[i]*sqrt((qchisq(w,(m[j]*(n-1))))/(m[j]*(n-1)))-shift)))))-0.5)*h)
EQL[i] <- integral3(int,0,1,0,1,0,3.5)
i<-i+1
UCL<-UCL+1
m[j]
EQLtot<-matrix(EQL,n,5)
EQLtot
```

D.4 ADJUSTED PARAMETERS UCL AND k_w

This section shows the k_w and UCL tables generated for different sample sizes during the np_x chart optimization process. Also the EQL value produced by each pair of k_w and UCL are presented.

Table 72 – k_w and UCL which satisfy the constraint $ATS_0 = \tau = 370$ for the np_x chart when the mean and the standard deviation are estimated for $n = 5, h = 1$ and different number of samples m.

1	2	3	4	5	6
UCL			k_w		
	$m = 25$	$m = 50$	$m = 100$	$m = 500$	$m = 1000$
0	3.3675	3.4132	3.4365	3.4554	3.4578
1	2.3294	2.3609	2.3769	2.3900	2.3917
2	1.7882	1.8101	1.8213	1.8304	1.8315
3	1.3848	1.3988	1.4059	1.4117	1.4124
4	1.0085	1.0157	1.0193	1.0223	1.0226

source: Elaborated by the author.

Table 73 – EQL for all combinations of k_w and UCL which satisfy the constraint $ATS_0 = \tau = 370$ for the np_x chart when the mean and the standard deviation are estimated for $n = 5$ and $h = 1$.

1	2	3	4	5	6
UCL			EQL		
	$m = 25$	$m = 50$	$m = 100$	$m = 500$	$m = 1000$
0	15.2582	*	15.2092	15.2060	15.2058
1	10.2639	*	10.1556	10.1341	10.1318
2	9.8807	*	9.7458	9.7152	9.7116
3	11.1019	10.9787	10.9223	10.8790	10.8738
4	14.9988	14.8331	14.7522	14.6900	14.6823

source: Elaborated by the author.

Table 74 – k_w and UCL which satisfy the constraint $ATS_0 = \tau = 370$ for the np_x chart when the mean and the standard deviation are estimated for $n = 10, h = 1$ and different number of samples m.

1	2	3	4	5	6
UCL			k_w		
	$m = 25$	$m = 50$	$m = 100$	$m = 500$	$m = 1000$
0	3.5941	3.6182	3.6303	3.6401	3.6414
1	2.6170	2.6364	2.6462	2.6541	2.6551
2	2.1409	2.1571	2.1653	2.1719	2.1728
3	1.8197	1.8330	1.8398	1.8452	1.8459
4	1.5688	1.5796	1.5851	1.5895	1.5900
5	1.3555	1.3640	1.3684	1.3718	1.3723
6	1.1628	1.1693	1.1726	1.1753	1.1756
7	0.9795	0.9842	0.9866	0.9885	0.9887
8	0.7948	0.7978	0.7993	0.8006	0.8007
9	0.5895	0.5911	0.5918	0.5924	0.5925

source: Elaborated by the author.

Table 75 – EQL for all combinations of k_w and UCL which satisfy the constraint $ATS_0 = \tau = 370$ for the np_x chart when the mean and the standard deviation are estimated for $n = 10$ and $h = 1$.

1	2	3	4	5	6
UCL			EQL		
	$m = 25$	$m = 50$	$m = 100$	$m = 500$	$m = 1000$
0	12.8833	12.8445	12.8275	12.8152	12.8139
1	7.8798	7.8393	7.8215	7.8084	7.8069
2	6.8425	6.8045	6.7869	6.7738	6.7722
3	6.6097	6.5675	6.5485	6.5341	6.5324
4	6.7175	6.6717	6.6505	6.6343	6.6324
5	7.0722	7.0209	6.9966	6.9778	6.9755
6	7.6955	*	7.6085	7.5867	7.5840
7	8.7229	8.6549	8.6216	8.5957	8.5925
8	10.5518	10.4702	10.4304	10.3986	10.3947
9	14.8122	14.7097	14.6593	14.6192	14.6143

source: Elaborated by the author.

Table 76 – k_w and UCL which satisfy the constraint $ATS_0 = \tau = 370$ for the np_x chart when the mean and the standard deviation are estimated for $n = 15, h = 1$ and different number of samples m.

1	2	3	4	5	6
UCL			k_w		
	$m = 25$	$m = 50$	$m = 100$	$m = 500$	$m = 1000$
0	3.7116	3.7285	3.7370	3.7439	3.7447
1	2.7660	2.7807	2.7880	2.7940	2.7947
2	2.3160	2.3290	2.3356	2.3409	2.3415
3	2.0196	2.0311	2.0370	2.0417	2.0423
4	1.7948	1.8049	1.8100	1.8141	1.8146
5	1.6101	1.6189	1.6233	1.6269	1.6273
6	1.4505	1.4581	1.4619	1.4649	1.4653
7	1.3075	1.3139	1.3172	1.3198	1.3201
8	1.1757	1.1810	1.1837	1.1859	1.1862
9	1.0513	1.0556	1.0579	1.0596	1.0599
10	0.9312	0.9347	0.9364	0.9379	0.9380
11	0.8126	0.8153	0.8166	0.8177	0.8179
12	0.6923	0.6942	0.6952	0.6960	0.6961
13	0.5652	0.5665	0.5671	0.5676	0.5677
14	0.4193	0.4199	0.4202	0.4205	0.4205

source: Elaborated by the author.

Table 77 – EQL for all combinations of k_w and UCL which satisfy the constraint $ATS_0 = \tau = 370$ for the np_x chart when the mean and the standard deviation are estimated for $n = 15$ and $h = 1$.

1	2	3	4	5	6
UCL			EQL		
	$m = 25$	$m = 50$	$m = 100$	$m = 500$	$m = 1000$
0	11.7782	11.7489	11.7347	11.7241	11.7230
1	6.9692	6.9432	6.9315	6.9225	6.9215
2	5.8851	5.8617	5.8512	5.8433	5.8424
3	5.5080	5.4853	5.4751	5.4674	5.4665
4	5.3893	5.3660	5.3554	5.3474	5.3464
5	5.4072	5.3822	5.3708	5.3620	5.3610
6	5.5184	5.4915	5.4790	5.4693	5.4681
7	5.7108	5.6817	5.6677	5.6572	5.6559
8	5.9896	5.9576	5.9421	5.9305	5.9291
9	6.3738	6.3379	6.3206	6.3073	6.3057
10	6.9006	*	6.8417	6.8267	6.8248
11	7.6448	7.6005	7.5785	7.5614	7.5593
12	8.7686	8.7176	8.6927	8.6730	8.6705
13	10.6857	10.6259	10.5966	10.5732	10.5703
14	15.0363	14.9636	14.9274	14.8986	14.8951

source: Elaborated by the author.

Table 78 – k_w and UCL which satisfy the constraint $ATS_0 = \tau = 370$ for the np_x chart when the mean and the standard deviation are estimated for $n = 20, h = 1$ and different number of samples m.

1	2	3	4	5	6
UCL			k_w		
	$m = 25$	$m = 50$	$m = 100$	$m = 500$	$m = 1000$
0	3.7906	3.8038	3.8105	3.8158	3.8165
1	2.8658	2.8777	2.8837	2.8886	2.8892
2	2.4312	2.4423	2.4479	2.4523	2.4529
3	2.1486	2.1587	2.1638	2.1679	2.1685
4	1.9369	1.9461	1.9507	1.9545	1.9550
5	1.7654	1.7737	1.7779	1.7813	1.7817
6	1.6195	1.6270	1.6308	1.6338	1.6342
7	1.4910	1.4977	1.5011	1.5038	1.5042
8	1.3750	1.3810	1.3840	1.3864	1.3867
9	1.2682	1.2735	1.2761	1.2783	1.2785
10	1.1682	1.1728	1.1751	1.1770	1.1772
11	1.0733	1.0773	1.0792	1.0809	1.0811
12	0.9821	0.9854	0.9871	0.9885	0.9887
13	0.8933	0.8961	0.8975	0.8987	0.8988
14	0.8059	0.8082	0.8094	0.8103	0.8105
15	0.7186	0.7205	0.7215	0.7222	0.7223
16	0.6302	0.6317	0.6324	0.6330	0.6330
17	0.5385	0.5396	0.5401	0.5405	0.5406
18	0.4401	0.4408	0.4412	0.4414	0.4415
19	0.3260	0.3263	0.3265	0.3266	0.3266

source: Elaborated by the author.

Table 79 – EQL for all combinations of k_w and UCL which satisfy the constraint $ATS_0 = \tau = 370$ for the np_x chart when the mean and the standard deviation are estimated for $n = 20$ and $h = 1$.

1	2	3	4	5	6
UCL			EQL		
	$m = 25$	$m = 50$	$m = 100$	$m = 500$	$m = 1000$
0	11.1008	11.0773	11.0660	11.0577	11.0566
1	6.4568	6.4381	6.4293	6.4230	6.4222
2	5.3830	5.3672	5.3601	5.3547	5.3541
3	4.9713	4.9567	4.9502	4.9453	4.9447
4	4.7931	4.7785	4.7720	4.7673	4.7668
5	4.7260	4.7112	4.7046	4.6995	4.6989
6	4.7285	4.7131	4.7061	4.7007	4.7000
7	4.7775	4.7612	4.7536	4.7479	4.7472
8	4.8670	4.8495	4.8412	4.8351	4.8343
9	4.9944	4.9757	4.9667	4.9601	4.9593
10	5.1595	5.1393	5.1295	5.1221	5.1212
11	5.3704	5.3483	5.3376	5.3293	5.3283
12	5.6358	5.6114	5.5995	5.5904	5.5892
13	5.9678	5.9416	5.9286	5.9186	5.9174
14	6.3992	6.3674	6.3531	6.3419	6.3405
15	*	*	6.9139	6.9015	6.8999
16	7.7438	7.7078	7.6902	7.6763	7.6745
17	8.8997	8.8590	8.8390	8.8232	8.8212
18	10.8489	10.8027	10.7796	10.7613	10.7590
19	15.2479	15.1913	15.1631	15.1406	15.1378

source: Elaborated by the author.

Table 80 – k_w and UCL which satisfy the constraint $ATS_0 = \tau = 370$ for the np_x chart when the mean and the standard deviation are estimated for $n = 25, h = 1$ and different number of samples m.

1	2	3	4	5	6
UCL			k_w		
	$m = 25$	$m = 50$	$m = 100$	$m = 500$	$m = 1000$
0	3.8499	3.8609	3.8664	3.8708	3.8714
1	2.9402	2.9504	2.9556	2.9597	2.9602
2	2.5165	2.5262	2.5311	2.5350	2.5355
3	2.2430	2.2521	2.2567	2.2603	2.2608
4	2.0397	2.0481	2.0524	2.0558	2.0563
5	1.8763	1.8841	1.8880	1.8912	1.8916
6	1.7384	1.7456	1.7493	1.7522	1.7525
7	1.6181	1.6247	1.6281	1.6307	1.6311
8	1.5105	1.5165	1.5196	1.5220	1.5223
9	1.4124	1.4179	1.4207	1.4229	1.4232
10	1.3218	1.3268	1.3293	1.3313	1.3316
11	1.2368	1.2413	1.2436	1.2454	1.2456
12	1.1565	1.1605	1.1625	1.1642	1.1644
13	1.0796	1.0832	1.0850	1.0865	1.0867
14	1.0057	1.0089	1.0105	1.0118	1.0119
15	0.9339	0.9367	0.9381	0.9393	0.9394
16	0.8638	0.8662	0.8674	0.8684	0.8685
17	0.7947	0.7967	0.7978	0.7986	0.7987
18	0.7260	0.7278	0.7287	0.7294	0.7295
19	0.6573	0.6587	0.6595	0.6601	0.6601
20	0.5877	0.5889	0.5895	0.5899	0.5900
21	0.5163	0.5172	0.5177	0.5181	0.5181
22	0.4417	0.4423	0.4426	0.4429	0.4429
23	0.3609	0.3614	0.3616	0.3618	0.3618
24	0.2668	0.2670	0.2671	0.2672	0.2672

source: Elaborated by the author.

Table 81 – EQL for all combinations of k_w and UCL which satisfy the constraint $ATS_0 = \tau = 370$ for the np_x chart when the mean and the standard deviation are estimated for $n = 25$ and $h = 1$.

1	2	3	4	5	6
UCL			EQL		
	$m = 25$	$m = 50$	$m = 100$	$m = 500$	$m = 1000$
0	10.6270	10.6080	10.5986	10.5916	10.5907
1	6.1167	6.1023	6.0953	6.0903	6.0897
2	5.0634	5.0518	5.0464	5.0426	5.0421
3	4.6432	4.6330	4.6284	4.6250	4.6246
4	4.4422	4.4324	4.4281	4.4249	4.4245
5	4.3433	4.3336	4.3293	4.3260	4.3256
6	4.3039	4.2940	4.2896	4.2862	4.2858
7	4.3025	4.2922	4.2875	4.2840	4.2836
8	4.3291	4.3183	4.3132	4.3095	4.3090
9	4.3790	4.3675	4.3621	4.3581	4.3576
10	4.4519	4.4395	4.4336	4.4293	4.4288
11	4.5436	4.5304	4.5240	4.5193	4.5188
12	4.6568	4.6427	4.6359	4.6308	4.6301
13	4.7915	4.7764	4.7690	4.7634	4.7628
14	4.9550	4.9387	4.9307	4.9247	4.9239
15	5.1509	5.1333	5.1246	5.1180	5.1171
16	5.3861	5.3668	5.3571	5.3497	5.3488
17	5.6716	5.6506	5.6406	5.6326	5.6316
18	6.0237	6.0010	5.9900	5.9813	5.9803
19	*	6.4450	6.4329	6.4232	6.4220
20	7.0521	7.0248	7.0115	7.0009	6.9996
21	7.8507	7.8205	7.8057	7.7939	7.7925
22	9.0244	8.9909	8.9743	8.9610	8.9594
23	10.9951	10.9567	10.9376	10.9224	10.9205
24	15.4179	15.3715	15.3484	15.3299	15.3276

source: Elaborated by the author.

Table 82 – k_w and UCL which satisfy the constraint $ATS_0 = \tau = 370$ for the np_x chart when the mean and the standard deviation are estimated for $n = 38, h = 1$ and different number of samples m.

1	2	3	4	5	6
UCL			k_w		
	$m = 25$	$m = 50$	$m = 100$	$m = 500$	$m = 1000$
0	3.9572	3.9650	3.9689	3.9720	3.9724
1	3.0742	3.0817	3.0855	3.0886	3.0890
2	2.6685	2.6760	2.6797	2.6828	2.6831
3	2.4097	2.4169	2.4206	2.4235	2.4239
4	2.2193	2.2263	2.2298	2.2326	2.2330
5	2.0680	2.0746	2.0780	2.0807	2.0810
6	1.9417	1.9480	1.9512	1.9537	1.9541
7	1.8326	1.8386	1.8416	1.8440	1.8443
8	1.7361	1.7418	1.7447	1.7470	1.7473
9	1.6493	1.6547	1.6574	1.6595	1.6598
10	1.5700	1.5751	1.5776	1.5796	1.5799
11	1.4967	1.5014	1.5039	1.5058	1.5061
12	1.4283	1.4328	1.4351	1.4369	1.4372
13	1.3641	1.3683	1.3704	1.3721	1.3723
14	1.3032	1.3071	1.3092	1.3108	1.3110
15	1.2453	1.2490	1.2508	1.2523	1.2525
16	1.1898	1.1933	1.1950	1.1964	1.1966
17	1.1365	1.1397	1.1413	1.1426	1.1428
18	1.0850	1.0880	1.0895	1.0907	1.0908
19	1.0351	1.0378	1.0392	1.0403	1.0404
20	0.9865	0.9890	0.9903	0.9913	0.9914
21	0.9390	0.9414	0.9425	0.9435	0.9436
22	0.8926	0.8947	0.8958	0.8967	0.8968
23	0.8469	0.8489	0.8499	0.8507	0.8508
24	0.8020	0.8037	0.8046	0.8053	0.8054
25	0.7575	0.7591	0.7599	0.7606	0.7606
26	0.7134	0.7149	0.7156	0.7162	0.7162
27	0.6696	0.6709	0.6715	0.6720	0.6721
28	0.6258	0.6269	0.6275	0.6280	0.6280
29	0.5820	0.5830	0.5834	0.5838	0.5839
30	0.5378	0.5387	0.5391	0.5394	0.5395
31	0.4932	0.4939	0.4943	0.4946	0.4946
32	0.4478	0.4484	0.4487	0.4489	0.4490
33	0.4012	0.4017	0.4019	0.4021	0.4022
34	0.3529	0.3533	0.3535	0.3536	0.3536
35	0.3019	0.3022	0.3023	0.3024	0.3024
36	0.2464	0.2466	0.2467	0.2467	0.2467
37	0.1815	0.1816	0.1816	0.1817	0.1817

source: Elaborated by the author.

Table 83 – EQL for all combinations of k_w and UCL which satisfy the constraint $ATS_0 = \tau = 370$ for the np_x chart when the mean and the standard deviation are estimated for $n = 38$ and $h = 1$.

1	2	3	4	5	6
UCL			EQL		
	$m = 25$	$m = 50$	$m = 100$	$m = 500$	$m = 1000$
0	9.8431	9.8297	9.8236	9.8188	9.8181
1	5.5850	5.5761	5.5717	5.5685	5.5681
2	4.5848	4.5783	4.5751	4.5730	4.5727
3	4.1699	4.1647	4.1622	4.1606	4.1604
4	3.9547	3.9501	3.9480	3.9466	3.9465
5	3.8305	3.8263	3.8244	3.8231	3.8230
6	*	3.7527	3.7509	3.7497	3.7496
7	*	3.7087	3.7069	3.7058	3.7056
8	*	3.6855	3.6837	3.6826	3.6824
9	*	*	3.6755	3.6742	3.6741
10	*	*	3.6787	3.6774	3.6772
11	*	*	3.6918	3.6903	3.6901
12	*	3.7136	3.7112	3.7096	3.7108
13	3.7468	3.7415	3.7390	3.7373	3.7385
14	3.7817	3.7760	3.7749	3.7730	3.7727
15	3.8246	3.8186	3.8156	3.8136	3.8133
16	3.8723	3.8659	3.8629	3.8607	3.8605
17	3.9270	3.9204	3.9173	3.9149	3.9147
18	3.9892	3.9822	3.9788	3.9763	3.9760
19	4.0591	4.0516	4.0480	4.0453	4.0449
20	4.1374	4.1294	4.1255	4.1226	4.1223
21	4.2251	4.2167	4.2125	4.2094	4.2090
22	4.3235	4.3145	4.3100	4.3067	4.3063
23	4.4341	4.4245	4.4197	4.4161	4.4157
24	4.5588	4.5485	4.5435	4.5397	4.5392
25	4.7003	4.6893	4.6840	4.6799	4.6793
26	4.8613	4.8495	4.8439	4.8395	4.8389
27	5.0472	5.0347	5.0286	5.0239	5.0233
28	5.2626	5.2501	5.2436	5.2384	5.2378
29	5.5177	5.5034	5.4964	5.4909	5.4902
30	5.8203	5.8050	5.7976	5.7917	5.7910
31	*	*	6.1678	6.1616	6.1608
32	*	6.6377	6.6288	6.6218	6.6209
33	7.2569	7.2377	7.2281	7.2205	7.2196
34	8.0747	8.0536	8.0431	8.0348	8.0337
35	9.2731	9.2497	9.2381	9.2288	9.2277
36	11.2623	11.2354	11.2219	11.2112	11.2099
37	15.7134	15.6820	15.6663	15.6538	15.6523

source: Elaborated by the author.

Table 84 – k_w and UCL which satisfy the constraint $ATS_0 = \tau = 370$ for the np_x chart when the mean and the standard deviation are estimated for $n = 5, h = 0.5$ and different number of samples m.

1	2	3	4	5	6
UCL			k_w		
	$m = 25$	$m = 50$	$m = 100$	$m = 500$	$m = 1000$
0	3.5341	3.5874	3.6147	3.6370	3.6398
1	2.4447	2.4815	2.5003	2.5156	2.5176
2	1.8848	1.9107	1.9240	1.9348	1.9362
3	1.4725	1.4896	1.4983	1.5053	1.5062
4	1.0921	1.1013	1.1060	1.1098	1.1103

source: Elaborated by the author.

Table 85 – EQL for all combinations of k_w and UCL which satisfy the constraint $ATS_0 = \tau = 370$ for the np_x chart when the mean and the standard deviation are estimated for $n = 5$ and $h = 0.5$.

1	2	3	4	5	6
UCL			EQL		
	$m = 25$	$m = 50$	$m = 100$	$m = 500$	$m = 1000$
0	12.3687	*	12.3287	12.3300	12.3311
1	*	7.8879	7.8565	7.8363	7.8342
2	7.5419	7.4484	7.4079	7.3790	7.3757
3	8.4270	8.3140	8.2613	8.2214	8.2166
4	11.3215	11.1695	11.0969	11.0401	11.0331

source: Elaborated by the author.

Table 86 – k_w and UCL which satisfy the constraint $ATS_0 = \tau = 370$ for the np_x chart when the mean and the standard deviation are estimated for $n = 10, h = 0.5$ and different number of samples m.

1	2	3	4	5	6
UCL			k_w		
	$m = 25$	$m = 50$	$m = 100$	$m = 500$	$m = 1000$
0	3.7610	3.7889	3.8030	3.8143	3.8158
1	2.7281	2.7502	2.7615	2.7706	2.7717
2	2.2304	2.2490	2.2583	2.2659	2.2669
3	1.8977	1.9131	1.9209	1.9272	1.9280
4	1.6398	1.6524	1.6588	1.6639	1.6645
5	1.4218	1.4318	1.4369	1.4409	1.4415
6	1.2256	1.2333	1.2372	1.2404	1.2408
7	1.0395	1.0452	1.0480	1.0503	1.0506
8	0.8524	0.8562	0.8581	0.8596	0.8598
9	0.6448	0.6469	0.6479	0.6487	0.6488

source: Elaborated by the author.

Table 87 – EQL for all combinations of k_w and UCL which satisfy the constraint $ATS_0 = \tau = 370$ for the np_x chart when the mean and the standard deviation are estimated for $n = 10$ and $h = 0.5$.

1	2	3	4	5	6
UCL			EQL		
	$m = 25$	$m = 50$	$m = 100$	$m = 500$	$m = 1000$
0	10.5116	10.4824	10.4691	10.4597	10.4587
1	6.0675	6.0284	6.0108	5.9980	5.9965
2	5.1304	5.0913	5.0736	5.0605	5.0590
3	4.9010	4.8602	4.8414	4.8273	4.8256
4	4.9684	4.9244	4.9036	4.8879	4.8860
5	5.2444	5.1941	5.1707	5.1527	5.1506
6	5.7358	5.6795	5.6527	5.6319	5.6294
7	6.5452	6.4808	6.4493	6.4251	6.4221
8	7.9671	7.8913	7.8539	7.8248	7.8212
9	11.1738	11.0835	11.0366	10.9993	10.9947

source: Elaborated by the author.

Table 88 – k_w and UCL which satisfy the constraint $ATS_0 = \tau = 370$ for the np_x chart when the mean and the standard deviation are estimated for $n = 15, h = 0.5$ and different number of samples m.

1	2	3	4	5	6
UCL			k_w		
	$m = 25$	$m = 50$	$m = 100$	$m = 500$	$m = 1000$
0	3.8770	3.8964	3.9062	3.9141	3.9151
1	2.8744	2.8909	2.8993	2.9060	2.9068
2	2.4020	2.4167	2.4242	2.4302	2.4309
3	2.0938	2.1069	2.1135	2.1188	2.1194
4	1.8616	1.8730	1.8788	1.8835	1.8841
5	1.6718	1.6818	1.6869	1.6909	1.6914
6	1.5085	1.5171	1.5215	1.5250	1.5254
7	1.3625	1.3699	1.3736	1.3766	1.3770
8	1.2283	1.2345	1.2377	1.2402	1.2405
9	1.1018	1.1070	1.1096	1.1117	1.1119
10	0.9800	0.9841	0.9862	0.9879	0.9881
11	0.8598	0.8630	0.8646	0.8658	0.8660
12	0.7378	0.7401	0.7413	0.7423	0.7424
13	0.6089	0.6105	0.6113	0.6119	0.6120
14	0.4609	0.4617	0.4622	0.4625	0.4625

source: Elaborated by the author.

Table 89 – EQL for all combinations of k_w and UCL which satisfy the constraint $ATS_0 = \tau = 370$ for the np_x chart when the mean and the standard deviation are estimated for $n = 15$ and $h = 0.5$.

1	2	3	4	5	6
UCL			EQL		
	$m = 25$	$m = 50$	$m = 100$	$m = 500$	$m = 1000$
0	9.6304	9.6059	9.5950	9.5866	9.5857
1	5.3288	5.3023	5.2905	5.2816	5.2806
2	4.3487	4.3257	4.3156	4.3070	4.3061
3	4.0038	3.9812	3.9708	3.9629	3.9620
4	3.8891	3.8660	3.8552	3.8470	3.8461
5	3.8941	3.8689	3.8575	3.8488	3.8478
6	3.9791	3.9524	3.9400	3.9306	3.9295
7	4.1333	4.1044	4.0906	4.0800	4.0788
8	4.3590	4.3276	4.3127	4.3011	4.2997
9	4.6692	4.6350	4.6189	4.6066	4.6051
10	5.0948	5.0570	5.0385	5.0242	5.0224
11	5.6947	5.6525	5.6318	5.6126	5.6106
12	6.5864	6.5382	6.5148	6.4963	6.4940
13	8.0827	8.0270	7.9997	7.9780	7.9753
14	11.3786	11.3106	11.2768	11.2498	11.2464

source: Elaborated by the author.

Table 90 – k_w and UCL which satisfy the constraint $ATS_0 = \tau = 370$ for the np_x chart when the mean and the standard deviation are estimated for $n = 20, h = 0.5$ and different number of samples m.

1	2	3	4	5	6
UCL			k_w		
	$m = 25$	$m = 50$	$m = 100$	$m = 500$	$m = 1000$
0	3.9545	3.9696	3.9773	3.9834	3.9842
1	2.9720	2.9855	2.9922	2.9977	2.9984
2	2.5151	2.5274	2.5336	2.5387	2.5393
3	2.2204	2.2317	2.2375	2.2421	2.2427
4	2.0012	2.0115	2.0167	2.0209	2.0214
5	1.8245	1.8338	1.8386	1.8424	1.8428
6	1.6747	1.6831	1.6874	1.6909	1.6913
7	1.5433	1.5508	1.5547	1.5577	1.5581
8	1.4249	1.4316	1.4350	1.4378	1.4381
9	1.3160	1.3220	1.3250	1.3274	1.3277
10	1.2143	1.2195	1.2222	1.2243	1.2246
11	1.1179	1.1224	1.1247	1.1265	1.1268
12	1.0252	1.0291	1.0311	1.0327	1.0329
13	0.9352	0.9385	0.9402	0.9415	0.9417
14	0.8466	0.8493	0.8507	0.8518	0.8520
15	0.7581	0.7604	0.7615	0.7624	0.7625
16	0.6685	0.6702	0.6711	0.6718	0.6719
17	0.5755	0.5768	0.5774	0.5779	0.5780
18	0.4756	0.4765	0.4769	0.4772	0.4773
19	0.3594	0.3599	0.3601	0.3603	0.3603

source: Elaborated by the author.

Table 91 – EQL for all combinations of k_w and UCL which satisfy the constraint $ATS_0 = \tau = 370$ for the np_x chart when the mean and the standard deviation are estimated for $n = 20$ and $h = 0.5$.

1	2	3	4	5	6
UCL			EQL		
	$m = 25$	$m = 50$	$m = 100$	$m = 500$	$m = 1000$
0	9.0808	9.0613	9.0520	9.0455	9.0447
1	4.9061	4.8872	4.8787	4.8722	4.8714
2	3.9354	3.9187	3.9113	3.9056	3.9050
3	3.5619	3.5463	3.5394	3.5342	3.5336
4	3.3973	3.3820	3.3752	3.3702	3.3696
5	3.3332	3.3176	3.3107	3.3054	3.3048
6	3.3280	3.3120	3.3047	3.3006	3.3000
7	3.3660	3.3492	3.3416	3.3357	3.3350
8	3.4376	3.4199	3.4117	3.4054	3.4046
9	3.5410	3.5221	3.5133	3.5065	3.5056
10	3.6771	3.6570	3.6473	3.6401	3.6392
11	3.8507	3.8272	3.8168	3.8089	3.8080
12	4.0667	4.0433	4.0318	4.0231	4.0220
13	4.3414	4.3156	4.3029	4.2932	4.2920
14	4.6895	4.6617	4.6480	4.6374	4.6361
15	5.1491	5.1185	5.1033	5.0915	5.0900
16	5.7792	5.7450	5.7284	5.7152	5.7136
17	6.7019	6.6636	6.6448	6.6299	6.6281
18	8.2348	8.1912	8.1695	8.1523	8.1501
19	11.5667	11.5135	11.4870	11.4659	11.4633

source: Elaborated by the author.

Table 92 – k_w and UCL which satisfy the constraint $ATS_0 = \tau = 370$ for the np_x chart when the mean and the standard deviation are estimated for $n = 25, h = 0.5$ and different number of samples m.

1	2	3	4	5	6
UCL			k_w		
	$m = 25$	$m = 50$	$m = 100$	$m = 500$	$m = 1000$
0	4.0125	4.0250	4.0313	4.0364	4.0370
1	3.0449	3.0564	3.0621	3.0667	3.0673
2	2.5987	2.6094	2.6149	2.6192	2.6198
3	2.3132	2.3232	2.3283	2.3324	2.3329
4	2.1022	2.1116	2.1164	2.1202	2.1207
5	1.9336	1.9423	1.9467	1.9502	1.9507
6	1.7919	1.7999	1.8039	1.8072	1.8076
7	1.6685	1.6759	1.6796	1.6826	1.6830
8	1.5585	1.5652	1.5687	1.5714	1.5717
9	1.4584	1.4646	1.4677	1.4702	1.4705
10	1.3660	1.3716	1.3745	1.3767	1.3770
11	1.2796	1.2847	1.2872	1.2893	1.2896
12	1.1979	1.2025	1.2048	1.2066	1.2069
13	1.1199	1.1240	1.1261	1.1277	1.1279
14	1.0449	1.0485	1.0504	1.0518	1.0520
15	0.9721	0.9753	0.9769	0.9782	0.9784
16	0.9010	0.9038	0.9052	0.9063	0.9065
17	0.8309	0.8333	0.8346	0.8355	0.8357
18	0.7614	0.7634	0.7645	0.7653	0.7654
19	0.6917	0.6934	0.6943	0.6950	0.6950
20	0.6212	0.6225	0.6232	0.6238	0.6239
21	0.5487	0.5498	0.5504	0.5508	0.5509
22	0.4729	0.4737	0.4741	0.4744	0.4745
23	0.3908	0.3913	0.3916	0.3918	0.3918
24	0.2948	0.2951	0.2952	0.2953	0.2954

source: Elaborated by the author.

Table 93 – EQL for all combinations of k_w and UCL which satisfy the constraint $ATS_0 = \tau = 370$ for the np_x chart when the mean and the standard deviation are estimated for $n = 25$ and $h = 0.5$.

1	2	3	4	5	6
UCL			EQL		
	$m = 25$	$m = 50$	$m = 100$	$m = 500$	$m = 1000$
0	8.6934	8.6776	8.6698	8.6643	8.6636
1	4.6244	4.6097	4.6027	4.5978	4.5971
2	3.6699	3.6574	3.6518	3.6476	3.6471
3	3.2892	3.2774	3.2725	3.2688	3.2683
4	3.1057	3.0950	3.0903	3.0868	3.0864
5	3.0148	3.0042	2.9996	2.9961	2.9957
6	2.9757	2.9651	2.9603	2.9568	2.9564
7	2.9710	2.9600	2.9551	2.9514	2.9510
8	2.9909	2.9795	2.9743	2.9704	2.9699
9	3.0306	3.0187	3.0132	3.0091	3.0086
10	3.0903	3.0776	3.0718	3.0673	3.0668
11	3.1659	3.1525	3.1461	3.1414	3.1408
12	3.2595	3.2453	3.2384	3.2333	3.2327
13	3.3727	3.3575	3.3500	3.3445	3.3439
14	3.5082	3.4921	3.4842	3.4783	3.4776
15	3.6682	3.6509	3.6425	3.6361	3.6353
16	3.8629	3.8444	3.8353	3.8284	3.8275
17	4.0992	4.0791	4.0692	4.0616	4.0606
18	4.3900	4.3682	4.3574	4.3490	4.3480
19	4.7541	4.7304	4.7189	4.7098	4.7087
20	*	5.2026	5.1900	5.1799	5.1787
21	5.8738	5.8451	5.8312	5.8201	5.8187
22	6.8120	6.7804	6.7648	6.7523	6.7508
23	8.3637	8.3277	8.3097	8.2954	8.2936
24	11.7186	11.6749	11.6535	11.6363	11.6342

source: Elaborated by the author.