A LETTERS JOURNAL EXPLORING THE FRONTIERS OF PHYSICS

LETTER

The point-charge self-energy in a nonminimal Lorentz-violating Maxwell electrodynamics

To cite this article: L. H. C. Borges *et al* 2018 *EPL* **122** 31002

A Letters Journal Exploring the Frontiers of Physics

May 2018

EPL, **122** (2018) 31002 www.epljournal.org
doi: 10.1209/0295-5075/122/31002

The point-charge self-energy in a nonminimal Lorentz-violating Maxwell electrodynamics

L. H. C. Borges[1], F. A. Barone[2] and A. F. Ferrari[3,4]

[1] *UNESP, Campus de Guaratinguetá, DFQ - Avenida Dr. Ariberto Pereira da Cunha 333, CEP 12516-410, Guaratinguetá, SP, Brazil*
[2] *IFQ, Universidade Federal de Itajubá - Av. BPS 1303, Pinheirinho, Caixa Postal 50, 37500-903, Itajubá, MG, Brazil*
[3] *Universidade Federal do ABC, UFABC - Avenida dos Estados, 5001, Santo André, SP, 09210-580, Brazil*
[4] *Indiana University Center for Spacetime Symmetries, Indiana University - Bloomington, IN, 47405-7105, USA*

received 23 March 2018; accepted in final form 1 June 2018
published online 22 June 2018

PACS 11.10.-z – Field theory
PACS 03.50.-z – Classical field theories
PACS 11.15.-q – Gauge field theories

Abstract – In this letter we study the self-energy of a point-like charge for the electromagnetic field in a non-minimal Lorentz symmetry breaking scenario in a $(n+1)$-dimensional space time. We consider two variations of a model where the Lorentz violation is caused by a background vector d^ν that appears in a higher derivative interaction. We restrict our attention to the case where d^μ is a time-like background vector, namely $d^2 = d^\mu d_\mu > 0$, and we verify that the classical self-energy is finite for any odd spatial dimension n and diverges for even n. We also make some comments regarding obstacles in the quantization of the proposed model.

Lorentz symmetry violations have been systematically studied in the past years in different scenarios encompassing low-energy, nuclear and high-energy physics, astrophysics and others, thus providing a very extensive view of possible physical effects arising from an assumed violation of Lorentz symmetry, which may indicate some new physics at very small length scales such as the Planck length [1]. As a result, besides a deeper understanding of the theoretical possibilities involving the spontaneous breaking of such an essential symmetry in our understanding of particle physics, also an extensive set of very high-precision bounds on Lorentz-violating (LV) parameters has been obtained [2]. Most of these works have been done in the context of the Standard Model extension (SME) [3] that incorporates in the Standard Model the full set of gauge-invariant, renormalizable LV interactions. Understanding the SME as an effective field theory that is derived from some more fundamental theory at very high energy, it becomes natural to incorporate also non-minimal terms, *i.e.*, those which are not renormalizable. As a matter of fact, a systematical study of the non-minimal LV operators that may be added to the SME (still maintaining gauge invariance) has begun to gain momentum more recently [4–6]. As non-minimal operators are expected to be suppressed by powers of the very high-energy scale of the fundamental theory triggering the Lorentz violation, it is a general expectation that their effects will be subleading comparing to those already present in the minimal SME. Also, from the technical viewpoint, the presence of additional derivatives in the non-minimal LV terms are linked to possible issues with unitarity, given the presence of ghost modes. However, in certain situations, a non-minimal LV operator might induce some effect in low-energy physics which cannot be replicated by any minimal LV operator of the SME. Therefore, one might entertain the hope of finding new interesting phenomena related to the non-minimal LV.

In a recent work [7], some of us have exposed a relation between non-minimal LV and axion physics, since a particular setting of LV couplings, generated by some Lorentz-violating high-energy dynamics, could contribute to the standard (Lorentz-invariant) axion-photon coupling. The complete modification induced in the Maxwell electrodynamics by the LV background considered in [7] was calculated in [8], and this result opens up the opportunity for investigating several aspects of photon physics that may be affected by the particular LV couplings considered in these works. As a first step in this direction, we explored in [9] the effects of one of the non-minimal terms found in [8], to wit,

$$d^\lambda d_\alpha \partial_\mu F_{\nu\lambda} \partial^\nu F^{\mu\alpha}, \tag{1}$$

in the classical electromagnetic interaction between sources, such as point charges, dipoles, lines of current and Dirac strings. Here, d^μ is a constant vector parametrizing the LV. Some new physical effects due the LV were unveiled, such as a spontaneous torque on an isolated electric dipole. An extensive study of dimension-six LV operators, of which eq. (1) can be considered as a particular case, was recently reported in [10], where the question of causality in such models is throughly discussed. In this letter we present another consequence of the presence of the interaction (1) in classical electrodynamics, more specifically the regularization of the self-energy of a point charge in a certain number of spatial dimensions.

The self-energy of an electrical charge is a well-known problem in classical electrodynamics, representing one of the early divergence problems that were faced by theoretical physics at the beginning of the twentieth century, since the self-energy diverges linearly with the ultraviolet cutoff. Dirac's quantum theory of the electron improved matters, reducing the divergence to a logarithmic one, and the problem of calculating the electron self-energy remained a central one during the key years of the development of the modern approach of quantum field theory and the renormalization program. Some historical perspective on the early attempts to solve the self-energy problem can be found in [11]. Podolsky [12–14] and later Lee-Wick [15,16] discussed a generalization of electrodynamics including higher derivatives in which no ultraviolet divergences appeared, in particular in the electron self-energy. Recently, some of us studied in detail the finiteness of the electron self-energy in the context of the Lee-Wick electrodynamics [17,18], showing that a finite result can be obtained for an odd number of spatial dimensions only. In this letter, we will show that a similar behavior occurs for the electrodynamics modified by the non-minimal LV coupling (1).

We consider first the simplest extension of the Maxwell theory incorporating the LV term given in eq. (1), which is defined by the following Lagrangian density in $(n+1)$-dimensional space-time:

$$\mathcal{L}_{(1)} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} - \frac{1}{2\gamma}(\partial_\mu A^\mu)^2 + \frac{1}{2}d^\lambda d_\alpha \partial_\mu F_{\nu\lambda}\partial^\nu F^{\mu\alpha} + J^\mu A_\mu, \tag{2}$$

where A^μ is the electromagnetic field coupled to an external source J^μ, with the associated field strength $F^{\mu\nu} = \partial^\mu A^\nu - \partial^\nu A^\mu$. Here, γ is a gauge fixing parameter, and d^λ is the background vector parametrizing the LV in our model.

The parameter d^λ has dimension of inverse of mass and is taken to be constant and uniform in the reference frame where the calculations are performed. Assumedly d^μ is of the order of the inverse of some very large mass related to some fundamental theory at the Planck scale from which the LV originates. As in [9], we restrict ourselves to the case of d^μ being a time-like background vector, namely

$$d^2 = d^\mu d_\mu = (d^0)^2 - \mathbf{d}^2 > 0, \tag{3}$$

where $\mathbf{d} = (d^1, d^2, \ldots, d^n)$. For other classes of d^μ vectors we are not able to perform the necessary integrals, so our results do not extend to those cases.

The propagator for our model, in the Feynman gauge $\gamma = 1$, is given by

$$D^{\mu\nu}(x,y) = \int \frac{\mathrm{d}^{n+1}p}{(2\pi)^{n+1}}\Bigg\{-\frac{\eta^{\mu\nu}}{p^2} + \frac{1}{[1 - d^2p^2 + (p\cdot d)^2]} \times\Bigg[-d^\mu d^\nu - \frac{(p\cdot d)^2}{p^4}p^\mu p^\nu + \frac{(p\cdot d)}{p^2}(p^\mu d^\nu + d^\mu p^\nu)\Bigg]\Bigg\}e^{-ip\cdot(x-y)}. \tag{4}$$

Since the Lagrangian (2) is quadratic in the field variables A^μ, the vacuum energy due to the presence of the external source is given by

$$E = \frac{1}{2T}\int\int \mathrm{d}^{n+1}x\ \mathrm{d}^{n+1}y J_\mu(x)D^{\mu\nu}(x,y)J_\nu(y), \tag{5}$$

where T is the time variable, and the limit $T \to \infty$ is implicit [19,20].

We consider the external source J corresponding to a point-like stationary charge q placed at position $\mathbf{a} = (a^1, a^2, \ldots, a^n)$,

$$J^\mu(\mathbf{x}) = q\eta^{\mu 0}\delta^n(\mathbf{x} - \mathbf{a}), \tag{6}$$

where δ is the Dirac delta function in n spatial dimensions. Substituting (6) in (5), using the explicit form of the propagator in eq. (4) and computing the integrals in the following order: $\mathrm{d}^n\mathbf{x}$, $\mathrm{d}^n\mathbf{y}$, $\mathrm{d}x^0$, $\mathrm{d}p^0$ and $\mathrm{d}y^0$, we obtain

$$E_{(1)} = \frac{q^2}{2}\Bigg[\int\frac{\mathrm{d}^n\mathbf{p}}{(2\pi)^n}\frac{1}{\mathbf{p}^2} - \frac{(d^0)^2}{d^2}\int\frac{\mathrm{d}^n\mathbf{p}}{(2\pi)^n}\frac{1}{\left(\mathbf{p}^2 + \frac{(\mathbf{d}\cdot\mathbf{p})^2}{d^2}\right) + \left(\frac{1}{d}\right)^2}\Bigg], \tag{7}$$

where $d = \sqrt{d^2}$.

In order to calculate the second integral in eq. (7), we shall carry out a change in the integration variables in n spatial dimensions, in the same way as in [21]. First we split the vector $\mathbf{p} = (p^1, p^2, \ldots, p^n)$ as follows:

$$\mathbf{p} = \mathbf{p}_\perp + \mathbf{p}_\parallel, \tag{8}$$

where the vectors $\mathbf{p}_\parallel$ and $\mathbf{p}_\perp$ are respectively parallel and perpendicular to the vector $\mathbf{d}$, *i.e.*,

$$\mathbf{p}_\parallel = \mathbf{d}\left(\frac{\mathbf{d}\cdot\mathbf{p}}{\mathbf{d}^2}\right), \quad \mathbf{p}_\perp = \mathbf{p} - \mathbf{d}\left(\frac{\mathbf{d}\cdot\mathbf{p}}{\mathbf{d}^2}\right). \tag{9}$$

We also define the vector $\mathbf{u}$ as follows:

$$\mathbf{u} = \mathbf{p}_\perp + \mathbf{p}_\parallel \sqrt{1+\frac{\mathbf{d}^2}{d^2}}, \tag{10}$$
$$= \mathbf{p} + \mathbf{d}\left(\frac{\mathbf{d}\cdot\mathbf{p}}{\mathbf{d}^2}\right)\left(\frac{|d^0|}{d}-1\right). \tag{11}$$

With the previous definitions, we can write

$$\mathbf{p}_\parallel = \frac{\mathbf{d}(\mathbf{d}\cdot\mathbf{u})}{\mathbf{d}^2}\frac{d}{|d^0|}, \quad \mathbf{p}_\perp = \mathbf{u} - \frac{\mathbf{d}(\mathbf{d}\cdot\mathbf{u})}{\mathbf{d}^2}, \tag{12}$$

which implies

$$\mathbf{p} = \mathbf{u} + \frac{(\mathbf{d}\cdot\mathbf{u})\mathbf{d}}{\mathbf{d}^2}\left(\frac{d}{|d^0|}-1\right), \tag{13}$$

and

$$\mathbf{u}^2 = \mathbf{p}^2 + \frac{(\mathbf{d}\cdot\mathbf{p})^2}{d^2}. \tag{14}$$

The Jacobian of the transformation from $\mathbf{p}$ to $\mathbf{u}$ can be obtained from eq. (13), resulting in

$$\det\left[\frac{\partial\mathbf{p}}{\partial\mathbf{u}}\right] = \frac{1}{\sqrt{1+\frac{\mathbf{d}^2}{d^2}}} = \frac{d}{|d^0|}. \tag{15}$$

With this change of variables, we obtain

$$E_{(1)} = \frac{q^2}{2}\left[\int\frac{\mathrm{d}^n\mathbf{p}}{(2\pi)^n}\frac{1}{\mathbf{p}^2} - \frac{|d^0|}{d}\int\frac{\mathrm{d}^n\mathbf{u}}{(2\pi)^n}\frac{1}{\mathbf{u}^2+\left(\frac{1}{d}\right)^2}\right]. \tag{16}$$

Both integrals in (16) are performed along the same n-dimensional space. To avoid misunderstandings, we rewrite them in the $\mathbf{k}$ variable, instead of $\mathbf{u}$ and $\mathbf{p}$, as follows:

$$E_{(1)} = \frac{q^2}{2}\int\frac{\mathrm{d}^n\mathbf{k}}{(2\pi)^n}\left[\frac{1}{\mathbf{k}^2} - \frac{|d^0|}{d}\frac{1}{\mathbf{k}^2+\left(\frac{1}{d}\right)^2}\right], \tag{17}$$
$$= \frac{q^2}{2}\left[\left(1-\frac{|d^0|}{d}\right)\int\frac{\mathrm{d}^n\mathbf{k}}{(2\pi)^n}\frac{1}{\mathbf{k}^2+\frac{1}{d^2}} + \frac{1}{d^2}\int\frac{\mathrm{d}^n\mathbf{k}}{(2\pi)^n}\frac{1}{\mathbf{k}^2\left(\mathbf{k}^2+\frac{1}{d^2}\right)}\right]. \tag{18}$$

Integrating in n-dimensional spherical coordinates, using that the integral in the solid angle of $\mathbf{k}$ gives $2\pi^{n/2}/\Gamma(n/2)$, we arrive at

$$E_{(1)} = \frac{q^2}{(4\pi)^{n/2}\Gamma(n/2)}\left[\left(1-\frac{|d^0|}{d}\right)\int_0^\infty \mathrm{d}k\frac{k^{n-1}}{k^2+\frac{1}{d^2}} + \frac{1}{d^2}\int_0^\infty \mathrm{d}k\frac{k^{n-3}}{k^2+\frac{1}{d^2}}\right], \tag{19}$$

with Γ standing for the Euler Gamma function. The remaining integral can be performed by means of the formula

$$\int_0^\infty \mathrm{d}r\frac{r^\beta}{(r^2+C^2)^\alpha} = \frac{\Gamma\left(\frac{1+\beta}{2}\right)\Gamma\left(\alpha-\frac{(1+\beta)}{2}\right)}{2(C^2)^{\alpha-(1+\beta)/2}\Gamma(\alpha)}, \tag{20}$$

leading to

$$E_{(1)} = \frac{q^2}{2^{n+1}\pi^{n/2}d^{n-2}}\left[\left(1-\frac{|d^0|}{d}\right)\Gamma\left(1-\frac{n}{2}\right) + \frac{\Gamma\left(\frac{n}{2}-1\right)\Gamma\left(2-\frac{n}{2}\right)}{\Gamma\left(\frac{n}{2}\right)}\right]. \tag{21}$$

This expression can be further simplified by using the basic properties of the Gamma function leading to

$$E_{(1)} = -\frac{q^2}{2^{n+1}\pi^{n/2}}\frac{|d^0|}{d^{n-1}}\Gamma\left(1-\frac{n}{2}\right). \tag{22}$$

It is interesting to notice that the self-energy of a point charge in this theory is finite for odd n and diverges for even n. For instance, we have the following results for $n = 1, 3, 5, 7$:

$$E_{(1)}(n=1) = -\frac{q^2}{4}|d^0|, \tag{23}$$
$$E_{(1)}(n=3) = \frac{q^2}{8\pi}\frac{|d^0|}{d^2}, \tag{24}$$
$$E_{(1)}(n=5) = -\frac{q^2}{48\pi^2}\frac{|d^0|}{d^4}, \tag{25}$$
$$E_{(1)}(n=7) = \frac{q^2}{480\pi^3}\frac{|d^0|}{d^6}, \tag{26}$$

while for $n = 2, 4, 6$ we have

$$E_{(1)}(n=2) = \frac{q^2}{8\pi}\frac{|d^0|}{d^2}\left[\frac{2}{n-2}+\gamma\right], \tag{27}$$
$$E_{(1)}(n=4) = -\frac{q^2}{32\pi^2}\frac{|d^0|}{d^3}\left[\frac{2}{n-4}+\gamma-1\right], \tag{28}$$
$$E_{(1)}(n=6) = \frac{q^2}{128\pi^3}\frac{|d^0|}{d^6}\left[\frac{1}{n-6}+\frac{\gamma}{2}-\frac{3}{4}\right], \tag{29}$$

$\gamma = 0.5772156649$ being the Euler constant.

The result obtained so far can be generalized for a slightly different model, in which the same LV vector d^μ appears also in a minimal LV operator. More concretely, we consider the model defined by

$$\mathcal{L}_{(2)} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} - \frac{1}{2\gamma}\left(\partial_\mu A^\mu\right)^2 - \frac{1}{2}\mu^2 d^\mu d_\nu F_{\mu\lambda}F^{\nu\lambda} + \frac{1}{2}d^\lambda d_\alpha \partial_\mu F_{\nu\lambda}\partial^\nu F^{\mu\alpha} + J^\mu A_\mu, \tag{30}$$

where μ is a mass scale that is introduced by dimensional reasons. For consistency with experimental observations,

μ can be considered as arbitrary, but not too large in order to keep the dimensionless combination $\mu^2 d^\mu d_\nu$ small. As in the previous case, we shall consider only the case where $d^2 > 0$. Clearly, in the limit $\mu \to 0$, the Lagrangian (30) reduces to (2) at the classical level, so in this sense this model can be seen as a generalization of the first one.

Again choosing the Feynman gauge $\gamma = 1$, and following similar steps employed previously to deal with d^μ appearing in the non-minimal coupling, we can arrive at the following expression for the self-energy of a steady point-like charge:

$$\begin{aligned} E_{(2)} &= \frac{q^2}{2}\int \frac{\mathrm{d}^n\mathbf{p}}{(2\pi)^n}\tilde{D}^{00}_{(2)}(p^0=0,\mathbf{p}) \\ &= \frac{q^2}{2}\frac{1-\mu^2\mathbf{d}^2}{1+\mu^2 d^2}\int \frac{\mathrm{d}^n\mathbf{p}}{(2\pi)^n}\frac{1}{\mathbf{p}^2-\mu^2(\mathbf{d}\cdot\mathbf{p})^2} \\ &\quad -\frac{q^2}{2}\frac{(d^0)^2}{1+\mu^2 d^2} \\ &\quad \times \int \frac{\mathrm{d}^n\mathbf{p}}{(2\pi)^n}\frac{1}{d^2\mathbf{p}^2+(\mathbf{d}\cdot\mathbf{p})^2+(1+\mu^2 d^2)}, \end{aligned} \tag{31}$$

where $\tilde{D}^{\mu\nu}_{(2)}(p)$ is the Fourier transform of the propagator corresponding to the Lagrangian (30). It is instructive to compare this expression with eq. (16), noticing that the effect of the quadratic piece involving d^μ contributes with the $1 \pm \mu^2 d^2$ factors and the new term $\mu^2(\mathbf{d}\cdot\mathbf{p})^2$ in the propagator.

We deal with this new factors and arrive at the final result with the help of an additional change of variables. For the first integral in (31), we perform the change of integration variable given by

$$\mathbf{p} \to \mathbf{k} = \mathbf{p} + \mathbf{d}\frac{(\mathbf{d}\cdot\mathbf{p})}{\mathbf{d}^2}\left[\sqrt{1-\mu^2\mathbf{d}^2}-1\right], \tag{32}$$

such that

$$\mathbf{k}^2 = \mathbf{p}^2 - \mu^2(\mathbf{d}\cdot\mathbf{p})^2, \tag{33}$$

and also

$$\left|\frac{\partial\mathbf{p}}{\partial\mathbf{k}}\right| = \frac{1}{\sqrt{1-\mu^2\mathbf{d}^2}}. \tag{34}$$

For the second integral in (31), we use (10). Collecting terms, and performing some simple manipulations, we can show that the self-energy (31) is given by

$$\begin{aligned} E_{(2)} &= \frac{q^2}{2}\frac{1}{1+\mu^2 d^2} \\ &\quad \times \left[\sqrt{1-\mu^2\mathbf{d}^2}\int \frac{\mathrm{d}^n\mathbf{k}}{(2\pi)^n}\left(\frac{1}{\mathbf{k}^2}-\frac{1}{\mathbf{k}^2+\frac{1+\mu^2 d^2}{d^2}}\right)\right. \\ &\quad \left. +\left(\sqrt{1-\mu^2\mathbf{d}^2}-\frac{|d^0|}{d}\right)\int \frac{\mathrm{d}^n\mathbf{k}}{(2\pi)^n}\frac{1}{\mathbf{k}^2+\frac{1+\mu^2 d^2}{d^2}}\right]. \end{aligned} \tag{35}$$

Integrating out in the solid angle, using eq. (20) and performing some simple manipulations, we obtain the self-energy

$$E_{(2)} = -\frac{q^2}{2^{n+1}\pi^{n/2}}\frac{|d^0|}{d^{n-1}}(1+\mu^2 d^2)^{n/2-2}\Gamma\left(1-\frac{n}{2}\right). \tag{36}$$

Again, for even n the self-energy is divergent, while it is finite for odd n. As expected, in the limit $\mu \to 0$, the energy (36) goes to the result presented in eq. (22).

A similar pattern was found in the Lee-Wick electrodynamics [17,18], in which a Lorentz-invariant, higher-derivative modification is introduced in QED to tame its divergences. We see that the LV parameter d^μ also acts as a regulator for the self-energy of a point-like charge, which in our case turns out to be finite, yet dependent on the particular reference frame used to describe the measurement experiment, as it is customary in Lorentz-violating theories.

The Lagrangian in eq. (2) was also considered to describe the electromagnetic interaction between different kinds of classical sources in [9]. We remark, however, that if this model is considered at the quantum level, with the Maxwell field coupling to other quantum fields, such that radiative corrections can appear, the presence of the LV coupling in eq. (1) alone does not guarantee finiteness at the quantum level. Actually, the problem of radiative corrections in a LV model is a highly non-trivial one, that has been discussed extensively in the last years. There are models in which specific LV interactions generate finite, well-defined corrections at one loop [22–24], however, in general, these corrections can be divergent (thus requiring some renormalization mechanism), and even ambiguous [25]. In fact, the basic LV coupling considered by us in this paper, given in eq. (1), was generated as a radiative correction arising from a fermion loop in a specific LV model: we refer the reader to refs. [7,8] for an extensive discussion of these quantum corrections.

Another well-known issue concerning higher-derivative theories has to do with the presence of classical instabilities and/or ghost states, related to the presence of additional poles in the propagator. These problems have been extensively discussed, for example, in connection with the Lee-Wick electrodynamics [15,16], see, for example, [26,27] and references therein. Even with the ongoing discussion on how to treat these issues from the theoretical point of view, a Lee-Wick extension of the Standard Model was proposed [28,29] and a study of several phenomenological aspects ensued. Regarding non-minimal LV models, questions of stability and unitarity are also non-trivial, and have been discussed in different contexts [30–36]. There is no known general prescription to discern which non-minimal LV models can still have a consistent quantum formulation, free of instabilities and unitary: each specific model has to be studied individually. On general grounds, however, these additional poles are expected to appear at very high mass scales, so from the point of view of effective field theories, suitable for phenomenological considerations in low energy (relative to the scale where Lorentz violation is generated, which assumedly is near the Planck scale), they might be ignored [4–6].

Concerning our specific model, the propagator in eq. (4) exhibits, besides the usual pole at $p^2 = 0$, additional poles at the zeros of the function $\Psi(p) = 1 - d^2p^2 + (p \cdot d)^2$. More explicitly, we are interested in the solutions in the complex p^μ plane for the equation

$$\Psi(p) = \mathbf{d}^2(p_0)^2 - 2(\mathbf{p}\cdot\mathbf{d})d_0p_0 + 1 + d^2\mathbf{p}^2 + (\mathbf{p}\cdot\mathbf{d})^2 = 0. \quad (37)$$

Taking into account that $d^2 = (d_0)^2 - \mathbf{d}^2 > 0$, one might be tempted to choose the preferred frame in which $d^\mu = (d, \mathbf{0})$ to simplify the calculations, but then one would find the condition $1 + d_0^2\mathbf{p}^2 = 0$, independent of p_0, which cannot define a consistent dynamics. Disregarding this particular choice as anomalous, we set $d_0 = \eta|\mathbf{d}|$ with $\eta > 1$, in which case $d_0/|\mathbf{d}| = \eta$ and $d^2/\mathbf{d}^2 = \eta^2 - 1 = \varepsilon > 0$. Then, we can solve the condition $\Psi(p)$ for p_0 as a function of $\mathbf{p}$ and d^μ, obtaining two solutions,

$$p_0^\pm = (\mathbf{p}\cdot\hat{\mathbf{d}})\eta \pm [\varepsilon(\mathbf{p}\cdot\hat{\mathbf{d}})^2 - \varepsilon\mathbf{p}^2 - 1/|\mathbf{d}|^2]^{1/2}, \quad (38)$$

where $\hat{\mathbf{d}} = \mathbf{d}/|\mathbf{d}|$. These are two additional poles in the propagator, appearing as a consequence of the LV term in eq. (1), and they represent instabilities in the theory: assuming $|\mathbf{d}| \sim 1/M$, where M is a very high mass scale related to the origin of the Lorentz violation (a natural assumption is $M \sim M_{\text{Planck}}$), and also that $|\mathbf{p}| \ll M$, we can write

$$p_0^\pm \approx (\mathbf{p}\cdot\hat{\mathbf{d}})\eta \pm iM. \quad (39)$$

This pole, corresponding to an imaginary energy, clearly signals an instability in the theory.

One might verify that relaxing the condition $|\mathbf{p}| \ll M$ does not solve this issue; also, the same general picture arises in the generalized model shown in eq. (30). As a result, the definition of a consistent quantum theory starting from the classical model considered in this letter depends on whether these unphysical poles can somehow be removed from the theory, as in the Lee-Wick electrodynamics [26,27] or Standard Model [28,29], or in the LV models considered in [36], and this is a question that deserves further investigation.

In summary, the emergence of additional poles for propagators in LV scenarios in theories with higher-order derivatives, which may jeopardize unitarity and/or stability, is a common problem and can be a way to determine restrictions that must be imposed in theories of this kind in order to quantize them [37]. The presence of problematic poles can also be used to distinguish between theories that are feasible to be quantized and theories that must be taken just in the classical context, or considered as effective theories valid up so some scale, smaller than the characteristic scale of these additional poles [4–6].

Our main objective was to exhibit another instance where higher-derivative terms, in this case arising from a Lorentz violation, can act as physical regulators for the classical self-energy of a point-like charge. Differently from the Lee-Wick electrodynamics considered in [17,18], it is expected that, in a Lorentz-violating model, the value of the self-energy can depend on the reference frame where it is measured. A general result for a moving charge with respect to the background field is a much more complicated problem that deserves to be further investigated.

* * *

This work was supported by Conselho Nacional de Desenvolvimento Cientfico e Tecnolgico (CNPq) and Fundao de Amparo a Pesquisa do Estado de São Paulo (FAPESP), via the following grants: CNPq 304134/2017-1 and FAPESP 2017/13767-9 (AFF), CNPq 311514/2015-4 (FAB), FAPESP 2016/11137-5 (LHCB).

REFERENCES

[1] Kostelecky V. A., *Seventh Meeting on CPT and Lorentz Symmetry*, Vol. **9** (World Scientific, Singapore) 2017.
[2] Kostelecky V. A. and Russell N., *Rev. Mod. Phys.*, **83** (2011) 11 (arXiv:0801.0287).
[3] Colladay D. and Kostelecky V. A., *Phys. Rev. D*, **58** (1998) 116002.
[4] Kostelecky V. A. and Mewes M., *Phys. Rev. D*, **80** (2009) 015020.
[5] Kostelecky V. A. and Mewes M., *Phys. Rev. D*, **85** (2012) 096005.
[6] Kostelecky V. A. and Mewes M., *Phys. Rev. D*, **88** (2013) 096006.
[7] Borges L. H. C., Dias A. G., Ferrari A. F., Nascimento J. R. and Petrov A. Yu., *Phys. Rev. D*, **89** (2014) 045005.
[8] Borges L. H. C., Dias A. G., Ferrari A. F., Nascimento J. R. and Petrov A. Yu., *Phys. Lett. B*, **756** (2016) 332.
[9] Borges L. H. C., Ferrari A. F. and Barone F. A., *Eur. Phys. J. C*, **76** (2016) 599.
[10] Casana R., Ferreira M. M., Lisboa-Santos L., dos Santos F. E. P. and Schreck M., *Maxwell electrodynamics modified by CPT-even and Lorentz-violating dimension-6 higher-derivative terms* (arXiv:1802.07890).
[11] Schweber S. S., *An Introduction to Relativistic Quantum Field Theory* (Dover Publications Inc.) 2005, p. 913.
[12] Podolsky Boris, *Phys. Rev.*, **62** (1942) 68.
[13] Podolsky Boris and Kikuchi Chihiro, *Phys. Rev.*, **65** (1944) 228.
[14] Podolsky Boris and Schwed Philip, *Rev. Mod. Phys.*, **29** (1948) 40.
[15] Lee T. D. and Wick G. C., *Nucl. Phys. B*, **9** (1969) 209.
[16] Lee T. D. and Wick G. C., *Phys. Rev. D*, **2** (1970) 103.
[17] Barone F. A., Flores-Hidalgo G. and Nogueira A. A., *Phys. Rev. D*, **91** (2015) 027701.
[18] Barone F. A., Flores-Hidalgo G. and Nogueira A. A., *Phys. Rev. D*, **88** (2013) 105031.
[19] Barone F. A. and Flores-Hidalgo G., *Phys. Rev. D*, **78** (2008) 125003.
[20] Barone F. A. and Flores-Hidalgo G., *Braz. J. Phys.*, **40** (2010) 188.

[21] Borges L. H. C., Barone F. A. and Helayël-Neto J. A., *Eur. Phys. J. C*, **74** (2014) 2937.

[22] Gomes M., Nascimento J. R., Petrov A. Yu. and da Silva A. J., *Phys. Rev. D*, **81** (2010) 045018 (arXiv:0911.3548); *On the Aether-like Lorentz-breaking action for the electromagnetic field*, arXiv:1008.0607.

[23] Gazzola G., Fargnoli H. G., Baeta Scarpelli A. P., Sampaio M. and Nemes M. C., *J. Phys. G*, **39** (2012) 035002 (arXiv: 1012.3291).

[24] Baeta Scarpelli A. P., Mariz T., Nascimento J. R. and Petrov A. Yu., *Eur. Phys. J. C*, **73** (2013) 2526 (arXiv:1304.2256).

[25] Jackiw R. and Kostelecky V. A., *Phys. Rev. Lett.*, **82** (1999) 3572 (hep-ph/9901358).

[26] Anselmi D. and Piva M., *Phys. Rev. D*, **96** (2017) 045009.

[27] Anselmi D. and Piva M., *Phys. JHEP*, **06** (2017) 066.

[28] Grinstein B., O'Connell D. and Wise M. B., *Phys. Rev. D*, **77** (2008) 025012.

[29] Carone C. D. and Lebed R. F., *Phys. Lett. B*, **668** (2008) 221.

[30] Casana R., Ferreira M. M. jr., Gomes A. R. and Pinheiro P. R. D., *Phys. Rev. D*, **80** (2009) 125040.

[31] Belich H., Ferreira M. M. jr., Helayel-Neto J. A. and Orlando M. T. D., *Phys. Rev. D*, **67** (2003) 125011; **69**, (2004) 109903.

[32] Baeta Scarpelli A. P., Belich H., Boldo J. L. and Helayel-Neto J. A., *Phys. Rev. D*, **67** (2003) 085021.

[33] Baeta Scarpelli A. P. and Helayel-Neto J. A., *Phys. Rev. D*, **73** (2006) 105020.

[34] Reyes C. M., *Phys. Rev. D*, **82** (2010) 125036.

[35] Reyes C. M., *Phys. Rev. D*, **87** (2013) 125028.

[36] Balart L., Reyes C. M., Ossandon S. and Reyes C., *Perturbative unitarity and higher-order Lorentz symmetry breaking*, arXiv:1802.06918.

[37] Reyes C. M., Urrutia L. F. and Vergara J. D., *Phys. Rev. D*, **78** (2008) 125011.