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Evidence of quantum gravity at low energies

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To my wife

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Resumo

Este trabalho teve como objetivo a investigação de efeitos de gravitação quântica a baixas energias dado que as tentativas de modelagem de uma teoria completa não foram bem sucedidas até os dias de hoje. Primeiramente apresentamos a primeira comprovação experimental de que a gravidade a baixas energias, de fato, interage com sistemas quânticos. Posteriormente, analizamos duas evidências de que o compo gravitacional, de fato, tem natureza quântica, uma associada ao emaranhamento de duas particulas interagentes e a outra associada a um aparente paradoxo que só pode ser resolvido levando em consideração as propriedades quânticas do campo que medeia as interações. Em seguida, analizamos dois efeitos propostos na escala de gravitação quântica. O primeiro está associada à indefinição de ordem causais em um espaço-tempo em que as fontes de curvatura tem natureza quântica. O segundo está associada à auto-interação gravitacional de partículas quânticas, que seria responsável pela transição do mundo quântico para o clássico. Em particular, argumentamos que assumir esta auto-interação gravitacional é não natural pois, seu análogo elétrico, leva a observáveis que não condizem com os experimentos.

Palavras Chaves: Gravitação quântica; Fundamentos de mecânica quântica; Ordens causais indefinidas; Auto-interação.

Áreas do conhecimento: Física; Mecânica quântica; gravitação quântica.

Abstract

This work has the goal of investigating low-energy quantum gravity effects since attempts to model a full theory have not been successful until now. First, we present the first experimental evidence that low-energy gravity, in fact, interacts with quantum systems. Subsequently, we analyze two pieces of evidence that the gravitational field indeed has a quantum nature, one associated with the entanglement of two interacting particles and the other associated with an apparent paradox that can only be solved by taking into account the quantum properties of the field that mediates the interactions. Next, we analyze two proposed effects on the quantum gravity scale. The first is associated with an indefinition in the space-time causal order when the source of curvature has a quantum nature. The second is associated with the gravitational self-interaction of quantum particles, which would be responsible for the transition from the quantum to the classical world. In particular, we argue that assuming this gravitational self-interaction is unnatural because its electric analog leads to observables that do not fit with the experiments.

Keywords: Quantum gravity; Quantum foundations; Indefinite causal order; Self-interaction.

Knowledge areas: Physics; Quantum mechanics; Quantum gravity.

Contents

Chapter 1

Introduction

The two fundamental pillars of physics are general relativity and quantum mechanics. Quantum mechanics dictates the rules of the dynamics of the objects that are over a space-time that general relativity says how it is curved. The problem is that these two theories do not agree with each other in the sense that we do not know how a quantum object curves the space-time since general relativity requires a classical description of matter and energy. Also, we still do not have a model to describe the quantum features of the space-time. In fact, it is not surprising that the usual approaches to field quantization do not work for general relativity since the commutation rules are defined in a spatial hyperficies, and the possible quantum structure of space-time turns this poorly defined.

The most famous attempts to describe a quantum theory of gravity started many years ago [1]. They have no success until the moment and seem less promising each day. In this sense, many authors started to seek low-energy effects of quantum gravity in order to gain some insight into the direction of the full theory and to comprehend if the gravitational field indeed has a quantum nature.

Bose et al. [2] analyzed a thought experiment in which gravity is responsible for creating an entangled state of two particles that are superposed in two distinct position states and interact gravitationally with each other. Based on a general quantum information theorem of local operation and classical communication (LOCC), they were able to conclude that the gravitational field must be quantized.

Also in a thought experiment, Belenchia et al. [3] probed an apparent paradox between the principles of quantum mechanics and causality. If the field that mediates the interaction is not quantized, one particle could affect the measurement probabilities of the other even if they were causally disconnected, and one could use this to transmit information faster than light. In this sense, the gravitational field must have quantum degrees of freedom.

In the search for general properties of a quantum gravity theory, Magdalena Zych et al. [4] analyzed another thought experiment in which a source of gravity is placed in a superposition of two different positions. They conclude that the casual order of events in space-time, which is classically defined given a classical mass-energy distribution, is undefined in this quantum scenario, which seems to be a general property that any quantum gravity theory must obey.

Another proposed effect in this low-energy regime of quantum gravity is due to Penrose [5,6]. He argues that a gravitational mechanism would be responsible for the decoherence of quantum particles, which would explain the transition from quantum to classical worlds. To realize it, he proposed that particles interact with themselves via their own gravitational field. Penrose proposes a modification of the standard Schrödinger equation, known in literature as the Schrödinger-Newton equation.

This work is organized as follows. In chapter 2 we show the details of the COW experiment, which was the first experiment showing the influence of the gravitational field over quantum systems. In chapter 3 we explore the entanglement due to gravitational interaction between two particles as evidence of the quantum nature of gravity. In chapter 4 we show the apparent paradox between quantum mechanics and causality also as an evidence for the quantization of the gravitational field. In chapter 5 we analyze the undefined causal order due to a quantum matter-energy distribution as what seems to be a general property for any quantum gravity model. In chapter 6 we present and investigate the Schrödinger-Newton equation and probe how promising it is to describe the quantum-classical transition and as a fundamental equation. In chapter 7 we present the conclusions of this work. In appendix A we present what is a LOCC operation mentioned in chapter 3. In appendix B we give some intuition about the fluctuations of a quantum field used in chapter 4. In appendix C we present the python code used in chapter 6 to solve the modified Schroedinger-Newton equation.

Chapter 2

COW Experiment

Since gravity has a different nature compared to the other fundamental interactions, its low-energy action on quantum systems was a theme of investigation in the 20th century. Also, its weakness makes any experimental detection challenging. In this chapter, we will analyze the COW experiment due to Colella, Overhauser, and Werner [7]. This was the first experimental evidence that quantum systems are affected by the gravitational field, and, of course, it is an important first step in the path to a complete quantum gravity theory. To explain the experiment, we first present the Mach-Zender interferometer since it is the experimental setup used in the COW experiment, and then we discuss the theoretical features of the experiment and show its conclusions.

2.1 Mach-Zender Interferometer

The Mach-Zehnder (MZ) interferometer plays a fundamental role in the probing of gravitational interaction since it uses interference phenomena as its basis. Such effects, even very weak ones, can be observed. Now we will present in detail how it works and define some notations that will be used in this chapter. We use massive particles since the COW experiment was made with neutrons. A scheme of the MZ interferometer setup is shown in figure 2.1. The MZ interferometer works by dividing a particle beam into two superposed paths, recombining the superpositions, and measuring the particle with two different detectors.

Following the directions defined in figure 2.1, we use the notation $|\nearrow\rangle$ to denote the particles that go up and right and $|\searrow\rangle$ for those that go down and right. Furthermore, the particle evolves according to the Schroedinger equation. Hence, an extra phase $e^{-ip^2t/(2m\hbar)}$ will be added to the state, where p is the particle momentum, m is its mass, and t is the time spent by the particle to cross some part of the interferometer. By making all parts of the interferometer the same length, this extra phase will be the same for the two branches of the superposition and, then, will be an ignorable global phase.

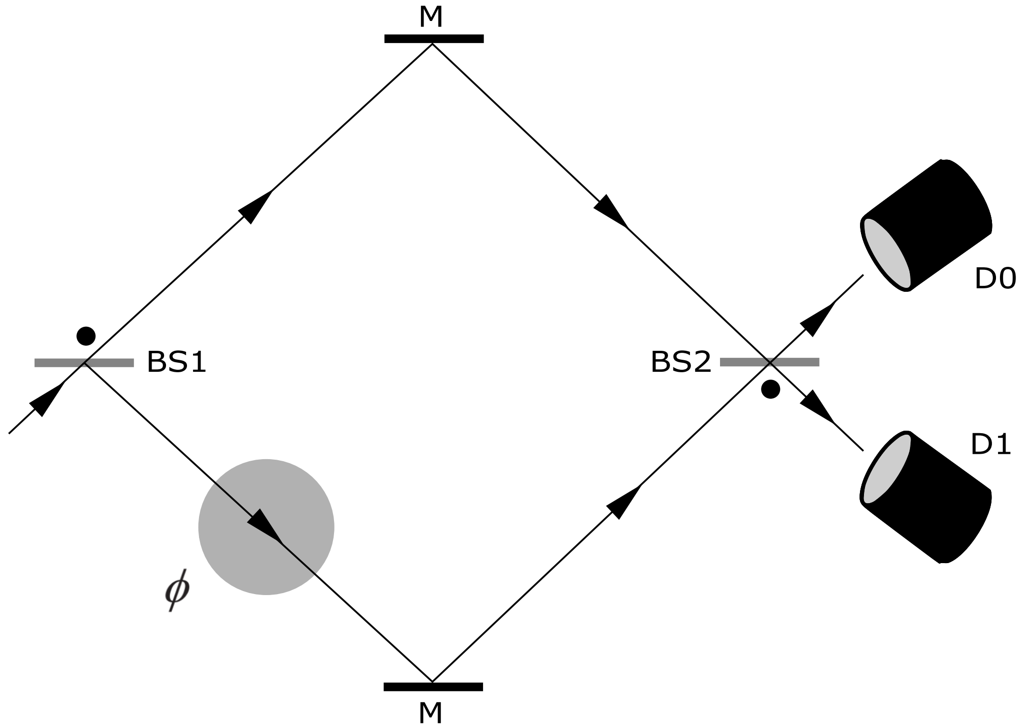

Figure 2.1: Schematic figure of the MZ interferometer. BS1 and BS2 are the beam-splliters. The faces of the beam-splliters that add the phase π in the reflected beam are indicated by $\bullet$. M are the mirrors, ϕ denotes some external agent adding a phase ϕ on the state, and the detectors are D0 and D1. Figure based on [8].

Let us now focus our attention on the precise description of the MZ interferometer. The mirrors M in figure 2.1 cause a total reflection of the particle such that the incident angle is equal to the reflected angle and a phase π is added to the state, that is, $|\alpha\rangle \rightarrow e^{i\pi}|\alpha\rangle = -|\alpha\rangle$. The beam splitter (or half-silvered mirror), BS in figure 2.1, is a device that divides the particle beam into two, each with half of the original amplitude. The first part crosses the BS, and the other is reflected. Moreover, one of the faces of the BS, indicated by $\bullet$ in figure 2.1, adds a phase π to the reflected particle if it collides directly with this part. If the beam does not collide directly with the $\bullet$ face, the phase is not added for the reflected particle. No extra phase is added to the beam that crosses the BS. The ϕ in figure 2.1, represents an external agent that acts in some part of the interferometer, adding a phase ϕ to the state of this part.

For further comparisons, we calculate what we call the "standard proportion" of finding a particle in some of the detectors. To do this, we follow figure 2.1 with $\phi = 0$. The initial state is $|\psi\rangle = |\nearrow\rangle$. After the interaction with BS1, since it does not collide directly on • face, the state becomes $|\psi\rangle \to (|\nearrow\rangle + |\searrow\rangle)/\sqrt{2}$. Next, the beams collide on the mirrors M such that $|\psi\rangle \to -(|\searrow\rangle + |\nearrow\rangle)/\sqrt{2}$. Finally, the beams are directed to BS2, and one of them collides with the • face. Hence, the final state is

$$|\psi\rangle \to -\frac{1}{\sqrt{2}}\left[\frac{1}{\sqrt{2}}(|\searrow\rangle + |\nearrow\rangle) + \frac{1}{\sqrt{2}}(|\nearrow\rangle - |\searrow\rangle)\right] = -|\nearrow\rangle . \tag{2.1}$$

In this way, there is a 100% of chance that the particle will be detected in D0 and 0% in D1. This is what we call the standard proportion. Of course, by changing the • parts of the BS, we would obtain different probabilities, but they would be essentially the same: 100% for one detector and 0% for the other.

One possible way to deviate from the standard proportions is to modify the length of one part of the interferometer. This turns the global phase $e^{-ip^2t/(2m\hbar)}$ to not be global anymore since the time t will not be equal in each part of the interferometer. Another way is to drive a gravitational phase difference between two paths of the interferometer, which is exactly what we will investigate in the next section.

2.2 The experiment

Now we present the first experimental evidence that gravity affects quantum systems, the COW experiment [7] mentioned in the beginning of the chapter. The authors used a version of the MZ interferometer with neutrons to detect the Earth's gravitational potential difference between parts of the interferometer.

For pedagogical reasons, we will use a different set-up of the MZ interferometer to explain the COW experiment, which is shown in figure 2.2 where the blue lines are beam splitters and the red lines are mirrors. We use this rectangular form to avoid complications in the calculations. The paths 1 and 4 have length L_1, the paths 2 and 3 have length L_2 and the green box represents a detector. Since opposite paths have the same length, a standard proportion is expected in the detector if no external agent acts in the interferometer.

To measure the gravitational effects over the interferometer, we need to tilt its plan by an angle θ relative to the Earth's gravitational field, as shown in figure 2.3. Therefore, the neutron in path 4 will feel a gravitational potential difference relative to path 1. Let us see how this difference affects the system.

First, we note that the system is in a low energy regime, such that we will assume that the neutron evolves following the Schrödinger equation and that gravity acts as an external potential. So, the wave function of the neutron in path 4, $\Psi_4(x,t) = \psi_4(x)e^{-iEt/\hbar}$, satisfies

$$-\frac{\hbar^2}{2m}\frac{d^2\psi_4}{dx^2} + mgL_2 \sin\theta\, \psi_4 = E\psi_4. \tag{2.2}$$

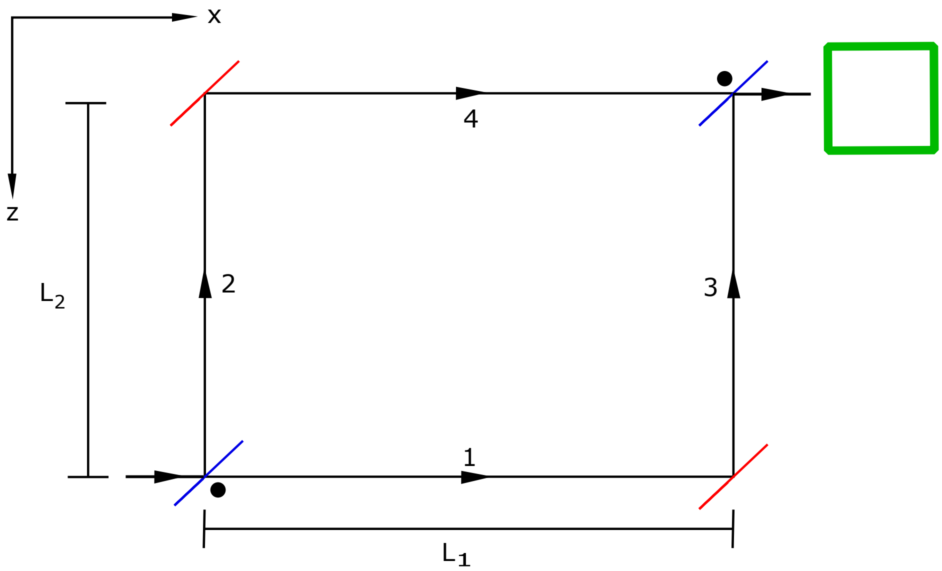

Figure 2.2: Schematic figure showing the MZ interferometer used in our analysis of the COW experiment. The red lines are mirrors, and the blue lines are beam splitters. The paths 1 and 4 have length L_1 and the paths 2 and 3 have length L_2. The green box represents a detector.

The beam in path 1 is a free particle that propagates to positive x. Than, we will approximate it as a plane wave $\psi_1(x) = e^{ipx/\hbar}$. Also, the kinetic energy is much greater than the gravitational potential energy in this experiment set up. A typical neutron interferometer has about 10 cm as its length scales. For a neutron of mass $\sim 10^{-24}$ g with typical speed $\sim 10^5$ cm/s we obtain that the potential energy is $\sim 10^{-9}$ eV and the kinetic energy is $\sim 10^{-2}$ eV. In this sense, the external potential do not dramatically affect the wave function in path 4. So, we will approximate it by $\psi_4(x) = \psi_1(x)\, e^{i\alpha(x)}$, in which $\alpha(x)$ changes slightly with x. Using these approximations, one can obtain from equation 2.2 [9,10]

$$i\frac{d^2\alpha}{dx^2} - \left(\frac{d\alpha}{dx}\right)^2 - \frac{2p}{\hbar}\frac{d\alpha}{dx} = \frac{2m^2 g L_2 \sin\theta}{\hbar^2}. \tag{2.3}$$

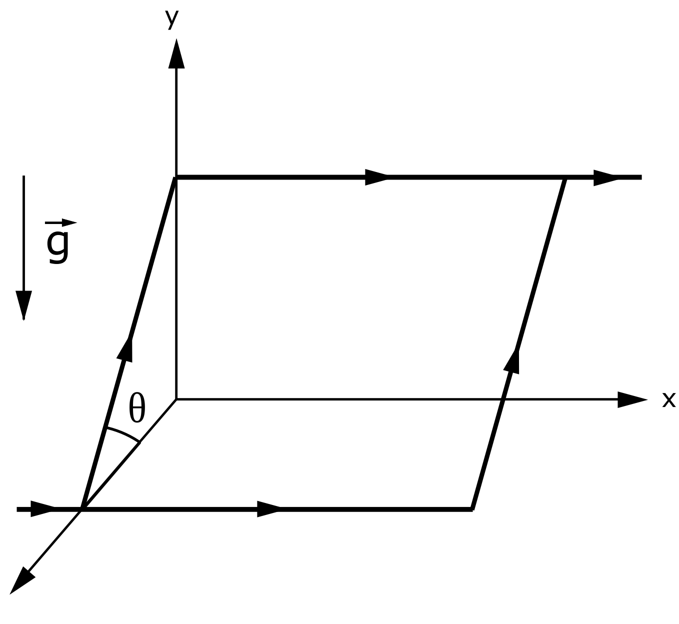

Figure 2.3: Interferometer tilted by an angle θ relative to the Earth's gravitational field $\vec{g}$ which is in direction $-\hat{y}$ in this coordinate system.

Discarding the high order two first terms of Eq. (2.3) we obtain

$$\alpha(x) = \alpha = -\frac{mgL_1L_2\sin\theta}{\hbar v}, \tag{2.4}$$

where $v = p/m$ is the speed of the neutron. It is important to note that the extra phase obtained by the neutron in paths 2 and 3 is not important to our experiment since it will be an ignorable global phase.

The probability P of finding a neutron in the detector after the recombination of the beams is

$$P = \frac{1 \pm \cos\alpha}{2} = \frac{1}{2}\left[1 \pm \cos\left(\frac{mgL_1L_2\sin\theta}{\hbar v}\right)\right]. \tag{2.5}$$

where the sign $\pm$ means that the probability alternates between the two possible results as we change how the beam splitters are positioned. The figure 2.4 shows the original data obtained in the COW experiment. The authors could verify the veracity of the model with less than 1% of error.

This experiment clearly shows the influence of the gravitational field over quantum systems, even though it is very weak, and also shows that we can consider the gravitational field as an external potential in the low energy regime. Of course, this is not an evidence of the quantum behavior of gravity, but it is an important step toward it. In the next two chapters, we will discuss evidences of this quantization.

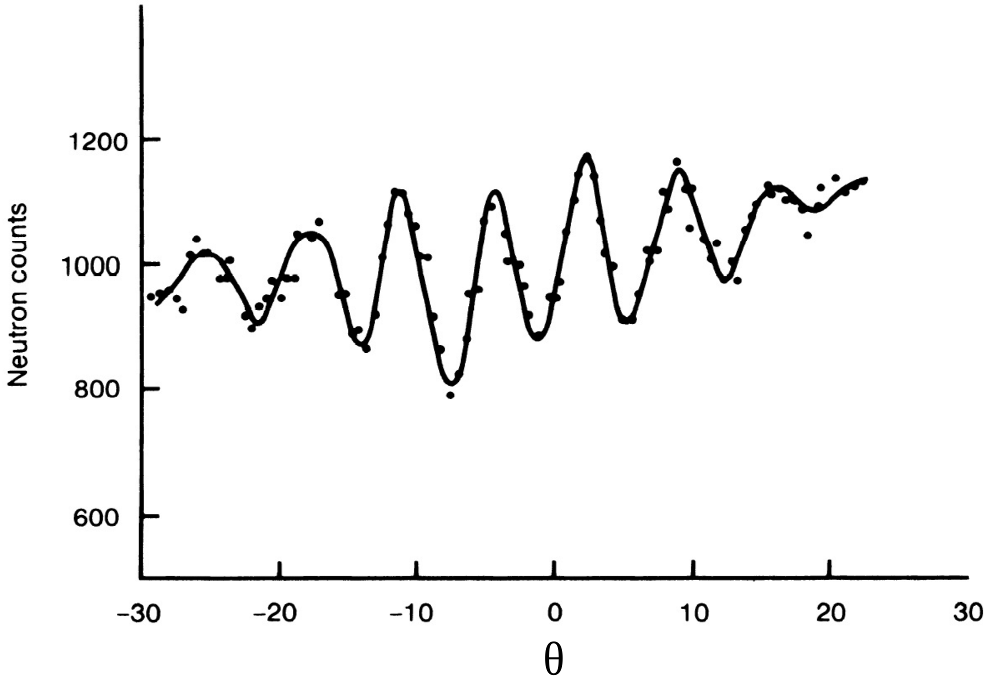

Figure 2.4: Original data of the COW experiment. Figure retrieved from [7].

Chapter 3

Entanglement via Gravity

Whether or not the gravitational field must be quantized has been a theme of debate until nowadays. One reason is the failure of the usual techniques for quantum field quantization when applied to gravity. In this chapter, we will analyze evidence for the quantization of gravity presented in Ref. [2]. This evidence is based on a proposed experiment that creates an entangled pair of particles due to their mutual gravitational interaction. Let us show the experiment in detail.

Consider the situation shown in figure 3.1. The particles 1 and 2 are placed in a superposition of two orthogonal position states where the distance between the centers of mass of these two states[1] is Δx and the distance between the centers of mass of the particles[2] is d. Considering that initially the particles did not interact, the state of the system in time $t = 0$ is the unentangled state

$$|\psi(t=0)\rangle = \frac{|L\rangle_1 + |R\rangle_1}{\sqrt{2}} \otimes \frac{|L\rangle_2 + |R\rangle_2}{\sqrt{2}}, \tag{3.1}$$

where $\langle L|R\rangle_i = 0$.

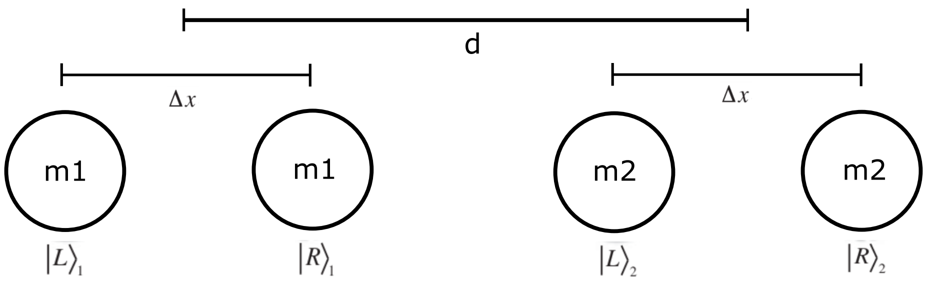

Figure 3.1: Two particles superposed in two orthogonal position states separated by a distance Δx whose centers of mass are separated by a distance d. Figure based on Ref. [2].

[1]Defined as the mean value of the position probability distribution of each branch of the superposition.
[2]Defined as the mean value of the position probability distribution of each particle.

Now we will consider that the particles interact gravitationally. To model this interaction, we will define the following Hamiltonian operator

$$\hat{H} = -Gm_1 m_2 \hat{D}, \tag{3.2}$$

where G is the newton constant and m_1 and m_2 are the masses of particles 1 and 2, respectively. This Hamiltonian operator is to be thought of as a low-energy limit coming from a full quantum gravity theory. The operator $\hat{D}$ has as its eigenstates the tensor product of the two particles eigenstates of the position, and the corresponding eigenvalues are the inverse of the distance between them:

$$\hat{D} \left| \vec{r}_1, \vec{r}_2 \right\rangle = \frac{1}{|\vec{r}_1 - \vec{r}_2|} \left| \vec{r}_1, \vec{r}_2 \right\rangle, \tag{3.3}$$

in which $|\vec{r}_1, \vec{r}_2\rangle \equiv |\vec{r}_1\rangle_1 \otimes |\vec{r}_2\rangle_2$ and $\vec{r}_1$ and $\vec{r}_2$ represent the 3-D position vector of particles 1 and 2, respectively. Note that we are modeling a Newtonian version of the gravitational interaction since we are in a low energy regime. Also, we assume with this Hamiltonian the approximation that the particles do not change their position state. This approximation is valid in the scales of this experiment as we show in the end of the chapter.

The total state of the system will evolve as $|\psi(t)\rangle = e^{-i\hat{H}t/\hbar} |\psi(t=0)\rangle$. Supposing that the system interacts for a time $t = \tau$ and defining $\sigma \equiv iGm_1 m_2 \tau/\hbar$, the final state will be

$$\begin{aligned}
|\psi(\tau)\rangle &= e^{-i\hat{H}\tau/\hbar} |\psi(t=0)\rangle \\
&= e^{-i\hat{H}\tau/\hbar} \left(\frac{|L,L\rangle + |L,R\rangle + |R,L\rangle + |R,R\rangle}{2} \right) \\
&= \frac{e^{\sigma/d} |L,L\rangle + e^{\sigma/(d+\Delta x)} |L,R\rangle + e^{\sigma/(d-\Delta x)} |R,L\rangle + e^{\sigma/d} |R,R\rangle}{2} \\
&= \frac{e^{i\phi}}{2} \left\{ |L\rangle_1 \left(|L\rangle_2 + e^{i\Delta\phi_{LR}} |R\rangle_2 \right) + |R\rangle_1 \left(e^{i\Delta\phi_{RL}} |L\rangle_2 + |R\rangle_2 \right) \right\},
\end{aligned} \tag{3.4}$$

where $\Delta\phi_{RL} = \phi_{RL} - \phi$, $\Delta\phi_{LR} = \phi_{LR} - \phi$ and

$$\phi_{RL} = \frac{Gm_1 m_2 \tau}{\hbar(d - \Delta x)}, \qquad \phi_{LR} = \frac{Gm_1 m_2 \tau}{\hbar(d + \Delta x)}, \qquad \phi = \frac{Gm_1 m_2 \tau}{\hbar d}. \tag{3.5}$$

For the authors, this entanglement is sufficient evidence (if measured) for the gravitational field quantization. To ensure that, they are underpinned by a theorem from quantum information. Quoting Bose et al.:

> "Our proposal relies on two simple assumptions: (a) the gravitational interaction between two masses is mediated by a gravitational field (in other words, it is not a direct interaction at a distance) and (b) the validity of a central principle of quantum information theory: entanglement between two systems cannot be created by local operations and classical communication (LOCC) ... Translating to our setting of two test masses in adjacent interferometers any external fields (including the gravitational fields from other masses around them) can only make LOs on their states, while a classical gravitational field propagating between the test masses can only give a CC channel between them. These LOCC processes cannot entangle the states of the masses. Thus it immediately follows that if the mutual gravitational interaction entangles the state of two masses, then the mediating gravitational field is necessarily quantum mechanical in nature."

For more about LOCC operations see the appendix A.

Measuring this entanglement is difficult because we need to measure the spatial degrees of freedom on different bases. To solve this problem, the authors propose a modification of this experiment in which one can detect the spins of the particles to witness the entanglement. The new experiment set up is shown in figure 3.2.

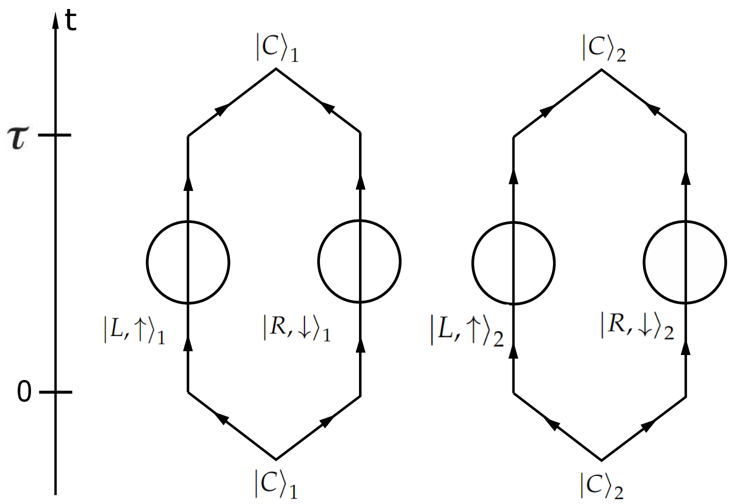

Figure 3.2: Experimental set-up to witness the entanglement via gravity. The particles are superposed with a Stern-Gerlach apparatus, interact gravitationally with each other, and are recombined after a time τ. Figure based on Ref. [2].

The initial state of the system is

$$|\psi_0\rangle = \frac{|C\rangle_1}{\sqrt{2}}(|\uparrow\rangle_1 + |\downarrow\rangle_1) \otimes \frac{|C\rangle_2}{\sqrt{2}}(|\uparrow\rangle_2 + |\downarrow\rangle_2), \tag{3.6}$$

where $|C\rangle_i$ is the position state of the particle i as shown in figure 3.2 and $|\uparrow\rangle_i$ and $|\downarrow\rangle_i$ are the spin eigenstates in the z direction of the particle i. Then, each particle is sent to a Stern-Gerlach (SG) apparatus, which changes the state to

$$|\psi_0\rangle \to |\psi_1\rangle = \frac{|L,\uparrow\rangle_1 + |R,\downarrow\rangle_1}{\sqrt{2}} \frac{|L,\uparrow\rangle_2 + |R,\downarrow\rangle_2}{\sqrt{2}}. \tag{3.7}$$

Then, the particles interact following the Hamiltonian (3.2) for a time τ and the position superpositions are brought back thought a reverse SG, which makes $|L,\uparrow\rangle_i \to |C,\uparrow\rangle_i$ and $|R,\uparrow\rangle_i \to |C,\uparrow\rangle_i$. Hence, the final state is

$$\begin{aligned}
|\psi(\tau)\rangle &= e^{-i\hat{H}\tau/\hbar}\,|\psi(t=0)\rangle \\
&= e^{-i\hat{H}\tau/\hbar}\left(\frac{|L,\uparrow,L,\uparrow\rangle + |L,\uparrow,R,\downarrow\rangle + |R,\downarrow,L,\uparrow\rangle + |R,\downarrow,R,\downarrow\rangle}{2}\right) \\
&= \frac{1}{2}\{e^{\sigma/d}\,|L,\uparrow,L,\uparrow\rangle + e^{\sigma/(d+\Delta x)}\,|L,\uparrow,R,\downarrow\rangle + \\
&\quad + e^{\sigma/(d-\Delta x)}\,|R,\downarrow,L,\uparrow\rangle + e^{\sigma/d}\,|R,\downarrow,R,\downarrow\rangle\} \\
&= \frac{e^{i\phi}}{2}\left\{|L,\uparrow\rangle_1\left(|L,\uparrow\rangle_2 + e^{i\Delta\phi_{LR}}\,|R,\downarrow\rangle_2\right) + |R,\downarrow\rangle_1\left(e^{i\Delta\phi_{RL}}\,|L,\uparrow\rangle_2 + |R,\downarrow\rangle_2\right)\right\} \\
&\to \frac{e^{i\phi}}{2}\left\{|C,\uparrow\rangle_1\left(|C,\uparrow\rangle_2 + e^{i\Delta\phi_{LR}}\,|C,\downarrow\rangle_2\right) + |C,\downarrow\rangle_1\left(e^{i\Delta\phi_{RL}}\,|C,\uparrow\rangle_2 + |C,\downarrow\rangle_2\right)\right\} \\
&= \frac{e^{i\phi}}{2}\left\{|\uparrow\rangle_1\,(|\uparrow\rangle_2 + e^{i\Delta\phi_{LR}}\,|\downarrow\rangle_2) + |\downarrow\rangle_1\,(e^{i\Delta\phi_{RL}}\,|\uparrow\rangle_2 + |\downarrow\rangle_2)\right\}|C\rangle_1\,|C\rangle_2\,.
\end{aligned} \tag{3.8}$$

By this token, the position degrees of freedom are not entangled anymore, but the spin degrees are. Therefore, according to the authors, measuring this entanglement is a proof of the quantum nature of the gravitational field. One possible observable that indicates the entanglement is $\mathcal{W} = |\langle\sigma_x^{(1)} \otimes \sigma_z^{(2)}\rangle + \langle\sigma_y^{(1)} \otimes \sigma_y^{(2)}\rangle|$, where $\sigma_j^{(i)}$ are the spin operator in the direction j for the particle i. If $\mathcal{W}$ exceeds unity, the state is an entangled one.

The authors estimate $\mathcal{W}$ based on possible experiments in the near future that must be realized with neutral objects with embedded electronic spin. To avoid decoherence, the experiment requires a very low crystal temperature and low pressures. For a microsphere of 10^{-14} Kg, $d - \Delta x \sim 200\ \mu$m to avoid the

Casimir-Polder interaction, $\Delta x \sim 250\ \mu$m and $\tau \sim 2.5$ s the result is $\Delta\phi_{LR} \sim -0.2$, $\Delta\phi_{RL} \sim 0.7$ and $\mathcal{W} \sim 1.16$.

Previously, we mentioned that we could assume that the position of the particles do not change during the experiment. In fact, we can classically estimate this deviation δs as

$$\delta s \sim \frac{Gm}{2(d-\Delta x)^2}\tau^2 \sim 5 \times 10^{-7}\ \text{m}, \tag{3.9}$$

that is around 400 times smaller than the typical distances of the experiment.

In this chapter, we showed possible evidence for the quantization of the gravitational field. If the correlation $\mathcal{W}$ will be measured, according to the author of Ref. [2], the gravitational field indeed has a quantum nature. However, we still do not know how to model it. In the next chapter, we will show further evidence of this quantization based on an apparent paradox.

Chapter 4

Paradox

In this chapter, we will present an evidence of the gravitational field quantization based on an apparent paradoxical situation involving two interacting particles presented in Ref. [3]. The authors used a thought experiment, shown in figure 4.1, to present this paradox, which we explain next. In this chapter, we use natural units such that $\hbar = c = 1$. We first present an electric version of the paradox and then the gravitational version of it.

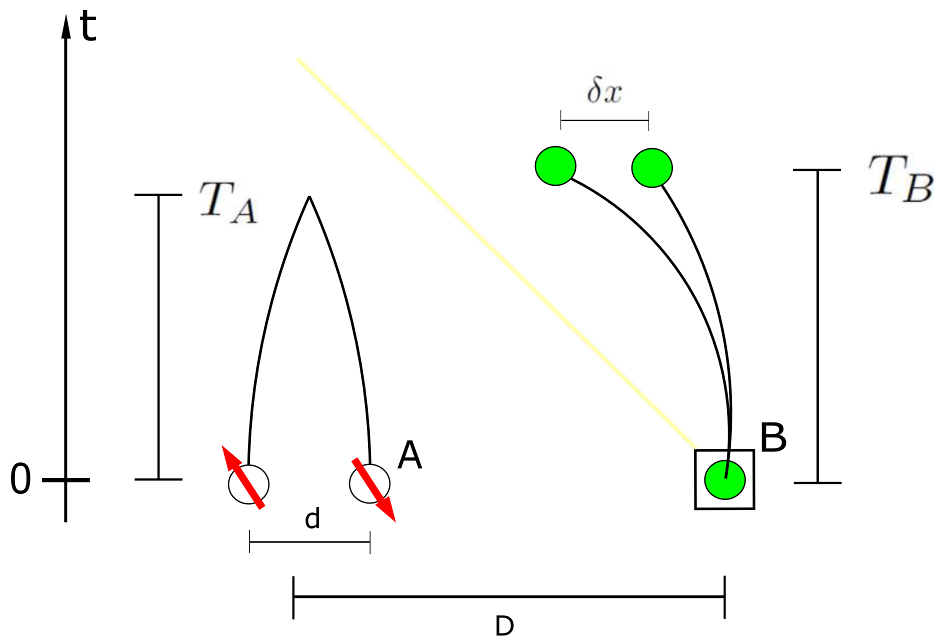

Figure 4.1: Thought experiment that shows the paradoxical situation presented in this chapter. Space is in the horizontal direction, and time is in the vertical direction. The distance between the superposition of Alice's particle is d, the distance between Alice and Bob is D, the displacement difference of Bob's particle due to the interaction with Alice's one is δx, the time that Alice ends her test is T_A, and the time that Bob measures the position of his particle is T_B. Figure based on Ref. [3].

Two experimentalists, Alice and Bob (A and B in the figure 4.1, respectively), are far apart by a distance D and carry, each one, a quantum body with masses

m_A and m_B, respectively, and charges q_A and q_B, respectively. Since we are only interested in the center of mass degree of freedom, we will refer to these bodies as particles, but they do not need to be elementary particles and can have large masses and charges. We also admit that Alice's particle has spin, and, in the distant past, she used a Stern-Gerlach apparatus (SG) to place her particle in the superposition

$$|\psi_0\rangle_A = \frac{1}{\sqrt{2}} \left(|L\rangle_A |\downarrow\rangle_A + |R\rangle_A |\uparrow\rangle_A \right) \tag{4.1}$$

separated by a distance d and avoiding any emission of radiation. The distant past mentioned above is enough time to guarantee that the field produced by the superposed Alice's particle propagates asymptotically and becomes static. In contrast, Bob holds his particle in a trap strong enough to neglect any interaction with Alice's one. The state in time $t = 0$ of Bob's particle is $|D\rangle_B$. So, the state of the system in time $t = 0$ is $|\psi_0\rangle_A \otimes |D\rangle_B$.

Bob releases his particle from the trap in the time $t = 0$, the same instant that Alice starts to recombine her particle with a reverse SG. Now, Bob's particle interacts with Alice's. After a time $t = T_B$, he measures the center of mass degree of freedom of his particle. On the other hand, Alice finishes her recombination and makes some experiment (a double slit, for example) to test if her particle is in a mixed or pure state in the time $t = T_A$.

The interaction between the particles depends on which position state, $|L\rangle_A$ or $|R\rangle_A$, is Alice's particle. So, it is natural to expect that the final state will be an entangled one since her particle is initially in a superposed state. If Alice's particle is in the state $|R\rangle_A$ the Bob's particle will be more deviated than when Alice's particle is in the state $|L\rangle_A$. We call this difference in the deviation of Bob's particle of δx. In this sense, when we trace out the degrees of freedom of Bob's particle, the state of Alice's particle will be in a mixed state if Bob releases his particle. But if Bob do not release his particle, the total system would never become entangled, and Alice's particle will be in a pure state. Therefore, according to the principle of complementarity of quantum mechanics described above in this paragraph, Bob's decision about whether or not to release his particle will affect Alice's measurements.

The paradoxical situation arises when $T_A, T_B < D$ (remember that we have $c = 1$). In this situation, Alice's and Bob's particles are casually disconnected throughout the process. So, according to the principle of causality, Bob's decision to release or not his particle could not affect Alice's measurements. In this sense, the principles of complementarity and causality would seem incompatible. To avoid this paradoxical situation, we need to consider the quantum nature of the field that mediates the interaction. We will first analyze the electric version of the experiment, and then we will show that the gravitational version solves the paradox in the same way. All the time, we will make only rough order of magnitude estimates to qualitatively understand the features of the problem, and we will also discard numerical factors of order unity when making these estimates.

4.1 Electric Interaction

To solve the paradox, we will use two important properties that quantum fields have. The first one is the vacuum fluctuations of the electric field. The magnitude of these fluctuations will be of order [3]

$$\Delta E = \sqrt{\langle [E(f)]^2 \rangle} \sim \frac{1}{R^2}, \tag{4.2}$$

when we average over a space-time region of dimension R. $E(f)$ is the electric field smeared with a smooth function f with support in a region of size R that is nearly constant in this region and normalized so that $\int f = 1$. See the appendix B for some intuition about the Eq. (4.2).

The influence of this fluctuation over a free non-relativistic particle will be governed by Newton's second law: $m\ddot{x} = qE$. Integrating this equation, we find that the displacement of the particle over the timescale R is

$$\Delta x \sim \frac{q}{m}, \tag{4.3}$$

that is independent of R. This shows that Bob's measurements do not have better precision than his particle charge-radius q_B/m_B. As a consequence, this implies a limited ability in the entanglement between his particle and Alice's. To create the entangled state and enable Bob to obtain information about Alice's particle, the following condition due to the vacuum fluctuation must be satisfied

$$\delta x > \frac{q_B}{m_B}. \tag{4.4}$$

We can estimate δx based on the configuration of Alice's particle as the difference in the deviations caused by each branch of the superposition

$$\begin{aligned}
\delta x &= |\Delta x_{|L\rangle_A} - \Delta x_{|R\rangle_A}| \\
&\approx \left| \frac{q_A q_B}{2m_B(D+d/2)^2} T_B^2 - \frac{q_A q_B}{2m_B(D-d/2)^2} T_B^2 \right| \\
&= \frac{q_A q_B}{2m_B} T_B^2 \left[\frac{1}{(D-d/2)^2} - \frac{1}{(D+d/2)^2} \right] \\
&= \frac{q_A q_B}{2m_B} T_B^2 \frac{1}{D^2} \left[(1-d/2D)^{-2} - (1+d/2D)^{-2} \right] \\
&\approx \frac{q_A q_B}{2m_B} T_B^2 \frac{1}{D^2} \left[1 + d/D - (1-d/D) \right] \\
&= \frac{q_B}{m_B} \frac{\mathcal{D}_A}{D^3} T_B^2,
\end{aligned} \tag{4.5}$$

where we have assumed that $d/D \ll 1$. Hence, effectively, δx is caused by a dipole moment $\mathcal{D}_A = q_A d$ of Alice's particle. Combining condition (4.4) and Eq. (4.5) we obtain

$$\frac{\mathcal{D}_A}{D^3} T_B^2 > 1. \tag{4.6}$$

This is the condition that must be satisfied in order for Bob to obtain information about Alice's particle and to entangle the system. If it is not satisfied, the system will remain unentangled, and Bob will not affect the measurements of Alice.

The second important effect of the quantum electric field is the quantized emission of radiation. Each branch of Alice's particle superposition corresponds to a charge-current source $j^{\mu}_{L(R)}$, which is assumed to generate the corresponding coherent states $\left|\alpha_{L(R)}\right\rangle$. In general, the overlap can be written as $\langle \alpha_R | \alpha_L \rangle = \langle 0 | \alpha_L - \alpha_R \rangle$, where $|\alpha_L - \alpha_R\rangle$ is the coherent state generated by the difference $j_L - j_R$, as shown in Ref. [3]. The latter corresponds to the effective dipole $\mathcal{D}(t)$. Classically, the energy flux radiated by a time-dependent dipole is $\sim (\ddot{\mathcal{D}}_A)^2$, so the total energy, $\mathcal{E}$, radiated when Alice "closes her dipole" will be given by

$$\mathcal{E} \sim \left(\frac{\mathcal{D}_A}{T_A^2} \right)^2 T_A. \tag{4.7}$$

In quantum mechanics, this energy will appear in the form of photons of frequency ν that is proportional to the particle acceleration $a = d/T_A^2$. Therefore, the number of photons N emitted by Alice's particle during the recombination will be of order

$$N = \frac{\mathcal{E}}{\nu} \sim \left(\frac{\mathcal{D}_A}{T_A^2}\right)^2 T_A \frac{T_A^2}{d} = \left(\frac{\mathcal{D}_A}{T_A}\right)^2 \frac{T_A}{d}. \tag{4.8}$$

The minimum number of emitted photons will occur when $d \sim T_A$ (of course $d < T_A$) because the acceleration would be enormous forcing the emission of the whole energy into few high-energetic photons. Considering this scenario, the number of emitted photons is

$$N \sim \left(\frac{\mathcal{D}_A}{T_A}\right)^2. \tag{4.9}$$

Therefore, Alice can only avoid the emission of radiation and keep her particle in a pure state if

$$\mathcal{D}_A < T_A. \tag{4.10}$$

If this condition is not satisfied, Alice's particle will be in a mixed state, regardless of whether Bob releases or not his particle.

Now we can analyze how these conditions affect the paradox. The first condition of the paradox is $T_A, T_B < D$ which guarantees that the particles are casually disconnected in all the process. But if condition (4.10) is satisfied, we have $\mathcal{D}_A < D$ and then condition (4.6) is not satisfied. The result is that if Alice makes her recombination slow enough to avoid the emission of radiation, the vacuum fluctuations will disable the entanglement of the system and then keep Alice's particle in a pure state, that is, Bob will not affect Alice's measurements. In this sense, complementarity and causality still hold.

We can see that the quantum properties of the quantized electric field solve the paradox. Without these properties, there will be a violation of either causality or complementarity. So, any interaction between the particles must be quantized since, if not, the paradox still holds. What we mean by "be quantized" is to consider all the quantum properties and degrees of freedom of the field that mediates the interactions. Next, we will see the gravitational version of the thought experiment and how a quantized gravitational field solves the paradox.

4.2 Gravitational Interaction

In the gravitational case, we set $q_A = q_B = 0$ and consider the quantum linearized gravitational field. Let us estimate the deviation in Bob's particle due to the vacuum fluctuation analogously to Eq. (4.3). In the absence of background space-time, the location of a particle is not a well defined concept. The best we can do is evaluate relative locations. Consider two bodies separated by a distance R. When averaged over a space-time region of dimension R, the magnitude of the vacuum fluctuations of the Riemann curvature tensor $\mathcal{R}$ is [3]

$$\Delta\mathcal{R} \sim \frac{l_P}{R^3}, \tag{4.11}$$

where $l_P \equiv (G\hbar/c^3)^{1/2} = \sqrt{G}$ is the Plank length. Integration of the geodesic deviation equation over time R then yields

$$\Delta x \sim l_P. \tag{4.12}$$

That's it, the precision in the localization of any object is smaller then the Planck length. Therefore,

$$\delta x > l_P, \tag{4.13}$$

in order to Bob obtain information about Alice's particle and entangle the system.

Now we estimate δx based on Alice's particle configuration. An important difference in the gravitational case compared to the electric case is that the dominant gravitational effect that operates on Bob's particle is the effective mass octopole $\mathcal{O}_A$. The dipole contribution is canceled due to the fact that the center of mass of the total system, Alice's particle and her laboratory (and everything that is attached to it, such as the Earth), is conserved. If Alice's particle goes $d/2$ to the right, the laboratory goes exactly the amount ϵ to the left to maintain the center of mass constant. The same is true if Alice's particle goes to the left. In both cases, the variables satisfy

$$m_A d - 2M\epsilon = 0, \tag{4.14}$$

where M is the laboratory mass. Also, the effective quadrupole contribution is null for this configuration, which can be shown with an explicit calculation. We note that this is not what the original paper Ref. [3] states, where they consider the quadrupole effective contribution. Let us calculate this deviation:

Consider δx_L and δx_R the deviations of the Bob's particle when the Alice's particle goes to the left and to the right, respectively. Then, similar to Eq. (4.5), we obtain

$$\begin{aligned} \delta x_L &\approx \frac{GT_B^2}{2}\left[\frac{m_A}{(D+d/2)^2}+\frac{M}{(D-\epsilon)^2}\right] \\ &= \frac{GT_B^2}{2D^2}\left[m_A(1+d/2D)^{-2}+M(1-\epsilon/D)^{-2}\right]; \end{aligned} \tag{4.15}$$

$$\begin{aligned} \delta x_R &\approx \frac{GT_B^2}{2}\left[\frac{m_A}{(D-d/2)^2}+\frac{M}{(D+\epsilon)^2}\right] \\ &= \frac{GT_B^2}{2D^2}\left[m_A(1-d/2D)^{-2}+M(1+\epsilon/D)^{-2}\right]. \end{aligned} \tag{4.16}$$

Now we expand δx_L and δx_R until the octopole contribution

$$\delta x_L \approx \frac{GT_B^2}{2D^2}\left[M+m_A+\frac{2M\epsilon-m_A d}{D}+\frac{3M\epsilon^2+\frac{3}{4}m_A d^2}{D^2}+\frac{4M\epsilon^3-\frac{1}{2}m_A d^3}{D^3}\right]$$

$$\delta x_R \approx \frac{GT_B^2}{2D^2}\left[M+m_A+\frac{-2M\epsilon+m_A d}{D}+\frac{3M\epsilon^2+\frac{3}{4}m_A d^2}{D^2}+\frac{-4M\epsilon^3+\frac{1}{2}m_A d^3}{D^3}\right]$$

Calculating $\delta x = |\delta x_R - \delta x_L|$ until the octopole approximation and using Eq. (4.14) we obtain

$$\begin{aligned} \delta x &\approx \frac{GT_B^2}{2D^2}\frac{m_A d^3-8M\epsilon^3}{D^3} \\ &= \frac{\mathcal{O}_A}{D^5}T_B^2. \end{aligned} \tag{4.17}$$

with

$$\mathcal{O}_A \equiv \frac{Gm_A d^3}{2}\left(1-\frac{m_A^2}{M^2}\right). \tag{4.18}$$

We can see that the effective dipole does not cause any deviation in the gravitational case, as well as the effective quadrupole.

Now using Plank units such as $G=1$, consequently, $l_P=1$, we obtain that the condition for Bob to obtain information about Alice's particle is

$$\frac{\mathcal{O}_A}{D^5} T_B^2 > 1. \tag{4.19}$$

As in the electric case, Alice's particle will radiate during the recombination. Classically, the energy flux radiated by a time dependent octopole is $\sim (\dddot{\mathcal{O}}_A)^2$, so the total energy, $\mathcal{E}$, radiated when Alice "closes her octopole" will be given by

$$\mathcal{E} \sim \left(\frac{\mathcal{O}_A}{T_A^4}\right)^2 T_A. \tag{4.20}$$

In quantum mechanics, this energy will appear in the form of gravitons of frequency ν that is proportional to the particle acceleration $a = d/T_A^2$. Therefore, the number of gravitons N emitted by Alice's particle during the recombination will be of order

$$N = \frac{\mathcal{E}}{\nu} \sim \left(\frac{\mathcal{O}_A}{T_A^3}\right)^2 \frac{T_A}{d}. \tag{4.21}$$

The minimum number of emitted gravitons will occur when $d \sim T_A$ (of course $d < T_A$) because the acceleration would be enormous forcing the emission of the whole energy into few high-energetic gravitons. Considering this scenario, the number of emitted gravitons is

$$N \sim \left(\frac{\mathcal{O}_A}{T_A^3}\right)^2. \tag{4.22}$$

Therefore, Alice can only avoid the emission of radiation and keep her particle in a pure state if

$$\mathcal{O}_A < T_A^3. \tag{4.23}$$

If this condition is not satisfied, Alice's particle will be in a mixed state, regardless of whether Bob releases or not his particle.

So, as well as in the electric case, if condition (4.23) is satisfied, Alice will not emit any radiation and will keep her particle in a pure state. So, since $T_A, T_B < D$, then $\mathcal{O}_A < D^3$. Hence, the condition (4.19) is not satisfied. Similar to the electric case, if Alice recombines her particle slowly enough to avoid the emission of radiation, the vacuum fluctuations will disable the entanglement between the particles, avoiding that Bob influences on Alice's measurements. In this token, the quantization of the gravitational field is necessary to avoid the paradox discussed in this chapter and keep causality and complementarity working together. Although our calculation differs from Ref. [3] in the gravitational case, the conclusion about the quantization of the gravitational field is still the same.

In this and the last chapter, we analyzed evidences for the quantization of the gravitational field, one related to the generation of entanglement between two interacting particles and the other related to a paradoxical situation. In the next chapters, we will investigate two proposed effects in the quantum gravity scale: one related to the difficulty of defining a causal order in a quantum space-time, and the other related to the self-interaction of quantum particles, which would explain the emergence of the classical world from the quantum paradigm.

Chapter 5

Indefinite Causal Order

Although quantum mechanics breaks our intuition about nature, at least the order of events should be fixed on it since the theory is defined over a classical space-time governed by the laws of general relativity. General relativity requires local classical variables to define the curvature of space-time, and this is a feature not shared with quantum mechanical systems. Therefore, it is natural to ask what will be the consequences over the space-time when we consider quantum matter-energy distributions. Answering this question is really difficult since it is equivalent to having a quantum gravity theory on hand. In this token, it is more interesting to investigate the low energy limit of both general relativity and quantum mechanics to find some interesting consequences. This has been done by Magdalena Zych, et.al [4] where the authors investigate a thought experiment, which we show next, that involves a quantum source of gravity. They found that the order between two events, that was previously defined, becomes indefinite.

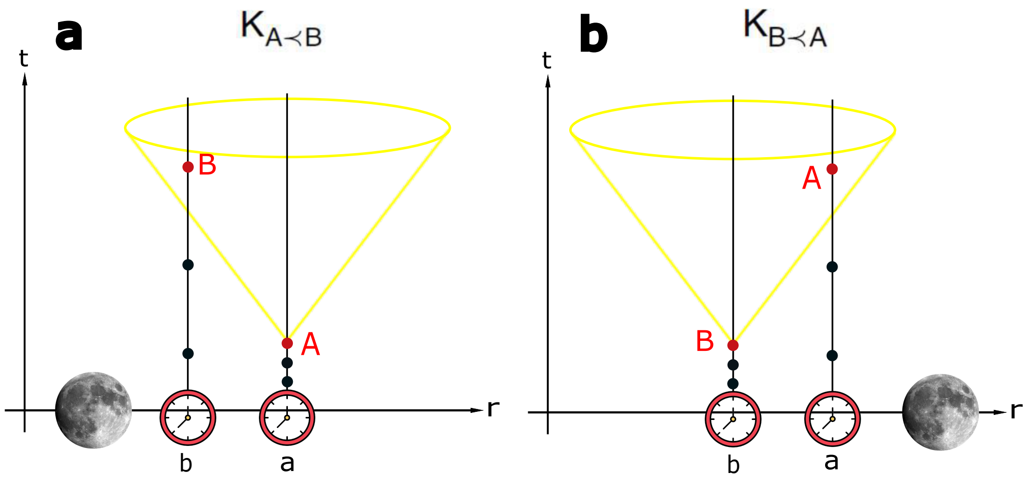

Figure 5.1: Thought experiment of indefinite causal orders. a and b are the agents carrying each one a clock. A and B are events in which agents a and b operate on the quantum auxiliary system, respectively. In figure **a** the source mass is on the left side of agent b and in figure **b** the source mass is on the right side of agent a. Figure based on Ref. [4].

The scheme of the experiment is shown in figure 5.1. Two agents, a and b, carry each one a clock that they assume to be initially synchronized such that $t_a = t_b = 0$ when $t = 0$, where t_a and t_b are the proper times of agents a and b, respectively, and t is the coordinate time. In figure 5.1(a) a point like mass is positioned on the left side of agent b. We call this configuration $K_{A \prec B}$. In figure 5.1(b) the source is on the right side of agent a. We call this configuration $K_{B \prec A}$. By $i \prec j$ we mean that event i is in the past light cone of event j.

We know from general relativity that the tick rate of clocks depends on the matter-energy distribution. So, for configuration $K_{A \prec B}$, it is always possible to find a time τ such that the event A, defined as the proper time of agent a marking $t_a = \tau$, will be in the past light cone of the event B, defined as the proper time of agent b marking $t_b = \tau$. To see this, we note that the structure of the space-time is determined by the metric tensor $g_{\mu\nu}$, in which we adopt the sign convention $(-,+,+,+)$. In isotropic coordinates, the metric components are

$$g_{00}(r) = -\left[1 + 2\frac{\Phi(r)}{c^2}\right]; \tag{5.1}$$

$$g_{rr}(r) = \left[1 + 2\frac{\Phi(r)}{c^2}\right]^{-1}, \tag{5.2}$$

where $\Phi(r) = -Gm/r$ is the gravitational potential and r is the spatial distance between the mass and the event where the metric is evaluated.

We consider that a and b remain at fixed coordinates r_a and r_b from the mass, respectively. An infinitesimal proper time element along a world line at a fixed distance r from the mass is given by

$$d\tau(r) = \sqrt{-g_{00}(r)}dt. \tag{5.3}$$

So, when the agent a clock marks $t_a = \tau$ the coordinate time is

$$T_A = \frac{\tau}{\sqrt{-g_{00}(r_a)}}. \tag{5.4}$$

A photon traveling in the radial direction from a point of coordinate r_a reaches r_b in the coordinate time interval

$$T = \frac{1}{c}\int_{r_a}^{r_b} \sqrt{-\frac{g_{rr}(r')}{g_{00}(r')}}dr'. \tag{5.5}$$

Hence, a photon emitted by agent a in $t_a = \tau$ reaches agent b in the coordinate time $T_B = T_A + T$. Then, the proper time t_b of the agent b is

$$t_b = \sqrt{-g_{00}(r_b)}T_B = \sqrt{-g_{00}(r_b)}\left(\frac{\tau}{\sqrt{-g_{00}(r_a)}} + T\right). \tag{5.6}$$

For

$$\tau > T\frac{\sqrt{-g_{00}(r_b)}}{1-\sqrt{\frac{g_{00}(r_b)}{g_{00}(r_a)}}} \equiv \tau^*, \tag{5.7}$$

we guarantee that $t_b < \tau$. This means that the agent b clock marks $t_b = \tau$ after the light sent by agent a reaches him. With this, we can be sure that the event A, defined by $t_a = \tau$, is in the past light cone of the event B, defined by $t_b = \tau$. In summary, τ^* is the proper time that agent a must wait in order to the event A to be in the past light cone of event B. An analogous analysis works for the configuration $K_{B\prec A}$.

Now, suppose that there is an auxiliary quantum system shared by the agents a and b that is initially in the state $|\psi_0\rangle^S$. Agents a and b are in charge of operating over the system in events A and B, respectively. These operations are represented by the unitary operators U_A for agent a in event A and by U_B for agent b in event B. If the configuration $K_{A\prec B}$ is built up, then the final quantum state is

$$|\psi_1\rangle^S = U_B U_A |\psi_0\rangle^S, \tag{5.8}$$

and if the configuration $K_{B\prec A}$ is built up, the final quantum state is

$$|\psi_2\rangle^S = U_A U_B |\psi_0\rangle^S. \tag{5.9}$$

Therefore, in general, one can carry out measurements on the auxiliary system to find out what is the temporal order of events in the space-time.

We now assume that these two distinguishable classical mass distributions, $K_{A\prec B}$ and $K_{B\prec A}$, can be associated with two orthogonal quantum states, $|K_{A\prec B}\rangle^M$ and $|K_{B\prec A}\rangle^M$, respectively, and also that the quantum superposition principle holds for all objects. With this, we can build the following state for the mass source

$$|K_+\rangle^M = \frac{1}{\sqrt{2}}\left(|K_{A\prec B}\rangle^M + |K_{B\prec A}\rangle^M\right). \tag{5.10}$$

Then, the initial total system state $|\Psi_0\rangle^{MS} = |K_+\rangle^M |\psi_0\rangle^S$ will evolve to the entangled state

$$|\Psi\rangle^{MS} = \frac{1}{\sqrt{2}}\left(|K_{A\prec B}\rangle^M |\psi_1\rangle^S + |K_{B\prec A}\rangle^M |\psi_2\rangle\right). \tag{5.11}$$

Tracing out the source mass degrees of freedom, we obtain the following mixed state for the auxiliary system

$$\rho^S = \frac{1}{2}\left(|\psi_1\rangle\langle\psi_1|^S + |\psi_2\rangle\langle\psi_2|^S\right). \tag{5.12}$$

In this situation, we see that the causal order of the events is indefinite since the probabilities resulting from Eq. (5.12) are, in general, different from the ones when the causal order is defined, Eq. (5.8) and Eq. (5.9). Eq. (5.12) can be seen as half of the times the causal structure comes from $K_{A\prec B}$ and the other half from $K_{B\prec A}$.

As an example for this thought experiment, let us use as the auxiliary system a spin $1/2$ particle that is initially in the spin eigenstate in the positive x direction, that is, $|\psi_0\rangle^S = |\rightarrow\rangle = (|\uparrow\rangle + |\downarrow\rangle)/\sqrt{2}$. As the unitary operations, we will use $U_A = e^{-i\alpha\hat{S}_z/\hbar}$ and $U_B = e^{-i\theta\hat{S}_x/\hbar}$, in which $\hat{S}_z$ and $\hat{S}_x$ are the spin operators in directions z and x, respectively. With this, the states become

$$\begin{aligned}
|\psi_1\rangle^S &= e^{-i\theta\hat{S}_x/\hbar}e^{-i\alpha\hat{S}_z/\hbar}|\rightarrow\rangle \\
&= \frac{1}{\sqrt{2}}e^{-i\theta\hat{S}_x/\hbar}\left(e^{-i\alpha/2}|\uparrow\rangle + e^{i\alpha/2}|\downarrow\rangle\right) \\
&= \frac{1}{2}\left[e^{-i\alpha/2}\left(e^{-i\theta/2}|\rightarrow\rangle + e^{i\theta/2}|\leftarrow\rangle\right) + e^{i\alpha/2}\left(e^{-i\theta/2}|\rightarrow\rangle - e^{i\theta/2}|\leftarrow\rangle\right)\right] \\
&= \frac{1}{2}\left[\left(e^{-i(\alpha+\theta)/2} + e^{i(\alpha-\theta)/2}\right)|\rightarrow\rangle + \left(e^{i(-\alpha+\theta)/2} - e^{i(\alpha+\theta)/2}\right)|\leftarrow\rangle\right]
\end{aligned} \tag{5.13}$$

$$\begin{aligned}
|\psi_2\rangle^S &= e^{-i\alpha\hat{S}_z/\hbar}e^{-i\theta\hat{S}_x/\hbar}|\rightarrow\rangle \\
&= \frac{1}{\sqrt{2}}e^{-i\theta/2}e^{-i\alpha\hat{S}_z/\hbar}|\rightarrow\rangle \\
&= \frac{1}{\sqrt{2}}\left(e^{-i(\alpha+\theta)/2}|\uparrow\rangle + e^{i(\alpha-\theta)/2}|\downarrow\rangle\right).
\end{aligned} \tag{5.14}$$

Now we can ask, what is the probability $P(\cdot, |\uparrow\rangle)$ of finding each final state (5.13), (5.14) and (5.12) in the spin eigenstate in the positive z direction? The answer is

$$\begin{aligned} P(|\psi_1\rangle^S, |\uparrow\rangle) &= |\langle\uparrow|\psi_1\rangle|^2 \\ &= \left|\frac{1}{2\sqrt{2}}\left(e^{-i(\alpha+\theta)/2} + e^{i(\alpha-\theta)/2} + e^{i(-\alpha+\theta)/2} - e^{i(\alpha+\theta)/2}\right)\right|^2 \\ &= \frac{1+\sin\alpha\sin\theta}{2} \end{aligned} \tag{5.15}$$

$$\begin{aligned} P(|\psi_2\rangle^S, |\uparrow\rangle) &= |\langle\uparrow|\psi_2\rangle|^2 \\ &= \frac{1}{2} \end{aligned} \tag{5.16}$$

$$\begin{aligned} P(\rho^S, |\uparrow\rangle) &= Tr\left(\rho^S |\uparrow\rangle\langle\uparrow|\right) \\ &= \langle\uparrow|\rho^S|\uparrow\rangle \\ &= \frac{P(|\psi_1\rangle^S, |\uparrow\rangle) + P(|\psi_2\rangle^S, |\uparrow\rangle)}{2} \\ &= \frac{2+\sin\alpha\sin\theta}{4}. \end{aligned} \tag{5.17}$$

Therefore, by measuring the spin in z direction of the auxiliary system, we can identify whether the causal order is defined or not.

We note that this experiment is hard to do in practice because, up to now, we can just create superpositions of small masses. In this sense, the time dilation is really small for typical laboratory experiments. For a mass $m = 10^{-14}$ Kg, $r_b = 0.01$ m, $r_a = 0.02$ m we have from Eq. (5.7)

$$\tau^* \sim 10^{29} \text{ s}, \tag{5.18}$$

which is bigger than the age of the universe by many orders of magnitude. (We recall that τ^* is the proper time that agent a must wait in order to the event A to be in the past light cone of the event B for the configuration $K_{A\prec B}$.)

The analysis of this chapter's thought experiment does not use any particular model of quantum gravity and takes into account only the very well-tested general relativity time dilation and quantum mechanics. In this sense, we can say that any complete theory of quantum gravity is likely to predict the indefinite causal order as a general property. In the next chapter, we will see another proposed effect in the quantum gravity scales that some authors expect to be responsible for the quantum-classical transition, but we will argue that this effect is not natural even though we cannot directly discard it.

Chapter 6

Self-Interaction

This chapter follows quite closely Ref. [11]. The transition from quantum to classical world remains not completely comprehended [12]. One of the proposals to solve this problem is the "quantum state reduction" mechanism presented by Penrose [5, 6]. In this mechanism, the quantum coherent superpositions are destroyed via a gravitational self-interaction. One may think of it as follows. Suppose a Schroedinger-cat state where a particle is set in a coherent superposition at two different locations:

$$|\psi\rangle_0 = \frac{1}{\sqrt{2}}\left(|\,^\bullet\,\rangle + |\,_\bullet\,\rangle\right). \tag{6.1}$$

Now, let us assume the *unorthodox* view that the two branches gravitationally interact with each other, driving the system to some sort of self-entanglement:

$$|\psi\rangle_T = \frac{1}{\sqrt{2}}\left(|\,^\bullet\,\rangle\,|\,_\circ\,\rangle + |\,_\bullet\,\rangle\,|\,^\circ\,\rangle\right). \tag{6.2}$$

with the transition time T being scaled by the gravitational interaction. This reflects Penrose's view that each chunk of the wave function would function as a sub-particle and "see" every other chunk as a distinct sub-particle. Then, by tracing out over the "immaterial" $\{|\,_\circ\,\rangle, |\,^\circ\,\rangle\}$, one would end up with the mixed state given by the density matrix

$$\hat{\rho}_T = \frac{1}{2}\,|\,^\bullet\,\rangle\,\langle\,^\bullet\,| + \frac{1}{2}\,|\,_\bullet\,\rangle\,\langle\,_\bullet\,| \tag{6.3}$$

In Penrose's view (if confirmed), this would be a factual self-reduction responsible for the emergence of classicality. In this way, Penrose raises the possibility that the Schroedinger equation for a particle is replaced by the so-called Schroedinger-Newton equation:

$$i\hbar\frac{\partial}{\partial t}\Psi(\vec{r},t) = -\frac{\hbar^2}{2m}\nabla^2\Psi(\vec{r},t) + [V(\vec{r},t) + U_g(\vec{r},t)]\Psi(\vec{r},t), \tag{6.4}$$

where m is the particle mass, $V(\vec{r},t)$ and

$$U_g(\vec{r},t) \equiv -Gm^2 \int d^3\vec{r}\,' \frac{|\Psi(\vec{r}\,',t)|^2}{|\vec{r}-\vec{r}\,'|} \tag{6.5}$$

are the external and self-interacting potentials, respectively, and G is the gravitational constant. According to the Schroedinger-Newton equation, $m|\Psi(\vec{r},t)|^2$ plays the role of *real* matter density in such a way that different parts of the wave function interact with each other. The results given by Eq. (6.4) would be indistinguishable from the ones delivered by the Schroedinger equation for typical laboratory experiments since the gravitational interaction is very weak, even though the Schroedinger-Newton equation is nonlinear. Continuous efforts have been made to solve it and find its properties (see, *e.g.*, Refs. [13–20] and references therein).

We note that, at low energies, the electric and gravitational potentials only differ from each other with respect to the coupling constant. Both have successfully been used in the Schroedinger equation to describe from the hydrogen atom [21] to the interference of free-falling neutrons [7] (as discussed in chapter 2). In this sense, it would be "natural" to expect that if particles self-interact via their gravitational potential, they would also self-interact via their electric potential [22]. Therefore, it seems worth revisiting the Schroedinger-Newton equation for the hydrogen atom with the inclusion of the self-interacting electric potential

$$U_e(\vec{r},t) \equiv k_e e^2 \int d^3\vec{r}\,' \frac{|\Psi(\vec{r}\,',t)|^2}{|\vec{r}-\vec{r}\,'|}, \tag{6.6}$$

where k_e is the Coulomb constant and e is the fundamental charge.

In Sec. 6.1, we introduce the Schroedinger-Newton equation for the hydrogen atom modified by the addition of the electric self-interacting potential. In Sec. 6.2, we compare its results with the experimental data. In Sec. 6.3, we analyze the difficulties of testing the Schroedinger-Newton equation using the hydrogen atom as a guide. In Sec. 6.4, we discuss the results and present our conclusions.

6.1 The hydrogen atom under self-interaction

The Schroedinger equation for the hydrogen atom is

$$i\hbar\frac{\partial}{\partial t}\Psi(\vec{r},t) = -\frac{\hbar^2}{2\mu}\nabla^2\Psi(\vec{r},t) + V_T(r)\Psi(\vec{r},t), \tag{6.7}$$

where $\mu \equiv m_e m_p/(m_e + m_p)$, m_e and m_p are the electron and proton masses, respectively, and

$$V_T(r) \equiv V_g(r) + V_e(r) \tag{6.8}$$

with $V_g(r) = -Gm_e m_p/r$ and $V_e(r) = -k_e e^2/r$. (We recall that the corresponding spectroscopy is completely determined by the spherically symmetric eigenfunctions.)

Our main goal is to compare the results given by Eq. (6.7) with the ones provided by the Schroedinger-Newton equation modified by the addition of the electric self-interacting potential:

$$i\hbar\frac{\partial}{\partial t}\Psi(\vec{r},t) = -\frac{\hbar^2}{2\mu}\nabla^2\Psi(\vec{r},t) + \left[V_T(r) + U_T(\vec{r},t)\right]\Psi(\vec{r},t), \tag{6.9}$$

where

$$U_T(\vec{r},t) \equiv U_g(\vec{r},t) + U_e(\vec{r},t). \tag{6.10}$$

Obviously, we could have already discarded the gravitational term in this step since it is smaller than the electric one by many orders of magnitude, but we will hold it for further discussions.

Eq. (6.9) with $V_g(r) = U_g(\vec{r},t) = 0$ has been investigated in Ref. [23] but finding no bound states for the hydrogen atom, in disagreement with our results and with the one presented in Refs. [24, 25] which used the Dirac equation with electric self-interaction to find out the hydrogen atom eigenstates and eigenenergies. Surprisingly, the results obtained by Refs. [24, 25] show that self-interaction plays a more significant role (by a factor around 5 in the eigenenergies) in the relativistic case than in the nonrelativistic one when compared with our results.

Nonetheless, Refs. [23–25] consider electric self-interaction in the hydrogen atom under a quite different mindset with respect to ours (that is close to the initial ideas of Schroedinger in the beginnings of quantum mechanics [17].); quoting Biguaa and Kassandrov:

(A) “From the point of view of quantum ideas, the electron in the

> hydrogen atom represents a spatially distributed system, according to the probability density $\Psi^\dagger\Psi$, whose elements interact both with the field of the nucleus and between each other."

In order to reconcile their results with experimental data, they invoke a sort of finite "classical renormalization" procedure. As expected, this is not enough to recover consistency with experiments. We shall stress that standard quantum mechanics does not assert (A). Instead, it states that $|\Psi(\vec{r},t)|^2$ is the probability density of finding the electron *when* its position is measured by some apparatus. Furthermore, statements that radioactive corrections of quantum field theory endorse the self-interaction of single particles are unfounded. Here we take quantum mechanics at its face value and compare its results with the ones delivered by Eq. (6.9).

Let us look for stationary spherically symmetric solutions

$$\Psi(\vec{r},t) = \psi(r)e^{-iEt/\hbar}$$

for Eq. (6.9). In this case, Eq. (6.9) can be cast as

$$-\frac{\hbar^2}{2\mu}\frac{1}{r}\frac{d^2}{dr^2}\left[r\psi(r)\right] + \left[V_T(r) + W_T(r)\right]\psi(r) = E\psi(r), \tag{6.11}$$

where

$$W_T(r) = 4\pi\alpha\left(\frac{1}{r}\int_0^r dr' r'^2|\psi(r')|^2 + \int_r^\infty dr' r'|\psi(r')|^2\right) \tag{6.12}$$

and $\alpha \equiv -Gm_em_p + k_ee^2$. In order to write it, we have used (see, *e.g.*, Ref. [26])

$$\frac{1}{|\vec{r}-\vec{r}\,'|} = 4\pi\sum_{l=0}^{\infty}\sum_{m=-l}^{l}\frac{1}{2l+1}\frac{r_<^l}{r_>^{l+1}}Y^*_{lm}(\theta',\phi')Y_{lm}(\theta,\phi),$$

where $r_< \equiv \min(r,r')$, $r_> \equiv \max(r,r')$, and $Y_{lm}(\theta,\phi)$ are the spherical harmonics. Equation (6.11) can be simplified by defining $\phi(r) \equiv r\psi(r)$, in which case it becomes

$$-\frac{\hbar^2}{2\mu}\frac{d^2}{dr^2}\phi(r) + \left[V_T(r) + W_T(r)\right]\phi(r) = E\phi(r), \tag{6.13}$$

where

$$W_T(r) = 4\pi\alpha\left(\frac{1}{r}\int_0^r dr'|\phi(r')|^2 + \int_r^\infty dr'\frac{|\phi(r')|^2}{r'}\right) \tag{6.14}$$

and we recall that the probability of finding the electron in a spherical shell with inner and outer radius r_i and r_o, respectively, is

$$P(r_i, r_o) = \int_{r_i}^{r_o} dr F(r) \quad \text{with} \quad F(r) \equiv 4\pi|\phi(r)|^2 = 4\pi r^2 |\psi(r)|^2. \tag{6.15}$$

Although the quest for stationary spherically symmetric solutions simplifies the problem, Eq. (6.13) is still nontrivial. Therefore, we solve it numerically by adapting the code of Ref. [14].

6.2 Numerical procedure and results

The code used to numerically solve Eq. (6.13) is an adaptation of the one developed in Ref. [14], where the external potential $V_T(r)$ was included and $U_g(\vec{r}, t)$ was replaced by $U_T(\vec{r}, t)$. (Furthermore, the convergence procedure was somewhat improved.) Briefly speaking, we depart from a test function, use it to calculate $W_T(r)$ in Eq. (6.14), and replace the result in Eq. (6.13) to numerically evaluate the eigenfunctions $\phi(r)$ and corresponding eigenvalues E. Then, depending on the eigenvalue E we are interested in, we select the associated eigenfunction $\phi(r)$, using it in the next round as a new test function to recalculate $W_T(r)$ and so on. The process is repeated until the eigenenergy E of interest converges. The full code is presented in Appendix C.

As a consistency check, we have verified that the obtained eigenfunctions and corresponding eigenenergies do satisfy Eq. (6.13). Moreover, Table 6.1 shows that the eigenenergies calculated by our program without the self-interacting term are in agreement with experimental data in good approximation. By "Level n", $n = 1, 2, 3, \ldots$ in Tables 6.1, 6.2 and 6.3, we implicitly mean level $n\, s^{1/2}$, since the solutions are spherically symmetric.

Level	Numerical (eV)	Experimental (eV)
1	-13.593	-13.598
2	-3.3993	-3.3996
3	-1.5109	-1.5109

Table 6.1: Comparison of the output of the three smallest eigenenergies given by Eq. (6.7) calculated with our program without the self-interacting term against the experimental data (restricted to 5 significant figures).

Now, we are in position to use our program to find the spectroscopy of the hydrogen atom as ruled by the modified Schroedinger-Newton equation (6.13). Of course, any deviation from the experiment must be addressed to the electric self-interaction since $k_e e^2$ is much larger than $Gm_e m_p$. In Table 6.2, we exhibit the three smallest eigenenergies for the hydrogen atom obtained solving numerically Eq. (6.13) against the experimental data. It is clear that the ionization energy is at odds with the experiments. Moreover, the whole hydrogen atom spectroscopy is ruined, as can be seen in Table 6.3.

Level	Numerical (eV)	Experimental (eV)
1	-1.2561	-13.598
2	-0.21601	-3.3996
3	-0.074618	-1.5109

Table 6.2: Comparison of the three smallest eigenenergies obtained numerically through Eq. (6.13) against experimental data (restricted to 5 significant figures).

Transition	Numerical (eV)	Experimental (eV)
$3 \to 2$	-0.14139	-1.8887
$2 \to 1$	-1.0400	-10.199
$3 \to 1$	-1.1814	-12.087

Table 6.3: Comparison of the energy gap for three transitions obtained numerically through Eq. (6.13) against experimental data (restricted to 5 significant figures). By "$n_i \to n_f$," we mean the electronic transition from the n_i-th to the n_f-th energy eigenstate.

Table 6.2 also shows for us that the energy levels do not follow the usual Bohr relationship $E_n = E_1/n^2$ as expected since the self-interacting term should screen the proton charge differently depending on the quantum state.

In Figs. 6.1, 6.2, and 6.3, we compare the radial probability densities $F(r)$ [see Eq. (6.15)] associated with the eigenfunctions of the three smallest eigenenergies given by the Schroedinger equation (6.7) and modified Schroedinger equation (6.13). We see that the electric self-interaction pushes the probability density farther from the nucleus, in agreement with the larger eigenenergies found for the corresponding eigenfunctions displayed in Table 6.2.

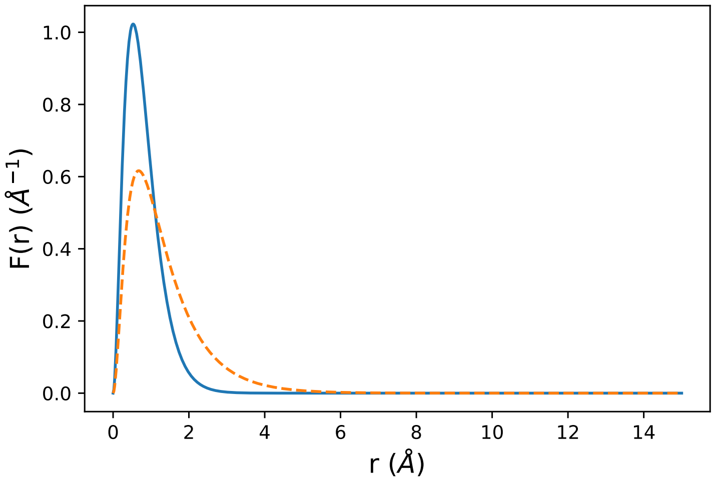

Figure 6.1: Plot of the radial probability density $F(r)$ for the ground state obtained from Eq. (6.7) (solid line) and Eq. (6.13) (dashed line).

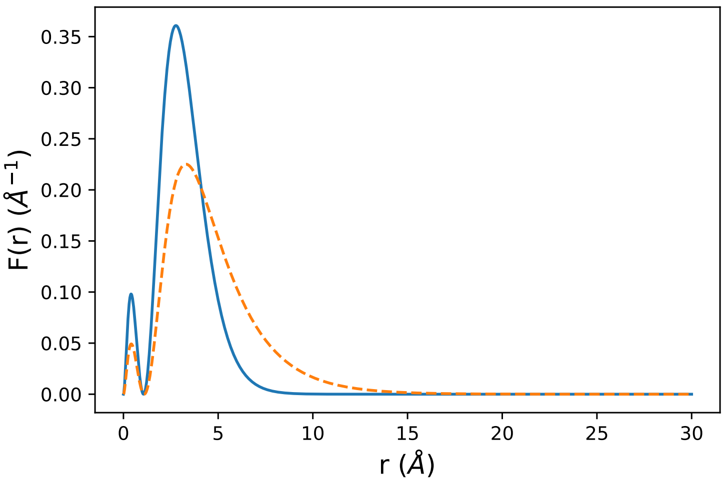

Figure 6.2: Plot of the radial probability density $F(r)$ for the first excited state obtained from Eq. (6.7) (solid line) and Eq. (6.13) (dashed line).

The previous results rule out the electric self-interaction and may be understood as disfavoring the gravitational self-interaction (based on the similarity of the Newton and Coulomb potentials), but they do not rule it out at all. To test the gravitational version, we must analyze situations where the gravitational self-interaction turns out to be dominant. In the next section, we use the hydrogen atom to gain insights into how challenging this can be.

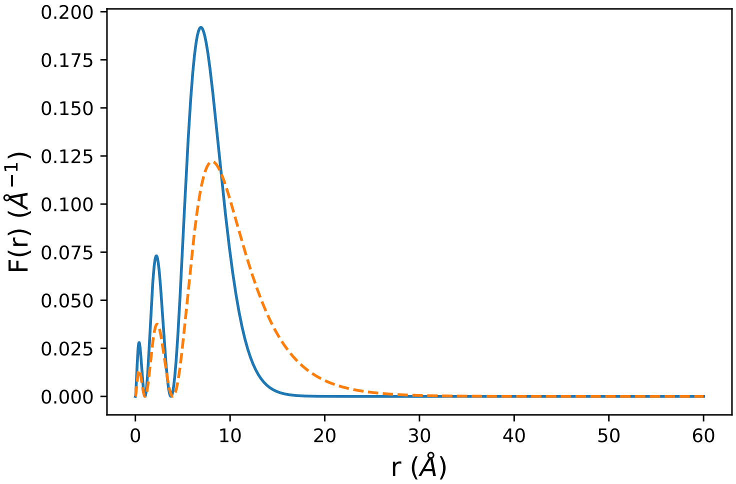

Figure 6.3: Plot of the radial probability density $F(r)$ for the second excited state obtained from Eq. (6.7) (solid line) and Eq. (6.13) (dashed line).

6.3 On gravitational self-interaction

Let us start replacing Eq. (6.9) by

$$i\hbar\frac{\partial}{\partial t}\Psi(\vec{r},t) = -\frac{\hbar^2}{2\mu}\nabla^2\Psi(\vec{r},t) + \left[V_T(r) + U_g(\vec{r},t)\right]\Psi(\vec{r},t), \tag{6.16}$$

where the electric self-interaction was promptly discarded in the face of the previous results. From Sec. 6.2 we see that, effectively, the energy levels are affected as the term $U_e(\vec{r},t)$ screens the proton charge by some factor of the electron charge that depends on the state. In the case of the ground state, replacing the proton charge by $q_P = e \to 0.3\,e$, where it means that the screening is around 0.3 of the electron charge divided by the proton charge, the ionization energy from Eq. (6.7)

$$E_1 = -\frac{\mu}{2\hbar^2}(k_e e^2 + G m_p m_e)^2 \tag{6.17}$$

would change about -13.598 eV $\to$ -1.2561 eV (see Table 6.2). In this sense, we expect that the presence of $U_g(\vec{r},t)$ would effectively increase the mass of the proton by no more than a factor of the electron mass divided by the proton mass, as in the electric case. The ionization energy, as ruled by Eq. (6.16), can be estimated to be of the order

$$E_1' \sim -\frac{\mu}{2\hbar^2}(k_e e^2 + G m_p' m_e)^2, \tag{6.18}$$

with $m'_p \equiv \gamma m_p$, where γ is the correction factor due to the gravitational self-interaction term $U_g(\vec{r}, t)$. Nevertheless, the correction term γ will be ~ 1 since $m_e/m_p \ll 1$. So, to measure some deviation in the energy levels of the hydrogen, we would have the electron and the proton with masses such that the gravitational interaction between them turns out to be of the same order of magnitude as the electric one.

Consider the ionization energy of the usual hydrogen atom

$$\epsilon = -\frac{\mu}{2\hbar^2}(k_e e^2)^2. \tag{6.19}$$

The relative difference R between the ionization energies (6.17) and (6.19) is

$$R \equiv \frac{E_1 - \epsilon}{\epsilon} \sim 2\left(\frac{G m_p m_e}{k_e e^2}\right), \tag{6.20}$$

which gives us a scale of how difficult is to measure some gravitational effect in the hydrogen atom. With the usual values of the masses and charges of the electron and the proton, R corresponds to 1 part in 10^{39} parts. Of course, measure this difference experimentally is really difficult, but another intrinsic problem arises when we consider the precision in the fundamental constants of the problem. All of them must to have, at least, the same precision of the observable to be measured. As an example, we recall that the value recommended by CODATA for the Newtonian constant has only six significant digits: $G = 6.674\,30(15)\ 10^{-11}\ \mathrm{m}^{-3}\ \mathrm{kg}^{-1}\mathrm{s}^{-2}$ [27].

When $R \sim 1$, the electric and gravitational interactions have the same order of magnitude, and we expect that it would be possible to measure some gravitational influence in the hydrogen atom. For this purpose, we should have $m_e m_p \sim (M_P/10)^2$, where $M_P \equiv \sqrt{\hbar c/G} \approx 2 \times 10^{-5}$g is the Planck mass.

Eventually, any attempt to test the gravitational self-interaction will face the quantum gravity regime by combining coherently a large number of particles to act as a single quantum particle with mass $m \lesssim M_P$. However, it is unclear in what sense Schroedinger and other quantum wave equations would be approximately valid close to the Planck scale. Relativistic,

$$\Box\phi + \lambda^{-2}\phi = 0, \quad i\,\partial\!\!\!/\,\psi - \lambda^{-1}\psi = 0, \tag{6.21}$$

and nonrelativistic,

$$i\partial_t\phi + (c\lambda/2)\nabla^2\phi = 0, \tag{6.22}$$

wave equations are solely characterized by the (reduced) Compton wavelength $\lambda\!\!\!^{-} = \hbar/(mc)$. The closer m approaches M_P, the closer $\lambda\!\!\!^{-}$ approaches the Planck length:

$$\lambda\!\!\!^{-} \sim L_P \equiv \sqrt{\hbar G/c^3} \approx 10^{-35}\text{m}.$$

Hence, in order to write Eqs. (6.21)-(6.22) for Planck-mass particles, one must presuppose that classical spacetimes are reliable at such distance scales, which does not seem to be the case.

In relativistic space-times, spatial distances can be solely measured using bona-fide classical clocks. By *bona-fide classical clocks* we mean "pointike" apparatuses that ascribe the same real number (time interval) to any given arbitrarily-close-causally-connected pair of events they visit regardless of the state of motion and past history (as defined by relativity). Thus, in order to measure $\lambda\!\!\!^{-} \sim L_P$, there must exist bona-fide clocks with size $L_{\text{clock}} < L_P$ and accuracy

$$\delta t \lesssim T_P \equiv \sqrt{\hbar G/c^5},$$

in which case one would end up with a ruler with accuracy $\delta l \equiv c\delta t < \lambda\!\!\!^{-}$. On the other hand, quantum mechanics teaches us that clocks with such accuracy would involve Planck-scale energies: $E_{\text{clock}} \gtrsim \hbar/\delta t$ [28], leading to a clock mass

$$M_{\text{clock}} \gtrsim M_P.$$

It happens, however, that according to Thorne's hoop conjecture, such clocks should collapse into black holes since they would be small enough to fit in spheres, with tipical size L_{clock}, into the corresponding Schwarzschild radii R^S_{clock} (see, *e.g.*, Ref. [29]):

$$L_{\text{clock}} < L_P < 2GM_P/c^2 \lesssim 2GM_{\text{clock}}/c^2 \equiv R^S_{\text{clock}}.$$

Not only are black holes not supposed to function as clocks, but such apparatuses would disturb the background space-time, raising doubts on the validity of Eq. (6.4) written for the Galilean spacetime.

As a result, any proposal to assess the Schroedinger-Newton equation must approach the Planck scale but still not cross the line where bona-fide clocks are absent and Galilean spacetime cannot be assumed. In Ref. [18], it is suggested the employment of Paul (ionic) traps to confine osmium disks of 10^{-9} g subjected to a harmonic potential. Traces of gravitational self-interaction would be fingerprinted in the corresponding energy spectrum. For this purpose, it would be necessary to reach temperatures of a few milikelvin. This has already been achieved in Paul traps for masses of 10^{-14} g [30–32] but not for ones of 10^{-9} g yet. (We address to Ref. [33] for an actual review about particle trapping technology.)

6.4 Overview

The Schroedinger-Newton equation was proposed as an attempt to explain the quantum to classical transition in which particles decohere due to gravitational self-interaction. We used the analogous electric self-interaction in the hydrogen atom to probe how promising this equation is. We were able to do so since, in a low-energy regime, these two interactions are formally the same (only differing due to the universality of the coupling constant in the gravitational case). In this scenario, we were able to find out the energies of the spherically symmetric eigenstates. Comparing the outputs with experimental data, we can rule out the possibility that quantum particles self-interact electrically. Although our results only refer to the electromagnetic interaction, they may be seen as disfavoring the Schroedinger-Newton equation in the sense that if a single electron would "perceive" itself through its mass distribution, it would be "natural" to expect that the same would be true concerning its charge distribution. In the end, we assessed some intrinsic difficulties in testing the Schroedinger-Newton equation and saw that the current technology has not reached the necessary level to test it yet.

Chapter 7

Conclusion

Due to technical and conceptual issues, the quantization of the gravitational field has not yet been achieved, raising doubts about its classical or quantum nature. This work has the goal of showing evidences of the quantization of the gravitational field and probing some proposed general effects on the quantum gravity scale. In chapter 2 we showed the first experimental evidence that gravity indeed affects quantum systems. Of course, this is the first step toward the full theory. In chapters 3 and 4 we presented two pieces of evidence in favor of the gravitational field quantization. The first one is based on a theorem of quantum information that states that any interaction between two particles that is mediated by a field and creates an entangled state implies that this field has a quantum nature. The second one is based on a paradoxical situation involving two interacting particles. If the field that mediates the interaction is not quantized (in the sense that the corresponding vacuum does not fluctuates and there is no emission of radiation), it is possible to violate the causality principle, one of the most tested principles of physics. With these two arguments, we can conclude that gravity indeed has a quantum nature. Next, we probed the two proposed effects on the quantum gravity scale. The first one was presented in chapter 5. The authors combined the time dilation of clocks and standard quantum mechanics to show that the causal order of the events, before defined, turns out to be undefined when the matter-energy distribution has a quantum nature. The second one was presented in chapter 6 where a proposed gravitational self-interaction that would explain the emergence of the classical world from quantum mechanics was analyzed. We used the hydrogen atom as a guide to investigate an electric version of this self-interaction and concluded that this interaction is at odds with the experiments. Since, at low energies, the gravitational and electric interactions have the same nature, this analysis disfavors the gravitational self-interaction, although it does not discard it directly.

Appendix A

Here we present what is a LOCC operation, a concept used in chapter 3. However, we need other definitions before it. First, we define what is a quantum general measurement. Quantum general measurements are described by a collection $\{M_i\}$ of measurement operators that act on the Hilbert space of the system that is being measured. The index i refers to the possible outcomes that may be measured. If the system is the in the pure state $|\psi\rangle$ immediately before the measurement, the probability that the result i occurs is

$$p(i, |\psi\rangle) = \langle\psi| M_i^\dagger M_i |\psi\rangle . \tag{A.1}$$

For a mixed state ρ the probability is

$$p(i, \rho) = Tr(M_i \rho M_i^\dagger) = Tr(M_i^\dagger M_i \rho). \tag{A.2}$$

To have the probabilities sum to one, $\{M_i\}$ must satisfy

$$\sum_i M_i^\dagger M_i = \mathbb{1}, \tag{A.3}$$

where $\mathbb{1}$ is the identity operator. The state after the measurement outcome i will be, for a pure state

$$|\psi\rangle \rightarrow \frac{M_i |\psi\rangle}{\sqrt{\langle\psi| M_i^\dagger M_i |\psi\rangle}}, \tag{A.4}$$

and for a mixed state

$$\rho \rightarrow \frac{M_i \rho M_i^\dagger}{Tr(M_i \rho M_i^\dagger)}. \tag{A.5}$$

We note that the well-known projective measurements are a particular case of the general measurement. The former are hermitian operators and satisfy $M_i M_j = \delta_{ij} M_i$. The general measurements have other interesting applications in quantum information. Let us show one of them.

First, we define what is a POVM (Positive Operator-Valued Measure). In some cases, the quantum system after the measurements is not of our interest (the particle was absorbed, for example). Since the probabilities above only depend on the product $M_i^\dagger M_i$ in both pure and mixed state cases, we define the POVM collection of positive operators $\{E_i\}$ such that $E_i \equiv M_i^\dagger M_i$ and $\sum_i E_i = \mathbb{1}$. So the probabilities will be given by

$$p(i, |\psi\rangle) = \langle\psi| E_i |\psi\rangle \,, \tag{A.6}$$

for pure states, and

$$p(i, \rho) = Tr(E_i \rho), \tag{A.7}$$

for mixed states.

Now suppose that an experimentalist, Alice, sends to another one, Bob, a quantum state among two different possibilities, $|\psi_1\rangle$ and $|\psi_2\rangle$. Bob's task is to identify the quantum state he received. If the states are not orthogonal, it is impossible to distinguish these two states with 100% accuracy. Nevertheless, in order to have some degree of precision, Bob can define the following POVM:

$$\begin{aligned} E_1 &= \frac{S_2}{k}; && \text{(A.8)} \\ E_2 &= \frac{S_1}{k}; && \text{(A.9)} \\ E_3 &= \mathbb{1} - E_1 - E_2, && \text{(A.10)} \end{aligned}$$

where $S_i \equiv \mathbb{1} - |\psi_i\rangle\langle\psi_i|$ and $k = 1 + |\langle\psi_1|\psi_2\rangle|$. Now, Bob uses this POVM to measure the state he receives from Alice. If he obtains the output that corresponds to E_1, he can be sure that he has $|\psi_1\rangle$ on hand since

$$\begin{aligned} p(1, |\psi_1\rangle) &= \langle\psi_1| E_1 |\psi_1\rangle = \frac{1}{k}; && \text{(A.11)} \\ p(1, |\psi_2\rangle) &= \langle\psi_2| E_1 |\psi_2\rangle = 0. && \text{(A.12)} \end{aligned}$$

If he obtains the output that corresponds to E_2, he can be sure that he has $|\psi_2\rangle$ on hand since

$$\begin{aligned} p(2, |\psi_1\rangle) &= \langle\psi_1| E_2 |\psi_1\rangle = 0; && \text{(A.13)} \\ p(2, |\psi_2\rangle) &= \langle\psi_2| E_2 |\psi_2\rangle = \frac{1}{k}. && \text{(A.14)} \end{aligned}$$

but if he obtains the output that corresponds to E_3, he cannot conclude anything since

$$p(3, |\psi_1\rangle) = \langle\psi_1| E_3 |\psi_1\rangle = 1 - \frac{1}{k}; \quad (A.15)$$

$$p(3, |\psi_2\rangle) = \langle\psi_2| E_3 |\psi_2\rangle = 1 - \frac{1}{k}. \quad (A.16)$$

Now we define a quantum map. They are positive linear maps acting on density matrices, leading to density matrices: $\rho \to \Lambda(\rho)$. We call a quantum map to be completely positive if $\Lambda(\rho) \otimes \mathbb{1}$ acting on $\mathcal{M}(\mathcal{H}) \otimes \mathcal{M}(\mathcal{H}')$ is positive for all $d \equiv dim(\mathcal{H}')$, where $\mathcal{M}(\mathcal{H})$ and $\mathcal{M}(\mathcal{H}')$ are the matrix set on the Hilbert spaces $\mathcal{H}$ and $\mathcal{H}'$, respectively. When a map is completely positive and preserves the trace, we call it a CPTP (completely positive trace-preserving) map.

Now we define a LOCC map. Suppose that two experimentalists, Alice and Bob, each with a quantum system on hands, have the following prescription: Alice uses a general measurement M_i to measure her system. Depending on the output i, she applies the CPTP map Λ_i^A on her system. Then, she uses a classical channel to communicate her output to Bob, who applies the CPTP map Λ_i^B on his system corresponding to Alice's output i. We can describe this situation with the operator

$$\mathcal{L}^{A\to B}(\rho_{AB}) \equiv \left(\sum_i \Lambda_i^A \otimes \Lambda_i^B\right) \rho_{AB}. \quad (A.17)$$

Analogously, we can also define the LOCC map where Bob makes the measurement and communicates it to Alice. Also, one can compose many sequences of the two types of LOCC maps to create a more general one. We note that if the initial state ρ_{AB} is unentangled, the final state after the LOCC map will also be unentangled.

Appendix B

Here we present some intuition about the Eq. (4.2):

$$\Delta E = \sqrt{\langle [E(f)]^2 \rangle} \sim \frac{1}{R^2}. \tag{B.1}$$

First, we show that, typically, the standard deviation of some set of random variables is inversely proportional to the number of variables. To achieve this goal, we used the following protocol:

Define k sets of N random variables between 0 and 1 uniformly distributed

$$\begin{gathered} \{x_1^{(1)}, x_2^{(1)}, ..., x_N^{(1)}\}^{(1)}; \\ \{x_1^{(2)}, x_2^{(2)}, ..., x_N^{(2)}\}^{(2)}; \\ \vdots \\ \{x_1^{(k)}, x_2^{(k)}, ..., x_N^{(k)}\}^{(k)}. \end{gathered}$$

Then, calculate the standard deviation

$$\sigma_N^{(i)} = \sqrt{\frac{1}{N}\sum_{j=1}^{N}(x_j^{(i)} - \langle x^{(i)} \rangle)^2} \tag{B.2}$$

for each one of the k sets, where $\langle x^{(i)} \rangle$ is the averege value of the set i. With this, we obtain a set of standard deviations

$$\{\sigma_N^{(1)}, \sigma_N^{(2)}, ..., \sigma_N^{(k)}\},$$

where each element is the standard deviation of the set i of random variables.

Now we calculate the mean value of the set of standard deviations, obtaining a mean standard deviation $\langle \sigma_N \rangle$ that depends only on N. By changing the number N in the range $[2, d]$ we can see how $\langle \sigma_N \rangle$, in general, depends on N.

Figure B.1 shows a typical plot of $\langle \sigma_N \rangle \times N$. We can see clearly that the standard deviation is inversely proportional to the size of the set of random numbers. This gives us some intuition about the behavior of R in Eq. (4.2).

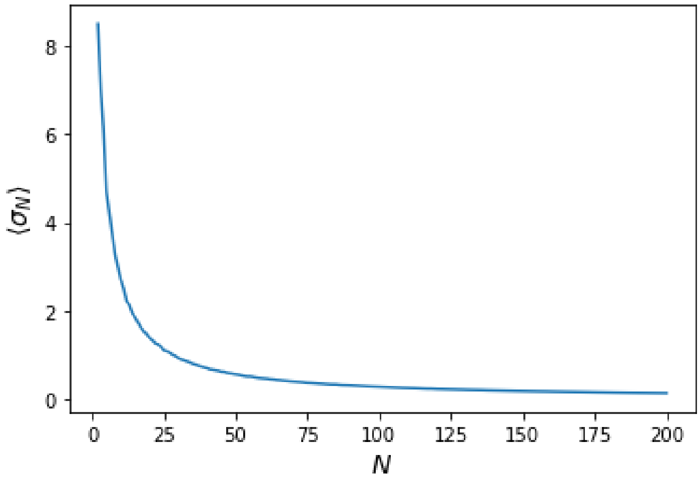

Figure B.1: Typical plot of $\langle\sigma_N\rangle \times N$

Below we present the python code used to make these calculations and to plot the graphic

```
import random
import numpy as np
import matplotlib.pyplot as plt



k=100

N=2

d=200-1

desviosmediosN = np.zeros(d)
desviomedio = np.zeros(k)

media = 0
media2 = 0

for m in range(d):
```

```
    x = np.zeros(N)
    y = np.zeros(k)
    for j in range(k):
        media = 0
        media2 = 0
        for i in range(N):
            x[i] = random.random()
        for p in range(N):
            media += x[p]/N
            media2 += (x[p]**2)/N
        y[j] = (media2-(media)**2)**(1/2)
    for c in range(k):
        desviosmediosN[m] += y[c]/N
    N += 1

Nx = np.linspace(2,2+d-1,d)

plt.plot(Nx,desviosmediosN)
```

Now we try to justify the power $1/R^2$ in Eq. (4.2). To this goal, we make a dimensional analysis. Let us look at the dimension of the Casimir effect. The volumetric energy density is

$$\rho \sim \frac{\hbar c}{d^4}, \tag{B.3}$$

where d is the distance between the plates. Classically, the electric field volumetric energy density is $\rho \sim E^2$, where E is the magnitude of the electric field. Hence the electric field will be (in unities where $c = \hbar = 1$)

$$E \sim \frac{1}{d^2}, \tag{B.4}$$

in analogy with Eq. (4.2).

Of course, the calculations in this appendix are not a proof of the Eq. (4.2), but can be seen as a justification of the order of magnitude of the effect.

Appendix C

Here we present the python code used in chapter 6 to solve the Eq. (6.13) which is an adaptation of the one developed in Ref. [14]:

```
import numpy as np
import matplotlib.pyplot as plt
from scipy.linalg import eigh_tridiagonal
from numba import njit, float64


# Physical parameters ==========


h = 1.05457181765e-34           # Reduced Planck constant
m = 9.1093837e-31               # Electron mass

k = 8.9875517923e9              # Coulomb constant
q = 1.60217663e-19              # Electron charge

a = (h**2)/(k*m*(q**2))         # Bohr radius

G = 6.6743e-11                  # Gravitational constant
M = 1.67262192e-27              # Proton mass

b = (m*M)/(m + M)               # Reduced mass


# User inputs ==========
```

```
print(80*'=')

#Energy level
l = int(input('Energy level (1, 2 or 3): '))

# Degree of self-interaction
f = int(input('Degree of self-interaction (0 or 1): '))

print(80*'=')


# Numerical grid ==========


if l == 1:

    N = 751                         # Number of grid points
    L = 1.5e-9                      # Spatial extent

    if f == 0:
        n = 10                      # Number of iterations

    if f == 1:
        n = 50                      # Number of iterations

if l == 2:

    N = 1501                        # Number of grid points
    L = 3e-9                        # Spatial extent

    if f == 0:
        n = 10                      # Number of iterations

    if f == 1:
        n = 100                     # Number of iterations
```

```
if l == 3:

    N = 3001                        # Number of grid points
    L = 6e-9                        # Spatial extent

    if f == 0:
        n = 10                      # Number of iterations

    if f == 1:
        n = 200                     # Number of iterations

r = np.linspace(0,L,N)              # Spatial discretization
dr = r[1] - r[0]                    # Space unit


# Electron-proton potential ==========


def V(r):

    N = len(r)
    V = np.zeros(N)

    dr = r[1] - r[0]

    V[1:] = (- G*(m*M) - k*(q**2))/r[1:]
    V[0] = (- G*(m*M) - k*(q**2))/(dr/2)

    return V


# Self-interaction potential ==========


@njit(float64[:](float64[:],float64[:]),nogil=True,fastmath=True)
def W(phi,r):
```

```
    N = len(phi)
    W = np.zeros(N)
    W1 = np.zeros(N)
    W2 = np.zeros(N)

    dr = r[1] - r[0]

    phi2 = phi**2

    for j in np.arange(1,N):
        for i in np.arange(j):
            W1[j] += phi2[i]*dr

        for i in np.arange(j,N):
            W2[j] += phi2[i]*(1/r[i])*dr

    W[1:] = W1[1:]/r[1:] + W2[1:]

    for i in np.arange(1,N):
        W[0] += phi2[i]*(1/r[i])*dr

    W = (- 4*np.pi*G*(m**2) + 4*np.pi*k*(q**2))*W

    return W


# Spatial derivative ==========


def second(A,dr):

    B = A.copy()
    N = len(A)

    B[0] = (35*A[0]-104*A[1]+114*A[2]
```

```
    -56*A[3]+11*A[4])/(12*(dr**2))

    B[1] = (11*A[0]-20*A[1]+6*A[2]
    +4*A[3]-A[4])/(12*(dr**2))


    B[N-1]=(11*A[N-5]-56*A[N-4]+114*A[N-3]
    -104*A[N-2]+35*A[N-1])/(12*(dr**2))

    B[N-2] = (-A[N-5]+4*A[N-4]+6*A[N-3]
    -20*A[N-2]+11*A[N-1])/(12*(dr**2))

    for i in np.arange(2,N-2):
        B[i] = (-A[i-2]+16*A[i-1]-30*A[i]
        +16*A[i+1]-A[i+2])/(12*(dr**2))

    return B


# Hydrogen atom fundamental state ==========


psi0_1 = np.exp(- r/a)/np.sqrt(np.pi*(a**3))
phi0_1 = r*psi0_1


# Hydrogen atom first excited state ==========


psi0_2 = (2 - r/a)*np.exp(- r/(2*a))
/np.sqrt(32*np.pi*(a**3))

phi0_2 = r*psi0_2


# Hydrogen atom second excited state ==========
```

```
psi0_3 = (27 - 18*r/a + 2*(r**2)
/(a**2))*np.exp(- r/(3*a))/np.sqrt(19683*np.pi*(a**3))

phi0_3 = r*psi0_3


# Iterative procedure ==========


print(80*'=')

V = V(r)                            # Electron-proton potential

S = [phi0_1,phi0_2,phi0_3]          # List of test functions
phi0 = S[l - 1]                     # Hydrogen atom eigenstate
phi = S[l - 1][1:-1]                # Test function

for i in np.arange(n + 1):

    A = phi.copy()
    B = np.append([0],A)
    B = np.append(B,[0])

    # Computation of the eigenfunctions and eigenvalues

    in1 = 1/(dr**2) + (b/(h**2))*V[1:-1]
    + (b/(h**2))*f*W(B,r)[1:-1]

    in2 = - 1/(2*(dr**2))*np.ones(len(in1)-1)

    u,v = eigh_tridiagonal(in1,in2)

    norm = 4*np.pi*dr*np.sum(v.T[l - 1]**2)
    v.T[l - 1] = v.T[l - 1]/np.sqrt(norm)
```

```
    # Improvement of the convergence

    phi = np.sum([v.T[l - 1],A],axis=0)
    norm = 4*np.pi*dr*np.sum(phi**2)
    phi = phi/np.sqrt(norm)

    # Computed eigenenergy

    print(((h**2)/(b*q))*u[l - 1])

print(80*'=')


# Final eigenfunction ==========


B = np.append([0],phi)
B = np.append(B,[0])
phi = B


# Plots of the radial probability densities ==========


print(80*'=')

plt.plot(r*1e10,(4*np.pi*((phi0)**2))/1e10)

plt.plot(r*1e10,(4*np.pi*(phi**2))/1e10,linestyle='dashed')

plt.xlabel('r ($\AA$)',fontsize=14)

plt.ylabel('F(r) ($\AA^{-1}$)',fontsize=14)

plt.savefig('phi_' + str(l) +
```

```
'.pdf',format="pdf",bbox_inches="tight")

plt.show()

print(80*'=')
```

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