DISCRETE AND CONTINUOUS
DYNAMICAL SYSTEMS–SERIES B
Volume **5**, Number **1**, February **2005**

Website: http://AIMsciences.org

pp. **51–66**

REVERSIBLE HAMILTONIAN LIAPUNOV CENTER THEOREM

Claudio A. Buzzi

IBILCE
UNESP
São José do Rio Preto, CEP 15054-000, Brazil

Jeroen S.W. Lamb

Department of Mathematics
Imperial College London
London SW7 2AZ, United Kingdom

Abstract. We study the existence of periodic solutions in the neighbourhood of symmetric (partially) elliptic equilibria in purely reversible Hamiltonian vector fields. These are Hamiltonian vector fields with an involutory reversing symmetry R. We contrast the cases where R acts symplectically and anti-symplectically.

In case R acts anti-symplectically, generically purely imaginary eigenvalues are isolated, and the equilibrium is contained in a local two-dimensional invariant manifold containing symmetric periodic solutions encircling the equilibrium point.

In case R acts symplectically, generically purely imaginary eigenvalues are doubly degenerate, and the equilibrium is contained in two two-dimensional invariant manifolds containing nonsymmetric periodic solutions encircling the equilibrium point. In addition, there exists a three-dimensional invariant surface containing a two-parameter family of symmetric periodic solutions.

1. **Introduction.** It is well known that (purely) reversible and Hamiltonian dynamical systems have many striking features in common, see for instance [18, 17, 13]. Surprisingly, despite the frequent occurrence in applications, only relatively few theoretical studies have focussed on the category of dynamical systems that are both Hamiltonian and reversible at the same time.

In this paper, we discuss the dynamics in the neighbourhood of an equilibrium with some purely imaginary pairs of eigenvalues. We call such an equilibrium (partially) elliptic. Importantly, such types of equilibria occur generically (with codimension zero) in reversible, Hamiltonian, and reversible Hamiltonian dynamical systems. We discuss the existence of periodic solutions in the immediate vicinity of such equilibria. It is well known [1] that generic (partially) elliptic equilibria in Hamiltonian vector fields are contained in two-dimensional manifolds containing one-parameter families of periodic solutions. Such families of periodic solutions are called *Liapunov Center families.* Devaney [7] was the first to prove a Liapunov Center theorem for (purely) reversible vector fields, see also Vanderbauwhede [19] and Sevryuk [18].

1991 *Mathematics Subject Classification.* 37C45.
Key words and phrases. Liapunov center theorem, time-reversal symmetry.