

Nonlinear control system for optical interferometry based on variable structure control and sliding modes

ROBERTA I. MARTIN,^{1,*} JOÃO M. S. SAKAMOTO,² MARCELO C. M. TEIXEIRA,¹ GUILHERME A. MARTINEZ,¹ FERNANDO C. PEREIRA,¹ AND CLÁUDIO KITANO¹

¹Department of Electric Engineering, School of Natural Sciences and Engineering, São Paulo State University (UNESP), Ilha Solteira, SP 15385-000, Brazil

²Division of Photonics, Instituto de Estudos Avançados (IEAv), Trevo Cel. Av. José A. A. do Amarante nº 1, São José dos Campos, SP 12228-001, Brazil

*robertagortan@gmail.com

Abstract: This work presents a novel nonlinear control system designed for interferometry based on variable structure control and sliding modes. This approach can fully compensate the nonlinear behavior of the interferometer and lead to high accuracy control for large disturbances, featuring low cost, ease of implementation and high robustness, without a reset circuit (when compared with a linear control system). A deep stability analysis was accomplished and the global asymptotic stability of the system was proved. The results showed that the nonlinear control is able to keep the interferometer in the quadrature point and suppress signal fading for arbitrary signals, sinusoidal signals, or zero input signal, even under strong external disturbances. The system showed itself suitable to characterize a multi-axis piezoelectric flexentional actuator, which displacements that are much smaller than half wavelength. The high robustness allows the system to be embedded and to operate in harsh environments as factories, bringing the interferometry outside the laboratory.

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1. Introduction

Linear control methods have been regularly employed to stabilize interferometers and accurately estimate phase [1–5]. Despite the wide variety of linear systems techniques, their application to nonlinear systems, such as interferometers, can bring limitations and do not ensure the necessary robustness in the system performance, since linear control methods rely on the key assumption of a small-value disturbance (not enough to remove the system from its equilibrium point) for the linear model to be valid. In cases of great disturbance (enough to remove the system from its equilibrium point), a linear controller is likely to perform very poorly or to be unstable, due to the nonlinearities in the system. Thus, the disturbance may not be properly compensated. Linear control systems that are able to handle strong disturbances usually comprises a reset circuit, which increases the complexity of the circuitry and can introduce spurious signals during reset [4–6].

On the other hand, in this work, we propose a nonlinear control system designed for interferometry, based on variable structure control and sliding modes (VS/SM). This approach can fully compensate the nonlinear behavior of the interferometer and lead to high-accuracy control for strong disturbances, featuring ease of implementation and high robustness [7]. A reset circuit is not necessary which means that there is no spurious signal generated by the reset itself, and the circuitry is very simple and low cost. The high robustness allows the system to be embedded and to operate in a place with great mechanical or environmental disturbances, being suitable to operate in harsh environments, as factories, taking the interferometry outside the laboratory.

The main components of the nonlinear control system were presented, and a deep stability analysis was accomplished. The experimental phase-plane was acquired, showing agreement with the theoretical one. Regarding the application of this nonlinear control system on the interferometer, we show that it was able to control it, keeping the quadrature point and suppressing the signal fading for arbitrary, sinusoidal, or zero input signals, even under strong external disturbances such as vibrations and temperature variations. Finally, the system was applied to characterize a multi-axis piezoelectric flextentional actuator, which displacements are much smaller than half light wavelength.

In this work, we are concerned about the detection of sub-nanometric displacements, whose phase variation is less than $\pi/2$ rad. However, the proposed nonlinear system, without modification, also works properly for detecting large signals (phase variation larger than $\pi/2$ rad, i.e., multi-fringe operation), with the same accuracy. Therefore, we focus on an active homodyne detection since it can offer a lower noise floor when compared to heterodyne techniques, which are usually limited by laser frequency, intensity and acoustic noise [8].

The nonlinear control system was designed based on the two-beam interferometer equation. In our specific case, we applied it on a Michelson interferometer in order to validate and prove that the system works properly. Therefore, the application to any other two-beam interferometer (e.g. Mach-Zehnder and Sagnac) or even other system similarly equated, is direct and does not require a new design project. Thus, the application of the nonlinear control system in each specific interferometer can be accomplished by varying the phase on the reference arm, driven by the control signal.

1.1. Michelson interferometer

The nonlinear control system proposed in this work was applied to a bulk homodyne Michelson interferometer in order to demonstrate its capability of suppressing signal fading and keeping the interferometer in quadrature. The Michelson interferometer was chosen since it is suitable to measure displacements generated by piezoelectric actuators.

The interferometer's photo-detected signal (after a transimpedance circuit) has the following general form:

$$v_{PD}(t) = A + AV \cos[\Delta\phi(t) + \phi_o], \quad (1)$$

where $\Delta\phi(t)$ is the signal of interest, ϕ_o the static phase shift between the two interferometer's arms, the constant A accounts for the laser power, photodiode responsivity, amplifier gain, and V is the fringe visibility.

The ideal static phase shift should be a constant. However, due to the spurious disturbances acting on the interferometer, the actual static phase shift is a quasi-static quantity (variation usually occurs below 20 Hz). This variation on ϕ_o makes the interferometer operation point vary with time and can cause signal fading.

The nonlinear control system is then used to keep ϕ_o constant and in the condition of phase quadrature, i.e., $\phi_o = \pi/2$ rad. Also, removing the first term from Eq. (1), we have:

$$v(t) = AV \cos\left[\Delta\phi(t) + \frac{\pi}{2}\right] = -AV \sin[\Delta\phi(t)]. \quad (2)$$

The AV factor can be determined experimentally by a self-calibration procedure [9], for displacement measurement. For low modulation depth, we have $\Delta\phi(t) \ll 1$ rad, and $\sin[\Delta\phi(t)] \approx \Delta\phi(t)$, consequently $\Delta\phi(t)$ can be measured directly from $v(t)$. On the other hand, if $\Delta\phi(t) \gg 1$ rad, some phase-unwrapping demodulation technique can be used to measure $\Delta\phi(t)$ [10–12].

2. Variable structure control and sliding modes for optical interferometry

The nonlinear control method used in this work was the VS/SM. This type of system consists of a switching control law to drive the system state trajectory onto a specified and user-chosen surface in the state space (called sliding or switching surface). Thus, the system state is constrained to remain on this surface for all subsequent time. This surface is called switching surface because the control path gain is determined by the state trajectory of the plant: the gain is switched if the trajectory is "above" or "below" this surface. The plant dynamics restricted to this surface represents the controlled system behavior [13, 14].

The VS/SM control law was chosen to control the interferometer due to its capability to deal with a nonlinear characteristic curve (as the interferometer cosine) and keep it on the maximum point of sensitivity (phase quadrature). In addition, this method presents easy implementation and high robustness [14].

The closed loop control system's block diagram is presented in Fig. 1. In the following

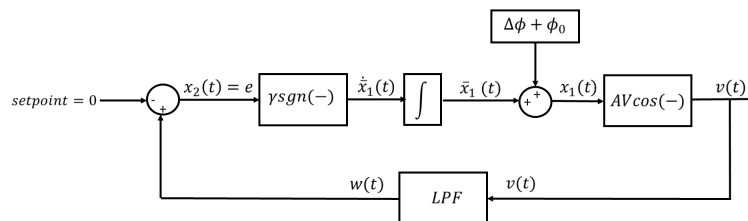


Fig. 1. Control system block diagram.

analysis, we considered $\Delta\phi = 0$, since the nonlinear control system actuates only on the disturbance (ϕ_o), through the low-pass filter (LPF). The analysis for $\Delta\phi \neq 0$, will be accomplished in Section 2.2. The block $AV \cos(-)$ represents the interferometer, whose output voltage is $v(t)$. This voltage is filtered by the LPF, which provides the signal $w(t)$. The difference between $w(t)$ and the *setpoint* provides the error signal, $x_2(t) = e$. The block $\gamma \text{sgn}(-)$ (where γ is the feedback gain and $\text{sgn}(-)$ is the nonlinear sign function) switches the feedback gain signal depending on the polarity of $x_2(t)$, providing the derivative of $\bar{x}_1(t)$, i.e. $\dot{\bar{x}}_1(t)$. Thus, the integrator yields $\bar{x}_1(t)$, which is summed to the disturbance ϕ_o , and inserted into the interferometer, as the signal $x_1(t) = \bar{x}_1(t) + \phi_o$.

The LPF is a first order butterworth filter with 20 Hz cutoff frequency, used to compensate only ϕ_o fluctuations, which occur at low frequencies (bellow 20 Hz). The LPF transfer function is given by: $H(s) = W(s)/V(s) = 1/(1 + \tau s)$, where $\tau = 1/40\pi$ seconds is the filter time constant and s is the complex variable. The function $\text{sgn}(-)$ operates as follows:

$$\text{sgn}(e) = \begin{cases} 1 & \text{if } e > 0, \\ 0 & \text{if } e = 0, \\ -1 & \text{if } e < 0. \end{cases} \quad (3)$$

The nonlinear control system's goal is to keep the interferometer in phase quadrature and to remove the low frequency spurious disturbance, thus avoiding signal fading. To accomplish that, the control system makes the signal $w(t)$ (the low frequency part of the output $v(t)$) equal to the *setpoint*, taking the error $e = w(t) - \text{setpoint}$ to zero. Since the *setpoint* is zero, the cosine argument ($x_1(t) = \bar{x}_1(t) + \phi_o$) should be an odd multiple of $\pi/2$ rad, leading to a phase quadrature operation point.

For example, regarding γ and AV positive, an initial condition $0 \leq \bar{x}_1(t) \leq \pi/2$ and the error $e > 0$, when the control system starts, the function $\text{sgn}(e)$ will switch to a positive value, making $\dot{\bar{x}}_1(t)$ positive, leading the integrator output $\bar{x}_1(t)$ to increase (at a positive rate). Consequently, $w(t)$ will decrease, leading the error to zero ($e = 0$). If the initial error is $e < 0$, the function $\text{sgn}(e)$ will switch to a negative value, making $\dot{\bar{x}}_1(t)$ negative, and $\bar{x}_1(t)$ will decrease (at a negative rate). Consequently, $w(t)$ will increase, leading again the error to zero ($e = 0$).

The output signal $v(t)$ will then be in phase quadrature and carry the information of interest ($\Delta\phi(t)$). The low-pass filter allows the compensation only for the low frequency (bellow 20 Hz) components of $v(t)$, while information of interest at higher frequencies (above 20 Hz) remains unaltered.

2.1. Stability analysis

2.1.1. State-space representation

From $H(s)$ and Eq. (3), the state equations that represent the system of Fig. 1 are the following:

$$\dot{x}_1 = \dot{\bar{x}}_1 + \dot{\phi}_o = \gamma \text{sgn}(x_2) + \dot{\phi}_o, \quad (4)$$

$$\dot{x}_2 = \frac{v(t)}{\tau} - \frac{x_2}{\tau} = \frac{AV \cos(x_1)}{\tau} - \frac{x_2}{\tau}, \quad (5)$$

where x_1 and x_2 are the system state variables. From Eq. (4), the control condition is given by

$$\gamma > |\dot{\phi}_o|, \quad (6)$$

which implies that $\text{sgn}(\dot{x}_1) = \text{sgn}[\gamma \text{sgn}(x_2) + \dot{\phi}_o] = \text{sgn}(x_2)$. Thus, $\dot{x}_1$ depends only on x_2 and the disturbance does not affect the time derivative of x_1 . It should be noted that, the higher the gain γ of the system, the faster the state x_1 converges to the equilibrium point, and more robust the system is. Consequently, the overshoot will be greater and the settling time, shorter.

From the state equations, the system equilibrium points are given by:

$$\dot{x}_1(t) = 0 \Rightarrow \operatorname{sgn}(x_2) = 0 \Rightarrow x_2 = 0, \quad (7)$$

$$\dot{x}_2(t) = 0 \Rightarrow AV \cos(x_1) = 0 \Rightarrow x_1 = k \frac{\pi}{2}, \quad (8)$$

for $k \in \{\dots, -3, -1, 1, 3, \dots\}$, independently of ϕ_o .

2.1.2. Stability analysis based on Lyapunov's linearization method

Lyapunov's linearization method allows the evaluation of local stability of a nonlinear system. According to [7], if an equilibrium point of the linearized system is strictly stable, then this point (and around it) is locally asymptotically stable for that particular nonlinear system.

The stability around the origin (the steady state point) will then be evaluated, choosing a specific equilibrium point ($x_1 = \pi/2$, $x_2 = 0$) to be the origin of the state-space. We regard that the control condition is met in the Lyapunov's analysis. In this way, we perform the following variable substitution:

$$y_1(t) = x_1 - x_1^* = x_1 - (\pi/2), \quad (9)$$

$$y_2(t) = x_2 - x_2^* = x_2 - 0, \quad (10)$$

where x_i^* is the state x_i at the equilibrium point. With the aid of Eqs. (4)–(5), the new system is described by the state equations:

$$\dot{y}_1(t) = \gamma \operatorname{sgn}(y_2(t)) = f_1(y_1(t), y_2(t)), \quad (11)$$

$$\dot{y}_2(t) = -\frac{AV}{\tau} \sin(y_1(t)) - \frac{y_2}{\tau} = f_2(y_1(t), y_2(t)). \quad (12)$$

Thus, the new equilibrium points are

$$\dot{y}_1(t) = 0 \Rightarrow \gamma \operatorname{sgn}(y_2(t)) = 0 \Rightarrow y_2 = 0, \quad (13)$$

$$\dot{y}_2(t) = 0 \Rightarrow \sin(y_1) = 0 \Rightarrow y_1 = k\pi, \quad (14)$$

for $k \in \{\dots, -2, -1, 0, 1, 2, \dots\}$. The system is therefore linearized around the equilibrium points ($y_1 = k\pi$, $y_2 = 0$) in order to analyze the system local stability. For the linearization of the system, the sgn function is approximated by the sigmoid function, given by $\dot{y}_1 = \operatorname{sgm}(y_2) = y_2/(|y_2| + \epsilon)$, $\epsilon > 0$ (sufficiently small). Thus, the linearization around the state variables [7], for an even k , results

$$\begin{bmatrix} \Delta \dot{y}_1 \\ \Delta \dot{y}_2 \end{bmatrix} = \begin{bmatrix} \frac{\partial f_1}{\partial y_1} & \frac{\partial f_1}{\partial y_2} \\ \frac{\partial f_2}{\partial y_1} & \frac{\partial f_2}{\partial y_2} \end{bmatrix}_{y_1=k\pi, y_2=0} = M_{2 \times 2} \begin{bmatrix} \Delta y_1 \\ \Delta y_2 \end{bmatrix} = \begin{bmatrix} 0 & \frac{\gamma}{\epsilon} \\ -\frac{AV}{\tau} & -\frac{1}{\tau} \end{bmatrix} \begin{bmatrix} \Delta y_1 \\ \Delta y_2 \end{bmatrix}. \quad (15)$$

The eigenvalues of matrix M , given by $\{\det(sI - M) = 0, s \in \mathbb{C}\}$, correspond to the closed loop poles of the linearized system. Regarding that $\tau = 1/40\pi$ seconds, $\epsilon = 0.01$, $\gamma = 54$ rad (experimental values) and $AV = 1$ V (set as 1 V in this analysis, without losing generality), the poles are $s_{1,2} = (-63 \pm j821)$ which, according to [7], correspond to a stable focus, i.e., this equilibrium point is strictly stable. Then, it is locally asymptotically stable for the nonlinear system. Similarly, for an odd k , we obtain the poles $s_1 = -889$ and $s_2 = 763$ which correspond to a saddle point [7]. Thus, these equilibrium points are unstable.

Therefore, the linear approximation is only locally stable while the nonlinear system is globally asymptotically stable, as we shall see in Section 2.1.3.

2.1.3. Phase-plane

In order to verify the stability of the nonlinear system, the phase-plane method was used. Thus, the phase-plane was numerically obtained (using the software Matlab[®]) from Eqs. (4)–(5), where the AV factor was experimentally measured and the gain γ comprises the complete feedback loop gain, given by $\gamma = G_{sgn}G_{AMP}G_{PZT_{fb}}$. The sign function gain is G_{sgn} , while the linear amplifier gain is G_{AMP} , and the feedback piezoelectric actuator gain is $G_{PZT_{fb}}$. The linear amplifier (AMP) and the feedback piezoelectric actuator (PZT_{fb}) will be described in Section 2.2.

The phase-plane was first obtained for $\gamma = 54$ rad, where $G_{sgn} = 15$ V, $G_{AMP} = 20$ V/V, and $G_{PZT_{fb}} = 0.18$ rad/V. The simulation result is shown in Fig. 2, for several initial conditions (x_o) at a circumference around the equilibrium point ($x_1 = \pi/2, x_2 = 0$), according to $x_o = r[(\cos \theta + (\pi/2r)) \sin \theta]^T$, where r is the circumference radius and $\theta \in \{0, \frac{\pi}{32}, \frac{2\pi}{32}, \dots, 2\pi\}$. All the trajectories inside the observation region converge to the point ($x_1 = \pi/2, x_2 = 0$) when time $t \rightarrow \infty$. For example, if the initial condition departs from point A, in Fig. 2(a), the solution will track the trajectory from the left to the right, mostly for $x_2 > 0$, towards the equilibrium point ($x_1 = \pi/2, x_2 = 0$). For an initial condition on point B, the solution will track the trajectory from the right to the left, mostly for $x_2 < 0$, towards the same equilibrium point. The point ($x_1 = \pi/2, x_2 = 0$) corresponds to a stable equilibrium point, according to state equation analysis. Regarding a larger initial-condition circumference, i.e., $x_o = r[\cos \theta \sin \theta]^T$, one can observe two stable equilibrium points: ($x_1 = \pi/2, x_2 = 0$), and ($x_1 = -3\pi/2, x_2 = 0$); and an unstable one: ($x_1 = -\pi/2, x_2 = 0$). This is due to the periodic and nonlinear characteristic of the interferometer, with infinite stable and unstable equilibrium points.

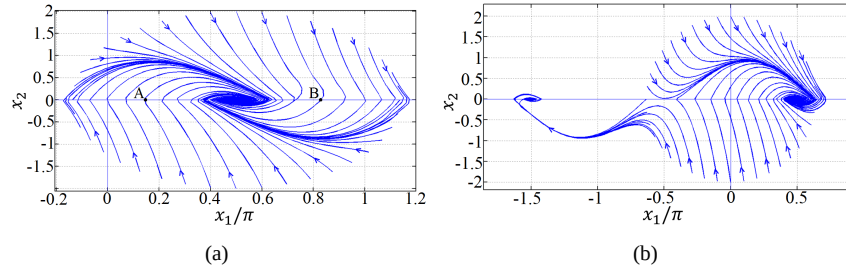


Fig. 2. Theoretical phase-plane for $\gamma = 54$ rad. (a) $x_o = r[(\cos \theta + (\pi/2r)) \sin \theta]^T$. (b) $x_o = r[\cos \theta \sin \theta]^T$.

As can be seen in Fig. 2 we evaluated a 2π region and, since the system is periodic, it always converges to a stable point, regardless the initial conditions. In addition, the system does not tend to a limit cycle nor to instability.

2.2. Closed loop interferometer simulation

The block diagram of the control system, shown in Fig. 1, was simulated on Simulink[®] as presented in Fig. 3. The phase variation $\Delta\phi$ in Eq. (1) was provided by the piezoelectric actuator under evaluation (PZT). Provided that this piezoelectric actuator operates on its linear region, the phase shift (in radians) in the interferometer is directly proportional to the voltage (volts) applied. Thus, the signal of interest $\Delta\phi(t)$ and the static phase shift ϕ_o were synthesized by voltage signal generators, and superimposed by the sum block. The signal of interest was sinusoidal with peak voltage of 1 V and 1 kHz frequency, while the static phase shift was sinusoidal with peak voltage of 2 V and at a frequency of 5 Hz. The resultant signal was amplified by amplifier 2 (AMP₂) ($G_{AMP_2} = 12$ V/V) and applied on the PZT ($G_{PZT} = 0.034$ rad/V), that provided the conversion from voltage to angle in radians. This

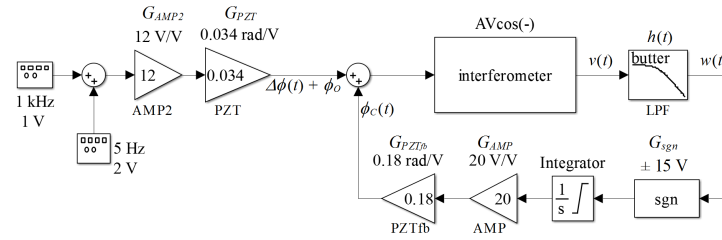


Fig. 3. Block diagram to simulate the closed loop Michelson interferometer.

voltage simulates the signal of interest $\Delta\phi(t)$ superimposed by the static phase shift ϕ_o spurious variation.

The feedback loop comprises the low-pass filter (LPF) $h(t)$ (gain at the pass-band $G_{LPF} = 1$ V/V), the sgn function (sign function with gain $G_{sgn} = \pm 15$ V); the integrator ($1/s$) followed by an amplifier ($G_{AMP} = 20$ V/V) and the feedback piezoelectric actuator (gain $G_{PZTfb} = 0.18$ rad/V). The “interferometer” block simulates the relation $v(t) = AV \cos[\Delta\phi(t) + \phi_o + \phi_c]$, where the signal of correction is $\phi_c = -\phi_o + \pi/2$; the AV product was set to 1 V in the simulation. Therefore, the output signal will become in phase quadrature, as stated in Eq. (2).

The simulation result for the open loop operation is presented in Fig. 4, where the input signal is shown in Fig. 4(a). One can observe that the output signal, represented by Fig. 4(b), was disturbed by the static phase shift ϕ_o spurious variation and the signal of interest $\Delta\phi(t)$ was not properly recovered, presenting a spurious low-frequency variation.

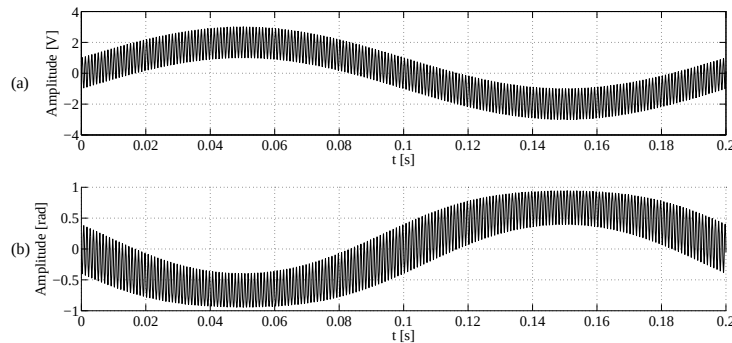


Fig. 4. Simulation results for open loop operation. (a) Input signal. (b) Output signal.

After that, the closed loop operation was simulated using the same parameters and the result is presented in Fig. 5. One can observe that the output signal was properly recovered in Fig. 5(b), where the static phase shift spurious variation was removed, i.e., the signal fading was suppressed and the output signal remained on quadrature. The behavior of the sign function is shown in Fig. 5(c).

3. Experimental setup

The experimental setup comprises the Michelson interferometer and the nonlinear control system, as shown in Fig. 6. The Michelson interferometer was assembled with a He-Ne laser (wavelength of 632.8 nm and power of 5 mW), a beam-splitter (BS, 50:50 split ratio), two tiny flexible mirrors (3M™ Scotchlite™ reflective sheet), and a PIN photodiode (Thorlabs PDA55). The nonlinear control system was implemented through an electronic circuit that comprises a

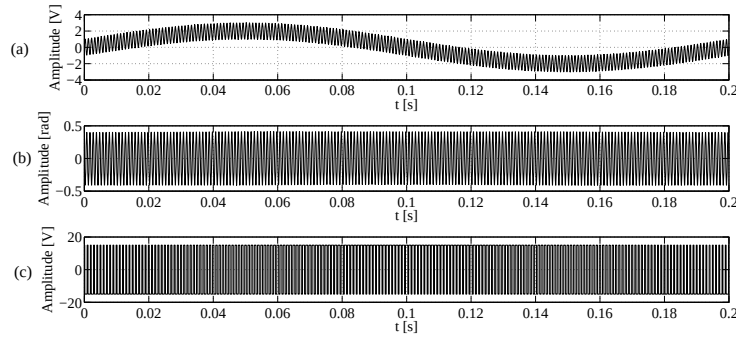


Fig. 5. Simulation results for closed loop operation. (a) Input signal. (b) Output signal. (c) sign function.

DC suppressor (to remove the term A in Eq. 1), a low-pass filter (LPF), a sign function (sgn), an integrator ($1/s$), and an amplifier (G_{AMP}) for the PZT (A. A. Lab Systems, A-301HS).

In the Michelson interferometer, one of the mirrors was fixed on the feedback piezoelectric actuator (PZT_{fb}) (Control Techniques), while the other one was fixed over the piezoelectric actuator (PZT) under evaluation. The laser beam was split in two by the beam-splitter; one of them, called reference beam (or reference arm), traveled towards the feedback piezoelectric actuator and was reflected back while the other one, called sensor beam (or sensor arm), traveled towards the piezoelectric actuator under evaluation and was also reflected back. The two beams were then recombined in the beam-splitter and directed towards the photodetector, which detects the interference signal.

The optical signal acquired from the photodetector was converted to voltage by a transimpedance circuit. Then, this voltage was processed by the electronic control circuit which, in turn, generated the nonlinear control signal to be applied on the feedback piezoelectric actuator to keep the interferometer in quadrature and suppress the signal fading.

It is noteworthy that this electronic circuit does not require a reset system, differently from usual control system circuits. In addition, the electronic circuit comprises low cost components, presents easy implementation and high robustness.

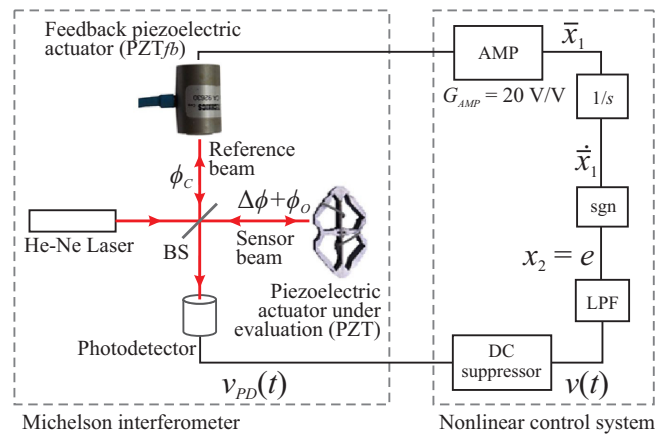


Fig. 6. Block diagram of the Michelson interferometer with the nonlinear control system.

4. Results

Every experiment was performed within a not-controlled environment, composed of spurious disturbances as ground-borne mechanical vibration, air current flow (air-conditioning and fan), and temperature variation (room temperature varying from approximately 290 K to 305 K). The first result concerns the experimental acquisition of the phase-plane in order to evaluate if the practical nonlinear control system approaches the theoretical one (simulated in Section 2). The second result regards the experimental detection, in time domain, of dynamic displacements by the interferometer in order to verify the actuation of the nonlinear control system in a practical (or real) situation. The input signals applied to the interferometer were zero input (without signal), sinusoidal, and arbitrary signals. The third result was the characterization of a multi-axis piezoelectric flexentional actuator [15], which generates small signal displacements (corresponding to a phase smaller than $\pi/2$ rad), in order to prove the interferometer's capability of detection those displacements with the nonlinear control.

4.1. Experimental phase-plane

The state variable used was named $\bar{x}_1$ and is located in the integrator circuit output, shown in Fig. 6. This variable was chosen because the state variable x_1 is not accessible in the implemented circuit plant. The state variable $\bar{x}_1$ is proportional to x_1 , since the equilibrium point is $(x_1 = \pi/2, x_2 = 0)$, $x_1 = \bar{x}_1 + \phi_o$, i.e., in terms of $\bar{x}_1$, the equilibrium point is $\bar{x}_1 = \pi/2 - \phi_o$, which corresponds to the voltage required to vanish the fading and take the system input to the defined equilibrium point (quadrature). It is noteworthy that, from variable $\bar{x}_1$, it is possible to measure ϕ_o in real-time.

Therefore, the experimental phase-plane was obtained by plotting $x_2 \times \bar{x}_1$, as shown in Fig. 7(a), where the state tracks a trajectory from the left to the right mostly over the positive semi-plane, $x_2 > 0$. In Fig. 7(b), the state tracks a trajectory from the right to the left, mostly over the negative semi-plane, $x_2 < 0$. Thus, one can observe that the state trajectories are in

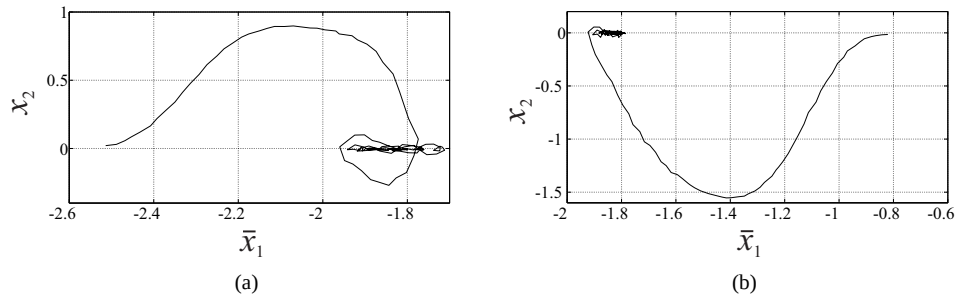


Fig. 7. Experimental phase-plane for different initial conditions. (a) The initial point tracks a trajectory mostly over the positive semi-plane, $x_2 > 0$. (b) The initial point tracks a trajectory mostly over the negative semi-plane, $x_2 < 0$.

agreement with the theoretical phase-plane, shown in Fig. 2, always converging to the stable equilibrium point as $t \rightarrow \infty$. In Fig. 7, one can observe that $\bar{x}_1$ does converges to the voltage required to compensate ϕ_o and it takes the state x_1 to a stable equilibrium point (quadrature).

4.2. Nonlinear control system applied to the interferometer

4.2.1. Zero input signal

This experiment starts with the nonlinear control system turned off (open loop operation) and without signal of interest ($\Delta\phi(t) = 0$). Thus, it can be seen in the first 4 s of Fig. 8, that

the interferometer output signal (top signal, in blue) presents random oscillations generated by spurious environmental disturbances (ϕ_o , spurious variation). Moreover, the sign function (bottom signal, in red) keeps constant, i.e., the control is not actuating. However, after turning the nonlinear control system on (closed loop operation), after 4 s, and keeping the input signal null ($\Delta\phi(t) = 0$), the output signal in Fig. 8 stabilizes itself in a constant value, which corresponds to the parameter A from Eq. (1), while the sign function starts switching. This result shows that the nonlinear control system operates properly, effectively removing the spurious disturbances from the output signal, even for zero input signal. This feature is important to some interferometer applications, e.g. laser ultrasonics.

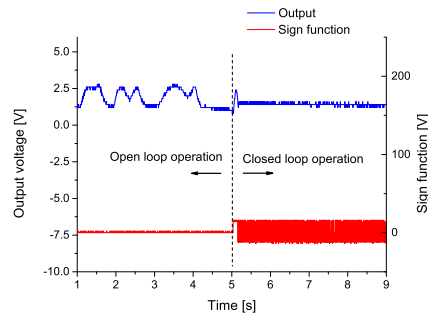


Fig. 8. Interferometer output signal (top) and sign function (bottom), before and after turning the nonlinear control system on.

4.2.2. Sinusoidal signal

Since the control system was working properly, the next step was apply a sinusoidal signal to the piezoelectric actuator (in the sensor arm) and verify if the interferometer with the nonlinear control system would be able to measure the corresponding displacement, and suppress the spurious signals. In this way, to generate the $\Delta\phi(t)$, we applied to the PZT under evaluation a sinusoidal input signal with peak voltage of 17.2 V and frequency of 1 kHz. It should be noted that this voltage range corresponds to $\Delta\phi(t) \ll 1$ rad, thus $\Delta\phi(t)$ can be acquired directly from $v(t)$, as stated in Section 1.1. In Fig. 9, the input signal is shown as the top solid blue line, while the interferometer output signal is shown as the bottom dashed red line. The sign function is shown in the bottom graphic, as a solid black line. The open loop operation is shown in Fig. 9(a), while the closed loop operation is shown in Fig. 9(b).

As can be observed, in open loop operation (Fig. 9(a)), the interferometer output signal presents fading, i.e., the operation point is out of quadrature and the sinusoidal displacement was not properly recovered. On the other hand, in closed loop operation (Fig. 9(b)), the interferometer output signal does not present fading and the sinusoidal signal was properly recovered. It is noteworthy that the closed loop output signal is shifted 180° in relation to the input signal, as predicted in Eq. (2).

4.2.3. Arbitrary signal

The final step is to check if the interferometer with the nonlinear control system works properly even for an arbitrary signal at the input. As shown in Fig. 10, the input signal, applied to the piezoelectric actuator (sensor arm) is on the top (solid blue line) and the interferometer output signal on the bottom (dashed red line). Regarding the interferometer in open loop operation, the output signal presents fading and does not represent the input displacement, as shown in Fig. 10(a). On the other hand, once again, when the closed loop operation actuates, the

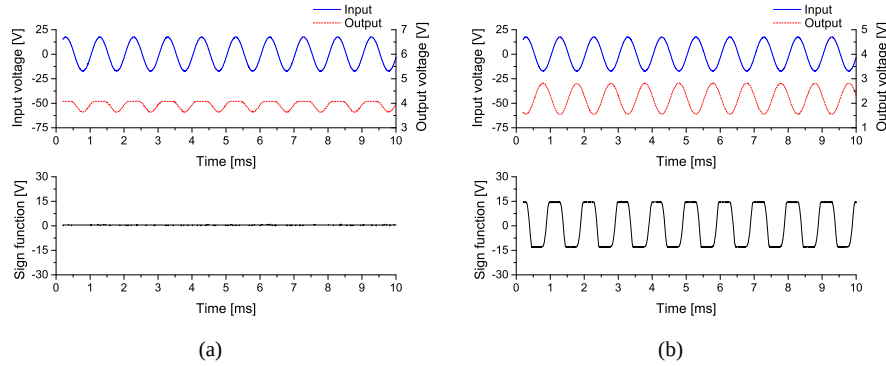


Fig. 9. Sinusoidal displacement detected by the interferometer. Input voltage (solid blue line), output voltage (dashed red line), and sign function on bottom (solid black line). (a) Open loop operation. (b) Closed loop operation.

interferometer output signal is in accordance with the displacement imposed by the piezoelectric actuator, as shown in Fig. 10(b). In this case, the input signal was shifted by 180° (on the oscilloscope) to provide a better comparison view between the input and output signals.

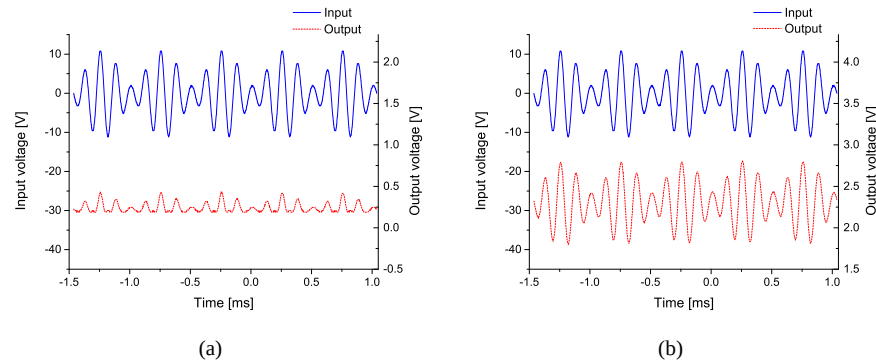


Fig. 10. Arbitrary displacement detected by the interferometer. Input on the top (solid blue line) and output on the bottom (dashed red line). (a) Open loop operation. (b) Closed loop operation (output shifted by 180°).

4.3. Application of the interferometer with the nonlinear control system: multi-axis piezoelectric flextentional actuator characterization

Once we proved that the interferometer with the nonlinear control system works properly, we are concerned on the application of this device, for example, on the characterization of piezoelectric actuators. Therefore, the multi-axis piezoelectric actuator characterized in this work, is of interest since it provides reduced displacements, corresponding to a phase variation smaller than $\pi/2$ rad, not enough to employ multi-fringe demodulation methods. Therefore, the interferometer proposed in this work is suitable to evaluate this actuator design performance, since it can detect those reduced displacements.

The multi-axis piezoelectric actuator comprises two piezo-ceramics and an aluminum structure, as shown in Fig. 11, designed by a topological optimization method with finite elements method [15, 16]. This structure was optimized such that each piezo-ceramic provides

mechanically amplified displacement in one direction, i.e., one piezo-ceramic (PZT B1) provides displacement in the x direction while the other (PZT B2), in the y direction. However, the actuator built in practice can generate displacements of two types: the direct one (PZT B1 causes a displacement in x , or PZT B2 causes a displacement in y) and the coupled one (PZT B1 causes a displacement in y , or PZT B2 causes a displacement in x).

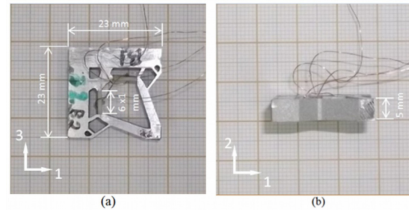


Fig. 11. Multi-axis piezoelectric actuator. (a) Top view. (b) Lateral view.

The “piezoelectric actuator under evaluation” shown in the setup of Fig. 6 was then replaced by the multi-axis piezoelectric actuator from Fig. 11, while the “feedback piezoelectric actuator” was kept the same. Thus, the multi-axis piezoelectric actuator was characterized by measuring its frequency response and linearity (at a resonance frequency and at a frequency out of resonance), for direct and coupled displacements.

4.3.1. Frequency response measurements

The frequency response curves were obtained from 100 Hz to 20 kHz, by applying sinusoidal signals to PZT B1 or to PZT B2 and measuring the direct or coupled displacements. This provides four different situations: i) PZT B1 direct displacement (x direction); ii) PZT B1 coupled displacement (y direction); iii) PZT B2 direct displacement (y direction); and iv) PZT B2 coupled displacement (x direction). The results are shown in Fig. 12, where the displacement was evaluated by $\Delta L(t) = \frac{\lambda}{4\pi} \Delta \phi(t)$. In Figs. 12(a) and 12(b) one can see a main resonance at 19.15 kHz, a secondary resonance followed by an anti-resonance at 12 kHz, and several micro-resonances around 8 kHz. This can also be readily seen from the phase graphics, Figs. 12(c) and 12(d).

One can observe that the coupling factor (ratio between the coupled and direct displacements) is higher in resonance frequencies and, in some cases, the coupled displacement can be higher than the direct one. Therefore, we emphasize the importance of measuring the resonance frequencies in actuators, in order to prevent unintended movements. In applications like sample positioning in an atomic force microscope, for example, an undesired displacement can cause permanent damage to the sample.

4.3.2. Linearity measurements

The linearity for the direct displacement (PZT B1) was acquired at two frequencies: 4.5 kHz and 19.3 kHz and the results are shown in Fig. 13. It can be noted that the actuator presents a linear behavior at the frequency of 4.5 kHz, and the curve slope (also called linear length-to-voltage sensitivity) was calculated and equates to 38.6 nm/V. On the other hand, at 19.3 kHz, an elliptical hysteresis curve was obtained, with slope of 400.8 nm/V (about ten times bigger than the one acquired at 4.5 kHz). The hysteresis and the bigger slope can indicate the presence of a resonance around 19.3 kHz.

5. Conclusion

In this work, we proposed a novel control system topology, using a nonlinear control technique, based on variable structure control and sliding modes to suppress the signal fading in an optical

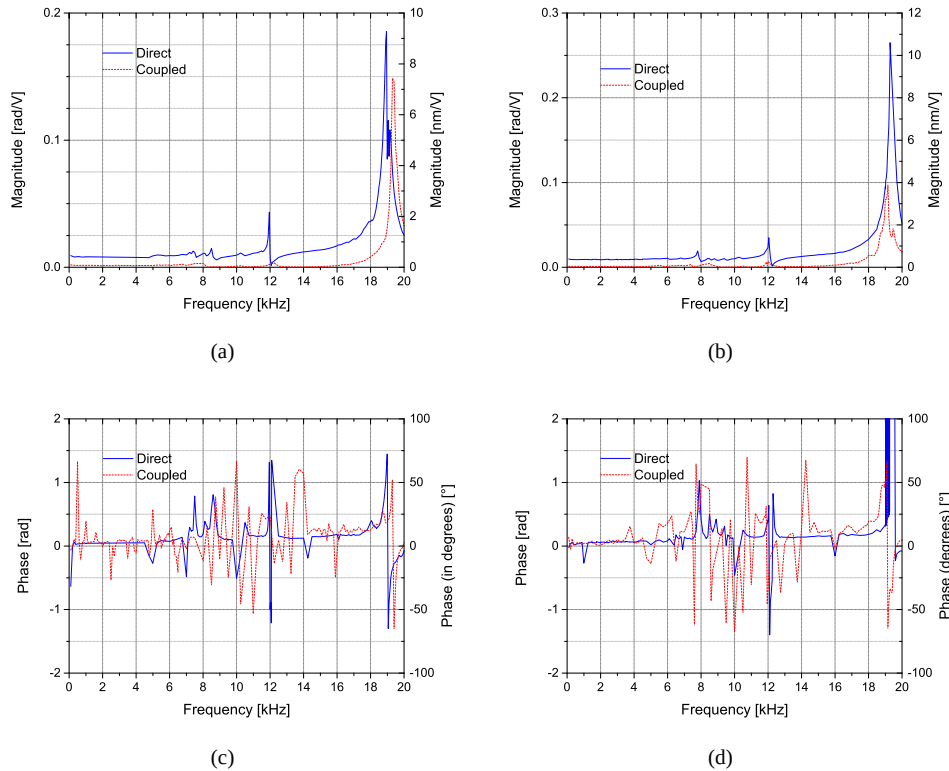


Fig. 12. Frequency response for PZB B1 and B2. (a) Magnitude (PZT B1). (b) Magnitude (PZT B2). (c) Phase (PZT B1). (d) Phase (PZT B2).

interferometer. We proved that the system is asymptotically stable by using two methods: the Lyapunov's linearization method and the phase-plane method. Through the phase-plane method, it was possible to consider all the nonlinearities from the system, and we proved that for any initial condition, the nonlinear system is stable and converges to the equilibrium point.

The proposed nonlinear control system presents important features as ease of implementation, low cost and robustness. In addition, the robustness could be easily improved by increasing the value of the feedback gain γ . During the experiments, strong external disturbances (in a non-controlled environment) were present around the interferometer setup such as vibration, temperature variation and air current flow. Even though, the system presented a suitable behavior, the nonlinear control system satisfied the interferometer quadrature condition and eliminated the signal fading in the cases of zero input signal, sinusoidal signal, and arbitrary signal.

The characterization of a multi-axis piezoelectric flexientional actuator was accomplished measuring sub-nanometric displacement amplitudes, linearity, hysteresis, frequency response, and mechanical resonances, showing the interferometer's capability of detecting those displacements with the nonlinear control system.

Although the proposed nonlinear control system was applied to a specific device (an interferometer), it can also be applied to control different systems whose characteristic equation is written in a similar way.

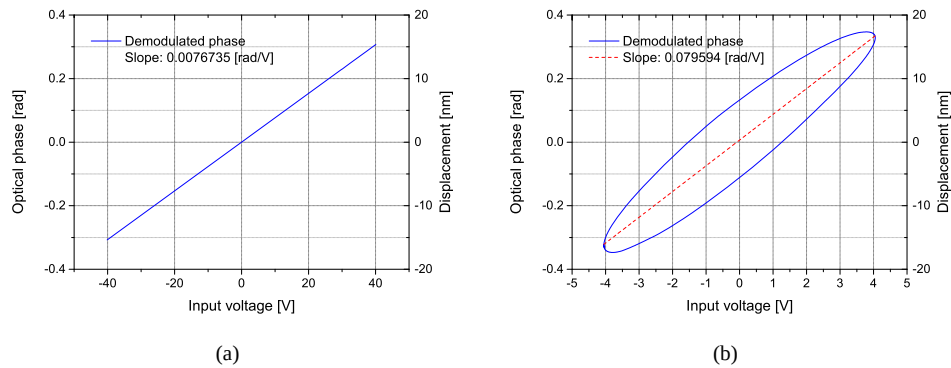


Fig. 13. Linearity of the actuator. (a) Linearity of the actuator at 4.5 kHz. (b) Hysteresis of actuator at 19.3 kHz.

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