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One-dimensional cutting stock problems with multiple  
periods applied to the precast slab industry

São José do Rio Preto  
2022

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Tese apresentada como parte dos requisitos para obtenção do título de Doutora em Matemática, junto ao Programa de Pós-Graduação em Matemática, do Instituto de Biociências, Letras e Ciências Exatas da Universidade Estadual Paulista “Júlio de Mesquita Filho”, Câmpus de São José do Rio Preto.

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*'All we have to decide is what to do with the time that is given us',  
John Ronald Reuel Tolkien*

## RESUMO

Este estudo analisa o planejamento da produção de lajes pré-moldadas decorrente de fábricas brasileiras especializadas. A primeira parte desta pesquisa considera o processo de produção de lajes alveolares, para o qual são propostos dois modelos matemáticos baseados no problema de corte de estoque unidimensional multiperíodo com aspectos inovadores relativos a múltiplos modos de manufatura que podem ser usados para produzir as lajes. Visando a minimização de custos de produção e de estoque, os dois modelos são resolvidos por métodos heurísticos. Com base em dados fornecidos por uma fábrica, resultados computacionais indicam a relevância de tais modelos como ferramentas auxiliares na tomada de decisão. A segunda parte deste estudo trata o processo de produção de lajes treliçadas, o qual compreende dois estágios: corte de treliças de aço e concretagem nos moldes. O modelo matemático proposto consiste em um problema de corte de estoque unidimensional multiperíodo de dois estágios de modo a minimizar custos de *setup*, produção e estoque sob restrições de capacidade e balanceamento de estoque. Diferentes estratégias de solução baseadas no método de geração de colunas são aplicadas a este modelo para gerar os padrões de corte que serão utilizados na produção, e um pacote computacional é utilizado para identificar soluções inteiras. As instâncias são baseadas em informações obtidas da fábrica e os resultados computacionais ilustram a aplicação do modelo proposto em um contexto realista.

Palavras-chave: Problemas de corte de estoque multiperíodo, Produção de laje alveolar, Produção de laje treliçada, Relax-and-fix, Geração de colunas.

## **ABSTRACT**

*This study looks at the production planning of precast slabs derived from specialized Brazilian factories. The first part of this research considers the production process of hollow-core slabs. For this, two mathematical models are proposed for the arising problem, which consists of a multi-period one-dimensional cutting stock problem with innovative aspects regarding the multiple manufacturing modes that can be used to produce the slabs. These models are solved using heuristic methods, both models aiming at the minimization of production and inventory costs. Based on real data provided by a company, the computational results indicate the relevance of such models as auxiliary tools in decision-making. The second part of this study addresses the production of lattice slabs, which comprises a process in two stages: cutting steel trusses and concreting them in the molds. The proposed mathematical model is expressed by a two-stage multi-period one-dimensional cutting stock problem and aims at the minimization of setup, production and inventory costs, subject to capacity and demand balance constraints. To solve this model, different strategies based on the column generation procedure are implemented to generate the cutting patterns used in the production and then different rounding strategies are applied to identify integer solutions. The instances are based on information obtained from the factory and computational results are used to evaluate the application of the proposed model in a realistic context.*

*Keywords: Multi-period cutting stock problems, Hollow-core slab production, Lattice slab production, Relax-and-fix, Column generation.*

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# List of Abbreviations

<b>KMPCSP</b>	Kantorovich-based multi-period cutting stock problem
<b>GGMPCSP</b>	Gilmore and Gomory-based multi-period cutting stock problem
<b>DWMPCSP</b>	Dantzig-Wolfe-based multi-period cutting stock problem
<b>MILP</b>	Mixed-integer linear programming
<b>RF</b>	Relax-and-fix
<b>CG</b>	Column generation
<b>RMP</b>	Restricted master problem
<b>O</b>	Optimal solution
<b>F</b>	Feasible solution
<b>NF</b>	Non-feasibility
<b>U</b>	Unknown solution status
<b>OF</b>	Objective function
<b>ICG</b>	Integrated column generation
<b>SCG</b>	Separated column generation

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# 1 Introduction

The advances in infrastructure conquered by humanity in the last two centuries are clear. In civil construction, industrial processes ease and speed up building while offering durability, sustainability and quality control. In the second industrial age, precast concrete combined with steel emerged as new technology to compose structural elements of buildings, including joists, beams, columns, modular boxes, stairways and walls, among other components. The use of precast elements enhances technological and social development since their application combines more sophisticated equipment, better working conditions and more efficient quality management. Consequently, an increasing use of precast concrete has been seen (Debs, 2000; Steinle et al., 2019). However, the construction industry in Brazil still has many challenges, since there is widescale waste of materials, low productivity, low quality, slow processes and an informal and low-skilled workforce. To face these challenges, this sector has been increasingly integrating innovative technologies and management tools into its production processes, aiming to increase productivity, quality, safety and client satisfaction (Debs, 2000; Barbosa and Vilnītis, 2017).

In the light of growing competitiveness, environmental responsibility and lack of resources, the industrial sector has had to develop decision-making tools intending to both design and produce materials economically and efficiently. As a scientific tool, mathematical modeling applied to industrial processes provides objective and measurable support for decision-making. In these circumstances, the cutting stock problem (CSP) can be a helpful approach to solve arising problems.

The cutting stock problem arises in many industries as a fundamental subproblem of production planning and establishes the best way to cut standard length objects (material units such as aluminum or paper coils, metal or wood sheets, and steel bars, among others) into smaller items with specified dimensions, minimizing material waste while meeting the demand (Dyckhoff, 1990, Wang and Wäscher, 2002, Wäscher et al., 2007, Morabito et al., 2009, Gomes et al., 2016). There are many studies of the CSP from different perspectives. For example, the residual length of a cutting pattern can be stocked for future use, i.e. handling the usable leftovers (Cui et al., 2017). A multi-objective approach can also be considered, aiming, for example, to minimize the

frequency of different cutting patterns to be used and, simultaneously, to minimize the number of cut objects (Aliano Filho et al., 2018, İhsan Erozan and Çalışkan, 2020, Oliveira et al., 2021). Another perspective in production planning is the approach of multiple periods, where the decision can be made looking not only at the demand of a single period but considering the demand of future periods of a planning horizon (Aktin and Özdemir, 2009).

The main objective of this thesis is to study mixed-integer linear programs applied to the production planning of hollow-core slabs and lattice slabs. According to the classification in Melega et al. (2018), the models presented here consist of a *Multi-Period Cutting Stock Problems* (MPCSP), also denoted by  $(- /L2/ - /M)$ , since they do not presuppose more than one production level and allow integration of periods by carrying inventory items from one period to another. Here, the one-dimensional case of the cutting process is considered.

In Chapter 2, there is a literature review on the cutting stock problems, including papers with similar applications to those addressed in this research, along with a discussion of the mathematical formulations and papers that address mixed-integer mathematical models with cutting processes of two stages. The next two chapters of this work are based on the production planning of precast slabs derived from two different companies. Chapter 3 addresses the hollow-core slab production planning problem of a plant, and Chapter 4 is based on the production process of a factory specialized in lattice slabs.

More specifically, Chapter 3 is based on the published paper by Signorini et al. (2021) that focuses on the production planning of hollow-core slabs at a specific plant, which is briefly described next. First, an order form is considered. It contains the client order specifications: prestressed wire configurations, the length of the slabs and the due date. Based on this information, the production manager plans weekly when, which and how many hollow-core slabs (items) should be made in each available mold. Following these instructions, the assembly sector conducts the preparation of the mold, prestressing, casting the concrete, curing, post-tensioning, wire release and item removal. A more detailed description of this process is presented in Section 3.1. Considering the previously-described production process and focusing on the weekly planning decision on which hollow-core slabs will be produced in each available mold, the main contribution of Chapter 3 is the proposal of two mathematical models based on the cutting stock problem literature, aiming at the minimization of inventory and production costs (mainly related to the use of steel wires) subject to demand balance and capacity constraints, as presented in Section 3.2. There is one aspect of these models which, to the best of our knowledge, has not appeared in other studies in the literature about cutting

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stock problems. In this study, the handling of wire configurations is more comprehensive, as it contemplates the combination of items with different wire configurations in the same production line (same mold) under an unusual restriction: to reduce the total costs, an item might be produced with a wire configuration whose load is equal to or higher than that required (multiple manufacturing modes). Moreover, a theoretical analysis of the compact and extended formulations is discussed in Section 3.3. The solution methods are described in Section 3.4. The first proposed model is solved by an exact method and heuristic methods, based on decomposition approaches, are used to solve both models. Considering a data set based on real information, Section 3.5 analyzes the computational results, which provides an interesting introspection for the industry. Section 3.6 summarizes the outcomes of Chapter 3 and some directions to amend this study.

Looking upon the lattice slab production process of a Brazilian factory, the main contribution of Chapter 4 is the proposal of a mathematical model for the production planning of lattice joist based on the one-dimensional multi-period cutting stock problem with two stages. In the joists' production process, a limited number of molds is used, which are divided into sections of equal length. The production planning of lattice slabs is also weekly and the production process starts with the preparation of the molds. Next, each mold is filled with concrete. The steel trusses previously cut into the length of the items are used as reinforcement, being inserted into the still liquid concrete base. Subsequently, the items are removed from the molds, resetting the production cycle. A more detailed description of the production process of lattice joists is presented in Section 4.1. The main concerns of the factory are the waste of steel trusses during the cutting process, the minimization of the setup and inventory costs and the increase in productivity. Considering a cutting process with two stages, Section 4.2 introduces a mathematical model based on the cutting pattern formulation that matches the factory objectives while meeting demand and satisfying capacity constraints. The first stage is the cutting of steel trusses of standard length into pieces of specific lengths, and the second stage consists of dividing the molds with separators that will outline the allocation of each item. Different cutting patterns in the multiple sections of each mold are allowed, which entails a setup cost related to this use. To the best of our knowledge, this approach has not been handled by any other studies in the literature on cutting stock problems, since we optimize the multi-period cutting stock decisions in a two-stage scenario simultaneously. Another contribution of this research is the proposition of different column generation-based methods with innovative aspects arising from the two-stage problem, where two types of subproblems have to be solved. Exploring decomposition by stages, different rounding strategies are

also proposed. Details are specified in Section 4.3. To evaluate the application of the proposed model in a realistic context, Section 4.4 presents the computational results based on information collected from the factory. Conclusions and opportunities for further research are discussed in Section 4.5.

Finally, Chapter 5 revises the contributions of this thesis and points out some perspectives for extending the research presented here. Other contributions of this thesis are presented in the appendices. Appendix A illustrate by a numerical example the application of the solution methods proposed in Chapter 3. Tables with more detailed computational results of Chapter 3 can be found in Appendix B. Variations of the solution strategies proposed in Chapter 4 are in Appendix C. Lastly, Appendix D contains a general example to illustrate the model proposed in Chapter 4 and the application of one of the proposed solution strategies to a numerical example.

## 5 Conclusion and future research

The main objective of this research was to develop mathematical-computational tools that assist the decision making for the production planning of hollow-core slabs and lattice slabs motivated by partnership with precast concrete producers in Brazil.

Considering the production planning of hollow-core slabs at a specific plant, two models based on compact and extended formulations of the one-dimensional multi-period cutting stock problems were proposed in the first part of this thesis. Both models aim at the minimization of inventory penalty and production costs (mainly related to the use of steel wires while satisfying demand balance and capacity constraints). Also, both models handle with wire configurations more comprehensively, since they foresee the combination of items with different wire configurations in the same production line provided that the wire configuration of a produced item is equal or higher than that required. To the best of our knowledge, this feature has not appeared in other studies. To solve the compact model KMPCSP, the MILP approach and the Relax-and-fix heuristic were applied, while a strategy based on the column generation procedure was used to solve the extended model GGMPSP. The computational results used fictional instances generated based on real data from the plant. A better efficiency of the compact model compared to the extended model was observed. This unusual result arises from some particularities of this problem that produces a demand matrix and cutting pattern matrix of higher dimensions, which increases the complexity in generating interesting columns, forcing the hardware. It was also noted that bin-packing based models (compact formulation) are generally considered by studies where the average demand for items is low and a weak lower bound to the integer problem is produced, along with symmetrical structures, decreasing the efficiency of branch-and-bound methods. On the other hand, models based on the cutting stock problems (extended formulation) perform better in cases where the average demand for items is high, granting a tight linear relaxation and stronger lower bounds through the elimination of some fractional solutions and removal of symmetries. In this study, the low demand rate in the instances tested caused a drawback in the performance of the column generation procedure used to solve the extended formulation, provoking its low efficiency. Furthermore, a theoretical analysis of the compact and extended

formulations was argued.

Another widely used precast slab in Brazil is the lattice slab, the focus of the second part of this research. Considering the production planning of lattice joists (which compose the slab) and keeping in view the central interests of the factory (the minimization of setup and production costs and waste of steel trusses, while managing the inventory level and increasing productivity), a model based on the compact formulation for the multi-period cutting stock problem with two stages was proposed. The first stage comprehends the cutting of steel trusses of standard length into smaller pieces, while the second stage involve the division of the molds with separators, outlining the placement of each item. Since this model optimizes cutting stock decisions in a multi-period and two-stage scenario concomitantly, to the best of our knowledge, it provides an innovative addition to the related literature. An additional contribution of this research is the proposal of three different solution strategies based on the column generation procedure that explore the two-stage feature, where two groups of subproblems are used to deal with the two cutting processes involved. The computational results used fictional instances based on existing orders from the factory in focus, and provide a very positive evaluation that supports the application of such research in the real-world problem.

Opportunities for further research include the incorporation of other production processes, e.g., the acquisition of raw material, packing, and distribution of final products. Readjustments in the proposed models and the use of alternative mathematical models, such as the arc-flow formulation, might be necessary. Other possibilities to extend this research are improving the solution methods addressed here, while comparing their results to others existent in the literature, implementing different techniques to generate interesting columns, and exploring combinations of different heuristic approaches.

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