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Mixed Charmonium-Molecule Interpretation for the $Y(3940)$ State

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Abstract. QCD sum rules are used to study the mass and the decay widths of the $Y(3940)$ state. We assume that it can be described by a mixed charmonium-molecule scalar current with $J^{PC} = 0^{++}$. Using a mixing angle $\theta = (76.0 \pm 5.0)^\circ$, we obtain $M_Y = (3.95 \pm 0.11)$ GeV, which is in good agreement with the experimental mass of the $Y(3940)$ state. We also evaluate the decay width in the channels $Y \rightarrow J/\psi \omega$ and $Y \rightarrow \gamma\gamma$ obtaining the values $\Gamma_{Y \rightarrow J/\psi \omega} \approx (1.7 \pm 0.6)$ MeV and $\Gamma_{Y \rightarrow \gamma\gamma} \approx (1.6 \pm 1.3)$ KeV, respectively. We also study the decay process of this state into channels containing $D\bar{D}$ mesons in the final state.

1. Introduction

Most of the new charmonium states discovered in recent years at the B factories, called X , Y , Z particles, do not seem to have a simple $c\bar{c}$ structure. Their production mechanism, masses, decay widths, spin-parity assignments and decay modes have been discussed in some reviews [1, 2, 3]. Although the masses of these states are above the corresponding thresholds of decays into a pair of open charm mesons, they decay into J/ψ or ψ' plus pions, which is unusual for $c\bar{c}$ states. Besides, their masses and decay modes are not in agreement with the predictions of potential models, which, in general, describe very well $c\bar{c}$ states. For these reasons, they are considered as candidates for exotic states such as hybrid, molecular or tetraquark states, with a more complex structure than the simple quark-antiquark states.

In particular, several states in the region of mass of about 3940 MeV has been observed in different processes of production and decay. The $Y(3940)$, observed by BELLE Collaboration in the decay $B \rightarrow (J/\psi \omega)K$ [4], was the first of these states. It has been also observed by BABAR Collaboration, with a slightly smaller mass and decay width[5]. BELLE Collaboration has also reported the observation of a state in the process $\gamma\gamma \rightarrow D\bar{D}$, which was called $Z(3930)$ [6, 7]. However, this state is generally linked to the charmonium state $\chi_{c2}(2P)$. Following these observations, the state $X(3915)$ was reported by BELLE Collaboration in the process $\gamma\gamma \rightarrow J/\psi \omega$ [8]. The mass and total decay width of these states are shown in in Table 1.

Although all these states could be connected to the same particle observed in different processes, there are evidences that, at least, two of these reported states, $Y(3940)$ and $X(3915)$,



Table 1. Mass and decay width of the states in the mass region 3940 GeV

state	mass (MeV)	total decay width (MeV)	ref.
Y(3940)	$3943 \pm 11 \pm 13$	$87 \pm 22 \pm 26$	[4]
Y(3940)	$3914.6^{+3.9}_{-3.4} \pm 2.0$	$34^{+12}_{-8} \pm 5.0$	[5]
Z(3930)	$3929 \pm 5 \pm 2$	$29 \pm 10 \pm 2$	[6, 7]
X(3915)	$3915 \pm 3 \pm 2$	$17 \pm 10 \pm 3$	[8]

could be interpreted as molecular states. In Ref. [1] these two states were interpreted as the same state and it was called $X(3915)$. However, the Particle Data Group [9] associates the label $X(3915)$ to the charmonium state $\chi_{c0}(2P)$. Therefore, to avoid misinterpretation, here we use the label $Y(3940)$ to identify the state observed in the decay mode $J/\psi \omega$.

In Ref. [10], it was proposed that the $Y(3940)$ could be a molecular state $D^* \bar{D}^*$, with quantum numbers $J^{PC} = 0^{++}$ or 2^{++} . In Ref. [11], the Y state was studied with QCD Sum Rules (QCDSR) method [12, 13, 14] as a $D^* \bar{D}^*$ molecule with quantum numbers 0^{++} and the mass obtained was $m_{D^* \bar{D}^*} = (4.13 \pm 0.10)$ MeV, failing to reproduce the experimental mass of the state. Here we study the $Y(3940)$, within QCDSR approach, using a mixed charmonium-molecule current, following the works in Refs. [15, 16, 17, 18] where the states $X(3872)$ and $Y(4260)$ were considered as molecule-charmonium states.

2. Mixed Hadronic Current

We use the QCDSR approach to describe the $Y(3940)$ as a mixing between the χ_{c0} charmonium and the $D^* \bar{D}^*$ molecule, with $J^{PC} = 0^{++}$. We use the following hadronic current

$$j = a \cos \theta j_{\chi_{c0}} + \sin \theta j_{D^* \bar{D}^*} \quad (1)$$

where θ is an arbitrary mixing angle and the meson and molecule currents are given by:

$$j_{\chi_{c0}} = \bar{c}_k c_k, \quad j_{D^* \bar{D}^*} = (\bar{q}_i \gamma_\mu c_i) (\bar{c}_j \gamma^\mu q_j) \quad (2)$$

The factor a is introduced in Eq. (2) in such way that the mixed current can be evaluated at the same Fock space. We use [15, 16, 17]

$$a = -\frac{\langle \bar{q} q \rangle}{\sqrt{2}} \quad (3)$$

2.1. Two-Point Correlation Function

The mass of a hadronic state is obtained, in the QCDSR approach, by considering the two-point correlation function

$$\Pi(q) = i \int d^4x e^{iq \cdot x} \langle 0 | T[j(x) j^\dagger(0)] | 0 \rangle \quad (4)$$

In the QCDSR approach, Eq. (4) is evaluated in two ways: the phenomenological side and the QCD side. In the QCD side, one uses the Wilson's operator product expansion (OPE), so the QCD side is also called the OPE side. Inserting Eq. (1) into the above equation, one obtains:

$$\Pi^{OPE}(q) = i \int d^4x e^{iq \cdot x} \left\{ \frac{1}{2} \langle \bar{q} q \rangle^2 \cos^2 \theta \Pi_{\chi_{c0}} + \sin^2 \theta \Pi_{D^* \bar{D}^*} - \frac{\langle \bar{q} q \rangle}{\sqrt{2}} \sin \theta \cos \theta \left[\Pi_{mix} + \Pi_{mix}^* \right] \right\} \quad (5)$$

where the $\Pi_{\chi_{c0}}(x)$ and $\Pi_{D^*D^*}(x)$ functions are, respectively, the correlation functions of the χ_{c0} meson and the D^*D^* (0^{++}) molecular state, which have been calculated in other works [13, 19]. Thus, one only has to calculate the $\Pi_{mix}(x)$ and $\Pi_{mix}^*(x)$ functions defined as follows:

$$\Pi_{mix}(x) = \langle 0 | T[j_{\chi_{c0}}(x) j_{D^*D^*}^\dagger(0)] | 0 \rangle = -\text{Tr} \left[S_{ji}^q(0) \gamma_\mu S_{ik}^c(-x) S_{kj}^c(x) \gamma^\mu \right] \quad (6)$$

$$\Pi_{mix}^*(x) = \langle 0 | T[j_{D^*D^*}(x) j_{\chi_{c0}}^\dagger(0)] | 0 \rangle = -\text{Tr} \left[S_{ji}^q(0) \gamma_\mu S_{ik}^c(x) S_{kj}^c(-x) \gamma^\mu \right] \quad (7)$$

where $S^c(x)$ and $S^q(x)$ are the charm- and light-quark propagators, respectively. The correlation function can be written in terms of a dispersion relation, such that

$$\Pi^{OPE}(q^2) = \int_{4m_c^2}^{\infty} ds \frac{\rho^{OPE}(s)}{s - q^2}, \quad (8)$$

where $\rho^{OPE}(s)$ is given by the imaginary part of the correlation function: $\pi \rho^{OPE}(s) = \text{Im}[\Pi^{OPE}(q^2 = s)]$. According to Eq. (5), the expression for the spectral density is:

$$\rho^{OPE}(s) = \frac{1}{2} \langle \bar{q}q \rangle^2 \cos^2 \theta \rho_{\chi_{c0}}(s) + \sin^2 \theta \rho_{D^*D^*}(s) - \frac{\langle \bar{q}q \rangle}{\sqrt{2}} \sin \theta \cos \theta \rho_{mix}(s) . \quad (9)$$

We calculate the sum rule at leading order in α_s considering the contributions from the condensates up to dimension-8 in the OPE. The expressions for the spectral density are given in ref. [20].

To evaluate the phenomenological side we insert, in Eq. (4), a complete set of intermediate states, Y , which couple to the hadronic current in Eq. (1). We get

$$\Pi^{PHEN}(q) = \frac{\lambda_Y^2}{M_Y^2 - q^2} + \int_0^{\infty} ds \frac{\rho^{cont}(s)}{s - q^2}, \quad (10)$$

where M_Y is the ground state mass, λ_Y , gives the coupling between the state and the current:

$$\langle 0 | j | Y \rangle = \lambda_Y , \quad (11)$$

and the second term in the RHS of Eq. (10) denotes the continuum (or higher resonance) contributions. Assuming that the continuum contribution to the spectral density, $\rho^{cont}(s)$ in Eq. (10), coincides with the one obtained in the OPE side above a certain threshold s_0 , we get [21]:

$$\rho^{cont}(s) = \rho^{OPE}(s) \Theta(s - s_0) \quad (12)$$

where $\Theta(s - s_0)$ is the Heaviside step function.

To improve the matching between the two sides of the sum rule, one performs the Borel transform. After transferring the continuum contributions to the OPE side and performing the Borel transform to the two sides of the sum rule, one gets:

$$\lambda_Y^2 e^{-M_Y^2/M_B^2} = \int_{4m_c^2}^{s_0} ds e^{-s/M_B^2} \rho^{OPE}(s) . \quad (13)$$

The ground state mass can be obtained from the following ratio:

$$M_B^2 = \frac{\int_0^{s_0} ds s e^{-s/M_B^2} \rho^{OPE}(s)}{\int_0^{s_0} ds e^{-s/M_B^2} \rho^{OPE}(s)} \cdot \frac{4m_c^2}{4m_c^2}. \quad (14)$$

2.2. Three-Point Correlation Function

To compute the form factor associated with the vertex $Y J/\psi \omega$ and to estimate the width of the channel $Y(3940) \rightarrow J/\psi \omega$ we consider the three-point function:

$$\Pi_{\mu\nu}(p, p', q) = \int d^4x d^4y e^{ip' \cdot x} e^{iq \cdot y} \Pi_{\mu\nu}(x, y), \quad (15)$$

where $p = p' + q$ and $\Pi_{\mu\nu}(x, y)$ is given by

$$\Pi_{\mu\nu}(x, y) = \langle 0 | T \{ j_\mu^\psi(x) j_\nu^\omega(y) j^\dagger(0) \} | 0 \rangle. \quad (16)$$

The interpolating currents for the J/ψ and ω mesons are given by:

$$j_\mu^\psi = (\bar{c}_a \gamma_\mu c_a), \quad j_\nu^\omega = \frac{1}{6} (\bar{u}_a \gamma_\nu u_a + \bar{d}_a \gamma_\nu d_a), \quad (17)$$

and the mixed $(\chi_{c0}) - (D^* \bar{D}^*)$ state is described by the current in Eq. (1).

In the OPE side, we calculate the correlation function at leading order in α_s and we consider condensates up to dimension 7. The three-point function includes a number of different Lorentz structures and we choose to work with the $q_\mu p'_\nu$ structure [20]. In general, for any given structure, the sum rule method is inapplicable at large Q^2 where the power corrections become large and uncontrollable. At small Q^2 , the situation is even worse since when approaching the physical region the operator expansion stops working. In this sense, one has to consider that the sum rule is valid up to a rather small Q^2 and the extrapolation from the values of Q^2 to the physical region can be obtained with a good accuracy.

In the phenomenological side we consider intermediate states for the J/ψ , ω and $Y(3940)$ mesons. We get:

$$\Pi_{\mu\nu}^{PHEN}(p, p', q) = \frac{\lambda_Y M_\psi f_\psi M_\omega f_\omega g_{Y\psi\omega}(q^2)}{(p^2 - M_Y^2)(p'^2 - M_\psi^2)(q^2 - M_\omega^2)} \times \left[q_\mu p'_\nu - (p' \cdot q) g_{\mu\nu} \right] + \dots, \quad (18)$$

where we have used the following relations:

$$\langle 0 | j_\mu^\psi | J/\psi(p') \rangle = M_\psi f_\psi \epsilon_\mu(p'), \quad \langle 0 | j_\nu^\omega | \omega(q) \rangle = M_\omega f_\omega \epsilon_\nu(q), \quad \langle Y(p) | j | 0 \rangle = \lambda_Y.$$

The dots in Eq. (18) stand for the contribution of all possible excited states.

The form factor, $g_{Y\psi\omega}(q^2)$, is defined by the generalization of the on-shell mass matrix element, $\langle J/\psi \omega | Y \rangle$, for an off-shell ω meson:

$$\langle J/\psi \omega | Y \rangle = g_{Y\psi\omega}(q^2) \left[(p' \cdot \epsilon^*(q)) (q \cdot \epsilon^*(p')) - (p' \cdot q) (\epsilon^*(p') \cdot \epsilon^*(q)) \right], \quad (19)$$

which can be extracted from the effective Lagrangian that describes the coupling between two vector mesons and one scalar meson:

$$\mathcal{L} = \frac{i}{2} g_{Y\psi\omega} V_{\alpha\beta} \Psi^{\alpha\beta} Y \quad (20)$$

where $V_{\alpha\beta} = \partial_\alpha \omega_\beta - \partial_\beta \omega_\alpha$ and $\Psi^{\alpha\beta} = \partial^\alpha \psi^\beta - \partial^\beta \psi^\alpha$, are the tensor fields of the ω and ψ fields respectively.

Matching both side of the sum rule, taking the approximation $p^2 \simeq p'^2 = -P^2$ and doing the Borel transform to $P^2 \rightarrow M_B^2$, we get the following expression in the $q_\mu p'_\nu$ structure:

$$\frac{\lambda_Y M_\omega f_\omega M_\psi f_\psi g_{Y\psi\omega}(Q^2)}{(M_Y^2 - M_\psi^2)(Q^2 + M_\omega^2)} \left(e^{-M_Y^2/M_B^2} - e^{-M_\psi^2/M_B^2} \right) + H(Q^2) e^{-s_0/M_B^2} = \Pi^{OPE}(M_B^2, Q^2), \quad (21)$$

where $Q^2 = -q^2$, and the function $H(Q^2)$ represents the contribution of the pole-continuum transitions [15, 22, 23, 24]. The $\Pi^{OPE}(M_B^2, Q^2)$ function is

$$\Pi^{OPE}(M_B^2, Q^2) = \sin \theta \int_{4m_c^2}^{+\infty} ds e^{-s/M_B^2} \rho(s, Q^2), \quad (22)$$

where $\rho = \rho^{pert} + \rho^{\langle \bar{q}q \rangle} + \rho^{\langle G^2 \rangle} + \rho^{\langle \bar{q}Gq \rangle} + \rho^{\langle \bar{q}q \rangle \langle G^2 \rangle}$ is given in ref. [20].

As in previous calculations [15, 16, 17, 18], only the molecule part of the mixed current defined in Eq. (1) contributes to the decay channel $Y(3940) \rightarrow J/\psi \omega$. This can be seen by the presence of the sine function in Eq. (22).

To extract the value of the coupling constant associated with the $Y \rightarrow J/\psi \omega$ vertex we first determine the form factor $g_{Y\psi\omega}(Q^2)$ in Eq. (21). To do that we follow the procedure described in [20].

3. Numerical Analysis

The numerical values for the quark masses and condensates are listed in Table 2.

Table 2. QCD input parameters.

Parameters	Values
$\bar{m}_c$	$(1.23 - 1.47) \text{ GeV}$
$\langle \bar{q}q \rangle$	$-(0.23 \pm 0.03)^3 \text{ GeV}^3$
$\langle g_s^2 G^2 \rangle$	$(0.88 \pm 0.25) \text{ GeV}^4$
$\langle g_s^3 G^3 \rangle$	$(0.58 \pm 0.18) \text{ GeV}^6$
$m_0^2 \equiv \langle \bar{q}Gq \rangle / \langle \bar{q}q \rangle$	$(0.8 \pm 0.1) \text{ GeV}^2$
$\rho \equiv \langle \bar{q}q\bar{q}q \rangle / \langle \bar{q}q \rangle^2$	$(0.5 - 2.0)$

We start our analysis discussing the possible values of the continuum threshold s_0 and the mixing angle θ . A reasonable initial value for the continuum threshold would be $\sqrt{s_0} \sim M_Y + 0.5 \text{ GeV} \sim 4.40 \text{ GeV}$, since we are interested in a state with a mass $M_Y \sim 3.9 \text{ GeV}$. To fix the value of the mixing angle θ , we search for a value which allows us to determine the best M_B^2 -stability inside of a valid Borel window. We find that the optimal choice is

$$\sqrt{s_0} = (4.40 \pm 0.10) \text{ GeV} \quad (23)$$

$$\theta = (76.0 \pm 5.0)^\circ. \quad (24)$$

We notice that the OPE does not converge for θ values outside this range.

Using these values, we analyze the relative contributions of the terms in the OPE. As one can see in Fig. 1, the contribution of the dimension-8 condensate is smaller than 20% of the total contribution for values of $M_B^2 \geq 2.40 \text{ GeV}^2$, which indicates the starting point for a good OPE

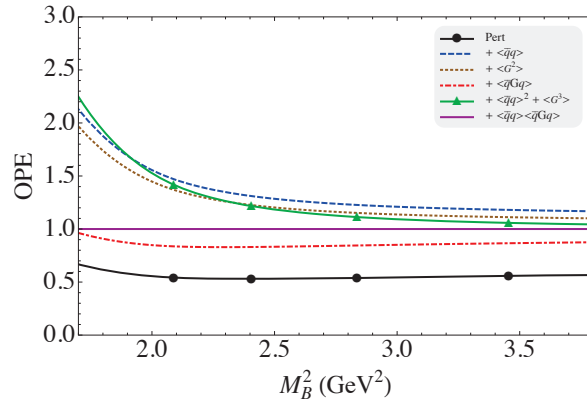


Figure 1. OPE convergence in the region $1.7 \leq M_B^2 \leq 3.8 \text{ GeV}^2$ for $\sqrt{s_0} = 4.40 \text{ GeV}$ and $\theta = 76.0^\circ$. One plots the relative contributions starting with the perturbative contribution (line with circles), and each other line represents the relative contribution after adding of one extra condensate in the expansion: $+\langle \bar{q}q \rangle$ (dashed line), $+\langle G^2 \rangle$ (dotted line), $+\langle \bar{q}Gq \rangle$ (dot-dashed line), $+\langle \bar{q}q \rangle^2 + \langle G^3 \rangle$ (line with triangles) and $\langle \bar{q}q \rangle \cdot \langle \bar{q}Gq \rangle$ (solid line). Extracted from ref. [20]

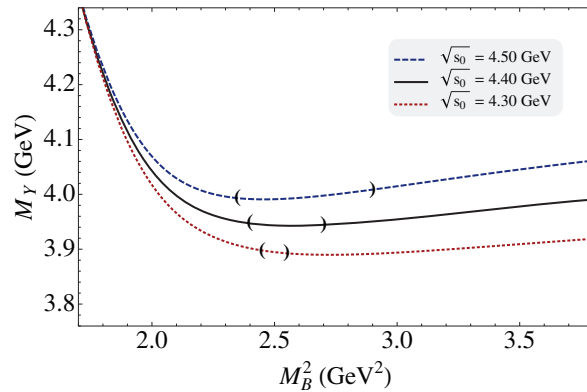


Figure 2. The mass as a function of the sum rule parameter M_B^2 for $\sqrt{s_0} = 4.30 \text{ GeV}$ (dotted line), $\sqrt{s_0} = 4.40 \text{ GeV}$ (solid line) and $\sqrt{s_0} = 4.50 \text{ GeV}$ (dashed line). The respective parentheses indicate the valid Borel window. Extracted from ref. [20]

convergence. We fix the maximum value of the Borel mass parameter as the value for which the pole contribution is greater than or equal to the continuum contribution. This condition is satisfied when $M_B^2 \leq 2.70 \text{ GeV}^2$ [20]. Therefore, the Borel window is set as $2.40 \leq M_B^2 \leq 2.70 \text{ GeV}^2$.

The ground state mass, as a function of M_B^2 , is shown in Fig. 2, for three different values of $\sqrt{s_0}$. We conclude that there is a good M_B^2 -stability in the determined Borel window.

Varying the value of the continuum threshold in the range $\sqrt{s_0} = (4.40 \pm 0.10) \text{ GeV}$, the mixing angle in the range $\theta = (76.0 \pm 5.0)^\circ$, and the other parameters as indicated in Table 2, we get:

$$M_Y = (3.95 \pm 0.11) \text{ GeV} . \quad (25)$$

This mass is compatible with the experimental mass of the $Y(3940)$ state observed by BELLE Collaboration [4]. Therefore, from a QCD sum rule point of view, a mixed scalar $(\chi_{c0})-(D^*\bar{D}^*)$ state could be a good candidate to explain the $Y(3940)$ state.

The coupling parameter, defined in Eq. (11), can be estimates by using the result obtained for the mass in Eq. (13). We obtain:

$$\lambda_Y = (2.1 \pm 0.6) \times 10^{-2} \text{ GeV}^5. \quad (26)$$

In the numerical analysis of the three-point function we use the experimental values of the meson masses and decay constants: $M_\psi = 3.10 \text{ GeV}$, $f_\psi = 0.405 \text{ GeV}$, $M_\omega = 0.782 \text{ GeV}$, $f_\omega = 0.046 \text{ GeV}$. For the Y mass, we use the experimental value in Ref. [4] and the meson-current parameter λ_Y given in Eq. (26).

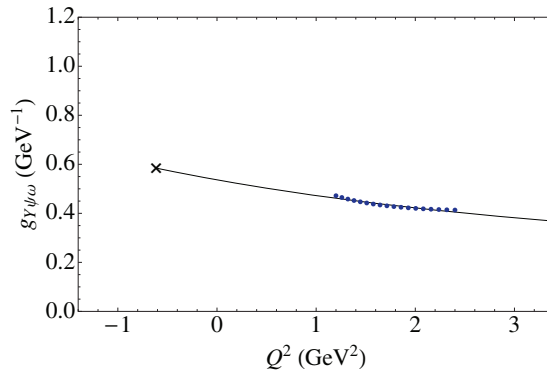


Figure 3. QCDSR results for the form factor $g_{Y\psi\omega}(Q^2)$, for $\sqrt{s_0} = 4.40 \text{ GeV}$ (circles). The solid line gives the parametrization of the QCDSR results through Eq. (27). The cross is the value of the coupling constant. Extraced from ref. [20]

We find that the form factor $g_{Y\psi\omega}(Q^2)$ is a very stable function of M_B^2 in Borel mass region $1.8 \text{ GeV}^2 \leq M_B^2 \leq 4.0 \text{ GeV}^2$ [20]. Therefore, the form factor dependence in Q^2 can be evaluated by taking the average of the M_B^2 values inside this stability region. The results are shown in Fig. 3.

The coupling constant is defined by the value of the form factor at the ω meson pole, $Q^2 = -M_\omega^2$. Therefore, to determine the coupling constant we have to extrapolate the form factor to the region of Q^2 where the QCDSR is not valid. This extrapolation can be done by parametrizing the QCDSR results, shown in Fig. 3 for $g_{Y\psi\omega}(Q^2)$, using a monopolar function:

$$g_{Y\psi\omega}(Q^2) = \frac{g_1}{g_2 + Q^2}, \quad (27)$$

and the results for the fitting parameters are:

$$\begin{aligned} g_1 &= (4.0 \pm 1.0) \text{ GeV}; \\ g_2 &= (7.4 \pm 0.2) \text{ GeV}^2. \end{aligned} \quad (28)$$

The theoretical errors are evaluated considering errors on the following parameters: $\sqrt{s_0} = 4.40 \pm 0.10 \text{ GeV}$, $\theta = 76.0^\circ \pm 5.0^\circ$, and also the error on the meson coupling parameter λ_Y , given by Eq. (26). We notice that the results do not depend much on the parameters $\sqrt{s_0}$ and θ , while the theoretical errors are mainly affected by the meson coupling λ_Y .

The solid line in Fig. 3 represents Eq. (27) with values given in Eq. (28). The coupling constant, $g_{Y\psi\omega}$, indicated by the cross in Fig. 3, is obtained by using $Q^2 = -M_\omega^2$ in Eq. (27). We get:

$$g_{Y\psi\omega} = g_{Y\psi\omega}(-M_\omega^2) = (0.58 \pm 0.14) \text{ GeV}^{-1}. \quad (29)$$

The decay width for this process $Y(3940) \rightarrow J/\psi \omega$ is given by

$$\Gamma_{Y(3940) \rightarrow J/\psi \omega} = \frac{g_{Y\psi\omega}^2}{3} \frac{p(M_Y, M_\omega, M_\psi)}{8\pi M_Y^2} \times \left(M_\psi^2 M_\omega^2 + \frac{1}{2} (M_Y^2 - M_\psi^2 - M_\omega^2)^2 \right), \quad (30)$$

where

$$p(a, b, c) \equiv \frac{\sqrt{a^4 + b^4 + c^4 - 2a^2b^2 - 2a^2c^2 - 2b^2c^2}}{2a}. \quad (31)$$

Inserting the value obtained for the coupling constant (29) in (30) we get:

$$\Gamma_{Y(3940) \rightarrow J/\psi \omega} = (1.7 \pm 0.6) \text{ MeV}. \quad (32)$$

This result is consistent with the experimental width of the state and the lower limit for the process $Y \rightarrow J/\psi \omega$ [4, 5, 25, 26]. It is also of the same order as other available theoretical evaluations [27, 28].

We can use the same technique discussed above to determine the $Y(3940) \rightarrow \gamma\gamma$ decay width. This was done in ref. [20] and the result obtained was:

$$\Gamma_{Y(3940) \rightarrow \gamma\gamma} = (1.6 \pm 1.3) \text{ KeV}. \quad (33)$$

Based on this decay width value and the one obtained for the channel $J/\psi \omega$ in Eq. (32), the product of the two partial widths of the $Y(3940)$ is given by

$$\Gamma_{\gamma\gamma} \times \Gamma_{J/\psi \omega} \sim \mathcal{O}(10^3) \text{ KeV}^2, \quad (34)$$

which is in an excellent agreement with the measurements from BELLE and BABAR Collaborations in Refs. [8, 29].

4. Conclusions

We have used the QCDSR approach to study the two-point and three-point functions of the $Y(3940)$ state, considering it as a mixed charmonium-molecule state. We have determined the mixing angle to be $\theta = (76.0 \pm 5.0)^\circ$. For this mixing angle the mass obtained is in a very good agreement with the experimental value for the $Y(3940)$ state.

We have also evaluated the decay width in the channels $Y(3940) \rightarrow J/\psi \omega$ and $Y(3940) \rightarrow \gamma\gamma$. The values obtained for the width in these channels are: $\Gamma_{J/\psi \omega} = (1.7 \pm 0.6) \text{ MeV}$ and $\Gamma_{\gamma\gamma} = (1.6 \pm 1.3) \text{ KeV}$. These results are consistent with the lower limit for the channel $Y(3940) \rightarrow J/\psi \omega$: $\Gamma > 1 \text{ MeV}$ [27, 28], and with the product $\Gamma_{\gamma\gamma} \times \Gamma_{J/\psi \omega} \sim \mathcal{O}(10^3) \text{ KeV}^2$, determined by BELLE and BABAR Collaborations in Refs. [8, 29].

Thus, according to the available experimental data, we can conclude that a mixing between the χ_{c0} charmonium and the $D^*\bar{D}^*$ molecule, with $J^{PC} = 0^{++}$ quantum numbers, could be a good candidate to explain the $Y(3940)$ state.

Acknowledgments

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