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Research Article

Effective Synchronization of a Class of Chua's Chaotic Systems Using an Exponential Feedback Coupling

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A robust exponential function based controller is designed to synchronize effectively a given class of Chua's chaotic systems. The stability of the drive-response systems framework is proved through the Lyapunov stability theory. Computer simulations are given to illustrate and verify the method.

1. Introduction

Shortly after Pecora and Carroll showed the possibility of synchronizing chaotic elements [1], applications stretched out in many fields [2–4] giving rise to interdisciplinary research, since this phenomenon appears in many systems in a variety of ways [5–12]. Much research was done developing different strategies in the quest of effective synchronization such as adaptive synchronization [13–15], inverse synchronization [16], and antisynchronization [17–24]. The robustness of many of these methods, as surprising as it may appear, has already been demonstrated in many cases in the presence of noise, perturbations, or parameter mismatches [25–27].

For applications such as telecommunications, where the transmission of messages is not possible unless transmitter and receiver are synchronized [10–12], the investigation of new chaotic systems as well as the most effective means of synchronization is always of great importance. Thus, mathematical models [28, 29], mechanical systems [30], and electronic circuits [10, 13] are continually built. One of the best known electronic circuits is the Chua's oscillator [31, 32]. Although Chua's circuit is one of the simplest circuits in the literature, it has various complex chaotic dynamics properties which has made it a topic of extensive study [31–34]. A modified version of the circuit has also been topic of attention [33–35]. Its theoretical analysis and numerical simulations agree very well with experimental results.

Recently, some authors proposed a nonlinear controller in order to force synchronization with the purpose of saving energy [22, 23]. The nonlinear controllers used are based on bounded nonlinear functions [22, 23]. In this work we apply the exponential function based nonlinear controller to achieve synchronization between the drive-response systems when disturbances are present. Our controller has certain properties which makes it more advantageous to use it, such as the following properties: (1) it is easy to implement in practice; (2) it needs no adaptation algorithm; hence its electrical circuit remains simple; (3) it is faster than the synchronization based on fixed feedback gain which is usually used.

This work is organized as follows. In Section 2, the problem is formulated and the assumptions are given. Section 3 presents the main results. We use Lyapunov stability theory to study the robustness of our proposed controller. We show

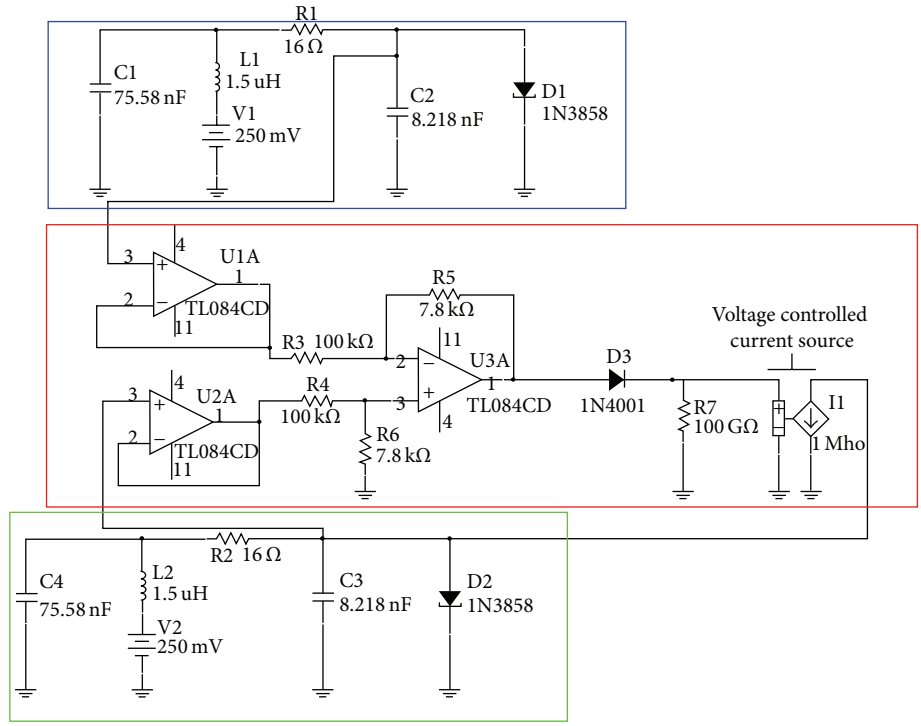


Figure 1: Scheme of the circuit for the whole system: transmitter (blue box), controller (red box), and receiver (green box).

that with this controller the drive and response systems are practically synchronized—the errors between the master system and the slave system do not tend to zero but to a limit value. In this case, it is shown that the derivative of Lyapunov function is contained in a closed domain to which the error between master and slave system converges. Since the error is sufficiently small, using the principle of the "ultimate boundedness property," we arrive to the conclusion that the system is globally stable because the derivative of the Lyapunov function is negatively definite. Ultimate boundedness is in particular compatible with local instability about zero and implies global stability. This was demonstrated by Ding and Cheng in [36]. They proposed a new criterion of globally uniformly ultimate boundedness for discrete-time nonlinear systems which helps to relax the condition of stability based on Lyapunov function. The same ideas were successfully applied by de la Sen and Alonso [37], while in [38], Bitsoris et al. work on the robust positive invariance and ultimate boundedness of nonlinear systems with unknown parameters and disturbances, where only their bounds of variance are known. In Section 4, numerical results are presented and we compare the given scheme with that using the simple fixed gain based controller. The conclusions are given in Section 5.

2. Formulation of the Problem

In this paper, we study the master-slave synchronization of a class of Chua's chaotic systems, represented in Figure 1 and described by the equations that follow.

The master system is given by:

$$\begin{aligned} \dot{x}_1(\tau) &= \alpha\left[x_2(\tau) - x_1(\tau) - Rf\left(x_1(\tau)\right)\right] + d(\tau), \\ \dot{x}_2(\tau) &= \beta\left[x_1(\tau) - x_2(\tau) + Rx_3(\tau)\right], \\ \dot{x}_3(\tau) &= \gamma\left[v(\tau) - x_2(\tau)\right], \end{aligned} \tag{1}$$

where τ is a dimensionless time, $x_i(t)$, $i = 1, 2, 3$, are the state variables, $v(\tau)$ is an external force, and α, β, γ, and R are positive constant parameters of the system. The function $f(x_1(\tau))$ represents the nonlinearity of the system and $d(\tau)$ the disturbances. The function $f(x_1(\tau))$ defines Chua's circuit, which is given by $f(x_1) = a_2x(\tau) + 0.5(a_1 - a_2)(|x_1(\tau) + 1| - |x_1(\tau) - 1|)$ [31, 33, 34], while the modified Chua's system is obtained using $f(x_1(\tau)) = ax_1^3$ [12] or $f(x_1(\tau)) = a_1(x_1 - b)^3 - a_2(x_1 - b) + a_3$ [32]. The latter represents the behavior of a tunnel diode [32]. For an autonomous system $v(\tau)$ is constant and $d(\tau) = 0$.

The slave system is given by

$$\begin{aligned} \dot{y}_1(\tau) &= \alpha\left[y_2(\tau) - y_1(\tau) - Rf\left(y_1(\tau)\right)\right] + U(\tau), \\ \dot{y}_2(\tau) &= \beta\left[y_1(\tau) - y_2(\tau) + Ry_3(\tau)\right], \\ \dot{y}_3(\tau) &= \gamma\left[v(\tau) - y_2(\tau)\right], \end{aligned} \tag{2}$$

where $y_i(\tau), i = 1, 2, 3$, is the slave state variables and $U(\tau)$ the feedback coupling.

Here we present a scheme to solve the synchronization problem for system (1). That is to say, if the uncertain system (1) is regarded as the drive system, a suitable response system

should be constructed to synchronize it with the help of the driving signal x. In order to do so, we assume the following:

(i) There is a bounded region $\mathcal{U} \subset R^3$ containing the whole basin of the drive system (1) such that no orbit of system (1) ever leaves it.

(ii) The disturbance $d(\tau)$ is bounded by an unknown positive constant D, namely,

$$\|d(\tau)\| \leq D, \tag{3}$$

where $\|\cdot\|$ denotes the euclidian norm of a vector.

(iii) All chaotic systems are supposed to be confined to a limited domain; hence there exists a positive constant L such that

$$\left\|f\left(y_1(\tau)\right) - f\left(x_1(\tau)\right)\right\| \leq L\left\|y_1(\tau) - x_1(\tau)\right\|. \tag{4}$$

We will now try to synchronize the systems described in (1) and (2) designing an appropriate control $U(\tau)$ in system (2) such that

$$\left\|y_i(\tau) - x_i(\tau)\right\| \leq r, \quad \text{for } \tau \longrightarrow \infty, \tag{5}$$

where r is a sufficiently small positive constant.

Let us define the state errors between the transmitter and the receiver systems as

$$e_i(\tau) = y_i(\tau) - x_i(\tau), \quad \text{with } i = 1, 2, 3, \tag{6}$$

and the feedback coupling as

$$U(\tau) = -\varphi\left(\exp\left(ke_1(\tau)\right) - 1\right), \tag{7}$$

where φ and k are positive fixed constants.

Introducing the definition of the systems (1), (2), and (7) into (2), the dynamics of the error states (6) becomes

$$\begin{aligned} \dot{e}_1(\tau) &= \alpha\left[e_2(\tau) - e_1(\tau) - R\left(f\left(y_1\right) - f\left(x_1\right)\right)\right] \\ &\quad - d(\tau) - \varphi\left(\exp\left(ke_1\right) - 1\right), \\ \dot{e}_2(\tau) &= \beta\left[e_1(\tau) - e_2(\tau) + Re_3(\tau)\right], \\ \dot{e}_3(\tau) &= -\gamma e_2(\tau). \end{aligned} \tag{8}$$

The problem is now reduced to demonstrating that with the chosen control law $U(\tau)$, the error states e_i, $i = 1, 2, 3$, in (8) are at most a sufficiently small positive constant r, which will prove the proposition.

3. Main Results

If we consider the master-slave chaotic systems (1) and (2) with all the aforementioned assumptions (3) and with the exponential function based feedback coupling given by the relation (7), we will show that the overall system will be practically synchronized, that is, $\|y_i(\tau) - x_i(\tau)\| \leq r$, where r is a sufficiently small positive constant for large enough τ.

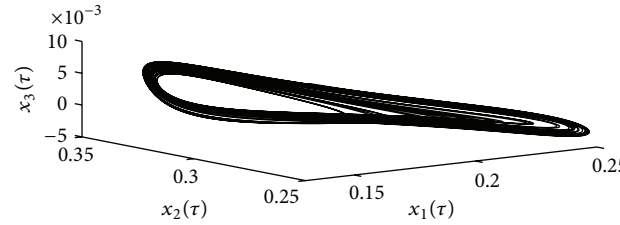


Figure 2: 3D chaotic attractor of the tunnel diode based modified Chua's system.

In order to do so, let us consider the following Lyapunov function:

$$V = \frac{1}{2}\left[\frac{e_1^2}{\alpha} + \frac{e_2^2}{\beta} + \frac{Re_3^2}{\gamma}\right]. \tag{9}$$

Differentiating the function V with respect to time yields

$$\begin{aligned} \dot{V} &= 2e_1e_2 - e_1^2 - e_2^2 - R\left(f\left(y_1\right) - f\left(x_1\right)\right)e_1 \\ &\quad - \frac{d(\tau)}{\alpha}e_1 - \frac{\varphi}{\alpha}\left[\exp\left(ke_1\right) - 1\right]e_1 \\ &= -\left(e_1 - e_2\right)^2 - R\left(f\left(y_1\right) - f\left(x_1\right)\right)e_1 \\ &\quad - \frac{d(\tau)}{\alpha}e_1 - \frac{\varphi}{\alpha}\left[\exp\left(ke_1\right) - 1\right]e_1. \end{aligned} \tag{10}$$

Expanding the exponential function as follows:

$$\begin{aligned} \exp\left(ke_1\right) - 1 \simeq ke_1 + \sum_{i=1}^{n}\frac{\left(ke_1\right)^{2i}}{2i!} + \sum_{i=1}^{n}\frac{\left(ke_1\right)^{2i+1}}{(2i+1)!} \\ + \theta\left(e_1\right) + \zeta\left(e_1\right), \end{aligned} \tag{11}$$

where $\theta(e_1)$ and $\zeta(e_1)$ constitute the rest of the expansion in order greater than n for odd part and for even part of the development, respectively, and substituting by the maximum value of the disturbance, D, it follows that

$$\begin{aligned} \dot{V} &\leq RLe_1^2 + \frac{D}{\alpha}\left|e_1\right| \\ &\quad - \frac{\varphi}{\alpha}\left(ke_1 + \sum_{i=1}^{n}\frac{\left(ke_1\right)^{2i}}{2i!} + \theta\left(e_1\right)\right)e_1. \end{aligned} \tag{12}$$

Hence, we have

$$\begin{aligned} \dot{V} &\leq \left(RL - \frac{\varphi}{\alpha}k\right)e_1^2 \\ &\quad + \frac{1}{\alpha}\left(D + \varphi\sum_{i=1}^{n}\frac{\left(k\left|e_1\right|\right)^{2i}}{2i!} + \left|\theta\left(e_1\right)\right|\right)\left|e_1\right|, \end{aligned} \tag{13}$$

$$\begin{aligned} \dot{V} &\leq \left(RL - \frac{\varphi}{\alpha}k\right)e_1^2 \\ &\quad + \frac{1}{\alpha}\left(D + \varphi\sum_{i=1}^{n}\frac{(kr)^{2i}}{(2i)!} + \Theta(r)\right)r, \end{aligned} \tag{14}$$

where $\Theta(r) \geq \max(\theta(e_1))$.

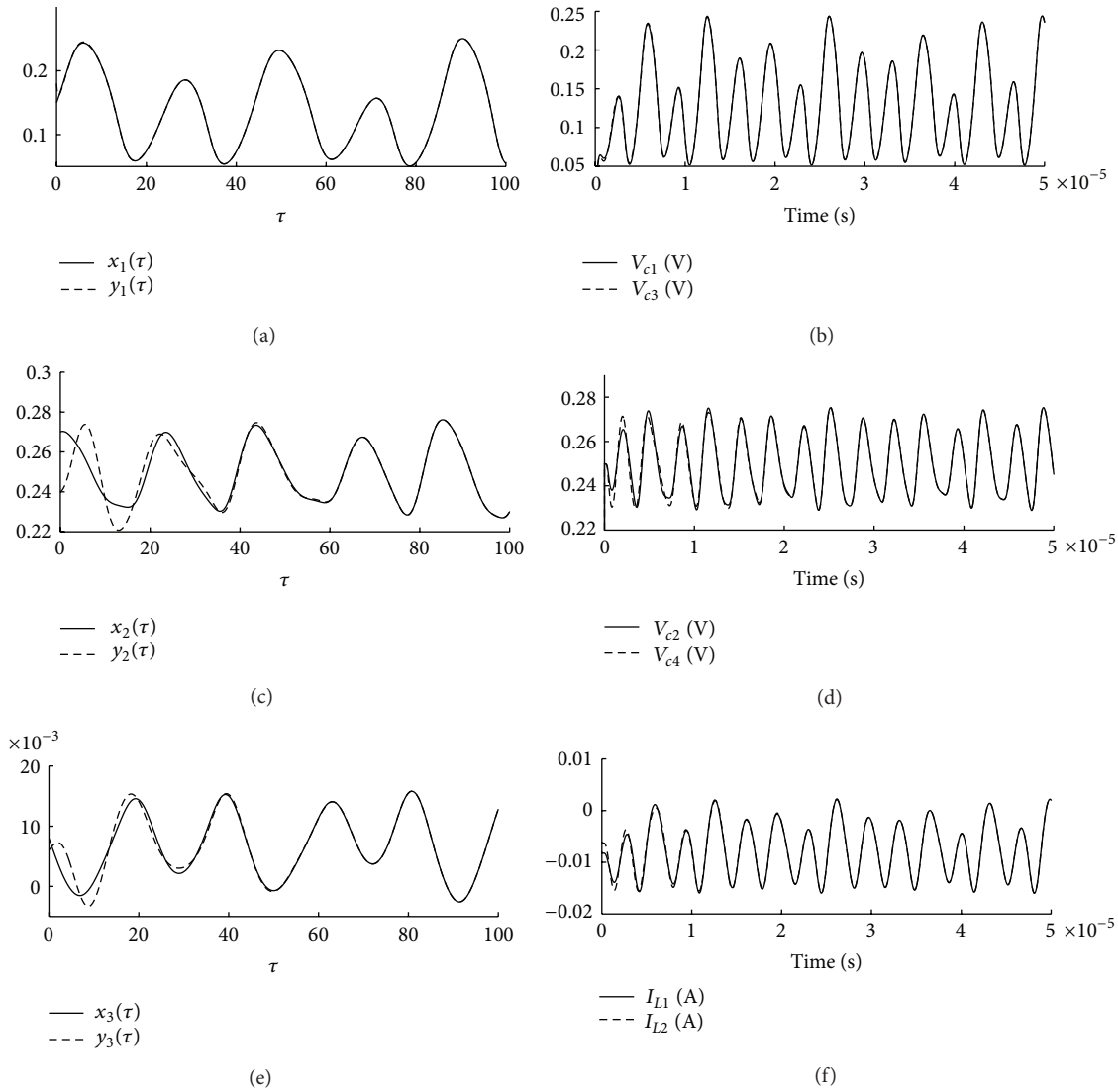


FIGURE 3: Time evolution of the master system (solid lines) and slave system (dashed lines) from Matlab simulations (left) and Pspice simulations (right).

Here we use r as an upper bound for the error in each axis. Then we see that the derivative of the Lyapunov function (12) is lower than that in (13), which in turn is smaller than the one given by (14). Thus expression (14) is maximized and the radius of the close domain to which the error is attracted is determined. Defining

$$\varphi \geq \frac{\alpha RL}{k}, \qquad \phi = \left(\frac{D + \Theta(r)}{\alpha} + \frac{RL}{k} \sum_{i=1}^{n} \frac{(kr)^{2i}}{(2i)!} \right) r, \quad (15)$$

one obtains

$$\dot{V} \leq -\psi e_1^2 + \phi, \quad \text{where } \psi = \left| RL - \frac{\varphi}{\alpha} k \right|. \quad (16)$$

Equation (16) is in principle a form of the ultimate boundedness property in the sense that if the error is sufficiently small, then the system is globally stable because the upper-bound is negative [36]. From (16), it follows that if

$$\|e_1\| > \sqrt{\frac{\phi}{\psi}} \quad (17)$$

therefore, $\dot{V}(\tau) < 0$; hence $V(\tau)$ decreases, which implies that $\|e_1\|$ decreases. It then follows from standard invariance arguments as in [23] that asymptotically for increasing time the error satisfies the following bound

$$\|e_1\| < C, \quad (18)$$

for any $C > \sqrt{\phi/\psi}$.

So if ϕ is sufficiently small, the bound for the synchronization error will also be sufficiently small. Therefore, the synchronization state error would be contained within a neighborhood of the origin, as we wanted to prove. In

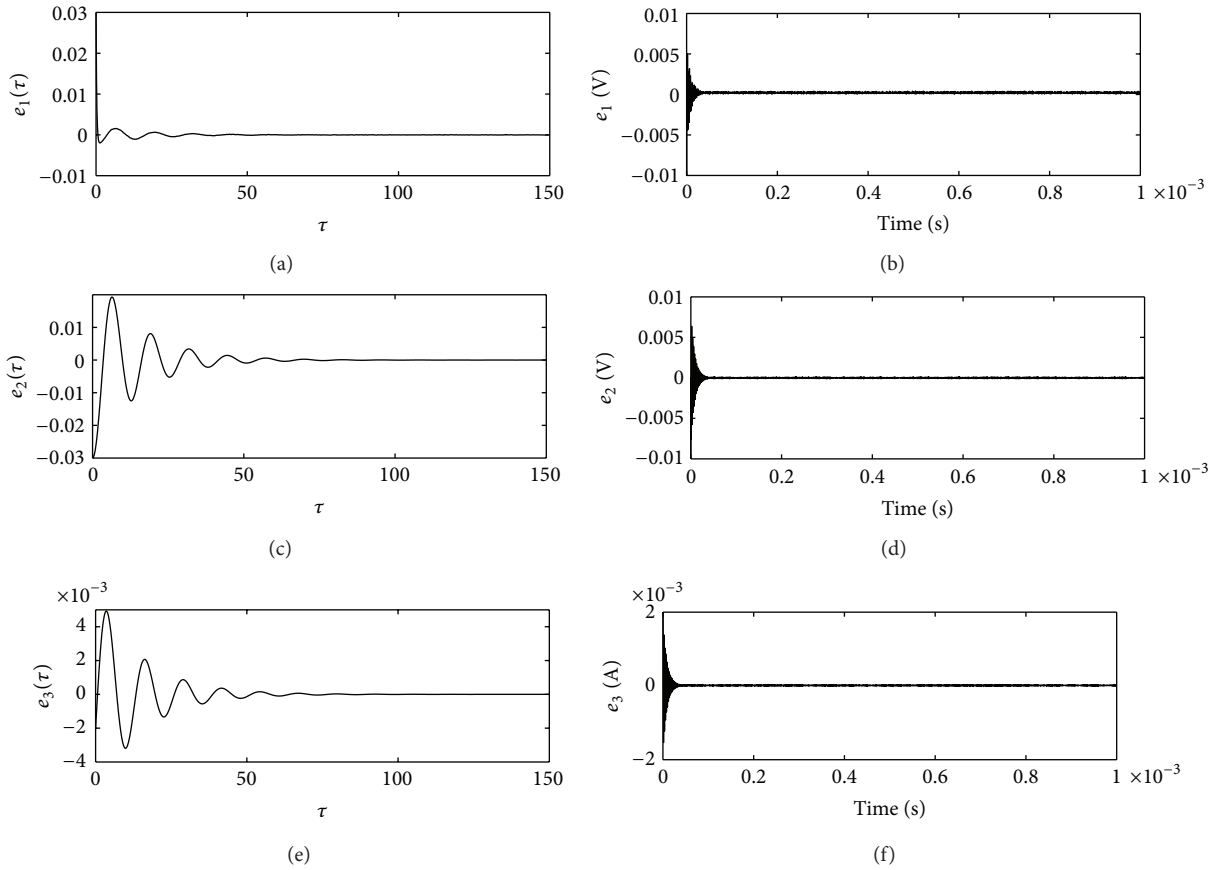


Figure 4: Time evolution of the synchronization errors from Matlab simulations (left) and Pspice simulations (right).

addition as $V(\tau)$ decreases, then there exists a continuous and strictly increasing function ϱ and a finite integer η, such that

$$V\left(e\left(\tau+\eta\right),\tau+\eta\right)-V\left(e\left(\tau\right),\tau\right)\leq-\varrho\left(\left\|e\left(\tau\right)\right\|\right), \tag{19}$$

where $e(\tau)=(e_1(\tau),e_2(\tau),e_3(\tau))$.

Thus, the Lyapunov function respects [36, Theorem 3.1] and then (8) is globally uniformly ultimate bounded near the origin.

4. Numerical Simulations

4.1. Chaotic Systems. In this section, we present some numerical results for the circuit shown in Figure 1, to illustrate the effectiveness of the proposed scheme, where the three-dimensional tunnel diode based modified Chua's system [35] is used as transmitter (blue box) and receiver (green box), the controller appears inside the red box. With the initial conditions selected as $(x_1(0),x_2(0),x_3(0))=(0.15,0.27,0.008)$ and $(y_1(0),y_2(0),y_3(0))=(0.18,0.24,0.006)$ and with the given system's parameters: $\alpha=2.507463$, $\beta=0.2985075$, $\gamma=0.20875$, $R=16$, $e=0.250$, $a_1=1.3242872$, $a_2=0.06922314$, $a_3=0.00539$, and $b=0.167$, the systems behave chaotically as shown in Figure 2. The disturbances $d(\tau)$ are given by the relation $d(\tau)=0.001\,\mathrm{wgn}(1,1,1)(x_1(\tau)+x_2(\tau))$, where wgn(1, 1, 1) is Matlab white gaussian noise generator.

4.2. Simulation Results and Discussion. The controller's parameters are $\varphi=10$ and $k=3$. The controller circuit was realized through the following relations: $k=R_5/V_TR_3=R_6/V_TR_4$ and $\varphi=R_7I_s$ where $V_T\simeq0.026$ Volt and $I_s\simeq10^{-12}$ are diode characteristics. The Voltage controlled current source (VCCS) is used to minimize as much as possible the mutual influence of between the slave system (Green box) and the controller (Red box) and to only generate the current which obliges the response system to follow the drive system. The graphs of Figures 3 and 4 show that the synchronization is reached around the dimensionless time $\tau=60$.

Remark 1. In Pspice simulations, the synchronization is reached for high values of R_7 particularly if $R_7>100\,\mathrm{k}\Omega$. R_7 role is to increase the value of the VCCS output current by increasing the value of the voltage at its landmarks.

Considering the case without disturbances, if we compare the proposed scheme with the one for which the controller is given by the following relation:

$$U\left(t\right)=-\zeta e_1\left(\tau\right), \tag{20}$$

where ζ is a positive constant chosen equal to φ, it appears that, as one can visually appreciate on the graphs of Figures 5 and 6, the exponential function based nonlinear controller is faster than the linear controller with fixed gain.

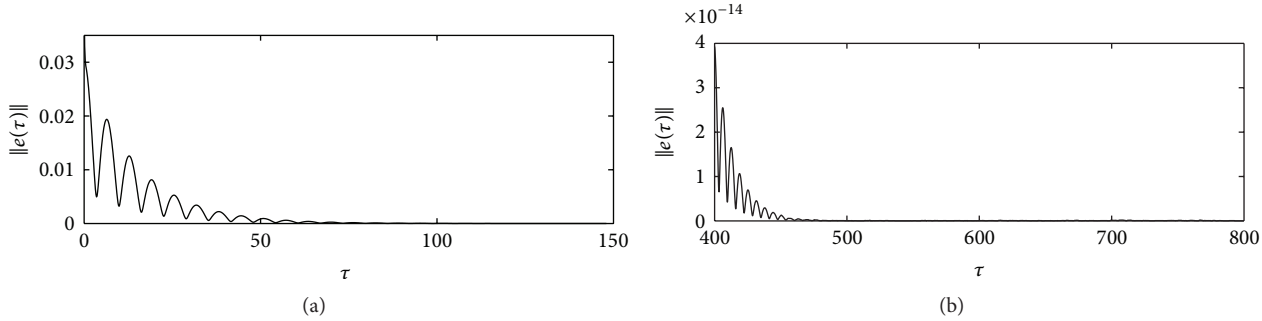


Figure 5: Time evolution of the synchronization errors norm with the propose scheme (7).

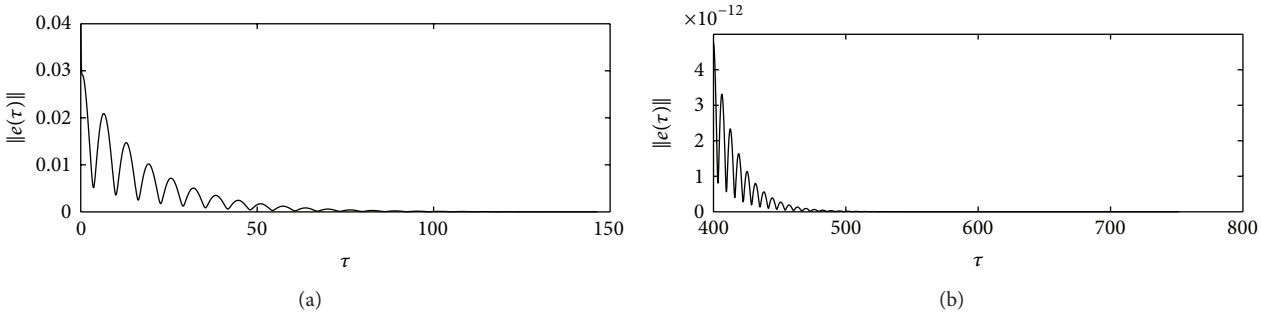


Figure 6: Time evolution of the synchronization errors norm with linear controller (20).

5. Conclusion

In this paper the synchronization between two different delayed chaotic systems is studied via a simple—exponential function based—nonlinear controller. Although different initial conditions and disturbances make synchronization more difficult, a simple exponential function based nonlinear controller is designed which facilitates the task. This is proven through the Lyapunov stability theory; it is shown that both master-slave systems should be practically synchronized. It is important to note that the proposed scheme improves the linear controller with fixed gain usually used. To show the effectiveness of the proposed strategy, some numerical simulations are given; they show the efficiency of the proposed strategy in front of the linear fixed gain based controller. The electronic circuit of the used controller is also given followed by some simulations.

Acknowledgments

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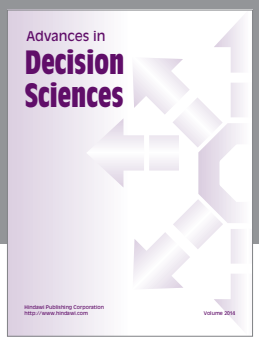
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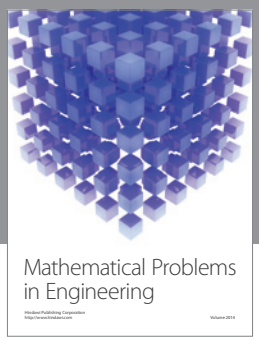
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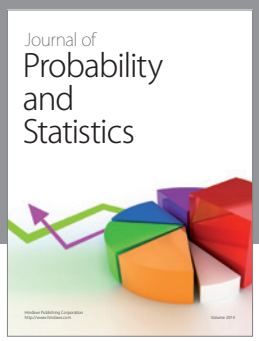
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