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Addressing uncertainty in sugarcane harvest planning through a revised multi-choice goal programming model

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ABSTRACT

In this paper a new revised multi-choice goal programming (RMCGP-LHS) model is proposed to deal with uncertainty in sugar cane harvest scheduling for sugar and ethanol milling companies. The RMCGP-LHS model uses a weekly decision-making horizon and takes into account the time and condition of land management, cane cutting decisions, and agricultural logistics. Its objective is to obtain information in order to harvest sugar cane plots in the period closest to the highest saccharose levels, while also minimizing agro-industrial costs. The RMCGP-LHS model was applied to a real case sugar and ethanol mill, and its optimization has provided harvesting policies that were validated by the company's managers. Besides that the RMCGP-LHS model is a very practical tool for simulating in a fast way different scenarios involving uncertainties on model parameters and helping the managers in decision making process in real time.

1. Introduction

In Brazil, the total sugarcane production in 2013 reached a national record of 652.02 million tons with an increase of 10.70% compared to results in 2012, however, the total sugarcane production in 2014 decreased 2.5%, according to the Brazilian National Supply Company (Conab) [1].

Since then, much research has been carried out with the aiming of improving the sector's operational and financial performance [2]. Colin [3] points out that Brazil has the largest fleet of vehicles running on ethanol in the world. In Brazil, the total production of ethanol during the season 2014/15 is estimated in 28.66 billion in liters, 2.53% increase with respect to season 2013/14 [1]. In fact, sugar and ethanol generation are important for global economics, particularly for Brazil, where the sector represents a relevant portion of the gross domestic product (GDP).

Paiva and Morabito [4] highlighted some important particularities of the sugarcane-based energy sector:

– Seasonal demand;
– Relatively high cost of raw material, accounting for 60% of the product's final cost;
– Lack of an adequate harvest-planning model, which takes into account the expenses of each plot on each farm, harvest strategies, the logistical transportation fleet, the maturation curve, and the perishability of the raw material after harvesting.

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The same authors state that it is important to properly define the specific moment at which to harvest each sugarcane plot, given the necessity of obtaining raw material with the greatest level of total reducing sugars (TRS) and a high level of purity. TRS are parameters of the sugarcane payment system. Further, the difference between the TRS and the losses incurred during the cane washing, to the filter cake or the presses, and to the final bagasse is considered the standard of average efficiency in these processes.

Thus, one important aspect in terms of agricultural stage is the sugarcane maturation curve, a graphical representation of the sugarcane's lifecycle, illustrated in Fig. 1. The curve is commonly found as a function of the polarity (POL) and the time to cutting, usually able to be divided into two phases:

- The first phase of vegetative growth in which the plant accumulates energy in the form of saccharose, thus increasing the POL value;
- In the following phase, the plant utilizes the energy accumulated in the previous period for the reproduction of the species, and a decrease in the POL value occurs.

Although the best harvest period is when the POL is at its maximum, it is unfeasible to harvest all canes at peak POL, given that this would generate an inefficient zigzag of machines along the cutting plots as individual plants matured, thus creating increased harvest costs. Therefore, sugarcane harvesting usually occurs in a period close to maximum POL, but not exactly pinpointed on this date [5].

The POL curve can be estimated using a second-order polynomial equation, considering time (in weeks) and POL (% of sugarcane) as variables. With this curve, by means of agricultural samples, one can determine when the optimum TRS are going to occur.

In order to better characterize in which period the sugarcane can be processed, Brazilian mills created the industry life cycle (ILC) standard, which establishes sugarcane POL of 13% as being satisfactory for the processing of different varieties. As the payment is determined per saccharose content (PSC), the POL should be converted from the percentage of sugarcane juice into POL in metric tons, according to the CONSECANA Manual (www.unica.com.br).

The milling company that is the object of this study, is a sugar and ethanol producer situated in the southeast of Brazil. It is able to produce many types of products such as very high POL (VHP) sugar, very very high POL (VVHP) sugar, crystal sugar, two types of ethanol fuel, and some sub-products such as filter mud, bagasse, vinasse, and fuel oil.

The milling company owns more than 200 farms; however, for the application of the model hereby proposed, only 16 out of the 200 farms were considered, as this was the data for this study made available by the company. On each of the 16 farms, there are, on average, 34 sugarcane plots, including 12 different varieties of sugarcane.

About 115 varieties of sugarcane are available in Brazil with different productivity features (TRS), which enables their maturation to be classified as early, intermediate, and late. Therefore, sugar mills can choose among the varieties available with a view to perform their cuts at the most suitable time, thus increasing TRS and production [6].

Observe that, in a typical harvesting season, the milling company crushes 3.5 million tons of sugarcane, of which, on average, 1.4 million tons of cane are acquired from partner farmers that do not belong to the company.

The case study involving the milling company is aimed at checking whether the proposed model could improve the corresponding sugarcane harvest, having been conducted using data from the 2012/2013 harvesting seasons. In fact, for reasons of confidentiality, we worked with proportional values associated with real data from the milling company.

According to Paiva and Morabito [4], agro-industrial costs account for 60% of a plant's total cost; therefore, good planning involving the choice of the sugarcane source (from within the company or partner farms acting as third parties), the plot to cut, the cutting technique (mechanized or manual), and the transport mode (own, third-party) is essential.

The main contribution of this study is to extend a revised multi-choice goal programming model [7] to treat uncertainties of left-hand side coefficients (RMCGP-LHS), which encompasses coefficients of uncertainty on the LHS in the restrictions to aid in decision-making processes for sugarcane harvest planning to produce sugar and ethanol. This uncertainty affects the plant's industrial efficiency, as the choice to cut sugarcane in plots with low TRS could impair the yield of alcohol and sugar production and increase agro-industrial costs.

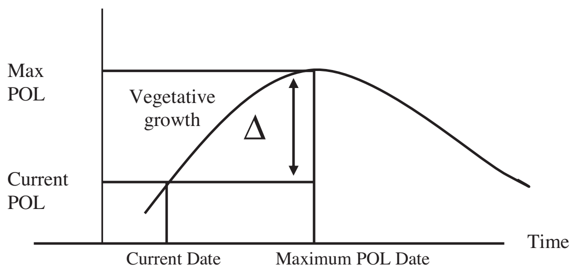

Fig. 1. Illustration of the maturation curve.

In fact, in real scenarios, several factors influence agribusiness parameters and costs. Among them, climate is the main source of uncertainty, with a high impact on costs and on the behavior of the maturation curve of the sugarcane in plots. This context justifies the objective of this paper, which is to investigate the effects of uncertainty on the LHS coefficients of the RMCGP-LHS model. This model includes constraints related to each agro-industrial cost function (generating LHS coefficients) and its associated goals (or right-hand side coefficients: RHS).

Silva and Marins [5] proposed a fuzzy goal programming (FGP) for a real aggregate production-planning problem considering uncertainty on RHS coefficients. This research focused on nine objectives to be achieved: minimize the costs of each agro-industrial phase (supply of sugar cane, sugar cane transportation and production process), minimize the total inventory cost, minimize the total distribution cost of the products (sugars and ethanol) to clients, maximize the production of crystal sugar, maximize the production of VHP sugar, maximize the production of VVHP sugar and maximize the production of ethanol.

The objective functions considered in this work were: total agricultural transportation costs for the sugarcane administered by the company and that supplied by partner farms; total cost related to the cutting strategy adopted for the sugarcane plantation administered and that supplied; total cost related to the planting process, soil treatment, fertilization, pesticides, and irrigation for the sugarcane administered by the company; total cost of the sugarcane supplied; and a function related to a POL measurement (% in sugarcane juice) obtained through an agricultural sampling of the cut plot. In this research, we opted to work with each individual cost function rather than only one aggregated agro-industrial cost function. This choice was made for two reasons:

- Each cost function's values use a different scale; thus, grouping them into a single total agro-industrial cost function tends to cause the highest values to predominate, making the model less sensitive to deviations (associated with uncertainties) on the cost coefficients of each particular function.
- It facilitates a sensitivity analysis of the coefficients of each cost function.

Uncertainty is incorporated in the RMCGP-LHS model by applying the deviation ranges of the LHS coefficients. These deviations should be chosen by the plant managers, as good knowledge of the real scenario is necessary to allow the simulation of useful scenarios for good planning for the plant operation. One should note the deviation in the different values for each LHS coefficient and the range of nonsymmetrical intervals, another advantage of adopting the RMCGP-LHS model.

The use of nonsymmetrical intervals to establish the uncertainty of each LHS coefficient offers managers higher flexibility when testing the sensitivity of the solution in the presence of uncertainties without increasing the computational complexity of the solution process. This approach is similar to the multi-segment model [8], the revised multi-segment model, and the multi-coefficients model [9,10]; however, once these models are used to evaluate the uncertainty of each LHS coefficient, their computational complexities are higher than those of the RMCGP-LHS model.

Chang [11] developed the multi-choice goal programming (MCGP) model to include the uncertainty of RHS coefficients by means of binary variables. In a different proposal, Bankain-Tabrizi et al. [12] combined the fuzzy set theory with MCGP, thus creating the fuzzy multi-choice goal programming model.

Silva et al. [13] developed a multi-choice mixed integer goal programming (MC-MIGP) model to solve a real, large-scale problem in a sugarcane mill, and the model covers decisions on the agricultural and cutting stages, sugar loading and transportation by suppliers and, specially, energy cogeneration.

Chang [7] later presented an alternative to his previous model [11], the revised multi-choice goal programming (RMCGP) model, which introduced the uncertainty of RHS coefficients by considering their variations in continuous intervals.

Pursuant to his important contributions to the GP model, Chang [14] developed the revised multi-choice model with utility functions in order to increase its sensitivity and practicality when addressing the management of real issues. In another approach, Ustun [15] proposed the aggregation of a scalarizing function in the MCGP models, and highlighted three contributions of his investigation:

- It enables decision makers to define multi-choice levels of aspiration for each goal.
- It reduces the number of auxiliary constraints and additional variables.
- It ensures the acquisition of an efficient solution.

Considering that the referenced models contemplated uncertainty in the RHS coefficients, next we present a review of research handling uncertainty in the LHS coefficients. Liao [8] and Chang et al. [9,10] investigated LHS uncertainty; however, they included auxiliary, binary, continuous, and discrete variables for each coefficient. Such approaches are less attractive for application to real, large-scale problems, mainly due to an increase in the computational time to find a solution.

The robust optimization was proposed by Ben-Tal and Nemirovski [16], addresses uncertainty in the LHS coefficients. As a basis for comparison, the RMCGP-LHS model proposed here has the advantage of not using probability functions to estimate uncertainty, thus reducing considerably the number of parameters to be taken into account.

Furthermore, the proposed RMCGP-LHS model introduces an algebraic structure to the GP literature that is a simpler and more innovative than the models seen in robust stochastic optimization and robust optimization [17,16,18–20] given that it is not necessary to calibrate parameters or to adopt statistical techniques to estimate them.

In short, the proposal described in this article includes uncertainty in the LHS coefficients by means of adopting continuous auxiliary variables, thus avoiding the need to add discrete and binary variables. This approach is the most suitable for real, large-scale problems, as compared to the previous alternatives, it will demand less computational time and will be more compatible with the necessity of making real-time decisions.

This work differs from Silva and Marins [5], and Silva et al. [13], because the focus here is the planning of the sugar cane harvest, which corresponds to 60% of total costs [4]. Thus, this important phase is explored in more detail, taking into account the time and condition of land management, cane cutting decisions, and agricultural logistics. The model includes information on plots of land, cutting strategy, state of sugarcane, cane condition, and cane varieties in different farms. The sugar and ethanol companies studied here, as well as the harvest periods analyzed are different to those considered in [5,13]. Observe, finally, that [5,13] considered uncertainty in the RHS coefficients and in this work the uncertainty is modeled in the LHS coefficients.

This article is organized as follows: Section 2 presents a description of the problem and the adopted research method. Section 3 refers to recently proposed GP models under uncertainty. In Section 4, the RMCGP-LHS is developed to address the process of condition choice management, cane plot harvest time, front-cutting dimensioning, and the agricultural logistics of the sugar and ethanol sector. Section 5 summarizes and discusses the main results obtained using the RMCGP-LHS model in a real, large-scale problem, and finally, in Section 6, the conclusions and recommendations for future research opportunities are presented, followed by the references.

2. Description of the problem and research method

In this paper, an RMCGP-LHS model is proposed to deal with uncertainty in planning sugarcane harvesting for a sugar and ethanol milling company. Its differential is the adaptation of the models proposed by Liao and Kao [21], Chang [11,7,14], and Silva et al. [13] in order to handle uncertainty in the LHS in a real, large-scale application.

The RMCGP-LHS model, described in detail in Section 5, handles the agricultural phase and enables decisions to be made on a weekly basis, while encompassing the process of decision-making management for harvest strategies and logistical issues at the same time. The main decisions, which are part of the sugarcane harvest-planning problem, are:

– How much of each variety is to be harvested weekly?
– How much manual and mechanized labor is to be used?
– Which mode should be utilized weekly?

Fig. 2 shows the research phases.

2.1. Identification of the problem

Some companies were visited in order to identify the parameters to compose the model. A Brazilian sugar and ethanol milling company located in the state of Minas Gerais was chosen as the study object.

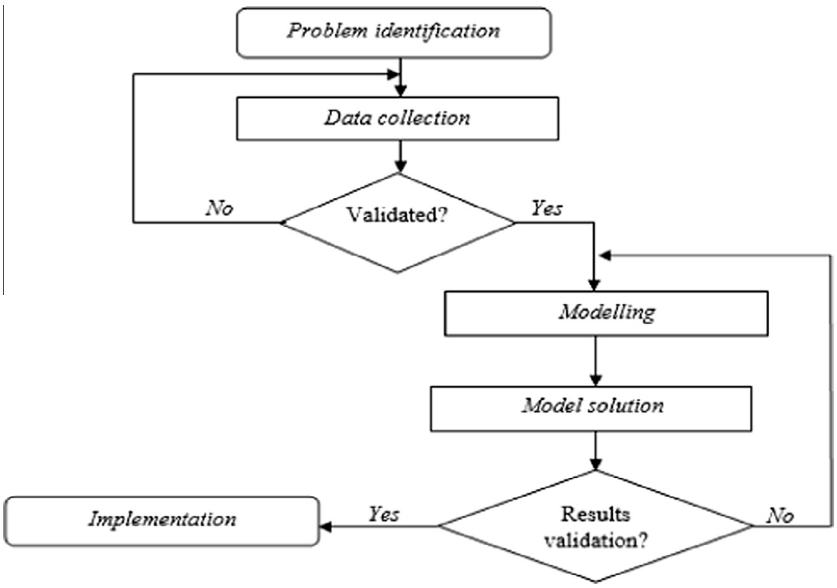

Fig. 2. Research steps. Source: Silva et al. [13].

2.2. Data collection

Internal reports on agricultural data from the chosen company were used, and interviews were conducted with professionals who deal with scheduling and operations on a daily basis in order to detail any necessary information for the problem modeling.

2.3. Modeling

The modeling phase took into account the harvest decisions over 32 weeks and included the design of a multi-objective, large-scale model in which the uncertainty in planning was considered to determine agro-industrial scheduling. This is shown in Section 4.

2.4. Model solution

The RMCGP-LHS model design was implemented in the General Algebraic Modeling System, version GAMS 23.6.5, and resolved using Solver CPLEX, version 12.2.1.1. The results are shown in Section 5.

2.5. Validation of results

The results of the model were validated with the mill's managers using a comparison of the results with real figures. Details are shown in Section 5.

2.6. Implementation

This stage depends on the sugar and ethanol milling company that was the study object, and does not fit into the scope of this article.

According to the criteria proposed by Bertrand and Fransoo [22] for the classification of scientific research, this study can be classified as having descriptive and empirical objectives, as the designed model describes causal relationships that may exist in fact, thus enabling a better understanding of real processes. This problem was approached in a quantitative way using optimization and mathematical modeling.

In the next section, the important GP models under uncertainty considered in the development of the new RMCGP-LHS model are discussed.

3. Goal programming models with uncertainty

The ideal in a decision-making problem is for all necessary information to be known at the planning stage [23]; however, besides the inherent uncertainties of information available at the moment of decision making, the future can often be unforeseeable at times, and is generally random and uncertain [24]. It is also known that many decision-making problems have many objectives to be optimized, and some may conflict with one another, which makes the solution more difficult.

In classical mathematical modeling models, it is assumed that the model parameters are precisely known and are set equal to some nominal values. For Charnes and Cooper [25], Chen et al. [26], Liang [27], and Chen and Xu [28], in practical situations, the goals and model inputs (parameters) for the decisions are normally imprecise/vague, as some relevant information is incomplete and unavailable. However, the data uncertainty is inherent in most practical situations and should be taken into account in any realistic mathematical model [29]. In sugarcane harvest scheduling models, the uncertainty is inherent in most parameters, including climate condition, cost, demand, yield process, and capacity [30].

Paiva and Morabito [4] applied a robust stochastic optimization model to incorporate uncertainty in some parameters of problems related to the sugarcane-based energy sector, such as product price, yield matrices, and global efficiency (in TRS), among others. These authors utilized GP combined with stochastic optimization [31].

In this context, and in order to handle a multiple objective situation, GP models have been applied with success [32] to solve optimization models under uncertainty [33].

3.1. Multi-choice goal programming

According to Chang [11], real multi-objective problems may have imprecise levels for the goals associated with the objectives, which makes it difficult for analysts to model the problem.

Due to such uncertainties, fuzzy goal programming (FGP) models were developed utilizing triangular and trapezoidal pertinence functions to deal with uncertainties associated with the values of the constants on the RHS in the model constraints Silva and Marins [5]. Moreover, according to Paksoy and Chang [34], situations may arise in which, from the decision maker's perspective, multiple attractive variables for the goals are associated with the objectives, that is, objectives can be attained when different specific desired levels are reached.

The MCGP model, developed by Chang [11], allows the uncertainties in the values of the RHS to be assessed:

$$Min \quad \sum_{i=1}^{n} |f_i(X) - g_{i1} \text{ or } g_{i2} \text{ or} \cdots \text{or } g_{im}|, \tag{1}$$

$$\text{s.t.} \quad X \in F(F \text{is a feasible set}), \tag{2}$$

where g_{ij} is the jth value of the m possible levels (or segments) desired for a goal associated with the ith objective.

According to Chang [11], with the incorporation of deviational variables, MCGP models can be rewritten as follows:

$$Min \sum_{i=1}^{n} (\alpha_i d_i^+ + \beta_i d_i^-), \tag{3}$$

$$\text{s.t.} \quad f_1(X) - d_1^+ + d_1^- = g_1 Z_1 + g_2(1 - Z_1), \tag{4}$$

$$f_2(X) - d_2^+ + d_2^- = g_3 Z_2 Z_3 + g_4 Z_2 (1 - Z_3) + g_5 (1 - Z_2) Z_3, \tag{5}$$

$$f_3(X) - d_3^+ + d_3^- = g_6 Z_4 Z_5 + g_7 Z_4 (1 - Z_5) + g_8 (1 - Z_4) Z_5 + g_9 (1 - Z_4)(1 - Z_5), \tag{6}$$

$$d_i^+, d_i^- \geqslant 0, d_i^+ . d_i^- = 0, \quad i = 1, 2, 3, \tag{7}$$

$$Z_1, Z_2, \ldots, Z_5 \in \{0, 1\}, \quad X \in F(F\text{is a feasible set}), \ X \text{ is unrestricted in sign.} \tag{8}$$

It can be observed that the objective function (3) aims at minimizing the values of the deviational variables d_i^+, d_i^- linked to goals g_i, α_i, and β_i, which are the respective positive weights attached to these deviations in the achievement function. The constraint (4) draws up a scenario in which goal i has two segments. When the value of Z_1 is zero, segment g_2 will be selected, and when the value of Z_1 is one, segment g_1 will be selected. Constraints (5) and (6) make up scenarios in which goal i has three and four segments, respectively, with a similar reasoning for combinations of values for binary variables Z_2, Z_3, Z_4, and Z_5, leading to the choice of values g_3, g_4, g_5, g_6, g_7, g_8, and g_9. Constraints (7) and (8) refer to the variables' domain.

3.2. Revised multi-choice goal programming

An MCGP model, in the case of two segments with the incorporation of deviational variables, will still be linear. However, it will not be linear for more than two segments. A relevant contribution was made by Chang [7,14], who proposed an RMCGP model to substitute the necessity for binary variables with the incorporation of auxiliary continuous variables y_i, which enable goals g_i to vary in a continuous space.

In the RMCGP model, shown in (9)–(14), where α_i and β_i are the weights attached to deviations d_i and e_i, $g_i^{\max}$ and $g_i^{\min}$ are, respectively, the estimated upper and lower bounds for goal g_i, and e_i^+ and $e_i^-$are deviational variables associated with the bounds (upper or lower) of goal g_i, to be optimized by $|y_i - g_i^{max}|$ or $|y_i - g_i^{max}|$. d_i^+ and $d_i^-$are the positive and negative deviations attached to the ith goal $|f_i(x) - y_i|$.

It is observed that, depending on the scope of the objective associated with goal g_i, it could be an interesting alternative to consider the upper or lower bound in constraint (11), as explained below:

$$Min \sum_{i=1}^{n} [\alpha_i (d_i^+ + d_i^-) + \beta_i (e_i^+ + e_i^-)], \tag{9}$$

$$\text{s.t.} \quad f_i(X) - d_i^+ + d_i^- = y_i, \quad i = 1, 2, \ldots, n, \tag{10}$$

$$y_i + e_i^- - e_i^+ = g_i^{max} \text{ or } g_i^{\min}, \quad i = 1, 2, \ldots, n \tag{11}$$

$$g_i^{\min} \leqslant y_i \leqslant g_i^{\max}, \quad i = 1, 2, \ldots, n, \tag{12}$$

$$d_i^+, d_i^-, e_i^+, e_i^- \geqslant 0, \quad i = 1, 2, \ldots, n, \tag{13}$$

$$X \in F(F \text{ is a feasible set}), X \text{ is unrestricted in sign.} \tag{14}$$

One important observation in relation to constraint (11) – which involves the choice between variables g_i^{max} and $g_i^{\min}$ as the RHS – is that this decision depends on what is desired for the analyzed goal: If the goal has a "the higher the better" perspective, as is the case with profits, then $g_i^{\max}$ is used; otherwise, $g_i^{\min}$ is used.

The main difference between the RMCGP and the MCGP models is that in the RMCGP, the desired levels of limitation of goal i are defined in continuous spaces, as shown in constraint (12), without the existence of binary auxiliaries.

Chang [7] comments that the MCGP and the RMCGP models provide better solutions than those provided by the weight goal programming (WGP), minmax goal programming (MA), and lexicographic goal programming (LGP) models. Paksoy and Chang [34] applied their RMCGP model to solve multi-stage, multi-product, and multi-period problems. Both the MCGP and RMCGP models are quite recent, and not much literature is available. For further details, consult Silva et al. [13], Chang [11,7], and Liao and Kao [21].

3.3. Multi-segment goal programming

Liao [8] developed a multi-segment goal programming (MSGP) model derived from the MCGP model proposed by Chang [11]. According to Liao and Kao [35], the MSGP is devised to approach decision-making problems involving multi-segment aspiration levels of evaluation criteria.

The main difference between these models lies in the context in which uncertainty is considered. MCGP models consider uncertainty on the RHS, whereas MSGP models consider uncertainty on the LHS. The MSGP model can be expressed by (15)–(17):

$$\text{Min} \sum_{i=1}^{n}(d_i^- + d_i^-), \tag{15}$$

$$\text{s.t.} \quad \sum_{i=1}^{n} s_{ij}X_i + d_i^- - d_i^+ = g_i, \quad j = 1,2,\ldots,m. \tag{16}$$

$$d_i^- \geqslant 0, d_i^+ \geqslant 0, i = 1,2,\ldots,n \; X \in F(F \text{ is a feasible set}), \tag{17}$$

where S_{ij} indicates the decision variable coefficients corresponding to the level established for the jth segment of the ith goal g_i. The other variables are the same as those defined for the previous models.

Because the MSGP model was created recently, there is little literature on its nature and utilization. For details, Liao [8] and Liao and Kao [35] are recommended.

The MSGP model evaluates the uncertainties on the LHS; however, its use is not advisable for large-scale problems, as its implementation is very complex from a computational point of view. This difficulty stems from the fact that the inclusion of the uncertainty in each LHS coefficient makes it a non-linear model, that is, it is a product of the binary auxiliary variables and the original decision variables. To illustrate this problem, considering a model with 10,000 coefficients in the technological matrix, it would not be difficult to imagine the inherent hurdles when handling 10,000 additional binary auxiliary variables in a nonlinear mixed optimization environment.

Chang et al. [9] developed a revised multi-segment goal programming (RMSGP) model that avoids the need for binary control variables. Its basic idea is to restrict the variation of the LHS in a continuous interval, while the MSGP model restricts the variation of the LHS in a discrete interval. Chang et al. [10] proposed a multi-coefficients goal programming (MCGP) model for the same purpose as the RMSGP, but utilizing discrete auxiliary variables linked to each LHS coefficient.

The next section describes the RMCGP-LHS model, the main proposal of this article, in detail and adopts an alternative strategy for handling the uncertainty in the technological matrix.

4. The RMCGP-LHS model to deal with uncertainty in the scheduling of sugarcane harvesting

In this section, details of the proposed RMCGP-LHS model are presented. This new approach deals with uncertainty in the LHS associated with the constraints of the GP model using an innovative proposal that enables quick scenario evaluation (with acceptable computational time), and allows the interaction of decision makers in the development stages of the model and in its optimization. Another advantage of this model is that it does not require a phase for calibration of the model parameters, which usually presents serious difficulties both for the analyst and the decision maker. The RMCGP-LHS model can be formulated as:

$$Min \sum_{i=1}^{n}\left[\alpha_i(d_i^+ + d_i^-) + \beta_i(e_i^+ + e_i^-)\right], \tag{18}$$

$$\begin{aligned} &\text{s.t.} \quad f_i(X)\cdot\omega - d_i^+ + d_i^- = y_i, \quad i = 1,2,\ldots,n. \\ &\text{or} \\ &\quad f_i(X)\cdot\tau - d_i^+ + d_i^- = y_i, \quad i = 1,2,\ldots,n. \end{aligned} \tag{19}$$

$$y_i - e_i^+ + e_i^- = g_i, \quad i = 1,2,\ldots,n, \tag{20}$$

$$f_i(X)\cdot\omega \leqslant y_i \leqslant f_i(X)\cdot\tau, \quad i = 1,2,\ldots,n, \tag{21}$$

$$d_i^+, d_i^-, e_i^+, e_i^- \geqslant 0, \quad i = 1, 2, \dots, n, \tag{22}$$

$$X \in F(F \text{ is a feasible set}), \tag{23}$$

where the parameters ω and τ, given by percentages, represent, respectively, a decrease and an increase on the original objective functions $f_i(X)$, and goal values g_i are defined based in the expertise of decision-makers, whereas deviational variables $d_i^+, d_i^-, e_i^+, e_i^-$ have a similar meaning as that explained in RMCGP model.

The constraint (21) is similar to constraint (12), with $f_i(X)\cdot\omega$ and $f_i(X)\cdot\tau$ replacing, respectively, $g_i^{\min}$ and $g_i^{\max}$. Note that due to constraint (12), the RMCGP model finds solutions in which no more than the desired goal value ($g_i^{\min}$ or $g_i^{\max}$) is reached; already RMCGP LHS-model, with the constraint (21) allows finding solutions beyond the value of the desired goal value (g_i), and thus is more flexible.

Uniquely, the alternative RMCGP-LHS achievement represents a linear form of MCGP that can easily be solved using common linear programming packages, without the need to use integer programming package; in addition, it is easily understood by industrial participants.

In order to illustrate this modeling, examples involving the MSGP [8] and RMCGP-LHS approaches are presented. The first problem, from Liao [8], has three objective functions, see (24)–(26), and three constraints, see (27)–(29). In these objective functions, the coefficient x_1 in the g_1 function may have a value of 3 or 6, the coefficient x_2 in the g_2 function may have a value of 5 or 9, and the coefficient x_3 in the g_3 function may have a value of 7 or 10; the goals for each objective function are, respectively, 115, 80, and 110. The objective functions and the constraints are:

$$(g_1) \quad (3 \text{ or } 6)x_1 + 2x_2 + x_3 = 115, \tag{24}$$

$$(g_2) \quad 4x_1 + (5 \text{ or } 9)x_2 + 2x_3 = 80, \tag{25}$$

$$(g_3) \quad 3.5x_1 + 5x_2 + (7 \text{ or } 10)x_3 = 110, \tag{26}$$

$$\text{s.t.} \quad x_2 + x_3 \geqslant 9, \tag{27}$$

$$x_2 \geqslant 5, \tag{28}$$

$$x_1 + x_2 + x_3 \geqslant 21. \tag{29}$$

The corresponding MSGP model [8] is:

$$Min\ (d_1^+ + d_1^- + d_2^+ + d_2^- + d_3^+ + d_3^-), \tag{30}$$

$$\text{s.t.}(3b_1 + 6(1 - b_1))x_1 + 2x_2 + x_3 + d_1^- - d_1^+ = 115, \tag{31}$$

$$4x_1 + (5b_2 + 9(1 - b_2))\,x_2 + 2x_3 + d_2^- - d_2^+ = 80, \tag{32}$$

$$3.5x_1 + 5x_2 + (7b_3 + 10(1 - b_3))x_3 = 110, \tag{33}$$

$$x_2 + x_3 \geqslant 9, \tag{34}$$

$$x_2 \geqslant 5, \tag{35}$$

$$x_1 + x_2 + x_3 \geqslant 21, \tag{36}$$

$$d_1^+ \geqslant 0, d_1^- \geqslant 0, d_2^+ \geqslant 0, d_2^- \geqslant 0, d_3^+ \geqslant 0, d_3^- \geqslant 0, b_1 \in \{0,1\}, b_2 \in \{0,1\}, b_3 \in \{0,1\}. \tag{37}$$

The optimal solution obtained by Liao [8] was (x_1, x_2, x_3, b_1, b_2, b_3) = (11.54, 5, 4.46, 0, 1, 0), and goal g_1 = 83.70 (the aspiration level was 115), goal g_2 = 73.60 (the aspiration level was 80), and goal g_3 = 109.85 (the aspiration level was 110).

The high level of complexity in this MSGP model can be observed even though it is a small model. In real problems, with millions of variables, the computational time spent to solve it would be prohibitive, as managers are often expected to solve problems in real time.

In a second example, using the same objective functions and constraints, a deviation of 20% is applied above (τ) and below (ω) in the technological matrix, the interest for the functions g_1 and g_3 is related to the upper bound, and the interest for the function g_2 is related to the lower bound.

The RMCGP-LHS model is:

$$Min\ (d_1^+ + d_1^- + d_2^+ + d_2^- + d_3^+ + d_3^- + e_1^+ + e_1^- + e_2^+ + e_2^- + e_3^+ + e_3^-), \tag{38}$$

$$\text{s.t.}\quad \begin{matrix} (3x_1+2x_2+x_3).\omega - d_1^+ + d_1^- = y_1 \\ \text{or} \\ (3x_1+2x_2+x_3).\tau - d_1^+ + d_1^- = y_1 \end{matrix}, \tag{39}$$

$$y_1 - e_1^+ + e_1^- = 115, \tag{40}$$

$$(3x_1+2x_2+x_3).\omega \leqslant y_1 \leqslant (3x_1+2x_2+x_3)\cdot\tau, \tag{41}$$

$$\begin{matrix} (4x_1+5x_2+2x_3).\omega - d_2^+ + d_2^- = y_2 \\ \text{or} \\ (4x_1+5x_2+2x_3).\tau - d_2^+ + d_2^- = y_2 \end{matrix}, \tag{42}$$

$$y_2 - e_2^+ + e_2^- = 80, \tag{43}$$

$$(4x_1+5x_2+2x_3).\omega \leqslant y_2 \leqslant (4x_1+5x_2+2x_3)\cdot\tau, \tag{44}$$

$$\begin{matrix} (3.5x_1+5x_2+7x_3).\omega - d_3^+ + d_3^- = y_3 \\ \text{or} \\ (3.5x_1+5x_2+7x_3).\tau - d_3^+ + d_3^- = y_3 \end{matrix} \tag{45}$$

$$y_3 - e_3^+ + e_3^- = 110, \tag{46}$$

$$(3.5x_1+5x_2+7x_3).\omega \leqslant y_3 \leqslant (3.5x_1+5x_2+7x_3)\cdot\tau, \tag{47}$$

$$x_2 \geqslant 5, \tag{48}$$

$$x_1+x_2+x_3 \geqslant 21, \tag{49}$$

$$x_2+x_3 \geqslant 9, \tag{50}$$

$$d_1^+ \geqslant 0, d_1^- \geqslant 0, d_2^+ \geqslant 0, d_2^- \geqslant 0, d_3^+ \geqslant 0, d_3^- \geqslant 0, e_1^+ \geqslant 0, e_1^- \geqslant 0, e_2^+ \geqslant 0, e_2^- \geqslant 0, e_3^+ \geqslant 0, e_3^- \geqslant 0 \tag{51}$$

It should be mentioned here that the solution would differ for varied

It should be mentioned here that the solution would differ for varied parameter values ω and τ; that is, different scenarios could be generated that would allow managers to carry out sensibility analysis and verify the effect of the uncertainty in practical terms without the need to include binary, discrete, or continuous auxiliary variables. Further, the computational time is irrelevant to solve this last model and allows for decision making in real time. For Fleskens and Graaff [36], the simulation of scenarios provides a tool to evaluate the effects of current and potential future policies.

For instance, optimization results from two scenarios are presented:

– If ω = 80% and τ = 120%, the optimal solution is:

$$(x_1, x_2, x_3, d_1^+, d_1^-, d_2^+, d_2^-, d_3^+, d_3^-, e_1^+, e_1^-, e_2^+, e_2^-, e_3^+, e_3^-, y_1, y_2, y_2) = (13.75, 9, 0, 0, 0, 0, 0, 1.75, 43.9, 0, 0, 0, 0, 0, 0, 71.10, 80, 110).$$

– If ω = 90% and τ = 180% the optimal solution is:

$$(x_1, x_2, x_3, d_1^+, d_1^-, d_2^+, d_2^-, d_3^+, d_3^-, e_1^+, e_1^-, e_2^+, e_2^-, e_3^+, e_3^-, y_1, y_2, y_2) = (12, 9, 0, 0, 0, 0, 0, 46.6, 17.8, 0, 0, 0, 3.7, 0, 0, 97.2, 83.7, 110).$$

An RMCGP-LHS model to handle uncertainty in the scheduling of sugarcane harvesting for a sugar and ethanol milling company is described in the sequence. The indices, sets, parameters, decision and auxiliary variables, objective function, and constraints utilized in the model are as follows:

4.1. Indices and sets

- t Periods, $t \in T$, $T = \{1, 2, \ldots, 32\}$;
- i Plots of land, $i \in I$, $I = \{1, 2, \ldots, 34\}$;
- f Agricultural transport, $f \in F$, F = {Own transport, Outsourced transport};
- k Cane from suppliers, $k \in K$, K = {A, B, C};
- j Cutting strategy $j \in J$, J = {Mechanized, Manual};
- q State of sugarcane, $q \in Q$, Q = {Burnt, Raw};

c Cane condition, $c \in C$, C = {Early, Intermediate, Late};
v Cane varieties, $v \in V$, $V = \{1,2,\ldots,12\}$;
π Farms, $\pi \in \Pi$, $\Pi = \{1,2,\ldots,16\}$.

4.2. Parameters

$M_t^{\min}$ Milling minimum/week in period t [tons/week];
$M_t^{\max}$ Milling maximum/week in period t [tons/week];
CT_j Weekly cutting capacity j [tons/week];
$R_{j,\pi}$ Cost of cutting by strategy j on farm π [$/ton];
$Rk_{j,k}$ Cost of cutting by strategy j at supplier k [$/ton];
CP_t Own transport capacity in period t [tons/week];
$\boldsymbol{\phi_t}$ Effective working time for industry during period t [%];
δ_{ft} Availability of transport supplier f in period t [%];
$Dispp_{i\ q\ c\ v\ \pi\ 0}$ Forecast for crop per plot i, in state q, in condition c, of variety v, on farm π; before the beginning of the scheduled period [tons];
$Dispz_{k\ 0}$ Forecast for crop per type from supplier k before the beginning of the scheduled period [tons];
$ATR_{i\ q\ c\ v\ \pi\ t}$ TRS (optimal) of plot i, in condition c, of variety v, on farm π, in period t [tons/week];
$U_{i\ q\ c\ v\ \pi\ t}$ TRS of plot i (at the moment of cutting), in condition c, of variety v, on farm π, in period t [tons/week];
$ATRK_{k\ t}$ TRS (optimal) of the sugarcane supplied by k in period t [tons/week];
$UK_{k\ t}$ TRS of sugarcane provided by k at the moment of cutting in relation to the TRS in the raw material in period t [tons/month];
$L_{f\ \pi}$ Variable cost for transport option f for farm π [$/ton];
$Lk_{f\ k}$ Variable cost for transport option f for the supplier k [$/ton];
$CK_{k\ t}$ Cost of transporting raw material coming from supplier k in period t [$/ton];
μ_t Effective milling time in period t [%];
$C_{\ \pi}$ Cost of raw material from farm π [$/ton].

4.3. Decision variables

$\boldsymbol{X_{i\ q\ c\ v\ \pi\ t}}$ Quantity of cane cut in plot i, in state q, in condition c, of variety v, on farm π, in period t [tons];
$Y_{f\,t}$ Quantity of sugarcane transported by transport option f in period t [tons];
$Z_{i\ q\ c\ v\ \pi\ t}$ Availability of raw material from plot i, in state q, in condition c, of variety v, on farm π, in period t [tons];
$W_{k\ t}$ Availability of raw material from supplier k in period t [tons];
$H_{j\ t}$ Variable associated with dimensioning the cutting strategy j in period t;
$N_{k\ t}$ Variable associated with the choice of sugarcane quantity provided by supplier k in period t.

4.4. Auxiliary variables

d_1^+ Positive deviational variable (total cost in agricultural transport);
d_1^- Negative deviational variable (total cost in agricultural transport);
e_1^+ Positive deviational variable (total cost in agricultural transport);
e_1^- Negative deviational variable (total cost in agricultural transport);
y_1 Continuous variable associated with the interval of cost variation in agricultural transport;
d_2^+ Positive deviational variable (total transport cost of sugarcane from supplier);
d_2^- Negative deviational variable (total transport cost of sugarcane from supplier);
e_2^+ Positive deviational variable (total transport cost of sugarcane from supplier);
e_2^- Negative deviational variable (total transport cost of sugarcane from supplier);
y_2 Continuous variable associated with the variation interval of the sugarcane total transportation cost from supplier;
d_3^+ Positive deviational variable (total cost of dimensioning cutting strategy)
d_3^- Negative deviational variable (total cost of dimensioning cutting strategy)
e_3^+ Positive deviational variable (total cost of dimensioning cutting strategy)
e_3^- Negative deviational variable (total cost of dimensioning cutting strategy)
y_3 Continuous variable associated with the interval of total cost variation for the dimensioning of the cutting strategy;
d_4^+ Positive deviational variable (total cost of dimensioning cutting strategy for sugarcane from supplier)
d_4^- Negative deviational variable (total cost of dimensioning cutting strategy for sugarcane from supplier);
e_4^+ Positive deviational variable (total cost of dimensioning cutting strategy for sugarcane from supplier)
e_4^- Negative deviational variable (total cost of dimensioning cutting strategy for sugarcane from supplier);

y_4 Continuous variable associated with the interval of total cost variation for dimensioning sugarcane cutting strategy from supplier;
d_5^+ Positive deviational variable (cost of raw material);
d_5^- Negative deviational variable (cost of raw material);
e_5^+ Positive deviational variable (cost of raw material);
e_5^- Negative deviational variable (cost of raw material);
y_5 Continuous variable associated with the interval of the total cost variation for raw material;
d_6^+ Positive deviational variable (cost of purchased raw material);
d_6^- Negative deviational variable (cost of purchased raw material);
e_6^+ Positive deviational variable (cost of purchased raw material);
e_6^- Negative deviational variable (cost of purchased raw material);
$y6$ Continuous variable associated with the interval of total cost variation for purchased raw material;
d_7^+ Positive deviational variable (TRS);
d_7^- Negative deviational variable (TRS);
e_7^+ Negative deviational variable (TRS);
e_7^- Positive deviational variable (TRS);
y_7 Continuous variable associated with the interval of variation for TRS.

4.5. RMCGP-LHS model

Company managers interested in exploring the possibility of deviations (increases and reductions) in the matrix of agro-industrial costs (raw material, cutting strategy, logistical questions) collaborated extensively in the model's development.

The deviations suggested by the managers were 5%, 10%, and 20%. The effect of the perturbations in the TRS matrix was also studied to determine the harvest period for each plot of land, since these matrices compose the database for the model. For more details, see Silva et al. [13].

As previously mentioned, the objectives considered in the model were:

- Objective 1 – Total agricultural transportation costs for company's own administered sugarcane, and the goal chosen by decision maker is g_1 = $ 3,013,424.
- Objective 2 – Total agricultural transportation costs for the supplied sugarcane (sugarcane from partner farms), and the goal chosen by decision maker is g_2 = $ 8,248,183.
- Objective 3 – Total cost related to the cutting strategy adopted for the company's own administered sugarcane plantation, and the goal chosen by decision maker is g_3 = $ 2,673,263.
- Objective 4 – Total cost related to the cutting strategy adopted for supplied sugarcane (sugarcane from partner farms), and the goal chosen by decision maker is g_4 = $ 35,247,380.
- Objective 5 – Total cost related to planting, soil treatment, fertilization, protection against insects and weeds (pesticides), irrigation, for sugarcane administered by the company, and the goal chosen by decision maker is g_5 = $ 1,050,000.
- Objective 6 – Total cost of the supplied sugarcane, usually established by means of payment through saccharose content, converting the POL in % of sugarcane juice into tons, according to the CONSECANA manual (www.unica.com.br) – CONAB [1], and the goal chosen by decision maker is g_6 = $ 7,919,525.
- Objective 7 – Related to a POL measurement (% in sugarcane juice) obtained through agricultural sampling of the cut plot, being that each should have been harvested at the moment in which the TRS was at its optimal levels.

A new objective function is related to the minimization of the unwanted deviational variables associated with these seven objectives functions, as given by (52), subject to the constraints (53)–(93): *Objective function*: Considering that all goals are of equal importance, the objective function can be expressed as in (52):

- Considering that all goals are of equal importance, the objective function can be expressed in (52):

$$Min\begin{pmatrix} d_1^+ + d_1^- + e_1^+ + e_1^- + e_2^+ + e_2^- + d_2^+ + d_2^- + d_3^+ + d_3^- + e_3^+ + e_3^- + e_4^+ + e_4^- + \\ +d_4^+ + d_4^- + d_5^+ + d_5^- + e_5^+ + e_5^- + e_6^+ + e_6^- + d_6^+ + d_6^- + d_7^+ + d_7^- + e_7^+ + e_7^- \end{pmatrix} \tag{52}$$

Constraints:

- Constraint (53) is associated with the availability of sugarcane in plot i, in state q, in condition c, in variety v, on farm π and in period t:

$$Z_{\mathrm{iqcv}\pi t} = \mathrm{dispp}_{\mathrm{iqcv}\pi 0} + Z_{\mathrm{iqcv}\pi(t-1)} - X_{\mathrm{iqcv}\pi(t-1)}, \quad \forall i \in I,\ \forall q \in Q,\ \forall c \in C,\ \forall p \in P,\ \forall t \in T. \tag{53}$$

- Constraint (54) is associated with the availability of sugarcane from supplier k in period t:

$$W_{\mathrm{kt}} = \mathrm{dispz}_{k0} + W_{\mathrm{k(t-1)}} - N_{\mathrm{k(t-1)}}, \quad \forall k \in K,\ \forall t \in T. \tag{54}$$

– Constraint (55) is associated with the quantity of sugarcane cut in plot i, in state q, in condition c, in variety v, on farm π in period t:

$$X_{iqcv\pi t} \leqslant Z_{iqcv\pi t}, \quad \forall i \in I,\ \forall q \in Q,\ \forall c \in C,\ \forall \pi \in \pi',\ \forall t \in T. \tag{55}$$

– Constraint (56) is associated with the quantity of sugarcane provided by supplier k in period t:

$$N_{kt} \leqslant W_{kt}, \quad \forall k \in K, t \in T. \tag{56}$$

– Constraint (57) is associated with the quantity of sugarcane cut in plot i, in state q, in condition c, in variety v, on farm π in period t, and the quantity of sugarcane cut by supplier k, in period t, which will be transported by mode f, in period t:

$$\sum_{i\in I}\sum_{q\in Q}\sum_{c\in C}\sum_{v\in V}\sum_{\pi\in\Pi} X_{iqcv\pi t} + \sum_{k\in K} N_{kt} = \sum_{f\in F} Y_{ft}, \quad \forall t \in T. \tag{57}$$

– Constraint (58) establishes the quantity transported in week t, which should be equal to the quantity cut using strategy j in period t:

$$\sum_{f\in F} Y_{ft} = \sum_{j\in J} H_{jt}, \quad \forall t \in T \tag{58}$$

– Constraint (59) establishes that there shall not be a stock of sugarcane for the following season's crop in plot i, in state q, in condition c, in variety v, and on farm π:

$$\sum_{i\in I}\sum_{q\in Q}\sum_{c\in C}\sum_{v\in V}\sum_{\pi\in\Pi} Z_{iqcv\pi t} = \sum_{i\in I}\sum_{q\in Q}\sum_{c\in C}\sum_{v\in V}\sum_{\pi\in\Pi} X_{iqcv\pi t}, \quad \forall t \in T. \tag{59}$$

– Constraint (60) establishes that there shall not be a stock of sugarcane from supplier k for the following season:

$$\sum_{k\in K}\sum_{t\in T} N_{kt} = \sum_{k\in K}\sum_{t\in T} W_{kt}. \tag{60}$$

– Constraints (61) and (62) established the bounds (upper and lower) for the company in period t:

$$\sum_{k\in K}\sum_{t\in T} N_{kt} + \sum_{i\in I}\sum_{q\in Q}\sum_{c\in C}\sum_{v\in V}\sum_{\pi\in\Pi} X_{iqcv\pi t} \geqslant M_t^{\min} \cdot \frac{\mu_t}{100} \cdot \frac{\phi_t}{100}, \quad \forall t \in T, \tag{61}$$

$$\sum_{k\in K}\sum_{t\in T} N_{kt} + \sum_{i\in I}\sum_{q\in Q}\sum_{c\in C}\sum_{v\in V}\sum_{\pi\in\Pi} X_{iqcv\pi t} \leqslant M_t^{\max} \cdot \frac{\mu_t}{100} \cdot \frac{\phi}{100}, \quad \forall t \in T. \tag{62}$$

– Constraint (63) is associated with the capacity of the company's own transport in period t:

$$Y_{ft} \leqslant \frac{\delta_{ft}}{100} \cdot \frac{\phi_t}{100} \cdot \mathrm{CP}_t, \quad \forall f \in F,\ \forall t \in T. \tag{63}$$

– Constraint (64) is associated with the capacity of the cutting strategy j in period t:

$$H_{jt} \leqslant CT_{jt}, \quad \forall j \in J,\ \forall t \in T. \tag{64}$$

– Constraints (65)–(68) model the uncertainty in the calculation of RTS, per ton, with the incorporation of a deviation of ±20%, according to calculation suggested by the Company's decision-makers:

$$\left(\sum_{i\in I}\sum_{q\in Q}\sum_{c\in C}\sum_{v\in V}\sum_{\pi\in\Pi}\sum_{t\in T}(X_{qcv\pi t} \cdot ATR_{iqcv\pi t} - X_{qcv\pi t} \cdot U_{iqcv\pi t}) + \sum_{k\in K}\sum_{t\in T} N_{kt} \cdot \mathrm{ATRK}_{kt} - \sum_{k\in K}\sum_{t\in T} N_{kt} \cdot \mathrm{UK}_{kt}\right) \cdot \omega - d_7^+ + d_7^- = y_7$$

or

$$\left(\sum_{i\in I}\sum_{q\in Q}\sum_{c\in C}\sum_{v\in V}\sum_{\pi\in\Pi}\sum_{t\in T}(X_{qcv\pi t} \cdot ATR_{iqcvpit} - X_{qcv\pi t} \cdot U_{iqcv\pi t}) + \sum_{k\in K}\sum_{t\in T} N_{kt} \cdot \mathrm{ATRK}_{kt} - \sum_{k\in K}\sum_{t\in T} N_{kt} \cdot \mathrm{UK}_{kt}\right) \cdot \tau - d_7^+ + d_7^- = y_7, \tag{65}$$

$$y_7 \geqslant 0.8\left(\sum_{i\in I}\sum_{q\in Q}\sum_{c\in C}\sum_{v\in V}\sum_{\pi\in\Pi}\sum_{t\in T}(X_{qcv\pi t} \cdot ATR_{iqcv\pi t} - X_{qcv\pi t} \cdot U_{iqcv\pi t}) + \sum_{k\in K}\sum_{t\in T} N_{kt} \cdot \mathrm{ATRK}_{kt} - \sum_{k\in K}\sum_{t\in T} N_{kt} \cdot \mathrm{UK}_{kt}\right), \tag{66}$$

$$y_7 \leqslant 1.2\left(\sum_{i\in I}\sum_{q\in Q}\sum_{c\in C}\sum_{v\in V}\sum_{\pi\in\Pi}\sum_{t\in T}(X_{qcv\pi t} \cdot ATR_{iqcv\pi t} - X_{qcv\pi t} \cdot U_{iqcv\pi t}) + \sum_{k\in K}\sum_{t\in T} N_{kt}.\mathrm{ATRK}_{kt} - \sum_{k\in K}\sum_{t\in T} N_{kt} \cdot \mathrm{UK}_{kt}\right), \tag{67}$$

$$y_7 + e_7^- - e_7^+ = 1,800,000. \tag{68}$$

– Constraints (69)–(72) model the uncertainty in the agricultural transportation cost for transport mode f:

$$\begin{array}{l}\left(\sum_{i\in I}\sum_{q\in Q}\sum_{c\in C}\sum_{v\in V}\sum_{\pi\in\Pi}\sum_{t\in T}\sum_{f\in F}X'_{iqcv\pi t}\cdot L_{f\pi}\right)\cdot\omega - d_1^+ + d_1^- = y_1\\ \text{or}\\ \left(\sum_{i\in I}\sum_{q\in Q}\sum_{c\in C}\sum_{v\in V}\sum_{\pi\in\Pi}\sum_{t\in T}\sum_{f\in F}X'_{iqcv\pi t}\cdot L_{f\pi}\right)\cdot\tau - d_1^+ + d_1^- = y_1\end{array}, \tag{69}$$

$$y_1 \geqslant 0.8\left(\sum_{i\in I}\sum_{q\in Q}\sum_{c\in C}\sum_{v\in V}\sum_{\pi\in\Pi}\sum_{t\in T}\sum_{f\in F}X_{iqcv\pi t}\cdot L_{f\pi}\right), \tag{70}$$

$$y_1 \leqslant 1.2\left(\sum_{i\in I}\sum_{q\in Q}\sum_{c\in C}\sum_{v\in V}\sum_{\pi\in\Pi}\sum_{t\in T}\sum_{f\in F}X_{iqcv\pi t}\cdot L_{f\pi}\right) \tag{71}$$

$$y_1 + e_1^- - e_1^+ = 3,013,425. \tag{72}$$

– Constraints (73)–(76) model the uncertainty in the agricultural cost related to the transportation of raw material provided by supplier k:

$$\begin{array}{l}\left(\sum_{f\in F}\sum_{k\in K}\sum_{t\in T}N_{kt}\cdot Lk_{fk}\right)\cdot\omega - d_2^+ + d_2^- = y_2\\ \text{or}\\ \left(\sum_{f\in F}\sum_{k\in K}\sum_{t\in T}N_{kt}\cdot Lk_{fk}\right)\cdot\tau - d_2^+ + d_2^- = y_2\end{array}, \tag{73}$$

$$y_2 \geqslant 0.8\left(\sum_{f\in F}\sum_{k\in K}\sum_{t\in T}N_{kt}\cdot Lk_{fk}\right), \tag{74}$$

$$y_2 \leqslant 1.2\left(\sum_{f\in F}\sum_{k\in K}\sum_{t\in T}N_{kt}\cdot Lk_{fk}\right), \tag{75}$$

$$y_2 + e_2^- - e_2^+ = 8,284,183. \tag{76}$$

– Constraints (77)–(80) model the uncertainty in the agricultural cost related to the dimensioning of the cutting strategy j on farm π:

$$\begin{array}{l}\left(\sum_{i\in I}\sum_{q\in Q}\sum_{c\in C}\sum_{v\in V}\sum_{\pi\in\Pi}\sum_{j\in J}\sum_{t\in T}X_{iqcv\pi t}\cdot R_{j\pi}\right)\cdot\omega - d_3^+ + d_3^- = y_3\\ \text{or}\\ \left(\sum_{i\in I}\sum_{q\in Q}\sum_{c\in C}\sum_{v\in V}\sum_{\pi\in\Pi}\sum_{j\in J}\sum_{t\in T}X_{iqcv\pi t}\cdot R_{j\pi}\right)\cdot\tau - d_3^+ + d_3^- = y_3\end{array}, \tag{77}$$

$$y_3 \geqslant 0.8\left(\sum_{i\in I}\sum_{q\in Q}\sum_{c\in C}\sum_{v\in V}\sum_{\pi\in\Pi}\sum_{j\in J}\sum_{t\in T}X_{iqcv\pi t}\cdot R_{j\pi}\right), \tag{78}$$

$$y_3 \leqslant 1.2\left(\sum_{i\in I}\sum_{q\in Q}\sum_{c\in C}\sum_{v\in V}\sum_{\pi\in\Pi}\sum_{j\in J}\sum_{t\in T}X_{iqcv\pi t}.R_{j\pi}\right), \tag{79}$$

$$y_3 + e_3^- - e_3^+ = 2,673,263. \tag{80}$$

– Constraints (81)–(84) model the uncertainty related to the dimensioning of cutting strategy j from supplier k:

$$\left(\sum_{k\in K}\sum_{j\in J}\sum_{t\in T}N_{kt}\cdot Rk_{jk}\right)\cdot\omega-d_4^{+}+d_4^{-}=y_4 \quad \text{or} \quad \left(\sum_{k\in K}\sum_{j\in J}\sum_{t\in T}N_{kt}\cdot Rk_{jk}\right)\cdot\tau-d_4^{+}+d_4^{-}=y_4, \tag{81}$$

$$y_4\geqslant 0.8\left(\sum_{k\in K}\sum_{j\in J}\sum_{t\in T}N_{kt}\cdot Rk_{jk}\right), \tag{82}$$

$$y_4\leqslant 1.2\left(\sum_{k\in K}\sum_{j\in J}\sum_{t\in T}N_{kt}\cdot Rk_{j\,k}\right), \tag{83}$$

$$y_4+e_4^{-}-e_4^{+}=35{,}247{,}380. \tag{84}$$

– Constraints (85)–(88) model the uncertainty associated with the cost of raw material for the company:

$$\left(\sum_{i\in I}\sum_{q\in Q}\sum_{c\in C}\sum_{v\in V}\sum_{\pi\in\Pi}\sum_{t\in T}X_{iqcv\pi t}\cdot C_{\pi}\right)\cdot\omega-d_5^{+}+d_5^{-}=y_5 \quad \text{or} \quad \left(\sum_{i\in I}\sum_{q\in Q}\sum_{c\in C}\sum_{v\in V}\sum_{\pi\in\Pi}\sum_{t\in T}X_{iqcv\pi t}\cdot C_{\pi}\right)\cdot\tau-d_5^{+}+d_5^{-}=y_5, \tag{85}$$

$$y_5\geqslant 0.8\left(\sum_{i\in I}\sum_{q\in Q}\sum_{c\in C}\sum_{v\in V}\sum_{\pi\in\Pi}\sum_{t\in T}X_{iqcv\pi t}\cdot C_{\pi}\right), \tag{86}$$

$$y_5\leqslant 1.2\left(\sum_{i\in I}\sum_{q\in Q}\sum_{c\in C}\sum_{v\in V}\sum_{\pi\in\Pi}\sum_{t\in T}X_{iqcv\pi t}\cdot C_{\pi}\right), \tag{87}$$

$$y_5+e_5^{-}-e_5^{+}=1{,}050{,}000. \tag{88}$$

– Constraints (89)–(92) model the uncertainty associated with the source of raw material from supplier k:

$$\left(\sum_{k\in K}\sum_{t\in T}N_{kt}\cdot CK_{kt}\right)\cdot\omega-d_6^{+}+d_6^{-}=y_6 \quad \text{or} \quad \left(\sum_{k\in K}\sum_{t\in T}N_{kt}\cdot CK_{kt}\right)\cdot\tau-d_6^{+}+d_6^{-}=y_6, \tag{89}$$

$$y_6\geqslant 0.8\left(\sum_{k\in K}\sum_{t\in T}N_{kt}\cdot CK_{kt}\right), \tag{90}$$

$$y_6\leqslant 1.2\left(\sum_{k\in K}\sum_{t\in T}N_{kt}\cdot CK_{kt}\right), \tag{91}$$

$$y_6+e_6^{-}-e_6^{+}=7{,}919{,}525. \tag{92}$$

– Constraint (93) expresses the non-negativity conditions:

$$X_{iqcv\pi t} \geqslant 0; Y_{ft} \geqslant 0; Z_{iqcv\pi t} \geqslant 0; W_{kt} \geqslant 0; N_{kt} \geqslant 0; H_{jt} \geqslant 0,$$
$$\forall i \in I,\ \forall q \in Q,\ \forall c \in C,\ \forall v \in V,\ \forall \pi \in \Pi,\ \forall t \in T,\ \forall f \in F,\ \forall k \in K,\ \forall j \in J$$
$$d_1^+, d_1^-, e_1^+, e_1^-, d_2^+, d_2^-, e_2^+, e_2^-, d_3^+, d_3^-, e_3^+, e_3^-, d_4^+, d_4^-, e_4^+, e_4^-, \tag{93}$$
$$d_5^+, d_5^-, e_5^+, e_5^-, d_6^+, d_6^-, e_6^+, e_6^-, d_7^+, d_7^-, e_7^+, e_7^-, \geqslant 0.$$

5. Results analysis

Considering a real problem with 16 contributing farms, four suppliers from whom the company purchases raw material, two agricultural transporters, two cutting strategies, 32 weeks, two states, and three conditions and 12 varieties of sugarcane, the RMCGP-LHS model presented 2,507,306 constraints and 16,245,741 non-negative variables.

For the model's optimization, a computer with an Intel Core i7 processor was used with max turbo frequency, 4 MB cache, and 8 GB of RAM DDR3 80 MHZ, with Windows 7, 64-bits as the operating system. The computational time was approximately 45 minutes for each scenario.

Thus, Table 1 shows how much sugarcane should be cut in plot *i*, in state *q*, in condition *c*, of variety *v*, on farm π for the simulated scenarios, as suggested by the company's managers, considering the recommended uniform percentage deviations of 5%, 10%, and 20% in the input parameters:

$R_{j,\ \pi,}$ Cost of cutting strategy *j* on farm π [$/ton];
$Rk_{j,k}$ Cost of cutting strategy *j* from supplier *k* [$/ton];
$ATR_{i\ q\ c\ v\ \pi\ t}$ TRS (optimal) of plot *i*, in condition *c*, of variety *v*, on farm π, in period *t* [tons/week];
$U_{i\ q\ c\ v\ \pi\ t}$ TRS for plot *i*, in condition *c*, of variety *v*, on farm π, in period *t* [tons/week];
$ATRK_{k\ t}$ TRS (optimal) in sugarcane from supplier *k* in period *t* [tons/week];
$UK_{k\ t}$ TRS of sugarcane from supplier *k* at the moment of cutting in relation to the TRS from a raw material source in period *t*;
$L_{f\pi}$ Variable transport cost for option *f* for farm π [$/ton];
$Lk_{f\ k}$ Variable transport cost for option *f* from supplier *k* [$/ton];
$CK_{k\ t}$ Raw material cost from supplier *k* in period *t* [$/ton];
C_π Cost of raw material on farm π [$/ton].

As already pointed out, the intervals for τ and ω can be nonsymmetrical and different from each other. In other words, the manager has full autonomy to set the value of these parameters, thus generating different optimal scenarios. Therefore, the adoption of deviations allowed managers to carry out a sensitivity analysis of the solution, thus enabling decisions more suitable to the company's daily reality. It is noted that the optimization for each scenario is aimed at minimizing the overall agro-industrial costs, while minimizing sugar loss.

Tables 1–3 show the quantity of sugarcane that should be harvested weekly for each considered level of deviation. As there were 32 weeks of scheduling, Tables 1–3 contain only partial results from an extensive report made available to managers, including the variable values for $X_{iqcv\,\pi\,t}$, and the quantity of sugarcane cut in plot *i*, in state *q*, in condition *c*, of variety *v*, on farm π, in period *t* [tons].

Per Table 1, with a deviation of 5%, the model indicates that the sugarcane harvest should be started in the fifteenth week. According to Table 2, with a deviation of 10%, the model indicates that the harvest should be started in the tenth week. Per Table 3, with a deviation of 20%, the model indicates that the sugarcane harvest should be started in the fifth week. It is also observed that the company should be supplied with additional purchased sugarcane, as can be seen in Table 4.

Table 4 shows only partial results for the 32 periods considered for purchasing the sugarcane to be processed in week *t*, corresponding to the variable values for $H_{j\ t}$, the variable associated with the dimensioning of cutting strategy *j* in period *t*.

Table 1
5% deviation: quantity of sugarcane cut in plot *i*, in state *q*, in condition *c*, of variety *v*, on farm π, in period *t* [tons].

5% deviation										
Plot	State	Condition	Variety	Farm	Period (Weeks)					
					4	7	9	11	13	14
1	Burnt	Late	8	12				2,275		
1	Burnt	Late	9	7					1,848	
1	Burnt	Late	12	9					1,716	
1	Raw	Intermediate	1	6				286		
1	Raw	Intermediate	5	2						
1	Raw	Intermediate	10	8				200		
1	Raw	Late	1	1				639		
1	Raw	Late	9	3				1,629		
2	Burnt	Early	9	7				94		
2	Burnt	Intermediate	12	9				641		

Table 2
10% deviation: quantity of sugarcane cut in plot i, in state q, in condition c, of variety v, on farm π, in period t [tons].

10% deviation									
Plot	State	Condition	Variety	Farm	Period (weeks)				
					1	4	6	8	13
1	Burnt	Intermediate	12	11					
1	Burnt	Late	7	13					
1	Burnt	Late	9	7					
1	Raw	Early	11	5					
1	Raw	Intermediate	1	10					
1	Raw	Intermediate	5	2				200	
1	Raw	Late	1	1					
2	Burnt	Intermediate	12	9					
2	Burnt	Late	8	14					
1	Burnt	Intermediate	12	11					
1	Burnt	Late	7	13					

Table 3
20% deviation: quantity of sugarcane cut in plot i, in state q, in condition c, of variety v, on farm π, in period t [tons].

20% deviation										
Plot	State	Condition	Variety	Farm	Period (Weeks)					
					5	6	7	12	14	15
1	Burnt	Late	7	13						234
1	Burnt	Late	9	7					1,848	
1	Burnt	Late	12	9						
1	Raw	Early	11	5					286	
1	Raw	Intermediate	1	6			1,329			
1	Raw	Intermediate	1	10						383
1	Raw	Intermediate	10	8						
1	Raw	Late	1	1						
1	Raw	Late	9	3						94
2	Burnt	Intermediate	12	9						365
2	Burnt	Late	1	16						

Table 4
Quantity of sugarcane purchased by supplier k in period t.

Supplier	Period (weeks)						
	1	2	3	4	5	6	7
5% deviation							
A		21,165	22,577.5		21,922.5		
B	46,456.55			20,427		20,687.5	
C							21,787.5
10% deviation							
A		21,165				17,565.36	
B					25,056.14		21,902.5
C	20,496.5		22,577.5	20,656.77		2,123.288	
20% deviation							
A	21,152.5	46,563		22,355			
B							
C			49,670.5		21,545.5	18,131.5	18,251.5

Illustrating contents in Table 4, with a deviation of 20%, the model indicated that:

– In week 1, the total purchased from Supplier A amounted to 21,152.5 ton.
– In week 2, the total purchased from Supplier A amounted to 46,563 ton.
– In week 3, the total purchased from Supplier C amounted to 49,670.5 ton.
– In week 4, the total purchased from Supplier A amounted to 22,355 ton.
– In week 5, the total purchased from Supplier C amounted to 21,545.5 ton.

Table 5
For all deviations: which cutting strategy j to use in week t.

Cutting strategy							
	1	2	3	4	5	6	7
5% deviation							
Mechanized	45,000	9,165	22,577.5	12,355	21,922.5	20,687.5	11,902.5
Manual	1,456.55	12,000		10,000			10,000
10% deviation							
Mechanized	21,152.5	9,165	22,577.5	12,355	25,056.14	20,687.5	11,902.5
Manual		12,000		10,000			10,000
20% deviation							
Mechanized	21,152.5	34,563	45,000	12,355	21,922.5	20,687.5	11,902.5
Manual		12,000	4,670.5	10,000			10,000

– In week 6, the total purchased from Supplier C amounted to 18,131.5 ton.
– In week 7, the total purchased from Supplier C amounted to 18,251.5 ton.

Table 5 illustrates that, for all deviations and for only eight of the first considered periods, the adequate dimensioning of cutting strategy j in week t, which is of utmost importance, as it leads to a global increase in productivity of all processes, corresponds to the value of N_{kt}, the variable associated with the choice of the sugarcane quantity purchased from supplier k in period t.

It can be observed that the use of deviations did not suggest alterations in decisions linked to the cutting strategy, as:

– In week 1, mechanized cutting was utilized to cut 21,153 tons.
– In week 2, mechanized cutting was utilized to cut 9,165 tons, while 12,000 tons were cut manually.

Observe that the case study had seven objectives and three possible levels of deviation, which amounts to $3^7 = 2{,}187$ possible combinations, making the disclosure of all results for each scenario unfeasible. To demonstrate the potential of the new RMCGP-LHS model, the managers chose 10 scenarios with their respective values of total agro-industrial costs, as shown in Table 6.

It is observed that, in Scenario 1 (line 2, Table 6), the uncertainty is not considered in the input data, and objective 7, which is associated with the drop in the sugar's POL owing to immeasurability with the other cost objectives, is disregarded without compromising the cost analysis.

Analysing results from Table 6, one can conclude that Scenario 1 presented the highest total cost ($86,382,405.00), as expected, once it had dealt with a single optimization objective, which evidenced the advantages of adopting a multi-objective model under uncertainty, as the RMCGP-LHS model herein presented. Furthermore, Scenario 7 showed the lowest total cost ($61,129,267) followed y Scenario 9 ($61,305,491).

Table 7 shows a more detailed display of results in Table 6, with scenarios and values (costs) of each objective function associated with the suitable considered deviations. In Table 7, it can be seen that:

– Scenarios 2 and 3 had the lowest cost for objective 1.
– Scenarios 7 had the lowest costs for objectives 2 and 4.
– Scenarios 2, 6, and 9 had the lowest costs for objectives 3.
– Scenario 9 had the lowest costs for objective 5.
– Scenarios 3 and 7 had the lowest costs for objective 6.

Table 6
Some selected scenarios analyzed using the RMCGP-LHS model.

Scenario	Obj-1 (%)	Obj-2 (%)	Obj-3 (%)	Obj-4 (%)	Obj-5 (%)	Obj-6 (%)	Total cost [$]
1	0	0	0	0	0	0	58,230,020.80
2	5	5	5	5	20	20	62,725,494.42
3	20	20	20	20	5	5	68,931,435.57
4	20	20	20	20	10	10	68,974,846.87
5	10	10	10	10	20	20	64,954,198.19
6	5	5	5	5	20	20	62,725,494.42
7	5	5	5	5	5	5	61,198,404.54
8	10	10	10	10	10	10	64,083,506.52
9	10	20	5	5	5	5	62,538,699.49
10	20	20	20	20	20	20	69,876,023.77

Table 7
Some selected scenarios analyzed using the RMCGP-LHS model for each cost objective function [$].

Scenarios	Obj-1	Obj-2	Obj-3	Obj-4	Obj-5	Obj-6
1	3,013,425.00	8,284,183.00	2,673,263.00	35,247,380.00	1,092,244.00	7,919,525.00
2	2,931,167.76	8,770,681.58	2,617,553.46	37,883,376.83	1,205,571.60	9,317,143.20
3	4,362,142.79	9,701,061.88	3,968,463.58	41,040,457.92	1,418,497.50	8,091,796.65
4	3,616,110.00	9,941,019.30	3,207,915.67	42,296,856.00	1,201,468.40	8,711,477.50
5	3,314,767.00	9,112,601.03	2,940,589.37	38,772,118.00	1,310,692.80	9,503,430.00
6	2,931,167.76	8,770,681.58	2,617,553.46	37,883,376.83	1,205,571.60	9,317,143.20
7	3,816,874.94	8,488,429.14	3,472,405.63	35,910,400.68	1,418,497.50	8,091,796.65
8	3,673,194.13	8,991,216.33	3,341,879.40	38,136,557.36	1,358,511.00	8,582,148.30
9	3,070,747.00	10,022,271.80	2,617,553.46	37,382,001.72	1,054,875.15	8,391,250.35
10	3,616,110.00	9,941,019.30	3,207,915.67	42,296,856.00	1,310,692.80	9,503,430.00

Table 7 presents analysis of the optimized scenarios, through which it is evidenced that:

- Objective 4 (Total cost linked to cutting strategy for purchased raw material) contributed most to an increase in total costs.
- Objective 2 (Total cost of agricultural transport for purchased sugarcane) and Objective 6 (Total cost of purchased raw material) significantly contributed to an increase in cost.

Based on this information, the managers were able to seek out solutions aimed at reducing possible oscillations resulting from inherent uncertainties, as well as minimizing the total agro-industrial cost of operations. Such actions could apply to:

- The adoption of agricultural sampling on each farm and plot, aiming at correctly determining the optimal POL of the sugarcane plots.
- The acquisition of more productive machinery.
- Training investments for non-specialized labor.

6. Conclusions and recommendations for new research

This study has presented a new procedure for handling decision making related to sugarcane harvesting, cutting strategies, and transportation from farms to company, aiming at minimizing loss of sugar POL, and reducing total agro-industrial costs. The RMCGP-LHS herein proposed was applied to a real, large-scale problem at a company located in the state of Minas Gerais, Brazil.

The model used in this study, the RMCGP-LHS, has different characteristics from those found in the literature, which deal with the inherent uncertainty in scheduling planting and sugarcane harvesting. One important contribution of the RMCGP-LHS model is that it incorporates the uncertainty in the LHS of the model parameters and avoids the use of discrete or binary variables by means of deviational variables and other continuous auxiliary variables. Based on this, the model produced excellent results when applied to a real-time, large-scale problem, while performing in computational time, which enables managers to make real-time decisions.

Furthermore, it allowed a sensitivity analysis of the solution, enabling managers to test more feasible scenarios for the day-to-day reality of the company, thus presenting itself as an important scientific and technological contribution to agricultural commerce in Brazil.

Additionally, based on preliminary tests carried out with the RMCGP-LHS model (which were not detailed in this paper, but made available to managers), it was possible to identify that it becomes more sensitive to the deviations when there are conflicting objectives, objectives to be maximized and minimized, or when reference values for the objectives are available.

For future research, the approach adopted in this article will be further investigated, especially in terms of the insertion of uncertainty in the data, while incorporating other functionalities in the model, such as:

- An analysis of the uncertainty effect on input parameters using robust optimization [31];
- An analysis of the uncertainty effect on input parameters using robust stochastic optimization [18,19].
- An analysis of the uncertainty using fuzzy logic [12].
- Application of the RMCGP-LHS in the complete problem of planting aggregate planning, harvest, transport, production, stock, logistics, and cogeneration of energy.

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