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THAMIS CORTES FREIRE DE CARVALHO FERREIRA

Planning constellations of small satellites around Io

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THAMIS CORTES FREIRE DE CARVALHO FERREIRA

Planning constellations of small satellites around Io

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I dedicate this work to Lord Almighty, the Beginning and the End...

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Abstract

The aim of this investigation was to study and map the initial orbital conditions around Io for the implementation of a constellation of small satellites. The most relevant dynamic scenario was considered. The study used two approaches: analytical and numerical. Previous studies have shown that, due to intense disturbance from Jupiter, probes around Io generally have a short lifetime. This scenario hinders the implementation of constellations. The best conditions were mapped considering inclination, semi-major axis, and longitude of the ascending node. The non-sphericity of Io, its ellipticity (C_{22}), and flattening (J_2) were considered. The third-body perturbation from Jupiter, Europa, Callisto, and Ganymede was also included. This mapping was performed using the IAS15 integrator for a total integration time of 2.5 years, with 20 radius of Io as the criterion for escape. Still, in all other simulations, the Hill radius was adopted. Orbits that survived the total integration time were obtained. Still, when considering orbits with an initial semi-major axis greater than $2.0R_{Io}$, the altitude varied significantly throughout the analysis period, meaning that if the constellations were implemented in these conditions, the probe would not maintain an average altitude relative to each other within the same constellation. In this sense, the study of frozen orbits was an option to be applied. For the study of frozen orbits, a simplified dynamic scenario was considered, excluding Galilean satellites and the ellipticity of Io, but including their third and fourth-order zonal harmonics. The results showed the existence of frozen orbits, with initial conditions that maintained acceptable altitude control for implementing low-altitude constellations, following numerical validation performed with IAS15, considering the complete dynamical scenario. Two constellation configurations were then proposed, with the objective of observing the paterae Loki, Fuchi, Amaterasu, and Surt. Probes in these constellations have quasi-circular orbits, with initial altitudes under 800 km, inclinations under 48 degrees, and $\omega = \frac{3\pi}{2}$. The constellation at 600 km performed best, and Loki, being the target nearest to the equator of Io, was most frequently observed.

keywords: Io, Jupiter, Orbits, Constellations, and Frozen Orbits.

Resumo

O objetivo deste trabalho foi estudar e mapear as condições orbitais iniciais em torno de Io para a implementação de uma constelação de pequenos satélites. Foi considerado o cenário dinâmico mais relevante. O estudo utilizou duas abordagens: analítica e numérica. Estudos anteriores demonstraram que, devido à intensa perturbação de Júpiter, as sondas em torno de Io geralmente têm uma vida útil curta. Esse cenário dificulta a implementação de constelações. As melhores condições foram mapeadas considerando a inclinação, o semieixo maior e a longitude do nó ascendente. A não esfericidade de Io, sua elipticidade (C_{22}) e achatamento (J_2) foram considerados. A perturbação de terceiro corpo de Júpiter, Europa, Calisto e Ganimedes também foi incluída. Esse mapeamento foi simulado usando o integrador IAS15 para um tempo total de integração de 2,5 anos, com 20 raios de Io como critério de escape. No entanto, em todas as outras simulações, o critério de escape utilizado foi o raio de Hill. Foram obtidas órbitas que sobreviveram ao tempo total de integração. Entretanto, ao considerar órbitas com um semieixo maior inicial superior a $2,0R_{Io}$, a altitude das sondas apresentaram uma variação significativa ao longo do período de analisado, significando que, caso as constelações fossem implementadas nessas condições, as sondas não manteriam uma altitude média relativa entre si. Nesse sentido, o estudo de órbitas congeladas mostrou-se uma opção viável a ser aplicada. Para o estudo de órbitas congeladas, foi considerado um cenário dinâmico simplificado, excluindo os satélites galileanos e a elipticidade de Io, mas incluindo seus harmônicos zonais de terceira e quarta ordem. Os resultados mostraram a existência de órbitas congeladas, com condições iniciais que mantinham um controle de altitude aceitável para a implementação de constelações de baixa altitude, após validação numérica realizada com o IAS15, considerando o cenário dinâmico completo. Duas configurações de constelações foram então propostas, com o objetivo de observar as crateras vulcânicas Loki, Fuchi, Amaterasu e Surt. As sondas nessas constelações apresentaram órbitas quase circulares, com altitudes iniciais abaixo de 800 km, inclinações abaixo de 48 graus e $\omega = \frac{3\pi}{2}$. A constelação a 600 km obteve o melhor desempenho, e Loki, o alvo mais próximo do equador de Io, sendo o mais frequentemente observado.

Paravras-chave: Io, Júpiter, Órbitas, Constelações e Órbitas Congeladas.

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1 Introduction

Ancient peoples studied the skies for various purposes. They recorded the dates of solstices, equinoxes, eclipses, and their durations (KRUPP, 2003). This data was used in agriculture to distinguish planting and harvesting seasons. It was also used to create calendars, as seen with the Mayans and Babylonians. Over time, advances in astronomy and mathematics led to the development of instruments such as the astrolabe. This device played a crucial role for explorers and navigators in the 15th century, marking the start of the Age of Discovery. The planets Mercury, Venus, Mars, Jupiter, and Saturn were known before the Christian era. The heliocentric theory, describing their movement around the Sun, was formalized in the 16th century by Nicolaus Copernicus in his work "De revolutionibus orbium coelestium." In 1609, Kepler published his three laws of planetary motion. The next year, Galileo Galilei published telescopic observations in the Latin translation by Albert Van Helden, "Siderius Nuncius":

Accordingly, on the seventh day of January of the present year 1610, 70 at the first hour of the night, when I inspected the celestial constellations through a spyglass, Jupiter presented himself. And since I had prepared for myself a superlative instrument, I saw (which earlier had not happened because of the weakness of the other instruments) that three little stars were positioned near him — small but yet very bright. (GALILEI, 1989)

Galileo made several observations of Jupiter and discovered four wandering stars that, according to him, moved around Jupiter. He verified that they were natural satellites of Jupiter, about which he wrote: "Since then, I have admitted that there were, without any doubt, stars in the heavens that orbited Jupiter in the same way that Mercury and Venus orbit the Sun..." (FERRAZ-MELLO, 2022). Only centuries later did Laplace discover that these natural satellites are in mean motion resonance, which hovers around 180 degrees, with a minimum restoring force due to the mutual actions of the satellites Io, Europa, and Ganymede, tending to bring this angle to the value 180° (MUSOTTO et al., 2002).

Jupiter and its moons would only be visited by probes on space exploration missions in the mid-20th century. Launched in 1972, Pioneer 10 was the first artificial satellite to see Jupiter and its natural satellites. During this mission, several essential measurements were taken, including the magnetic field of Jupiter, measurements of the Io atmosphere (KLIORÉ et al., 1975), and measurements of the gravitational field and mass of Jupiter, as well as the masses of Io, Europa, Ganymede, and Callisto (ANDERSON et al., 1974). Pioneer 11, launched one year after Pioneer 10, also flew by Jupiter and captured the first images of its polar region. Voyager 1 brought an important discovery about Io, which was the detection of intense volcanic activity (JOHNSON et al., 1979), (SMITH et al., 1979). Currently, the scientific community considers Io to be the body with the highest volcanic activity in the Solar System. The Galileo probe measured the moment of inertia of Io,

and more detailed images were obtained. Mountains and evidence of volcanic lava flows were observed on them (SHOWMAN et al., 1999). The extensive volcanic activity on Io, captured by Galileo, may be 100 times greater than that found on Earth (NASA, 2025). One of the most recent data acquisitions available for Io was made by Juno mission in 2023. Approximately 400 images were collected from the surface of this volcanic moon, and the event lasted approximately six hours. Previous studies and data indicate that Io has numerous irregular craters containing lava lakes (MCEWEN et al., 2000). New data provided by Juno/JIRAM reveal a common set of thermal characteristics for at least ten craters, including thermal rings. Four of these craters, or paterae, will be the targets of this work. These new data suggest that hot rings around craters are a common phenomenon and indicative of active lava lakes (MURA et al., 2024). The JUICE and Europa Clipper missions have already been launched and will study the Jupiter system. They are expected to arrive at Jupiter in the early part of the next decade, demonstrating the community interest in exploring this system.

1.1 Literature Review

Since the first space launches, obtaining alternative approaches for global coverage of celestial bodies has proven to be highly useful and necessary for various purposes, including military, telecommunications, and monitoring specific conservation areas. A space vehicle can view a particular region of the target body. Still, the size of the covered area depends on several factors, such as its altitude and the scanning angle of its sensor, among others (BESTE, 1978). Therefore, the use of satellite constellations was proposed. A satellite constellation can be defined as a group of spacecraft that have an interconnected system and cooperate for a specific purpose. Small satellite constellations offer several valuable advantages for reconnaissance and mapping, including improved coverage, shorter revisit times, ongoing observation of dynamic phenomena, and adaptability for updates or reconfiguration (WERTZ, 2011).

Systems such as the Navstar Global Positioning System (GPS), which began operating at the end of the 20th century, provide continuous, three-dimensional global navigation coverage of the Earth. Building on the success of such navigation technologies, many other systems subsequently came into operation, such as Iridium and, more recently, Starlink, which are used for communication and internet access. Given the growing demand for these services, improving the design and maintenance of these constellations is becoming increasingly important in the private sector.

In the 1960s and 1970s, some studies proposed constellations in which the bodies were in circular orbits and had a configuration that allowed them to have orbits with common periods (DRAIM, 1985). Configurations with elliptical and more comprehensive orbits can be seen in DONIANZ and ULYBYSHEV (2002), MORTARI et al. (2004a), ARNAS

et al. (2018), and ARNAS et al. (2017). The use of constellations was not limited to Earth and was applied to other bodies in the Solar System. The study conducted by Ely and Lieb (2006) proposed a constellation model around the Moon, consisting of six satellites covering both the south and north poles. The results showed that this configuration remained stable for approximately 10 years. For this study, disturbances from the Earth and Sun, radiation pressure, and the non-sphericity of the Moon were considered.

In the work by Gabriele Montesia et al. (2024), a constellation was proposed that would provide global longitudinal coverage of Mars at a low maintenance cost. The restricted three-body problem was employed, and for the Mars-Phobos and Mars-Deimos systems, the zonal harmonics J_2 and J_4 of the bodies were taken into account in the dynamics. Periodic and resonant solutions were obtained, and among the cases analyzed is the implementation of four satellites. The first three will be in periodic orbits around the Lagrangian points L_3 , L_4 , and L_5 of the Mars-Deimos system, and the fourth will be in a periodic orbit around Phobos.

Sending probes to study celestial bodies requires detailed planning and prior knowledge of the dynamic environment surrounding them. Hoolst et al. (2020) studied the gravitational torque that Io experiences due to Jupiter and how this effect deforms the natural satellite, periodically altering its rotation. There was also discussion about Io's volcanism and its internal structure, as well as the role of Europa and Ganymede in this mechanism. The rate of tidal energy dissipation between these natural satellites controls the stability of the resonant orbital configuration in this system. Io's gravitational field was studied by Anderson et al. (2001), who estimated second-order harmonics using data from the Galileo mission. Zharkov (2004) studied the higher-order components of Io's gravitational field using equilibrium figure theory. For the homogeneous model, the values of J_4 , C_{42} , C_{44} , among others, were estimated. The importance of Jupiter's J_2 and its influence on Io and Europa were studied by Lari and Saillenfest (2024) and Huang et al (2024).

Kozai (1973), Broucke (2003), and Prado (2003) studied third-body perturbations in the motion of artificial satellites and highlighted their relevance to the lifetime of orbits, while works such as Marchal (2000), Liu and Wang (2000), and Santos et al. (2017) indicate that the effects due to the flattening of the central body tend to compensate for the effects due to a disturbing body, helping to maintain more stable orbits. Ferreira et al. (2022) mapped the ideal initial orbital conditions for orbits around Io, taking into account the perturbing effects of Jupiter and the non-sphericity of Io.

To generate these lifetime maps, the restricted three-body circular problem was solved numerically. The results showed that most orbits around Io have lifetimes of less than six months. However, some regions were identified where the initial orbital conditions resulted in longer lifetimes of approximately two years. Caritá et al. (2023) obtained lifetime maps of probes around Io considering a more realistic scenario, taking into account

the J_2 gravitational coefficient of Jupiter, the gravitational effects of Saturn, and the Sun. No surviving orbits were recorded, considering that the total integration time was approximately 300 days, for the interval between 50 and 130 degrees for the initial inclination of the probes. This study was conducted for initially eccentric orbits, considering different orbital parameters for probes such as the initial argument of perihelion and the longitude of the ascending node.

Xavier et al. (2022) studied the lifetime of probes around Titania, the largest satellite of Uranus. The probe was subject to perturbation due to the non-sphericity of Titania, represented by its second-order zonal gravitational harmonic J_2 and its ellipticity, represented by the sectorial harmonic C_{22} . This work also considered the perturbation due to Uranus. Among the results, it was seen that quasi-circular and low-altitude orbits have a longer lifetime compared to other conditions. This is because these probes are more influenced by Titania's harmonics, thus softening the effect of the third body. Another significant result was the role of the ascending node longitude and the pericenter longitude. It was found that choosing a non-zero value for these two parameters can increase the useful lifetime of the probes.

Frozen orbits have been investigated by several prominent authors (LARA et al., 1995), (ELIPE; LARA, 2003), (CONDOLEO et al., 2016), and (LEI et al., 2019). These works analyze the theoretical foundations for deriving families of frozen orbits, their characteristics and stability, as well as their application to important missions to targets such as Titan and the Galilean moons, including Ganymede and Callisto.

Frozen orbits are very useful in reconnaissance missions, as shown in Abad et al. (2009), where analytical approaches based on Lie-Deprit series are used to obtain frozen orbits for a probe around the Moon. Analyses were performed for low and moderate altitudes, considering the zonal harmonics J_2 and J_7 of the Moon. These conditions were mapped by obtaining a surface of frozen conditions that depends on the inclination, eccentricity, and semi-major axis of the probes.

Tresaco et al. (2018) conducted an analytical study of the dynamics of a probe around Mercury, considering an average model, i.e., without the short-period terms of the probe or a third body. After the averaging process, the frozen conditions were obtained considering Mercury's zonal harmonics up to the sixth order, the effect of C_{22} , the gravitational influence of the Sun, and the radiation pressure. Finally, propagations were made for the mean model, using the planetary equations of Lagrange, over a long period of time, approximately 25 years, where the variations in eccentricity and argument of perihelion were analyzed.

A comparison between analytical and numerical models for frozen orbits was performed by Carvalho et al. (2012) for probes around Europa. In this work, the author considered the perturbation of Jupiter in a circular orbit and the J_2 and C_{22} terms of Europa. Only an average of the perturbing potential was calculated. In this work, the authors considered

the perturbation of Jupiter in a circular orbit and the J_2 and C_{22} terms of Europa. Only an average of the perturbing potential was calculated. Numerical simulations for the model without averages were performed using the Mercury integrator. As a result, frozen orbits were recorded at low altitudes and low inclinations, which maintained their behavior when simulated using a non-averaged model.

1.2 Goals and Outline of the Work

In this context, the main objective of this work was to investigate the initial orbital parameters suitable for designing small body constellations around Io under different scenarios. Frozen orbits were analyzed using an analytical model and a simplified dynamic scenario, which considered perturbations from Jupiter and zonal gravitational harmonics up to the fourth degree of Io. For the numerical model, simulated using the IAS15 integrator, lifetime maps were obtained, considering J_2 and C_{22} of Io, Jupiter, and the other Galilean moons. Subsequently, the propagation of frozen orbits were analyzed with all perturbations. Finally, the best conditions were applied to design two configurations of constellations, in order to visualize the paterae of Loki, Fuchi, Amaterasu, and Surt over a period of 2.5 years. Therefore, this document is divided as follows: Chapter 2 outlines the mathematical formalism and methodology for the N-body perturbation, the non-sphericity of the central body, and an analytical model for obtaining frozen orbits around Io. This chapter also discusses the observational data for the targets, the visibility geometry, and the constellation design parameters. Chapter 3 presents the simulation results, and Chapter 4 summarizes the main findings.

4 Conclusion

This study aimed to investigate and map the optimal initial orbital conditions that enable orbits with durations and configurations suitable for implementing small satellite constellations around Io. An analytical and numerical model was used in this study. Frozen orbits were obtained from the analytical model, whose dynamic scenario considered Io's non-sphericity represented by zonal harmonics up to fourth order, and the gravitational disturbance from Jupiter. Stable frozen orbits were obtained and applied to the design of constellations, with numerical validation performed using the numerical model. The numerical model was used at two different times. Initially, lifetime maps were simulated to investigate the influence of inclination, ascending node longitude, and semi-major axis on the orbit of the probes. In this initial analysis, the dynamic model considered the perturbations of Jupiter, Europa, Ganymede, and Callisto, the flattening of Io (J_2), and its ellipticity (C_{22}). The collision criterion used was the altitude of the probe relative to the surface of Io, and the escape criterion was for the probe to reach approximately 20 radii of Io. The second part consisted of propagating the frozen orbits for 2.5 years that were used in the design of the proposed constellations. In this stage, the criterion used to define the escape of the probe was the radius of gravitational influence of Io (Hill radius).

The propagation was first performed considering the same dynamic scenario as the analytical model, and then the simulation was performed using the complete model, which also took into account $C_{22(Io)}$ and the oblateness of Jupiter ($J_{2(Jup)}$). When propagating the stable frozen orbits for the semi-major axis of $1.22R_{Io}$ for the complete model, it was observed that these conditions provided acceptable altitude control for implementing constellations around Io at low altitudes for the entire mission time established in this work, which was 2.5 years. Therefore, two configurations were proposed for small satellite constellations to observe significant volcanic depressions located north of Io, which were: Loki, Fuchi, Amaterasu, and Surt. The first configuration, case 1, was implemented at an altitude of approximately 400 km ($a_0 = 2221.49$), while case 2 was implemented at approximately 600 km ($a_0 = 2421.49$). All probes were in quasi-circular orbits, with an initial inclination of 15, 20, 30, 40, and 45 degrees for case 1 and up to 40 degrees for case 2, with five probes in the first case and four probes in the second. The orbital planes considered were 0 degrees and 180 degrees, and ω was 270 degrees. The target with the longest viewing time was Loki, the target closest to Io's equator, which was observed for over 110 days. The most efficient configuration was case 2, with probes SAT 3 and 4 performing best when observing Fuchi, Amaterasu, and Surt.

4.1 Next Steps

The following steps involve analyzing constellations with a greater number of satellites and more complete dynamic models. Since Io has many regions that can be studied, as Pale Patera, enabling the inclusion of various targets, this research could be extended to other bodies within the Solar System in future projects.

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