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Steady, Spatially-Periodic Fully-Developed Turbulent Flow in Interrupted-Plate Channels

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Abstract

This paper presents numerical results for steady, spatially-periodic, two-dimensional, fully-developed turbulent flows in interrupted-surface passages. The geometry of interest consists of an array of interrupted plates, in an in-line arrangement, placed parallel with the main flow direction. Such a configuration can be regarded as a two-dimensional idealization of the cores of some offset-fin compact heat exchangers. The performance of several low-Reynolds number, two-equation linear eddy-viscosity models of turbulence is discussed in the context of a time-averaged formulation. Simulations were carried out for a range of Reynolds numbers and one value of the plate thickness. Numerical predictions of the friction factor and skin friction coefficient along the plates are compared with available experimental results.

Keywords: Turbulence, Periodic Flows, Control-Volume Method, Heat Exchangers.

Introduction

Flow passages with spatially-periodic interruptions are commonly encountered in heat transfer equipment, such as compact heat exchangers (Shah, 1981; Kays and London, 1984). Compact heat exchangers are used in the chemical, material processing, and power-generation industries, as well as in automobiles and aircrafts. The interruptions in the flow passages are intended to produce enhanced heat transfer rates, but they also cause higher pressure drops than those encountered in uninterrupted configurations. Thus, the design of such heat exchangers involves an optimization problem in which the objective is to obtain the maximum heat transfer rate between the hot and cold fluids, for fixed values of the associated pumping power. Other examples of fluid flow in interrupted passages are provided by cooling arrangements for electronic equipment (Kakaç, et al., 1993; Kelkar et al., 1993; Patankar, 1993; Wang et al., 1997). Effective cooling of circuit boards in electronic devices is crucial to ensure reliability of such systems.

The understanding of the fluid flow and heat transfer phenomena that occurs in compact heat exchangers has motivated researchers for many years. Experimental data on overall heat transfer coefficients and pressure drops obtained from tests on either actual or large-scale models of heat exchangers that incorporate interrupted passages have been reported in the literature, examples include the works of Wieting (1975), Shah (1981), and Kays and London (1984). Although this data can be used directly in the design of interrupted-surface heat exchangers, it cannot be used as definitive checks on numerical predictions because of the lack of necessary local details. Many numerical and experimental studies of fluid flow and heat transfer in simplified periodic geometries have also been reported in the literature. Most of the investigations were concerned with steady-state laminar flows (Patankar et al., 1977; Patankar and Prakash, 1981; Sebben and Baliga, 1996a; Fowler et al., 1997). However, typical ranges of the Reynolds number in compact heat exchangers extend from a laminar flow regime to a low Reynolds number turbulent flow regime (Kays and London, 1984).

Over the last decade, experimental and numerical studies of statistically steady, spatially-periodic turbulent flows have also appeared in the literature. Examples include the works of McBrien and Baliga (1988), McBrien (1989), Faghri and Asako (1990), and Kim and Anand (1994). The numerical solution of the problem in conjunction with the time-averaged form of the governing equations has become a common practice in these type of turbulent flows, mainly due to the high computational effort required by unsteady formulations. For this reason, relatively few numerical investigations of unsteady turbulent flows in interrupted-surface passages have been reported. Some examples are the works of Ciofalo and Collins (1992), Ciofalo et al. (1996), and Sebben and Baliga (1996b).

The present paper describes the analysis and results for statistically steady, spatially-periodic, fully-developed turbulent flows in the interrupted-plate rectangular duct illustrated in Fig. 1(a). Two-

dimensional numerical simulations were performed using seven low-Reynolds numbers, two-equation, linear eddy-viscosity models of turbulence (EVMs). The intention in this work was to compare the numerical predictions of the friction factor and skin friction coefficient to the available experimental data from McBrien (1989), and discuss the relative performance of several low-Reynolds number EVMs in the context of a steady Reynolds time-averaged formulation. A concise description of the rationale for the choice of eddy-viscosity based, low-Reynolds number time-averaged models is also discussed. The work reported here is part of an effort of parallel investigations of unsteady, self-sustained oscillatory flows in interrupted surfaces of the type encountered in compact heat exchangers.

Mathematical Formulation

The problem of interest involves statistically steady, two-dimensional, spatially-periodic fully-developed turbulent flows in rectangular interrupted-plate ducts akin to that shown in Fig. 1(a). Such ducts are characterized by geometrically identical and spatially-periodic modules as illustrated in the cross-sectional view presented in Fig. 1(b). In this figure, a spatially-periodic geometric module is the domain indicated as *ABCD*. Attention is focused in the region *ABCD* because, with the assumption of statistically steady flows, the channel centerline becomes a symmetry line and, therefore, the calculation domain can be limited to one-half of the channel height (region *ABCD*). It is to be noted that effects of vortex shedding have not been considered in the present investigation, even though vortex shedding from the plates can occur for some combinations of the Reynolds number and geometrical parameters of the duct. This assumption implies that the frequencies of vortex shedding are treated in the same manner as flow turbulence, and are time-averaged to allow steady time-mean flows. As previously mentioned, this consideration is often made to reduce the computational costs involved with an unsteady approach to the problem. It is also assumed that the fluid is Newtonian, incompressible, and with constant thermophysical properties. This assumption is commonly invoked in design calculations for plate-fin heat exchangers, with the fluid properties values based on mean bulk temperature.

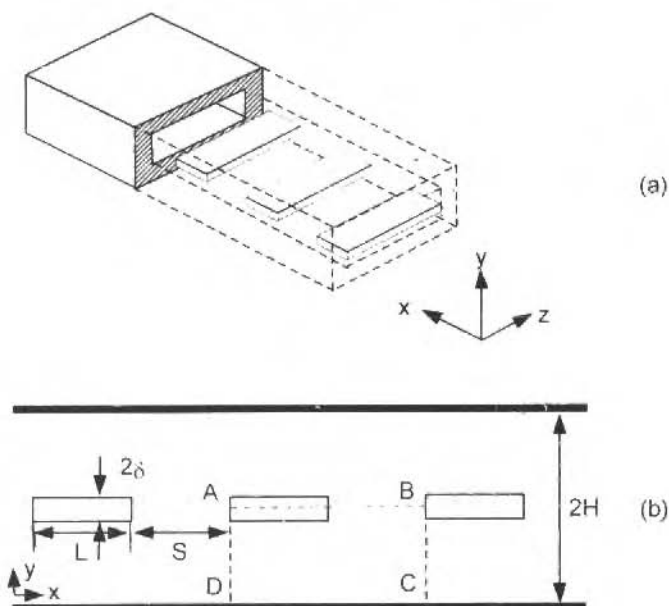


Fig. 1 (a) Rectangular Interrupted-Plate Duct; and (b) Cross-Section View, Associated Nomenclature and a Representative Geometric Module *ABCD*

Typically, for steady flows in interrupted passages, a spatially periodic fully-developed behavior is attained after a short entrance region, about 5 to 10 geometrical modules downstream from the inlet

plane (Sparrow et al., 1977; Sparrow and Hajiloo, 1980; Loehrke and Lane, 1982). In the fully-developed regime, the distributions of the velocity components repeat identically in successive geometrical modules (Sparrow et al., 1977). With reference to Fig. 1(b), the periodic behavior of the time-averaged velocity components can be expressed as:

$$u_i(x, y) = u_i(x + (L + s), y) = u_i(x + 2(L + s), y) = \dots \quad (1)$$

where $(L + s)$ is the length of one module.

A procedure for the solution of the flow in a single module in the fully developed region has been described by Patankar et al. (1977). As explained by Patankar et al. (1977), the reduced pressure, p , can be expressed as a linear combination of an overall pressure drop, which drives the flow in the streamwise direction, and a pressure P which is related to the details of the local motions in the geometric module. Thus, the reduced pressure can be expressed as follows:

$$p(x, y) = -\beta x + P(x, y) \quad (2)$$

where the term β is a uniform value for the entire periodic region, and is responsible for the overall mass flow rate through the module. In the present work, β is treated as an adjustable parameter so as to obtain a desired, or specified Reynolds number in the problem. The pressure P behaves in a periodic manner from module to module. Akin to the velocity field, the periodic condition for $P(x, y)$ can be written as follows:

$$P(x, y) = P(x + (L + s), y) = P(x + 2(L + s), y) = \dots \quad (3)$$

Using Eq. (2), the time-averaged continuity and momentum equations can be written as:

$$\frac{\partial u_i}{\partial x_i} = 0 \quad (4)$$

$$\rho u_j \frac{\partial u_i}{\partial x_j} = \beta \delta_{ii} - \frac{\partial P}{\partial x_i} + \frac{\partial}{\partial x_j} \left(\mu \frac{\partial u_i}{\partial x_j} - \overline{\rho u_i' u_j'} \right) \quad (5)$$

where δ_{ii} is the Kronecker delta, and the correlation $\overline{\rho u_i' u_j'}$ is interpreted as the turbulent stresses, commonly referred to as the Reynolds stresses. They are unknown elements in the time-averaged equations and need to be modeled in order to "close" the set of equations.

In this work, six variations of the low-Reynolds number $\kappa - \epsilon$ model were assessed to predict the flows of interest: The Jones and Launder (1973) and the Launder and Sharma (1974) models, which will be denoted as JL and LS, respectively; the Jones and Launder (1973) and Launder and Sharma (1974) models with a modification proposed by Hanjalic and Launder (1980), which will be denoted as JLH and LSH, respectively; and the Launder and Kato (1993) model with the closure coefficients proposed by Jones and Launder (1973) and Launder and Sharma (1974), which will be denoted as LKJ and LKS, respectively. The low-Reynolds number $\kappa - \omega$ of Wilcox (1988, 1993), WL, was also assessed in the present investigation.

Within the framework of Reynolds averaging, eddy-viscosity based models remain the most widely used "fast" engineering methods for computing complex turbulent flows (Wilcox, 1993; Hanjalic, 1994; Launder, 1995). Computationally, EVMs offer advantages over Reynolds Stress Models (RSM), Direct Numerical Simulations (DNS), and Large Eddy Simulations (LES), due to relatively simple implementation and lower computational costs. For these reasons EVMs have been used in this work. Because the flows of interest involve interrupted-plates, separation and reattachment regions, low-Reynolds number EVMs were preferred as it would be difficult to justify the use of high-Reynolds

EVMS with wall functions in the modeling of such flows. With regard to the different low-Reynolds number EVMS tested, the modifications included in the JLH and LSH models were proposed by Hanjalic and Launder (1980) in order to improve the predictions obtained with the JL and LS models in flows with adverse pressure gradients. The model of Launder and Kato (1993), initially proposed for periodic unsteady flows with vortex shedding, is tested here to evaluate its performance in a steady formulation of the governing equations.

The time-averaged equations for these various low-Reynolds number $\kappa-\varepsilon$ models can be written as follows:

$$\rho u_j \frac{\partial k}{\partial x_j} = \frac{\partial}{\partial x_j} \left[\left(\mu + \frac{\mu_t}{\sigma_k} \right) \frac{\partial k}{\partial x_j} \right] + P_k - \rho \tilde{\varepsilon} - 2\mu \left(\frac{\partial \sqrt{k}}{\partial x_j} \right)^2 \quad (6)$$

$$\rho u_j \frac{\partial \tilde{\varepsilon}}{\partial x_j} = \frac{\partial}{\partial x_j} \left[\left(\mu + \frac{\mu_t}{\sigma_\varepsilon} \right) \frac{\partial \tilde{\varepsilon}}{\partial x_j} \right] + P_\varepsilon - C_{2\varepsilon} f_2 \rho \frac{\tilde{\varepsilon}^2}{k} + \frac{2\mu\mu_t}{\rho} \left[\frac{\partial}{\partial x_j} \left(\frac{\partial u_i}{\partial x_k} \right) \right]^2 \quad (7)$$

It is to be noted that Eq. (7) is written in terms of $\tilde{\varepsilon}$, rather than the dissipation ε . The dissipation, ε , is related to the quantity $\tilde{\varepsilon}$ by $\varepsilon = \varepsilon_o + \tilde{\varepsilon}$, where ε_o is the finite value of ε at the wall. The practice of writing the rate of dissipation of k , and its governing equation in terms of $\tilde{\varepsilon}$, rather than ε , has the computational advantage that $\tilde{\varepsilon}$ becomes zero at a solid wall. In the above equations, the turbulent viscosity, μ_t , is given by $\mu_t = f_\mu C_{\mu} \rho k^{1/2} / \tilde{\varepsilon}$, and the production term of the $\tilde{\varepsilon}$ equation, P_ε , is given by $P_\varepsilon = f_1 [C_{1\varepsilon} P_k (1 - \delta_{ij}) + C_{3\varepsilon} P_k \delta_{ij}] (\tilde{\varepsilon} / k)$.

Overall, the six $\kappa-\varepsilon$ models used here, named: JL, LS, JLH, LSH, LKJ, and LKS, contain five closure coefficients that originated from the high-Reynolds number version, plus other dumping

Table 1 Constants from the High-Reynolds $k-\varepsilon$ Model

c_μ	σ_k	σ_ε	$C_{1\varepsilon}$	$C_{2\varepsilon}$
0.09	1.0	1.3	1.44	1.92

functions due to low-Reynolds number modifications. A summary of these various closure coefficients and dumping functions is given in Tables 1 and 2, respectively. The expression for the turbulence production term, P_k , for these various models is given in Table 2.

Table 2 Constants and Expressions Used in the Low-Reynolds $k-\varepsilon$ Models

$k-\varepsilon$	f_μ	f_1	f_2	$C_{3\varepsilon}$	P_k
JL	$e^{\frac{-2.5}{1+0.02 Re_t}}$	1	$1 - 3e^{-Re_t^2}$	1.44	$\mu_t \left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} \right) \frac{\partial u_i}{\partial x_j}$
LS	$e^{\frac{-3.4}{(1+0.02 Re_t)^2}}$	1	$1 - 3e^{-Re_t^2}$	1.44	$\mu_t \left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} \right) \frac{\partial u_i}{\partial x_j}$
JLH	$e^{\frac{-2.5}{1+0.02 Re_t}}$	1	$1 - 3e^{-Re_t^2}$	4.44	$\mu_t \left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} \right) \frac{\partial u_i}{\partial x_j}$
LSH	$e^{\frac{-3.4}{(1+0.02 Re_t)^2}}$	1	$1 - 3e^{-Re_t^2}$	4.44	$\mu_t \left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} \right) \frac{\partial u_i}{\partial x_j}$
LKJ	$e^{\frac{-2.5}{1+0.02 Re_t}}$	1	$1 - 3e^{-Re_t^2}$	1.44	$\mu_t \sqrt{\frac{1}{2} \left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} \right)^2} \sqrt{\frac{1}{2} \left(\frac{\partial u_i}{\partial x_j} - \frac{\partial u_j}{\partial x_i} \right)^2}$
LKS	$e^{\frac{-3.4}{(1+0.02 Re_t)^2}}$	1	$1 - 3e^{-Re_t^2}$	1.44	$\mu_t \sqrt{\frac{1}{2} \left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} \right)^2} \sqrt{\frac{1}{2} \left(\frac{\partial u_i}{\partial x_j} - \frac{\partial u_j}{\partial x_i} \right)^2}$

The time-averaged equations for the low-Reynolds number $k-\omega$ model are now presented. Following Wilcox (1993), they can be written as:

$$\rho u_j \frac{\partial k}{\partial x_j} = \frac{\partial}{\partial x_j} \left[\left(\mu + \frac{\mu_t}{\sigma_k} \right) \frac{\partial k}{\partial x_j} \right] + P_k - \rho \epsilon \quad (8)$$

$$\rho u_j \frac{\partial \omega}{\partial x_j} = \frac{\partial}{\partial x_j} \left[\left(\mu + \frac{\mu_t}{\sigma_\omega} \right) \frac{\partial \omega}{\partial x_j} \right] + C_{1\omega} \frac{\omega}{k} P_k - C_{2\omega} \rho \omega^2 \quad (9)$$

where the specific dissipation rate, ω , is defined as:

$$\omega = \frac{\epsilon}{k}$$

The turbulent viscosity, μ_t , is given by $\mu_t = c_\mu \rho k / \omega$, and P_k is modeled as in the JL, LS, JLH, and LSH models. The values of the five closure coefficients of the $k-\omega$ model are presented in Table 3.

Table 3 Constants in the $k-\omega$ Model

c_μ	σ_k	σ_ω	$C_{1\omega}$	$C_{2\omega}$
0.09	2.0	2.0	5/9	5/6

With reference to Fig. 1(b), the boundary conditions for the problem are the following: Periodic conditions across AD and BC ; $v = 0$ and symmetry conditions for u , k , $\bar{\epsilon}$, and ω along line AB ; and $u = v = k = \bar{\epsilon} = 0$ at the surfaces of solid boundaries. For the $k-\omega$ model, the boundary condition on ω at the solid boundaries is given by (Wilcox, 1988):

$$\omega = \frac{6(\mu/\rho)}{C_{2\omega}\Delta^2} \quad (10)$$

where Δ represents the physical normal distance to the solid surface of the first adjacent ω grid-point.

Wilcox (1988) recommends that the location of the mesh points closest to the solid surfaces be chosen so that $\Delta^+ \leq 1$; and that adjacent to each solid surface, at least five mesh points along each wall-normal grid line lie within $\Delta^+ \leq 2.5$. The quantity Δ^+ represents a dimensionless normal distance from the wall, defined as: $\Delta^+ = \rho u_\tau \Delta / \mu$.

With reference to the notation in Fig. 1(b), the computations were performed for the following nondimensional geometrical parameters:

$$\frac{\delta}{H} = 0.11, \quad \frac{L}{H} = 177, \quad \frac{s}{H} = 176 \quad (11)$$

The results are presented in terms of nondimensional quantities: the Reynolds number, Re , the friction factor, ff , and the skin friction coefficient on the surface of the plate, τ_w . These quantities are defined as:

$$Re = \frac{\rho \bar{U} (2U)}{\mu}, \quad ff = \frac{\beta(2U)}{0.5 \rho \bar{U}^2}, \quad \tau_w^* = \frac{\tau_w}{0.5 \rho \bar{U}^2} \quad (12)$$

where $\bar{U}$ is the time-mean streamwise average velocity.

Four Reynolds numbers were considered: $Re = 4997$, 10980, 16930, and 27840. These values of the geometrical parameters and Reynolds number were chosen so as to match those considered by McBrien (1989) in his experimental investigation.

Computational Details

The computational domain was confined to the region denoted by ABDCA in Fig. 1(b). It was discretized using staggered grids for the velocity components and pressure, as discussed by Patankar (1980). As can be seen in Fig. 1(b), the calculation domain includes a solid region (plate), as well as a fluid flow region. A convenient method to treat internal flow blockages (plates) has been discussed by Patankar (1980): It consists of setting the fluid viscosity equal to a very large number; in addition, in the present computations, the coefficients of the discretized equations were overwritten so as to obtain identically zero for the dependent variables (u , v , P , k , $\tilde{\epsilon}$) in the blocked zones. The coefficients of the discretized equation for ω were set so as to satisfy the boundary condition given by Eq. (10) close to the solid surfaces.

All computations were performed with nonuniform grids. When using low-Reynolds number turbulence models, fine grids are needed in order to adequately resolve the very steep gradients of the dependent variables through the viscous sublayer, all the way to the wall. Furthermore, in order to place the first grid point adjacent to a solid boundary within the desirable normal nondimensional distance of $\Delta^+ \leq 1$, the corresponding required physical distance, Δ , can be very small, specially for higher Reynolds numbers. This restriction made the use of uniform grids prohibitive.

The momentum equation, Eq. (5), was discretized using the QUICK scheme of Leonard (1979). The SIMPLEC algorithm of Van Doormaal and Raithby (1984) was used to solve the discretized momentum and pressure correction equations. The transport equations for the turbulent quantities k , $\tilde{\epsilon}$, and ω were discretized using the hybrid scheme (Patankar, 1980). The use of higher order schemes, such as the QUICK scheme of Leonard (1979), in the discretized equations of k , $\tilde{\epsilon}$, and ω can produce negative coefficients, which in turn, can lead to negative effective turbulent transport coefficients and numerical instability (Lien and Leschziner, 1994). Such negative values can appear particularly in regions where the actual (positive) k and $\tilde{\epsilon}$ values are close to zero. In most circumstances, however, the turbulent transport equations are dominated by their source terms. Thus, the order of the scheme used in the discretization of the convective terms in these equations is of relatively little importance, provided it is done consistently (Lien and Leschziner, 1994). Based on the aforementioned justifications, in this work, the QUICK scheme was used for the discretization of the u and v equations, while the hybrid scheme was used for the k , $\tilde{\epsilon}$ and ω equations.

Four line-by-line iterative algorithms were used to solve the nominally linear and decoupled set of equations: the tridiagonal and pentadiagonal matrix algorithms (TDMA and PDMA), and the cyclic tridiagonal and pentadiagonal matrix algorithm (CTDMA and CPDMA). The cyclic algorithms are required for lines of nodes along which the periodic boundary conditions apply (AD and BC). Detailed descriptions of these and related algorithms are available in the work of Sebben and Baliga (1995), for example. Generalized block-correction procedures (Settari and Aziz, 1973) were used for the velocity components and pressure correction, in order to speed up the convergence of the line-by-line algorithms. The block-correction procedure was not used in the iterative solution of the discretized equations for k , $\tilde{\epsilon}$, and ω , as it did not improve the overall rate of convergence of these equations.

To start the calculations, an initial value of β had to be specified. The experimental values of the Reynolds number and friction factor results of McBrien (1989) were used to obtain initial values of β using Eq. (12). For each run, β was adjusted at each iteration of the overall solution procedure, so as to maintain the Reynolds number at the desired value. The procedure for adjusting β was the same as that proposed by Kelkar et al. (1993). Following the recommendations of Kelkar et al. (1993), the value of β at the end of each SIMPLEC iteration, was modified in the following manner:

$$\beta = \beta^o \gamma^\alpha \quad \text{where} \quad \gamma = \bar{U} / \bar{U}^o$$

where $\bar{U}$ is the averaged velocity that will generate the desired Reynolds number, $\bar{U}^o$ is the average velocity of the last available flow field, corresponding to the pressure gradient β^o , and the exponent α is an underrelaxation factor set equal to 0.9. In the end of each iteration, after adjusting β , the velocity field was adjusted as follows:

$$u_i = u_i^o \gamma$$

Convergence was considered to be achieved when the sum of the absolute values of the normalized residues of the discretized equations for u , v , k , $\bar{\epsilon}$, and ω were less than 10^{-4} . The underrelaxation factors for k , $\bar{\epsilon}$, and ω were set equal to 0.9, while for u and v they were calculated at each iteration using the following expression:

$$\alpha = \text{MIN} \left(\frac{|\Sigma a_{nb} - Sp \Delta V|}{1.3 \Sigma |a_{nb}|} \right) \quad (13)$$

where ij are nodal indices that cover the entire solution domain; a_{nb} are the coefficients of the discretized equations; Sp is part of the linearized source term (as discussed by Patankar (1980)); and ΔV is the volume of the control volume.

Results and Discussion

Validation of Computations

The finite volume method (FVM) code was subjected to numerous verification tests, in accordance with well-established guidelines, such as those discussed by Patankar (1980). The capabilities of the code to solve spatially-periodic, fully-developed flows is well documented in Sebben and Baliga (1996a). Steady, fully-developed turbulent flow in a two-dimensional straight channel was the problem considered to test the implementation, in the computer code, of the low-Reynolds number versions of the $\kappa-\epsilon$ and $\kappa-\omega$ models (the JLH, LSH, and WL models were tested). Steady, fully-developed flows in straight channels can be predicted by solving a one-dimensional formulation. However, for testing purposes, two-dimensional simulations were carried out to check the two-dimensional implementations of the aforementioned turbulence models. In the simulations, periodic conditions were imposed at the inlet and outlet flow boundaries. The results of these computations were checked against the Prandtl-Jones correlation (Jones, 1976) which gives the friction factor as a function of the Reynolds number. Calculations for Reynolds numbers in the extreme range of the Re values investigated in the interrupted-plate channel flow problem were considered. The friction factor results, particularly those obtained with the JLH model, agreed well with the corresponding values yielded by the Prandtl-Jones correlation. Specifically for the JLH model, the values were within 2% of the Prandtl-Jones correlation values. Detailed description of these results can be found in Sebben (1996).

The solution of the aforementioned test problems allowed an effective evaluation of the validity and capabilities of the proposed FVM computer code for predicting spatially-periodic, fully-developed turbulent flows in interrupted-plate geometries.

Grid Independence Study

Friction factor results obtained with two nonuniform grids are presented in Table 4 for all seven turbulence models used in this work. The number of control volumes in the two nonuniform grids were Grid 1 = 72x100 and Grid 2 = 144x200, in the x and y directions, respectively. A larger number of grid points were concentrated near the solid walls and plate surfaces. A typical grid pattern is illustrated in Fig. 2.

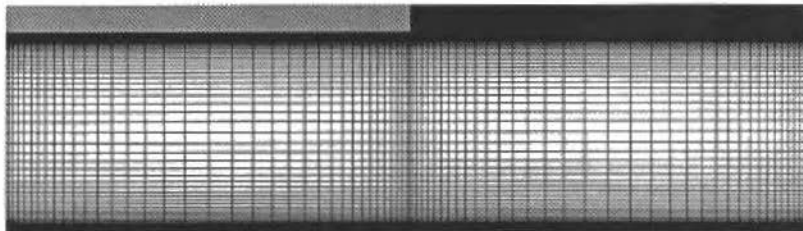


Fig. 2 Representation of the Grid Layout Corresponding to Grid 1 = 72x100 Control Volumes

From Table 4 it can be observed that the friction factor values calculated with the JLH model show the smallest differences in the results obtained with the two grids considered: These differences range from 0.25% to 4.5% for $Re = 4997$ and 27840 , respectively. On the other hand, the results obtained with the WL model show the biggest differences in the values obtained with the two grids: These differences range from 13% to 25% for $Re = 4997$ and 27840 , respectively. For the other models, these differences in the friction factor results are all within 10%. It is also noted that the JL, JLH, and LKJ models are less sensitive to grid refinement than the LS, LSH, and LKS models, respectively. The only difference between the JL and LS models is in their expression for the damping function, f_{μ} . This statement also applies to the JLH and LSH models, and to the LKJ and LKS models. As expected, for each model, the difference between the results obtained with the two grids increases as the Reynolds number increases. This is because with any given grid, as the Reynolds number increases, the gradients that occur near the solid regions are steeper.

**Table 4 Friction Factor Values Obtained With Seven Turbulence Models:
Grid 1 = 72x100 and Grid 2 = 144x200 Control Volumes**

Re	Grid	$ff \times 10^2$						
		JL	LS	JLH	LSH	LKJ	LKS	WL
4997	1	8.309	8.240	7.144	6.673	7.460	7.078	8.836
	2	8.127	7.691	7.126	6.462	7.338	6.776	10.13
10980	1	8.995	8.924	6.693	6.449	7.660	7.590	6.729
	2	8.482	8.258	6.479	6.095	7.281	7.065	8.360
16930	1	9.121	9.005	6.588	6.407	7.732	7.638	6.246
	2	8.498	8.261	6.327	6.004	7.401	7.222	7.954
27840	1	9.090	8.971	6.407	6.272	7.688	7.607	5.758
	2	8.401	8.183	6.129	5.857	7.333	7.127	7.666

In general, the number of grid points used in the y-direction, in both grids, was large enough to guarantee that the mesh points closest to the solid wall and plate surfaces were below the $\Delta^+ = 1$ values, for all models and Reynolds numbers studied. In the x-direction, however, this condition was satisfied only in the lower range of Reynolds number. For $Re = 27840$ and Grid 2, the normal distance of the grid points closest to the plates in the streamwise direction yielded $\Delta^+ \leq 2$, rather than the desired $\Delta^+ \leq 1$.

Friction Factor Results

Plots of the ff versus Re results, along with the experimental data of McBrien (1989), are shown in Figs. 3(a) to (g). In these figures, the filled circles ($\bullet$) represent the experimental data of McBrien (1989), the triangles (Δ) represent the numerical results obtained with Grid 1, and the squares ($\square$) represent the numerical results calculated with Grid 2.

The plots in Figs. 3(a) to (f) show that the results obtained with the JL, LS, JLH, LSH, LKJ, and LKS models are quite unsatisfactory, even with the finer grid (denoted by $\square$). The absolute values of the difference between the computed results and the experimental results of McBrien (1989) can be as high as 30%. Furthermore, the experimental results show that ff values decrease appreciably with Re , in the range of parameters studied. However, the ff values obtained with the JL, LS, LKJ, and LKS models increase with Re , albeit only slightly. The ff values obtained with the JLH and LSH models show a trend that is similar to that of the experimental results, but, quantitatively, these computed results decrease only very slightly with Re . It should also be noted that the differences between the ff values obtained with the JL and LS models are rather small ($\leq 5\%$ in most cases), with the LS model predictions slightly lower than those of the JL model. This observation also applies to the ff values obtained with the LKJ and LKS models, and the JLH and LSH models.

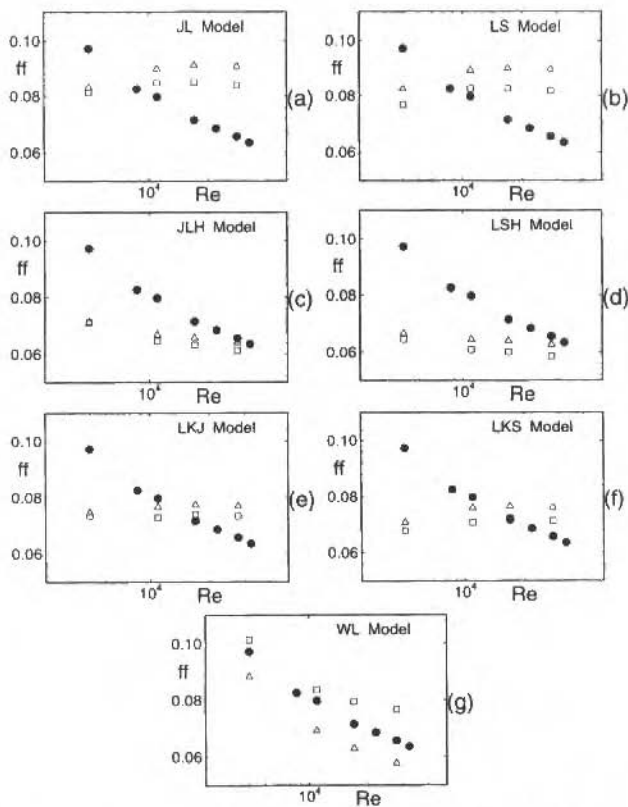


Fig. 3 Comparison Between the Experimental Friction Factors of McBrien (1989) and the Numerical Values Obtained with Seven Different Turbulence Models

The effect of using the Hanjalic and Launder (1980) modification is quite significant, as can be observed with reference to Figs. 3(a) and (c), and (b) and (d). The JLH and LSH friction factor results are lower than the corresponding values obtained with the JL and LS models. The effect of this modification is more prominent at higher values of Re , with a difference of as much as 40% in the ff values at corresponding Reynolds numbers.

The results obtained with the WL model (Fig. 3(g)) show the best agreement with the experimental results, both qualitatively (trend) and quantitatively. However, the differences between the ff values obtained with Grid 1 (Δ) and Grid 2 ($\square$) are rather large, relative to these grid-related differences in the results obtained with the other turbulence models. Furthermore, it was found that the WL model results are very sensitive to the placement of the grid points adjacent to the solid boundaries. This statement applies even when the recommendations of Wilcox (1988) (namely, normal distance from a solid boundary of the first adjacent grid point should give $\Delta^+ < 1$, and the next four nodes should satisfy $\Delta^+ < 2.5$) are met. It should also be noted that in order to meet these recommendations, very fine grids are required in regions adjacent to solid boundaries. As mentioned before, for the case of $Re = 27840$, the recommendation of the first grid point within $\Delta^+ < 1$ was not satisfied in the streamwise direction even with the finer grid (Grid 2).

Streamline Plots

A good qualitative appreciation of the flow field behavior can be obtained from the streamline plot presented in Fig. 4. This streamline plot is for $Re = 10980$ and was obtained with the WL model and

Grid 2. In Figure 4, a very small recirculation zone is observed at the plate leading edge as the flow impinges the plate. Downstream of the plate trailing edge a recirculation zone of approximately 5δ in length is also observed. The streamlines in the core region of the channel are more or less aligned with the mean flow direction.

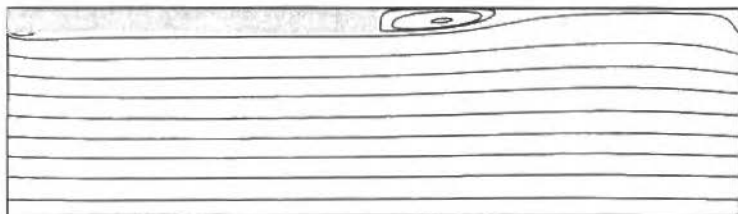


Fig. 4 Streamline Plots for $Re=10980$ Obtained with the WL Model

Skin Friction Coefficient Distributions

For $Re = 10980$, distributions of skin-friction coefficient, $\tau_w^* = \tau_w / 0.5\rho U^2$, on the surface of the plate are presented in Fig. 5. Results obtained with the JLH model and the WL model are presented in this figure. It is seen in Fig. 5 that, up to $x/H=0.68$, the two models behave quite differently, with the JLH model given rise to a much longer separation zone at the plate leading edge. After $x/H=0.68$ there is a pretty good agreement between the results obtained with the two models.

For the WL model, the τ_w^* plot in Fig. 5 shows the following features: (i) a high positive value of τ_w^* right at the leading edge of the plate ($x/H=0$); (ii) a sharp high negative peak of τ_w^* in the region of $0 \leq x/H \leq 0.04$; (iii) a second negative smooth peak of τ_w^* that extends up to $x/H = 0.12$; (iv) a relatively smooth rise in τ_w^* from $\tau_w^* = -0.012$ to $\tau_w^* = 0.012$, roughly, in the region $0.12 \leq x/H \leq 0.68$; (v) a constant value of τ_w^* equals to 0.012 almost all the way to the plate trailing edge; (vi) a sharp rise in τ_w^* to a value of 0.24 at $x/H = 1.77$. (The nondimensional length of the plate is $L/H = 1.77$).

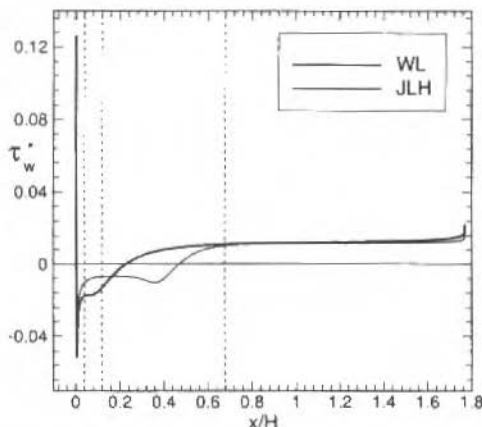


Fig. 5 Distribution of the Skin-Friction Coefficient on the Surface of the Plate for $Re = 10980$

The aforementioned features of the skin friction coefficient distribution obtained with the WL model compare rather well with the τ_w^* distribution that can be inferred from the plate-surface oil-flow visualization obtained experimentally by McBrien (1989), with exception to the plate trailing edge region. In particular, the smooth oil-flow pattern in the vicinity of the trailing edge of the plate in the experimental results of McBrien (1989) is entirely missed by the numerical calculations, which

predicted a constant value of τ_w^* almost all the way to the end of the plate. A photograph of the surface streamline pattern obtained by McBrien (1989) for the same interrupted-plate duct and $Re = 10980$ is given in Fig. 6.

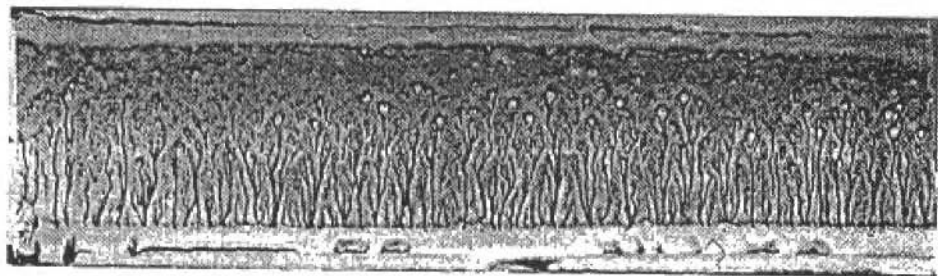


Fig. 6 Plate-Surface Streamline for $Re = 10980$. Main Flow Direction is From top to Bottom of the Picture: Experimental Results of McBrien (1989).

Experimentally, the flow undergoes strong accelerations at the plate trailing edge region, which are caused by an oscillating wake and possible vortex shedding. This flow acceleration gives rise to the smooth oil-flow pattern seen in Fig. 6 in the vicinity of the trailing edge of the plate. In the present steady-state simulations, however, there are no wake oscillations, or vortex shedding, thus the flow over the plate in the vicinity of the trailing edge does not undergo any significant acceleration, except at $x/H = 1.77$.

Conclusions

A numerical investigation of steady, spatially-periodic fully-developed turbulent flow in interrupted-plate ducts has been presented in this paper. Seven versions of the low-Reynolds number, two-equation linear eddy-viscosity models of turbulence were assessed, and the results obtained were compared with available experimental data.

Comparisons of the friction factor values between the available experimental results and the numerical results obtained with the seven turbulence models showed that none of the models are entirely satisfactory. The calculated friction factor results showed good qualitative agreement (trend) with the experimental results with three of the models tested, namely: the JLN, LSH, and WL models. The results obtained with the WL model showed the best quantitative agreement with the experimental data. However, it was found that the WL model results are very sensitive to the number of grid points, and also to their location adjacent to solid boundaries.

The skin friction coefficient distribution on the surface of the plate showed good agreement with the distribution that can be inferred from plate-surface oil-flow visualizations obtained experimentally, except close to the trailing edge of the plate. In this region, unsteady effects due to wake oscillations or vortex shedding can be significant. Such effects, which can only be captured with an unsteady formulation of the problem, may explain the poor agreement obtained in the plate trailing edge zone.

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Local Measurements in Two-Phase Flows Using a Resistivity Double Probe Technique

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Abstract

Local measurements are of primary importance for the characterization of gas-liquid two-phase flows, both for processes control and numerical modeling validation. It is a very active research field due to the increasing number of applications in the thermohydraulics of heat exchangers, nuclear plants, chemical processes and oil industries.

This paper presents the local measurements in a vertical upward air-water flow using the electrical resistivity double probe technique. The test section was a 80 mm i.d. and 160 cm long Plexiglas pipe. Five different gas superficial velocities, ranging from 0.02 to 0.10 m/s, were used in combination with two liquid superficial velocities of 0 and 0.10 m/s. A resistivity double probe was employed for measurements of the radial profiles of void fraction, bubble frequency, bubble interface velocity, interfacial area concentration and Sauter mean diameter.

The electrical resistivity probe method consists of a instantaneous measurement of the local electrical resistivity in the two-phase flow by means of a sensor electrode. Since the circuit is opened or closed depending on whether the sensor tip is in contact with gas or liquid, the probe behaves in principle like a switch, yielding a two-stage signal. However, to obtain a true square wave type signals, a proper threshold voltage has to be used as a triggering criterion. Herein the signal conditioning is discussed and the influence of the threshold level is analyzed.

With a probe with two sensors displaced axially, the bubble interface velocity could be determined from the time delay which gave maximum correlation between the sensor responses. These values of gas velocity in conjunction with void fractions could be integrated to give average gas superficial velocities. Values determined in this manner were compared to values from the inlet gas flowrate measurements and showed an average deviation of less than 6% for bubbly flow.

Keywords: Two-Phase Flow, Instrumentation, Local Measurement, Resistivity Probe, Double Probe

Introduction

Two-phase flows consisting of simultaneous flow of liquid-vapor or liquid-gas are encountered quite frequently in power generation systems, heat exchangers, chemical reactors, oil industry and other industrial processes. In these two-phase flows, several flow regimes are encountered where the gas phase may occur in various patterns, from small bubbles to large slugs or as a continuous phase with or without liquid films or droplets. The bubble size and their distributions in these regimes vary with the liquid and gas superficial velocities, flow geometry and local conditions. A detailed knowledge of the flow regimes and local flow characteristics is very important in developing predictive tools. In addition, theoretical modeling of two-phase flow studies is often based on very specific local hydraulic conditions. An experimental verification of such analytical descriptions thus requires a very fine and detailed determination of the relevant local parameters.

In the study of two-phase flow there has been a strong need for instruments able to measure the detailed distribution of various local parameters, such as the distribution of the two phases, the bubble size distribution and the bubble frequency.

One of the requirements of a suitable measuring method would be the absence of obstructions in the flow channel. At least three methods meet this requirement: laser-Doppler anemometry, ultrasonic pulse transmission and particle image velocimetry. However, these methods are only applicable where the dispersed phase is sufficiently dilute, i.e., for low void fractions.

In view of the intention to measure local variables in gas-liquid flows with void fraction possibly ranging from zero to unity it is inevitable to use a probe technique. In a recent review work, Cartellier and Achard (1991) have shown that the most powerful probe techniques are electrical resistivity probe, optical probe and hot film anemometry. Among these techniques, the advantage of electrical resistivity

probe is that both the sensor and the detecting electronic circuit are easy to be realized. In this work, a double sensor resistivity probe was used for the measurement of the radial profiles of void fraction, bubble frequency, bubble interface velocity, interfacial area concentration and Sauter mean diameter.

Since the fundamental works of Serizawa et al (1975) and Herringe and Davis (1976), continuous progress has been made with respect to the application of electrical resistivity probe technique to studying local two-phase flow parameters. Progress has been done both on the sensor geometry and on the threshold procedure. The experimental work of Van der Welle (1985) was dedicated to the local measurement of the void fraction, bubble velocity and bubble size in air-water flows, using a double probe. Kataota et al. (1986) introduced a local formulation of the interfacial area concentration and proposed a three double-sensor probe. Teyssedou and Tapacu (1988) used a single probe for the measurement of the void fraction profile in air-water flows. In the work of Barnea and Shemer (1989) a single probe was used for the measurement of the void fraction at the centerline of a vertical pipe in upward air-water flow. Kocamustafaogullari and Wang (1991) presented an extensive work on the local measurement of the void fraction, interfacial area concentration, mean bubble diameter and bubble interface velocity in a horizontal air-water flow, using a double probe. Liu and Bankoff (1993) developed a miniature double probe for the local measurement of the void fraction, bubble velocity and bubble size in air-water bubbly flow. In the work of Leung et al. (1995) a double probe was used to the study of the axial development of the interfacial area and void fraction profiles.

The Electrical Resistivity Probe Technique

Measuring Principle

In principle the electrical resistivity probe method consists of a instantaneous measurement of the local electrical resistivity in the two-phase flow by means of a sensor electrode. A typical resistivity probe is depicted in Fig. 1.

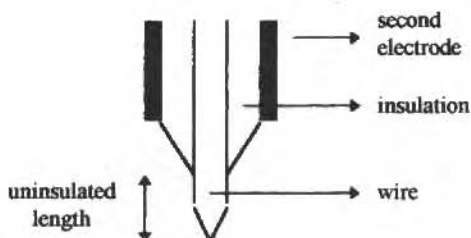


Fig. 1 Typical Resistivity Probe

Basically the sensor works as an identifier of a phase surrounding the probe tip. Since the circuit is opened or closed depending on whether the sensor is in contact with gas or liquid, the probe behaves in principle like a switch, yielding a two-stage signal. Such a signal shows a nearly immediate response to water contact with the probe, but a delayed response to bubble contact, due to the required dewetting time of the probe tip. Indeed, the sensor does not penetrate the interface without deforming it. Moreover, the instantaneous resistivity depends on the fraction of the sensitive tip area wetted by one phase, and therefore, long sensitive length induces smooth signal transitions. Minimization of this delay, i.e., approximation of a square wave shape, is desirable for signal conditioning. However, to obtain a true square wave type signal, a proper threshold voltage has to be used as a triggering criterion. The value of threshold voltage can be obtained by processing the data for void fraction and by comparing it with other reference measuring method.

Signal Conditioning

The most common method of signal conditioning is based on a single threshold level whose intersections with the raw signal determine the start and the end of rectangular waves, producing thus a

succession of gas residence times. This approach has the inherent disadvantage that signals which do not reach the threshold level will be undetected. Hence the threshold level must be set as close to the liquid signal level as possible, in order to minimize the influence of the dewetting time and to take account of small bubbles.

No agreement exists in the literature, neither for the recommended threshold level, nor for the resulting performance. Nevertheless, the threshold level usually ranges from 10% to 50% of the static high level signal.

The problem is further augmented in practice by shifts in the liquid signal level. This problem can be avoided by comparing the samples with a self-adjusting threshold level. First the data were divided in many data blocks (typically 16000 samples). Before a data block entered a phase discrimination routine, the maximum M and the minimum N values of signal were determined in advance. The threshold level T is given by:

$$T = (M - N) \cdot S + N \quad (1)$$

where S ranges from zero to one.

The main advantage of this phase discrimination method is that every data block has a flexible threshold level, even for the same value of S . This is of practical importance if the liquid signal level drifts.

Signal Processing

As the conditioned signal consists of a train of square waves, the signal has to be processed such that the local parameters can be obtained.

The local void fraction is defined as the time average of the phase indicator function X by:

$$\alpha = \lim_{T \rightarrow \infty} \frac{1}{T} \int_0^T X(x, t) dt \quad (2)$$

in which X as a function of position x and time t equals one for the gas phase and zero for the liquid phase.

As the conditioned signal is given in discrete binary form, Eq. (2) can be written as:

$$\alpha = \frac{1}{N} \sum_{i=1}^N X(i) \quad (3)$$

in which N is the total number of samples and $X(i)$ the binary signal.

The residence time for a bubble follows from the block length of the square wave signal, i.e., from the number of continuous samples n in the gas phase, and from the sampling frequency f :

$$T_b = \frac{n}{f} \quad (4)$$

The local bubble interface velocity may be determined from the signals of two probes placed in the flow direction. A bubble which contacts the front probe will, in general, subsequently make contact with the rear probe. The time delay between these two contact signals is a measure for the velocity of

the bubble. It is of course possible that a bubble is only pierced by one of the probes; this error source should not cause any problems provide this stochastic process is observed over an adequate length of time. A correlation technique will be required to determine the most probable time delay between two stochastic signals. The cross-correlation function of the two probe signals a and b is defined by:

$$F_{ab}(x, \tau) = \lim_{T \rightarrow \infty} \frac{1}{T} \int_0^T a(x, t) b(x+d, t+\tau) dt \quad (5)$$

where d is the distance between the probes and τ is the time delay. The maximum of the correlation function yields the most probable time delay τ_0 , from which the bubble interface velocity component in the axial direction is determined as:

$$v_i = \frac{d}{\tau_0} \quad (6)$$

For two identical signals shifted relative to each other the correlation function yields a triangular shaped function. In case of nonidentical signals, i.e., when the width of the blocks differs, the top of the correlation function is broadened, which may generate difficulties in determining τ_0 . In practice, however, the correlation functions show a single maximum. It can be shown that the error in determining τ_0 is inversely proportional to the number of samples within the time delay τ_0 , that is, if the sampling frequency is sufficiently large, the error in the measured velocity can be kept sufficiently low.

The chord length l of a bubble follows from the block length of the square wave signal, i.e., from the number of continuous samples n in the gas phase, and from the bubble interface velocity v_i obtained from Eq. (6):

$$l = \frac{n v_i}{f} \quad (7)$$

where f is the sampling frequency.

The transformation of a chord length into a bubble diameter remains problematic, as a certain chord length can correspond to a small chord in a large bubble or a large chord in a small bubble. Moreover, the velocity of the bubbles may differ. This problem may be overcome if the following assumptions are made:

- the bubbles are spherical;
- the probe has equal probability of piercing any point on the projected frontal area of the bubbles;
- all bubbles travel in the same direction with the same average velocity.

If the probability density function of the measured chord lengths is denoted by $g(l)$ and that of the detected bubbles diameter by $f(d)$, it is shown by Herring and Davis (1976) that:

$$f(d) = \frac{1}{2} (g(l) - l g'(l)) \quad (8)$$

The use of this equation to obtain the function $f(d)$ requires differentiation of function $g(l)$, which can cause substantial errors, due to inaccuracies in the experimentally determined distribution $g(l)$. On the other hand, some investigators report that bubbles which are not centrally pierced will tend to displace their center in such a way that anyhow the diameter is measured. This point of view makes the

second one of the above assumptions rather doubtful, and in some works the bubble diameter is assumed to be equal to the measured chord length, so that $f(d) = g(l)$.

It must be noted that the distribution functions and mean diameters obtained are only representative of *detected* bubbles. The spectrum of detected bubbles will in general not be representative of the spectrum of existing bubbles, as the probability for the probe to detect a bubble is inversely proportional to the cross-sectional area of the bubble, i.e., d_b^2 . If we denote the probability density function of the detected bubbles by $f(d)$ and that of all bubbles with center passing through a unit area of the cross section by $j(d)$, it is shown by Herringe and Davis (1976) that:

$$j(d) = k \frac{f(d)}{d_b^2} \quad (9)$$

where

$$k = \frac{4}{\pi} \frac{n_b}{N_b}$$

in which n_b is the number of detected bubbles and N_b is the total number of bubbles passing through a unit area. The function $j(d)$ will yield a much narrower bubble size range and much smaller mean diameters than those obtained from the function $f(d)$ due to the division by d_b^2 .

The time-averaged interfacial area concentration a_i can be obtained by counting the number of interfaces passing the probe per unit time N_i and knowing the interfacial velocity v_i and the angle ϕ between this velocity and the normal vector of the interface (Kataota et al., 1986):

$$a_i(x) = 2N_i \frac{l}{|v_i| \cos \phi} \quad (10)$$

In practice, it is difficult to determine the angle ϕ . However, with the assumption that the interfaces are composed of spherical bubbles, the probe passes every part of bubble with an equal probability and the interfacial velocity has the same direction of the probe tips, the interfacial area concentration can be determined (Vetau, 1981) by:

$$a_i(x) = 4N_i \frac{l}{|v_i|} \quad (11)$$

The profiles of the interfacial area concentration and the void fraction can be used to determine the Sauter mean bubble diameter variations along the cross section. The definition of the Sauter mean bubble diameter assumes spherical bubbles and is given by:

$$D_{sm} = \frac{\sum_{k=1}^{N_k} n_k D_k^3}{\sum_{k=1}^{N_k} n_k D_k^2} \quad (12)$$

where n_k is the number of bubbles of size D_k and N_k is the total bubble size classes.

From the definitions of void fraction and interfacial area concentration for spherical bubbles, it can be shown that:

$$D_{sm}(x) = \frac{6}{a_i(x)} \frac{\alpha(x)}{a_i(x)} \quad (13)$$

Experimental Results

Description of the Experimental Facility

A schematic diagram of the experimental facility is illustrated in Fig. 2. The test section is made of a 80 mm ID Plexiglas tube which is 1600 mm long. The local measurement station is located at $L/D=15$.

The air flow was supplied from the building central air system. The air flowrate was controlled and measured with a mass flow controller BROOKS-5851-E. The tap water flowrate was measured with a rotameter. Both air and water were injected into the mixing chamber placed at the bottom of the test section. The bubble generator consisted of a porous plate. At the top of the test section a constant level system allowed the air to be exhausted and the water to be drained.

The experimental conditions are summarized in Table 1. The gas flowrate and superficial velocity are always expressed at the calibration conditions (0°C and 101 KPa).

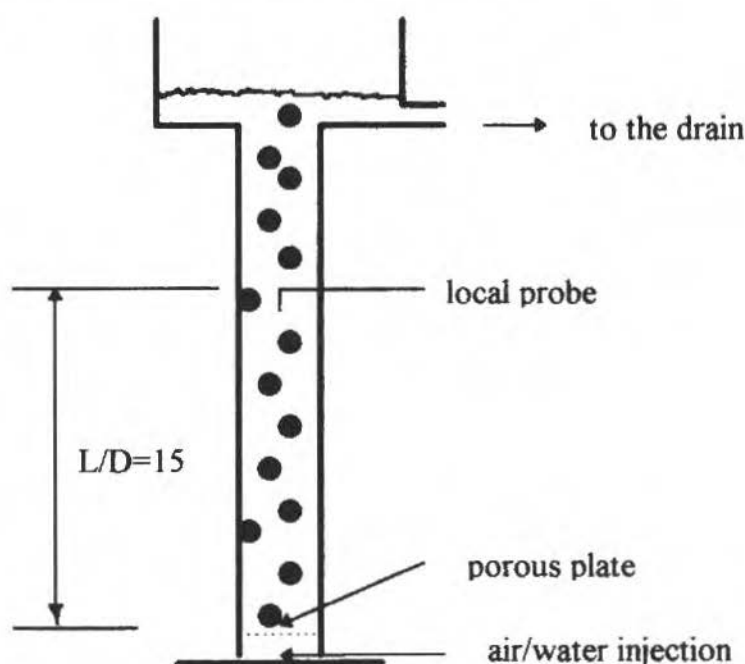


Fig. 2 Schematic of the Experimental Facility

Table 1 Experimental Flow Conditions

Liquid flowrate (m^3/h)	0	1.81
Superficial liquid velocity (m/s)	0	0.10

Gas flowrate (m^3/h)	0.36	0.72	1.09	1.45	1.81
Superficial gas velocity (m/s)	0.02	0.04	0.06	0.08	0.10

The Electrical Resistivity Double Probe

The local measurement station consists of a probe displacement mechanism, the electrical resistivity double sensor probe and the driven voltage-sensitive circuit. The double sensor probe was inserted in the test section through a probe support tube (3.0 x 2.0 mm) fixed to the displacement mechanism. A Vernier, with graduations to an accuracy of 0.05 mm, was used to traverse the probe in the radial direction. A high resolution mechanism was necessary to evaluate probe positions in flow stream accurately and to ensure reproducible results.

The design of the electrical resistivity sensor is shown in Fig. 3. For each sensor, one electrode is the exposed tip of an otherwise isolated wire and the return electrode is the supporting tube. The inner electrode is a 150 μm diameter stainless steel wire, accurately cut under a microscope at the tip. The conical tip of the sensor was made as short as possible (30 μm) to minimize the effect of bubble deformation. The stainless steel wire was inserted into a 0.2 mm i.d. thin plastic sleeve. This plastic tube containing the sensor was then inserted into a 0.6 mm i.d. x 1.0 mm o.d. stainless steel tube. Epoxy insulation was applied to the entire sensor and allowed to run back whilst drying, to expose only a small tip area. The exposed length was about 250 μm . The axial distance between the tip of the two sensors was 3.8 mm.

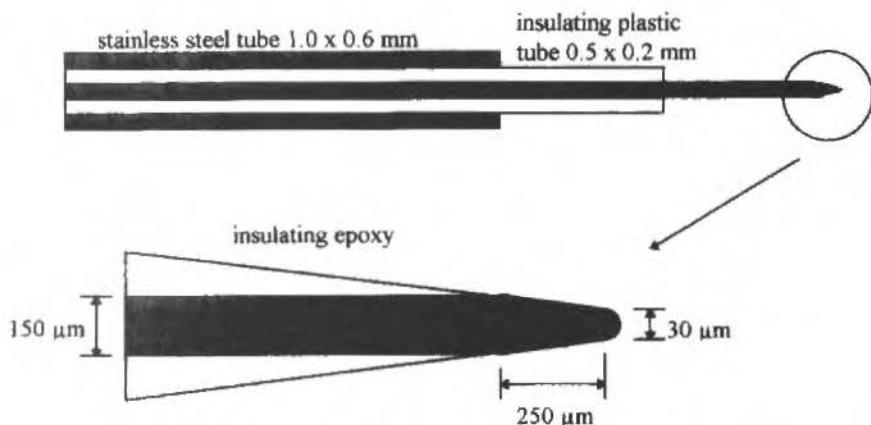


Fig. 3 Design of the Electrical Resistivity Double Probe

Each sensor was driven by a voltage-sensitive circuit consisting of a 1.5 volt battery and a 2.2 $M\Omega$ potentiometer connected in series with the probe to the ground. The potentiometer was adjusted to give an optimum output voltage for measurements. If the sensor tip is in the liquid phase, the circuit will be closed and the voltage output will be lower. When a gas bubble hits the sensor tip, the circuit continuity will be broken and the output will read a high voltage. The voltage drop across the probe during closed circuit (liquid signal) approximated 0.3 to 0.4 volts. This low voltage effectively reduced electrochemical phenomena at the sensor.

For each preset experimental condition the probe signals were digitized by a data acquisition system utilizing a personal computer and a Keithley-MetraByte DAS-1401 high-speed analogue-digital I/O expansion board.

A typical resistivity double probe response in two-phase bubbly flow (4 kHz sample rate) is shown in Fig. 4. It can be observed the time shift between the signal of the two sensors as a bubble passes through the probe.

Owing to the large volume of data generated, the sampling rate of the data acquisition was set at 4 kHz, and the total sampling time was 40 s. It was found that this combination provided a sufficiently number of bubbles for the statistical analysis of the flow. On average, 500 to 2000 bubbles were detected in each acquisition of raw data.

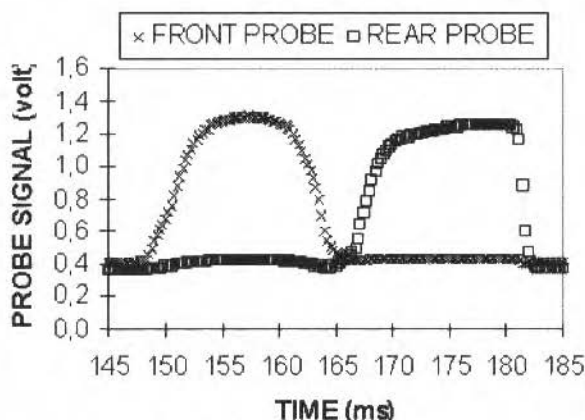


Fig. 4 A Typical Resistivity Double Probe Response in Bubbly Flow

It is to be noted that the sampling rate may seem to be very low when compared with some investigations carried out on vertical bubbly two-phase flows. However, it is important to note that for the present experimental conditions the bubble velocities are very low, and thus it becomes essential to have a total sampling time as long as possible to get enough bubbles. This simultaneously leads to a low sampling rate due to the overall limitations of the data acquisition system.

In principle, the resistivity probe is able to detect only bubbles greater than 250 μm because the exposed length of the sensor tip. Then, for a typical bubble relative velocity of 20 cm/s, even with such low sampling rate, a bubble passing the sensor tip will produce at least five samples at high level voltage.

Influence of Threshold Level

As mentioned above, a proper threshold level has to be applied to the raw signal in order to obtain a true square wave type signal. When the threshold level S , defined as a percentage of the voltage gap between the static levels, evolves from 0% to 100%, so does the local void fraction. Hence, for a given experiment, it is always possible to find an optimum threshold in order to retrieve the void fraction determined from another technique.

Figure 5 shows the influence of the threshold level on the void fraction determination. Consider now the sensitivity of the void fraction to the choice of the threshold. A typical value of this sensitivity for the experimental results shown in Fig. 5 is given by:

$$\frac{\Delta\alpha}{\Delta S} \approx -0.07 \quad (14)$$

where both void fraction and threshold level are expressed in percentages.

For example, if the threshold level is increased from 0.2 to 0.3, the void fraction will typically decrease by 0.007. This value of sensitivity may be compared to the better sensitivity results for resistivity probe techniques reported in the work of Cartellier and Achard (1991).

All the experimental results presented in this paper were obtained by using a threshold level $S=0.1$. The experimental uncertainty on the void fraction measurement associate with the threshold level was about 0.007. The uncertainty associate to the data acquisition sample rate was around 0.013 (bubble diameter = 5 mm, bubble velocity = 0.25 m/s and sample rate = 4 kHz). Then the uncertainty on the void fraction measurement was estimated to be 0.02, that is 2 %.

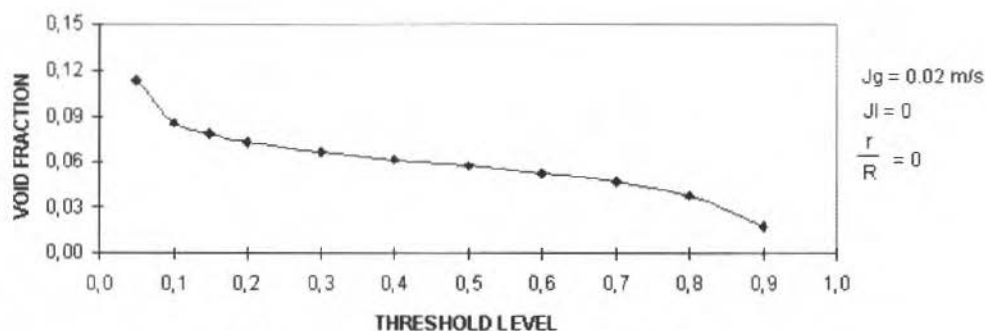


Fig. 5 Influence of the Threshold Level on the Void Fraction

Residence Time Distribution

The bubble residence time histograms at the pipe center ($r/R=0.0$) and near to the pipe wall ($r/R=0.75$), for two different gas superficial velocities ($J_g=0.02$ m/s and $J_g=0.10$ m/s) are shown in Fig. 6.

For the lower gas superficial velocity ($J_g=0.02$ m/s), the bubble residence time distribution is quite uniform both at the pipe center and near the pipe wall, as it was expected for a bubbly flow.

On the other hand, for a higher gas superficial velocity ($J_g=0.10$ m/s), there are much more small bubbles at the pipe center than near the pipe wall. This flow configuration is a characteristic of churn turbulent flow regime, where the large bubbles are broken down due to the turbulent liquid flow. The bubble residence time distribution at the pipe center shows a large number of bubbles with a residence time of less than 5 ms. For a typical bubble relative velocity of 20 cm/s, it corresponds to a bubble diameter of less than 1 mm.

It should be noted the relatively lower number of bubbles with a residence time less than 0.5 ms. As it was mentioned above, the resistivity probe is able to detect only bubbles greater than 250 μ m due to the exposed length of the sensor tip. Then, only a few number of very small bubbles (<0.1 mm) were detected by the probe.

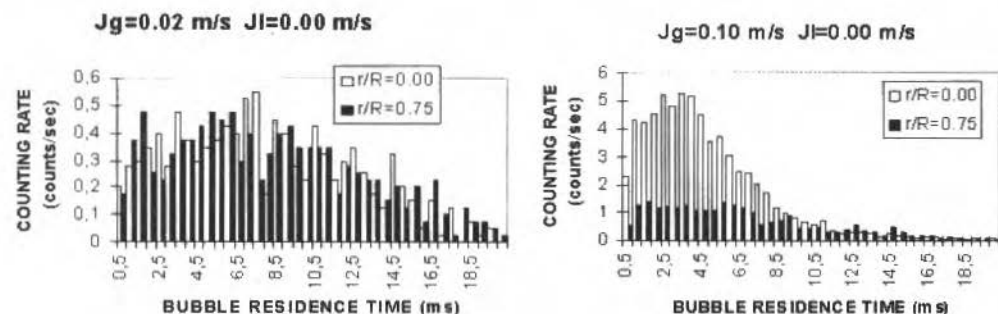


Fig. 6 The Bubble Residence Time Histogram

Bubble Frequency

The bubble frequency radial profiles, for two different liquid superficial velocities, are shown in Fig. 7. As it was expected, the number of bubbles passing by the same radial position increases as the gas superficial velocity increases. Generally, the bubble frequency decreases from the center to the pipe wall. Nevertheless, for low gas superficial velocities, the bubble frequency is quite uniform over the pipe radius. For a liquid superficial velocity of 0.10 m/s, a flat bubble frequency profile may be

observed even for a gas superficial velocity as high as 0.06 m/s. This fact could be explained by the transition between bubbly flow and churn flow regime. If for no water flow ($J_l = 0$) this transition is observed at a gas superficial velocity of about 0.03 m/s, for $J_l = 0.10$ m/s the flow transition is observed only for $J_g = 0.07$ m/s.

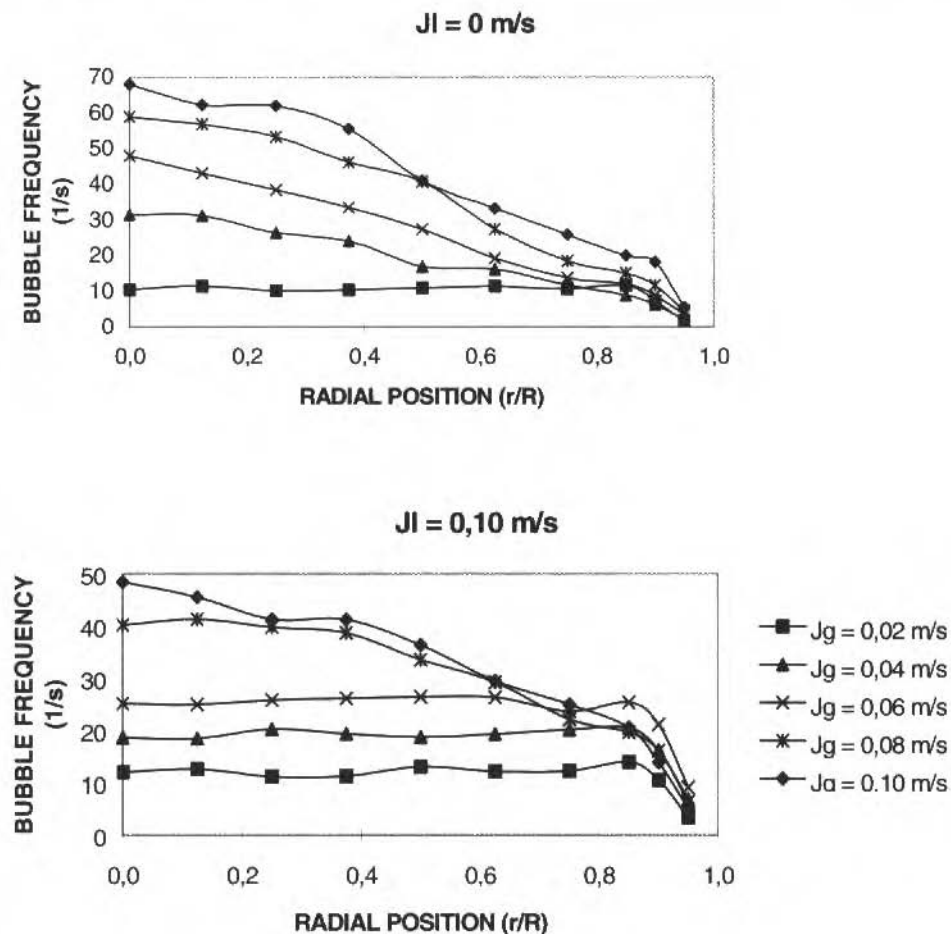


Fig. 7 The Radial Profiles of Bubble Frequency

Void Fraction Distribution

The void fraction radial profiles, for two different liquid superficial velocities, are shown in Fig. 8. For all experimental conditions, the void fraction profiles are similar to the bubble frequency profiles. Moreover, the bubble frequency distributions are proportional to the void fraction.

As it was expected for bubbly flow regime, i.e., for low gas superficial velocities, there is a peak in the void fraction distribution near the pipe wall. It disappears for higher gas superficial velocities (churn flow regime) and the void fraction profiles become close to a parabolic shape.

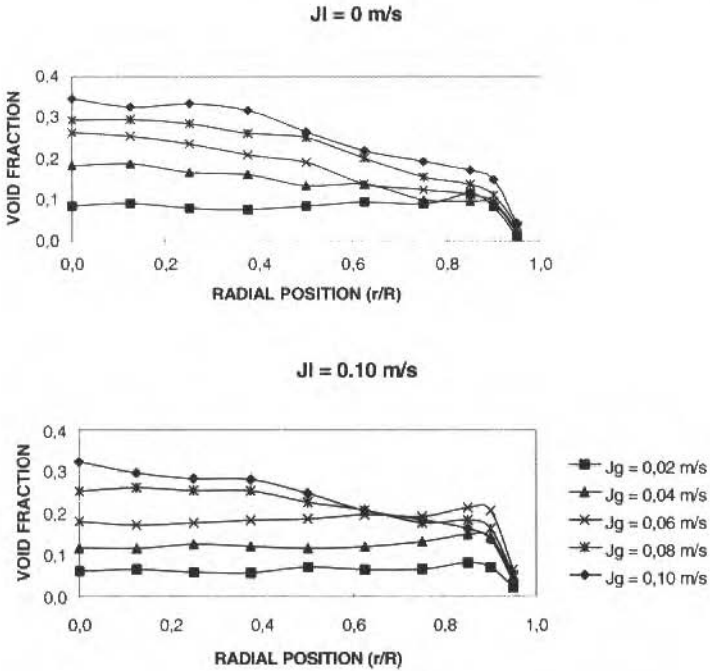


Fig. 8 The Radial Profiles of Void Fraction

The Double Sensors Response

Since the double probe sensors are separated by a finite distance and a bubble is free to move in any direction, a bubble that hits the upstream sensor is not always intercepted by the downstream sensor. Figure 9 shows a comparison between the bubble frequency measured by the front probe and by the rear probe. It is clear that some bubbles (about 10 %) were deflected by the front probe and were not intercepted by the rear probe. The same trends were observed in the void fraction measurements from the two probes.

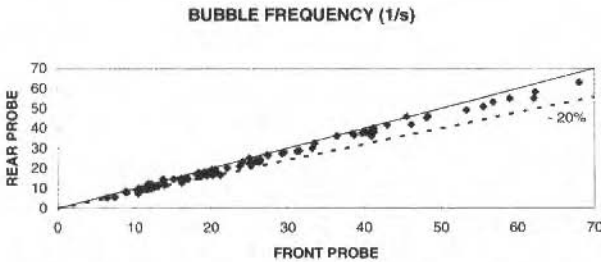


Fig. 9 The Bubble Frequency Detected by the Front and the Rear Probe

Bubble Interface Velocity

As it was mentioned above, the bubble interface velocity may be determined from the time delay between the signals of two probes placed in the flow direction. A correlation function, Eq. (5), was used to determine the most probable time delay between two stochastic signals. The maximum of the

correlation function yields the most probable time delay, from which the bubble interface velocity component in the axial direction is determined by Eq. (6).

Figure 10 shows a typical correlation function distribution for different radial positions. It corresponds to a bubbly flow regime for which a very well defined maximum was observed for all radial positions. For churn flow regime, the correlation functions were broadened because the presence of some large bubbles with greater interface velocity than the small bubbles. Nevertheless, it was always possible to find the function maximum that corresponds to the most probable time delay.

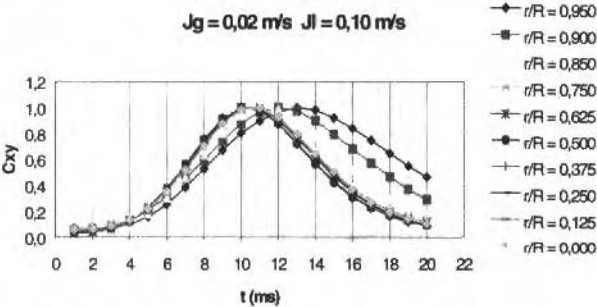


Fig. 10 The Dimensionless Cross Correlation Functions

The radial bubble velocity profiles are presented in Fig. 11. For both liquid superficial velocities ($J_l=0$ and $J_l=0.10 \text{ m/s}$), the bubble velocity decreases from the center to the pipe wall, except for the lower gas superficial velocities that correspond to the bubbly flow regime, where a flat profile was observed. The measured bubble velocities are close to the values predicted by the drift flux model (Zuber and Findlay, 1965). For example, the bubble velocity predicted by the drift flux model, for $J_l=0$ and $J_g=0.02 \text{ m/s}$, is about 0.25 m/s ; for $J_l=0.10$ and $J_g=0.02 \text{ m/s}$, the bubble velocity is about 0.37 m/s .

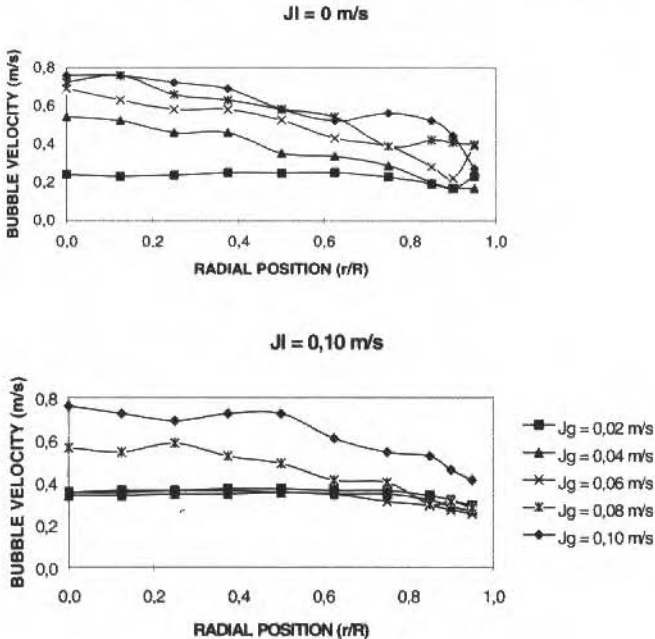


Fig. 11 The Radial Profiles of Bubble Interface Velocity

Interfacial Area Concentration

The local interfacial area concentration may be determined from the bubble interface velocity and bubble frequency measurements, by using Eq. (11). This equation was obtained with the assumption that the interfaces are composed of spherical bubbles, the probe passes every part of bubble with an equal probability and the interfacial velocity has the same direction of the probe tips. The radial profiles of interfacial area concentration are presented in Fig. 12. Generally, the interfacial area concentration increases as the gas superficial velocity increases. On the other hand, it was not influenced by the liquid superficial velocity. It should be noted that the interfacial area decreases near the pipe wall, certainly because the bubble frequency reduction in this region.

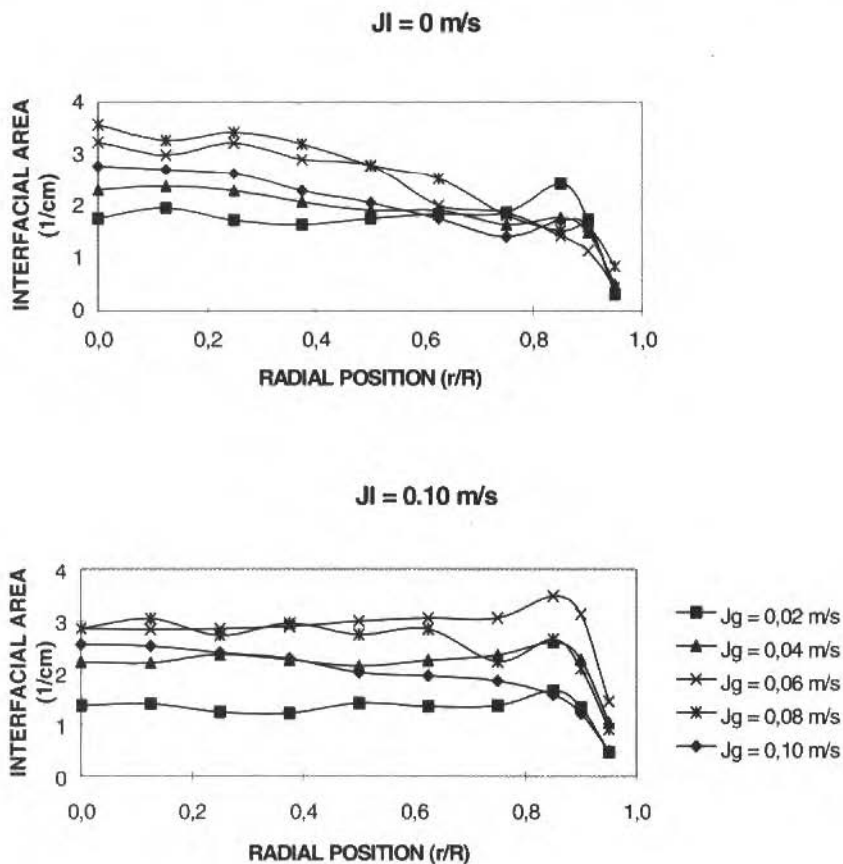


Fig. 12 The Radial Profiles of Interfacial Area Concentration

Sauter Mean Diameter

The Sauter mean diameter, defined by Eq. (12), may be determined from the void fraction and interfacial area concentration, by using Eq. (13). The radial profiles of the Sauter mean diameter are presented in Fig. 13. As it was expected, the bubble diameter increases as the gas superficial velocity increases. For bubbly flow regime the mean bubble diameter is approximately 3 mm, which is in agreement with flow visualization.

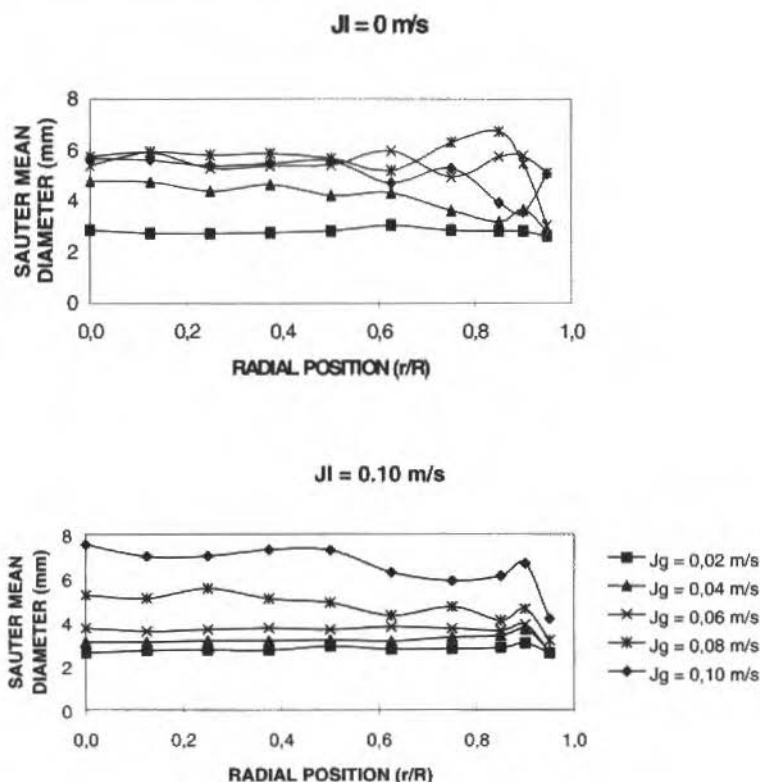


Fig. 13 The Radial Profiles of Sauter Mean Diameter

Average Gas Superficial Velocity

The average gas superficial velocity may be obtained from the local measurements of the void fraction and the bubble interface velocity. First, the pipe area is divided into concentric rings according to the radial position of the local measurement. The area averaged gas superficial velocity is obtained by multiplying the local void fraction and bubble interface velocity by the area ratio and summing them together:

$$J_g = \frac{1}{A} \int_A v_b \alpha dA = \sum_k \left[\left(\frac{r_o}{R} \right)^2 - \left(\frac{r_i}{R} \right)^2 \right] v_{bk} \alpha_k \quad (15)$$

where r_o , r_i and R are respectively the outer and the inner radius of the concentric ring and the pipe radius.

Figure 14 shows the comparison of the area average gas superficial velocities obtained by the probe method and by the inlet measurements. Most of data were with a 10% error range compared to the inlet flow measurement. The disagreement for high gas superficial velocity is probably due to the effect of the overprediction of the bubble interface velocity for the churn flow regime, as it was mentioned above.

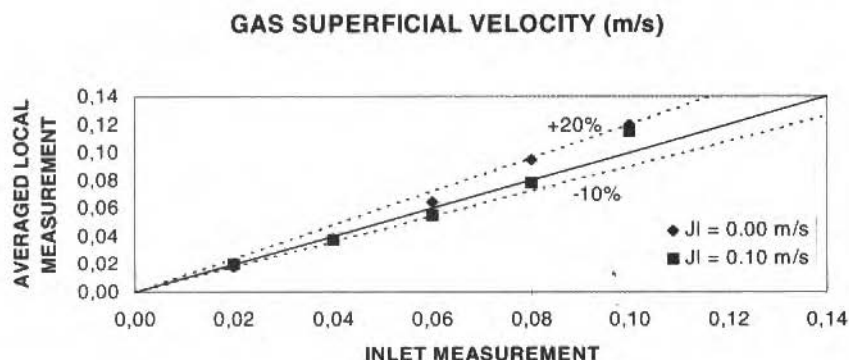


Fig. 14 Comparison Between Area Average and Inlet Gas Superficial Velocity

Conclusions

The electrical resistivity double probe technique for measuring local parameters in a two-phase flow has been described. Data on the local void fraction, bubble frequency, bubble interface velocity, interfacial area concentration and Sauter mean diameter were obtained for different gas and liquid superficial velocities in a air-water vertical flow. For the present experimental conditions, bubbly flow and churn flow regime were observed.

For bubbly flow the void fraction and the bubble frequency radial profiles showed a distinct peak near the wall and a relatively flat pattern in the core ($r/R < 0.8$). Increasing the gas superficial velocity, for a constant liquid superficial velocity, increased the void fraction and the bubble frequency, both in the core region and in their peaking region near the wall. For churn flow regime the void fraction and bubble frequency profiles become close to a parabolic shape.

The bubble interface velocities, obtained from the cross correlation function of the signal of the double sensor probe, are close to the values predicted by the drift flux model. It ranges from 0.25 m/s for bubbly flow, up to 0.75 m/s for churn flow. The interfacial area concentration and Sauter mean diameter radial profiles were obtained from the void fraction, bubble frequency and bubble interface velocity.

Finally, the area average gas superficial velocities, obtained from the local measurements of void fraction and bubble velocity, were compared to the inlet gas flow rate and a very good agreement was found.

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On the Fluctuations in a Random Suspension of Sedimenting Particles

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Abstract

This paper is concerned with velocity fluctuations in a monodisperse dilute suspension of sedimenting particles and in particular with the divergence problem of the variance with the system size. After various scaling arguments it is described a model for suspension flow in which macroscopic mass and momentum balances are constructed to be solved simultaneously. The general model is adapted to study the problem of fluctuations in sedimentation under conditions of low Reynolds numbers and finite Stokes numbers. The important contribution to the particle stress associated with velocity fluctuations induced by body forces is described using an effective viscosity. The coefficients of the scaling arguments for both limiting case examined (inertialess and when the inertia of the particle is important) are also calculated.

Keywords: Velocity Fluctuations, Sedimentation, Random Suspension.

Introduction

The sedimentation of a monodisperse, dilute suspension of non-Brownian, rigid spheres under creeping flow conditions is one of the basic simple flows in suspensions. If the suspension is infinitely dilute the hydrodynamic interactions between particles may be neglected. In this case each particle settles with the Stokes velocity, $U_s = 2/9 \Delta \rho a^2 / \mu g$ where $\Delta \rho$ denotes the difference between the density of the solid particles and fluid, a is the particle radius, μ is the fluid viscosity, and g is the acceleration due to gravity. At a finite particle concentration, however, the mean sedimentation velocity $\langle U \rangle$ of the spheres is noticeably proportional to the Stokes settling velocity. The proportionality factor f , known as the hindered function, is generally assumed to be dependent upon the volume concentration of the particles $\phi (= 4/3 a^3 n)$ and a monotonically decreasing function with $f(0)=1$. In this paper n is the number density of the particles (i.e. the number of particles N per unit of volume). The mean sedimentation velocity in the suspension is then assumed to be $\langle U \rangle = U_s f(\phi)$, where $\langle \rangle$ indicates an ensemble average over all possible particle configurations weighted by the probability of their occurrence. Even in a dilute suspension, the departure of $f(\phi)$ from unity is determined by the suspension microstructure.

A random free suspension structure is the most important one for sedimentation problems, although yet is in some way the most difficult to handle theoretically. In this problem the conditional probability function $P(x+r|x)$ which denotes the probability of finding a sphere with its centre at $x+r$ given that there is one centred at x , is not known a priori and can change from system to system depending on the condition of the real experiments. The theoretical progress that has been made, takes into account only the well-stirred particles distribution in a dilute monodisperse suspension. In general, the suspension is supposed to be statistically homogeneous, requiring uniform probability for all physically accessible position of one sphere relative to another. Specifically speaking, the centre of a test sphere cannot be located within an excluded volume shell $a < |r| < 2a$ of any other sphere. Under this condition $P(x+r|x)$ is constant ($= n$) for $|r| \geq 2a$ and vanishes for $0 \leq |r| < 2a$. The problem is still complicated because the first effect of particles interactions on the mean or variance of the sedimentation velocity cannot be found simply by adding up the contributions from all the pair interactions in which the test sphere takes part. The procedure results in sums or integrals which are often divergent. This difficulty

comes from the fact that in the Stokes flow, hydrodynamic interactions are long-range. Evidently, a sedimenting suspension containing an infinite number of particles makes

the integral $\langle U \rangle = \int U(x, x+r) P(x+r|x) dr$, divergent, as $U(x, x+r) = O(1/|r|)$ and $P(x+r|x) \rightarrow n$ as $|r| \rightarrow \infty$.

The convergence problem can be avoided in a number of ways; at present the most popular is the renormalization technique first introduced by Batchelor (1972). The essence of his procedure is to look for a quantity that has the same asymptotic dependence on the position of one particle in the suspension configuration, and whose mean value is known precisely involving all spheres in the configuration. Batchelor was able to show that the mean settling speed relative to zero-volume flux axes (that is, relative the containing box) in a statistically homogeneous dilute suspension of rigid particles is proportional to ϕ , finding $f(\phi) = (1 - 6.55\phi) + O(\phi^2)$.

The $O(\phi)$ correction consists of a small positive effect due to the finite size of the particles $(+0.5\phi)$, i.e. a degenerate quadrupole, $\frac{1}{6} a^2 \nabla^2 u(x, x+r)|_{x_0}$, a contribution due to the near field hydrodynamic which contains the fine details of the interaction between pairs of particles (-1.55ϕ) , and a large negative contribution from the back flow effect (-5.5ϕ) (which corresponds to the lowest level of point force approximation). These considerations have been clarified and amplified by Hinch (1977), who established a systematic approach of ensemble average in which an infinite hierarchy of linked bulk equations are constructed by repeatedly averaging with one more fixed particle at each level, and the hierarchy can be truncated by noting the linking terms become small for small particle concentration. Adding contribution from higher reflections Hinch was able to recover Batchelor's result, getting $f(\phi) = 1 - \frac{845}{128} \phi + O(\phi^2)$. Complementary techniques of renormalization are given in a number of other important articles on determination of transport properties of suspensions (e.g. Jeffrey, 1974, O'Brien, 1979), and all have confirmed Batchelor's calculation.

The origin, significance and interpretation for overcoming the convergence difficulties with calculating the sedimentation velocity in a random monodisperse dilute suspension are now well understood. However a dramatic illustration of the convergence problem in such suspension is formed by the divergence of the variance $\langle U'^2 \rangle$ (the simplest measure of the particle velocity fluctuations) of the sedimentation velocity. This paradoxical situation was first noticed by Caffish and Luke (1985), who showed that Batchelor's (1972) renormalization does not resolve the divergence difficulties associated with calculating the variance of the sedimentation velocity. Indeed they found that the variance of the particle velocity in a monodisperse suspension of spheres whose positions were randomly distributed with uniform probability would depend on the linear dimension L of the container, i.e. the variance would be $O(U_s \phi L/a)$. Here $U' = U - \langle U \rangle$ is the deviation of the particles velocity from its average value. A first scaling analysis based on arguments of buoyancy-driven convection in sedimentation was proposed by Hinch (1988); volume elements of the suspension with the lowest density rise relative to volume elements with highest density. The scaling confirmed the result predicted by Caffish and Luke.

The fluctuations in the motion of a particle in a sedimenting suspension are an inherent property of such a system, but by no means the result of hydrodynamic instability of the mean flow. If the latter occurs, additional large-scale perturbations, similar to secondary flows or turbulence, in the hydrodynamics of one-phase media, may be important. In the present work we are interested in suspensions under conditions of low Reynolds number, in which the variations in the individual particle velocities are due to varying configurations of neighbouring particles, i.e. statistical density fluctuations caused by viscous hydrodynamic interactions. The disordered motion of the particles gives the particles

a statistical tendency to migrate, that at long time behaviour can be characterized by means of a self-induced hydrodynamic dispersion diffusivity. In order to understand the problem of fluctuation in sedimentation in a reasonably simple manner, and to bring out the essential features, we shall present next scaling arguments for the velocity fluctuations produced by horizontal density fluctuations in a random suspension. Basically, variations in the number density of particles over length scales of order of the settling box size L drive convection currents. We will find that these convection currents can dominate the motion of particles during sedimentation.

Scaling Analysis

Let us consider the situation of a dilute suspension, where it is natural to suppose that, for the purpose of considering the influence of neighbouring particles on a test particle, it suffices to replace all other particles in the suspension by point singularities, such as point forces. Thus, the phase space operator corresponding to the density number of N point particles randomly distributed in the suspension can be defined as

$$n(\mathbf{x}, t) = \sum_{\alpha=1}^N \delta(\mathbf{x} - \mathbf{x}_{\alpha}(t)) \quad (1)$$

where $\mathbf{x}_{\alpha}(t)$ is the position of the α^{th} particle at time t , N is the number of particles, and $\delta(\mathbf{x})$ (the Dirac's "delta function") is labeled a distribution or a generalised function, defined by the sequences equivalent to $\delta_p = e^{-p\lambda^2} (p/\pi)^{1/2}$ (Lighthill, 1958) such that

$$\int_{-\infty}^{\infty} \delta(\mathbf{x} - \mathbf{x}_0) G(\mathbf{x}) d\mathbf{x} = \lim_{p \rightarrow \infty} \int_{-\infty}^{\infty} \delta_p(\mathbf{x} - \mathbf{x}_0) G(\mathbf{x}) d\mathbf{x} = G(\mathbf{x}_0) \quad (2)$$

Note that the unconditional average at a point $\langle n(\mathbf{x}, t) \rangle = n$ defines the mean number density of particles, with the average over all positions of $\mathbf{x}_{\alpha}$. On the other hand, the mean density given there is a particle at $\mathbf{x}_{\beta}$ (conditional average), is $\langle n(\mathbf{x}, t) \rangle_{\beta} = n [1 - h(\mathbf{x} - \mathbf{x}_{\beta})]$ with $h \rightarrow 0$ as $(\mathbf{x} - \mathbf{x}_{\beta}) \rightarrow \infty$.

Using Equation (1), the space-time correlation $G(\mathbf{x}, t)$ can be written as follows (van Hove, 1954)

$$G(\mathbf{x}, t) = \frac{1}{N} \left\langle \sum_{\alpha, \beta} \delta[\mathbf{x} + \mathbf{x}_{\alpha}(0) - \mathbf{x}_{\beta}(t)] \right\rangle \quad (3)$$

where $G(\mathbf{x}, t)$ splits naturally into two parts: one that describes the correlation of one particle at different times and one that describes the correlation between distinct particles at the same times. We interpret this function as the conditional probability that some test particle is located in $d\mathbf{x}$ at $\mathbf{x}$ time t given that it was at the origin at $t = 0$. It is immediate to see that $\langle n(\mathbf{x}, 0) n(\mathbf{x}, t) \rangle = NG(\mathbf{x}, t)$. We now define the scattering function $F(\mathbf{k}, t)$ (Rallison and Hinch, 1986) as the Fourier transform of the space-time correlation function. Then $F(\mathbf{k}, t)$ and $G(\mathbf{x}, t)$ are Fourier related by

$$G(\mathbf{x}, t) = \frac{1}{8\pi^3} \int F(\mathbf{k}, t) e^{-2\pi i \mathbf{k} \cdot \mathbf{x}} d\mathbf{k} \quad \text{and} \quad F(\mathbf{k}, t) = \int G(\mathbf{x}, t) e^{2\pi i \mathbf{k} \cdot \mathbf{x}} d\mathbf{x} \quad (4)$$

The corresponding Fourier transform sum of $n(\mathbf{x}, t)$ and its complex conjugate are given respectively by

$$\hat{n}(\mathbf{k},t)=\sum_{\alpha=1}^N e^{2\pi i \mathbf{k} \cdot \mathbf{x}_{\alpha}(t)} \text{ and } \hat{n}^*(\mathbf{k},0)=\sum_{\beta=1}^N e^{-2\pi i \mathbf{k} \cdot \mathbf{x}_{\beta}(0)} \tag{5}$$

Here the notation $\hat{n}^*(\mathbf{k},0)=n^*(-\mathbf{k},0)$. If a dynamical quantity like $e^{2\pi i \mathbf{k} \cdot \mathbf{x}(t)}$ is complex, the time correlation function is defined as $\langle A^*(0)A(t) \rangle$ (van Hove, 1954). This is equivalent to multiply $\hat{n}^*(\mathbf{k},0)$ by $\hat{n}(\mathbf{k},t)$ and average over an equilibrium ensemble, $\langle \hat{n}(\mathbf{k},t)\hat{n}^*(\mathbf{k},0) \rangle$. Hence the scattering function $F(\mathbf{k},t)=\frac{1}{N}\langle \hat{n}(\mathbf{k},t)\hat{n}^*(\mathbf{k},0) \rangle$ may be evaluated in according to the following expression

$$F(\mathbf{k},t)=\frac{1}{N}\left\langle \sum_{\alpha,\beta} e^{-2\pi i \mathbf{k} \cdot [\mathbf{x}_{\alpha}(t)-\mathbf{x}_{\beta}(0)]} \right\rangle \tag{6}$$

Physically, $F(\mathbf{k},t)$ may be interpreted as the autocorrelation of fluctuations in the number density $\hat{n}(\mathbf{k},t)$ at wave number $\mathbf{k}$, and it is often called in the literature (e.g. Rallison and Hinch, 1986) the dynamic structure factor of the suspension.

We consider the initial condition for $\langle n(\mathbf{x},t)n(\mathbf{x},0) \rangle = NG(\mathbf{x},t)$ and $\langle \hat{n}^*(\mathbf{k},t)\hat{n}^*(\mathbf{k},0) \rangle = NF(\mathbf{k},t)$ as being the “thermodynamic limit”, which corresponds $N \rightarrow \infty, V \rightarrow \infty$ with $(N/V) \rightarrow constant$. V is the suspension volume. Then, following Rallison and Hinch (1986), we deduce that

$$\langle n(\mathbf{x},t)n(\mathbf{x},0) \rangle = n\delta(\mathbf{x}) + n^2 \left[g(\mathbf{x}_2 - \mathbf{x}_1) - 1 \right] \tag{7}$$

and

$$\langle \hat{n}^*(\mathbf{k},t)\hat{n}^*(\mathbf{k},0) \rangle = N + N^2 \delta(\mathbf{k}) + \frac{N^2}{V} \int \left[g(\mathbf{x}_2 - \mathbf{x}_1) - 1 \right] e^{2\pi i \mathbf{k} \cdot \mathbf{x}} d\mathbf{x} \tag{8}$$

where $g(\mathbf{x}_2 - \mathbf{x}_1) = g(\mathbf{x}_2|\mathbf{x}_1)/n$ is the equilibrium pair-distribution function and $n = N/V$ is the uniform particle density number. Note that the function $g(\mathbf{x}_2 - \mathbf{x}_1) \rightarrow 1$ rapidly as $|\mathbf{x}_2 - \mathbf{x}_1|/a \rightarrow \infty$ (dilute limit).

In particular, we are interested in considering the simple case in which the distribution of point particles exists with uniform probability. Hence one assumes that the particles are randomly and independently positioned in the suspension domain, so that the equilibrium configuration is simply described by making $g(\mathbf{x}_2 - \mathbf{x}_1) = 1$ into (7) and (8). Then one obtains

$$n\langle (x,0)(x,0) \rangle = n\delta(x) \text{ and } \langle \hat{n}^*(\mathbf{k},0)\hat{n}^*(\mathbf{k},0) \rangle = N$$

With this assumption $G(\mathbf{x},0)=n$, and $F(\mathbf{k},0)$, that is now called the static structure factor of the suspension $S(\mathbf{k})$, takes the unit value.

Based on the above microstructure restriction, we wish to look at order of magnitude of the characteristic density number fluctuation n , occurring on a length scale l of the suspension in terms of an volume integral of the the density number variance $\langle n^2(\mathbf{x},0) \rangle$ (i.e. an volume average). Hence a typical fluctuation in number density may be estimated as a volume average,

$$n_l^2 \sim \frac{l}{l^3} \int_{|x|<l} \langle n^2(x,0) \rangle dx \quad (10)$$

Substituting the first equation from (9) into (10) and using Dirac's delta definition we obtain

$$n_l = O\left(\frac{\sqrt{N}}{l^3}\right) \quad (11)$$

If m is the particle mass, the statistical density fluctuation ρ_l in the region of size l is expected to be $\rho_l = O(mn_l)$. This result suggests fluctuations in the number of particles as being $O(\sqrt{N})$. Hence we could imagine that if a box of volume $O(l \times l \times l)$ containing N particles is divided into two equal parts by a vertical plane (see Fig. 1), one half of the box will contain $(\frac{N}{2} - \sqrt{N})$ particles, whereas the other half will contain $(\frac{N}{2} + \sqrt{N})$. This unbalance drives convection currents during the sedimentation process.

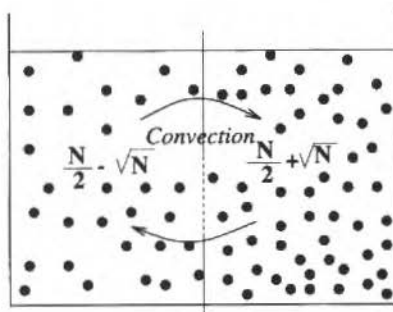


Fig. 1 Physical Mechanism of Fluctuations in Sedimentation

Velocity Fluctuations

We shall now consider two limiting cases for the velocity fluctuations: I) Inertialess or small box condition and II) Inertial or large box condition. The first limit is defined for length scale l sufficiently small (large wave numbers), which corresponds to be much less than the screening length, χ , but much greater than the particle radius, i.e. $a \ll l \ll \chi$. It will be show that this is the case when the fluctuations are going to depend on the size of the box. On the other, the inertial limit will occur for a length scale l , that is much greater than the screening length (small wave numbers), $l \gg \chi$. Under the latter condition scaling will show that the velocity fluctuation should be independent of the size of the box. The screening mechanism here is associated with particle inertia just as occurs in a dusty gas (Saffman, 1962 and Koch, 1990) instead of fluid inertia, so that the problem is still treated at low particle Reynolds numbers. Next is argued the steady state of the velocity fluctuations for both limiting cases above.

Inertialess case: $a \ll l \ll \chi$

Turning to expression (11), the statistical fluctuations in the density number of particles will lead to a fluctuation in the weight of $O(mg\sqrt{n_l^3})$. At the time scale it takes vorticity to diffuse over the length

$l, t = O(\rho_{eff} l^2 / \mu_{eff})$, the fluctuating buoyancy force is just balanced by the viscous drag associated with the driven flow u'_l due to the statistical density fluctuations. Hence $O(mg \sqrt{nl^3}) = O(u_{eff} l u'_l)$. Then, one obtains

$$u'_l = O\left(\frac{mg}{\mu_{eff}} n^{\frac{1}{2}} l^{\frac{1}{2}}\right) \tag{12}$$

Here ρ_{eff} is the effective density, ρ_{fluid} in a liquid-suspension and mn in a dusty gas, μ_{eff} is the effective viscosity, e.g. $\mu_{fluid} (1 + \frac{5}{2} \phi)$ in an infinitely dilute liquid-suspension (Einstein, 1956) and μ of the gas for a dusty gas suspension (Saffman, 1962).

Inertial case: $l \gg \chi$

The time the particles takes to accelerate across l is $t = (\frac{l}{\alpha'})$, where α' is the fluctuation in the acceleration $\alpha' \sim O(u'^2_l / l)$ caused by the fluctuating buoyancy force (convection currents). If the scaling is restrict for the case of large fluctuations, that is $u'_l \gg \langle u_l \rangle$, the fluctuating buoyancy force is only balanced by the inertial force like follows $O(\rho_{eff} \alpha' l^3) = O(mg \sqrt{nl^3})$. Substituting the scale of α' we find that

$$u'_l = O\left(\sqrt{\frac{mg}{\rho_{eff}} n^{\frac{1}{4}} l^{-\frac{1}{4}}}\right) \tag{13}$$

It should be noted that the scaling results show that in the inertialess limit the velocity disturbance increases with the linear scale l like $l^{1/2}$, whereas in the inertial length scale such a disturbance decreases with $l^{-1/4}$. This aspect is summarized in Fig. 2.

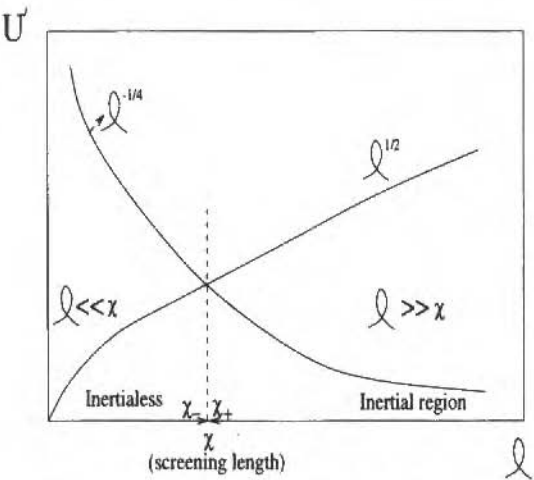


Fig. 2 Schematic Diagram of the Velocity Fluctuations for the Two Limiting Cases Investigated.

Screening Length

There are several ways of presenting the order of magnitude for the screening length χ . Figure 2 indicates that the maximum velocity fluctuation should occur where the viscous and inertial stress are comparable in the vicinity of the length l corresponding to the screening length χ . Note that, in the present context, the inertial stress (i.e. related to the velocity fluctuations) is associated with particle interactions rather than gas-phase turbulence. We shall therefore find the order of magnitude of χ for $(u'_i)_{\max}$, and see whether the Reynolds number based on the screening length and this velocity, Re_l , is actually $O(1)$. It should be important to have in mind that the restriction which is made to low particle Reynolds number does not necessarily imply that the bulk flow on the length scale l cannot have a Reynolds number $O(1)$. By noting the point of maximum velocity fluctuation in Fig. 2 we can write that, $O(u'_i(\chi_+)) = O(u'_i(\chi_-))$. Then using (12) and (13) we find that

$$\chi = O\left(\frac{\mu_{eff}^4}{(\rho_{eff} mg)^2 n}\right)^{\frac{1}{3}} \quad (14)$$

Substituting Eq. (14) into Eq. (12) we find that $u'_{\max} = O\left(\frac{m^{\frac{1}{2}} \epsilon^{\frac{2}{3}} n}{\rho_{eff} u'_{eff}}\right)^{\frac{1}{2}}$, which is independent of the linear size l . In other words it should be independent of the size of box L . Finally, note that $Re_l = \frac{\rho_{eff} \chi u'_{\max}}{\mu_{eff}} = O(1)$, such as we wanted to show.

Hydrodynamic Self-Diffusivity

The order of magnitude of the self-diffusivity may be estimated as the product of the variance of the particles mean velocity and the correlation time (i.e. the time over which the particles velocity correlation decays). We suppose that the particles velocity will remain correlated by a time $O(l/u'_i)$, that it takes a particle to fall through the interaction volume of linear dimension l . Hence $D_i = O\left(v_i^2 \frac{l}{u'_i}\right) = O(l u'_i)$. Now, calculating the order of magnitude of the diffusivity over the two limiting case discussed before, we find

$$D_i = O\left(\frac{mg}{\rho_{eff}} n^{\frac{1}{2}} l^{\frac{3}{2}}\right) \quad \text{for } a \ll l \ll \chi \quad \text{and} \quad D_i = O\left(\sqrt{\frac{mg}{\rho_{eff}}} n^{\frac{1}{4}} l^{\frac{3}{4}}\right) \quad \text{for } l \gg \chi \quad (15)$$

The scale for the self-diffusivity corresponding to the screening length from Eq. (14) is simply found to be $D_\chi = O\left(\frac{\mu_{eff}}{\rho_{eff} v_{eff}}\right) = O(v_{eff}^{-1})$. In the present context both momentum and particle mass are transported due to particle velocity fluctuations on a length scale l . This justifies the fact that the nonlocal diffusivity D_i and the nonlocal kinematic viscosity ν_{eff} will have the same order of magnitude.

A Dusty Gas Approach

An interesting way of presenting these general scalings is by considering the particular case of dusty gas suspension, where $\rho_{eff} = mn = \phi \rho_s$ and $\mu_{eff} = \mu$ (Saffman, 1962 and Koch, 1990). For common dusty materials the density ratio between particle-fluid, ρ_s / ρ , may be of the order of

several thousands. This corresponds to the situation where particle inertia is much more significant than fluid inertia. Moreover, qualitative difference in the motion of a dusty gas and suspension in a liquid may be expected. In the case of spherical particles, for example, Einstein's increased viscosity $\frac{2}{3}\mu\phi$ is negligible for a dusty gas, although it may be relevant for a liquid-suspension.

The important time scale for characterizing particles inertia is the relaxation time of the particle τ_p . It is a measure of the time for the particle to adjust to changes in the local fluid velocity, $\tau_p = O(\frac{m}{\mu}) = O(\frac{\rho_p a^2}{\mu})$. In nondimensional terms, this time defines the important parameter associated with particles inertia, the Stokes number

$$St = \frac{mU_s}{6\pi\mu a^2} = \frac{2}{9} \frac{aU_s\rho_s}{\mu} \quad (16)$$

The ratio between particle relaxation time and time vorticity takes to diffuse across a , $\tau = \rho_f a^2 / \mu$, gives $\frac{\tau_p}{\tau} = O(\frac{\rho_p}{\rho_f}) = O(\frac{Re}{R_e}) \gg 1$, for a dusty gas. Here $Re = \rho_f U_s a / \mu$ is the particle Reynolds number. Clearly, this condition suggests that in a dusty gas suspension the relevant inertial contribution due to the motion of particles and fluid comes from the particle inertia rather than fluid inertia. Thus in the limiting case $l \gg \chi$ the velocity fluctuations must be controlled by the inertia associated with the fluctuating motion of the particles. Physically the limit case $a \ll l \ll \chi$ corresponds to the situation of very small relaxation time (or Stokes number) of which $\tau_p = O(\frac{m}{\mu}) = O(\frac{\rho_p a^2}{\mu}) \ll O(\frac{a^2}{l}) \Rightarrow St \ll 1$. It follows that a test particle responds instantaneously to the viscous disturbances produced by the hydrodynamic interactions. This is the case in which particles following the fluid are equivalent to increasing the fluid density by an amount $\phi\Delta\rho$. On the other hand, for a moderate relaxation time $(\frac{a}{k_\infty}) \ll \tau_p \ll \frac{\chi}{l_{\text{int}}} \Rightarrow l \ll St \ll \phi^{-1}$, the test particle has not sufficient time to react in the same way to the disturbances caused by the neighbouring particles. It should cross a distance l with a smaller contribution from the disturbance for its motion. In the extreme case $St \gg \phi^{-1/4}$, the particle relaxation time is much larger than the time it takes a particle to translate through a fluid dynamic interaction. The fluid dynamic interaction for sufficiently high Stokes numbers of a gas-suspension is like the one that occurs in a fixed bed, so that a particle pressure plays an important role in the dynamics of particle volume fraction waves. The latter case will not be treated here (for details, see Koch, 1990).

The use of a dusty gas approach permits formulate the governing equations without involving any contribution from fluid inertia. So it is possible to treat the problem at low Reynolds numbers (particularly the fluid equation) with the Lagrangian equation of the particles motion. An application of this has been proposed by Cunha (1995) in which the dusty hypothesis justifies the assumption of ignoring fluid inertia.

Now the approximated forms taken by the previous general scaling are considered for the case where the dusty gas assumption is incorporated into the equations. The results can be expressed in terms of the particle concentration ϕ , making $n = O(\phi / a^3)$, the terminal velocity of the particle (i.e. the fall speed of a single sphere at low Reynolds number), $U_s = O(mg / \mu a)$, and for the limit case $l \gg \chi$, the effective density is made $\rho_{\text{eff}} = mn$, which gives $m / \rho_{\text{eff}} = O(a^3 / \phi)$. Applying these conditions into (12) and (13) we obtain,

$$u'_l = O\left(U_s \sqrt{\phi} \sqrt{\frac{l}{a}}\right) \quad \text{for } a \ll l \ll \chi \quad (17)$$

and

$$u'_l = O\left(\sqrt{ga} \frac{l}{\sqrt[4]{\phi}} \sqrt{\frac{a}{l}}\right) \quad \text{for } l \gg \chi \quad (18)$$

Again, it is seen that the velocity disturbance controlled by viscous effect increases with the linear size l , whereas that controlled by inertia decreases with l . A more convenient form to write (18) is determined if one notes that $\sqrt{ga} = O\left(U_s \frac{l}{\sqrt{St}}\right)$. Substituting this expression into (18) gives

$$u'_l = O\left(U_s \frac{l}{\sqrt{St}} \frac{l}{\sqrt[4]{\phi}} \sqrt{\frac{a}{l}}\right) \quad \text{for } l \gg \chi \quad (19)$$

The screening length can be estimated just as before. We therefore use (17) and (19) to rewrite (14) as follows

$$\chi = O\left(a \sqrt[3]{a^2 g^2} \frac{l}{\phi} \frac{l}{\sqrt[3]{U_s^4}}\right) = O\left(a St^{-\frac{2}{3}} \phi^{-1}\right) \quad (20)$$

The corresponding velocity fluctuation is found to be $u'_{max} = O(\sqrt[3]{agU_s}) = O(U_s St^{-\frac{1}{3}})$, which produces a finite value of the variance $\langle u'^2 \rangle = O(U_s^2 St^{-\frac{2}{3}})$. Note that if the particles are inertia free ($St \equiv 0$) the screening length is not defined (i.e. $\chi \rightarrow \infty$) for any particle concentration $\phi \neq 0$. This aspect holds that velocity fluctuations will always depend on the linear dimension $l, \forall l$. An obvious choice is to make $l = L$ (size of the box) and apply (17), what immediately recovers the result $u'^2_L = O(U_s^2 \phi L/a)$, predicted by Caflish and Luke (1985).

The new scaling expressions for the self-diffusivity D_l becomes

$$D_l = O\left(a U_s \sqrt{\left(\frac{l}{a}\right)^3 \sqrt{\phi}}\right) \quad \text{for } a \ll l \ll \chi \quad (21)$$

and

$$D_l = O\left(a U_s \frac{l}{\sqrt{St}} \frac{l}{\sqrt[4]{\phi}} \sqrt{\left(\frac{l}{a}\right)^3}\right) \quad \text{for } l \gg \chi \quad (22)$$

When the above expressions are evaluated at $l = \chi$, we find $D_\chi = O(\phi^{-1} St^{-1})$, which is independent of the box size.

As a final remark we shall say that the scaling analysis described in the present section has identified a fundamental mechanism of dispersion and mixing operating in a sedimenting suspension, and provided an important foundation upon which to base the current research.

A Simple Model for the Inertialess Case

We propose now a simple theory for evaluating the scaling coefficient for the limiting case $a \ll l \ll \chi$ (i.e. inertialess).

Let us consider a settling container H high, L wide and l deep, with $l < L/2$. Let the particle radius be a and the volume concentration ϕ . The suspension is random and the particles are positioned independently. We adapt the scalings to non-square container with no slip conditions, a situation close to real experiments.

Now the number of particles in the box is given by $N = \frac{l}{4\pi a} \vartheta l^1$, where $\vartheta = (lHL)$ is the volume of the settling box.

We have found above that typical fluctuations in the number of particles are just $\sqrt{N}$ statistical fluctuations. Hence the density fluctuations in one half of the box is found to be

$$\rho' = \sqrt{\frac{8}{3} \frac{\pi a^3 \phi}{\vartheta}} \Delta \rho \quad (23)$$

We suppose that the flow driven by this fluctuation in density is limited by the viscous flow in narrowest direction of l , say x . Hence, $\mu_{eff} \frac{d^2 u}{dx^2} = \rho' g$. Here we use the dusty gas hypothesis, with $\mu_{eff} = \mu$ (gas viscosity). The governing equation takes the form

$$u \frac{d^2 u}{dx^2} = \sqrt{\frac{8}{3} \frac{\pi a^3 \phi}{\vartheta}} \Delta \rho \quad (24)$$

Integrating (24) with no slip boundary conditions: $u'(0) = (0)$ and $u'(l) = (0)$ we obtain

$$u'(x) = \frac{l}{2\mu} \sqrt{\frac{8}{3} \frac{\pi a^3 \phi}{\vartheta}} \Delta \rho g x (l - x) \quad (25)$$

The variance is immediately calculated by noting that

$$\langle u'^2 \rangle = \frac{l}{\vartheta} \iiint_{\vartheta} u'(x) u'(x) dx dy dz \quad (26)$$

therefore

$$\langle u'^2 \rangle = \frac{9\pi}{20} \phi U_s^2 \frac{l^3}{aHL} \quad (27)$$

For typical experimental data (e.g. Ham and Homsy, 1988 and Nicolai et al., 1995), $H \approx \frac{25}{2} l$ and $L \approx \frac{5}{2} l$. Putting into (27) these data we find

$$\langle u'^2 \rangle = \frac{9\pi}{625} \phi U_s^2 \frac{l}{a} \quad \text{for } a \ll l \ll \chi \quad (28)$$

Note that the above estimation, even considering conditions closer to the practical experiments, depends on the size of the box, as found by the scaling arguments. We will compare this result with the results from numerical simulations by Cunha and Hinch (1996).

A Macroscopic Model for Suspension-Flow

Macroscopic models for suspension flows are still in early development. Such modeling is important, as it serves to unify the subject as well as extend our understanding of the phenomena for parameter values which can not be covered experimentally. We will assume that the particles are characterized by a size (radius) a . Most often they are treated as if they were spheres. The solid phase occupies a fraction of the total volume ϕ , and the fluid phase a fraction $1 - \phi$.

Both, fluid and particles, have velocities which are complicated function of space and time. It is impossible to attempt to describe these variables in exact detail, so it is adopted the continuum hypothesis, which assumes that the variables may be described by smooth fields which are defined as averages of velocities, etc. The fluid is Newtonian in its stress behaviour. Locally, the suspension is statistically homogeneous, in the sense that it is possible to find a length l which is larger compared with the average particle spacing and over which the statistical properties of the suspension, defined as ensemble averages, do not vary appreciably. The velocity fields are defined at a point, which has infinitesimal volume. Clearly this cannot be correct if one is interested in describing phenomena on the length scale of the particle or inter-particle distance, but it is a useful hypothesis in dealing with the averaged motion and transport between phases.

When the governing equation of particle is examined one notice, that there is no explicit reference to the fluid, although there is to the bulk material. The particles evolve according to Newton's laws of motion, just as molecules. Here the fluid determines the nature of interactions between particles, and once these interactions are known, no explicit reference to the fluid is necessary. From a statistical mechanics perspective there are no degrees of freedom associated with the fluid. This is not true, of course, at finite Reynolds numbers where the degrees of freedom of the fluid enter explicitly. It appears, therefore, that a macroscopic particle description as a homogenized continuum should be possible, just as in a molecular system.

Basic Equations

At the microstructural level is assumed that the fluid is Newtonian and the the suspended particles may be described by Continuum Mechanics. In the usual notation let the density be ρ , the Eulerian velocity $\mathbf{u}$, the Cauchy stress $\boldsymbol{\sigma}$ and the body force per unit volume or non-hydrodynamic $\mathbf{f}$. At the interface between the two phases the velocities should be continuous and the surface stress $\boldsymbol{\sigma} \cdot \mathbf{n}$ discontinuous only to accomodate any surface tension. The laws in the two phases may be conveniently combined into one as (Hinch, 1975)

$$\sigma_{ij}(\mathbf{x}) = -p(\mathbf{x})\delta_{ij} + 2\mu e_{ij}(\mathbf{x}) + \mathfrak{I}\sigma_{ij}^s(\mathbf{x}, C) \quad (29)$$

where p is the pressure field, $\mathbf{e} = \frac{1}{2}(\nabla \mathbf{u} + \nabla^T \mathbf{u})$ is the Eulerian strain rate and $\mathfrak{I}\sigma^s$ is a generalized function, with the phase indicator $\mathfrak{I} = 0$ if $\mathbf{x}$ is a point in the fluid and $\mathfrak{I} = 1$ if $\mathbf{x}$ is in the particle. C

denotes the full particle configurations. One will deal with the situation of point particles where there is no internal angular momentum such that the conservation of angular momentum requires the stress tensor to be symmetric, i.e. $\sigma_{ij} = \sigma_{ji}$.

Assuming that classical continuum mechanics holds at the microscopic level, the microscopic mass and momentum equations

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \mathbf{u}) = 0 \quad \text{and} \quad \rho \frac{D\mathbf{u}}{Dt} = \mathbf{f} + \nabla \cdot \boldsymbol{\sigma} \quad (30)$$

applied to any material point (in the particle or fluid) can be averaged to yield the macroscopic equations. Using the phase indicator function $\mathfrak{I}$ defined above it is possible to average (30) over the particles (Drew, 1983; Drew and Lahery, 1993). This is closely the continuum mechanics version of the Irving-Kirkwood (1950) procedure for point particles. The macroscopic conservation equations for particle mass and momentum are simply

$$\frac{\partial \phi}{\partial t} + \nabla \cdot (\phi \langle \mathbf{u} \rangle_s) = 0 \quad \text{and} \quad \rho_s \phi \frac{D_s \langle \mathbf{u} \rangle_s}{Dt} = \langle \mathbf{f} \rangle_s + \langle \mathbf{F} \rangle_s + \nabla \cdot \langle \boldsymbol{\varepsilon} \rangle_s \quad (31)$$

assuming that the density of particle, ρ_s , is constant for each particle. The material derivative $\frac{D_s}{Dt} \equiv \frac{\partial}{\partial t} + \langle \mathbf{u} \rangle_s \cdot \nabla$ is that following the average particle motion. Here $\langle \mathbf{F} \rangle_s$ is the average hydrodynamic force per unit volume exerted on the particles, $\langle \boldsymbol{\varepsilon} \rangle_s$ is the average particle stress tensor. This macroscopic particle stress can be antisymmetric owing to both the external couple and to non-central interparticle forces.

A Constitutive Equation for the Average Stress of the Particles

The present section is concerned with the investigation in which a transition is made from the individual particles to the bulk material. The relevant particle behaviour must be identified. The particles can rotate, deform or interact with one another. To begin it is assumed that the macroscopic observed stress tensor, i.e. the effective stress in the suspension, is the ensemble average of the stress distribution in all realizations of the suspension. As mentioned before, for homogeneous suspensions, this ensemble average is equivalent to a volume average over a volume which is larger enough to contain a statistically significant number of particles, but smaller than the scale of variations of interest in the macroscopic system. The formal expression for the effective stress of a suspension can be then written as proposed by Batchelor (1967) and Landau and Lifshitz (1959), in the form $\langle \varepsilon_{ij} \rangle = \frac{1}{\vartheta} \int_{\vartheta} \varepsilon_{ij} d\vartheta$.

Now, the volume integral may be decomposed into the portion over the fluid domain in which we may use the constitutive equation for the fluid stress and the portions over each rigid particle, so that $\vartheta = \vartheta_f + \sum_{\alpha=1}^N \vartheta_\alpha$. Here ϑ_α is the volume of one particle and N is the number of particles in ϑ . Hence

$$\begin{aligned} \langle \varepsilon_{ij} \rangle = & \frac{1}{\vartheta} \int_{\vartheta_f} \left[-p \delta_{ij} + \mu \left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} \right) - \rho u'_i u'_j \right] d\vartheta + \\ & + \frac{1}{\vartheta} \sum_{\alpha=1}^N \int_{\vartheta_\alpha} (\sigma_{ij} - \rho u'_i u'_j) d\vartheta \end{aligned} \quad (32)$$

The volume average velocity gradient for the entire material or suspension as a whole can be expressed as

$$\left\langle \frac{\partial u_i}{\partial x_j} \right\rangle = \frac{1}{\vartheta} \int_{\vartheta_f} \frac{\partial u_i}{\partial x_j} d\vartheta = \frac{1}{\vartheta} \int_{\vartheta_f} \frac{\partial u_i}{\partial x_j} d\vartheta + \frac{1}{\vartheta} \sum_{\alpha=1}^N \int_{\vartheta_\alpha} \frac{\partial u_i}{\partial x_j} d\vartheta \quad (33)$$

Applying the divergence theorem for the last term on the right hand side we deduce that

$$\frac{1}{\vartheta} \int_{\vartheta_f} \frac{\partial u_i}{\partial x_j} d\vartheta = \left\langle \frac{\partial u_i}{\partial x_j} \right\rangle - \frac{1}{\vartheta} \sum_{\alpha=1}^N \oint_{S_\alpha} u_i n_j dS \quad (34)$$

By the same procedure one obtains

$$\frac{1}{\vartheta} \int_{\vartheta_f} \frac{\partial u_i}{\partial x_j} d\vartheta = \left\langle \frac{\partial u_i}{\partial x_i} \right\rangle - \frac{1}{\vartheta} \sum_{\alpha=1}^N \oint_{S_\alpha} u_i n_i dS \quad (35)$$

The stresses in the particle are indeterminate, but we may use the following relation,

$$\begin{aligned} \int_{\vartheta_\alpha} \sigma_{ij} d\vartheta &= \int_{\vartheta_\alpha} \frac{\partial}{\partial x_k} (\sigma_{ik} x_j) d\vartheta - \int_{\vartheta_\alpha} \frac{\partial \sigma_{ik}}{\partial x_k} x_j d\vartheta \\ &= \int_{S_\alpha} \sigma_{ik} x_j n_k dS - \int_{\vartheta_\alpha} \frac{\partial \sigma_{ik}}{\partial x_k} x_j d\vartheta \end{aligned} \quad (36)$$

to rewrite the stresses in terms of the surface tractions plus the first moment of $\nabla \cdot \sigma$. In Equation (36), $\frac{\partial \sigma_{ik}}{\partial x_k}$, can be expressed from Cauchy's equation as $\frac{\partial \sigma_{ik}}{\partial x_k} = \rho a_i - f_i$, where $a_i = \frac{Du_i}{Dt}$ is the acceleration of the particle. Combining the results (34), (35) and (36) together, we find that

$$\langle \varepsilon \rangle = -\langle p \rangle_f I + 2\mu \langle e \rangle - \rho_f \langle u' u' \rangle_f + \langle \varepsilon \rangle_s \quad (37)$$

The first term in (37) is a purely isotropic contribution of no particular interest, and $\langle p \rangle_f$ is the average pressure in the fluid; the second term is the deviatoric stress that would exist even in the absence of the particles. The third term represents the inertial contribution associated with the turbulent fluctuations of the fluid motion (i.e. Reynolds stress) and the fourth term represents the contribution of the bulk stress due to the presence of the particles. From these calculations we conclude that the average particle stress is expressed by:

fluctuations, like $p_s = \frac{1}{3} \rho_s f(\phi) \langle \mathbf{u}' \cdot \mathbf{u}' \rangle$. Attention should be paid to the fact that the transport mechanism here has a different physical origin, compared with the Reynolds stress in fluid turbulence. Buyevich (1972) has attempted to examine the hydrodynamic dispersion in a suspension using an analogy with turbulent flow. Numerical simulations of shearing and sedimenting suspensions (Ladd, 1993; Cunha and Hinch, 1996 a, b; Cunha, 1993, 1995; and Cunha and Hinch, 1995 a, b, c) and experimental evidence about fluctuations and hydrodynamic dispersion in suspensions (Leighton and Acrivos, 1987; Nicolai et al., 1995; and Ham and Homsy, 1988) have given support for the understanding of fluctuations in suspensions at low Reynolds numbers.

It is instructive in this stage to give a physical interpretation of the stresslet $\langle S \rangle_s$ because it has no counterpart in molecular system. It is considered the simple case of computing the bulk stress tensor in a homogeneous infinitely dilute suspension of rigid spherical particles. This case corresponds to study the dynamics of an isolated particle, since for $\phi = \frac{1}{V} \sum \frac{4}{3} \pi a^3 \rightarrow 0$, all interparticle hydrodynamic interactions are neglected. For the condition of non-colloidal particles, which are torque and force free, undergoing shear at low Reynolds number (based on the particle size), the bulk stress tensor of the suspension becomes $\langle \epsilon \rangle = -\langle p \rangle_f \mathbf{I} + 2\mu \langle \epsilon \rangle + \langle S \rangle$. Now, the third Faxén law (Batchelor and Green, 1972), gives the expression for the stresslet $\langle S \rangle_\alpha$ exerted by an isolated sphere on the fluid $\langle S \rangle_\alpha = \frac{20}{3} \pi \mu a^3 \epsilon$. Then

$$\langle S_s \rangle = \frac{1}{\vartheta} \sum_{\alpha=1}^N S_\alpha = \frac{20}{3} \pi \mu a^3 n \langle \epsilon \rangle = 5 \mu \phi \langle \epsilon \rangle \quad (41)$$

where n is the particle density number $\left(= \sum_{\alpha=1}^N \frac{4\pi a^3}{3} \right)$. We find therefore that

$$\langle \epsilon \rangle = -\langle p \rangle_f \mathbf{I} + 2\mu \left[\mathbf{I} + \frac{5}{2} \phi + O(\phi^2) \right] \langle \epsilon \rangle \quad (42)$$

The above result shows that the resistance of rigid particles to straining motions leads to an increased rate of viscous dissipation which, for the equivalent homogeneous material, may be characterized as an increase in the bulk viscosity, first found by Einstein (1956). In view of this result, we argue that the general representation of the constitutive equation for $\langle \epsilon \rangle_s$ may be better represented on the form

$$\langle \epsilon \rangle_s = \frac{1}{2} \epsilon : \langle L \rangle_s - \rho_s \phi \frac{1}{2} \langle \mathbf{r} \mathbf{a} + \mathbf{a} \mathbf{r} \rangle - \langle x_\alpha f_\alpha \rangle_s + 2\mu \mu_s(\phi) \langle \epsilon \rangle - \rho_s \phi \langle \mathbf{u}' \mathbf{u}' \rangle_s \quad (43)$$

where $\mu_s(\phi)$ is the relative viscosity of the particle phase. Note that in the dilute suspension limit $\mu_s(\phi) = \frac{5}{2} \phi$.

The Average Hydrodynamic Force

The mass and momentum balances are valid for any material and for motion at any Reynolds number. What distinguishes one system from another is the form of the constitutive relations (for

hydrodynamic drag and stress, for example). For our applications to low-Reynolds number flows the average hydrodynamic force is simply given by

$$\langle \mathbf{F} \rangle_s = -\phi \nabla \langle p \rangle_f + 6\pi\mu a n \zeta^{-1}(\phi) (\langle \mathbf{u} \rangle_s - \langle \mathbf{u} \rangle) \tag{44}$$

The first term on the right is the Archimedean force and the last is the average viscous drag.

Fluctuations in Sedimentation at Moderate Stokes Numbers

In this section the models described above are adapted to describe the steady state of velocity fluctuations occurring in the large box limit, predicted by the scaling arguments. It is studied the case of fluctuations much larger than the mean motion, so that if U_s is a typical average velocity of the particles occurring on a length scale l of the suspension and U' is a typical fluctuation about the average on the same length scale, then $\frac{D\langle \mathbf{u} \rangle_s}{Dt} = O(U_s^2/l) \ll \langle \mathbf{u}' \mathbf{u}' \rangle = O(U'^2/l)$.

For typical particles of radius less than about $100\mu\text{m}$ in air, at standard atmospheric conditions, the particle Reynolds number $Re = \rho_f U_s a / \mu$ is of order one or smaller, indicating that inertial effect in the gas is small. On the other hand, the particles Stokes number defined as $St = \frac{\rho_f U_s l}{6\pi\mu a \tau}$ is greater than one for particles larger than a few micra, indicating that the inertia of the particle is significant. In view of this situation and the relative theoretical tractability of low Reynolds number suspensions, it is natural to seek a theory valid in the asymptotic limit of $Re \ll 1$ and moderate St . Note that this condition is physically consistent with the dusty gas hypothesis that we have assumed in our scaling arguments. In addition, we consider the limit case where the Péclet number is high which corresponds to a suspension of non-Brownian particles, $Pe = \frac{\rho_f U_s l}{\eta} = \frac{\rho_f \pi l \omega^2 l^3}{kT}$. In the present context Péclet number expresses the ratio of particle advection by the imposed flow (sedimentation) to Brownian diffusion. D is the ordinary diffusivity of a dilute dispersion of independent spheres of radius a as first derived by Einstein (1956).

Although the suspension here will be considered dilute, the effect of the viscous hydrodynamic interactions between the particles will play an essential role. Because of the slow decay (like $1/r$) of the velocity disturbance in sedimentation, the most fluid-dynamic interactions will be long range (between particles separated by a distance large compared with the radii a). As a result, a point particle approximation will be assumed for the description of the fluctuations.

As in the scaling arguments, the model will be based on considering that density fluctuations drive convection currents in sedimentation. We use an extension of Boussinesq's approximation, stating that variations of the particle phase density $\rho_s \phi$ are ignored, except insofar as they give rise to a gravitational force $\langle \mathbf{f} \rangle_s = \phi_a \rho_s \mathbf{g} + \langle \Delta p \rangle \mathbf{g}$. Thus the continuity equations may be used in its constant concentration form $\nabla \cdot \langle \mathbf{u} \rangle_s = 0$.

Since our interest is in sedimentation of non-colloidal point-particles at low Reynolds number in a dilute monodisperse suspension, we shall discard the term of acceleration $\frac{1}{\tau} \rho_s \phi (r\mathbf{a} + a\mathbf{r})_s$, and the term associated with non-colloidal forces $\langle \mathbf{x}_\alpha \mathbf{f}_\alpha \rangle_s$. Furthermore in the present application the average velocity of the suspension $\langle \mathbf{u} \rangle = \phi \langle \mathbf{u}_s \rangle + (1-\phi) \langle \mathbf{u}_f \rangle = \mathbf{0}$ (which imposes that the fluid must move upwards in order to compensate for descending particles), and the deviatoric stress tensor (defined in

$$\langle \varepsilon \rangle_s = \frac{1}{\vartheta} \sum_{\alpha=1}^N D_{\alpha} - \frac{1}{\vartheta} \sum_{\alpha=1}^N \oint_{S_{\alpha}} \mu(nu + un) dS - \frac{1}{\vartheta} \sum_{\alpha=1}^N \int_{\vartheta_{\alpha}} \rho r a d\vartheta - \langle x_{\alpha} f_{\alpha} \rangle_s - \rho_s \phi \langle u' u' \rangle_s \quad (38)$$

where $r = x - x_{\alpha}$, with x_{α} the location of the centre of particle α . It was assumed that the particle force f_{α} is constant within the particle and $\langle f \rangle_s = \frac{1}{V} \sum_{i=1}^N f_{(i)}$ is the body force per unit volume exerted on the particles. The sum is made over all N particles in an averaging volume.

The dipole $D_{\alpha} = \int_{S_{\alpha}} (\alpha n) r dS$ can be grouped as a combination of an antisymmetric part connected with the hydrodynamic torque; and a symmetric dipole field intimately identified with straining motions, i.e. $D = S_{\alpha}^* + T_{\alpha}^*$. Hence one finds

$$S_{\alpha}^* = \frac{1}{2} \oint_{S_{\alpha}} [(\sigma \cdot n)r + r(\sigma \cdot n)] dS \quad \text{and} \quad T_{\alpha}^* = \frac{1}{2} \oint_{S_{\alpha}} [(\sigma \cdot n)r - r(\sigma \cdot n)] dS \quad (39)$$

These symmetric and antisymmetric portions are called, respectively, the stresslet and the rotlet of the particle α (see Batchelor, 1970). The term $\int_{\vartheta_{\alpha}} \rho r a d\vartheta$ can be also written in terms of a symmetric and antisymmetric part in the following manner:

$$\int_{\vartheta_{\alpha}} \rho r a d\vartheta = \frac{1}{2} \int_{\vartheta_{\alpha}} \rho (ra + ar) d\vartheta + \frac{1}{2} \int_{\vartheta_{\alpha}} \rho (ra - ar) d\vartheta$$

Thus the antisymmetric part T_{α}^* along with the antisymmetric part of the acceleration term, is related to the total external torque exerted L_{α} on a particle α . Hence $T_{\alpha} = T_{\alpha}^* + \frac{1}{2} \int_{\vartheta_{\alpha}} \rho (ra - ar) d\vartheta = \frac{1}{2} \varepsilon : L_{\alpha}$ or $T_{jk}^{\alpha} = \frac{1}{2} \varepsilon_{jki} L_i^{\alpha}$. Then the equation for the average particle stress becomes

$$\langle \varepsilon \rangle_s = \langle S \rangle_s + \frac{1}{2} \varepsilon : \langle L \rangle_s - \rho_s \phi \frac{1}{2} \langle ra + ar \rangle_s - \langle x_{\alpha} f_{\alpha} \rangle_s - \rho_s \phi \langle u' u' \rangle_s \quad (40)$$

where $\langle S \rangle_s = \frac{1}{\vartheta} \sum_{\alpha=1}^N S_{\alpha}$ and $\langle L \rangle_s = \frac{1}{\vartheta} \sum_{\alpha=1}^N L_{\alpha}$. Note that $S_{\alpha} = S_{\alpha}^* - \int_{S_{\alpha}} \mu (nu + un) dS$, and for rigid particles the velocity at particle surface is a rigid-body motion. Consequently, the velocity terms in the integral vanish identically (i.e. $S_{\alpha} = S_{\alpha}^*$). Equation (40) is similar to the equations for a molecular system (see McQuarrie pp. 411-412, for example). Both the interparticle force $\langle x_{\alpha} f_{\alpha} \rangle_s$ (which is an average stress that arises in molecular or colloidal systems) and the fluctuation terms are present for molecular system. The acceleration and torque contributions would also be present for finite-size molecules. The only new term is the hydrodynamic stresslet $\langle S \rangle_s$.

The inertial stress $\rho_s \phi \langle u' u' \rangle_s$ for the particles, in the present context, is associated with the transport of particles momentum disturbances resulting from a randomly and fluctuating velocity, caused by all the neighborhood via the viscous hydrodynamic interactions. It describes the spread of momentum by fluctuation-fluctuation interactions associated with particle inertia, which can be described by an effective or non-local viscosity. This mechanism of fluctuation is different from the random motion of particles due to touching collisions that are characteristic of a gas-fluidized bed. The random motion in a gas-fluidized could be described by a "particle temperature". As the particles collide they exchange momentum. This effect may be described by a "particle pressure", determined locally by continual

both fluid and solid parts of the suspension) is an stationary random function with constant mean and $\nabla\langle e \rangle = 0$ because the suspension is assumed to be statistically homogeneous. Taking into account the above conditions into (43) and (44) and using (30), the equation which expresses the balance of force, per unit volume, in the flow driven by density fluctuations is deduced to be

$$-\rho_s \phi_o \nabla \cdot \langle u' u' \rangle_s + \langle \Delta p \rangle g + 6 \pi \mu a n U_s - \nabla \langle P \rangle = 0 \tag{45}$$

For convenience, the effect of the body force $\phi_o \rho_s g$ was incorporated into the modified pressure P defined as $P = \phi_o \left(\langle p \rangle_f - \rho_s g \cdot x \right)$.

A self-consistent analysis (Koch, 1992) for a solid-gas suspension at moderate Stokes number ($1 \ll St \ll \frac{1}{\phi}^{-\frac{1}{4}}$) predicted that the average effect of the particle stress associated with the fluctuations can be described by using a non-local viscosity, which depends on the wave number of the disturbance. That theory seems to be equivalent to use velocity fluctuations outside a test particle in the equivalent medium with this eddy viscosity. In particular, we suppose that $\rho_s \phi_o \langle u' u' \rangle_s$ is approximately a linear function of various components of the mean motion gradient driven by density fluctuations. For sufficiently small magnitudes of these components the hypothesis may be expressed as $\rho_s \phi_o \langle u'_i u'_j \rangle_s = -\eta_{ijkl} \frac{\partial \langle u_k \rangle_s}{\partial x_l}$. Here η_{ijkl} is fourth-order. The anisotropy of this tensor is associated with the gravity direction. In order to simplify the present analysis we shall neglect the anisotropy in the velocity fluctuations and consider the simpler form, when the dynamic of hydrodynamic is isotropic. Thus the governing equations take the form

$$\langle \Delta p \rangle g + \eta \nabla^2 \langle u \rangle_s - \nabla \langle P \rangle + 6 \pi \mu a n U_s = 0 \quad \text{and} \quad \nabla \cdot \langle u \rangle_s = 0 \tag{46}$$

The system studied here should be dimensionless. Now, we shall look for the best way of doing it. We consider that a , U_s and $6 \pi \mu a U_s$ represent respectively the characteristic references scales for length, velocity and force. Introducing dimensionless quantities into (46) ones obtains

$$\frac{n'}{n_o} \frac{g}{g} + \Lambda \frac{1}{6 \pi n_o} \nabla^{*2} u^* - \nabla^* p^* = 0 \quad \text{and} \quad \nabla \cdot u^* = 0 \tag{47}$$

where $u^* = \frac{u}{U_s}$, $p^* = \frac{p}{6 \pi \mu a^2 n_o U_s} - \frac{g \cdot x^*}{\mu}$, $\Lambda = \frac{\eta}{\mu}$, n' is the density number fluctuation (which for a dusty gas $\Delta p = m n'$), and $\frac{\partial \langle u_k \rangle_s}{\partial x_l} = \frac{g_o}{4 \pi}$. In order to simplify the notation we have dropped all symbols denoting averaging.

We will investigate the fluctuations on the wave number of a container $l \times L \times H$ by imposing periodic boundary conditions. The solution of (47) for the periodic fluctuation field may be written in the form (see Cunha, 1995, for details)

$$u(x) = \sum_{k\beta} \int \frac{\left(1 - \frac{k\beta}{k^2}\right) \cdot \hat{n}(k) g \delta(k - k_\beta) e^{-2\pi i k \cdot x}}{\frac{2}{3} g \Lambda(k) k^2 \vartheta} dk \tag{48}$$

where $\sum'$ indicates a summation over all reciprocal lattice vector $\mathbf{k}_\beta$, except where $|\mathbf{k}_\beta|=0$. The vector $\mathbf{k}_{||}$ is the continuous wave vector (in the Fourier space) and $\mathbf{k}_\beta$ is the inverse lattice vector which only takes on values commensurate with the inverse lattice. Thus the vector $\mathbf{k}_\beta$ represents a wave number which can be defined as $\mathbf{k}_{||} = \left(\frac{\beta_1}{l}, \frac{\beta_2}{l}, \frac{\beta_3}{l} \right)$, with $\beta_1, \beta_2, \beta_3$ integers positive or negative ($0, \pm 1, \pm 2, \dots$) and ϑ the volume of the periodic box. Again δ denotes Dirac's "delta function". Note that we have drop the symbol (*) of the dimensionless quantities in order to simplify the notation.

From Equation (48) and using the properties of the δ function we can write that

$$u(x) = \frac{1}{\vartheta} \sum'_{\mathbf{k}_\beta} \frac{\left(1 - \frac{\mathbf{k}_\beta \mathbf{k}_\beta}{k_\beta^2} \right) \cdot g \hat{n}(\mathbf{k}_\beta) e^{-2\pi \mathbf{k}_\beta \cdot \mathbf{x}}}{\frac{2}{3} g \Lambda(\mathbf{k}_\beta) k_\beta^2} \quad (49)$$

Here, $\Lambda(\mathbf{k}_\beta)$ denotes the Fourier transform of the dimensionless nonlocal viscosity associated with the wave number of the periodic box. Since our interest is in wavelengths much larger than the screening distance χ (i.e. the inertial range of the scaling arguments) the ratio $\Lambda(\mathbf{k}_\beta)$ is found to be a function of the box wave number, Stokes number and a function of the concentration of the particles $\Lambda(\mathbf{k}_\beta) \approx 2.6145 (2\pi \mathbf{k}_\beta \chi)^{-\frac{1}{2}}$ for $k_\beta a St^{-\frac{1}{2}} \phi_o^{-1}$ (Koch, 1992).

With the hypothesis of isotropic fluctuations (i.e. $gg = gI$) we examine the behaviour of the velocities fluctuations parallel ($u_{||}'$) and perpendicular ($u_{\perp}'$) to the gravity direction in a similar way. Using the variance as the simple... measure of the velocity fluctuations and the previous result in which for a suspension whose particles were randomly distributed with uniform probability $\langle \hat{n}(\mathbf{k}_\beta) \hat{n}^*(\mathbf{k}_\beta) \rangle = N$ ones obtains from (49) that

$$\langle u_{||}'^2 \rangle = \frac{9}{4} U_s^2 \frac{N}{\vartheta^2} \sum'_{\mathbf{k}_\beta} \left(1 - \frac{k_{\beta||}^2}{k_\beta^2} \right)^2 \frac{1}{\Lambda^2(\mathbf{k}_\beta) k_\beta^4} \quad (50)$$

and

$$\langle u_{\perp}'^2 \rangle = \frac{9}{4} U_s^2 \frac{N}{\vartheta^2} \sum'_{\mathbf{k}_\beta} \left(\frac{k_{\beta\perp} k_{\beta\perp}}{k_\beta^2} \right)^2 \frac{1}{\Lambda^2(\mathbf{k}_\beta) k_\beta^4} \quad (51)$$

where the notation $\hat{n}^*(\mathbf{k}_\beta)$ denotes the complex conjugate ($= \hat{n}(\mathbf{k}_\beta)$). The lattice sums (50) and (51) must be calculated numerically.

Results and Discussion

A theory based on the mechanism that horizontal density fluctuations leads to vertical velocity fluctuations in sedimentation, has been presented, to provide a statistical description of the fluctuating motion of slightly heavy spheres as they sediment through a monodisperse dilute suspension, under creeping flow conditions. To illustrate the behaviour and theory predictions some results are now presented. Whenever possible, they are compared with recent numerical simulations and also with the limited experimental data available.

In order to examine the prediction of Eq. (27), that describes the behaviour of velocity fluctuations for the limiting case $a \ll l \ll \chi$, the quantity $\langle U'^2 \rangle_{||}$ (i.e. the dimensionless vertical variance) was

evaluated and then plotted versus the box parameter $\phi \frac{L}{a}$. Figure 3 shows a comparison of the theory prediction with recent numerical simulations carried out by Cunha and Hinch (1996 b). The simulation results correspond to the vertical variance of sedimenting particles in a impenetrable box with side periodicity of dimensions $L \times L \times H$, and aspect ratio $\frac{H}{L} = 3$. All simulations were performed with the particles located randomly and independently in the impenetrable box, just as assumed by the present theory. The comparison shows that Eq. (27) predicts well the behaviour of the vertical variance, although giving results slightly larger than the simulations. It is seen that the variance is a monotonically increasing function of the scaling $\phi \frac{L}{a}$ when particle inertia is assumed very small. The discrepancy between the theory and simulation is attributed to the small deviation (i.e. statistical fluctuations) from unity of the suspension structure factor in the computer simulations, while it was assumed exactly equal to unity by our simple theory. Figure 3 shows that the values of the relative vertical fluctuations, $\sqrt{\langle u'^2 \rangle} / U_s$, are appreciably large, $O(1)$, and are thus in qualitative agreement with the experiments carried out by Ham and Homay (1988) (where the fluctuations were ranging between 25% to 46% of the mean for a dilute suspension), and with those measurements of fluctuations recently reported by Nicolai et al. (1995), who found a relative fluctuation of 77% of the mean for $\phi = 5\%$. On the other hand, this order of magnitude is substantially less than the theoretical prediction, $\sqrt{\langle u'^2 \rangle} = 2.2 U_s$, found by Koch and Shaqfeh (1991) for particle and fluid free of inertia.

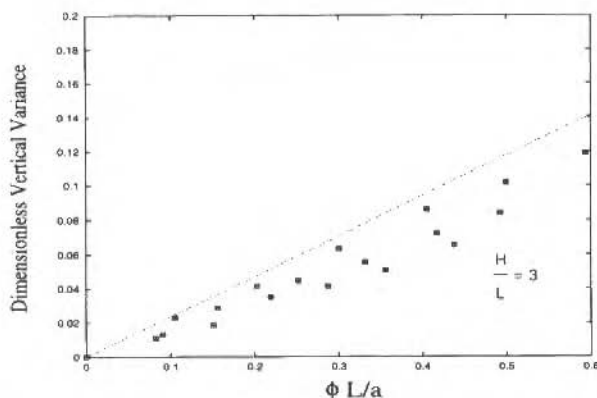


Fig. 3 Dimensionless Vertical Variance as a Function of the Box Parameter. Numerical simulation of Cunha and Hinch (1996 b); dashed line theoretical prediction of the present work

The most striking aspect of the result shown in Fig. 3 is related with the variance in the settling velocity. When particle inertia is negligible, it depends upon the size of the box with the variance increasing linearly when the size of the box is increased. In view of this, Cunha and Hinch (1996 b) also performed numerical simulations of sedimenting particles in a finite box with periodic sides to investigate if an initial random independently distribution of particles could evolve in time to a suspension structure that produces fluctuations, independent of box size. In particular, they wondered whether the convection currents which dominates the sedimentation process on the length scale of the settling box, would be able to produce horizontal uniform density so that fluctuations in velocity would decay in time. Surprisingly, they discovered that the velocity variance will always be limited by

the box size, because the probability density in the bulk of the suspension changes just slightly from the random and independent initial configuration for a monodisperse suspension of rigid particles.

Both the present theory and scaling arguments have shown velocity fluctuations increasing in a predictable way that agrees with Caflish and Luke's (1985) theory and the numerical simulations of Cunha and Hinch (1996 b), Ladd (1993) and Koch (1994), while the recent experiments by Nicolai and Guazzelli (1995) have shown that the fluctuations are independent of the settling box size. Eventually, for the understanding of this embarrassing feature, some effort should be made to investigate the difference between placing the particles at random and independently in the theory and in the procedure of stirring the suspension in laboratory.

Now it is shown a quantitative evaluation of the present theory that predicts velocity fluctuations for the limiting case $l \gg \chi$ (i.e. inertia range at moderate Stokes number). To this end the lattice sums (50) and (51) were evaluated numerically. It was considered the simple case in which the ratio of viscosity, Λ , is isotropic. Again the density number fluctuations $\langle \hat{n}(\mathbf{k}_\beta) \hat{n}^*(\mathbf{k}_\beta) \rangle$ were supposed to be constant, with its value equal to the number of particles N that corresponds to a random suspension with uniform probability. The numerical calculation of the lattice sums were then performed using a number of boxes for the periodic lattice system corresponding to 125. This number of boxes was sufficient for producing a rapid convergence of the series, with error less than 1% in comparison with test results using larger number of boxes. The scaling $St^{-1/2} \phi^{-1/4} (L/a)^{-1/4}$ was evaluated for various combinations of the Stokes numbers, concentration of particles and the linear size $\frac{L}{a}$, keeping the aspect ratio of the box $H/L=3$.

Figure 4 shows the steady state of the vertical variance as a function of the scaling $St^{-1/2} \phi^{-1/4} (L/a)^{-1/4}$. The theory indicates that the vertical variance is going to be independent of the

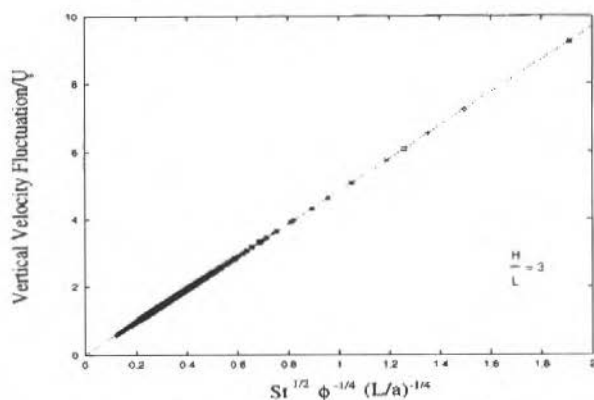


Fig. 4 Dimensionless Vertical Velocity Fluctuation vs Scaling Factor. $\diamond$ St = 10 and ϕ = 2%; + St = 10 and ϕ = 3%;

St = 10 and ϕ = 5%; $\times$ St = 20 and ϕ = 3%; Δ St = 5 and ϕ = 3%. Dashed line $4.84 St^{-1/2} \phi^{-1/4} (L/a)^{-1/4}$.

box size and it scales like $\sqrt{\langle \hat{u}^2 \rangle} \sim 4.84 St^{-1/2} \phi^{-1/4} (L/a)^{-1/4}$. The results are qualitatively similar with Cunha's simulations (1995). The theory has also predicted that particle inertia can diminish velocity fluctuations in sedimentation even with no change in the suspension structure. In view of this, the effect of particle inertia can be seen as a screening mechanism, which yields finite values of the variance in a random sedimenting suspension as the size of the container becomes large, $L \rightarrow \infty$. In

addition, Fig. 5 shows the ratio of velocity fluctuations, $\sqrt{\langle u'^2 \rangle_{\parallel}} / \sqrt{\langle u'^2 \rangle_{\perp}}$ function of the scaling $St^{-1/2} \phi^{-1/4} (L/a)^{-1/4}$. According to this result the ratio of fluctuations is insensitive to the variations of the scaling; it is roughly constant and equals to 3.72. This behaviour is in qualitative agreement with numerical simulation results for limited box size at moderate Stokes number of Cunha (1995), who found values for the ratio of fluctuations approximately equal to 2.5. Clearly, more work shall be done in this part in order to examine more precisely the ratio of viscosity Λ , for anisotropic situation. Experiments to measure the effective viscosity are needed to fully understand and explain fluctuation behaviour when particle inertia is important. We are unaware of experimental results available for a suitable comparison with our predictions.

Hopefully, the work presented in this article will provide a useful tool for understanding the mechanism of fluctuations in sedimentation. The predictions of the simplified models should help to design an efficient numerical simulation. Proceeding in this way, predicting mixing in sedimentation should be possible.

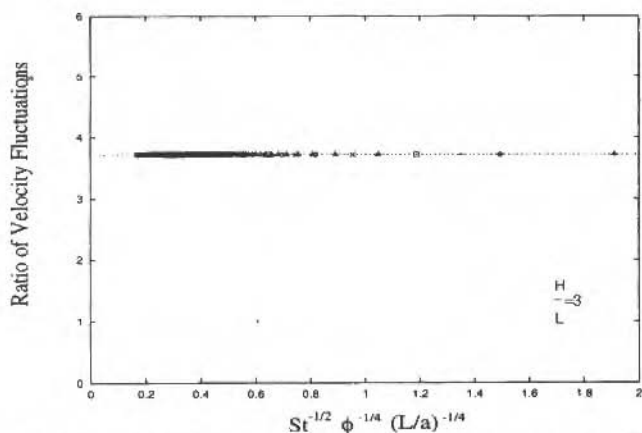


Fig. 5 Ratio Between the Vertical and Horizontal Velocity Fluctuations vs the Scaling Factor. $\diamond$ $St = 10$ and $\phi = 2\%$; $+$ $St = 10$ and $\phi = 3\%$; $\cdot$ $St = 10$ and $\phi = 5\%$; $\times$ $St = 20$ and $\phi = 3\%$; Δ $St = 5$ and $\phi = 3\%$. Dashed line constant value 3.72.

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Approximate Solution for Non-Darcy Transient Film Condensation in a Porous Medium

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Abstract

The problem of non-darcian transient film condensation adjacent to a vertical flat plate embedded in a porous medium has been considered. The governing equation for the boundary layer thickness was obtained by an integral method and solved approximately by the method of integral relations. It is shown that the results are in good agreement with those obtained exactly by the method of characteristics.

Keywords: Transient Film Condensation, Porous Medium, Convection, Boundary Layer.

Nomenclature

a = dimensionless time function	k_c = thermal conductivity (kW/m °C)	δ = film thickness (m)
A = shape factor defined by Eq. 7.b	K = permeability (m ²)	Δ = dimensionless film thickness
B = shape factor defined by Eq. 7.c	l = height of the plate (m)	ε = porosity
C = parameter defined by Eq. 23	m = local mass flux of condensate (kg/m ² s)	μ = dynamic viscosity (kg/m s)
c_p = specific heat at constant pressure (kJ/kg C)	n = 1 for the method of characteristics 0 for method of integral relations	θ = dimensionless temperature
D = shape factor defined by Eq. 10.b	Nu_x = Nusselt number (Eq. 24)	ρ = density (kg/m ³)
E = empirical constant associated with the porous inertia	Ra = modified Rayleigh number (Eq. 9.b)	σ = shape factor defined by Eq. 7.a
f = weighing function	t = time (s)	σ^o = heat capacity relation defined by Eq. 1
F = shape factor defined by Eq. 17.a	T = temperature (°C)	τ = dimensionless time
g = acceleration of gravity (m/s ²)	u = longitudinal velocity component (m/s)	ξ = dimensionless normal coordinate
G = shape factor defined by Eq. 17.b	v = normal velocity component (m/s)	
Gr = Grashof number (Eq. 2)	x = longitudinal coordinate (m)	
h_{fg} = latent heat of condensation (kJ/kg)	X = dimensionless longitudinal coordinate	
H = shape factor defined by Eq. 17.c	y = normal coordinate (m)	
Ja = Jacob number		

Greek Symbols:

α = thermal diffusivity (m²/s)

Subscripts:

l = denotes quantities associated with liquid film
c = denotes composites quantities of the fluid filled porous medium
s = saturation
ss = steady state
w = wall

Introduction

Cheng and Pop (1984) considered the problem of Darcian transient free convection adjacent to a vertical semi-infinite flat plate embedded in a porous medium with a step increase in the wall temperature. Manuscript received: January 1995. Technical Editor: Carlos Alberto Carrasco Attemani

temperature and surface heat flux. The governing equation for the boundary layer thickness was obtained by an integral method. The equation was solved exactly by the method of characteristics and, approximately, by the method of integral relations. The results based on the method of integral relations are in good agreement with those based on the method of characteristics.

The problems of transient film condensation over a vertical surface in a porous medium with constant wall temperature have been treated in a similar fashion by Cheng and Chui (1984) when the flow is Darcian, while the non-Darcian transient film condensation was investigated by the authors (Ebinuma and Nakayama, 1990a,b), using the method of characteristics.

In this work, consideration is given to the non-Darcian transient film condensation in a porous medium adjacent to a vertical surface, when its wall temperature is suddenly cooled below the saturation temperature and remains constant thereafter. The governing equations for the growth of the boundary layer film thickness are obtained by the Karman-Pohlhausen integral methods using the Ergun model (1952), which accounts for the porous inertia effects by a velocity squared term, known as the Forchheimer term (1901). These equations for the film thickness turn out to be first-order partial differential equations of the hyperbolic type and solved approximately by the method of integral relations (Cheng and Pop, 1984). The results show that the boundary layer thickness increases monotonically with time and approaches asymptotically the steady state condition, where the film thickness remains constant.

Analysis

Figure 1 shows the physical model and the coordinate system considered to analyze the problem of non-Darcy transient film condensation over a vertical surface.

Under the same assumptions made by Cheng and Chui (1984) and the authors (1990a,b), the energy equation with the aid of the continuity equation can be integrated across the condensate film thickness to give:

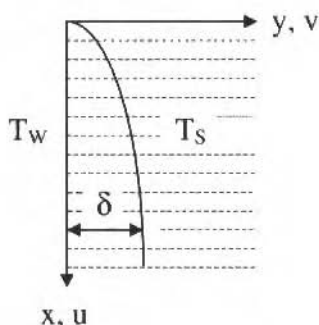


Fig. 1 Model and Coordinate Systems

$$\sigma^o \frac{\partial}{\partial t} \int_0^l (T - T_s) dy + \frac{\partial}{\partial x} \int_0^l u (T - T_s) dy = \alpha_c \left(\frac{\partial T}{\partial y} \right)_{y=0}^{y=\delta} \quad (1)$$

where x and y are the coordinates along and perpendicular to the vertical surface; t , the time; T and T_s are the temperature and saturation temperature of the condensate, respectively; $\sigma^o = (\rho c_p)_c / (\rho c_p)_l$ and $\alpha_c = k_c / (\rho c_p)_l$; $(\rho c_p)_l$ and k_c are the heat capacity and thermal conductivity, respectively. The subscript l denotes quantities associated with the liquid film and c denotes composite quantities of the fluid filled porous medium defined by the porosity relation between the quantities associated with the porous medium and the saturated liquid. The Darcian velocity in the x -direction given by the Ergun model (1952) is:

$$u = \left(\mu / \rho EK^{1/2} \right) \left[(1 + 4Gr)^{1/2} - 1 \right] \quad (2)$$

where ρ and μ are the density and the viscosity of the condensate, respectively; K , permeability; $Gr = \rho(\rho - \rho_v)gEK^{1/2} / \mu^2$ is the modified Grashof number representing the relative significance of the porous inertia; E , the empirical constant associated with the porous inertia which becomes quite significant as the Rayleigh number increases, and g is the acceleration of gravity.

The initial and the boundary conditions are:

$$t = 0 \quad \delta = 0 \quad T = T_s \quad (3.a)$$

$$t > 0 \quad y = 0 \quad T = T_w \quad (3.b)$$

$$y = \delta \quad T = T_s \quad (3.c)$$

and

$$mh_{fg} = k_c \left(\frac{\partial T}{\partial y} \right)_{y=\delta} \quad (3.d)$$

where h_{fg} is the latent heat of condensation and m is the local mass flux of the condensate related to the film thickness through:

$$m = \frac{\partial}{\partial x} \int_0^\delta \rho u dy + \varepsilon \rho \frac{\partial \delta}{\partial t} \quad (4)$$

where ε is the porosity. Substituting Eq. (4) into Eq. (3.d):

$$k_c \left(\frac{\partial T}{\partial y} \right)_{y=\delta} = h_{fg} \rho \left(u \frac{\partial \delta}{\partial x} + \varepsilon \frac{\partial \delta}{\partial t} \right) \quad (5)$$

Substituting Eqs. (5) and (2) into Eq. (1), the following dimensionless form can be obtained:

$$\left(A + \frac{1}{\sigma Ja} \right) \frac{\partial \Delta}{\partial \tau} + D \left(A + \frac{1}{Ja} \right) \frac{\partial \Delta}{\partial X} = - \frac{B}{\Delta} \quad (6)$$

where

$$\sigma = \sigma^o / \varepsilon, \quad A = \int_0^l (\xi) d\xi, \quad B = -\theta'(0) \quad (7a,b,c)$$

$$\xi = y / \delta, \quad \Delta = \delta / l, \quad X = x / l Ra, \quad \tau = \alpha_c t / \sigma l^2 \quad (8a,b,c,d)$$

$$Ja = c_p (T_s - T_w) / h_{fg}, \quad Ra = K(\rho - \rho_v) gl / \mu \alpha_c \quad (9a,b)$$

and

$$\theta = \frac{T - T_s}{T_w - T_s}, \quad D = \frac{(1 + 4Gr)^{1/2} - 1}{2Gr} \quad (10a,b)$$

The prime denote differentiation with respect to ξ ; l , the height of the plate; Ja , the Jacob number; Ra , the modified Rayleigh number; T_w , the wall temperature, and D a shape parameter determined for a given Grashof number.

The initial and the boundary conditions (3a,b,c) become:

$$\tau = 0 \quad \Delta = 0 \quad \theta = 0 \quad (11a,b)$$

$$\tau > 0 \quad \xi = 0 \quad \theta = 1 \quad (12)$$

$$\xi = 1 \quad \theta = 0 \quad (13)$$

Now, we shall adopt the same temperature profile assumed by Cheng and Chui (1984) and by the authors (1990a,b), namely:

$$\theta(\xi) = 1 - C\xi + (C-1)\xi^2 \quad (14)$$

The shape factors A and B can be expressed in term of C as:

$$A = (4 - C)/6 \quad B = -C \quad (15a,b)$$

The dimensionless parameter C is negative, and associated with the temperature gradient at the wall, which remains to be determined from Eq. (5) as Eq. (6) is integrated.

Substituting Eqs. (15.a,b) into Eq. (6) the result can be expressed in the following form:

$$\frac{\partial \Delta}{\partial \tau} + DF \frac{\partial \Delta}{\partial X} = \frac{C}{G \Delta} \quad (16)$$

where,

$$F = G/H, \quad G = (4 - C)/6 + 1/Ja, \quad H = (4 - C)/6 + 1/\sigma Ja \quad (17a,b,c)$$

In applying the method of Integral Relations (Cheng and Pop, 1984), we first multiply Eq. (16) by a weighting function $f(X)$. The resulting equation is integrated with respect to X from $X=0$ to $X=1$, to give:

$$\frac{d}{d\tau} \int_0^1 f(X) \Delta(X, \tau) dX + DF \left[f(1) \Delta(1, \tau) - \int_0^1 f'(X) \Delta(X, \tau) dX \right] = -\frac{B}{G} \int_0^1 \frac{f(X)}{\Delta(X, \tau)} dX \quad (18)$$

We now assume that the weighting function f and dimensionless film thickness Δ are given by (Cheng and Pop, 1984):

$$f(X) = X \quad (19)$$

$$\Delta(X, \tau) = a(\tau) X^{1/2} \quad (20)$$

Substitution of Eqs. (19) and (20) into Eq. (18) and integrating yields:

$$a(\tau) \frac{da(\tau)}{d\tau} + \frac{5}{6} DF a(\tau) = \frac{5C}{3H} \quad (21)$$

Equation (21) with the initial condition Eq. (11) has a closed form solution. With this expression for $a(\tau)$, Eq. (20) gives:

$$\Delta(X, \tau) = [2C/DG]^{1/2} \left\{ 1 - \exp[-(5/3) D (G/H) \tau] \right\}^{1/2} X^{1/2} \quad (22)$$

In order to determine the value of parameter C , we substitute Eq. (22) into Eq. (5) in consideration of the steady state condition (i.e., $\tau \rightarrow \infty$), and utilizing Eq. (14) to obtain the same expression as in Cheng and Chui (1984) and the authors (1990a), namely:

$$C = \left\{ 3 (Ja + 2) - \sqrt{9 (Ja + 2)^2 - 4 Ja (2 Ja + 3)} \right\} / Ja \quad (23)$$

The surface heat flux is easily obtained by utilizing Eq. (22) and represented in terms of local Nusselt number as:

$$Nu_x / Ra_x = \left\{ (C D G / 2) / [1 - \exp\{-(5/3) D (G/H) \tau\}] \right\} \quad (24)$$

It is interesting to note that as $\tau \rightarrow \infty$, Eqs. (22) and (24) become identical to those obtained by the authors (1990a), for the steady state condition, using the method of characteristics.

The growth of the film thickness with time (represented by Eq. (22)) is shown in Figs. 2 and 3 for $\sigma = 1.0$ and 0.6, respectively. In each figure the results obtained for two values of Ja (1.0 and 0.1) and three values of Gr (0.0, 1.0, 10.0) are presented in solid lines. In similar fashion, the variation of the local heat flux with time (represented by Eq. (24)) is plotted in Figs. 4 and 5, for the same values of σ , Ja and Gr . Figures. 6 and 7 show the growth of the film thickness along the vertical surface during the transient period until the steady state condition (τ_{ss}) is attained for $\sigma = 1.0$ and 0.6, respectively. In each figure the cases for $Ja = 0.1$ and $Gr = 0.0, 1.0$ and 10.0 are considered. The exact solution obtained by the method of characteristics is also presented in dashed line, for comparison.

Both methods show that, for fixed values of Ja , the dimensionless time required for the steady state condition increases with increasing Gr values and decreasing σ values. It is important to note that, for fixed values of Ja and σ , the exact solution by the method of characteristics fits the same points as in transient period and the influence of Gr parameter is on the time required to reach steady state. For the approximate solution by the method of integral relations, the time required is obtained asymptotically. So, the approximate solution needed only one equation to represent the transient film condensation in porous medium, while the exact solution needed two equations, for the transient and steady state periods. Thus, for the transient film condensation problem, the results based on the method of integral relations agree fairly well with the exact solution obtained by the method of characteristics.

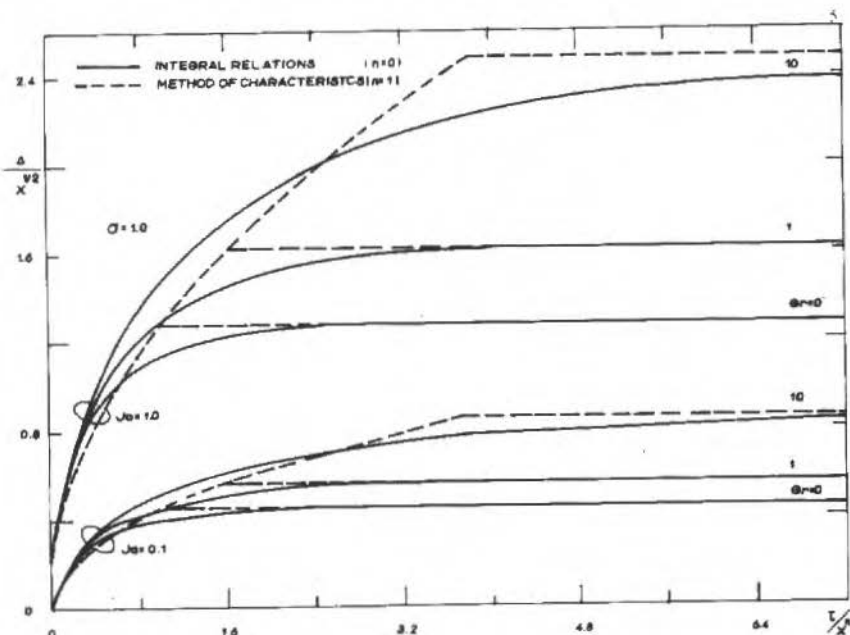
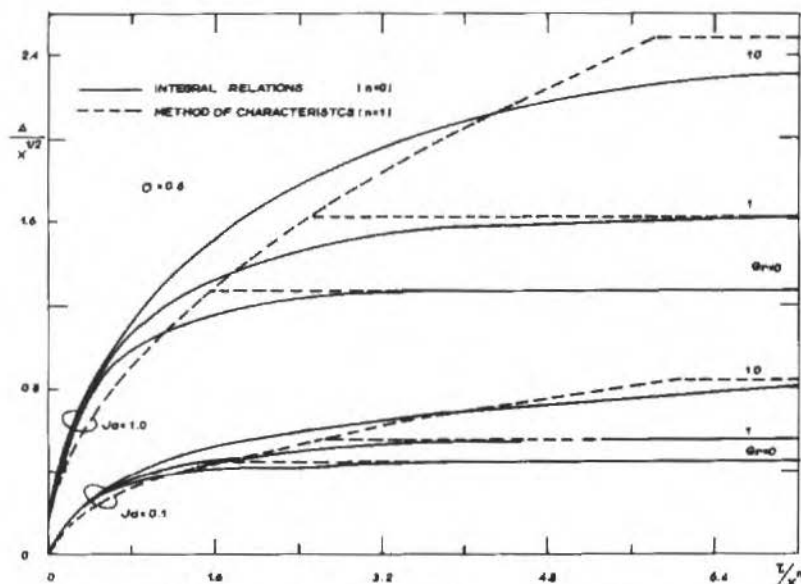
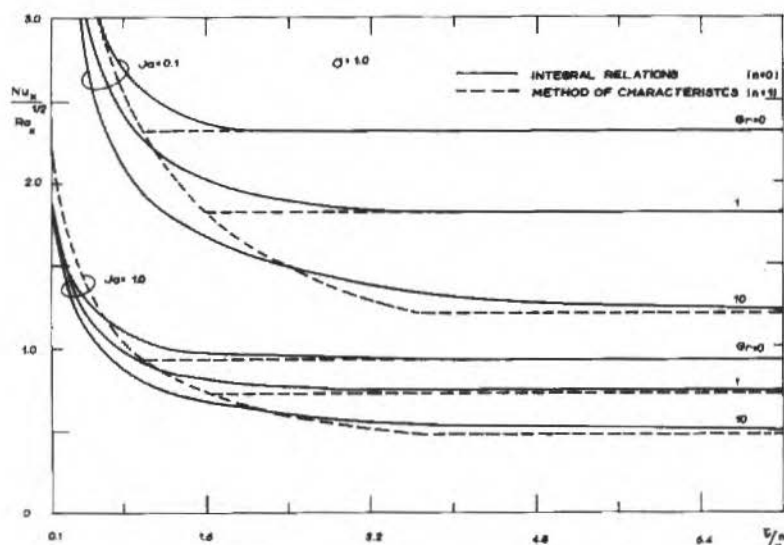


Fig. 2 Growth of Film Thickness With Time for $\sigma = 1.0$

Fig. 3 Growth of Film Thickness With Time for $\sigma = 0,6$

Concluding Remarks

The problem of non-Darcy film condensation in a porous medium has been solved approximately by the method of integral relations. The governing equation for the film thickness (Eq. (22)) has been reduced to an analytical function of X and τ . The equation reveals that the leading edge effects (Cheng and Pop, 1984) increase with the inertia effects. Thus, the time to reach the steady state condition increases as the inertia effects become significant.

Fig. 4 Local Heat Flux as a Function of Time for $\sigma = 1,0$

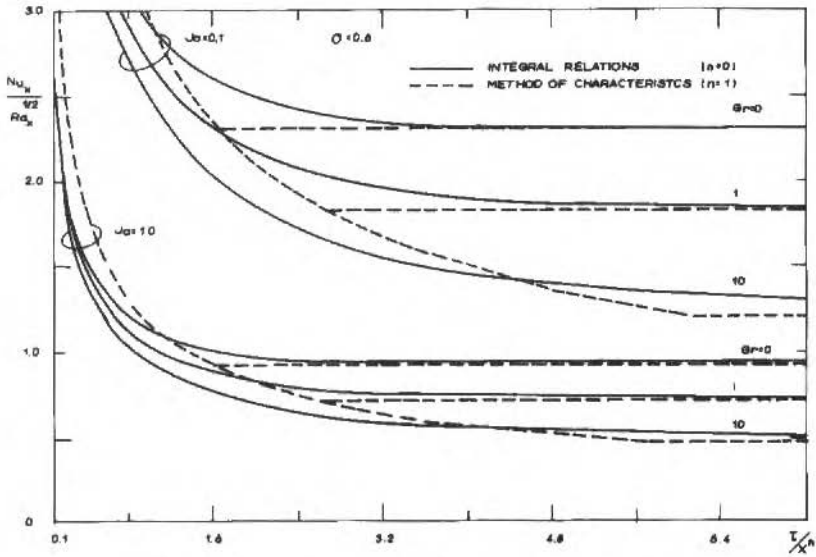


Fig. 5 Local Heat Flux as a Function of Time for $\sigma = 0,6$

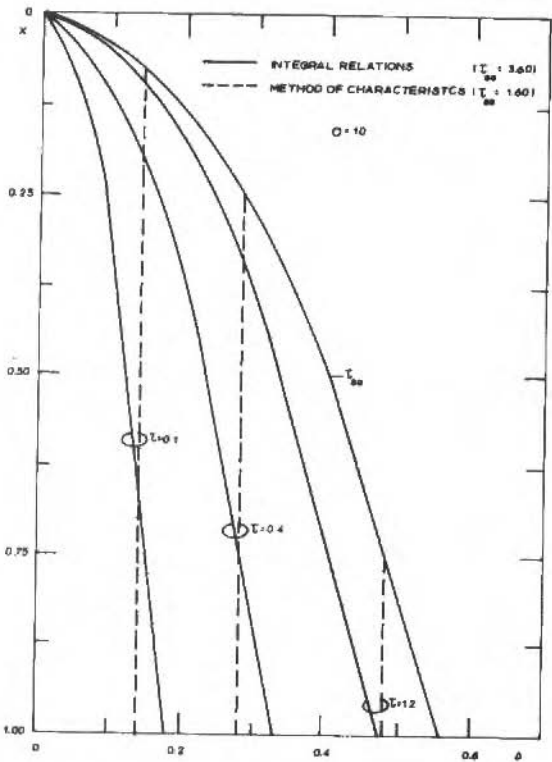


Fig. 6 Growth of Film Thickness Along the Vertical Surface for $\sigma = 1,0$

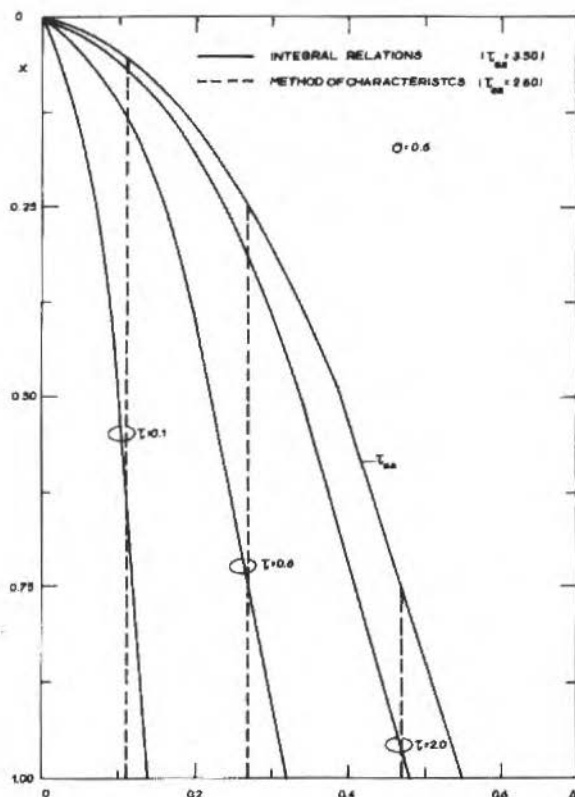


Fig. 7 Growth of film Thickness along the vertical surface for $\sigma = 0,6$

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Procedimento Adaptativo para Geometria Definida por B-Splines Aplicado ao Método dos Elementos de Contorno

Adaptive Geometrical Procedure Defined by B-Splines Applied to the Boundary Element Method

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Abstract

This paper presents an adaptive procedure to minimize errors that occur in the discretization of curved boundaries. Initially the boundary is represented exactly by a b-spline function. Then, the curve is approximated by a series of quadratic elements. An error measure avails the difference between the approximated curve and the original one, refining areas of major error.

Keywords : Numerical Methods, Boundary Element Methods, B-Splines, Adaptive Procedures.

Resumo

Este artigo apresenta um procedimento adaptativo para minimizar os erros obtidos na discretização de contornos curvos. Inicialmente o contorno é representado exatamente por uma função b-spline. A curva é então aproximada por uma série de elementos quadráticos. Um medidor de erro avalia a diferença entre a curva discretizada e a original, refinando as áreas de maior incidência de erro.

Palavras-chave: Métodos Numéricos, Método dos Elementos de Contorno, B-Splines, Procedimentos Adaptativos.

Introdução

Um dos fatores causadores de erro no MEC consiste na avaliação incorreta da geometria, especialmente nos pontos de descontinuidade de tangente. Em geral, a malha é refinada por inspeção visual, nem sempre levando a uma boa representação. Este tipo de erro é muito freqüente no MEC, onde funções de interpolação Lagrangeanas para a geometria podem introduzir descontinuidades de tangente que por não existirem, devem ser minimizadas (Ver Fig. 1).

Quando o canto realmente existe, ele deve ser considerado como tal. Várias técnicas foram desenvolvidas para se considerar descontinuidades de tangente. Inicialmente elementos contínuos foram utilizados. Neste caso, mesmo um refinamento excessivo na vizinhança do canto não levava a respostas boas, pois forçava a continuidade de forças de superfície na vizinhança dos cantos, gerando erros inaceitáveis. Estes erros foram inicialmente reduzidos através de procedimentos nos quais os cantos eram substituídos por curvas suaves de raio de curvatura pequenos (Jaswom e Symm, 1977).

Em uma tentativa de ser mais rigoroso, Brebbia (1978) desenvolveu uma técnica na qual se utilizam dois nós muito próximos e posicionados entre 2 a 5% da extremidade do elemento de canto. Esta técnica, além de aumentar a dimensão do sistema de equações, gera especiais dificuldades quanto à colocação ótima desses nós. Na época, como os procedimentos de integração para elementos quasi-singulares eram poucos conhecidos, erros de integração se somavam aos introduzidos pelo deslocamento dos nós geométricos de sua posição correta, tornando os resultados muito pobres.

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Mais recentemente, Cabral (1992) incorporou as b-splines no MEC, utilizando elementos iso-paramétricos com a subsequente inclusão das b-splines como funções de interpolação. Essa consideração, apesar de retirar os cantos, requer a alteração da formulação iso-paramétrica original do MEC.

Neste artigo, apresenta-se uma maneira de se utilizar uma spline no MEC sem alterar a sua formulação original. Mostra-se um pré-processador geométrico que gera uma spline que passa por pontos especificados. Posteriormente o procedimento aproxima essas splines por meio de funções quadráticas, para processamento em um programa do MEC iso-paramétrico, quadrático convencional.

Splines

O conceito de spline de forma genérica designa uma curva formada por uma série de funções polinomiais concatenadas umas às outras, e que obedecem a determinadas condições de continuidade entre as diversas partes. Essas funções requerem para sua representação apenas o armazenamento dos diversos coeficientes das funções e dos pontos específicos por onde ela aproxima ou passa. Por outro lado a utilização de uma função trigonométrica, exponencial ou logarítmica, requeria um esforço computacional maior.

Existem vários tipos de splines na literatura, incluindo-se a Padrão (ou Natural), Overhauser, Bezier e B-splines. As duas primeiras são splines de interpolação e as outras são de aproximação, conforme a curva passe ou não pelos pontos fornecidos. Para um estudo mais amplo sobre o tema o leitor pode consultar De Boor (1978).

Definição

Seja $X=(X_0, X_1, \dots, X_N)$ um vetor real, tal que X_i é menor ou igual que X_{i+1} (De Boor (1978)).

A função S é chamada spline de grau $M-1$ (ordem M) se ela satisfaz as seguintes condições:

- S é uma função polinomial de grau $M-1$ em cada sub-intervalo (X_i, X_{i+1})
- S e suas derivadas de ordem 1, 2, ..., $M-2$ são todas contínuas, ou seja, $S \in C_{m-2}$.

Os pontos por onde a spline passa são chamados nós e os sub-intervalos entre estes são chamados segmentos da spline, conforme a Fig. 2.

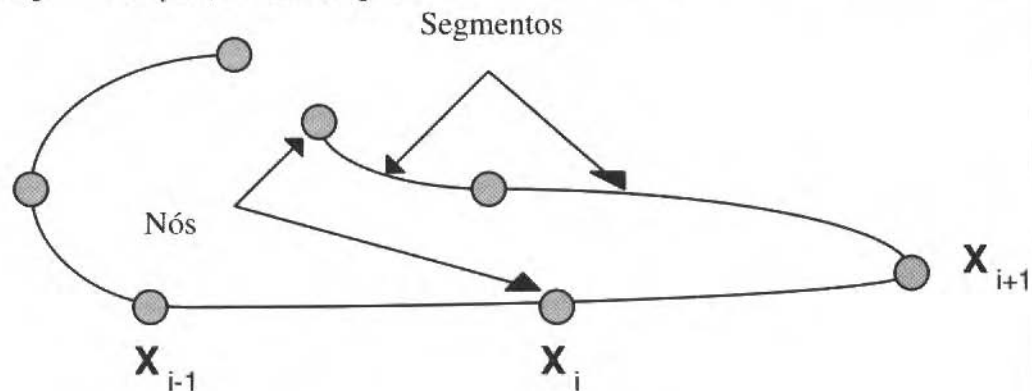


Fig. 2 Exemplo de Spline de Interpolação

As splines quadráticas são usadas para representar curvas com curvatura constante, ao passo que as cúbicas são as mais simples para curvas concatenadas com várias curvaturas diferentes e por isso são as mais usadas na computação gráfica e nos sistemas CAD/CAM.

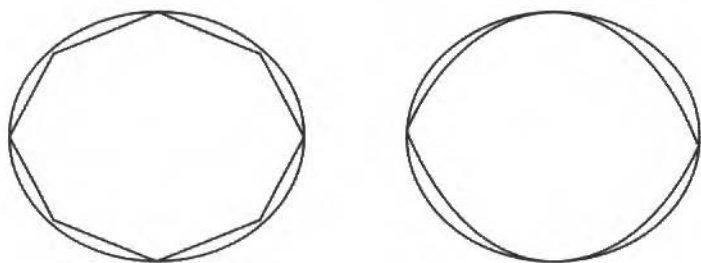


Fig. 1 Ilustração de Má Representação de Geometria por Elementos Lineares e Quadráticos

Posteriormente, foram desenvolvidos procedimentos que permitiam considerar corretamente cantos, desde que as forças de superfície à esquerda e à direita, ou pelo menos uma delas, fosse conhecida. Este procedimento foi denominado *nó duplo* e é discutido, por exemplo, em Mansur, Halbritter e Telles (1978).

O primeiro procedimento geral para considerar corretamente o problema de forças de superfície descontínuas em cantos foi desenvolvido por Chaundonneret (1978), para elasticidade bi-dimensional. Duas equações extras para pontos onde forças de superfície podem ser descontínuas foram obtidas através da hipótese da unicidade do tensor de tensões, juntamente com a condição de invariância do traço do tensor de deformações. Desta forma foi possível considerar as incógnitas extras que aparecem quando descontinuidades de forças de superfície existem. Um procedimento equivalente para problemas de potencial foi proposto por Alarcón, Martín e Paris (1979) que obtiveram equações extras da condição de unicidade do fluxo total em cantos.

O procedimento mais difundido hoje em dia é o proposto por Patterson e Sheik (1981), no qual pontos de colocação e nós funcionais são deslocados para o interior dos elementos. Estes elementos são denominados *descontínuos*.

Pontos de colocação não coincidentes com nós funcionais também têm sido utilizados para gerar as equações extras necessárias. Venturini (1983) adotou um procedimento computacionalmente simples que consiste em usar pontos de colocação fora do domínio. Uma outra alternativa é utilizar o procedimento sugerido por Mustoe, Volait e Zienkiewicz (1982) onde os nós geométricos e funcionais coincidem e os pontos de colocação são deslocados para o interior do elemento. Estes elementos têm sido referidos na literatura (Marques e Mansur (1987) e Mansur e Ferreira (1982)) como *elementos de colocação não nodal*.

A preocupação demonstrada pelos pesquisadores no decorrer dos anos com o tratamento correto de cantos decorre da sensibilidade da análise numérica a uma má representação dos mesmos. Cantos inexistentes devem a todo custo ser evitados, ou pelo menos minimizados. Isto pode ser conseguido utilizando-se procedimentos que permitam que duas curvas diferentes tenham a mesma tangente. Os polinômios de Hermite, amplamente utilizados no Método dos Elementos Finitos (MEF), podem garantir a continuidade de derivadas.

Em processos de fabricação industrial utilizam-se curvas que evitam a introdução artificial de cantos decorre da sensibilidade da análise numérica a uma má representação dos mesmos. Cantos inexistentes devem a todo custo ser evitados, ou pelo menos minimizados. Isto pode ser conseguido utilizando-se procedimentos que permitam que duas curvas diferentes tenham a mesma tangente. Os polinômios de Hermite, amplamente utilizados no Método dos Elementos Finitos (MEF), podem garantir a continuidade de derivadas.

A utilização de splines (b-splines inicialmente) no MEC contudo, demorou bastante, uma vez que os programas inicialmente desenvolvidos aproximavam geometria, deslocamentos e forças de superfície em termos de polinômios de Lagrange.

Algumas Variantes das Splines Cúbicas

Existem diversos tipos de splines geradas pela teoria matemática, de acordo com as diferentes funções polinomiais e as diversas condições de continuidade entre os segmentos:

Hermite: São polinômios definidos pela especificação dos nós por onde a spline atravessa e pelas suas derivadas nesses pontos. Possuem, portanto, uma continuidade da derivada primeira ao longo de toda a curva e são splines de interpolação.

Natural ou Padrão: São formadas pela condição de que os sucessivos segmentos possuam derivada primeira e segunda contínuas, e nos nós inicial e final a derivada segunda seja igual a zero. É um caso especial dos polinômios de Hermite. São pouco usadas porque não permitem uma perfeita modelagem da curva. Alterando-se a posição de um ponto se modifica toda a curva.

Overhauser: É uma spline de interpolação na qual se tem um controle local, e que é formada pela utilização de duas funções de ponderação parabólicas f_1 e f_2 . Na sua forma paramétrica, um segmento Overhauser entre dois nós P_1 e P_2 é definido como sendo:

$$s(t) = (1-t)f_1 + t \cdot f_2 \quad (1)$$

Bezier: Este tipo de spline é muito semelhante à definida pelos polinômios de Hermite diferindo destas na definição da tangente no nó inicial e final. Essas são determinadas a partir do primeiro e último segmento calculado. Da mesma maneira que a Overhauser, necessita de 4 pontos de controle para definir cada segmento, sendo uma spline de aproximação.

B-Splines: Podem ser de aproximação ou interpolação e sempre as derivadas de primeira e de segunda ordem são contínuas. Cada segmento é definido por quatro pontos de controle, com os quais se pode modelar a curva. Possui também um controle local, isto é, a posição de um trecho da curva depende somente das coordenadas dos pontos de controle mais próximos. As b-splines são mais suaves que as outras splines, e possuem mais flexibilidade para representar as características das diferentes curvas, não permitindo excessivas oscilações ou flutuações, que muitas vezes as outras apresentam. Por este motivo é a que será utilizada.

Existem outras formas de splines: Beta-Splines, Rational-Splines, Taut-Splines, etc, para diferentes formas e aplicações que não são tratadas neste artigo.

B-Splines

As b-splines podem ser divididas entre splines de interpolação e splines de aproximação. Na primeira, a curva atravessa os nós especificados, ao passo que na segunda, esses nós servem como pontos de controle para se modelar o comportamento da mesma.

A b-spline é definida como sendo uma função polinomial que é uma combinação linear de funções de base:

$$P_i(w) = V_{i-1}E_0(w) + V_iE_1(w) + V_{i+1}E_2(w) + V_{i+2}E_3(w) \quad (2)$$

sendo:

$P_i(w)$ = coordenada (x ou y) de um ponto da spline localizado no segmento i .

w = coordenada natural que varia entre 0 e 1 dentro de cada segmento.

E_0, E_1, E_2 e E_3 são as funções de base linearmente independentes.

V_j são as coordenadas dos pontos de controle para cada segmento ($j=i-1, i, i+1, i+2$).

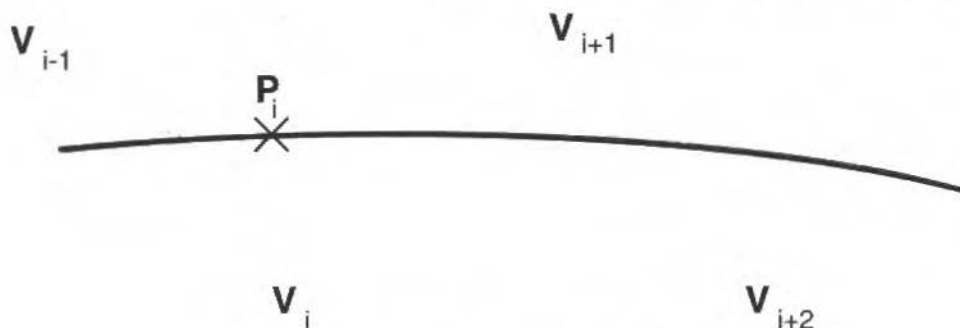


Fig. 3 B-Splines Geradas a Partir de Pontos de Controle

B-Splines Cúbicas e Uniformes

Neste artigo serão utilizadas as b-splines cúbicas com espaçamento uniforme entre os nós. Nesse caso, os nós são pontos correspondentes aos valores inteiros da coordenada natural ($w=0$ ou $w=1$), com o parâmetro w variando de 0 a 1 de um nó para o outro. A razão disso é a sua longa utilização na engenharia pelas propriedades que apresenta.

Para esse caso, definem-se as funções de forma como sendo:

$$\begin{aligned}
 E_0(w) &= -\frac{w^3}{6} + \frac{w^2}{2} - \frac{w}{2} + \frac{1}{6} \\
 E_1(w) &= \frac{w^3}{2} - w^2 + \frac{2}{3} \\
 E_2(w) &= -\frac{w^3}{2} + \frac{w^2}{2} + \frac{w}{2} + \frac{1}{6} \\
 E_3(w) &= \frac{w^3}{6}
 \end{aligned} \tag{3}$$

Para o cálculo de uma b-spline de aproximação, escolhem-se 4 pontos de controle para cada segmento, varia-se w de 0 a 1 e calculam-se os pontos da spline dentro do segmento. Para segmentos consecutivos, 3 pontos de controle permanecerão os mesmos do anterior. Essa condição garante a continuidade da curva. Para m segmentos existem $m+1$ nós e são necessários $m+3$ pontos de controle distintos.

Propriedades

Uma característica muito importante da b-spline é que qualquer usuário com um mínimo de treinamento pode modelar uma curva de uma forma prática e intuitiva, o que explica o seu uso nos sistemas CAD/CAM. Além do mais, a curva possui uma série de propriedades que garantem a sua larga utilização nos problemas de engenharia a saber:

Pontos de Controle: são pontos existentes fora da curva e que afetam a forma da mesma de uma maneira intuitiva e natural, permitindo uma perfeita modelagem.

Controle Local: Uma característica importante das funções b-splines é que os pontos de controle permitem uma alteração local da curva. Alterando-se a posição de um ponto de controle, somente se altera a posição do trecho da curva mais próximo à este, conforme a Fig. 4.

$$P_1(w) = V_0 E_0(w) + V_1 E_1(w) + V_2 E_2(w) + V_3 E_3(w) \quad (4)$$

Particularizando-se a Eq. (4) para os pontos P_1 ($w=0$) e P_2 ($w=1$) e colocando-se sob a forma matricial, vem:

$$\begin{Bmatrix} P_1 \\ P_2 \end{Bmatrix} = \begin{bmatrix} 1/6 & 2/3 & 1/6 & 0 \\ 0 & 1/6 & 2/3 & 1/6 \end{bmatrix} \begin{Bmatrix} V_0 \\ V_1 \\ V_2 \\ V_3 \end{Bmatrix} \quad (5)$$

Da mesma forma para o segundo segmento:

$$\begin{Bmatrix} P_2 \\ P_3 \end{Bmatrix} = \begin{bmatrix} 1/6 & 2/3 & 1/6 & 0 \\ 0 & 1/6 & 2/3 & 1/6 \end{bmatrix} \begin{Bmatrix} V_1 \\ V_2 \\ V_3 \\ V_4 \end{Bmatrix} \quad (6)$$

Agrupando na mesma matriz, vem:

$$\begin{Bmatrix} P_1 \\ P_2 \\ P_3 \end{Bmatrix} = \begin{bmatrix} 1/6 & 2/3 & 1/6 & 0 & 0 \\ 0 & 1/6 & 2/3 & 1/6 & 0 \\ 0 & 0 & 1/6 & 2/3 & 1/6 \end{bmatrix} \begin{Bmatrix} V_0 \\ V_1 \\ V_2 \\ V_3 \\ V_4 \end{Bmatrix} \quad (7)$$

No método inverso, são dados os pontos P_1 , P_2 e P_3 e deseja-se calcular os vértices, para posteriormente ser desenhada a b-spline. Porém, tal procedimento leva à indeterminação do sistema de equações, pois têm-se 5 incógnitas para três equações.

O problema da indeterminação se resolve introduzindo o critério de Yamaguchi (1978), que coloca duas condições extras para tornar o sistema equilibrado. Para splines abertas, o critério consiste em tornar:

$$\begin{aligned} V_0 &= V_1 \\ V_{n+1} &= V_n \end{aligned} \quad (8)$$

Para splines fechadas:

$$\begin{aligned} V_0 &= V_n \\ V_{n+1} &= V_1 \end{aligned} \quad (9)$$

O sistema então se torna perfeitamente determinado, e no caso considerado, $n=3$ e spline aberta, vem:

$$\begin{Bmatrix} P_1 \\ P_2 \\ P_3 \end{Bmatrix} = \begin{bmatrix} 1/6 + 2/3 & 1/6 & 0+0 \\ 0 + 1/6 & 2/3 & 1/6 + 0 \\ 0+0 & 1/6 & 2/3 + 1/6 \end{bmatrix} \begin{Bmatrix} V_1 \\ V_2 \\ V_3 \end{Bmatrix} \quad (10)$$

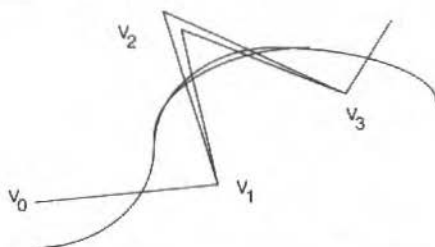


Fig. 4 Controle Local da B-Spline com a Alteração da Posição do Vértice V_2

Independência de Eixos: As curvas b-splines são independentes do sistema de coordenadas utilizado para descrever os pontos de controle.

Ordem de Continuidade: As b-splines cúbicas apresentam continuidade da primeira e da segunda derivada.

Representação Múltipla: Com a b-spline pode-se representar curvas fechadas ou abertas. Para as curvas fechadas os últimos 2 pontos de controle deverão coincidir com o primeiro e o segundo.

Varição Mínima: A b-spline consegue representar funções lineares de forma exata, e garante que a curva seja suave, não apresentando mais oscilações ou ondulações além das quais apresenta a própria função.

Nas extremidades, a curva pode ser modelada conforme o desejo do projetista, e assumir diferentes ângulos, suavizando ou introduzindo pequenos vértices nessas regiões.

Método Inverso

A presente seção discute a implementação de splines de interpolação, uma vez que o problema em questão consiste em representar o contorno de um determinado corpo ou de uma curva suave com equação conhecida, partindo de um ponto inicial conhecido e chegando ao ponto final igualmente conhecido.

A modelagem da curva pode ser feita através da definição de um terceiro ponto entre os dois, ou fazendo-se uso dos pontos de controle.

Para se transformar a b-spline em uma curva de interpolação, é necessário fazer uso do chamado *Método Inverso*, proposto por Yamaguchi (1978). Por este método, são fornecidas as coordenadas por onde se deseja passar a spline, e este calcula a partir desses pontos os pontos de controle.

Seja o problema no qual, a partir de 5 pontos de controle, deseja-se construir uma spline dados os pontos P_1 , P_2 e P_3 de cada trecho, conforme a Fig. 5.

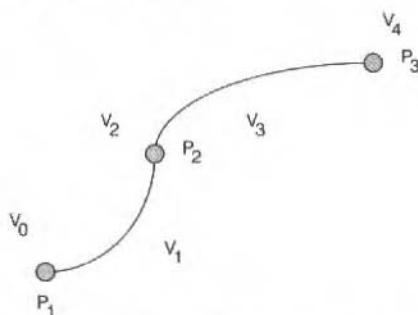


Fig. 5 Traçado de uma Spline com 5 Pontos de Controle e Dois Segmentos P_1 - P_2 e P_2 - P_3

Para o segmento P_1 - P_2 a equação que rege a b-spline é dada por:

Para se resolver o sistema, deve-se inverter a matriz do sistema indicado pela Eq. (10), o que é facilmente realizado para matrizes de ordem 3 ou 4. A existência da inversa é garantida pelo fato das funções de forma constituírem uma base para o espaço das splines.

Quando o critério de Yamaguchi é aplicado para splines fechadas, pode aparecer uma descontinuidade na derivada de segunda ordem entre o último trecho e o primeiro, isto pela própria condição de se fazer coincidir os pontos de controle.

Auto-Adaptatividade de Geometria

Usualmente, utilizam-se como funções de interpolação no MEC as funções de Lagrange com elementos isoparamétricos lineares, quadráticos ou cúbicos para a interpolação da geometria e das condições de contorno. Estas funções, conforme já foi comentado, não garantem continuidade de tangente. Se esta continuidade for importante para a análise numérica, a geometria será melhor representada através de splines.

Sendo assim, foi desenvolvido um pré-processador geométrico que gera uma b-spline pelo método inverso passando por pontos pré-determinados. Posteriormente, após ser gerada, a curva b-spline é aproximada sucessivamente por meio de funções de aproximação de segundo grau, cada uma contendo 3 pontos da spline.

Há um erro de aproximação, pois uma curva b-spline é capaz de representar com exatidão uma curva de 2° grau, porém a inversa não é verdadeira, conforme pode-se ver na Fig. 6.

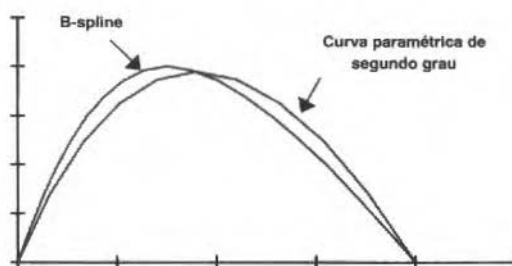


Fig. 6 Aproximação de uma Spline por Meio de uma Função de 2° Grau

Uma maneira de se minimizar essa diferença consiste em se utilizar um medidor de erro que identifique o grau de discordância entre as curvas e aproxime a b-spline por quantas funções sejam necessárias para minimizar essa discordância, conforme pode-se ver nas Figs. 7 e 8.

Implementou-se, portanto um estimador de erro que mede o quanto as funções de 2° grau se afastam da spline. Caso essa diferença seja maior que uma determinada tolerância, novas funções de 2° grau são inseridas, até se conseguir uma margem de erro pequena.

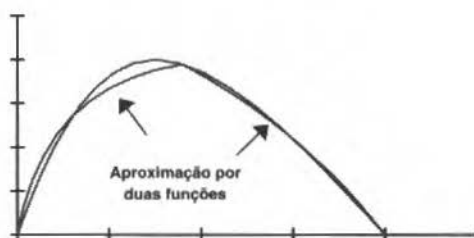


Fig. 7 Aproximação por Duas Funções de 2° Grau

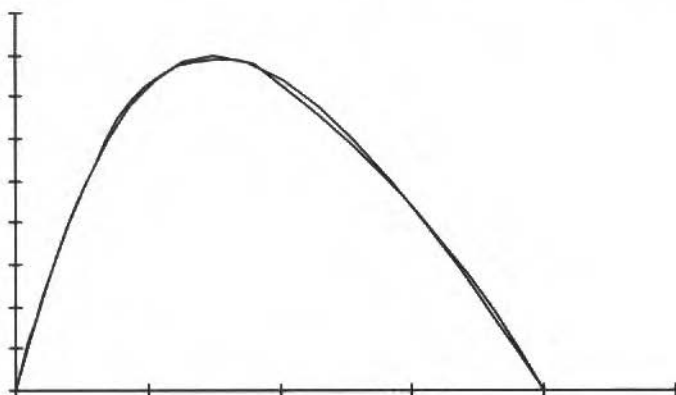


Fig. 8 Aproximação por Várias Funções de 2° Grau

O medidor utilizado consiste em se efetuar nas extremidades e no meio do elemento quadrático o produto escalar entre os vetores unitários tangentes às parábolas (funções de 2° grau) e às splines, verificando desse modo a diferença angular, conforme a Fig. 9. Se este for maior que uma determinada tolerância, deve-se redividir a spline com mais elementos quadráticos, caso contrário, pode-se passar à resolução do problema pelo MEC. A tolerância foi fixada em $1/10$, ou seja, a diferença angular máxima entre os vetores deve ser de aproximadamente 5 graus.

Pode-se resumir o procedimento nos seguintes passos.

- 1: Entrada de n pontos por onde se deseja que a spline passe.
- 2: Modelagem da curva por b-splines até se obter uma forma satisfatória. Esta modelagem pode ser feita introduzindo-se mais pontos de interpolação dentro da curva ou alterando-se a posição dos pontos de controle.
- 3: Calcula-se para a b-spline a coordenada do ponto médio do trecho da spline a ser aproximado pelo elemento quadrático.
- 4: De posse de 3 pontos, gera-se uma função quadrática para cada segmento.
- 5: Para cada função de 2° grau, calculam-se os vetores tangentes à spline e à função nos pontos inicial, do meio e final. Obtém-se o cosseno do ângulo entre eles pelo produto escalar.
- 6: Se houver algum valor que esteja acima de uma tolerância previamente definida, rediscritiza-se o segmento da spline a partir do nó central, dividindo-o em dois, e retorna-se ao passo 3, gerando duas novas funções.

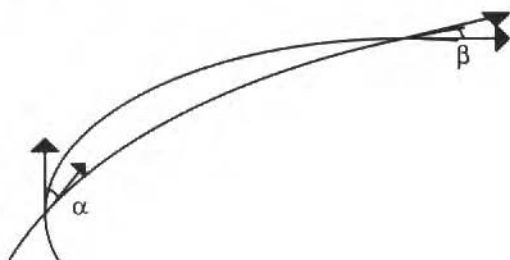


Fig. 9 Medidor de Erro para Geometria Entre B-Spline e uma Função Isoparamétrica de 2° Grau

Exemplo 1

Como demonstração da técnica apresentada, seja a spline que foi gerada por interpolação de quatro nós indicados na Fig. 10. Deseja-se aproxima-la por elementos quadráticos, porém pelo fato da curva ser assimétrica, deve-se tomar especiais cuidados na sua discretização.

Calcula-se a coordenada do ponto central da spline, e com esta gera-se uma função de 2º grau, conforme indica a Fig. 11a, e compara-se com a curva, verificando-se a grande distância entre elas. A partir dos pontos centrais dos dois segmentos da b-spline, outras duas funções são geradas, conforme indica a Fig. 11b, e comparadas.

Nas Figuras 12 e 13 indicam-se os outros refinamentos e a aproximação final. Deve-se observar que a partir da 4ª iteração, o programa se dedicou única e exclusivamente a gerar novas funções na região de curvatura máxima, ao redor do ponto (2.77, 7.98), que é a que apresenta maior dificuldade para as aproximações. Na 6ª iteração, pode-se ver que as funções de 2º grau geradas representam já com grande precisão a curva b-spline original, provando a eficiência do medidor de erro de geometria.

Na Tabela 1 indicam-se as coordenadas inicial, do meio e final dos elementos quadráticos gerados, bem como a indicação da medida de erro nesses pontos. A tolerância utilizada foi 0,10. Note-se que na 6ª iteração, o medidor ainda identifica a necessidade de se refinar mais a região de curvatura máxima, apesar de visualmente a aproximação já ser satisfatória.

O programa efetua então as iterações 7, 8 e 9, gerando mais 6 elementos quadráticos na região do elemento 4 da Fig. 13 (b). Os elementos com as suas medidas de erro estão mostrados na Tabela 2. Apesar de o erro ter diminuído consideravelmente, este ainda é superior à tolerância. Porém, não será feita uma nova iteração, para não se gerar elementos muito pequenos, o que poderia ocasionar erros numéricos na resolução do problema pelo MEC.

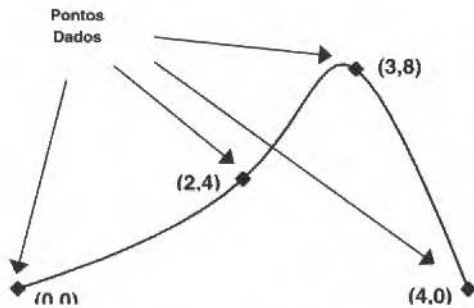


Fig. 10 B-Spline Traçada a Partir da Interpolação de 4 Pontos Dados

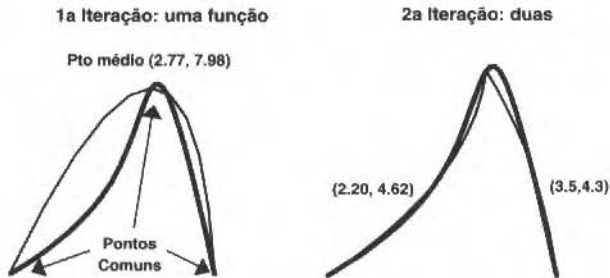


Fig. 11(a) Primeira Aproximação com uma Função de 2º grau, e (b) 2ª Iteração com a Subdivisão da Primeira Função em Duas



Fig. 12(a) 3ª iteração com 4 Funções de 2º Grau. Indica-se as regiões de maior erro que provocam a discretização das funções 2 e 3 gerando e (b) a 4ª aproximação com 6 funções. Indica-se a região mais crítica onde será necessário um novo refinamento, que é a região do ponto de máximo.

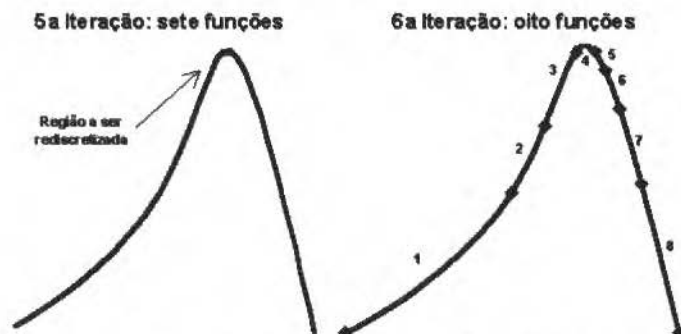


Fig. 13(a) 5ª Discretização com 7 Funções. Indica-se a Necessidade de uma Nova Subdivisão na Região do Ponto Máximo e (b) a 6ª Aproximação com 8 Funções.

Tabela 1 Medidor de Erro para os Pontos das Funções Quadráticas Geradas, com a Indicação dos Pontos onde o Erro é Maior que a Tolerância Adotada

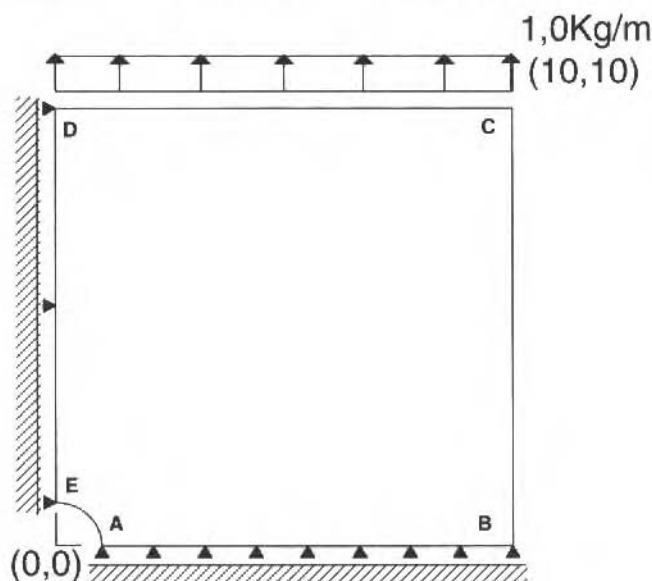
Ponto	1ª Iteração	2ª Iteração	3ª Iteração	4ª Iteração	5ª Iteração	6ª Iteração
(0,0)	.413	.039	.025			
(1.15,1.79)			.001			
(1.94,3.81)		.061	.01	.046	.008	
(2.19,4.83)				.009		
(2.39,5.88)			.028	.009	.021	.001
(2.47, 6.39)					.002	
(2.56, 6.96)				.015	.004	.022
(2.65, 7.46)					.011	
(2.77,7.98)	.950	.257	.555	.156	.70	.105
(2.92, 8.16)				.979	.076	.567
(3.00, 7.97)						.158
(3.07, 7.7)					.107	.729
(3.12,7.45)						.016
(3.21, 6.29)				.135	.107	.009
(3.29,6.38)			.071	.08	0	.004
(3.42, 5.37)				.002		
(3.55,4.24)		.041	.056	.002	0	
(3.78,2.11)			.01			
(4,0)	.069	.039	.002			

Tabela 2 Iterações 7, 8 e 9, com a Discretização da Região de Curvatura Máxima

	7ª Iteração	8ª Iteração	9ª Iteração
(2.77,7.98)	.142	.042	.006
(2.78,8.03)			.035
(2.79,8.05)		.014	.048 .029
(2.80,8.09)			.040
(2.82,8.12)	.144	.103 .194	.062 .135
(2.84,8.15)			.048
(2.86,8.17)		.019	.303 .039
(2.89,8.17)			.297
(2.92,8.16)	.697 .126	.334 .018	.018
(2.95,8.11)		.068	
(2.98,8.05)	.091	.005 .007	
(3.00,8.00)		.011	
(3.00,7.97)	.119	.022	

Exemplo 2

Para demonstrar a aplicabilidade da formulação em problemas do MEC, seja o problema de uma placa com furo no centro, uniformemente tracionada, como mostra a Fig. 14.

**Fig. 14** Exemplo 2. Raio do Furo: 1m

A discretização do furo deve ser feita com muito cuidado. Sendo o furo circular e o elemento quadrático, há uma pequena diferença entre as duas curvas, e se não há uma boa avaliação, podem se introduzir erros devido a não continuidade da normal entre as curvas. Especialmente importante é a manutenção do ângulo de 90° junto às extremidades. Aplicando a formulação e colocando uma tolerância de 0,001 chega-se à necessidade de se colocar tres elementos quadráticos de tamanhos diferentes para representar a região do furo, conforme se ilustra na Fig. 15.b. Repare-se que a utilização de dois elementos de menor tamanho nas extremidades, deve-se ao fato de se querer preservar o ângulo de 90° . Na Figura 15.a mostra-se como seria a aproximação com um elemento quadrático. Nota-se a diferença entre as curvas.

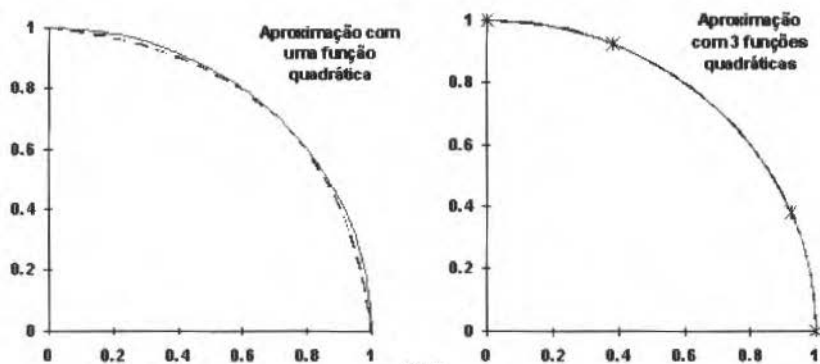


Fig. 15(a) Aproximação com um Elemento Quadrático e (b) Três Elementos Distribuídos de Maneira Não Uniforme

A seguir na Fig. 16 mostra-se uma comparação da solução (Pessolani (1996)) do MEC para 75 elementos quadráticos com malhas idênticas, mas tomando como base as discretizações das Fig. 15(a) e 15(b) para a região do furo. Repare-se que a utilização da discretização recomendada pela formulação conduz à uma maior precisão da resposta.

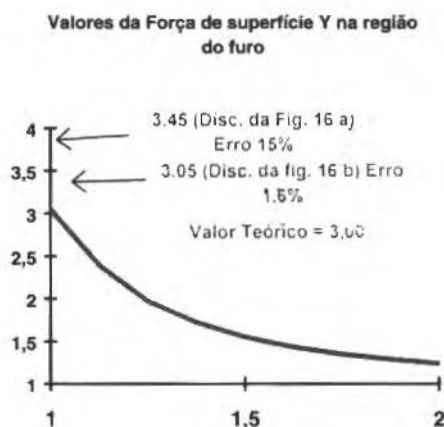


Fig. 16 Comparação entre a Solução com a Discretização do Furo Tomando como Base a Fig. 16(b), e a Linha Mais Fina a com a Discretização do Furo Tomando como Base a Fig. 16(a).

Conclusão

O Método dos Elementos de Contorno é bastante sensível a uma má representação de cantos. Atualmente existem técnicas muito simples e precisas para lidar com o problema, já sobejamente conhecidas pela comunidade que trabalha no assunto. Entretanto, a questão de pequenos ângulos inexistentes introduzidos pela discretização com elementos Lagrangeanos não tem sido suficientemente explorada. Em geral, as discretizações são verificadas por inspeção visual, não havendo nenhum procedimento matemático padronizado que indique se a discretização é suficiente.

Neste artigo um procedimento simples foi proposto, fácil de automatizar e que se aplica à aproximação de qualquer curva por meio de um conjunto de elementos Lagrangeanos.

Em particular, foi mostrado um pré-processador geométrico, que transforma uma curva b-spline em um conjunto de elementos quadráticos.

O exemplo analisado evidenciou a simplicidade, eficácia e aplicabilidade do indicador proposto.

Como sugestão para aprimoramentos, seria interessante ter algum controle, como por exemplo a medida da área entre as curvas, que parasse as iterações no momento em que o tamanho do elemento fôr muito pequeno relativamente aos demais. Isto seria útil para evitar erros numéricos no MEC devido ao mau condicionamento do sistema de equações ou ao cálculo das integrais de contorno.

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Intelligent Control Based Vertical-Axis Wind Turbine System

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Abstract

Intelligent control techniques, such as fuzzy logic, neural network and genetic algorithm are recently showing a lot of promise in the application of various engineering systems. The paper describes the control strategy development, design and experimental performance evaluation of a fuzzy logic based variable speed wind generation system that uses cage type induction generator and double-sided PWM converters. The system can feed a utility grid maintaining unity power factor at all conditions, or can supply to an autonomous load. The fuzzy logic based control of the system helps to optimize the efficiency and enhance the performance. A complete 3.5 kW generation system has been developed, designed and thoroughly evaluated by laboratory tests in order to validate the predicted performance improvements. The system gives excellent performance, and can easily be translated to a larger size in the field.

Keywords: Wind Generation, Fuzzy Control, Vertical Axis Turbine, Intelligent Control.

Introduction

A wind electrical generation system is the most cost-competitive of all the environmentally clean and safe renewable energy sources in the world. Of course, the installation of wind powered electrical systems depends on the local wind resources, its availability is somewhat statistical in nature and must be supplemented by additional sources to supply the demand curve. Wind systems are also competitive with fossil fuel generated power, and much cheaper than nuclear power. Although the history of wind power goes back more than two centuries, its potential to generate electrical power began to get attention from the beginning of this century. During the last two decades, wind power has been seriously considered to supplement the power generation by fossil fuel and nuclear methods. In recent years, wind power is gaining more acceptance because of environmental and safety problems of conventional power plants and advancement of wind electric generation technology. The world has enormous resources of wind power; it has been estimated that even if 10% of raw wind potential could be put to use, all the electricity needs of the world would be met (Ewea, 1991). There are currently over 1700 MW of wind generators installed worldwide with generation of 6 billion kWh of energy annually, the generation will grow up to 60 billion kWh by the year 2000.

Traditionally, wind generation systems used variable pitch constant speed wind turbines (horizontal or vertical axis) that were coupled to squirrel cage induction generators or wound-field synchronous generators and fed power to utility grids or autonomous loads. The recent evolution of power semiconductors and variable frequency drives technology has aided the acceptance of variable speed generation systems. In spite of the additional cost of power electronics and control, the total energy capture in a variable speed wind turbine (VSWT) system is larger and, therefore, the life-cycle cost is lower. The following generator-converter systems have been popularly used:

- Doubly fed induction generator with cascaded converter slip power recovery;
- Doubly fed induction generator with cycloconverter slip power recovery;
- Synchronous generator with line-commutated and load-commutated thyristor converters.

In addition to the above schemes, squirrel cage generators with shunt passive or active VAR (volt ampère reactive) generators have been proposed which generate constant voltage constant frequency power through a diode rectifier and line-commutated thyristor inverter, variable reluctance machines and doubly stator-fed induction machines have also been proposed in wind generation systems. The major problems in such traditional power conversion schemes are the poor line power factor and harmonic distortion in line and machine currents. The recent IEEE Standard 519 severely restrict to line harmonic injection. Therefore, to satisfy the stringent harmonic standard and poor power factor problem, active type VAR and harmonic compensators can be installed with large additional cost.

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Again, the conventional control principles used in these systems make the response sluggish and give non-optimum performance. Double-sided pulse width modulated (PWM) converter system has been proposed in order to overcome some of the above problems (Sukegawa, 1991).

There are several types of wind turbine designs which are usually classified in (1) horizontal-axis, somewhat similar to the "Dutch" windmills, because the rotor spins around a horizontal axis, and in (2) vertical-axis, where the most common is the "Darrieus" configuration, shown in Fig. 1. Unlike conventional wind turbines, which must be reoriented as the wind changes direction, vertical-axis wind turbines are omnidirectional. Their vertical axis of rotation also permits placing the drivetrain at the bottom so as to transmit power directly to the ground level and much easier to service. Despite their advantages, Darrieus rotors have inherent characteristics that have been discouraging a widespread use. There are alternated tensions on the blades, with torque pulsations, leading to fatigue of the turbine material. Such pulsations also cause dynamic instabilities for the control system. The turbine is not self-starting and the power electronics system must be able to motor up the turbine to the operating speed. During the past few years, there has been a lot of technological advances that might bring Darrieus turbines to the public attention again. There are new materials for mechanical design, like CFRP (carbon fiber reinforced polymer) and GFRP (glass fiber reinforced polymer), and several sort of polyester fibers for strength reinforcement. Today, the power electronic systems are much more efficient, and able to smoothly vary the generator speed. Very fast microprocessor systems allow the implementation of new control techniques, which can be used for optimization and enhancement of the control system.



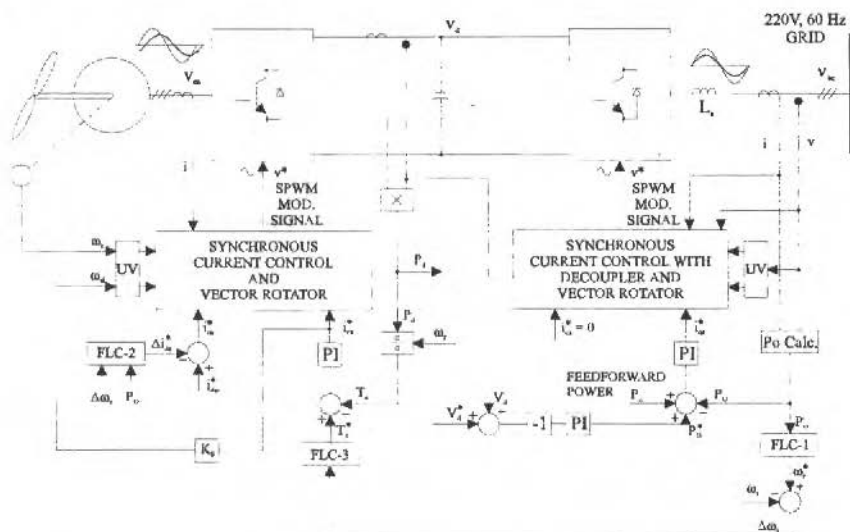
Fig. 1 Vertical-Axis Wind Turbine

This paper describes a variable speed wind generation system with squirrel cage induction generator and double-sided PWM converters, where fuzzy logic based intelligent control has been used extensively to optimize efficiency and to enhance the system performance. An extensive simulation study, in the beginning, validated all the control algorithms. Finally, a 3.5 kW laboratory drive system was designed and tested to evaluate the performance.

Power Circuit and Control Strategy

A qualitative explanation of the block diagram in Fig. 2 is given here in order to show the functions for the power circuit and the control system. The turbine (vertical type) is coupled to the induction

generator through a speed-up gear ratio. The variable frequency variable voltage power generated by the machine is rectified to dc by an IGBT PWM bridge rectifier that also supplies lagging excitation current to the machine. The dc link power is inverted to 60 Hz, 220 V ac through a IGBT PWM inverter and fed to utility grid at unity power factor, as indicated. The line power factor can also be programmed to be leading or lagging for static VAR compensation, if desired. The generator speed is controlled by indirect vector control with torque control and synchronous current control in the inner loops (Bose, 1986). The machine flux is controlled in open loop by control of i_d current, but in normal condition, the rotor flux is set to the rated value for fast transient response. The line-side converter is also vector-controlled using direct vector control and synchronous current control in the inner loops (Sukegawa, 1991). The output power P_o is controlled by the feedback loop of the dc link voltage V_d . Since increase of P_o causes decrease of V_d , the voltage loop error polarity has been inverted. The tight regulation of V_d within a small tolerance band requires a feedforward power injection in the power loop, as indicated. The insertion of filter inductance L_s creates some coupling effect which is eliminated by a decoupler in the synchronous current control loops. The system uses three fuzzy controllers (FLC-1, FLC-2 and FLC-3) which will be explained later. The power can be controlled to flow easily in either direction. A vertical wind turbine requires start-up motoring torque, as the speed develops, the machine goes into generating mode. The machine can be shut down by regenerative braking.



Fuzzy Logic Control System

Neglecting losses in the system, the line power (i.e., turbine power) with generator speed at different wind velocity can be given by the curves in Fig. 3. If, for example, the wind velocity is at a certain level, the maximum output power will be at the peak value for that conditions, but if the wind velocity changes to another level, the generator speed must be changed in order to correspondingly track the maximum power point i.e., maximize the turbine aerodynamic efficiency. This control function is performed by the fuzzy controller FLC-1 based on real time search. Since the wind velocity is unknown (no anemometers are used), FLC-1 changes the generator speed incrementally and observes the power P_o . By searching operation, it settles down to the optimum generator speed condition. Figure 4 shows the block diagram of fuzzy controller FLC-1.

With incrementation (or decrementation) of speed, the corresponding incrementation (or decrementation) of output power P_o is estimated. If ΔP_o is positive with the last positive variation of generator speed ($\Delta \omega_r^+$), the search is continued in the same direction. If, on the other hand, $+\Delta \omega_r^+$ causes $-\Delta P_o$, the direction of search is reversed. The speed oscillates with small increment when it reaches the optimum condition. The variables ΔP_o (variation of power), $\Delta \omega_r^+$ (variation of speed) and

variation of speed) are described by triangular membership functions shown in Fig. 5, and the control laws are given by the rule matrix in Table 1. A typical rule reads as:

IF ΔP_0 is PM (positive medium) AND $L\Delta\omega_r^*$ is P (positive) THEN $\Delta\omega_r^*$ is PM (positive medium)

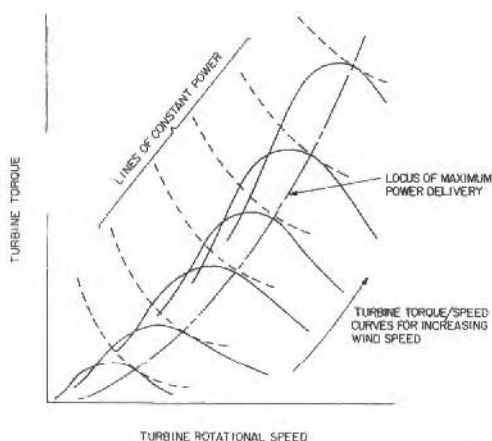


Fig. 3 A Typical Family of Torque/Speed Curves for a Fixed Pitch Wind Turbine

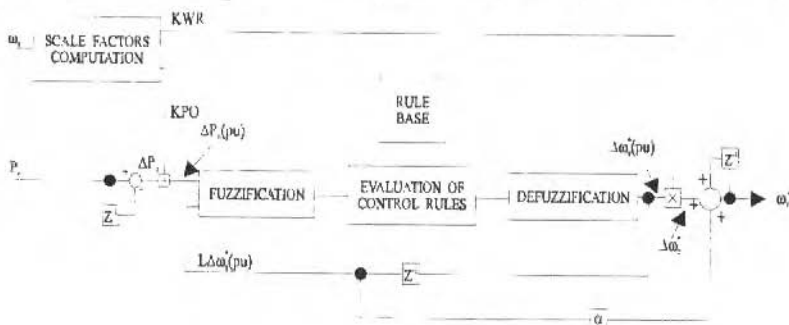


Fig. 4 Block Diagram of Fuzzy Controller FLC-1

Table 1 Rule Matrix for Fuzzy Controller FLC-1

$L\Delta\omega_r^*_{r(PU)}$				
		P	ZE	N
$\Delta P_{0(PU)}$				
	PVB	PVB	PVB	NVB
	PB	PB	PVB	NB
	PM	PM	PB	NM
	PS	PS	PM	NS
	ZE	ZE	ZE	ZE
	NS	NS	NM	PS
	NM	NM	NB	PM
	NB	NB	NVB	PB
	NVB	NVB	NVB	PVB

The membership functions for ΔP_0 and $\Delta\omega_r^*$ are asymmetrical, giving more sensitivity as the variables approach the zero value. Note that at any instant, more than one (at the most four) rule will be valid. In the implementation of fuzzy control, the input variables are fuzzified, the valid control rules are evaluated and combined, and finally the output is defuzzified to convert to the crispy value. The fuzzy controller output $\Delta\omega_r^*$ signal values are accumulated to generate the actual speed command

signal. The output $\Delta\omega_r^*$ is added to some amount of $L\Delta\omega_r^*$ signal in order to avoid local minima due to wind vortex and torque ripple. The controller operates on per-unit basis so that the response is insensitive to system variables and the algorithm is universal to any system. The scale factors KPO and KWR, shown in Fig. 4, are function of generator speed so that the control becomes somewhat insensitive to speed variation. The whole fuzzy controller is designed from the heuristic knowledge of the system. It is extensively iterated by system simulation study and then fine-tuned by experimentation. The advantages of fuzzy control in this case are: (1) it provides adaptively decreasing step size in the search that leads to fast convergence, (2) the controller will accept noisy and inaccurate signals, (3) it does not need wind velocity information, and (4) the system parameter variation does not affect the search. The turbine-generator can be looked upon as the inverse of a motor-blower system where the generated power is given by the equation of cubic relation of speed ($P_0 = K\omega_r^3$). Therefore, the generator is running at light load most of the time with variable wind speed. At light load, the rotor flux can be reduced from the rated value to decrease the excessive core loss and thereby improving the efficiency of the machine-converter system. The efficiency optimization can be done on the basis of on-line search of rotor flux (Kirschen, 1984) and it is implemented here by fuzzy controller FLC-2 which is described in (Sousa, 1995). At a certain steady state wind velocity V_w and at the corresponding optimum speed ω_r^* established by the controller FLC-1, the rated rotor flux is reduced by decreasing the excitation current i_{ds} . This causes the increase of the torque component of current (i_{qs}) by the speed loop for the same developed torque. As the flux is decreased, the machine iron loss decreases with the attendant increase of copper loss. However, the total converter-machine system loss decreases, resulting in an increase of total generated power P_0 . The search is continued until the system settles down at the maximum power point. Any attempt to search beyond will force the controller to return to the maximum power point. It has been shown that efficiency improvement as much as 10%-15% is possible by the fuzzy controller FLC-2. The principle of FLC-2 is somewhat similar to that of FLC-1 and it starts when FLC-1 has completed its search at the rated flux condition. Figure 6 explains the block diagram of FLC-2. The membership functions of the input and output variables and the rule table are not shown. The system output power $P_0(k)$ is sampled and compared with the previous value to determine the increment ΔP_0 . In addition, the last excitation current decrement ($L\Delta i_{ds}^*$) is reviewed. On these basis, the decrement step of i_{ds} is generated through fuzzy inference and defuzzification.

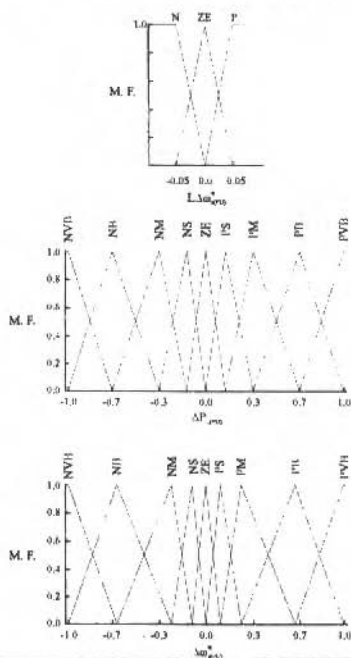


Fig. 5 Membership Functions of Fuzzy Controller FLC-1

As before, FLC-2 handles the signals in per-unit basis and the scale factors K_P and K_{IDS} are given by the linear equations as follows:

$$K_P = a\omega_r + b \tag{1}$$

$$K_{IDS} = c_1\omega_r - c_2T_e + c_3 \tag{2}$$

where a, b, c_1, c_2 and c_3 are constants and T_e is the estimated torque. The FLC-2 control will induce pulsating torque (Sousa, 1995) in the generator shaft but it will be highly attenuated by the torque control loop and the fuzzy controller FLC-3, which will be explained later. If the wind velocity changes during or at the end of FLC-2, FLC-2 control is abandoned, the rated flux is established, and the control returns to FLC-1 in steady state.

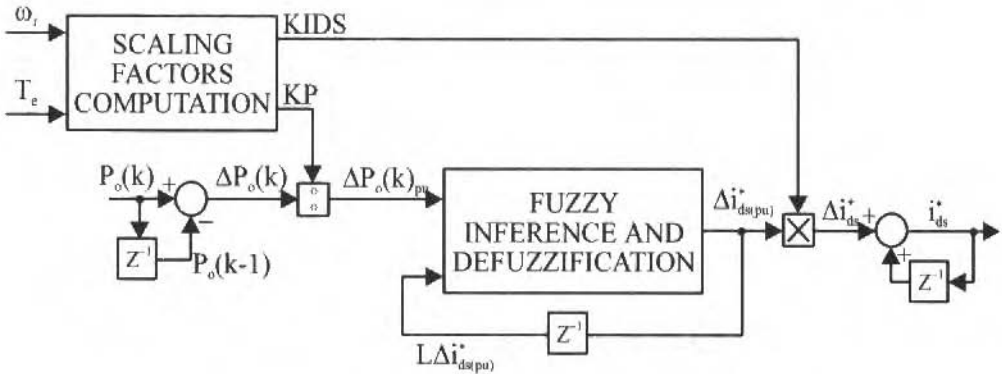


Fig. 6 Block Diagram of Fuzzy Controller FLC-2

As indicated in Fig. 2, the generator speed control loop uses fuzzy controller FLC-3 to get robust performance against turbine oscillatory torque, effect of wind vortex and pulsating torque induced by FLC-2. There is also the possibility of mechanical resonance of turbine-generator system in the absence of robust control. All the disturbance torque components are essentially modulated inversely so that their effect on the system is minimal. With fuzzy control, the speed control loop also gives deadbeat response (Sousa, 1994) when the speed command is changed. The block diagram of FLC-3 is shown in Fig. 7, and the corresponding rule matrix is given in Table 2. The membership functions of the fuzzy variables are not shown. The fuzzy control is essentially a proportional-integral (PI) control where the gain factors are nonlinear and are adaptively controlled to get the robust response. The speed loop error ($E\omega_r$) and the error change ($\Delta E\omega_r$) signals are converted to per-unit signals by the constant scale factors K_E and $K_{E\Delta}$, respectively, and then processed through fuzzy control to generate the signal $\Delta T_e^* (p.u.)$, as indicated. It is then multiplied by the constant scale factor K_T and accumulated to generate the torque command signal T_e^* . A typical control rule can be read as

I F error $E\omega_{r(p.u.)}$ **i** s **PS** (positive small) **A** **N** **D** change in error $\Delta E\omega_{r(p.u.)}$ **i** s **PM** (positive medium) **T** **H** **E** **N** the torque increment $\Delta T_e^* (p.u.)$ **i** s **PL** (positive large)

The membership functions of the fuzzy variables and the rule matrix were extensively iterated by simulation study until the performance was best.

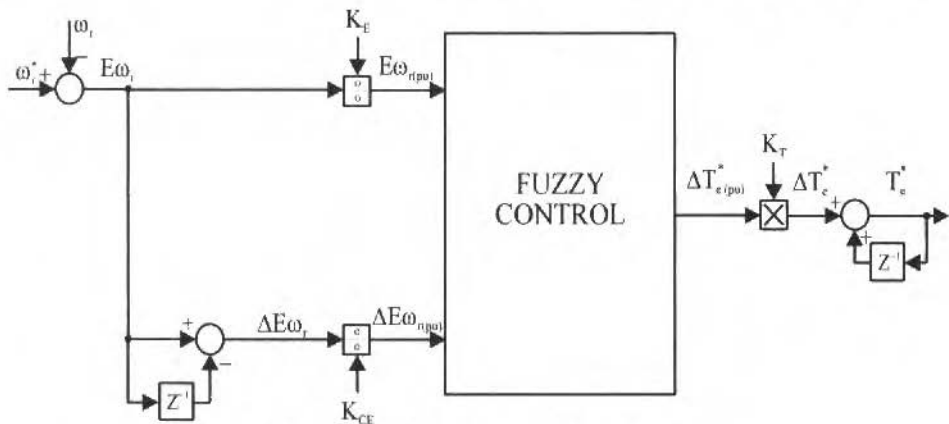


Fig. 7 Block Diagram of Fuzzy Controller FLC-3

Table 2 Rule Matrix for FLC-3

$E\omega_r$										
		NVL	NL	NM	NS	ZE	PS	PM	PL	PVL
$\Delta E\omega_r$	NVL					NVL	NL	NM	NS	ZE
	NL					NL	NM	NS	ZE	PS
	NM					NM	NS	ZE	PS	PM
	NS			NL	NM	NS	ZE	PS	PM	PL
	ZE		NL	NM	NS	ZE	PS	PM	PL	
	PS	NL	NM	NS	ZE	PS	PM	PL		
	PM	NM	NS	ZE	PS	PM	PL			
	PL	NS	ZE	PS	PM	PL				
	PVL	ZE	PS	PM	PL	PVL				

Coordination of System Control

The system has a complex start-up, control sequencing and shut-down operations, as indicated by the diagram of Fig. 8. As the wind velocity develops beyond a threshold value, the system goes through start-up procedure as follows:

- Close the line circuit breaker. The dc link capacitor charges through the diode rectifier that is integral with the line-side converter and a series resistor, which is by-passed when the capacitor is fully charged.
- Start the machine-side converter and build the rated flux in the machine.
- Start the turbine-generator in speed control mode with the fuzzy controller FLC-3. As the generated power flows in the dc link, its voltage begins to rise. The fuzzy controller FLC-3, from now on, operates all the time until the system is shut down.
- As the dc link voltage V_d reaches beyond the threshold value, the line-side converter is enabled and the dc voltage control loop is activated.

After the start-up sequence, as the system tends to attain steady state condition ($|\Delta P_0| < \epsilon$), the fuzzy controller FLC-1 is activated. The generator speed gradually builds up until the maximum power is generated for the given wind velocity condition. If the wind velocity changes slowly, the generator speed tracks it due to the FLC-1 control. When the FLC-1 control reaches the steady state, as indicated by small oscillation of ω_g about the optimum P_0 point, the control is transitioned to FLC-2 mode. At steady state of FLC-2, the current i_{ds} oscillates about the maximum power point. If wind velocity

changes in either control mode, as indicated by large increment (or decrement) of power ΔP_o , or there is a corresponding increment (or decrement) of torque ΔT_e^* , the control transfers to non-fuzzy operation mode with the rated machine flux, as indicated in the figure. When steady state wind velocity is reached in non-fuzzy mode ($|\Delta P_o| < \epsilon_1$), the control is sequenced through FLC-1 and FLC-2 modes, respectively. If fault is developed during any operation, or the generation system is to be stopped, the shut-down mode is activated. Note that if the line side converter is tripped, a dynamic brake (chopper with load resistor) is switched into the dc link to limit the dc voltage within the safe range.

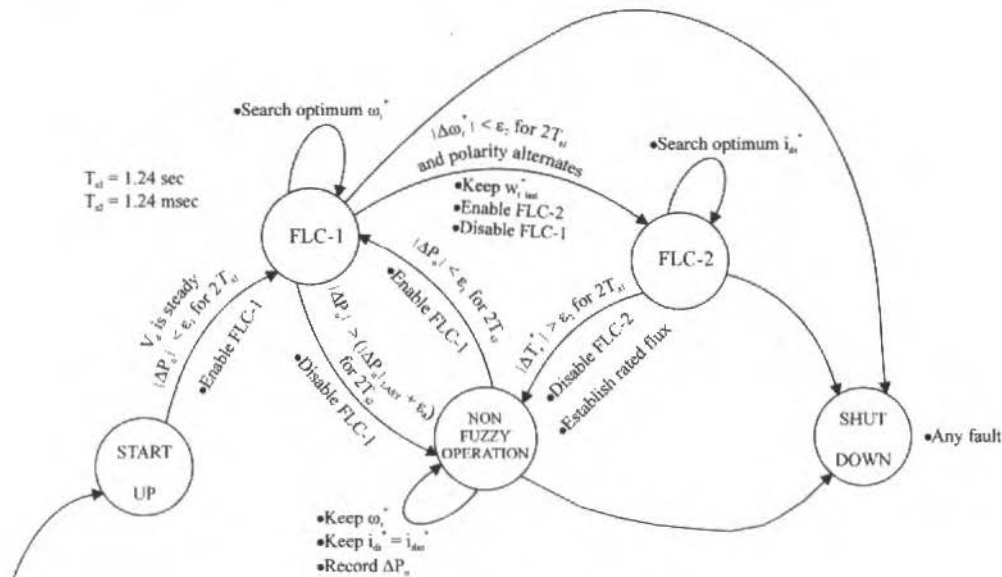


Fig. 8 Control Sequencing Diagram

Development of Hardware and Software

As mentioned before, a complete 3.5 kW laboratory breadboard wind generation system was designed to validate all the control laws before building the 200 kW field prototype. For field application, the converter-machine system is to be scaled up, but the control hardware and software remain the same. The control hardware and software for the project are quite complex and the detailed description is beyond the scope of this paper. Basically, the control hardware is based on two Texas Instruments TMS320C30 type digital signal processor boards which are placed in the PC slots with the I/O hardware (Simões, 1996). The 32-bit floating point DSP has 60 nsec instruction cycle time, and the advantage of floating point computation is that no scaling is needed. The multi-tasking software, principally based on C language, is strategically distributed between the two DSPs. The system control parameters are transported between the two DSPs using the two independently working serial ports. The system uses a PWM chip that is basically a hybrid ASIC that incorporates dedicated digital hardware and RISC microprocessor. It receives the voltage commands from the synchronous current control loops, vector rotates into stationary d^s-q^s frame with the help of vector angle information, converts the signals into a-b-c frame, and finally sinusoidally modulated PWM signals are generated for gating the bridge converter. The lock-out time, device switching times and current polarity signals for elimination of waveform distortion can be programmed in the chip. Since a wind turbine could not be available in laboratory, it was simulated in real time in DSP by solving the turbine model given on Fig. 9 (Simões, 1997).

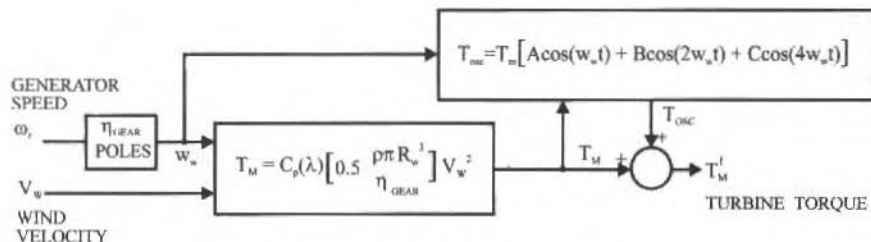


Fig. 9 Turbine Model Based Dynamometer

By giving command of wind velocity V_w (a software variable), and knowing the generator speed ω_g , the equations are solved to generate the turbine torque T_M ($= T_M + T_{OSC}$). This signal is fed to a 7.5 hp four-quadrant laboratory dynamometer through a D/A converter. The dynamometer generates this programmable torque on the shaft of the generator under test; the wind turbine inertia was not considered on the model, because it would lead to extremely slow responses, not practical for the laboratory implementation. The system inertia is the one actually present on the machine plus the dynamometer shaft.

Experimental Study

Figure 10 shows the experimental set-up used to validate the controls. The converters were built using POWEREX IGBT intelligent power modules type PM50RSA060. Basically, it is a six-pack plus brake hybrid module incorporating the gate drive and protection circuitry, and has the ratings of 600 V, 50 A (inverter peak current). The induction machine was an ordinary NEMA Class B type with 220 V,

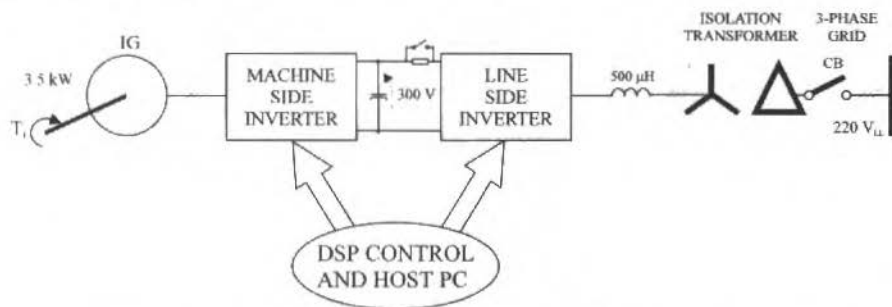


Fig. 10 Experimental Set-Up Used to Validate the Controls

3.5 hp rating. The dc link voltage was designed to be 300 V considering the voltage rating constraint of the IGBTs. Since the line-side converter has to always run in PWM (step-down) mode, the ac line voltage was appropriately reduced by a transformer. A 7.5 hp four-quadrant laboratory dynamometer was used to emulate the wind turbine (programmable shaft torque), as mentioned before. Figure 11 shows the static characteristics of the wind turbine at different wind velocity where the turbine oscillatory torque components were intentionally suppressed. Basically, these are families of curves for turbine output power, turbine torque and line-side output power as function of wind velocity and sets of wind velocities. For example, for a fixed wind velocity, as the generator speed increases, the torque and power outputs first increase and then decrease. If the generator speed remains constant and the wind velocity increases, the turbine power, turbine torque and line power will increase and then will tend to saturate. The slope of increase is higher with higher generator speed. The fuzzy controllers were fine-tuned during experimentation. Figure 12 shows the performance of the system with FLC-1, FLC-2, and FLC-3 when the wind velocity is ramped up and down. The turbine was modeled with oscillatory torque and some turbulence was added with the wind velocity to verify the robustness of controller FLC-3. As the generator speed is increased by FLC-1, the line output power gradually increases, but the line power indicates some dips which require explanation. As generator speed command is incremented

by FLC-1, the machine accelerates to the desired speed with the power extracted from the turbine output power. As a result, line power temporarily sags until boosted by the turbine power at steady state. With a large increment of speed command, the direction of P_O can even reverse. In order to prevent such conditions, the maximum speed command increment was limited to a reasonably small value (75 RPM) (that increases the search time) and had a ramp shape. The slope of the ramp can be adjusted to control the power dips. Note that the speed command decrement will have an opposite effect; i.e., the generator tends to decelerate, giving bumps in the output power. In order to test the robustness of the fuzzy speed controller FLC-3, the oscillatory torque components were added to the dynamometer wind turbine model. Figure 13 shows the smooth speed profile (top) at 900 rpm with oscillatory torque (bottom) that swings from 4.35 Nm to 5.65 Nm with average value of 5 Nm. The additional pulsating torque introduced by FLC-2 is also highly attenuated by the FLC-3 controller. Next, the turbine-generator system was operated at constant speed (940 rpm) and the wind velocity was varied. At each operating point, the FLC-1 and FLC-2 controllers were operated in sequence and the corresponding boost of power was observed. From these data, the respective efficiency improvement was calculated and plotted in Table 3. The table indicates that the efficiency gain is significant with FLC-1 control compared to that of FLC-2. The gain due to FLC-1 falls to zero near 0.7 pu wind velocity where the generator speed is optimum for that wind velocity, and then rises. The gain due to FLC-2 decreases as the wind velocity increases because of higher generator loading. During all the operation modes, the line current was sinusoidal with unity power factor, as shown in Fig. 14. The out-of-phase current wave indicates that the system is in generation mode.

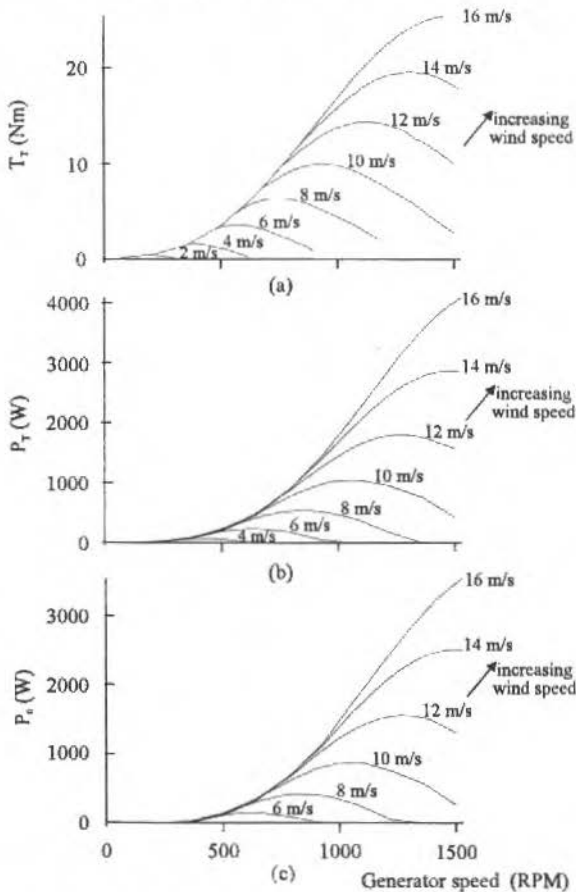


Fig. 11 Wind Turbine Static Characteristics (a) Turbine Power, (b) Turbine Torque, (c) Line Power

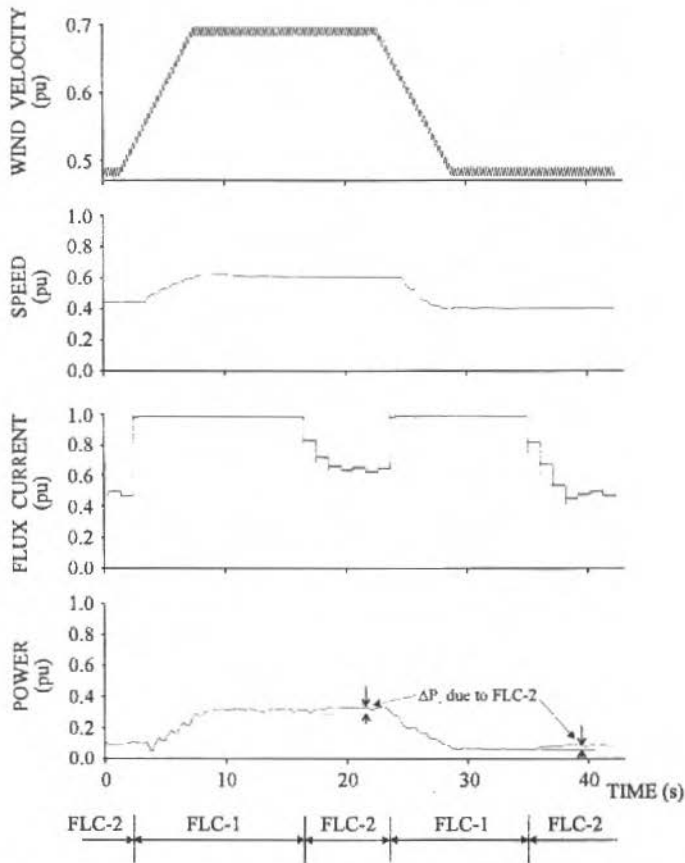


Fig. 12 Operation of Fuzzy Controls: (a) Wind Velocity, (b) Generator Speed, (c) Flux Current, (d) Output Power

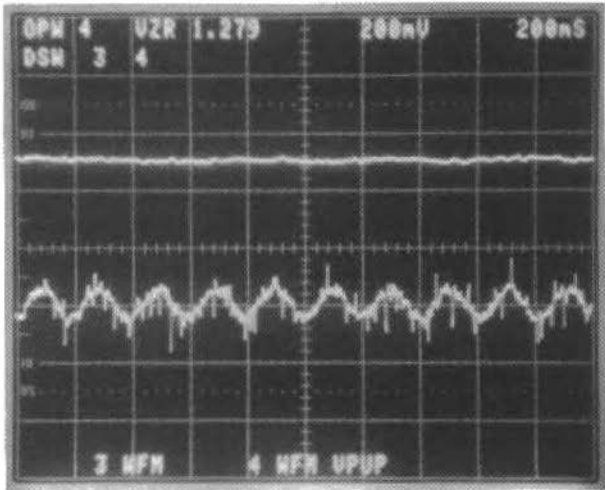


Fig. 13 Robust Performance of Fuzzy Controller FLC-3 Against Pulsating Turbine Torque

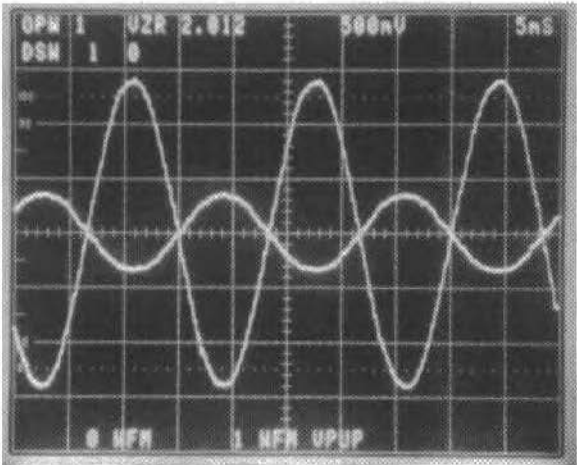


Fig. 14 Line Side Voltage and Current Waves Showing Unity Power Factor Operation

Table 3 Power Enhancement Due to Fuzzy Logic Control

Wind Velocity (pu)	Power for fixed ω_r^* (0.575 pu)	ΔP_o due to FLC-1 (pu)	ΔP_o due to FLC-2 (pu)	Efficiency due to FLC-1 (%)	Efficiency due to FLC-2 (%)	Total efficiency due to fuzzy control (%)	Average efficiency due to FLC-1 (%)	Average efficiency due to FLC-2 (%)	Total average efficiency (%)
0.5300	0.0581	0.0626	0.0343	107.67	28.45	136.12			
0.5691	0.1250	0.0990	0.0466	79.25	20.81	100.06			
0.6083	0.1740	0.0893	0.0390	51.36	14.82	66.18			
0.6475	0.2383	0.0643	0.0321	27.01	10.63	37.64			
0.6866	0.2941	0.0269	0.0250	9.17	7.78	16.95			
0.7258	0.3302	0.0024	0.0193	0.73	5.81	6.54	35.525	8.616	44.141
0.7650	0.3604	0.0064	0.0183	1.77	5.00	6.77			
0.8041	0.4330	0.0399	0.0151	9.21	3.20	12.41			
0.8433	0.4500	0.0890	0.0145	19.78	2.69	22.47			
0.8825	0.5000	0.1522	0.0132	30.44	2.02	32.46			
0.9216	0.5300	0.2135	0.0124	40.28	1.67	41.95			
0.9608	0.6250	0.3099	0.0041	49.58	0.44	50.02			

Conclusion

A complete fuzzy logic based 3.5 kW variable speed wind generation system has been designed and the performance was evaluated thoroughly in laboratory. The power circuit was based on induction generator and double-sided IGBT PWM converters, and the control hardware was based on dual TMS320C30 DSPs. The system uses three fuzzy controllers: a controller that tracks the generator speed with the varying wind velocity to optimize the turbine aerodynamic efficiency, the second controller programs the machine flux at light load to optimize the generator-converter system efficiency, and the third controller gives robust performance of the generator speed control system. The advantages of the fuzzy control are that the control algorithms are universal, give fast convergence, parameter insensitive and they accept noisy and inaccurate signals. The performance of the system was found to be excellent with all the controls. A higher power unit for field installation can use the same controller as long as the power circuit rating is boosted.

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Stability and Dynamics of Offshore Single Point Mooring Systems

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Abstract

The Floating Production Storage and Offloading System (FPSO's) is a modern concept for floating offshore oil exploration units, moored in deep water. 'Turret' and 'Mono-Buoys' are similar types of Single Point Mooring systems (SPM) envisaged for the stationkeeping task. Nevertheless, the highly non-linear dynamic nature of this kind of system may give rise to a rich behaviour scenario that may comprise from simple pitchfork point equilibrium bifurcations to Hopf bifurcations (limit cycles), or even chaotic regimes. Standard linearised stability analysis may be not sufficient anymore to deal with the design problem. Bifurcation theory and modern system dynamics form then a proper theoretical basis for the analysis. This paper addresses the stability problem and discusses a number of interesting dynamic behaviors that arise in steady current. A self-excited autonomous and dissipative non-linear system of equations governs the system dynamics. A classical 'hydrodynamic derivatives' model form the core for hydrodynamic forces description. Following Papoulias and Bernitsas (1988), some classical results on the stability problem are recovered. Then, reinterpreting the equilibrium analysis, it is also shown that bifurcation theory enables one not only to predict but also to qualify equilibrium pitchfork bifurcation scenarios, if super- or sub-critical. It is shown that the algebraic sign of the third-order derivative of the lateral force with respect to the lateral component of relative velocity governs the type of bifurcation scenario. When super-critical pitchfork bifurcation scenario is present a condition for structural stability loss is established and discussed. Hopf bifurcations (limit cycles) are also presented and discussed.

Keywords: System Dynamics, Bifurcation Theory, Mooring System

Introduction

The Floating Production Storage and Offloading System (FPSO's) is a modern concept for floating offshore oil exploration units, moored in deep water. A tanker is moored offshore and oil is stored before being transported by shuttle tankers that periodically are connected to the mother ship in a tandem formation. The vessels are subject to the environmental loads, due to the concomitant action of ocean currents, waves and wind. Single Point Mooring (SPM) systems are alternatives to conventional spread systems, envisaged for the stationkeeping task. The primer motivation of such systems is to allow the ship to be aligned with the 'resultant' of the environmental forces, diminishing motions and structural loads on the mooring lines, hawsers (the cable through which the ship is attached to the mono-buoy) and risers.

The conventional SPM system is a mooring system composed by a mono-buoy, moored to the sea bed by means of cables and chains and to which the FPSO, a specially converted tanker, is attached through a 'hawser cable'. Not only the length but also the elastic characteristics of the cable and the attachment point position are important control parameters, concerning the stability and the dynamics of the system. In this paper we shall refer to mono-buoy systems (or 'hawser cable' systems) simply as SPM.

The 'Turret' type is a special kind of single point mooring system composed by a huge bearing system, fixed directly to the ship, the hawser cable being eliminated, and moored to the sea-bed, as shown in Fig. 1.

If wind and waves actions are not considered and if ocean current is taken as steady, a self-excited autonomous non-linear system of ordinary differential equations can be shown to govern the system dynamics. The highly non-linear dynamic nature of this kind of system gives rise to a rich behaviour scenario that may comprise from simple pitchfork point equilibrium bifurcations to Hopf bifurcations (limit cycles), or even chaotic regimes. Bifurcation theory and modern system dynamics may form a proper theoretical basis for the analysis, although standard linearised stability analysis remains suitable for preliminary design purposes.

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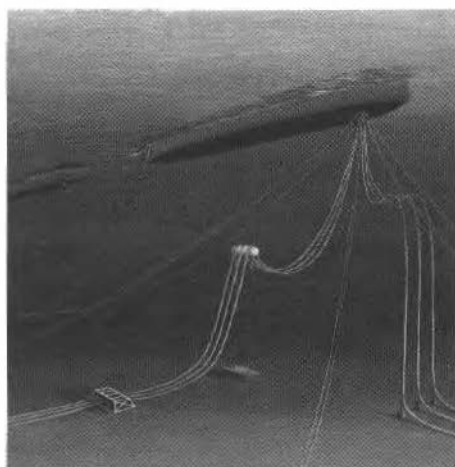


Fig. 1 FPSO Moored at a 'Turret' System With a Shuttle Tanker in Tandem Formation (Cortesia: Orcina Cable Protection)

Under the bifurcation theory approach some research work has been done in this subject, primarily motivated by a close-related problem of a towed ship in either a straight course or maneuvering in a port site. Bernitsas and Kekridis (1985) treat the towed-ship problem. Papoulias and Bernitsas (1988) take the single point mooring problem into attention. Recently, Bernitsas and Garza-Rios (1995) have studied the dynamics of some offshore spread but slacken mooring systems, that exhibit the same sort of dynamic behavior. Under such an approach the general equation of motion is derived on a "hydrodynamic derivative" model basis, in which the hydrodynamic forces due to the relative motion with respect to the water are represented through Taylor series expansions given in terms of relative velocity components.

Following Papoulias and Bernitsas (1988) this paper recovers and enlarges the analysis, addressing the stability problem again and discussing a number of interesting dynamic behaviors that arise for a 'Turret' or a SPM in steady current. Under a third-order model of the "hydrodynamic derivatives" type, it is shown that bifurcation theory enables one not only to predict but also to qualify point equilibrium instability. Two excludent *pitchfork bifurcation* scenarios are shown to exist: *sub-* and *super-critical*. As well known, the position of mooring line attachment at the ship is the control parameter governing equilibria bifurcation. It is also shown, in this paper, that this parameter can be responsible for a loss of structural stability of the system, switching bifurcation scenarios, whose type is controlled primarily by the 'hydrodynamic derivatives' coefficients. The sign of the third derivative of the lateral force with respect to the lateral velocity, for instance, is shown to govern the type of bifurcation scenario that would appear. The occurrence of Hopf bifurcations (limit cycles) are also exemplified and discussed.

The Governing Equations

We follow closely Papoulias and Bernitsas (1988). A classical "hydrodynamic derivatives model", extracted from maneuvering theory, see, e.g., Abkowitz (1972), is used in order to simulate the action of the relative current on the ship. Waves and wind are not considered in the present paper, neither are the hydrodynamic forces acting directly on the mooring lines of the 'Turret', for instance. It is also out of the scope of the present work to discuss the pertinence of this type of hydrodynamic model, although, as we shall show, the stability and dynamic scenarios are strongly dependent on the hydrodynamic coefficients. A number of alternative approaches does exist; see e.g., Faltinsen et al., (1979) and Whichers (1988). However, we are primarily interested in discussing the stability problem, rather than the robustness of the hydrodynamic model, at least at the present moment.

Let, then $Oxyz$ be a right-handed fixed reference frame, x being oriented in the opposite sense of the current velocity vector and z pointing upwards. Let GXY be a coordinate system attached to the

floating unit, where G is the center of mass, X being oriented towards the bow. We restrict ourselves to the motion in the horizontal plane. Let be, also,

u, v : the relative velocity components of G with respect to the water, in the AX e AY directions, respectively;

ψ : the yaw angle (xGX)

$\dot{x}, \dot{y}$: the relative velocity components of G with respect to the fixed frame;

P : the point of mooring;

U : the current velocity intensity;

x_p : the distance GP ;

l : the distance OP , where O is the fixed point where the hawser is attached (SPM case);

m : the floating unit mass;

I_z : the moment of inertia with respect to GZ ;

T : the tension in the cable (SPM case) or the mooring restoring force ('turret' case).

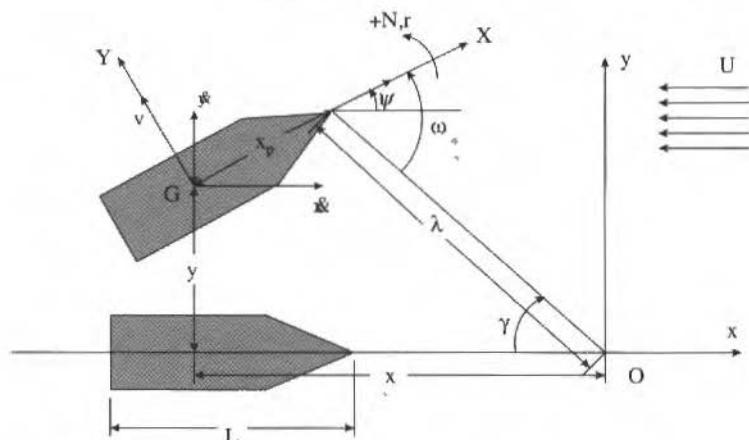


Fig. 2 Reference Frames and Geometric Definitions

Being $\mathbf{v}$ the velocity vector of G we have:

$$\mathbf{v} = (u \cos \psi - v \sin \psi - U) \mathbf{i} + (u \sin \psi + v \cos \psi) \mathbf{j} \quad (1)$$

or else,

$$\begin{aligned} \dot{x} &= u \cos \psi - v \sin \psi - U \\ \dot{y} &= u \sin \psi + v \cos \psi \end{aligned} \quad (2)$$

For further reference, we also write the inverse form of (2).

$$\begin{aligned} u &= (\dot{x} + U) \cos \psi + \dot{y} \sin \psi \\ v &= -(\dot{x} + U) \sin \psi + \dot{y} \cos \psi \end{aligned} \quad (3)$$

Let also, by definition,

$$r = \dot{\psi} \quad (4)$$

and the following geometrical relations (see Fig. 2)

$$\omega = \gamma + \psi \quad (5)$$

$$l^2 = (y + x_p \sin \psi)^2 + (x + x_p \cos \psi)^2 \quad (6)$$

$$\sin \gamma = \frac{l}{l} (y + x_p \sin \psi) \quad (7)$$

$$\cos \gamma = -\frac{l}{l} (x + x_p \cos \psi) \quad (8)$$

noticing that (5), (7) and (8) are strictly valid only for $l \neq 0$, being γ not defined for $l = 0$.

The acceleration of the center of mass, with respect to an inertial reference frame, but written in the vessel's frame, reads $\mathbf{a} = (\ddot{u} - vr)\mathbf{I} + (\dot{v} + ur)\mathbf{J}$,

The equations of motions are then written,

$$\begin{aligned} -A^{11}\ddot{u} + A^{22}vr + T \cos \omega + X(u, v, r) &= m(\ddot{u} - vr) \\ -A^{33}\dot{v} - A^{11}ur - A^{23}\dot{r} - T \sin \omega + Y(u, v, r) &= m(\dot{v} + ur) \\ -A^{33}\dot{r} - A^{32}(\dot{v} + ur) + N(u, v, r) - Tx_p \sin \omega &= l_Z \dot{r} \end{aligned} \quad (9)$$

where $X(u, v, r)$, $Y(u, v, r)$ e $N(u, v, r)$ are velocity dependent hydrodynamic forces acting on the floating unit and A^{mm} is the added inertia tensor, symmetric by construction (see, Newman, 1978, e.g.).

The SPM Case

In the SPM case let T , the tension in the cable, be given (Papoulias and Bernitsas, 1988) by

$$T = \begin{cases} S_b p \left(\frac{l - l_w}{l_w} \right)^q & l \geq l_w \\ 0 & l < l_w \end{cases} \quad (10)$$

where l_w is the original length of the cable (unloaded) and S_b the limit strength, being p and q empirical constants. By defining the state vector

$$\mathbf{x} = (u, v, r, l, \gamma, \omega)^T \quad (11)$$

Equations (10), (11) and (9) are transformed into a system of six first-order ordinary differential equations (Papoulias and Bernitsas, 1988),

$$\begin{aligned}\dot{x}_1 &= \frac{I}{m+A^{11}} \left(F_1(x_1, x_2, x_3) + T \cos x_6 + (m+A^{22})x_2x_3 - A^{23}x_3^2 \right) \\ \dot{x}_2 &= \frac{(I_2+A^{33})}{D} \left(F_2(x_1, x_2, x_3) - T \sin x_6 - (m+A^{11})x_1x_3 \right) \\ &\quad - \frac{A^{23}}{D} \left(F_3(x_1, x_2, x_3) - A^{23}x_1x_3 - Tx_p \sin x_6 \right) \\ \dot{x}_3 &= -\frac{A^{32}}{D} \left(F_2(x_1, x_2, x_3) + (m+A^{11})x_1x_3 - T \sin x_6 \right) \\ &\quad + \frac{(m+A^{22})}{D} \left(F_3(x_1, x_2, x_3) - A^{23}x_1x_3 - Tx_p \sin x_6 \right) \\ \dot{x}_4 &= x_2 \sin x_6 - x_1 \cos x_6 + U \cos x_5 + x_px_3 \sin x_6 \\ \dot{x}_5 &= \frac{I}{x_4} \left(x_1 \sin x_6 + x_2 \cos x_6 - U \sin x_5 + x_px_3 \cos x_6 \right) \\ \dot{x}_6 &= x_3 + \frac{I}{x_4} \left(x_1 \sin x_6 + x_2 \cos x_6 - U \sin x_5 + x_px_3 \cos x_6 \right)\end{aligned}\tag{12}$$

where $D = (m+A^{22})(I_2+A^{33}) - (A^{23})^2$, and

$$\begin{aligned}F_1(x_1, x_2, x_3) &= X_{\mu}x_1 + \frac{I}{2}X_{\mu\mu}x_1^2 + \frac{I}{6}X_{\mu\mu\mu}x_1^3 + mx_2x_3 \\ F_2(x_1, x_2, x_3) &= Y_{\nu}x_2 + \frac{I}{6}Y_{\nu\nu}x_2^3 + Y_rx_3 + \frac{I}{6}Y_{rrr}x_3^3 - mx_1x_3 \\ F_3(x_1, x_2, x_3) &= N_{\nu}x_2 + \frac{I}{6}N_{\nu\nu}x_2^3 + N_rx_3 + \frac{I}{6}N_{rrr}x_3^3\end{aligned}\tag{13}$$

are the “generalized” hydrodynamic forces given in terms of the well known hydrodynamic derivatives, up to third order, where we take the subscripts for partial derivatives. We notice that (12) is a nonlinear dissipative and autonomous dynamic system.

The ‘Turret’ Case

Another set of state variables, appropriate for the ‘Turret’ case, could be used instead. Let

$$\mathbf{x} = (u, v, r, x, y, \psi)^t\tag{14}$$

From (2), (3), (4), we get,

$$\begin{aligned}
\dot{x}_1 &= \frac{l}{m + A^{11}} \left(F_1(x_1, x_2, x_3) + T \cos \omega + (m + A^{22})x_2x_3 - A^{23}x_3^2 \right) \\
\dot{x}_2 &= \frac{(I_Z + A^{33})}{D} \left(F_2(x_1, x_2, x_3) - T \sin \omega - (m + A^{11})x_1x_3 \right) \\
&\quad - \frac{A^{23}}{D} \left(F_3(x_1, x_2, x_3) - A^{23}x_1x_3 - Tx_p \sin \omega \right) \\
\dot{x}_3 &= -\frac{A^{32}}{D} \left(F_2(x_1, x_2, x_3) + (m + A^{11})x_1x_3 - T \sin \omega \right) \\
&\quad + \frac{(m + A^{22})}{D} \left(F_3(x_1, x_2, x_3) - A^{23}x_1x_3 - Tx_p \sin \omega \right) \\
\dot{x}_4 &= x_1 \cos x_6 - x_2 \sin x_6 - U \\
\dot{x}_5 &= x_1 \sin x_6 + x_2 \cos x_6 \\
\dot{x}_6 &= x_3
\end{aligned} \tag{15}$$

with, again, $D = (m + A^{22})(I_Z + A^{33}) - (A^{23})^2$ and

$$\left\{ \begin{array}{l} \text{when } l > 0, \quad \omega = \begin{cases} x_6 + \arcsen\left(\frac{x_5 + x_p \sin x_6}{l}\right); & \text{if } (x_4 + x_p \cos x_6) < 0 \\ x_6 - \arcsen\left(\frac{x_5 + x_p \sin x_6}{l}\right) + \pi; & \text{if } (x_4 + x_p \cos x_6) \geq 0 \end{cases} \\ \text{when } l = 0, \quad \omega(t_k) = \omega(t_{k-1}) \end{array} \right. \tag{16}$$

$$l = \sqrt{(x_5 + x_p \sin x_6)^2 + (x_4 + x_p \cos x_6)^2} \tag{17}$$

Stability Analysis

Standard Linear Analysis

Let the governing equations be written

$$\dot{x} = f(x) \tag{18}$$

If $\bar{x}$ is a fixed point then,

$$f(\bar{x}) = 0 \tag{19}$$

Let $\xi(t) = x(t) - \bar{x}$ be a perturbation around the fixed point. Then

$$\dot{\xi} = A\xi, \quad \xi \in \mathbb{R}^6, \quad A \in \mathbb{R}^{6 \times 6} \tag{20}$$

with

$$\mathbf{A} = \mathbf{Df}(\bar{\mathbf{x}}) \quad (21)$$

being the Jacobian of $\mathbf{f}$ at $\bar{\mathbf{x}}$. From linear systems theory, asymptotic stability exists if all the eigenvalues of $\mathbf{A}$ are in the complex left plane. We could use the Routh Hurwitz criterion, for instance, in order to determine the conditions under which this necessary and sufficient condition is fully satisfied; see, e.g., Fernandes and Aratanha (1996).

For a given floating unit, being known the hydrodynamic coefficients, the control parameters for the SPM case are (U, x_p, l_w) . For the 'turret' case (U, x_p, β) are the parameters, where β indicates a subset of parameters characterizing the mooring restoring function. Besides changes in $\bar{\mathbf{x}}$, these parameters are responsible for qualitative variations in the stability and in the dynamic behavior of the system. As we have noticed before, we treat a nonlinear dissipative and autonomous dynamic system, in a hexa-dimensional space. Therefore equilibrium bifurcations, Hopf bifurcations, and even chaotic regimes can be expected. We should remember that nonlinear dissipative terms can lead to Hopf bifurcations, making the system structurally unstable, as well as nonlinear, restoring forces to equilibrium bifurcations. Different scenarios, where a number of attractors compete with each other, can lead to chaos. We also notice that, in the SPM case the desirable equilibrium is obviously given by

$$\bar{\mathbf{x}} = (U, 0, 0, l_0, 0, 0)^T \quad (22)$$

where

$$l_0 = l_w \left\{ 1 + \left(\frac{X(U, 0, 0)}{S_b p} \right)^{\frac{1}{q}} \right\} = l_w \left\{ 1 + \left(\frac{T_0}{S_b p} \right)^{\frac{1}{q}} \right\} \quad (23)$$

is the stretched length of the cable in this position, whereas, in the 'Turret' case the desirable fixed point is

$$\bar{\mathbf{x}} = (U, 0, 0, 0, 0, 0) \quad (24)$$

Equilibrium Bifurcations

We take U as invariant. The array of control parameters will be denoted by

$$\mu = (x_p, l_w)^T \quad (25)$$

in the SPM case and by

$$\mu = (x_p, \beta)^T \quad (26)$$

in the 'Turret' case. The dynamical system will be written in the form

$$\dot{\mathbf{x}} = \mathbf{f}(\mathbf{x}, \mu) \quad , \quad \mathbf{x} \in \mathbb{R}^6 \quad \mu \in \mathbb{R}^2 \quad (27)$$

Supercritical Pitchfork Bifurcation

There is an obvious plane of symmetry, condition under which pitchfork bifurcations usually arise. According to Papoulias and Bernitsas (1988), the equilibrium equation can be reduced to (see Appendix A)

$$\frac{1}{6} (N_{vvv} - x_p Y_{vvv}) \bar{v}^3 + (N_v - x_p Y_v) \bar{v} = 0 \quad (28)$$

This equation can be rewritten in the form of a cubic equation in $\bar{v}$

$$f(\bar{v}, \lambda) = \bar{v}^3 - \lambda \bar{v} = 0 \quad (29)$$

where, as defined in Papoulias and Bernitsas (1988),

$$\lambda = -6 \frac{(N_v - x_p Y_v)}{(N_{vvv} - x_p Y_{vvv})} \quad (30)$$

We should notice that this control parameter depends on a ratio between two differences. Thus, errors in evaluating the hydrodynamic coefficients can be highly amplified. At the bifurcation point $(\bar{v}_0, \lambda_0)$ both

$$\begin{aligned} f(\bar{v}, \lambda) &= 0 \\ f_{\bar{v}}(\bar{v}, \lambda) &= \frac{\partial f}{\partial \bar{v}} = 0 \end{aligned} \quad (31)$$

must be satisfied. Then, from (31)

$$\bar{v} = \begin{cases} 0 \\ \pm \sqrt{\lambda} \end{cases} \quad (32)$$

and, from (31), $\lambda=0$. Therefore

$$(\bar{v}_0, \bar{\lambda}_0) = (0, 0) \quad (33)$$

and hence, from (30)

$$x_{p0} = \frac{N_v}{Y_v} \quad (34)$$

that is the critical value below which supercritical pitchfork bifurcation occurs.

On the other hand, taking $(\dot{x}, \dot{y}, \dot{\psi}) = (0, 0, 0)$ in (3) we have the equilibrium values for the relative velocity components given by

$$\begin{aligned} u &= \bar{u} = U \cos \bar{\psi} \\ v &= \bar{v} = -U \sin \bar{\psi} \end{aligned} \quad (35)$$

After equilibrium bifurcation occurs, $\bar{v} = \pm\sqrt{\lambda}$, and so,

$$\text{sen}\bar{\psi} = \pm \frac{\sqrt{\lambda}}{U} \tag{36}$$

gives the new equilibrium positions. If λ is negative there is no real stable bifurcated solutions. Notice, also, that $\lambda \leq U^2$ must be satisfied, as well. In other words, at $\bar{\psi} = \pm\pi/2$ structural stability is broken. We shall return to this point later on.

Structural Stability and Subcritical Pitchfork Bifurcation

The previous analysis, recovered from Papoulias and Bernitsas (1988), takes the hydrodynamic derivatives given in terms of the relative velocity component v , defined by Eqs. (2) and (3), as usually done in marine hydrodynamics. We can think of reconducting such a reasoning, but now in terms of the heading angle ψ . perhaps enhancing our physical understanding. As a direct result, a condition for stability loss concerning supercritical pitchfork equilibrium bifurcations is discussed, and a new type, the subcritical one (or cathastrophic) appears vividly. For, taking (3), solved in terms of (u,v) we get,

$$\begin{aligned} \frac{\partial v}{\partial \psi} &= -u \\ \frac{\partial u}{\partial \psi} &= v \end{aligned} \tag{37}$$

valid for all states. If we assume $(\dot{x}, \dot{y}) = (0,0)$ - as in a *captive model experiment*, where a *restrained* small-scale model of the vessel is driven by a constant current velocity U , being measured the hydrodynamic forces and moment $(X^{(c)}, Y^{(c)}, N^{(c)})$, acting upon the model, from Eq. (3) we get¹

$$\begin{aligned} u &= U \cos \psi^{(c)} \\ v &= -U \text{sen} \psi^{(c)} \end{aligned} \tag{38}$$

We can think of $(X^{(c)}, Y^{(c)}, N^{(c)})$ as functions of $\psi = \psi^{(c)}(u,v) = \arctan(-v/u)$ and U , such that

$$\begin{aligned} X^{(c)} &= X^{(c)}(\psi; U) \\ Y^{(c)} &= Y^{(c)}(\psi; U) \\ N^{(c)} &= N^{(c)}(\psi; U) \end{aligned} \tag{39}$$

Accordingly, with the use of (37), we get

$$Y_v^{(c)} = \frac{\partial Y^{(c)}}{\partial v} = \frac{\partial Y^{(c)}}{\partial \psi} \frac{\partial \psi}{\partial v} = -\frac{1}{u} Y_{\psi}^{(c)}$$

that, after (38) is used, reads

¹ The subscript c stands for captive

$$Y_v^{(c)} = -\frac{I}{U \cos \psi^{(c)}} Y_\psi^{(c)} \quad (40)$$

leading, as can be easily verified, to

$$Y_{vv}^{(c)} = -\frac{I}{U^3 \cos^3 \psi^{(c)}} Y_{\psi\psi\psi}^{(c)} \quad (41)$$

Analogously,

$$N_v^{(c)} = -\frac{I}{U \cos \psi^{(c)}} N_\psi^{(c)} \quad (42)$$

and also,

$$N_{vv}^{(c)} = -\frac{I}{U^3 \cos^3 \psi^{(c)}} N_{\psi\psi\psi}^{(c)} \quad (43)$$

Equations (40)-(43) are valid only under $(\dot{x}, \dot{y}) = (0, 0)$, i.e. if (38) holds. Substituting (40)-(43) in (28) it follows at equilibrium $(\psi = \psi^{(c)}(\bar{u}, \bar{v}) = \bar{\psi})$, that²

$$x_p \{ Y_\psi \tan \bar{\psi} + \frac{I}{6} Y_{\psi\psi\psi} \tan^3 \bar{\psi} \} = N_\psi \tan \bar{\psi} + \frac{I}{6} N_{\psi\psi\psi} \tan^3 \bar{\psi} \quad (44)$$

Notice that (44) is satisfied for any $\bar{\psi}$, particularly at $\bar{\psi} = 0$ and $\bar{\psi} = \pi$, fixed points. For these two fixed points we may have super- (as previously presented) or sub-critical pitchfork bifurcations. But, as we shall see, confirming results presented in the last section, if super-critical pitchfork bifurcation scenario exists, $\bar{\psi} = \pm \pi/2$ will be an unstable solution, leading to a former (locally) sub-critical bifurcation, when the dynamic system loses structural stability.

Firstly we shall restrict ourselves to a local analysis around $\psi = 0$. Obviously, the same reasoning can be applied around $\psi = \pi$ as well. Equation (44) can then be written,

$$x_p \{ Y_\psi \bar{\psi} + \frac{I}{6} Y_{\psi\psi\psi} \bar{\psi}^3 \} = N_\psi \bar{\psi} + \frac{I}{6} N_{\psi\psi\psi} \bar{\psi}^3 \quad (45)$$

or else,

$$\bar{\psi}^3 - \lambda_c \bar{\psi} = 0 \quad (46)$$

where

$$\lambda_c = -6 \frac{(N_\psi - x_p Y_\psi)}{(N_{\psi\psi\psi} - x_p Y_{\psi\psi\psi})} \quad (47)$$

² From now on, $Y_\psi = Y_\psi^{(c)}|_{(\psi = \bar{\psi})}$, etc., are implied.

plays the role of the λ parameter, previously defined. As before, Eq. (46) admits up to three roots

$$\begin{aligned}\bar{\psi} &= 0 \\ \bar{\psi} &= \pm\sqrt{\lambda_c}\end{aligned}\quad (48)$$

Let, now, $M(\psi)$ be a cubic restoring moment, such that

$$M(\psi) = \frac{I}{6}\psi^3 (x_p Y_{\psi\psi\psi} - N_{\psi\psi\psi}) + \psi (x_p Y_{\psi} - N_{\psi}) \quad (49)$$

and take a non-dissipative one-dimensional dynamic system (one degree of freedom)

$$I\ddot{\psi} + M(\psi) = 0 \quad (50)$$

as representing our system around the considered fixed point. Equation (50) can be put in the form

$$\ddot{\psi} + \alpha(\psi^3 - \lambda_c \psi) = 0 \quad (51)$$

where

$$\alpha = \frac{x_p Y_{\psi\psi\psi} - N_{\psi\psi\psi}}{6I} \quad (52)$$

Two possibilities do exist: (i) $\alpha > 0$; (ii) $\alpha < 0$. Let us analyze the alternatives that appear:

(i) If $\alpha > 0$; or, equivalently, $\begin{cases} x_p > N_{\psi\psi\psi}/Y_{\psi\psi\psi} \text{ if } Y_{\psi\psi\psi} > 0 \\ x_p < N_{\psi\psi\psi}/Y_{\psi\psi\psi} \text{ if } Y_{\psi\psi\psi} < 0 \end{cases}$; two different situations arise:
(ia) if $\lambda_c < 0$:

In this case $\bar{\psi} = 0$ is the only fixed point, stable, a center.

As $\alpha > 0$, and $x_p < v$ always, so that, from (40), $x_{\psi} > v$ if $v > v$ and $\cos \psi > v$, we then have the following condition,

$$x_p > \frac{N_{\psi}}{Y_{\psi}} = \frac{N_v}{Y_v} \quad (53)$$

(ib) if $\lambda_c > 0$, conversely, we then have,

$$x_p < \frac{N_{\psi}}{Y_{\psi}} = \frac{N_v}{Y_v} \quad (54)$$

In this case three are the fixed points,

$\bar{\psi} = 0$: a saddle point (unstable, therefore)

$\bar{\psi} = \pm\sqrt{\lambda_c}$: nodes (stable)

This situation corresponds to the super-critical bifurcation, previously studied.

(ii) If $\alpha < 0$; or, equivalently, $\begin{cases} x_p < N_{\psi\psi\psi}/Y_{\psi\psi\psi} : \text{if } Y_{\psi\psi\psi} > 0 \\ x_p > N_{\psi\psi\psi}/Y_{\psi\psi\psi} : \text{if } Y_{\psi\psi\psi} < 0 \end{cases}$; again, two different situations

arise:

(iia) If $\lambda_c < 0$:

$\bar{\psi} = 0$ is the only fixed point, unstable, a *saddle point*.

As $\alpha < 0$, and, as mentioned before, $\gamma < 0$ always, so that, from (40), $Y_{\psi} > 0$ if $U > 0$ and $\cos\psi > 0$, we then have valid the following condition.

$$x_p < \frac{N_{\psi}}{Y_{\psi}} = \frac{N_v}{Y_v} \quad (55)$$

(iib) If $\lambda_c > 0$, conversely, we then have,

$$x_p > \frac{N_{\psi}}{Y_{\psi}} = \frac{N_v}{Y_v} \quad (56)$$

In this case three are the fixed points,

$\bar{\psi} = 0$: a *center* (stable)

$\bar{\psi} = \pm\sqrt{\lambda_c}$: *saddle points* (unstable).

Notice that $\lambda_c = 0$ is a bifurcation point, therefore. The *stability condition* $x_p > N_{\psi}/Y_{\psi}$ is a necessary condition, either for $\alpha > 0$ or $\alpha < 0$. This condition (53), is the same one that has been previously achieved, (34). It is also the same necessary condition that emerges if the Routh-Hurwitz criterion is applied to the linearised system, (see Fernandes, 1995).

If $x_p < N_{\psi}/Y_{\psi}$ and if $\alpha > 0$, the pitchfork bifurcation is of the *supercritical* (smooth) type, whereas, if $\alpha < 0$ it is of the *subcritical* (catastrophic) type. Notice that the *sign of the third-order derivative* r_{vvv} , that under equilibrium hypotheses is related to $Y_{\psi\psi\psi}$ as given by (41), controls the bifurcation scenario, if super- or sub-critical.

Summarizing, stability around $\bar{\psi} = 0$ exists if the necessary condition (53) holds. Otherwise ($x_p < N_{\psi}/Y_{\psi}$), two bifurcation scenarios arise:

(i) $\alpha > 0$: *supercritical bifurcation* and then a new equilibrium point is reached:

$$\bar{\psi} = \pm\sqrt{\lambda_c} \text{ (local analysis); } \text{sen}\bar{\psi} = \pm\frac{\sqrt{\lambda}}{U} \text{ (global analysis);}$$

(ii) $\alpha < 0$: *subcritical bifurcation*. The system would not reach, (smoothly) another point of equilibrium. As a matter of fact, looking at the original equation, $\bar{\psi} = \pi$ would be reached as the new stable fixed point, of the center type.

Notice that, although somewhat simplified, the present analysis do consider terms up to the third-order, retaining all the features of the system regarding equilibrium point stability analysis. This kind of feature is unaccessible under the Routh-Hurwitz criterion applied to the linearised equations. Notice also that dissipation terms, not considered in this simplified analysis, will transform *centers* in *stable foci*.

Another important point is that, even when a supercritical (smooth) bifurcation occurs, there is a limit value for a stable bifurcated equilibrium, namely $\bar{\psi} = \pm\pi/2$. This corresponds to the condition $\lambda = U^2$, as already mentioned. In fact, (44) can be written in the form (50), with the restoring moment given as

$$M(\psi) = \frac{I}{6} \tan^3 \psi (x_p Y_{\psi\psi\psi} - N_{\psi\psi\psi}) + \tan \psi (x_p Y_{\psi} - N_{\psi}) \tag{57}$$

Whenever $\psi \rightarrow \pm \pi/2$ the *restoring moment* changes sign suddenly, transforming itself into a *repulsive moment*. Thus stability is broken at $\psi \rightarrow \pm \pi/2$ and, usually $\bar{\psi} = \pi$ will be the new attractor point. This is in fact a loss in the structural stability for the dynamic system, considering up to the third-order hydrodynamic coefficients. Notice also that, from (40)-(43) in (30), and comparing it to (47), that,

$$\lambda = U^2 \cos^2 \bar{\psi} \lambda_c(\bar{\psi}) \tag{58}$$

where the parameter λ_c is determined for $\psi = \bar{\psi}$ from (47). The bifurcated equilibrium Eq. (36) can then be transformed into

$$\tan \bar{\psi} = \pm \sqrt{\lambda_c(\bar{\psi})} \tag{59}$$

This equation, together with the definition (47), shows clearly that, for each $\psi = \bar{\psi}$, there is a “turning point”, $x_{p,f} = x_{p,f}(\psi) = N_{\psi\psi\psi}/Y_{\psi\psi\psi}$, concerning the loss of stability for supercritical pitchfork bifurcations. Additionally, this latter form has the advantage, if compared to (36), of being (at least explicitly) independent of U . The above discussion will be clearly exemplified in the next section.

Figures 3 and 4 illustrate the present local analysis, where $M(\psi)$ is the restoring moment.

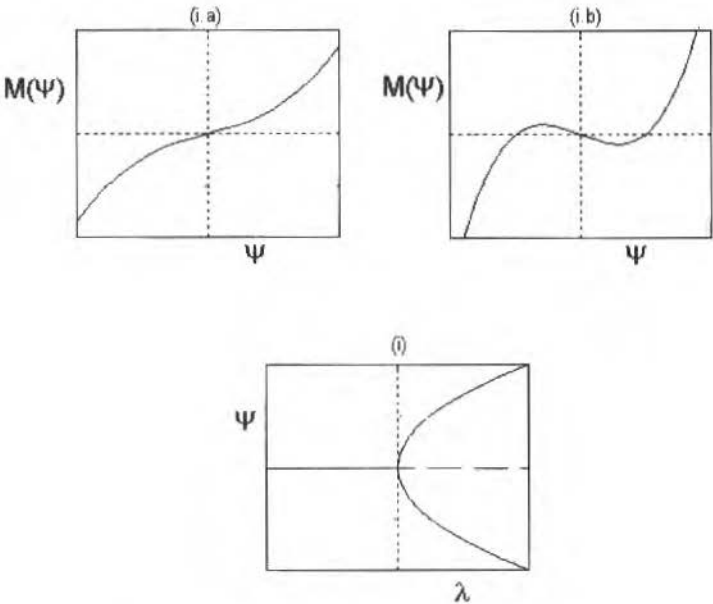


Fig. 3 Pitchfork Bifurcation Diagram.
Supercritical Case: $\alpha > 0$; $x_p > N_{\psi\psi\psi}/Y_{\psi\psi\psi}$; $Y_{\psi\psi\psi} > 0$. **Local Analysis Around $\bar{\psi} = 0$.**
(i.a) $\lambda_c < 0$; $x_p > N_{\psi\psi\psi}/Y_{\psi\psi\psi} = N_{\psi\psi\psi}/Y_{\psi\psi\psi}$. (i.b) $\lambda_c > 0$; $x_p < N_{\psi\psi\psi}/Y_{\psi\psi\psi} = N_{\psi\psi\psi}/Y_{\psi\psi\psi}$.

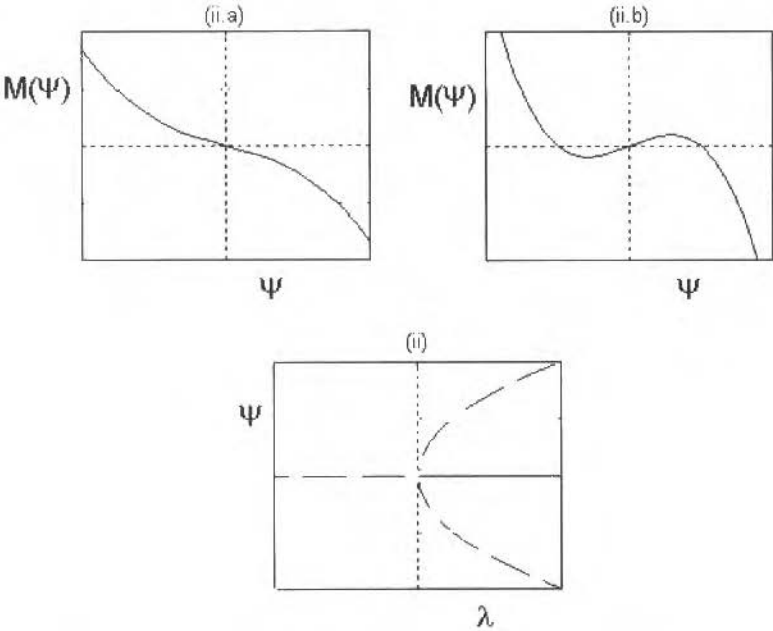


Fig. 4 Pitchfork Bifurcation Diagram.

Subcritical Case: $\alpha < 0$; $x_D < N_{\psi\psi\psi}/Y_{\psi\psi\psi}$; $Y_{\psi\psi\psi} > 0$. Local Analysis Around $\bar{\psi} = 0$.
(ii.a) $\lambda_c < 0$; $x_p < N_{\psi\psi}/Y_{\psi\psi} = N_{\psi}/Y_{\psi}$. (ii.b) $\lambda_c > 0$; $x_D > N_{\psi\psi}/Y_{\psi\psi} = N_{\psi}/Y_{\psi}$.

Examples on Stability Analysis and Bifurcations

The Studied Cases And Their Hydrodynamic Derivatives

Two different tankers moored at either a SPM or a 'Turret' type systems have been taken. Table 1 shows the particulars of these vessels. The characteristics of both the hawser cable, for the SPM case, and the restoring mooring force function, for the 'Turret' case, are shown in Table 2. Table 3 presents the nondimensional hydrodynamic derivatives, extracted, respectively from Takashina (1986) and Bernitsas and Kekridis (1985). It should be noticed that the Takashina coefficients were obtained by means of captive model experiments, in either static drift tests at Froude number 0.066, or yaw rotating tests (for four different constant yaw rates $\dot{\psi} = r = constant$). It should also be pointed out that Takashina's nondimensionalized parameters differ somewhat from Bernitsas and Kekridis, as shown in Table B1, Appendix B. For comparison purposes Table 3 shows, for the Takashina tanker, the nondimensional hydrodynamic derivatives in both forms. Notice, however, that equations of motions are invariant in form, under both nondimensionalization procedures, the velocity being always equal to unity. Both have been considered as linear.

Table 1 Tankers Particulars

	Tanker (1)	Tanker (2)
Length (L _{bp}) (m)	270	325
Breadth (m)	44.8	-
Draft (m)	10.8	-
Displacement (m ³)	90699	310669
Mass (t)	92967	318436

Table 2 Hawser Breaking Strength for the SPM and Restoring Force Characteristics for the 'Turret' System

	Hawser Breaking Strength ³ S_B	Turret System Restoring Coefficient K
	12200 kN	33.0 kN/m
Dimensionless (A)	8.0	6.0
Dimensionless (B)	0.2252	0.194

Table 3 Tankers' Dimensionless Parameters and Hydrodynamic Derivatives⁴

	Tanker 1 (A) ⁵	(B) ⁶	Tanker 2 (B)
m	2.258	0.0903	0.0181
I_z	0.14112	0.00564	-
$I_z + A^{rr}$	-	-	0.00222
A^{uu}	0.050xm	0.004516	0.0009
A^{vv}	0.650xm	0.05871	0.0171
A^{rr}	0.043xm	0.003884	-
A^{vr}	-	-	~0.0
A^{rv}	-	-	~0.0
X_u	-0.01	-0.0004	-0.003
Y_v	-0.285	-0.0114	-0.0261
Y_r	-	-	0.00365
N_r	-0.030	-0.0012	-0.0048
N_v	-0.0028	-0.00011	-0.0105
Y_{vvv}	-0.894	-0.21456	-0.045
N_{vvv}	-0.0093	-0.002232	0.00611
N_{rrr}	-	-	0.00611

Stability, Bifurcations And Dynamic Behaviours

Consider the fixed point corresponding to $\overline{\psi} = 0$. The necessary condition for stability is given by Eq. (53). If applied to *tanker* (I), irrespective if SPM or 'Turret' cases are considered, we get $x_p/L > N_r/Y_v = 0.0028/0.285 = 0.009824$. But, in the present case, $N_{vvv}/Y_{vvv} = 0.0093/0.894 = 0.0104$, and so

³ (q = 1.0; p = 5.0); i.e., linear and 20% stretching at rupture.

⁴ Only shown hydrodynamic derivatives actually used in the present time domain simulations.

⁵ Values according to Takashina (1986), and Table 2, column (A). Notice that in that paper the Taylor expansion coefficients (n!) that appear in the definition of the hydrodynamic derivatives are implicitly considered in the equation of motion. In other words, these values should be corrected by the corresponding factor if the hydrodynamic forces are calculated as in Eq. (13).

⁶ Values calculated from column (A) but already corrected according to Table 2, column (B).

$N_v/Y_v < N_{vv}/Y_{vv}$. Notice, also that, in the present case, $Y_{vv} < 0$, such that, for $\cos\psi > 0$, we have $Y_{\psi\psi\psi} > 0$. This implies that the stability condition (ia) ($\alpha > 0$; $\lambda_c < 0$), can be written as ($x_p/L > N_{vv}/Y_{vv}$; $x_p/L > N_v/Y_v$), i.e., $x_p/L > 0.0104$. Condition (ib) ($\alpha > 0$; $\lambda_c > 0$), for its turn, would be written ($x_p/L > N_{vv}/Y_{vv}$; $x_p/L < N_v/Y_v$), i.e., this would never occur, in this case. In words, only the subcritical pitchfork scenario may appear, and supercritical bifurcation will never occur. In fact, condition (iib) ($\alpha < 0$; $\lambda_c > 0$) or ($x_p/L < N_{vv}/Y_{vv}$; $x_p/L > N_v/Y_v$), Fig. 4, corresponding to a 'marginal stability' around $\bar{\psi} = 0$, holds whenever $0.009824 < x_p/L < 0.0104$, and condition (iia), corresponding to ($\alpha < 0$; $\lambda_c < 0$), or ($x_p/L < N_{vv}/Y_{vv}$; $x_p/L < N_v/Y_v$) (corresponding to be $\bar{\psi} = 0$ an unstable fixed point) applies if $x_p/L < 0.009824$. Table 5 summarizes the stability analysis. Figure 5 shows a number of time domain simulations, starting from the desired equilibrium position by means of a small perturbation in the transversal relative velocity v . The time domain simulations have been performed by means of MATLAB/SIMULINK code, using a fifth-order Runge-Kutta integration scheme. We see that $\psi = 0$ is a stable equilibrium point for $x_p/L = 0.011$, in accordance to Table 4 (i.a). We see also that $\psi = \pi$ is the attractor for the two other conditions exemplified. It should be noticed that 100 units of dimensionless time corresponds to about 7.5 hours in real scale, for a current speed of 1 m/s.

As a conclusion, *tanker (1) can be considered 'stable'*, since for practical reasons $x_p/L \geq 0.25$, in general. If the connection point is moved aft, only subcritical bifurcation shall appear, therefore.

Table 4 Stability Scenarios for the Tanker (1) Concerning $\bar{\psi} = 0$

	Stability parameters	Bifurcation scenario	Situation
$x_p/L < 0.00982$	$(\alpha < 0; \lambda_c < 0)$	(ii.a)	Unstable
$0.00982 < x_p/L < 0.0104$	$(\alpha < 0; \lambda_c > 0)$	(ii.b)	Subcritical bifurcation ⁷
$x_p/L > 0.0104$	$(\alpha > 0; \lambda_c < 0)$	(i.a)	Stable

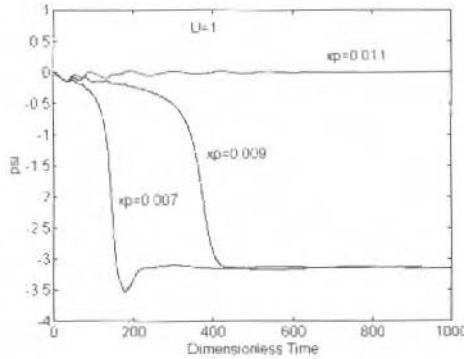


Fig. 5 Subcritical Pitchfork Bifurcations for Tanker (1) in SPM; x_p/L as a Parameter. Dimensionless Time $t' = tU/L$. Hawser Length: $l_w/L = 2$.

Initial Conditions: $x_1 = u/U = 1$; $x_2 = v/U = 0.1$; $x_3 = rL/U = 0$; $x_4 = l_w/L = 2$; $x_5 = \gamma = 0$; $x_6 = \omega = 0$.

⁷ "Marginally stable"

For the *tanker* (2), however, we get $N_v/Y_v = 0.0105/0.0261 = 0.4023$ and $N_{vvv}/Y_{vvv} = -0.0061/0.045 = -0.1358$. Hence, we have $N_{vvv}/Y_{vvv} < N_v/Y_v$. Notice that, even in the present case, $Y_{vvv} < 0$, such that, for $\cos\psi > 0$, we have $Y_{vvv\psi} > 0$. The stability condition (ia) ($\alpha > 0$; $\lambda_c < 0$), or ($x_p/L > N_{vvv}/Y_{vvv}$; $x_p/L > N_v/Y_v$), leads to $x_p/L > 0.4023$. Condition (ib) ($\alpha > 0$; $\lambda_c > 0$), or ($x_p/L > N_{vvv}/Y_{vvv}$; $x_p/L < N_v/Y_v$), gives $-0.1358 < x_p/L < 0.4023$, in this case. In words, supercritical bifurcation scenario is now present. On the other hand, condition (iib) ($\alpha < 0$; $\lambda_c > 0$) or ($x_p/L < N_{vvv}/Y_{vvv}$; $x_p/L > N_v/Y_v$), that would correspond to a 'marginal stability' around $\bar{\psi} = 0$, cannot hold, and condition (iia) ($\alpha < 0$; $\lambda_c < 0$), or ($x_p/L < N_{vvv}/Y_{vvv}$; $x_p/L < N_v/Y_v$) (corresponding to be $\bar{\psi} = 0$ an unstable fixed point) applies if $x_p/L < -0.1358$. Table 5 summarizes the stability analysis.

Table 5 Stability Scenarios for the Tanker (2) Concerning $\bar{\psi} = 0$

	Stability parameters	Bifurcation scenario	Situation
$x_p/L < -0.1358$	$\alpha < 0$; $\lambda_c < 0$	(ii.a)	Unstable
$-0.1358 < x_p/L < 0.4023$	$\alpha > 0$; $\lambda_c > 0$	(i.b)	Subcritical bifurcation
$x_p/L > 0.4023$	$\alpha > 0$; $\lambda_c < 0$	(i.a)	Stable

Condition (iib) cannot hold and so, subcritical bifurcation will never occur in this case.

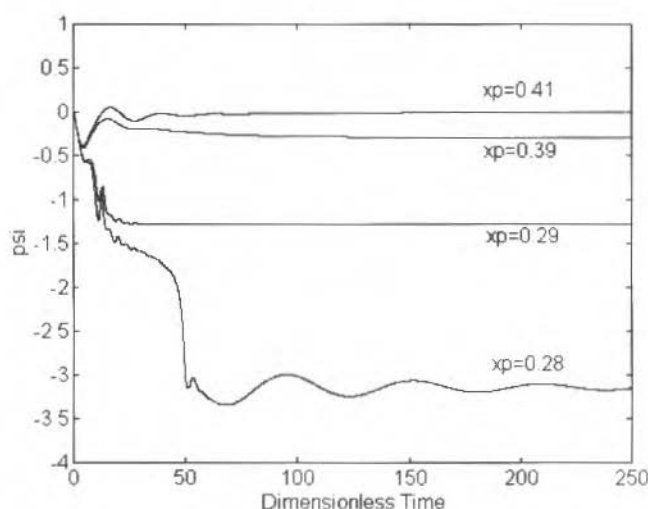


Fig. 6 Supercritical Pitchfork Bifurcations (Showing Some Cases With Subsequent Loss of Structural Stability) for Tanker (2) in SPM; x_p/L as a Parameter. Dimensionless Time $t' = tU/L$. Hawser Length: $l_w/L = 2.5$.

Initial Conditions: $x_1 = u/U = 1$; $x_2 = v/U = 0.1$; $x_3 = r/LU = 0$; $x_4 = \delta = 2.5$; $x_5 = \gamma = 0$; $x_6 = \omega = 0$.

Figure 6 shows a number of time domain simulations, also starting from the desired equilibrium position by means of a small perturbation in the transversal relative velocity v .

Notice, however, that if supercritical bifurcation occurs, i.e. condition (ia) transforms into (ib), stability around the bifurcated equilibrium position breaks down whenever $\lambda > U^2$, i.e. if $\bar{\psi} \rightarrow \pm\pi/2$ (otherwise Eq. (36) would have no real roots). This fact can be also interpreted as a loss of structural stability, regarding the system equations. Looking at Fig. 6 we see that this situation happens for $x_p/L \equiv 0.28$, as could be predicted, and now $\bar{\psi} = \pi$ is the new attractor.

In fact, from (30) and (36), under the condition $\lambda = U^2$, we get from Table 3,

$$\left(\frac{x_p}{L}\right)^* = \frac{(N_{vvv} + 6N_v)}{(Y_{vvv} + 6Y_v)} = \frac{(0.00611) + 6 \times (-0.0105)}{(-0.045) + 6 \times (-0.0261)} = 0.2822,$$

confirming the numerical simulations (see Fig. 6). Table 5 must then be corrected, in order to account for the loss of stability of supercritical pitchfork bifurcations. This is shown in Table 6. Figure 7 refers to Eq. (36), showing, as in Papoulias and Bernitsas (1988), $\bar{\psi}$ as a function of x_p/L .

As mentioned before the present stability analysis, concerning the desirable equilibrium position $\bar{\psi} = 0$, does not depend on the type of mooring system used, if SPM or 'Turret'. Figures 8 and 9 show the time domain simulations for the corresponding 'Turret' cases. Conditions are the same. Stability behavior is unchanged, as predicted but, the time histories, although similar, are not. Particularly, for tanker 2 a limit-cycle (Hopf bifurcation), around the attractor $\bar{\psi} = \pi$, is got for $x_p/L = 0.28$.

Table 6 Stability Scenarios for the Tanker (2) Concerning $\bar{\psi} = 0$, Considering 'Unstable' Supercritical Bifurcations

	Stability parameters	Bifurcation scenario	Situation
$x_p/L < -0.1358$	$\alpha > 0; \lambda_c > 0$	(ii.a)	Unstable
$-0.1358 < x_p/L < 0.2822$	$\alpha > 0; \lambda_c > 0$ $\lambda > U^2$	(i.b) unstable	"Unstable" supercritical bifurcation
$0.2822 < x_p/L < 0.4023$	$\alpha > 0; \lambda_c > 0$ $\lambda < U^2$	(i.b) stable	"Stable" supercritical bifurcation
$x_p/L > 0.4023$	$\alpha > 0; \lambda_c < 0$	(i.a)	Stable

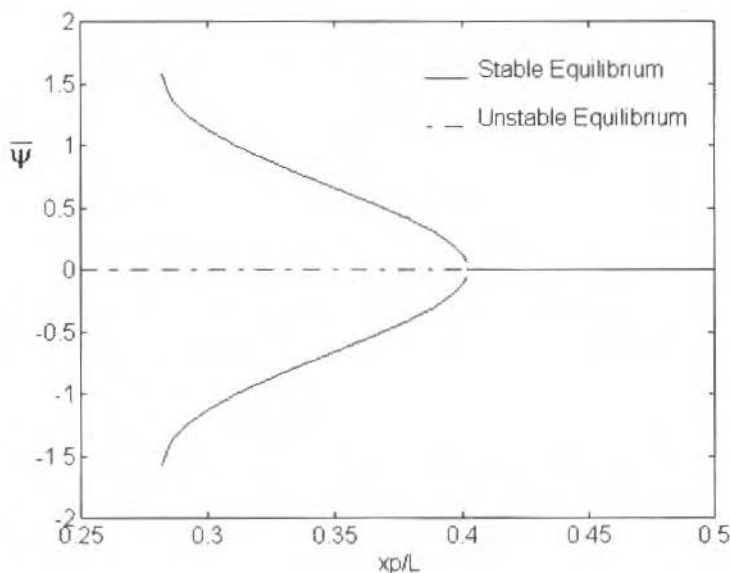


Fig. 7 Diagram of Supercritical Pitchfork Bifurcation for Tanker (2)

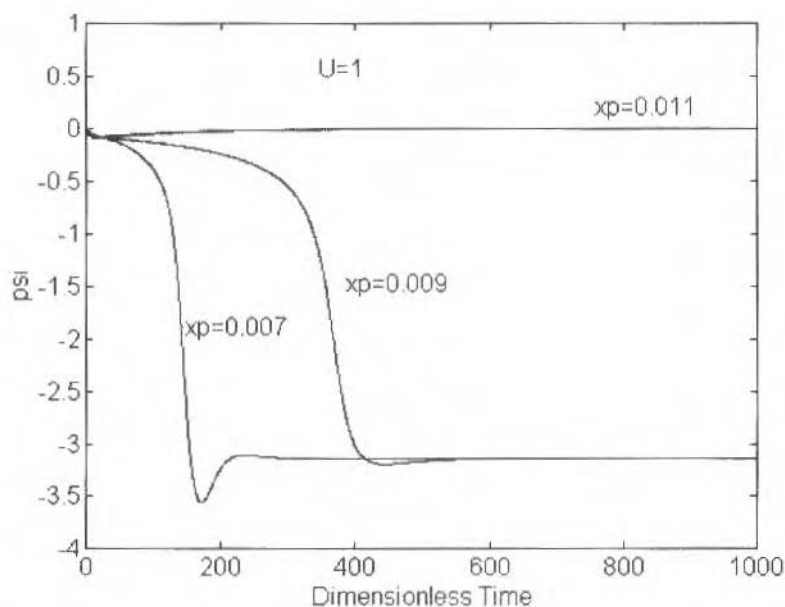


Fig. 8 Subcritical Pitchfork Bifurcations for Tanker (1) in 'Turret'; x_p/L as a Parameter. Dimensionless Time $t' = tU/L$. Initial Conditions: $x_1 = u/U = 1$; $x_2 = v/U = 0.1$; $x_3 = r/L = 0$; $x_4 = x/L = -x_p$; $x_5 = y/L = 0$; $x_6 = \psi = 0$.

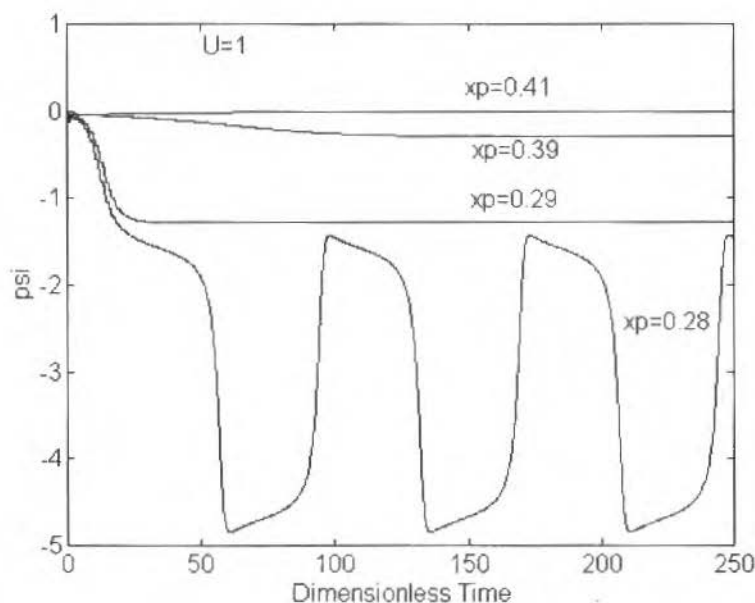


Fig. 9 Supercritical Pitchfork Bifurcations for Tanker (2) in 'Turret'; x_p/L as a Parameter. Dimensionless Time $t' = tU/L$. Initial Conditions: $x_1 = u/U = 1$; $x_2 = v/U = 0.1$; $x_3 = r/L = 0$; $x_4 = x/L = -x_p$; $x_5 = y/L = 0$; $x_6 = \psi = 0$.

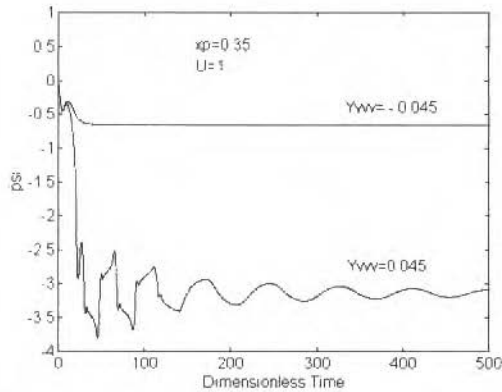


Fig. 10 Effect of Y_{vvv} Sign in the Pitchfork Bifurcations Scenario for Tanker (2) in SPM; $x_p/L = 0.35$. Dimensionless Time $t' = tU/L$. Initial Conditions: $x_1 = u/U = 1$; $x_2 = v/U = 0.1$; $x_3 = rL/U = 0$; $x_4 = l = 2.5$; $x_5 = \gamma = 0$; $x_6 = \omega = 0$.

Figure 10 exemplifies as the sign of the third-order derivative Y_{vvv} may change the whole picture. If sign is reversed such that $Y_{vvv} = 0.045$, the *supercritical scenario*, for which condition (i.a) ($\alpha > 0$; $\lambda_c < 0$) is now written ($N_c/Y_v < x_p/L < N_{vvv}/Y_{vvv}$), will not be possible anymore. In fact, now $N_c/Y_v = 0.0105/0.0261 = 0.4023$ and $N_{vvv}/Y_{vvv} = 0.0061/0.045 = 0.1358$. Hence, we have $N_{vvv}/Y_{vvv} < N_c/Y_v$, such that (i.a) cannot hold. Instead, subcritical pitchfork bifurcation scenario will appear, conversely. Explicitly, condition (iib) ($\alpha < 0$; $\lambda_c > 0$) or ($x_p/L > N_{vvv}/Y_{vvv}$; $x_p/L > N_c/Y_v$), would correspond to a ‘marginal stability’ around $\bar{\psi} = 0$, and condition (iia) ($\alpha < 0$; $\lambda_c < 0$), or $N_{vvv}/Y_{vvv} < x_p/L < N_c/Y_v$ (corresponding to be $\bar{\psi} = 0$ an unstable fixed point) applies if $0.1358 < x_p/L < 0.4023$. Notice, however, that this is a local analysis and gives no information concerning the existence or not of other attractors and if structural stability is preserved.

Finally, Fig. 11 shows time-histories corresponding to tanker (2), for different values of hawser cable length, taking $x_p/L = 0.7$ (the attachment point on a bridge at the bow). According to Table 7, $\psi = 0$ is a stable fixed point. Nevertheless, a Hopf bifurcation may appear, leading to a steady limit cycle. This behavior can depend also on hawser cable length. A “dynamic behaviour map”, as shown, for example in Papoulias and Bernitsas (1988), could be constructed, describing as a function of x_p/L and l_w/L , the kind of dynamic behavior that might be expected.

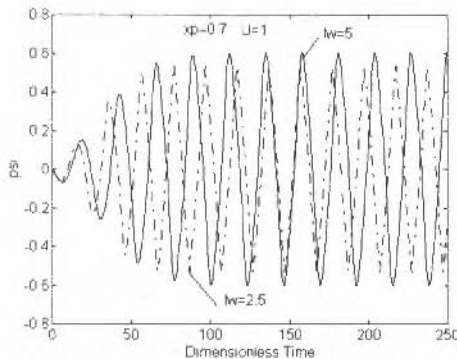


Fig. 11 Hopf Bifurcations (Limit Cycles) for Tanker (2) in SPM; l_w/L as a Parameter. Dimensionless Time $t' = tU/L$. $X_u = -0.001$; $x_p/L = 0.7$. Initial Conditions: : $x_1 = u/U = 1$; $x_2 = v/U = 0.1$; $x_3 = rL/U = 0$; $x_4 = l_w/L$; $x_5 = \gamma = 0$; $x_6 = \omega = 0$.

Conclusions

Stability analysis of Single Point (SPM) and 'Turret' Mooring Systems for Floating Production Storage and Offloading Systems, under the action of steady ocean currents, has been performed by means of bifurcation theory. Theoretical results previously published by Papoulias and Bernitsas (1988), have been recovered and discussed. The most important result is the necessary condition for stability, governed by the longitudinal position of the attachment point. Stability analysis has been enlarged somewhat, showing how bifurcation theory applied to a third-order model, based on the standard "hydrodynamic derivatives" type, is able to qualify two distinct equilibria bifurcation scenarios of the super- and sub-critical pitchfork type. *It has been shown that the sign of the third-order hydrodynamic derivative of lateral force with respect to the lateral component of relative velocity governs the type of bifurcation scenario.* Usually third-order hydrodynamic coefficients are small, and experimental errors can easily lead to changes in algebraic sign, conducting to a totally different bifurcation scenario.

Additionally, when super-critical pitchfork bifurcation scenario is present, a condition for structural stability loss has been established and discussed as well. Finally, some practical examples, taking two tankers of different size, moored either in SPM or in 'Turret' configurations have been presented, illustrating the features predicted by bifurcation analysis, through a number of time-domain simulations, performed with a fifth-order Runge-Kutta integration scheme. Some examples, concerning the appearance of Hopf bifurcations have also been presented and discussed.

Acknowledgements

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Appendix A

In fact, using (10), (11) and (9) under equilibrium assumption, Eqs. (19) can be written,

$$\begin{aligned} X(\bar{u}, \bar{v}, \bar{r}) + m \bar{r} \bar{v} + \bar{T} \cos \bar{\omega} &= 0 \\ Y(\bar{u}, \bar{v}, \bar{r}) - m \bar{r} \bar{u} - \bar{T} \sin \bar{\omega} &= 0 \\ N(\bar{u}, \bar{v}, \bar{r}) - T x_p \sin \bar{\omega} &= 0 \end{aligned} \tag{60}$$

The second and third equations are then combined, leading to

$$x_p \{ Y(\bar{u}, \bar{v}, \bar{r}) - m \bar{r} \bar{u} \} = N(\bar{u}, \bar{v}, \bar{r}) \tag{61}$$

As equilibrium is assumed, the angular velocity is null ($\bar{r} = 0$). Taking the hydrodynamic derivatives up to third order, we then have, at equilibrium,

$$x_p \{ Y_v \bar{v} + \frac{I}{6} Y_{vvv} \bar{v}^3 \} = N_v \bar{v} + \frac{I}{6} N_{vvv} \bar{v}^3 \tag{62}$$

Appendix B

Table B1 Nondimensionalizers Parameters

Type of Variable	(A) According to Takashina (1986)	(B) According to Bernitsas and Kekridis (1985)
Velocity	U	U
Length	L	L
Mass ⁸	0.5 ρ L ² T	0.5 ρ L ³
Inertia of mass	0.5 ρ L ⁴ T	0.5 ρ L ⁵
Force	0.5 ρ L T U ²	0.5 ρ L ² U ²

⁸ T is the draft

Experimental Evaluation of Wave Forces on a Circular Cylinder

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Abstract

Vertical forces were measured on a horizontal circular cylinder in a laboratory wave flume. The flume, at the time of measurements, was 14 m long, 1 m wide and circa 0.75 m deep. The length of the cylinder was circa 1 m. It was positioned horizontally along the transverse direction of the tank. The diameter was 0.15 m. Two immersions of the centerline were considered: 0.15 and 0.185 m. Regular surface waves were generated by a plunger. The range of frequencies was approximately from 0.5 to 2.3 Hz. Their amplitudes were between approximately 0.015 and 0.03 m. These dimensions of the cylinder and characteristics of the waves are typical of a semi-submersible pontoon in real conditions, once a geometric scale factor of about 1:100 is considered. The vertical force was measured with a strain-gage transducer, and the wave elevation with a resistance wave probe. The analogical signals were converted into digital and plotted. The experimental results were compared against Morison's equation (1950) predictions. As expected, it was observed that the Morison's equation results adheres to the experimental ones in its frequency range of validity. The depth of the tank is small compared to the wave length in the lower range of frequencies. This water depth limitation has been considered when applying Morison's equation.

Keywords: Wave Forces, Immersed Bodies.

Introduction

Morison et alii (1950) proposed an equation to predict the transverse force on a fixed vertical cylinder submitted to regular waves.

This formula was generalized, and nowadays it is usual to apply it to moving cylinders under the action of currents and waves.

The formula has one term proportional to the flow acceleration, related to inertial effects, and other term proportional to the flow velocity squared, related to drag.

For a fixed body under the effect of waves, it reads:

$$F = (\rho \nabla + m_a) U' + C_D \frac{1}{2} \rho S U |U| \quad (1)$$

where ρ is the water density, ∇ is the displaced volume of the submerged part of the body, m_a is the added mass, U' is the flow acceleration, C_d is the drag coefficient, S is the projected area of the body in the normal plane to the flow, U is the velocity of the flow.

The inertia term includes the non-disturbed incident wave effect, given by the displaced mass term, and the diffraction effect, given by the added mass term.

The expression for the drag term is consequence of empirical observation. For a certain flow regime (laminar or turbulent), and for bluff bodies, the force divided by the velocity squared does not depend significantly on the Reynolds number. In fact, for circular cylinders the coefficient C_d is typically constant for turbulent flows, equal to 0.6.

As for the inertia term, its expression can be obtained by Potential Theory, assuming that all viscous effects are considered in the drag term. Possible interferences between inertial and viscous effects are ignored. This derivation is presented in the following sections.

Formulation of the Potential Flow Problem

The problem of determining wave forces on a fixed body acted on by regular waves is formulated in the present section.

Viscous effects are here ignored.

The fluid is assumed to be incompressible and inviscid, and the flow potential.

Let ϕ be the total potential of the flow, ϕ^I the incident waves potential, and ϕ^D the perturbation potential due to the presence of the body, that is, the diffraction potential:

$$\phi = \phi^I + \phi^D \quad (2)$$

Mass conservation must apply throughout the fluid domain, and this is assured by Laplace's equation:

$$\nabla^2 (\phi^I + \phi^D) = 0 \quad (3)$$

Momentum conservation enables the calculation of the pressure by Bernouille's equation:

$$p(x, y; t) = -\rho \frac{\partial \phi(x, y; t)}{\partial t} - \rho \frac{1}{2} |\nabla \phi(x, y; t)|^2 - \rho g y + C(t) \quad (4)$$

where the third term of the second member is hydrostatic and will thus not be considered when evaluating the hydrodynamic forces, and the constant $C(t)$ will be assumed null.

Since the incident waves are themselves a solution of the problem in the absence of the body, potential ϕ^I alone must satisfy Laplace's equation, and consequently so must potential ϕ^D alone.

Just steady-state solutions are sought.

As for boundary conditions, one has:

- (1) cinematic free-surface condition, requiring that no water "splits" out of the free-surface, and that no "holes" are created;
- (2) dynamic free-surface condition, requiring the pressure on that surface to be uniform and constant (atmospheric pressure);
- (3) bottom condition, applied to a plane bottom, requiring the normal component of the flow velocity to be null (impermeable bottom);
- (4) body condition, similar to the bottom condition, but applied to the body surface.

In addition, a radiation condition must be applied, requiring that the diffraction potential is due to waves that propagate from the body outwards.

Conditions (1) and (2) are expressed in terms of two variables: the wave potential, ϕ , and the wave elevation in respect to the undisturbed free-surface plane, η . These conditions may be combined to give another one in terms solely of the wave potential.

Conditions (1) and (2), as well the combined free-surface condition, apply independently to potentials ϕ^I and ϕ^D and to corresponding wave elevations η^I and η^D , here the superscripts I and D meaning "incident waves" and "diffracted waves". Condition (3) also applies independently to potentials ϕ^I and ϕ^D . Condition (4) applies just for their sum.

The problem is difficult to solve, mainly because part of the boundaries, specifically the free-surface is unknown, and because the free-surface conditions are non-linear.

A simplifying hypotheses is adopted, restricting the solution to that of "small" wave amplitudes. The problem is then linearized around the non-deformed free-surface plane.

Consider two-dimensional problems, with axis x being the direction of propagation of waves, and axis y being vertical, positive upwards. The undisturbed free-surface is supposed to be at $y=0$.

The linearized hydrodynamic pressure is restricted to the first term in the second member of (4).

The linearized free-surface conditions read:

Dynamic condition:

$$\eta = -\frac{1}{g} \frac{\partial \phi^{I,D}}{\partial t} \quad \text{on } y = 0 \quad (5)$$

Combined cinematic and dynamic condition:

$$\frac{\partial^2 \phi^{I,D}}{\partial t^2} + g \frac{\partial \phi^{I,D}}{\partial y} = 0 \quad \text{on } y = 0 \quad (6)$$

The bottom boundary condition reads:

$$\frac{\partial \phi^{I,D}}{\partial y} = 0 \quad \text{on } y = -h \quad (7)$$

being h the water depth.

The body boundary condition is:

$$\frac{\partial (\phi^I + \phi^D)}{\partial n} = 0 \quad \text{on the body surface} \quad (8)$$

being " n " the normal direction to the body surface. (Below the normal vector $\vec{n}$ will be supposed to be directed outwards of the body).

Solution for the Problem in the Absence of Body: Airy Wave

Since just harmonic steady solutions are here considered, one may use complex notation, and write a generic variable $\theta(t) = \theta_a \cos(-\omega t + \delta_\theta)$, where θ_a is its amplitude, ω its frequency and δ_θ its phase, as:

$$\begin{aligned} \theta(t) &= \operatorname{Re} \left\{ \theta_a \exp[-i(\omega t - \delta_\theta)] \right\} \\ &= \operatorname{Re} \left\{ \theta_a \exp(i\delta_\theta) \exp(-i\omega t) \right\} = \operatorname{Re} \left\{ \theta_A \exp(-i\omega t) \right\} \end{aligned}$$

being $\theta_A = \theta_a \exp(i\delta_\theta)$ the so called complex amplitude, with its absolute value equal to the real amplitude, and its argument equal to the phase.

One steady solution for the linearized problem in the absence of body is the linear small amplitude progressive wave, also called Airy wave. It is presented next.

The incident wave elevation is:

$$\eta_A^I(x; t) = A \exp(ikx) \quad (9)$$

where A is the wave amplitude, $k = \frac{2\pi}{\lambda}$ is the wave number, with λ the wave length, and where k and ω are related by the so called dispersion relation:

$$k \tanh(kh) = \frac{\omega^2}{g} \quad (10)$$

The above wave propagates in the positive x -direction. In fact, its phase remains constant for $(kx - \omega t) = \text{constant}$, that is, when the differential of this quantity is null: $d(kx - \omega t) = 0$, resulting in a phase velocity equal to

$$c = \frac{dx}{dt} = \frac{\omega}{k} = \sqrt{\frac{g \tanh(kh)}{k}} \quad (11)$$

The incident wave potential is:

$$\phi_A^I(x, y; t) = -i \frac{gA}{\omega} \frac{\cosh[k(y+h)]}{\cosh(kh)} \exp(ikx) \quad (12)$$

Fluid particles velocities may be evaluated from $\vec{v} = u\vec{i} + v\vec{j} = \nabla\phi^I$, giving for instance for the y -direction component:

$$v_A^I = -i \frac{kgA}{\omega} \frac{\sinh[k(y+h)]}{\cosh(kh)} \exp(ikx) \quad (13)$$

Fluid particles accelerations may be evaluated from $\vec{a} = \ddot{u}\vec{i} + \ddot{v}\vec{j} = \frac{\partial \vec{v}}{\partial t}$, giving for instance for the y -direction component:

$$a_{y,A}^I = -kgA \frac{\sinh[k(y+h)]}{\cosh(kh)} \exp(ikx) \quad (14)$$

For unlimited deep waters ($kh \rightarrow \infty$), one has:

• in (10): $\tanh(kh) \equiv 1$, so that the dispersion relation reads $k = \frac{\omega^2}{g}$;

• in (11) the wave velocity resulting in $c = \sqrt{\frac{g}{k}}$;

• in (12): $\frac{\cosh[k(y+h)]}{\cosh(kh)} = \frac{\exp[k(y+h)] + \exp[-k(y+h)]}{\exp(kh) + \exp(-kh)} \equiv \frac{\exp[k(y+h)]}{\exp(kh)} = \exp(ky)$

in (13) and (14), similarly to the above approximation: $\frac{\sinh[k(y+h)]}{\cosh(kh)} \cong \exp(ky)$.

The Linearized Wave Force Acting on the Body

As for the steady solution of the problem in the presence of the body, the force exerted by the waves on the body may be evaluated considering the linear hydrodynamic term of the Bernoulli's equation, as below:

$$\vec{F}_A = F_{x,A} \vec{i} + F_{y,A} \vec{j} = \iint_{\substack{\text{body's} \\ \text{surface}}} (-p_A) \vec{n} dS = -i\omega\rho \iint_{\substack{\text{body's} \\ \text{surface}}} \phi^I \vec{n} dS - i\omega\rho \iint_{\substack{\text{body's} \\ \text{surface}}} \phi^D \vec{n} dS \quad (15)$$

Approximation Leading to the Inertia Term of Morison's Equation: the Froude-Krylov Term

The first term in the last member of (15) is called "Froude-Krylov" term. It may be readily evaluated since the incident wave potential is known.

If the body is "small" in the x-direction, say, less than one fifth of the wave length, the phase of the wave over the body is approximately constant, and its value on the center of the body, $x = X_c$, may be taken.

If, in addition, the body is assumed to be "small" in the y-direction, another simplification may be done.

For the y-component of the force, for instance, the body may be approximated by thin vertical strips, each one of width Δx . The upper face of each strip is at $y = y_u$, and the lower face at $y = y_l$. Its height is $\Delta y = y_u - y_l$ and its center at $y_c = \frac{y_u + y_l}{2}$. The normal to the upper face of the strip is $n_y = 1$ and to the lower face is $n_y = -1$. For each strip, one has:

$$\begin{aligned} \Delta F_{y,A}^{\text{Froude-Krylov}} &\cong -\omega\rho \left[\phi_A^I(x_c, y_u) - \phi_A^I(x_c, y_l) \right] \Delta x = \\ &= -\rho g A \frac{1}{\cosh(kh)} \exp(ikx_c) \left\{ \cosh[k(y_u + h)] - \cosh[k(y_l + h)] \right\} \Delta x = \\ &= -\rho g A \frac{1}{\cosh(kh)} \exp(ikx_c) \frac{1}{2} \cdot \\ &\quad \left\{ \exp \left[k \left(y_c + \frac{\Delta y}{2} + h \right) \right] + \exp \left[-k \left(y_c + \frac{\Delta y}{2} + h \right) \right] - \right. \\ &\quad \left. \left[-\exp \left[k \left(y_c - \frac{\Delta y}{2} + h \right) \right] - \exp \left[-k \left(y_c - \frac{\Delta y}{2} + h \right) \right] \right] \right\} \Delta x = \\ &= -\rho g A \frac{1}{\cosh(kh)} \exp(ikx_c) \frac{1}{2} \cdot \\ &\quad \left\{ \exp[k(y_c + h)] - \exp[-k(y_c + h)] \right\} \left[\exp \left(k \frac{\Delta y}{2} \right) - \exp \left(-k \frac{\Delta y}{2} \right) \right] \Delta x \\ &= -\rho g A \frac{1}{\cosh(kh)} \exp(ikx_c) \sinh[k(y_c + h)] \left[\exp \left(k \frac{\Delta y}{2} \right) - \exp \left(-k \frac{\Delta y}{2} \right) \right] \Delta x \end{aligned} \quad (16)$$

Now, the approximation

$$\exp\left(k\frac{\Delta y}{2}\right)-\exp\left(-k\frac{\Delta y}{2}\right)\equiv k\Delta y \tag{17}$$

may be adopted for $k\frac{\Delta y}{2}=\frac{\pi\Delta y}{\lambda}\ll 1$.

Substituting (17) in (16), and recognizing the acceleration $\dot{v}_A^I$ (expression 14) in the resulting expression, it results:

$$\overset{Force}{\Delta F_{y,A}^{Krylov}}\equiv \rho\Delta v\Delta v_A^I(x_c,y_c) \tag{18}$$

Summing up all the contributions from all strips, and admitting that the strips centers are all at about the same height $y=y_c$, the sectional force results:

$$\overset{Force}{F_{y,A}^{Krylov}}\equiv \rho A\dot{v}_A^I(x_c,y_c) \tag{19}$$

where A is the sectional area of the body, and where the acceleration is determined at the geometric center (x_c,y_c) of this section. This force corresponds to the $\rho\nabla U'$ term in (1), once a 2D flow and the y-direction are considered.

Approximation Leading to the Inertia Term of the Morison's Equation: the Diffraction Term

The second term in (15) is difficult to evaluate, since the potential ϕ^D is unknown.

However, it is possible to relate this problem to that of a freely oscillating body in the absence of incident waves, called radiation problem.

Suppose, for instance, that the 2D body considered so far is oscillating harmonically with frequency ω and unit amplitude along the y-direction. One is concerned about the determination of the corresponding radiation potential, $\phi_{2,A}^R$. This problem can be formulated, up to first order approximations, by the Laplace equation (3), the combined free-surface boundary condition (6) and the bottom boundary condition (7). As for the body boundary condition, one has in this case:

$$\frac{\partial\phi_{2,A}^R}{\partial n}=v_n \tag{20}$$

where $v_n=\vec{j}\cdot\vec{n}=u_2$ is the component of the velocity of the body normal to its surface.

Returning to the diffraction problem, the y-component of the second term in (15) may be thus written:

$$F_{y,A}^{diffraction}=-i\omega\rho\int\limits_{body's\ contour}\phi_A^Dn_2ds=-i\omega\rho\int\limits_{body's\ contour}\phi_A^D\frac{\partial\phi_{2,A}^R}{\partial n}ds \tag{21}$$

Applying Green's second identity¹, and afterwards the body boundary condition (8), one has:

$$F_{y,A}^{diffraction} = -i\omega\rho \int_{\text{body's contour}} \phi_{2,A}^R \frac{\partial \phi_A^D}{\partial n} ds = i\omega\rho \int_{\text{body's contour}} \phi_{2,A}^R \frac{\partial \phi_A^I}{\partial n} ds = i\omega\rho \int_{\text{body's contour}} \phi_{2,A}^R \nabla \phi_A^I \cdot \vec{n} ds \quad (22)$$

Being the 2D section symmetric in respect to y, as is the case of for instance a circular section, there will be no contribution for the y-component of the force from the $\nabla \phi_A^I \cdot (n_j \vec{i})$ term. In this case, (22) reads, after expressing n_2 again in terms of the radiation potential ϕ_2 :

$$F_{y,A}^{diffraction} = i\omega\rho \int_{\text{body's contour}} (\nabla \phi_A^I \cdot \vec{j}) \phi_{2,A}^R \frac{\partial \phi_{2,A}^R}{\partial n} ds \quad (23)$$

The above expression is known as Haskind's relation, and it enables the determination of the diffraction force based on the knowledge of the radiation potential instead of the diffraction potential.

Consider now the same approximations adopted in the last section, that is, that the body width and height are small relatively to the wave length, enabling the incident wave phase and immersion decay to be taken as uniform for the whole body section.

In this case, $\nabla \phi_A^I \cdot \vec{j}$ may be put out of the integral in (23).

The coefficient $f_{22} = -\rho \int_{\text{body's contour}} \phi_{2,A}^R \frac{\partial \phi_{2,A}^R}{\partial n} ds$ is related to the radiation problem concerning oscillation in

the y-direction and force also in the y-direction. It may be expressed by the sum of a term in phase with the body acceleration and of a term in phase with the body velocity:

$$f_{22} = \omega^2 a_{22} + i\omega b_{22} \quad (24a)$$

where

$$a_{22} = \frac{I}{\omega^2} Re \left\{ -\rho \int_{\text{body's contour}} \phi_{2,A}^R \frac{\partial \phi_{2,A}^R}{\partial n} ds \right\} \quad (24b)$$

¹ $\int_{\text{body's contour}} \left(\phi^D \frac{\partial \phi_2^R}{\partial n} - \phi_2^R \frac{\partial \phi^D}{\partial n} \right) ds = 0$, once $\lim \phi_2^R \rightarrow 0$ and $\lim \phi^D \rightarrow 0$ at large distances from the body, by

$$(6), \left(\phi^D \frac{\partial \phi_2^R}{\partial n} - \phi_2^R \frac{\partial \phi^D}{\partial n} \right)_{\text{free surface}} = \left(\phi^D \frac{\omega^2}{g} \phi_2^R - \phi_2^R \frac{\omega^2}{g} \phi^D \right)_{\text{free surface}} = 0,$$

$$\int_{\text{bottom}} \left(\phi^D \frac{\partial \phi_2^R}{\partial n} - \phi_2^R \frac{\partial \phi^D}{\partial n} \right) ds = 0$$

. $T_r \ll T_i$ ($T_r / T_i \ll 1$): the flow reverses before the fluid particles can reach the other side of the cylinder, not enabling the occurrence of separation and wake, and, consequently, viscous effects can be ignored;

. $T_r \gg T_i$ ($T_r / T_i \gg 1$): there is enough time for the fluid particles to surround the cylinder before flow reversion, and separation and wake do occur, being the viscous drag significant.

One has:

$$\frac{T_r}{T_i} \approx 0 \left(\frac{\pi}{\omega A} \right) = 0 \left(\frac{A}{d} \right) \quad (29)$$

Therefore the relative importance between inertial and viscous terms can be evaluated by the A/d quotient, which in fact is the Keulegan-Carpenter number.

For $A \ll d$ one has the inertial regime, and the viscous drag can be neglected.

For $A \gg d$ one has the viscous drag regime, and inertial effects may be neglected, due to the fact that in this case the body is "small", causing a "small blockage" of the flow.

For intermediate values of A/d , both effects may be important. The formula is generally used in this case simply considering the validity of superposition of effects.

Frequency Range of Validity of Morison's Formula

The flow acceleration U' and velocity U used in Morison's formula are to be taken as typical values for the undisturbed incident wave (not diffracted) along the section of the cylinder. These values are generally taken on the centerline of the cylinder.

Therefore its validity is restricted to situations where the wave characteristics do not present an intense variation along the section.

The criterion most frequently used is that the formula may be applied for $\frac{d}{\lambda} < 0.2$.

Taking account of the dispersion relation (10), the above criterion reads in terms of frequency in Hz ($f = \frac{\omega}{2\pi}$), for unlimited depth:

$$\frac{2\pi f^2 d}{g} < 0.2,$$

or, in terms of frequency range, Morison's equation holds for sufficiently low values of frequencies, such that

$$f < \sqrt{\frac{g}{10\pi d}} \equiv \frac{0.559}{\sqrt{d}} \quad (30)$$

The other restriction refers to "small" body heights in respect to the wave length λ .

Imagining that "d" and " Δy " are of the same order, the consequence of adopting

$$f < \frac{0.559}{\sqrt{\Delta y}} \quad (31)$$

is the so called "added-mass", and

$$b_{22} = \frac{I}{\omega} \operatorname{Re} \left\{ (-i) \left(-\rho \int_{\text{body's contour}} \phi_{2,A}^R \frac{\partial \phi_{2,A}^R}{\partial n} ds \right) \right\} \quad (25)$$

is the so called "damping".

The added mass and the damping may be calculated just considering the radiation problem. This is a much more "treatable" problem, since the radiation potentials are null at large distances from the body, which is not the case of the incident wave potential in the diffraction problem. For 2D numerical solutions in infinitely deep waters see for instance Frank (1967) and de Conti (1980).

With this approximation, (23) results:

$$\begin{aligned} F_{x,A}^{\text{diffraction}} &\equiv \frac{I}{(-i\omega)} (\omega^2 a_{22} - i\omega b_{22}) \left[\nabla \phi_A^I \cdot \vec{j} \right]_{(x_c, y_c)} \\ &= (-i\omega a_{22} + b_{22}) \left[\nabla \phi_A^I \cdot \vec{j} \right]_{(x_c, y_c)} \\ &= a_{22} v_A^I(x_c, y_c) + b_{22} v_A^I(x_c, y_c) \end{aligned} \quad (26)$$

The added mass term in (26) corresponds to the $m_a U'$ term in (1). The damping term in (26) is not present in Morison's equation, which admits that the potential damping is negligible face to the viscous damping.

The Inertia and the Drag Regimes

The quotient between the wave amplitude and the cylinder diameter may indicate whether the most important effect is inertial or viscous, as exposed below.

Consider the typical time for a fluid particle to surround the cylinder in the wave flow. This can be evaluated by dividing half the cylinder perimeter by the particle velocity:

$$T_l \approx 0 \left(\frac{\frac{\pi d}{2}}{\frac{\omega A}{2}} \right) = 0 \left(\frac{\pi d}{\omega A} \right) \quad (27)$$

where d is the cylinder diameter, and $\frac{\omega A}{2}$ was assumed to be the order of the particle velocity in the wave flow.

The flow reverses after each time lag equal to half the wave period, that is:

$$T_r = \frac{1}{2} \frac{2\pi}{\omega} = \frac{\pi}{\omega} \quad (28)$$

Comparing T_l and T_r , one has the following possibilities:

similarly to (30), could be sought. One would have:

$$k \frac{\Delta y}{2} = \frac{\pi \Delta y}{\lambda} < 0.2\pi \approx 0.63 \quad ,$$

so that the approximation

$$\exp\left(k\left(\frac{\Delta y}{2}\right)\right) - \exp\left(-k\left(\frac{\Delta y}{2}\right)\right) \approx k\Delta y \quad (32)$$

would lead to a maximum error of $\frac{|1.34 - 1.38|}{1.34} \approx 3\%$.

This shows that the restriction in (31) also guarantees small deviations from the exact predictions.

The Experimental Tests

The experimental tests were conducted in a wave flume. There was a plunger-type electrically driven servo-controlled generator in one of its extremities, and a wave absorber in the other (see Fig. 1).

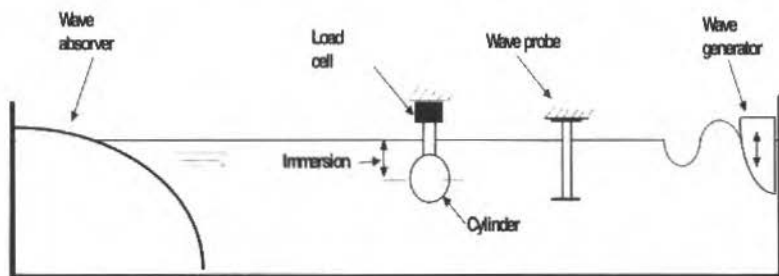


Fig. 1 Experimental Arrangement

The flume was at the time of measurements 14 m long, 1 m wide and circa 0.75 m deep².

Regular waves in the range of 0.5 to 2.3 Hz with amplitudes up to 0.03 m were generated.

The tested model was a circular cylinder with a 0.15 m diameter, and circa 1 m length, whose dimensions are typical of a semi-submersible pontoon in real conditions, once a scale factor of about 1:100 is considered.

This model was arranged horizontally in the transverse direction of the flume. The flow resulted thus approximately 2-dimensional. Two immersions of the centerline of the cylinder were considered: 0.15 and 0.185 m.

The vertical component of the force was measured by a strain-gage transducer and the wave elevation by a resistance wave probe. The analogical signals were converted into digital and plotted.

It was observed a linear behavior of the force in respect to the wave amplitude, in the range of frequencies considered. This indicated that the drag should not have had a significant role.

The quotient between vertical sectional force and wave amplitude versus wave frequency was plotted for both immersions (see Figs. 2 and 3). These figures also present estimations using Morison's

² Nowadays the flume-length has been extended to about 25m.

formula. Two Morison's plots (theoretical) are shown: one considering unlimited water depth, and the other considering limited water depth.

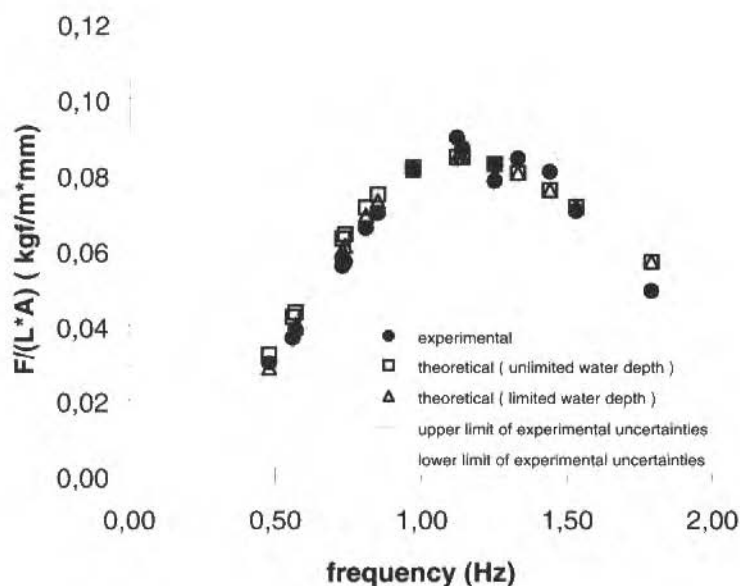


Fig. 2 Vertical Force Versus Wave Frequency for Immersion Equal to 0.15 m

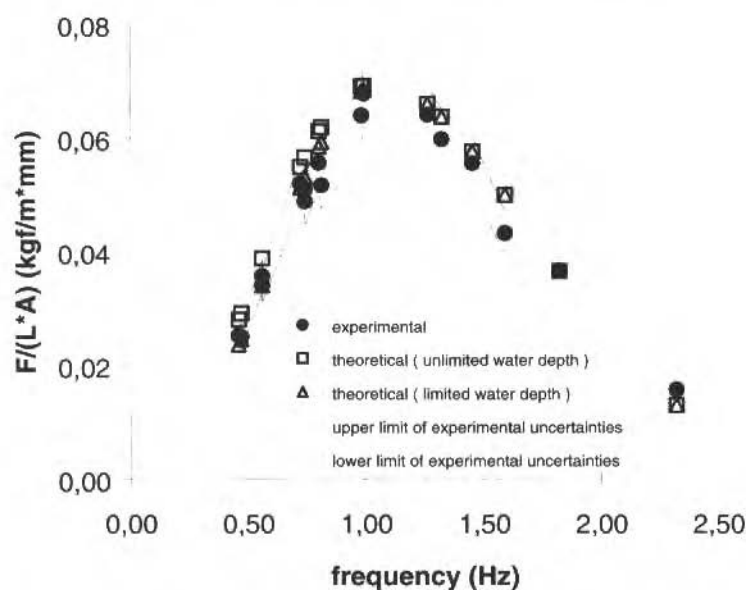


Fig. 3 Vertical Force Versus Wave Frequency for Immersion Equal to 0.185 m

It shall be noted that just the inertial term of the Morison's equation has been calculated, being the drag term neglected. This was due to the fact that for the present tests the quotient A/d did not exceed $(0.033/0.15) \ll 1$, which is, as seen above, a typical inertial regime.

Analysis of the Results

The range of measurement uncertainties, which amounts $\pm 8.5\%$, is indicated in Figs. 2 and 3. They were estimated considering the following effects:

- errors associated to instruments calibration;
- errors associated to data acquisition;
- errors due to spurious wave reflection;
- errors due to perturbations in the wave profile.

However it is known that there is another important uncertainty for the lower limit of the frequency range. This is associated to the shortness of the flume, with consequent wave reflection, and limitation of the wave register length. This error has not been indicated in Figs. 2 and 3, but certainly enlarges the uncertainty range for frequencies below 0.6 Hz.

The criteria for the range of validity of Morison's equation (Eqs. 30 and 31) state that the upper frequency for its application should be about

$$f_{max} = \frac{0.559}{\sqrt{0.15}} \text{ Hz} \cong 1.45 \text{ Hz} \quad ,$$

It should be also noted that the formula for unlimited waters is a good approximation for wave lengths typically smaller than twice the water depth, that is, for wave frequencies such that

$$f = \sqrt{\frac{g}{2\pi\lambda}} > \sqrt{\frac{g}{4\pi h}} \cong \frac{0.883}{\sqrt{h}} \cong 1 \text{ Hz}$$

Then, for smaller frequencies, the curve for limited waters is theoretically more adequate.

In fact, Figures, 2 and 3, respectively, relative to the smaller and higher immersions, indicate:

- that the experimental results are adherent to Morison's predictions until at least 1.5 Hz;
- that it does make a difference taking account of the depth limitation for low frequencies.

As for the added mass, it was evaluated by the analytical-numerical methodology described in Frank (1967), which applies to infinite water depth. Kelvin source singularities are distributed along the body contour. These singularities satisfy the linearized free surface conditions, the infinite condition and the radiation condition. The intensity of the singularities is determined by the imposition of the body boundary condition in various points of its contour.

Graphics for the sectional added mass coefficient $(a_{22} / \rho\pi d^2 / 4)$ and for the damping coefficient $b_{22} / \omega\rho\pi d^2 / 4$ versus frequency are presented in Figs. 4 and 5, respectively for the immersions 0.15 and 0.185 m.

The added mass coefficient is seen to vary considerably with the frequency, and this variation is in fact significant when estimating the diffraction force.

Sometimes the added mass is approximated for its zero frequency - infinite flow value, instead of considering the actual frequency of oscillation and the existence of a free-surface. This approximated value for the circular cylinder is $(a_{22} / \rho\pi d^2 / 4) = 1.0$.

The above approximation implies in errors, for instance for the lower immersion at a frequency around 0.9 Hz (see Fig. 4), of about $\frac{2.0-1.75}{1.75} \cong 14\%$. It was observed that errors like that may cause theoretical results to drop out of the experimental uncertainty range. Therefore, this approximation was not adopted in the present paper.

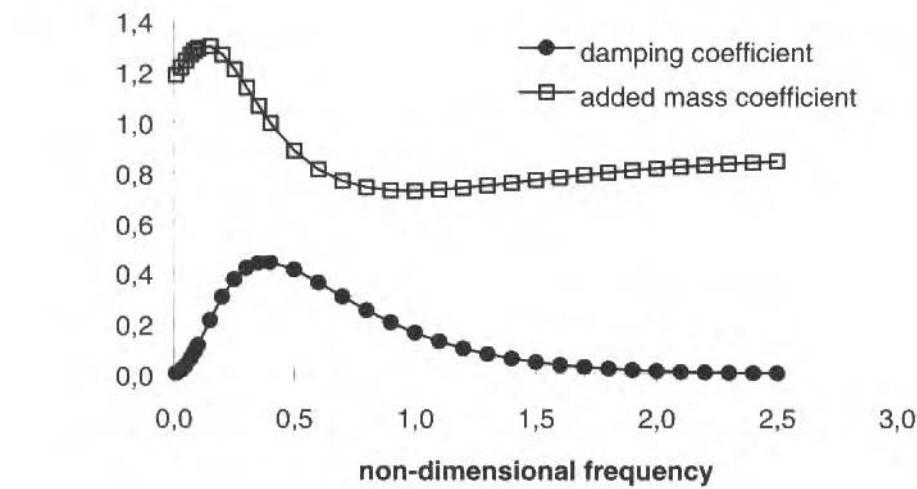


Fig. 4 Added Mass and Damping Coefficients for $h=0.15$ m

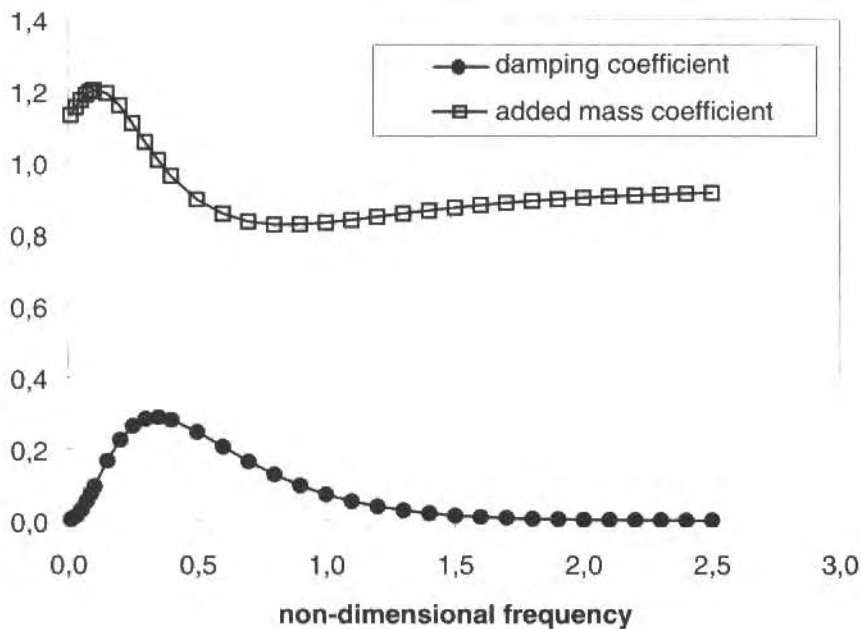


Fig. 5 Added Mass and Damping Coefficient for $h=0.185$ m

Moreover, in the case of having a circular cylinder in a free-surface flow, the limit added mass coefficient for zero frequency is circa 1.2 for a immersion-diameter ratio of 1.0 (Fig. 4), and circa 1.15 for a immersion-diameter ratio of 1.23 (Fig. 5), and not 1.0, as in infinite flow.

This is explained by the fact that the free surface condition (6) reads at the zero frequency limit as:

$$\frac{\partial \phi_2^R}{\partial y} \equiv 0 \quad \text{on } y=0 \quad ,$$

and the free surface acts as a rigid impermeable plane. The flow limited by this "rigid free-surface" is equivalent to that of the original cylinder and of another cylinder which is the specular image of the former, in respect to plane $y=0$, both oscillating with opposite phases. This is not, of course, the infinite flow approximation.

As for the potential damping effect, despite having been neglected, it is not that small in comparison to the added mass effect for the lower frequencies, specially around 0.4 Hz, as may be seen in Figs. 4 and 5.

Conclusions

Despite the limitations of Morison's equation, it has proven to give reasonable predictions, concerning vertical forces, for pontoon-like structures, at least for wave lengths larger than 4.6 times the body width. This is in accordance to the usual criterion, which recommends a factor of 5.0 times.

It was observed that it is important to take into account the variation of added mass with frequency.

The flume length is not adequate for experimentation with low frequencies, especially in the range up to 0.6 Hz. In this sense, the lengthening of the flume (which has just been provided) shall be convenient.

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O Tempo de Corte com Discos Abrasivos Pode Ser Reduzido pelo Controle da Velocidade de Corte

The Cutting Time with Abrasive Discs Can Be Reduced by the Control of the Cutting Speed

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Abstract

This work presents an experimental study on the life of abrasive discs in cut-off type operations with two conditions of cutting speed. The cutting time is shown with a comparative analysis of the cutting speed.

Keywords: Abrasive Discs. Cut-Off Type Operations, Cutting Speed.

Resumo

Este trabalho apresenta um estudo experimental sobre a vida de discos abrasivos em operações do tipo "cut-off" com duas condições de velocidade de corte. O tempo de corte é apresentado com uma análise comparativa da velocidade de corte.

Palavras-chave: Discos Abrasivos, Operações do Tipo "Cut-Off", Velocidade de Corte.

Introdução

O corte com discos abrasivos é um dos métodos mais antigos e devido à sua simplicidade operacional, ainda é um dos mais utilizados na indústria que trabalha com materiais metálicos.

Atualmente, os discos abrasivos são disponíveis em todos os níveis de preço e qualidade. O segredo na obtenção do bom desempenho do disco abrasivo está em quando e como utilizar o disco abrasivo correto para o trabalho. Este desempenho pode ser determinado pelo volume de material que estes são capazes de remover, em relação ao volume de disco abrasivo gasto num determinado tempo e segundo determinadas condições de trabalho (Snee, 1991).

As diferenças de qualidade na fabricação de um mesmo tipo de disco abrasivo pelos diversos fabricantes, e o desconhecimento de seu desempenho, agravam a determinação das condições ótimas de trabalho, tornando difícil a escolha de uma nova operação de corte, em função do tipo de disco abrasivo e do material a ser cortado. Estas dificuldades fazem com que os critérios para utilização de discos abrasivos no meio industrial não estejam devidamente estabelecidos, como na usinagem com ferramentas de arestas cortantes definidas, além de serem pouco estudados. Portanto, há pouca literatura formal sobre este assunto.

Muitos usuários utilizam discos abrasivos de forma inadequada por falta de informações técnicas sobre este processo, o que eleva, evidentemente, o custo do corte e prejudica seu desempenho. A avaliação do custo de discos abrasivos é difícil de ser feita e seu desempenho depende das condições de trabalho.

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Normalmente, o corte em operações do tipo "cut off" são realizadas por máquinas com relação de transmissão única entre o motor de acionamento e o eixo árvore, que suporta o disco abrasivo. Desta forma a velocidade de corte V_c [m/s], quando o disco abrasivo é novo, é alta e diminui na mesma proporção que ocorre a perda diametral do disco abrasivo. Isto afeta substancialmente os fenômenos de corte, comprometendo a capacidade de remoção do disco abrasivo (Bianchi et al., 1996).

Este trabalho tem como objetivo apresentar os resultados experimentais obtidos sobre a otimização do tempo de corte com discos abrasivos, quando submetidos a diferentes condições de velocidade de corte. Controlando-se a rotação do motor de acionamento, foram realizados dois ensaios. Em um deles manteve-se a velocidade de corte constante (aumentando-se a rotação do disco abrasivo proporcionalmente à sua perda diametral) e no outro utilizou-se a velocidade de corte variável e proporcional à perda diametral do disco abrasivo.

Fenomenologia do Corte com Discos Abrasivos e Parâmetros de Avaliação de Desempenho

Para a constatação da otimização do tempo de corte com discos abrasivos em função da velocidade de corte, a análise de alguns parâmetros relacionados aos processos abrasivos facilita a estimativa das vantagens econômicas que podem ser obtidas de acordo com as condições de trabalho utilizadas, além de mostrar a sua adequação no meio industrial. Por falta de literatura técnica específica sobre discos de corte, diversos conceitos utilizados no processo de retificação foram adequados a este trabalho.

O corte de materiais em operações do tipo "cut-off" consiste em pressionar um disco abrasivo contra o material a ser cortado, usualmente fixado numa morsa. A velocidade de penetração do disco abrasivo no material é denominada taxa temporal de penetração do disco abrasivo no material V_f [m/s]. Na Figura 1 é apresentado o desenho esquemático da fenomenologia de corte com discos abrasivos para operações do tipo "cut-off" e as principais variáveis envolvidas (Bianchi et al., 1996).

A taxa de remoção de material no tempo Q_w [mm³/s] representa o volume de material removido por unidade de tempo, que é determinada pela equação:

$$Q_w = A \cdot V_f \quad (1)$$

onde A [mm²] é a área de contato num dado instante entre o disco abrasivo e o material.

Nomenclatura

A = área de contato num dado instante entre o disco abrasivo e o material [mm ²]	Q_w = taxa de remoção específica de material no tempo [mm ³ /mm.s]	N = número de cavacos arrancados por unidade de tempo [N ² /s]
b = largura média do topo dos grãos abrasivos [mm]	H_{eq} = espessura equivalente de corte [mm]	t = largura do disco abrasivo [mm]
C = número de grãos abrasivos por unidade de área [N ² /mm ²]	h_{max} = espessura teórica máxima do cavaco [mm]	V_s = velocidade de corte [m/s]
d_s = diâmetro do disco abrasivo [mm]	K_s = pressão específica de corte [N/mm ²]	V_f = taxa temporal de penetração do disco abrasivo no material [m/s]
d_c = diâmetro do material [mm]	K_n = tensão de escoamento do metal da peça [N/mm ²]	Z_w = volume de material removido [mm ³]
F_{tg} = força tangencial de corte num grão abrasivo [N]	l_c = comprimento de contato entre o disco abrasivo e o material [mm]	Z_s = volume de disco abrasivo gasto [mm ³]
G = relação [--]	L = espaçamento médio entre os grãos abrasivos [mm]	δ = penetração do disco abrasivo no material [mm]
Q_w = taxa de remoção de material no tempo [mm ³ /s]		μ = coeficiente de atrito entre o grão e a peça [--]

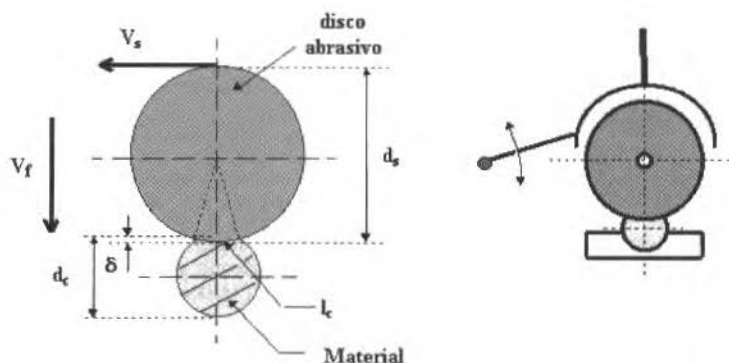


Fig. 1 Fenomenologia de Corte para Operações do Tipo "Cut-Off" (Bianchi et al., 1996)

A densidade de grãos abrasivos da superfície de corte do disco abrasivo, num determinado instante do corte, depende desta área de contato. No corte de materiais com seção transversal circular a área de contato varia com o diâmetro do disco abrasivo d_s [mm], diâmetro do material d_c [mm], largura do disco abrasivo t [mm] e com a penetração do disco abrasivo no material δ [mm], segundo a equação (Abrão, 1991):

$$A = 2 \cdot t \cdot \left(\frac{d_s}{2} \right) \cdot \arccos \left[\frac{\left[\left(\frac{d_s}{2} \right) + \left(\frac{d_c}{2} \right) - \delta \right]^2 - \left[\left(\frac{d_c}{2} \right)^2 - \left(\frac{d_s}{2} \right)^2 \right]}{\left[2 \cdot \left(\frac{d_c}{2} \right) \cdot \left(\left(\frac{d_s}{2} \right) + \left(\frac{d_c}{2} \right) - \delta \right) \right]} \right] \quad (2)$$

O comprimento de contato l_c [mm], entre o disco abrasivo e o material, influencia no número de grãos abrasivos, que afetam a geometria e a remoção dos cavacos arrancados. O comprimento de contato é obtido pela relação entre a área de contato e largura do disco abrasivo, conforme a equação:

$$l_c = \frac{A}{t} \quad (3)$$

Sendo a largura do disco abrasivo constante, o comprimento de contato é diretamente proporcional à área de contato.

Durante o corte de materiais cilíndricos, a área de contato inicialmente tende a crescer proporcionalmente à penetração do disco abrasivo no material, até um comprimento de contato máximo. Posteriormente a área de contato decresce até o final do corte. O disco abrasivo tende progressivamente a desgastar-se, diminuindo seu diâmetro.

No início do corte, quando o disco abrasivo ainda é novo, há uma maior densidade de grãos abrasivos ativos na superfície de corte deste, devido ao maior comprimento de contato. Desta forma, cada grão abrasivo remove menos material e o cavaco arrancado possui menor espessura.

O disco abrasivo desgasta-se durante a remoção de material, acarretando a diminuição do número de grãos ativos na sua superfície. Com menor densidade de grãos ativos, cada grão abrasivo remove um maior volume de material com espessura maior, comparativamente ao caso anterior.

Para um mesmo tipo de disco abrasivo, a manutenção do número de cavacos arrancados por unidade de tempo N [N^0/s] depende do número de grãos abrasivos por unidade de área C [N^0/mm^2], largura do disco abrasivo e da velocidade de corte do disco abrasivo. O número de cavacos arrancados por unidade de tempo é obtido pela equação (Malkin, 1989):

$$N = C \cdot t \cdot V_s \quad (4)$$

Com a perda diametral do disco abrasivo, há uma diminuição contínua da velocidade de corte. Isto provoca também a diminuição progressiva do número de cavacos arrancados por unidade de tempo.

A espessura equivalente de corte h_{eq} [mm] representa a espessura da camada de material que é arrancada pelo disco abrasivo numa volta completa. Trata-se de um parâmetro que permite quantificar uma condição de trabalho. A espessura equivalente de corte é definida como sendo a relação entre a taxa de remoção específica de material no tempo Q_w' [$mm^3/mm.s$] e a velocidade de corte do disco abrasivo, expressa por (Peters e Decneut, 1975):

$$h_{eq} = \frac{Q_w'}{V_s} \quad (5)$$

sendo a taxa de remoção específica de material no tempo Q_w' obtida pela relação entre taxa de remoção de material Q_w [mm^3/s] e a largura do disco abrasivo t . Dividindo-se Eq. (1) por t e combinando-a com as Eqs. (3) e (5), a espessura equivalente de corte pode ser expressa por:

$$h_{eq} = l_c \cdot \frac{V_f}{V_s} \quad (6)$$

O parâmetro h_{eq} foi escolhido para que se possa realizar uma análise comparativa dos resultados entre as duas condições de trabalho (V_s constante e V_s variável).

A espessura teórica máxima do cavaco h_{max} [mm] é diretamente proporcional ao parâmetro espessura equivalente de corte, conforme a equação (Bianchi et al., 1992):

$$h_{max} = h_{eq} \cdot (2 \cdot L / l_c) = 2 \cdot L \cdot (V_f / V_s) \quad (7)$$

onde L [mm] é o espaçamento médio entre os grãos abrasivos.

Portanto, aumentando-se V_f ou diminuindo-se V_s há um aumento na espessura máxima do cavaco arrancado. Todavia, variando-se a espessura do cavaco arrancado, altera-se a força tangencial de corte num grão abrasivo F_{tg} [N]. Esta força tangencial de corte pode ser expressa pela equação (Bianchi, 1992):

$$F_{tg} = b \cdot h_{max} \cdot K_s + \mu \cdot b \cdot \pi \cdot K_n / 4 \quad (8)$$

sendo b [mm] a largura média do topo dos grãos abrasivos, K_s [N/mm^2] a pressão específica de corte, μ o coeficiente de atrito entre o grão e a peça e K_n [N/mm^2] a tensão de escoamento do metal da peça.

Outro parâmetro utilizado para a determinação do comportamento de discos abrasivos é a relação G , definida como sendo a relação entre volume de material removido e volume de disco abrasivo gasto (ferramenta), determinada pela equação (Hahn, 1955):

$$G = \frac{Z_w}{Z_s} \quad (7)$$

sendo Z_w [mm³] e Z_s [mm³] o volume de material removido e o volume de disco abrasivo gasto, respectivamente.

Esta relação caracteriza o desgaste de um disco abrasivo sob determinadas condições de trabalho. Se o desgaste do disco abrasivo for grande com relação ao desgaste de material, a relação G é pequena. Isto significa que a perda volumétrica do disco abrasivo é grande. Assim, o custo do corte é maior. No caso do disco abrasivo desgastar-se lentamente, a relação G é alta, sendo o desempenho do disco abrasivo melhor, tomando o corte mais econômico (Bianchi, 1992).

Mecanismos de Desgaste em Discos Abrasivos

Durante a operação de corte ocorre o macro e micro desgaste dos grãos abrasivos e ligante da superfície de corte do disco abrasivo, decorrentes da interação do disco abrasivo com a peça.

Na Figura 2 são apresentadas as formas de ocorrência do macro e micro desgaste da superfície de corte dos discos abrasivos.

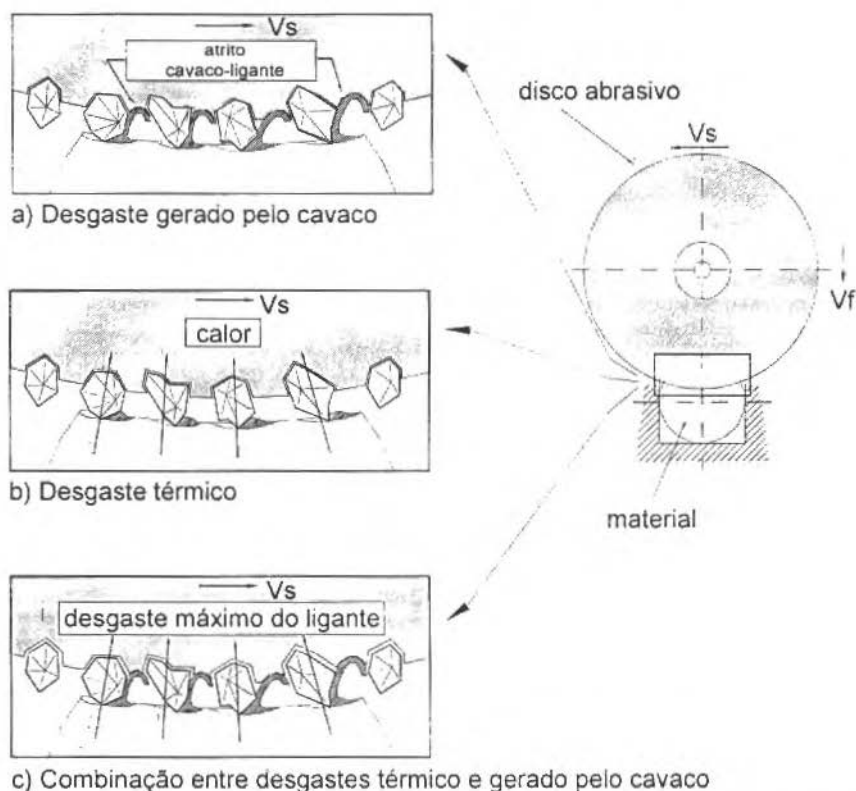


Fig. 2 Formas de Ocorrência do Macro e Micro Desgaste da Superfície de Corte dos Discos Abrasivos.

O micro desgaste corresponde ao desgaste das arestas ativas no topo dos grãos abrasivos durante o corte, em decorrência do atrito entre as arestas dos grãos abrasivos com o material. Durante o corte, os grãos abrasivos vão se desgastando e lentamente aumentando suas áreas de topo, aumentando as forças de corte e geração de calor.

O macro desgaste ocorre pelo desgaste do ligante, que retém os grãos abrasivos na superfície de corte do disco abrasivo. O desgaste do ligante pode ocorrer termicamente, pela geração do cavaco arrancado ou pela combinação entre ambos.

O desgaste térmico (Fig. 2b) ocorre pela ação do micro desgaste. As arestas no topo dos grãos abrasivos vão se desgastando e, conseqüentemente, há o aumento das forças de corte e temperatura na região de contato. Uma parte desta energia térmica é conduzida em direção ao centro do disco abrasivo, passando pelos grãos abrasivos e ligante. Quando a temperatura de trabalho é elevada, há a deterioração térmica do ligante que perde parte de sua capacidade de retenção dos grãos abrasivos. Assim, a força máxima de retenção, que o ligante exerce sobre o grão abrasivo, diminui. No instante em que as forças de corte são maiores que a de retenção do ligante sobre o grão abrasivo, há a liberação deste. Os discos abrasivos utilizados neste trabalho, são confeccionados com ligante resínico, que é muito sensível à ação da erosão do ligante pelo cavaco.

O cavaco arrancado pela interação do grão abrasivo com o material incide, ao sair, na parte do ligante que está à frente do grão abrasivo, na direção de movimento do disco abrasivo (Fig. 2a).

O atrito entre o cavaco e o ligante provoca, continuamente, o desgaste do ligante e este desgaste diminui a capacidade de retenção do grão abrasivo.

O macro desgaste combinado com o micro desgaste (Fig. 2c) é a combinação entre os tipos de desgaste já descritos.

Descrição do Banco de Ensaios

O banco de ensaios desenvolvido é composto de uma máquina de corte do tipo "cut-off", com capacidade para discos abrasivos de até 254,0 mm de diâmetro, montada sobre uma base rigidamente fixada ao solo. Foi utilizado um motor de indução trifásico com potência de 2 cv e rotação nominal de 3.480 rpm (frequência 60 Hz). A rotação do motor é transmitida ao eixo árvore, com relação de transmissão de 2,2:1 para a obtenção das rotações necessárias, de acordo com as condições de trabalho.

Um sistema de polia e peso foi montado para auxiliar o controle da força aplicada do disco abrasivo contra o material a ser cortado. A força aplicada era controlada manualmente pelo operador de tal forma que a rotação do motor fosse mantida constante durante o corte. Este sistema é constituído por um peso que traciona um cabo de aço, ligado à máquina através de uma polia.

O controle da rotação do disco abrasivo foi realizado por um conversor de frequência digital, que indicava a frequência instantânea do motor em todos ensaios. A força de corte aplicada, do disco abrasivo sobre o material, foi controlada de tal forma que a rotação do motor, para uma determinada velocidade de corte, ficasse mantida constante. Assim, as forças de corte foram controladas para que não houvesse perda volumétrica do disco abrasivo pela ação de sobrecarga nos grãos abrasivos, que poderiam provocar a liberação antecipada destes. Este fato é indesejável uma vez que pretende-se analisar a influência da velocidade de corte no desempenho do disco abrasivo e não submetê-lo à forças que provoquem a expulsão do grão abrasivo da superfície de corte do disco abrasivo. Isto provocaria uma perda diametral maior do disco abrasivo pelo aumento da espessura do cavaco arrancado (aumento V_f) alterando a relação G , principalmente por se tratar de discos abrasivos confeccionados com ligante resínico, que é sensível ao desgaste provocado pela erosão do cavaco arrancado sobre o ligante e a ação da temperatura na região de corte.

Para a realização deste trabalho, foram utilizados dois discos abrasivos idênticos (AR 302) e barras de aço trefilado ABNT 1045 (com diâmetro de 12,7 mm e 1 metro de comprimento), como material a ser cortado.

No primeiro ensaio manteve-se constante a rotação do disco abrasivo, variando-se assim sua velocidade de corte através da perda diametral do disco abrasivo.

No outro ensaio houve aumento na rotação do disco abrasivo, proporcionalmente à perda diametral do disco abrasivo, mantendo-se constante a velocidade de corte.

Em ambos os casos, foram realizadas séries de 10 cortes e em cada uma delas cronometrado o tempo de corte. No final de cada série, foi medida a perda diametral do disco abrasivo e calculado o tempo de corte médio.

Sendo constante o espaço percorrido pelo disco abrasivo durante o corte, este foi dividido pelo tempo de corte médio para a determinação da taxa temporal de penetração do disco abrasivo no material (V_f).

Estas séries foram repetidas até que o diâmetro do disco abrasivo chegasse a 170,00 mm, que corresponde à máxima utilização possível do disco abrasivo para cortar o material.

A relação G foi determinada medindo-se o volume de disco abrasivo gasto, a partir da perda diametral, e o volume de material removido, pelo número de cortes.

Resultados e Discussão

A Figura 3 apresenta as relações entre velocidade de corte e volume de material removido e disco abrasivo gasto durante os ensaios.

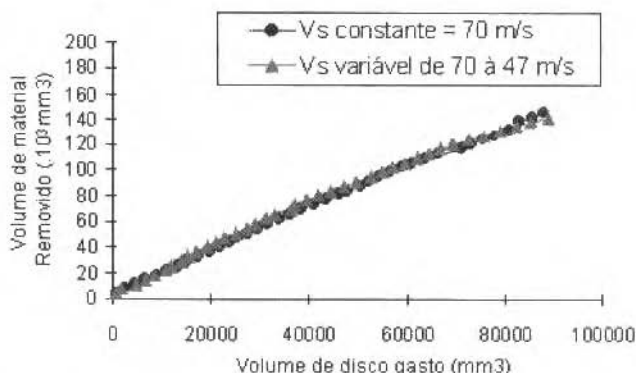


Fig. 3 Relação entre Volume de Material Removido e Volume de Disco Abrasivo Gasto

Nesta figura, pode-se notar que o volume de material removido e volume de disco abrasivo gasto são praticamente iguais quando utilizou-se velocidade de corte constante ou variável. Quando a velocidade de corte é variável, há uma tendência de decrescimento das curvas de forma quase linear, tanto de material removido como de disco abrasivo gasto. A relação G é obtida através da tangente das curvas, que corresponde ao coeficiente angular das retas.

Observa-se um comportamento crescente e quase sobreposto para ambas as curvas, mostrando uma semelhança muito próxima na relação do volume de material removido com o volume dos discos abrasivos gastos, mesmo sendo os discos abrasivos submetidos à diferentes velocidades de corte. O fato da velocidade de corte ser variável, não afetou a relação entre o volume de material e disco abrasivo gasto.

Os coeficientes angulares das retas da Fig. 3 são praticamente os mesmos, significando que os desgastes (material e disco abrasivo) são proporcionais e o comportamento da relação G é constante e idêntica para ambos os casos. Desta forma, não houve influência da velocidade de corte no comportamento da relação G , quando foram controladas as forças de corte pela rotação do motor de acionamento. Este fato já era esperado pois não houveram sobrecargas nos grãos abrasivos que proporcionassem a liberação antecipada destes da superfície de corte do disco abrasivo.

Com velocidade de corte variável, o número de cavacos arrancados por unidade de tempo é progressivamente diminuído. Um número menor de grãos abrasivos remove menos cavacos com maior espessura. Portanto, o volume de material removido diminui com o tempo de corte.

Quando a velocidade de corte é mantida constante, o número de cavacos arrancados por unidade de tempo também é constante e com espessura de cavacos menor. Desta forma o tempo de corte é menor e as forças por grão abrasivo são melhor distribuídas. Isto pode ser confirmado através da análise da Fig. 4, onde o volume de material removido na unidade de tempo é maior quando a velocidade de corte é constante.

O comportamento da velocidade é idêntico até o diâmetro do disco abrasivo (254 mm inicial) chegar a 241 mm. A partir deste valor, o tempo de corte passou a sofrer alterações pela ação das modificações nos fenômenos de corte. A diferença no tempo de corte entre as duas situações ensaiadas (velocidade de corte constante e variável) é de 49,63%.

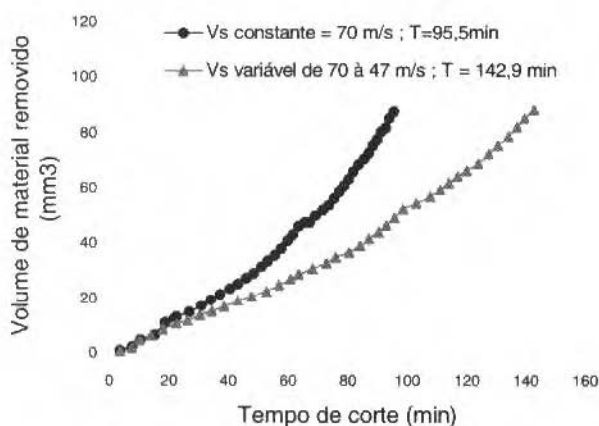


Fig. 4 Relação entre Volume de Material Removido e Tempo de Corte

Na Figura 5 é apresentada a relação entre V_f/V_s com o tempo de corte.

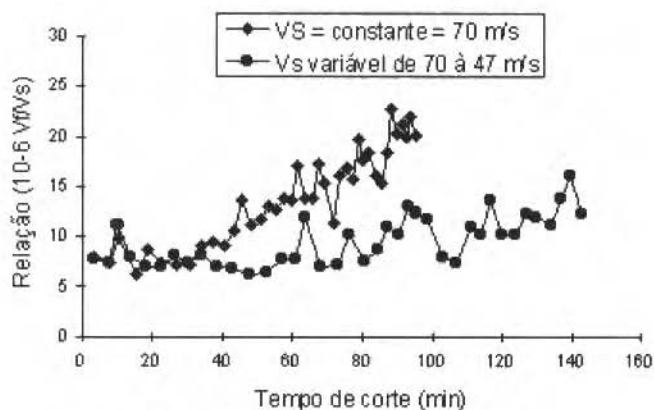


Fig. 5 Relação entre V_f/V_s e Tempo de Corte

Pode-se notar que para ambos os casos as curvas tendem a ser crescentes, porém de formas distintas. Diminuindo-se progressivamente o diâmetro dos discos abrasivos, o número de grãos abrasivos ativos no comprimento de contato, diminui. Ocorre então a geração de cavacos com maior espessura. Desta forma, as relações V_f/V_s tendem a ser crescentes para ambos os casos pelo aumento da espessura do cavaco, em consequência da perda diametral.

Quando a velocidade de corte é variável, a inclinação da curva é menor e o tempo para remover um determinado volume é maior. Isto ocorre pelo menor número de cavacos arrancados por unidade de tempo. A relação V_f/V_s cresce predominantemente pela diminuição de V_s . O comprometimento

térmico do material cortado é maior e a taxa temporal de penetração do disco abrasivo no material é menor.

Para velocidade de corte constante, a inclinação da curva é maior e o tempo de corte menor, pelo maior número de cavacos arrancados por unidade de tempo. Neste caso, a relação V_f/V_s cresce de forma mais acentuada. O crescimento da relação V_f/V_s ocorre somente pelo aumento de V_f , que proporciona o aumento da espessura equivalente de corte, e consequentemente da espessura do cavaco arrancado, e necessita de um menor tempo para a remoção de cavacos. Isto evita aquecimento excessivo da peça pelo menor tempo de contato.

Aumentando-se a taxa temporal de penetração do disco abrasivo no material, aumenta-se a espessura equivalente de corte, que é proporcional a espessura máxima do cavaco arrancado.

Conclusões

Deste trabalho pode-se concluir que o volume de material removido e a perda diametral dos discos abrasivos são praticamente iguais quando se utiliza velocidade de corte constante ou variável.

A velocidade de corte influencia significativamente o tempo de corte, pelas modificações da fenomenologia de corte provocada pela superfície de corte do disco abrasivo no material. Para velocidade de corte constante, o número de cavacos arrancados por unidade de tempo é constante e com espessura menor. Para velocidade de corte variável, o número de cavacos arrancados diminui com o tempo de corte e a espessura do cavaco é maior.

A velocidade de corte não influencia no volume de material removido e volume de disco abrasivo gasto, mas afeta diretamente o número de cavacos arrancados por unidade de tempo.

A espessura equivalente de corte h_{eq} , que está diretamente relacionada com a espessura do cavaco arrancado pelo disco abrasivo, foi observada durante os ensaios através de relação V_f/V_s . Portanto, trata-se de uma relação que, neste trabalho, foi analisada experimentalmente e pode-se verificar que exerce influência no comportamento do disco abrasivo.

A relação V_f/V_s aumenta com o tempo de corte para as duas condições de velocidade de corte. Para velocidade de corte variável, a relação V_f/V_s aumenta de forma lenta. O tempo de corte é maior, aquecendo mais a peça pelo maior tempo de contato. Para velocidade de corte constante, a relação V_f/V_s cresce de forma mais acentuada, que corresponde ao aumento de V_f . Neste caso os danos térmicos são menores.

A otimização da vida dos discos abrasivos, controlando-se a rotação do motor de acionamento, é de fácil implementação no meio industrial e permite ganhos significativos no tempo de corte (49,63%).

Usualmente as máquinas utilizadas em operações do tipo "cut-off" não possuem controle da rotação do eixo árvore. Isto faz com que a velocidade de corte seja sempre variável, aumentando o tempo de corte. Isto torna o corte oneroso pela diminuição do número de cavacos arrancados por unidade de tempo. Uma análise financeira específica para cada caso industrial, pode ser útil para economia e melhoria no desempenho de discos abrasivos, já que para a utilização de velocidade de corte constantes são necessários investimentos na aquisição de um conversor de frequência e equipamentos de proteção para a máquina e, principalmente, seu operador.

Agradecimentos

Manifestamos nossos agradecimentos ao CNPq (Conselho Nacional de Pesquisa e Desenvolvimento), FUNDUNESP (Fundação Para o Desenvolvimento da UNESP) e às empresas NORTON e KOHLBACH pelo apoio material e técnico gentilmente cedido para a realização deste trabalho.

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Fresotorneamento: Alguns Conceitos e Resultados Experimentais

Turn-Milling: Some Concepts and Experimental Results

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Abstract

This work presents the concepts and definitions of a combined machining process, so called turn-milling. Several methods of turn-milling are presented with major emphasis on the orthogonal process, studied experimentally. Test specimens made of ABNT 52100 bearing steel, with average hardness of 60-61 HRC were machined. The cutting tool material applied was mixed ceramic inserts. Several feed rate, tool diameter and rotational speeds of workpiece and tool were tested. Tests were conducted on a CNC lathe which had a milling head coupled to it. This coupling was made on the transverse carrier, figuring an orthogonal exposure of the rotating axes for workpiece and tool, thus characterizing orthogonal turn-milling. The achieved results showed that work quality and low roughness values can be obtained; however, the technology must be better investigated.

Keywords: Machining, Turn-Milling, Mixed Ceramics.

Resumo

Este trabalho procura apresentar os conceitos e definições a respeito de um processo de usinagem combinado, entre torneamento e fresamento, denominado de fresotorneamento. Desta forma, são apresentados os métodos de fresotorneamento, dando-se ênfase ao método ortogonal, que foi objeto de análise experimental. Neste aspecto, foram usinados corpos de prova de aço rolamento ABNT 52100, com 60-61 HRC de dureza. Utilizou-se pastilhas de cerâmica mista, variando-se os parâmetros: avanço, diâmetro da fresa, rotações da peça e da ferramenta. A máquina empregada nos ensaios foi um torno CNC, ao qual foi acoplado um cabeçote fresador. Este acoplamento foi feito no carro transversal, configurando os eixos de rotação da peça e ferramenta na disposição ortogonal, caracterizando, portanto, um fresotorneamento ortogonal. Os resultados alcançados mostraram ser possível atingir boa qualidade de trabalho e rugosidade baixa, todavia, ainda há necessidade de um melhor entendimento do fenômeno.

Palavras-chave: Usinagem, Fresotorneamento, Cerâmica Mista

Introdução

Fresotorneamento é um processo combinado de usinagem, em que uma fresa usina uma peça rotativa, segundo várias possibilidades (Fig. 1).

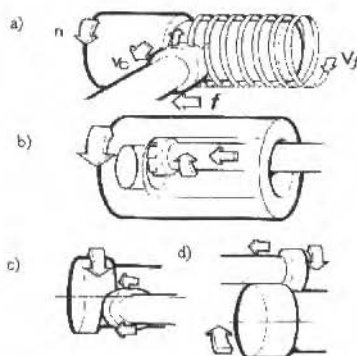


Fig. 1 Fresotorneamento: a) Ortogonal Externo Concordante; b) Fresotorneamento Coaxial Interno Discordante; c) Fresotorneamento Excêntrico e d) Fresotorneamento Periférico (Sandvik, 1994)

Este processo pode ser utilizado para usinar peças excêntricas, superfícies cônicas, cilíndricas e furos internos. O processo requer máquinas ferramenta, com funções peculiares ao processo, como por exemplo, tornos adaptados para este fim.

O princípio do fresotorneamento é obtido através dos movimentos relativos da fresa e peça simultaneamente, onde a velocidade de corte (v_c) é dada pela velocidade periférica da fresa. A rotação da peça (n_p), em combinação com o avanço axial da ferramenta (f), resulta na velocidade de avanço (v_f). Portanto, o fresotorneamento é um processo de usinagem composto por dois movimentos: o de corte e o de avanço, sendo que, o movimento de avanço é subdividido em movimentos axial (devido ao deslocamento linear da ferramenta) e rotacional (devido à rotação da peça).

Pode-se dividir o fresotorneamento em dois métodos (Fig. 2):

- Ortogonal: o eixo da ferramenta é disposto de forma ortogonal em relação ao eixo de rotação da peça.
- Coaxial: o eixo da ferramenta é disposto de forma paralela em relação ao eixo de rotação da peça.

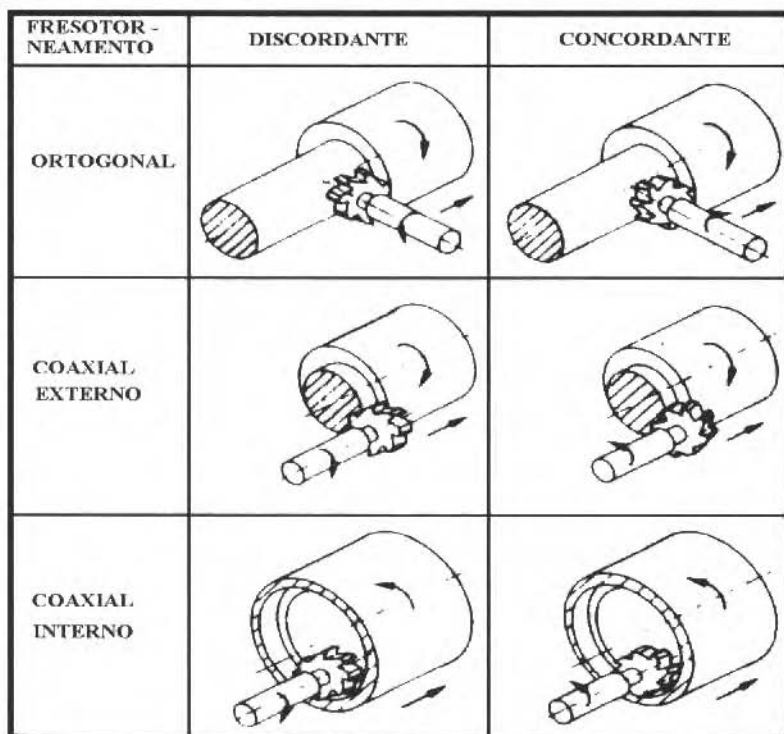


Fig. 2 Possibilidades de Arranjo para Fresotorneamento (Schulz, 1990)

Os cavacos formados no processo de fresotorneamento ortogonal, objeto do presente trabalho, diferenciam-se daqueles formados por outros processos. No torneamento os cavacos apresentam largura e espessura constantes e no fresamento apresentam a forma de vírgula, com largura constante e espessura variável. No fresotorneamento ortogonal os cavacos formados apresentam larguras e espessuras variáveis (Wand, 1985; Schulz e Daniel, 1993) (Fig. 3). O volume de material usinado por ambas as arestas (principais e secundárias) é dependente dos parâmetros de usinagem, tais como excentricidade, número de dentes, velocidade de avanço, velocidade de corte, etc..

Com a finalidade de se determinar a secção de corte, desenvolveu-se na TH Aachen (König e Wand, 1985; Wand, 1985; König e Wand, 1986) um programa para simulação do processo, que determina a espessura e a largura do cavaco, em função do ângulo de rotação da fresa (Fig. 3).

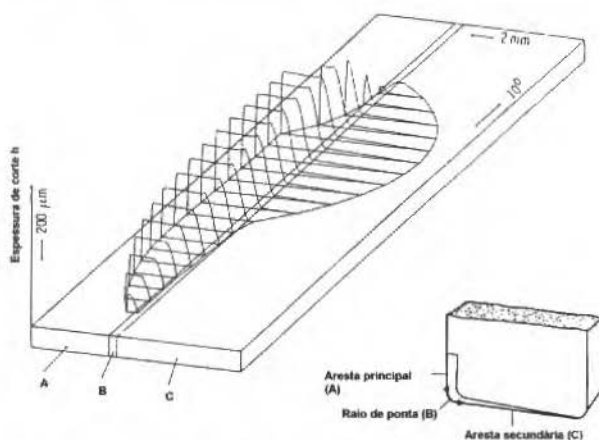


Fig. 3 Variação da Espessura do Cavaco ao Longo da Aresta, Durante o Giro da Fresa (Adaptado de Wand, 1985)

Wedeniowski (1984) descreve o processo de formação do cavaco, no qual a espessura aumenta até um máximo, para diminuir no final. Para a largura, há um comportamento semelhante. Além disto, ele apresenta a influência dos vários parâmetros de usinagem, na formação do cavaco. Sorge (1983) descreve a geometria do cavaco, de maneira qualitativa, e despreza o volume formado pelas arestas secundárias. Ele apresenta uma fórmula para a determinação do comprimento do cavaco, em função do número de dentes da ferramenta e da relação de rotações entre a ferramenta e a peça. Lehman (1992) descreve também a geometria do cavaco, formada pelo processo de fresotorneamento (Fig. 4).

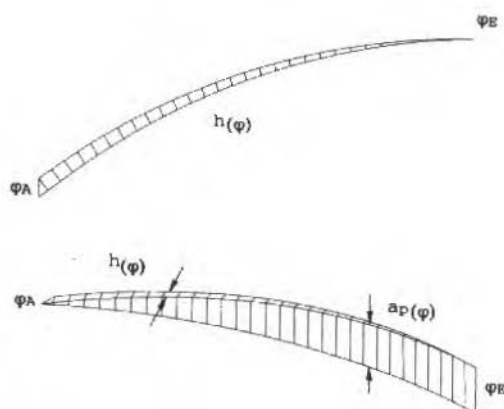


Fig. 4 Geometria do Cavaco Formado pela Aresta Principal de Corte (Adaptado de Lehman, 1992)

Fresotorneamento Ortogonal

O fresotorneamento ortogonal pode ser conduzido de maneira centralizada ou excêntrica (Fig. 5).

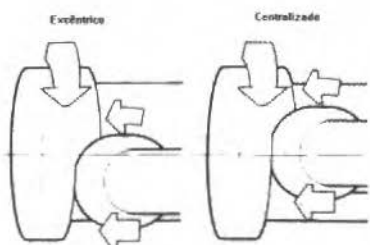


Fig. 5 Método Excêntrico e Centralizado de Fresotorneamento (Sandvik, 1994)

O fresotorneamento centralizado faz com que o avanço por volta (f) seja limitado pelo comprimento da aresta cortante (l) e ao se empregar o método excêntrico, pode-se utilizar avanços maiores, que atingem seus valores máximos na configuração mostrada na Fig. 6, caso C, onde "e" é a excentricidade, permitindo a retirada de maiores volumes de cavaco. Portanto, o avanço axial pode ser aumentado, até alcançar um valor igual ao comprimento projetado da aresta secundária na peça, sem que a mesma perca sua forma cilíndrica. Neste caso, atinge o seu valor máximo, porque o contato entre a peça e as arestas secundárias, alcança seu comprimento máximo. Pode-se calcular, de maneira aproximada o avanço máximo possível, como sendo (König e Wand, 1984):

$$f_{máx} = 2 \cdot l \cdot (d/2)^2 \cdot l^{1/2}$$

Aumentando-se ainda mais a excentricidade (Fig. 6-D), o avanço máximo possível de ser empregado volta a um valor mínimo, pois nesta excentricidade, já não é possível se obter superfícies cilíndricas, uma vez que praticamente não há mais contato entre a superfície cilíndrica da peça e as arestas secundárias da ferramenta.

Diretamente dependente do valor da excentricidade é o valor do raio do contorno (R) (Fig. 6). No modo centralizado, o raio obtido na peça é a reprodução do próprio raio da pastilha, que aumenta de valor à medida que se aumenta a excentricidade.

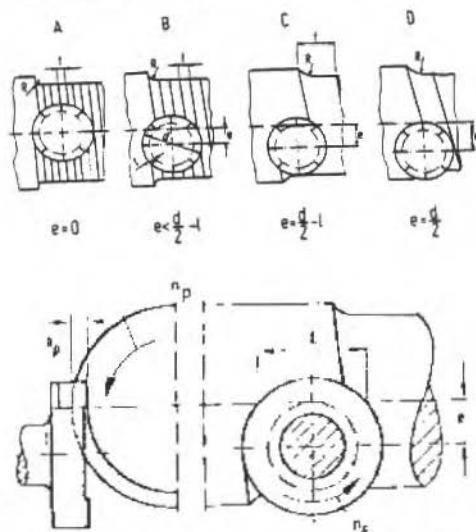


Fig. 6 Possibilidades de Arranjo no Fresotorneamento Ortogonal (König e Wand, 1984)

Dos quatro casos apresentados na Fig. 6, o caso "C" é, portanto, o que possibilita a maior taxa de remoção de material. Desta forma, tem seu emprego mais recomendado para peças de dimensões grandes, onde se necessita alta taxa de remoção de material (Strate, 1984).

Influência do Desvio Axial na Qualidade da Superfície

Normalmente as arestas de uma fresa possuem tolerância de posicionamento das pastilhas nos seus respectivos alojamentos. Assim, ocorrem irregularidades na estrutura das superfícies. A geometria da superfície é gerada somente pela aresta de corte mais próxima da peça (Fig. 7). Em ensaios realizados por Daniel (1994) com fresas de dois dentes, o desvio de posicionamento axial relativo de um dente a outro influenciou a rugosidade, conforme mostra a Fig. 8.

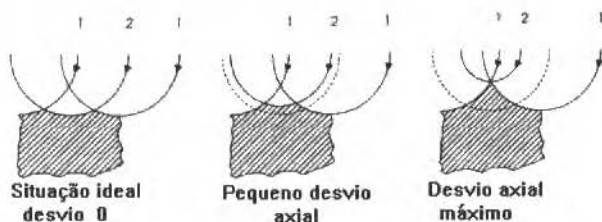


Fig. 7 Influência do Desvio Axial na Qualidade de Superfície (Daniel, 1994)

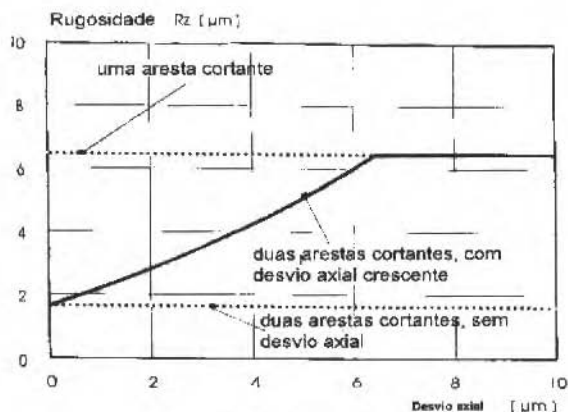


Fig. 8 Influência do desvio axial entre dois dentes na qualidade de superfície (Daniel, 1994)

Percebe-se da Fig. 8, que se bem posicionados os dentes da fresa, uma ferramenta com duas arestas cortantes proporciona um melhor nível de rugosidade do que uma fresa de apenas um dente. Na pior das hipóteses, proporciona uma rugosidade semelhante à uma fresa de apenas um dente.

Influência da Velocidade de Corte no Processo

A dureza da ferramenta de corte deve ser cerca de três vezes maior que a dureza da peça (Nakayama, 1988). Isto implica que, para se usinar aços endurecidos, um dos campos promissores para a utilização de fresotorneamento, deve se utilizar ferramentas de dureza elevada. Porém, tais ferramentas tem a desvantagem de serem frágeis (Ferraresi, 1989), tendendo a trincar sob velocidades

de corte similares às usadas pelo metal duro sem cobertura. Portanto, deve-se usar velocidades de corte para a usinagem de aços endurecidos em torno de 200 a 600 m/min, o que caracteriza a operação como usinagem em alta velocidade (Schulz e Moriwaki, 1992). As tensões de escoamento do material diminuem com o aumento da temperatura, e a deformação plástica, que envolve escorregamento dos contornos de grão, faz com que o material se comporte como um fluido viscoso, facilitando o corte. A temperatura, já elevada pelo efeito da deformação plástica, aumenta ainda mais com o aumento da velocidade de corte, facilitando as condições de deformação, na região de aderência, diminuindo o esforço de corte (Machado, 1991 a,b).

As velocidades de corte que proporcionam uma vida econômica da ferramenta devem estar dentro de campos relativamente estreitos, pois para o corte de materiais duros as ferramentas empregadas são em sua maioria extremamente frágeis, e os materiais frágeis apresentam um aumento da tenacidade e uma diminuição da dureza com o aumento da temperatura. Em decorrência, a faixa de velocidades na qual é viável a usinagem de materiais endurecidos é mais sensível à variação da velocidade de corte, do que em casos de usinagem de materiais não endurecidos.

Influência do Avanço por Dente na Vida da Ferramenta

Os avanços recomendados, para a operação de fresotorneamento, se situam entre 0,10 e 0,15 mm/v, segundo relatado por Schulz e Kneisel (1994).

Justificativas para o Emprego do Fresotorneamento

O fresotorneamento pode ser empregado com vantagens nos casos em que (Sandvik, 1994):

- é difícil o controle de cavacos provenientes da usinagem de grandes diâmetros, em materiais dúcteis, uma vez que, o fresotorneamento produzirá cavacos curtos, em qualquer material;
- usinagem em peças com estruturas instáveis (peças delgadas);
- necessidade de se ter uma vida da ferramenta que dure pelo menos a usinagem completa de um componente;
- operações de acabamento em peças de grandes diâmetros, tratadas termicamente, e não possíveis de serem usinadas por torneamento, pois permite que a rotação da peça seja pequena.

Além disso, apresenta as seguintes características (Schulz, 1990):

- possibilita altas velocidades de corte, superando os fatores limitantes do torneamento e fresamento, como por exemplo a força centrípeta na placa e na ferramenta, respectivamente;
- há pequenas oscilações de frequência no eixo árvore devido à baixa rotação da peça, possibilitando a usinagem de peças de parede fina, sem deformações por forças centrípetas.

A origem do desenvolvimento do processo de fresotorneamento se deu no fim do século dezenove. Tilghman (1889) substituiu a ferramenta de barra por uma fresa frontal, a fim de reduzir o efeito da temperatura, no corte interrompido. Na primeira metade deste século, o fresotorneamento continuou a ser desenvolvido. Nos anos 50 foi empregado pela primeira vez o fresotorneamento ortogonal. A partir do início dos anos 80, o processo começou a ser pesquisado nas universidades e a partir dos anos 90, o fresotorneamento ortogonal começou a ser empregado com altas velocidades. As primeiras publicações apresentando resultados de pesquisas conduzidas em fresotorneamento surgiram em 1983 (Sorge, 1983), resultado de pesquisas iniciadas na TH Darmstadt. Posteriormente surgiram outras publicações analisando diversos aspectos do processo (Strate, 1984; König e Wand, 1985; Svenningsson, 1989).

Procedimento Experimental

Materiais e Equipamentos Utilizados

Os corpos de prova usados ao longo do ensaio foram previamente manufaturados em um torno paralelo universal mecânico. Usou-se, para isto, barras laminadas de aço ABNT 52100, com 50 mm de

liâmetro, classificado como aço rolamento, para aplicações onde se necessita de têmpera profunda, tais como rolos, esferas, agulhas, e pistas internas e externas (Silva e Mei, 1988).

Após a usinagem, os corpos de prova foram submetidos a tratamento térmico de têmpera e revenimento para a dureza de 60-61 HRC.

O material possui a composição química apresentada na Tabela 1, obtida através de análise química.

Tabela 1 Composição Química do Aço ABNT 52100

C	Si	Mn	P	S	Cr	Ni	Mo	Cu	Al
1.040	0.190	0.310	0.016	0.010	1.500	0.090	0.030	0.130	0.030

A Figura 9 mostra a geometria do corpo de prova, bem como suas dimensões em milímetros (mm).

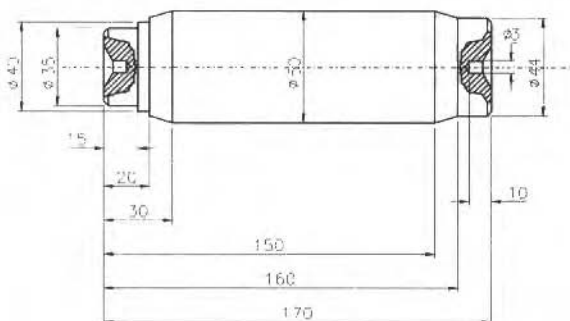


Fig. 9 Forma Geométrica e Dimensões do Corpo de Prova em mm (Sem Escala)

Para a realização dos ensaios foram utilizadas pastilhas de cerâmica SPK - Feldmühle, com formas tamanho SNGN 12 04 12 T 020 20, à base de óxido de alumínio com carvão de titânio disperso na matriz (Al_2O_3 / TiC) (Catálogo SPK, 1990).

Foram utilizadas fresas específicas para o processo. Para isto, procurou-se seguir as recomendações de Schulz e Moriwaki (1992) (Tabela 2). O material empregado para confeccionar as fresas foi aço ABNT 52100, temperado e revenido à 58 HRC.

As fresas são duplo negativas, ou seja os ângulos de saída dorsal γ_p e de saída lateral γ_r são negativos. Neste tipo de fresa são usadas pastilhas neutras, tornando-as adequadas à usinagem de materiais endurecidos ou que requeiram maior resistência ao impacto. Este tipo de locação da pastilha é exigência dos materiais cerâmicos, que desta forma trabalham sob compressão, situação em que são mais resistentes à fratura.

As características das fresas utilizadas estão apresentadas na Tabela 2.

Tabela 2 Características Geométricas das Fresas

Procedência	ϕ [mm]	Ângulo de saída γ_0 (°)	Ângulo de inclinação λ_0 (°)	Ângulo de posição χ_i (°)	Número de arestas [Z]	Fixação
Confeccionada	24	- 6	- 4	90	1	cone morse nº 3
Confeccionada	52	- 6	- 4	90	1	cone morse nº 3

Os seguintes equipamentos foram utilizados:

- Torno ROMI CNC ECN 40 II (Potência do motor principal - 15 kW);
- Cabeçote fresador adaptado de um cabeçote retificador, com:

- Motor balanceado dinamicamente (Potência - 7,2 kW)
- Mancal com rolamentos axiais (Rotação crítica - $n_{max}=10.000$ rpm)
- Rugosímetro modelo SURFTRONIC (Cut-off = 0,8 mm)
- Durômetro, marca BRIRO.

Metodologia dos Ensaios Realizados

O cabeçote fresador foi montado sobre o carro transversal do torno, de maneira que o centro do mancal ficasse perpendicular ao eixo árvore, configurando assim, o fresotorneamento ortogonal. Da mesma maneira, os eixos da ferramenta e da peça eram coplanares, ou seja, a excentricidade "e" (valor dimensional entre os planos que contém o eixo de rotação da peça e ferramenta, respectivamente) foi nula.

As condições dos ensaios foram as indicadas na Tabela 3.

Tabela 3 Condições dos Ensaios Realizados

Ensaio	Fresa	Parâmetros de corte	
1	$\phi = 52$ mm Z = 1 $\gamma_o = -6^\circ / \lambda_s = -4^\circ$ pastilha - Al_2O_3/TiC	$l_f = 50$ mm $v_c = 288$ m/min $a_p = 0,1$ a 2 mm $f = 0,12$ mm/v $v_f = 10,8$ mm/min	$n_p = 90$ RPM $n_f = 1760$ RPM concordante $n_f/n_p = 19,6$
2	$\phi = 52$ mm Z = 1 $\gamma_o = -6^\circ / \lambda_s = -4^\circ$ pastilha - Al_2O_3/TiC	$l_f = 50$ mm $v_c = 288$ m/min $a_p = 0,2$ mm $v_f = 10,8$ mm/min $f = \text{variável de } 0,96 \text{ à } 0,012$ mm/v	$n_p = 11,2$ a 850 RPM $n_f = 1760$ RPM discordante $n_f/n_p = 157,1$ a 2,1
3	$\phi = 24$ mm Z = 1 $\gamma_o = -6^\circ / \lambda_s = -4^\circ$ pastilha - Al_2O_3/TiC	$l_f = 50$ mm $v_c = 309$ m/min $a_p = 0,2$ mm $v_f = 10,8$ mm/min $f = 0,06$ mm/v	$n_p = 180$ RPM $n_f = 4098,5$ RPM discordante $n_f/n_p = 22,8$
4	$\phi = 24$ mm Z = 1 $\gamma_o = -6^\circ / \lambda_s = -4^\circ$ pastilha - Al_2O_3/TiC	$l_f = 50$ mm $v_c = 309$ m/min $a_p = 0,05$ mm $f = 0,01$ a 0,1 mm/rot.	$n_p = 600$ RPM $n_f = 4098,5$ RPM discordante $n_f/n_p = 6,8$

Resultados e Discussão

Os ensaios foram conduzidos utilizando-se duas fresas com diâmetros de 52 e 24 mm, possuindo apenas um dente, visando eliminar possíveis influências do desvio axial. A princípio os ensaios foram conduzidos com a fresa de $\phi 52$ mm e, em vista dos resultados obtidos, projetou-se outra fresa com $\phi 24$ mm.

Os ensaios foram realizados na sequência apresentada na Tabela 3. À medida que se obteve benefícios, este foi repassado para o ensaio posterior, visando ao final, resultados melhores. Não houve uma necessária dependência entre os ensaios, sendo que alguns deles tiveram caráter observatório, de forma isolada.

Ensaio 1

Este ensaio objetivou verificar o comportamento da rugosidade em função do aumento da profundidade de corte. Os parâmetros velocidade de corte, avanço e rotação da peça, foram determinados após uma série de ensaios preliminares, tendo-se como base o trabalho de Schulz e Kneisel (1994).

A Figura 10 mostra que a rugosidade média R_a aumentou de maneira aproximadamente linear até cerca de 0,52 mm de profundidade de corte. A partir deste ponto as características da curva mudaram, o que leva a crer que até 0,52 mm a rugosidade foi função predominantemente do processo e, a partir deste ponto passou a assumir características originadas da rigidez do cabeçote fresador e de uma possível elevação das forças de usinagem e da maior área de contato ferramenta-peça.

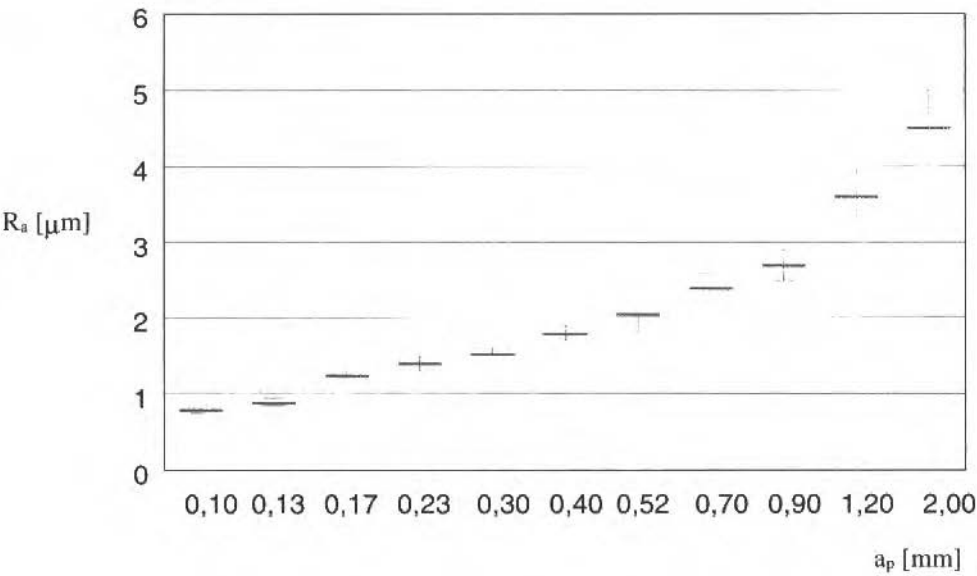


Fig. 10 Influência da Profundidade de Corte a_p na Rugosidade Média R_a

Ensaio 2

Neste ensaio foram alterados os parâmetros: sentido do movimento (discordante), a rotação da peça foi variável e a velocidade de avanço v_f foi mantida constante em 10,8 mm/min.

Para as condições utilizadas neste ensaio, observou-se que à medida que se aumentou a rotação da peça, a rugosidade diminuiu (Fig. 11). Este fato se dá tendo em vista que, mantida a velocidade de avanço constante e variando-se a rotação, variou-se o avanço por volta. Neste caso, o avanço por volta variou de 0,96 à 0,012 mm/v (rotação da peça variando entre 11,2 e 850 rpm, respectivamente).

A velocidade de avanço foi mantida constante em função das baixas velocidades de rotação na peça, pois com avanço constante, os tempos de corte seriam muito altos. Foi usado o mesmo valor da velocidade de avanço, utilizado no ensaio anterior, ou seja, $v_f = 10,8$ mm/min

Optou-se pelo método discordante para se investigar o efeito deste movimento, uma vez que, no fresamento convencional, consegue-se melhores resultados (R_a menor) com o sentido concordante (ver Tabela 4).

Tabela 4 Efeito do método discordante

Ensaio	f (mm/volta)	a_p (mm)	R_a (μ m)
1- conc.	0.12	0,2	1,25 (interpolação)
2 - disc.	0,12 ($v_f = 10,8$ a 90 RPM)	0,2	2,3 (interpolação)

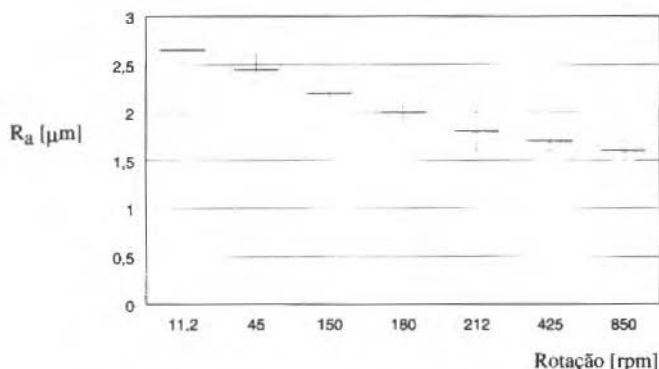


Fig. 11 Influência da Rotação da Peça na Rugosidade Média R_a

Ensaio 3

Para as mesmas condições do ensaio anterior apenas o diâmetro da ferramenta foi alterado a fim de se verificar o comportamento da rugosidade. A velocidade de corte foi alterada em 7%, em função do jogo de polias disponível. A rotação da peça neste caso foi de 180 rpm. Observou-se, que com o decréscimo do diâmetro da ferramenta em 117% a rugosidade decresceu em 17% (Tabela 5).

Tabela 5 Influência do diâmetro da fresa

Rotação da peça [mm]	Diâmetro da ferramenta [mm]	Rugosidade R_a [μm]
180	52	2
180	24	1,7

Ensaio 4

Neste ensaio procurou se utilizar uma profundidade de usinagem menor que 0,1 mm, uma vez que, observou-se no ensaio 1, que a rugosidade média é menor quanto menor for esta grandeza (Fig. 10). Assim, utilizando-se um $a_p = 0,05$ mm (a peça sofreu uma pré-usinagem para a uniformização geométrica da superfície), procurou-se, mantidos os demais parâmetros do ensaio 3, verificar o comportamento da rugosidade média, quando se altera o avanço.

A rotação da peça foi aumentada para 600 rpm, tendo em vista os resultados observados no ensaio 2 (quanto maior a rotação da peça, menor a rugosidade).

Os resultados estão mostrados na Fig. 12. Observa-se desta figura que, à medida que se aumenta o avanço, piora a rugosidade média da peça.

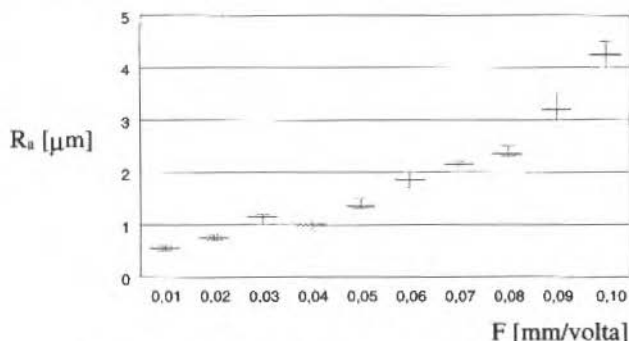


Fig. 12 Influência do Avanço Axial na Rugosidade Média R_a

Com relação à influência da profundidade de corte, nota-se que, comparando os resultados do ensaio 3, para as condições $f = 0,06 \text{ mm/v}$ ($v_f = 10,8 \text{ mm/min}$ à 180 rpm) e $a_p = 0,2 \text{ mm}$ (Tabela 5) com os resultados do ensaio 4, para as condições $f = 0,06 \text{ mm/v}$ ($v_f = 36 \text{ mm/min}$ à 600 rpm) e $a_p = 0,05 \text{ mm}$ (Fig. 12), que os resultados obtidos para a rugosidade foram semelhantes ($R_a \sim 1,7 \mu\text{m}$). Este fato necessita ser melhor investigado, pois, em primeira instância indicaria que, para o método discordante, a profundidade não alteraria os valores finais de rugosidade, todavia, as rotações da peça foram diferentes, sendo que, no último caso, a velocidade efetiva foi maior.

Em resumo, os ensaios de 1 a 4, realizados com várias combinações de avanço, velocidade de corte, rotação da peça, diâmetro da fresa, etc., procuraram verificar o comportamento da qualidade superficial, aqui caracterizada como R_a , quando se utiliza o processo de fresotorneamento. Destes ensaios observou-se que:

- A profundidade de corte a_p mostrou ser um parâmetro de grande importância no fresotorneamento ortogonal concordante. Observou-se uma tendência de piora da rugosidade à medida que a profundidade de corte aumentou (Fig. 10). Isto pode ter ocorrido devido à uma possível elevação das forças de usinagem e da maior área de contato ferramenta-peça, e também devido ao fato que o cabeçote utilizado não tinha rigidez suficiente para usinar em profundidades maiores. Em trabalhos futuros, pretende-se projetar um cabeçote mais rígido.
- O fresotorneamento discordante, da mesma maneira que o fresamento convencional, piorou a rugosidade (Tabela 4).
- À medida que, mantidas a velocidade de avanço e a profundidade de corte constantes, se aumentou a rotação da peça, diminuindo portanto a relação n/n_p , diminuiu também a rugosidade. Isto ocorreu porque, à medida que se aumentou a rotação da peça, uma vez mantida a velocidade de avanço constante, o avanço por volta diminuiu, diminuindo, conseqüentemente, a espessura do cavaco e as forças de usinagem, melhorando o acabamento, mostrando que a variação do avanço, da mesma maneira que no fresamento normal, melhora a rugosidade.
- A diminuição do diâmetro da fresa mostrou ser benéfica ao acabamento superficial, pelo fato que, fresas de menor diâmetro, permitem rotações maiores (maior rigidez) e este fato parece estar relacionado com a melhoria da rugosidade, embora a melhoria no acabamento não tenha sido substancial. Tal fato também foi citado por Daniel (1994).

Embora todos os ensaios tenham sido conduzidos sem fluido de corte, após o término da usinagem era possível a manipulação da peça sem o auxílio de qualquer proteção, sugerindo que a menor porção de calor gerada fica com a peça.

Conclusões

A pesquisa sobre o processo de fresotorneamento é recente e o estudo da sua tecnologia está ainda em desenvolvimento, merecendo ser melhor compreendido e estudado, objetivos estes parcialmente atingidos neste trabalho. Face às potencialidades do processo, procedeu-se a alguns ensaios experimentais, envolvendo alguns parâmetros do processo e o estudo de sua influência no acabamento da peça, aqui caracterizada como rugosidade. À semelhança de outros processos de usinagem conduzidos com ferramentas tendo geometria de corte definida, verificou-se a piora da rugosidade com o aumento do avanço, aumento da profundidade e o aumento do diâmetro da ferramenta.

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Caracterização do Comportamento da Curva-R de Refratários de MgO-C Obtida por Diferentes Métodos de Ensaio

Characterization of the R-Curve Behavior of MgO-C Refractories Applying Different Measurement Methods

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Abstract

The influence of carbon addition and the geometry of the applied bending test on the fracture behavior of magnesia-carbon refractory bricks, with 5, 8 e 17 wt% of carbon, were studied. The R Curve behavior was evaluated in samples having a single-edge notch, under three and four point bending test conditions. The tests were undertaken under controlled displacement and stable crack propagation conditions. A comparison between two different calculation procedures, were made. It is presented a set procedure criterium concerning the geometry of bending test and the calculation approach applied for the determination of R-Curve, with the objective of decreasing the uncertainty in the measured values. Explanations for the R-Curve shape were also given.

Keywords: Refractory, MgO, Carbon, R-Curve.

Resumo

Amostras a partir de tijolos refratários de MgO-C, com 5, 8 e 17% em peso de carbono, foram usadas para um estudo da influência do teor de carbono e da geometria de ensaio empregada. O comportamento de Curva-R foi avaliado para corpos de prova, com entalhe plano, em ensaios de flexão a três e quatro pontos. As curvas de carga versus deslocamento foram obtidas sob condição de propagação estável de trinca e com controle de deslocamento. Foi feita uma comparação entre diferentes procedimentos de cálculo da Curva-R. É apresentado um conjunto de critérios para a determinação da Curva-R para refratários cerâmicos, no que se refere ao tipo de ensaio e ao cálculo empregado, visando diminuir a incerteza nos valores medidos. É também explicada a forma das Curvas-R obtidas.

Palavras-chaves: Refratário, MgO, Carbono, Curva-R.

Introdução

Refratários de MgO-C tem sido bastante utilizados na indústria de fabricação de aço devido a sua alta resistência ao choque térmico e à corrosão. O carbono, devido as suas características de não molhabilidade, confere ao refratário uma maior resistência à penetração da escória e do metal líquido. O aumento na resistência ao dano por choque térmico deve-se ao aumento da energia total de fratura, provocado pela ação de mecanismos de tenacificação com a sua adição. A influência da adição de carbono no comportamento mecânico dos refratários de MgO-C, quanto à fratura, pode ser caracterizada com a determinação do comportamento de Curva-R, definida como a curva de resistência à propagação de trinca, $R(a)$, em função do seu tamanho, a (Pandolfelli et al., 1995). Além disso, é necessário estabelecer um critério, referente ao tipo de ensaio e ao procedimento de cálculo a ser utilizado.

Métodos e Materiais

O comportamento de Curva-R indica a evolução da tenacidade de um material durante a propagação lenta e estável de uma trinca, ou seja quase-estaticamente. Esta condição pode ser obtida através do controle da taxa de deflexão imposta ao corpo de prova, durante um ensaio de flexão, assim como das geometrias do entalhe e do teste. O critério usado para a obtenção desta condição encontra-se de acordo

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com as adaptações feitas à norma ASTM-399, acrescidas ao estabelecido por Nascimento, Pandolfelli e Rodrigues (1993). Foram empregados dois tipos de ensaios de flexão, isto é, a 3 e 4 pontos e comparados diferentes procedimentos de cálculo para a obtenção da Curva-R.

A Curva-R foi determinada baseando-se na Mecânica de Fratura Linear Elástica, especificamente no balanço energético. A taxa de liberação de energia elástica armazenada, G , e a resistência à propagação de trinca, R , são definidos como (Broek, 1987):

$$G = \frac{K_I^2}{E'} = \left[\left(\frac{P^2}{2b} \right) \cdot \left(\frac{dC}{da} \right) \right] \quad (1)$$

e

$$R = G \quad (2)$$

onde, C é a flexibilidade, P é a carga aplicada, a é o tamanho de trinca, b é a largura do corpo de prova, K_I é o fator de intensidade de tensão e $E' = E / (1 - \nu^2)$, sendo E o módulo de Young e ν o coeficiente de Poisson. Define-se K_I como:

$$K_I = Y(a/w) \cdot \sigma \cdot \sqrt{a} \quad (3)$$

onde, $Y(a/w)$ é um fator de forma, w é a altura do corpo de prova, σ é a tensão aplicada, definida de acordo com o tipo de ensaio empregado.

O procedimento de cálculo empregado utiliza a função $Y(\alpha)$ indicada por Wilson (1970), a seguir:

$$Y(\alpha) = 0,665 \cdot [\alpha \cdot (1 - \alpha)^3]^{-1/2} \quad (4)$$

onde, $\alpha = a/w$, é o comprimento relativo da trinca. Os valores mais indicados para a aplicação da equação 4 encontram-se na região de α entre 0,50 e 0,80, podendo ser usada até $\alpha = 0,9$, com um mínimo de erro. A função $Y^2(\alpha)$ possui integração analítica.

Nomenclatura

a	= tamanho da trinca	G	= taxa de liberação de energia elástica armazenada	S	= distância entre apoios
A	= área projetada da fratura	K_I	= fator de intensidade de tensão	U	= energia consumida na fratura
b	= largura do corpo de prova	K_{Ic}	= fator crítico de intensidade de tensão	w	= altura do corpo de prova
C	= flexibilidade	P	= carga aplicada	Y	= fator de forma
C_c	= carbono	P_i	= carga aplicada	$Y(\alpha)$	= fator de forma
C_0	= flexibilidade calculada para um corpo sem entalhe	$\overline{P_{max}}$	= valor médio da carga máxima	$Y(a/w)$	= fator de forma
D_i	= deslocamento do ponto de carga	R	= resistência à propagação de trinca	α	= tamanho relativo da trinca
$\overline{D_{max}}$	= valor médio do deslocamento máximo	$R(a)$	= resistência à propagação de trinca	α'	= tamanho relativo da trinca
E	= módulo de Young	$R(\alpha)$	= resistência à propagação da trinca	γ_{wof}	= energia total de fratura
e	= distância entre os pontos de carga	$\overline{R}$	= valor médio de R	$\overline{\gamma_{wof}}$	= valor médio da energia total de fratura
E'	= módulo elástico			ν	= coeficiente de Poisson
				σ	= tensão aplicada

Substituindo-se a Eq. (3) na Eq. (1), rearranjando-se os termos e integrando de um lado em α e de outro em C , obtém-se que:

$$\int_0^{\alpha} Y^2(\alpha') \cdot \alpha' \cdot d\alpha' = \int_{C_0}^C \left[\frac{P^2}{\sigma^2 \cdot 2 \cdot b \cdot w^2} \cdot \left(\frac{E}{I - v^2} \right) \right] \cdot dC \quad (5)$$

Substituindo-se a Eq. (4) na Eq. (5), integrando-se e resolvendo-se para α , obtém-se uma expressão geral para o tamanho relativo de trinca, em função da flexibilidade, como se segue:

$$\alpha(C) = 1 - \left[1 + \left(\frac{2 \cdot P^2}{(0,665)^2 \cdot \sigma^2 \cdot 2 \cdot b \cdot w^2} \right) \cdot \left(\frac{E}{I - v^2} \right) \cdot (C - C_0) \right]^{-1/2} \quad (6)$$

onde C representa os valores experimentais instantâneos da flexibilidade e C_0 simboliza a flexibilidade calculada para o mesmo corpo, porém sem entalhe. Os valores de C são obtidos experimentalmente, aplicando-se a técnica, aqui denominada de Linhas Radiantes (Hübner, 1991). Nesse caso, os valores de C são determinados a partir dos recíprocos da inclinação das retas, traçadas da origem até os diferentes pontos da curva de carga versus deslocamento, (P_i, D_i) . Essa curva é obtida através de ensaio de flexão, com carregamento único e propagação quase-estática de trinca. Define-se $C(P_i, D_i)$ através da relação: $C(P_i, D_i) = D_i / P_i$. Os valores de C_0 são calculados, de acordo com a geometria do ensaio de flexão empregado, através da seguinte relação:

$$C_0 = \left[\frac{(S - e)^2 \cdot (S + 2 \cdot e)}{4 \cdot E' \cdot b \cdot w^3} \right] \quad (7)$$

onde, S e e são as distâncias entre apoios e entre os pontos de carga, respectivamente. O valor de e é igual a zero para ensaios de flexão a 3 pontos.

Os valores de módulo elástico utilizados neste trabalho foram obtidos através de um programa computacional iterativo, no qual combinam-se as Eqs. (6) e (7). Partindo-se de um valor sugerido para E' , verifica-se se o valor calculado de C_0 (Eq. (7)) gera, através da Eq. 6, o correto valor do tamanho relativo do entalhe, α_0 . Para isso, a Eq. 6 tem de ser ocupada com o valor de C igual à flexibilidade inicial do corpo de prova, com entalhe. Esse valor de C é obtido do recíproco da inclinação da região elástica da curva carga versus deslocamento para o corpo com entalhe. Sendo o valor de α_0 não satisfatório, varia-se o valor de E' e se repete o procedimento até que se obtenha o valor correto para o tamanho relativo do entalhe. Assim, o correspondente valor de E' será adotado como o valor correto para aquele corpo de prova.

Os valores de R são, finalmente, obtidos combinando-se as Eqs. (2), (1), (3) e (4), que resulta na seguinte expressão:

$$R(\alpha) = \left[\frac{(0,665)^2 \cdot \sigma^2 \cdot w \cdot (I - v^2)}{E} \right] \cdot \frac{1}{(1 - \alpha)^3} \quad (8)$$

onde σ é substituído de acordo com o tipo de ensaio de flexão empregado. O valor de α é calculado através da Eq. 6, como explicado anteriormente. Deste modo obtém-se finalmente a Curva-R.

Outro parâmetro muito importante dentro do conceito da fratura quase-estática é a energia total de fratura, γ_{wof} , que é definida por (Pandolfelli et al. (1995); Nakayama (1965)):

$$\gamma_{wof} = \left[\frac{U}{2 \cdot A} \right] = \frac{\bar{R}}{2} \quad (9)$$

onde, U é a energia consumida na fratura, A é a área projetada da fratura (secção transversal do corpo de prova no plano do entalhe) e $\bar{R}$ é o valor médio de R calculado sobre todos os pontos da Curva- R (Pandolfelli et al., 1995).

Os corpos de prova utilizados foram cortados e retificados a partir de tijolos de $MgO-C$, com 5, 8, e 17%-p de carbono, para os ensaios de flexão a 3 e 4 pontos, com dimensões de $35 \times 35 \times 180$ mm³. Parte do carbono presente no material vem do piche, que é usado como ligante, e parte vem da adição de grafite na forma de flocos. Os tijolos empregados neste estudo foram curados a uma temperatura de 400 °C. Os entalhes com profundidades relativas entre 0,40 e 0,47, foram feitos com o auxílio de um disco de corte diamantado de 400 μ m de espessura. Os ensaios foram realizados numa máquina universal de testes mecânicos, MTS-810, Modelo-458.20, configurada para ensaios de flexão a 3 e 4 pontos, de acordo com adaptações feitas à norma ASTM-D-790. Usou-se uma célula de carga de 50 kN, reduzida eletronicamente para 2,5 kN. A distância entre apoios foi de $S = 140$ mm. Para a flexão a 4 pontos utilizou-se $e = 70$ mm de distância entre os pontos de aplicação de carga. As velocidades de deslocamento do atuador usadas para cada composição são mostradas na Tabela 1. Essas velocidades são aquelas que permitiram a propagação quase-estática da trinca, em cada caso.

Tabela 1 Velocidades de deslocamento usadas durante os ensaios de flexão a 3 e/ou 4 pontos, para a condição de propagação estável de trinca, para corpos de prova com diferentes teores de carbono.

Teor de C (%)	Velocidade de deslocamento (μ m/min)
5	0,5
8	1,0
17	10,0

De maneira a deduzir das curvas resultantes, de carga versus deslocamento, a influência das acomodações e deformações dos acessórios de ensaio, foram realizados ensaios de máquina versus máquina para caracterizar esses deslocamentos parasitas. Esses ensaios são similares aos de flexão a 4 pontos, sendo que, neste caso, os roletes de apoio são ajustados de tal modo que se alinham com os roletes de aplicação de carga. Entre os pares de roletes superior e inferior é colocado um corpo de prova de mesma composição daquela em estudo. As velocidades de deslocamento empregadas foram as mesmas dos ensaios de flexão a 3 e 4 pontos (ver Tabela 1). Com isso, possíveis afundamentos dos roletes na superfície do corpo de prova também fica caracterizado.

Portanto, as curvas finais de carga versus deslocamento que foram utilizadas para o cálculo da Curva- R , foram corrigidas no eixo dos deslocamentos, do deslocamento extraído da curva de máquina contra máquina, para mesmos valores de carga aplicada. Com isso a parte superior das curvas se deslocam ligeiramente para a esquerda, gerando valores de flexibilidade menores do que aqueles se não houvesse tal correção.

Resultados e Discussão

Os valores médios calculados do módulo elástico, E , são mostrados na Tabela 2.

As Curvas- R apresentadas nos resultados deste trabalho são curvas médias obtidas a partir de quatro curvas relativas a corpos de mesma composição. Portanto, para cada composição, caracterizada por um tipo de ensaio, obtêve-se a Curva- R média representativa do comportamento daquela composição. Como exemplo, a Fig. 1 mostra as quatro Curvas- R referentes ao material com 17 %-p de C e ensaiado por flexão a 3 pontos. A Curva- R média dessa composição, para ensaio de flexão a 3 pontos, aparece na Fig. 2, em comparação explicada abaixo.

A Figura 2 apresenta os resultados sobre a influência dos ensaios de flexão a 3 e a 4 pontos na Curva- R , exemplificada através da composição com 17 %-p de C. Foi empregado na sua obtenção o critério indicado por Magon, Rodrigues e Pandolfelli (1995), de desprezar os pontos da curva carga versus deslocamento, para os quais os valores de carga obtidos representassem uma queda maior do que 92% do valor da carga máxima atingida pelo corpo de prova. O que se vê na Fig. 2 são as Curvas- R médias, como descrito anteriormente.

Tabela 2 Valores Médios de E

Teor de C (%)	E(GPa)
5	$24 \pm 2,1$
8	$28 \pm 7,7$
17	$13 \pm 9,6$

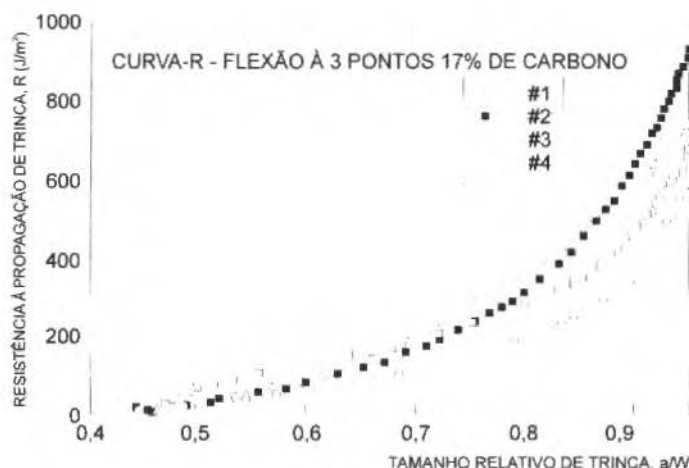


Fig. 1 Curvas-R para a Composição com 17 % de C, para Ensaio de Flexão a 3 pontos. Cada Curva é Referente a um Novo Corpo de Prova.

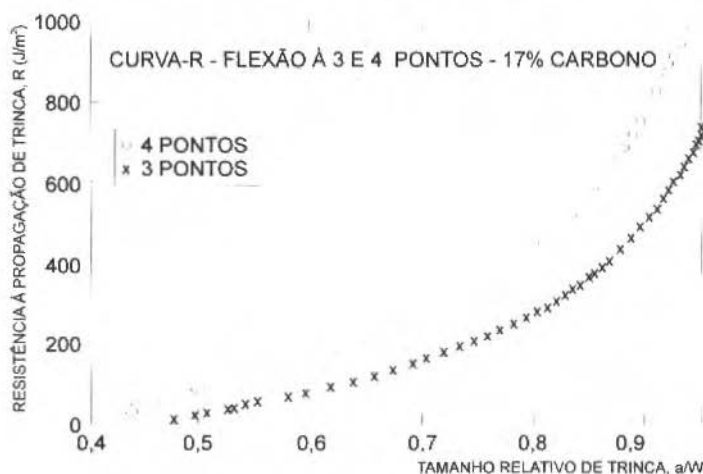


Fig. 2 Curvas-R Médias para Ensaios de Flexão a 3 e 4 Pontos, para a Composição com 17% de C.

De acordo com Magon, Rodrigues e Pandolfelli (1995), os mecanismos de dissipação de energia provocados por interação mecânica entre as superfícies durante a fratura, quando em ensaios de flexão, levam a um aumento falso na energia de fratura, no final da curva. Acredita-se que este efeito é mais acentuado para ensaios a 4 pontos. Nota-se através da Fig. 2, um aumento nos valores de R em toda a extensão da curva, quando em ensaios de flexão a 4 pontos. Este efeito é mais acentuado para a composição com maior teor de carbono, porém foi verificado em todas as composições.

A influência do teor de carbono, é mostrada nas Curvas-R médias apresentadas nas Figs. 3(a) e 3(b), referentes aos ensaios de flexão a 3 e 4 pontos, respectivamente.

A Tabela 3 apresenta os valores médios de $R(\alpha)$, $\bar{R}$, obtidos através das Curvas-R médias. Define-se R como a área sob a Curva-R dividido pelo intervalo correspondente de α . São também apresentados os valores médios da carga máxima e os valores médios do deslocamento máximo (total), P_{max} e D_{max} , respectivamente, obtidos através da média aritmética entre os valores determinados para 4 diferentes amostras, de cada composição e tipo de ensaio. A partir das mesmas curvas carga versus deslocamento usadas para o cálculo das Curva-R foram também avaliados os valores da energia total de fratura, γ_{wof} . Os valores de $2 \cdot \gamma_{wof}$ também estão mostrados na Tabela 3.

Tabela 3 Valores de $\bar{R}$, $2 \cdot \gamma_{wof}$, P_{max} e D_{max} , Obtidos nos Ensaio de Flexão a 3 e 4 Pontos, para os Diferentes Teores de C.

Tipo de teste	% de C	P_{max}	D_{max}	$\bar{R}$	$2 \gamma_{wof}$
3P	5	1010 ± 73	0,5 ± 0,05	215 ± 19	216 ± 13
	8	953 ± 13	0,5 ± 0,08	243 ± 56	246 ± 50
	17	498 ± 80	1,0 ± 0,10	250 ± 48	261 ± 47
4P	5	2234 ± 15	0,3 ± 0,04	307 ± 16	295 ± 20
	8	1853 ± 32	0,3 ± 0,05	292 ± 7	308 ± 18
	17	1145 ± 65	0,8 ± 0,08	338 ± 97	370 ± 99

De acordo com a Tabela 3, os valores de $2 \gamma_{wof}$ e R apresentam boa concordância entre si, quando utilizado o método das linhas radiantes para determinação da Curva-R, como discutido por Pandolfelli et al. (1995).

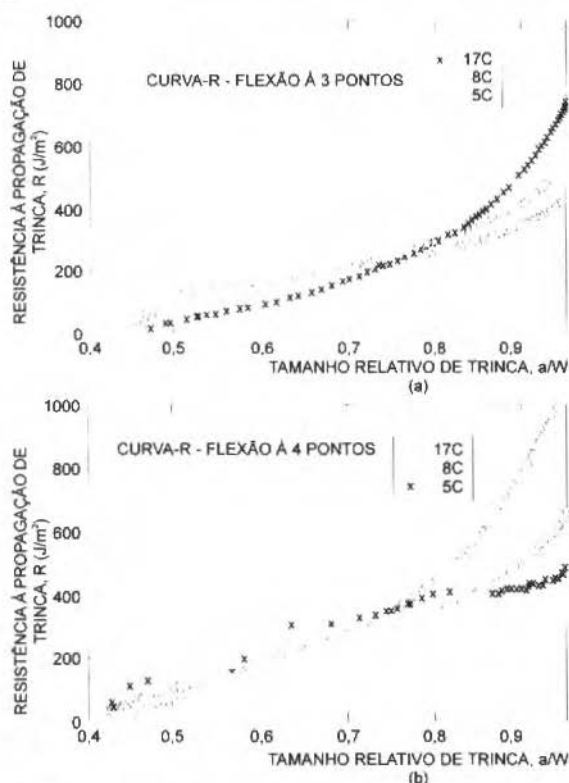


Fig. 3 Curvas-R Médias para Composições com 5, 8 e 17 % de C, para os Ensaio de Flexão a: (a) 3 Pontos e (b) 4 Pontos.

Como pode ser observado na Fig. 3, a composição com maior teor de carbono apresenta valores de R , no início da propagação da trinca, menores que os com menor teor de carbono. Pode-se, no entanto, verificar através dos valores mostrados na Tabela 3, um aumento de γ_{wof} com a adição de carbono, ocorrendo tanto para ensaios de flexão a 3 como a 4 pontos. Ao mesmo tempo, ocorre uma diminuição tanto no valor da carga máxima alcançada pelos corpos de prova, quanto nos valores de E (ver Tabela 2), com a adição de carbono. Os valores menores de E e de P_{max} são os responsáveis pelos valores menores de R , para o início de propagação da trinca, de acordo com a Eq. 8. Isso ocorre porque σ entra ao quadrado naquela fórmula. No entanto os valores de γ_{wof} e de R , são maiores para composições com maiores teores de carbono. Além disso, de acordo com Pandolfelli et al. (1995), composições com maiores teores de carbono apresentam menores valores de K_{Ic} , fator crítico de intensidade de tensão, confirmando a tendência para valores menores de R , no início da propagação da trinca, para a composição com 17% de carbono, conforme indicam as Eqs. 1 e 2.

A Tabela 3 também mostra que o aumento do teor de grafite causa um aumento na deflexão máxima. Isso revela a "plasticidade" emprestada pelo grafite, que é o principal agente tenacificador. Mesmo diminuindo o valor de P_{max} , o aumento de D_{max} supera este efeito causando o aumento de γ_{wof} , para composições com maiores teores de grafite.

Esses resultados indicam ser o carbono um agente enfraquecedor, apesar de seu efeito tenacificador, nos refratários de MgO-C. Da natureza anisotrópica do carbono decorre o favorecimento dos seguintes mecanismos de tenacificação: deflexão e ramificação de trincas, além do esfolheamento. Esse esfolheamento, observável na superfície de fratura, é o agente enfraquecedor da resistência mecânica. Por outro lado, o ganho da energia total de fratura pode ser, em determinadas situações, muito mais desejável.

Os valores de R obtidos de acordo com o procedimento de cálculo aqui empregado, são similares aos encontrados anteriormente por Magon, Rodrigues e Pandolfelli (1995), para composições com 5% de carbono, estendendo-se essa similaridade também para as outras composições. Naquele trabalho referenciado (Magon, Rodrigues e Pandolfelli, 1995), a função $Y(\alpha)$ usada, indicada por Srawley e Gross (1976), não possuía integração analítica, mas indicava uma faixa de α maior de aplicação. O gráfico traçado com a função de Srawley difere ligeiramente do gráfico da função de Wilson e além disso, o programa de computação para o cálculo de α em função da flexibilidade experimental tem de realizar uma integração numérica daquela função Y . A função aqui usada e indicada por Wilson (1970), facilita os cálculos devido ao fato de ser integrável analiticamente; no entanto, os valores gerados de α são bastante sensíveis ao valor do módulo elástico adotado.

Conclusões

A adição de carbono em refratários de MgO-C tem um efeito ligeiramente tenacificador, que se revela numa Curva-R mais inclinada, dificultando o crescimento da trinca, mesmo tendo seu início abaixo daquele do material com baixa concentração de carbono. Esse resultado justifica a utilização de refratários desta classe com percentuais de carbono próximos a 17 % quando se procura maximizar a resistência mecânica retida após choque térmico. Neste trabalho considerou-se apenas os efeitos termomecânicos decorrentes da presença do carbono, excluindo-se os aspectos de oxidação.

A Curva-R para estes refratários, determinada através de ensaios de flexão a 4 pontos, apresenta valores maiores de R em toda a extensão da curva, quando comparada aos ensaios a 3 pontos.

O procedimento de cálculo aqui empregado facilita o método para a determinação de Curva-R, mas é bastante sensível ao valor de módulo elástico empregado.

Agradecimentos

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Abstracts

Sebben, S., 1997, "Steady, Spatially-Periodic Fully-Developed Turbulent Flow in Interrupted-Plate Channels", RBCM - J. of the Braz. Soc. Mechanical Sciences, Vol. 19, No. 4, pp. 445-457.

This paper presents numerical results for steady, spatially-periodic, two-dimensional, fully-developed turbulent flows in interrupted-surface passages. The geometry of interest consists of an array of interrupted plates, in an in-line arrangement, placed parallel with the main flow direction. Such a configuration can be regarded as a two-dimensional idealization of the cores of some offset-fin compact heat exchangers. The performance of several low-Reynolds number, two-equation linear eddy-viscosity models of turbulence is discussed in the context of a time-averaged formulation. Simulations were carried out for a range of Reynolds numbers and one value of the plate thickness. Numerical predictions of the friction factor and skin friction coefficient along the plates are compared with available experimental results.

Keywords: Turbulence, Periodic Flows, Control-Volume Method, Heat Exchangers.

Moura, L. M. M., and Marvillet, C., 1997, "Local Measurements in Two-Phase Flows Using a Resistivity Double Probe Technique, RBCM - J. of the Braz. Soc. Mechanical Sciences, Vol. 19, No. 4, pp. 458-473.

Local measurements are of primary importance for the characterization of gas-liquid two-phase flows, both for processes control and numerical modeling validation. It is a very active research field due to the increasing number of applications in the thermohydraulics of heat exchangers, nuclear plants, chemical processes and oil industries.

This paper presents the local measurements in a vertical upward air-water flow using the electrical resistivity double probe technique. The test section was a 80 mm i.d. and 160 cm long Plexiglas pipe. Five different gas superficial velocities, ranging from 0.02 to 0.10 m/s, were used in combination with two liquid superficial velocities of 0 and 0.10 m/s. A resistivity double probe was employed for measurements of the radial profiles of void fraction, bubble frequency, bubble interface velocity, interfacial area concentration and Sauter mean diameter. The electrical resistivity probe method consists of an instantaneous measurement of the local electrical resistivity in the two-phase flow by means of a sensor electrode. Since the circuit is opened or closed depending on whether the sensor tip is in contact with gas or liquid, the probe behaves in principle like a switch, yielding a two-stage signal. However, to obtain a true square wave type signals, a proper threshold voltage has to be used as a triggering criterion. Herein the signal conditioning is discussed and the influence of the threshold level is analyzed.

With a probe with two sensors displaced axially, the bubble interface velocity could be determined from the time delay which gave maximum correlation between the sensor responses. These values of gas velocity in conjunction with void fractions could be integrated to give average gas superficial velocities. Values determined in this manner were compared to values from the inlet gas flowrate measurements and showed an average deviation of less than 6% for bubbly flow.

Keywords: Two-Phase Flow, Instrumentation, Local Measurement, Resistivity Probe, Double Probe

Cunha, F. R., 1997, "On the Fluctuations in a Random Suspension of Sedimenting Particles", RBCM - J. of the Braz. Soc. Mechanical Sciences, Vol. 19, No. 4, pp. 474-495.

This paper is concerned with velocity fluctuations in a monodisperse dilute suspension of sedimenting particles and in particular with the divergence problem of the variance with the system size. After various scaling arguments it is described a model for suspension flow in which macroscopic mass and momentum balances are constructed to be solved simultaneously. The general model is adapted to study the problem of fluctuations in sedimentation under conditions of low Reynolds numbers and finite Stokes numbers. The important contribution to the particle stress associated with velocity fluctuations induced by body forces is described using an effective viscosity. The coefficients of the scaling arguments for both limiting case examined (inertialess and when the inertia of the particle is important) are also calculated.

Keywords: Velocity Fluctuations, Sedimentation, Random Suspension

Ebinuma, C. D., and Nakayama, A., 1997, "Approximate Solution for Non-Darcy Transient Film Condensation in a Porous Medium", RBCM - J. of the Braz. Soc. Mechanical Sciences, Vol. 19, No. 4, pp. 496-503.

The problem of non-darcian transient film condensation adjacent to a vertical flat plate embedded in a porous medium has been considered. The governing equation for the boundary layer thickness was obtained by an integral method and solved approximately by the method of integral relations. It is shown that the results are in good agreement with those obtained exactly by the method of characteristics.

Keywords: Transient Film Condensation, Porous Medium, Convection, Boundary Layer.

Pessolani, R. B. V., and Mansur, W., J., 1997, "Adaptive Geometrical Procedure Defined by B-Splines Applied to the Boundary Element Method", RBCM - J. of the Braz. Soc. Mechanical Sciences, Vol. 19, No. 4, pp. 504-517 (In Portuguese).

This paper presents an adaptive procedure to minimize errors that occur in the discretization of curved boundaries. Initially the boundary is represented exactly by a b-spline function. Then, the curve is approximated by a series of quadratic elements. An error measure avails the difference between the approximated curve and the original one, refining areas of major error.

Keywords : Numerical Methods, Boundary Element Methods, B-Splines, Adaptive Procedures.

Simões, M. G., 1997, "Intelligent Control Based Vertical-Axis Wind Turbine System", RBCM - J. of the Braz. Soc. Mechanical Sciences, Vol. 19, No. 4, pp. 518-530.

Intelligent control techniques, such as fuzzy logic, neural network and genetic algorithm are recently showing a lot of promise in the application of various engineering systems. The paper describes the control strategy development, design and experimental performance evaluation of a fuzzy logic based variable speed wind generation system that uses cage type induction generator and double-sided PWM converters. The system can feed a utility grid maintaining unity power factor at all conditions, or can supply to an autonomous load. The fuzzy logic based control of the system helps to optimize the efficiency and enhance the performance. A complete 3.5 kW generation system has been developed, designed and thoroughly evaluated by laboratory tests in order to validate the predicted performance improvements. The system gives excellent performance, and can easily be translated to a larger size in the field.

Keywords: Wind Generation, Fuzzy Control, Vertical Axis Turbine, Intelligent Control

Pesce, C. P., and Tannuri, E. A., 1997, "Stability and Dynamics of Offshore Single Point Mooring Systems", RBCM - J. of the Braz. Soc. Mechanical Sciences, Vol. 19, No. 4, pp. 531-552.

The Floating Production Storage and Offloading System (FPSO's) is a modern concept for floating offshore oil exploration units, moored in deep water. 'Turret' and 'Mono-Buoys' are similar types of Single Point Mooring systems (SPM) envisaged for the stationkeeping task. Nevertheless, the highly non-linear dynamic nature of this kind of system may give rise to a rich behaviour scenario that may comprise from simple pitchfork point equilibrium bifurcations to Hopf bifurcations (limit cycles), or even chaotic regimes. Standard linearised stability analysis may be not sufficient anymore to deal with the design problem. Bifurcation theory and modern system dynamics form then a proper theoretical basis for the analysis. This paper addresses the stability problem and discusses a number of interesting dynamic behaviors that arise in steady current. A self-excited autonomous and dissipative non-linear system of equations governs the system dynamics. A classical 'hydrodynamic derivatives' model form the core for hydrodynamic forces description. Following Papoulias and Bernitsas (1988), some classical results on the stability problem are recovered. Then, reinterpreting the equilibrium analysis, it is also shown that bifurcation theory enables one not only to predict but also to qualify equilibrium pitchfork bifurcation scenarios, if super- or sub-critical. It is shown that the algebraic sign of the third-order derivative of the lateral force with respect to the lateral component of relative velocity governs the type of bifurcation scenario. When super-critical pitchfork bifurcation scenario is present a condition for structural stability loss is established and discussed. Hopf bifurcations (limit cycles) are also presented and discussed.

Keywords: System Dynamics, Bifurcation Theory, Mooring System.

Conti, M. B., and Souza, C. A. G. F., 1997, "Experimental Evaluation of Wave Forces on a Circular Cylinder", RBCM - J. of the Braz. Soc. Mechanical Sciences, Vol. 19, No. 4, pp. 553-566.

Vertical forces were measured on a horizontal circular cylinder in a laboratory wave flume. The flume, at the time of measurements, was 14 m long, 1 m wide and circa 0.75 m deep. The length of the cylinder was circa 1 m. It was positioned horizontally along the transverse direction of the tank. The diameter was 0.15 m. Two immersions of the centerline were considered: 0.15 and 0.185 m. Regular surface waves were generated by a plunger. The range of frequencies was approximately from 0.5 to 2.3 Hz. Their amplitudes were between approximately 0.015 and 0.03 m. These dimensions of the cylinder and characteristics of the waves are typical of a semi-submersible pontoon in real conditions, once a geometric scale factor of about 1:100 is considered. The vertical force was measured with a strain-gage transducer, and the wave elevation with a resistance wave probe. The analogical signals were converted into digital and plotted. The experimental results were compared against Morison's equation (1950) predictions. As expected, it was observed that the Morison's equation results adheres to the experimental ones in its frequency range of validity. The depth of the tank is small compared to the wave length in the lower range of frequencies. This water depth limitation has been considered when applying Morison's equation.

Keywords: Wave Forces, Immersed Bodies.

Bianchi, E. C., Yokoyama, E. M., Fernandes, O. C., Valarelli, I. D., and Aguiar, P. R., 1997, "The Cutting Time with Abrasive Discs Can Be Reduced by the Control of the Cutting Speed", RBCM - J. of the Braz. Soc. Mechanical Sciences, Vol. 19, No. 4, pp. 567-576 (In Portuguese).

This work presents an experimental study on the life of abrasive discs in cut-off type operations with two conditions of cutting speed. The cutting time is shown with a comparative analysis of the cutting speed.

Keywords: Abrasive Discs, Cut-Off Type Operations, Cutting Speed.

Novaski, O., and Lisboa, E. S., 1997, "Turn-Milling: Some Concepts and Experimental Results", RBCM - J. of the Braz. Soc. Mechanical Sciences, Vol. 19, No. 4, pp. 577-588 (In Portuguese).

This work presents the concepts and definitions of a combined machining process, so called turn-milling. Several methods of turn-milling are presented with major emphasis on the orthogonal process, studied experimentally. Test specimens made of ABNT 52100 bearing steel, with average hardness of 60-61 HRC were machined. The cutting tool material applied was mixed ceramic inserts. Several feed rate, tool diameter and rotational speeds of workpiece and tool were tested. Tests were conducted on a CNC lathe which had a milling head coupled to it. This coupling was made on the transverse carrier, figuring an orthogonal exposure of the rotating axes for workpiece and tool, thus characterizing orthogonal turn-milling. The achieved results showed that work quality and low roughness values can be obtained; however, the technology must be better investigated.

Keywords: Machining, Turn-Milling, Mixed Ceramics.

Magon, M. F. A., Rodrigues, J. A., and Pandolfelli, V. C., 1997, "Characterization of the R-Curve Behavior of MgO-C Refractories Applying Different Measurement Methods", RBCM - J. of the Braz. Soc. Mechanical Sciences, Vol. 19, No. 4, pp. 589-596 (In Portuguese).

The influence of carbon addition and the geometry of the applied bending test on the fracture behavior of magnesia-carbon refractory bricks, with 5, 8 e 17 wt% of carbon, were studied. The R Curve behavior was evaluated in samples having a single-edge notch, under three and four point bending test conditions. The tests were undertaken under controlled displacement and stable crack propagation conditions. A comparison between two different calculation procedures, were made. It is presented a set procedure criterium concerning the geometry of bending test and the calculation approach applied for the determination of R-Curve, with the objective of decreasing the uncertainty in the measured values. Explanations for the R-Curve shape were also given.

Keywords: Refractory, MgO, Carbon, R-Curve.

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INDIVIDUAL

Por favor, preencha os dois lados do formulário.

Nome _____

Endereço Residencial _____

CEP _____ Cidade _____ Estado _____

País _____ Tel.: () _____ Fax: () _____

E-mail _____

Empresa _____

Dept./Divisão _____ Posição _____

Endereço Comercial _____

CEP _____ Cidade _____ Estado _____

País _____ Tel.: () _____ Fax: () _____

E-mail _____

Candidato-me a: ☐ Admissão ☐ Mudança de Categoria

Na categoria de: ☐ Sócio efetivo ☐ Sócio Aspirante

Solicito enviar correspondência para o seguinte endereço:

☐ Comercial ☐ Residencial

Data _____ Assinatura _____

Para uso da ABCM

Aprovado _____ Data _____ Sócio nº _____

FORMAÇÃO E EXPERIÊNCIA PROFISSIONAL

Por favor, liste em ordem cronológica os dados completos de sua formação e experiência profissional. A falta desses dados impedirá o processo de admissão. Obrigado.

FORMAÇÃO ACADÊMICA

Graduação - Área _____ Anos _____ a _____
Instituição _____ País _____

Mestrado - Área _____ Anos _____ a _____
Instituição _____ País _____

Doutorado - Área _____ Anos _____ a _____
Instituição _____ País _____

Outro - Área _____ Anos _____ a _____
Instituição _____ País _____

EXPERIÊNCIA PROFISSIONAL

Empresa _____ Anos _____ a _____
Natureza da atividade _____ Posição _____

Empresa _____ Anos _____ a _____
Natureza da atividade _____ Posição _____

Empresa _____ Anos _____ a _____
Natureza da atividade _____ Posição _____

Indique até um máximo de 8 áreas de acordo com os códigos numéricos do Anexo.

Áreas de Especialização _____

Aplicação _____

Comentários _____

Área de Especialização

Especifique no Formulário de Afiliação os códigos numéricos das suas Áreas de Especialização e de Aplicação (verso).

1000 Fundamentos e Métodos Básicos em Mecânica Teórica e Aplicada

- 1010 Mecânica do Contínuo
- 1110 Método dos Elementos Finitos
- 1120 Método dos Elementos de Contorno
- 1130 Métodos Assintóticos
- 1140 Método das Diferenças Finitas
- 1150 Outros Métodos em Mec. Computacional
- 1210 Métodos Estocásticos e Estatísticos
- 1310 Modelagem
- 1410 Fundamentos de Análise Experimental
- 1510 Metrologia
- 1610 Gerência de Projetos

2000 Dinâmica e Vibrações

- 2110 Cinemática e Dinâmica
- 2210 Vibrações de Sólidos - Fundamentos
- 2310 Vibrações - Elementos de Estruturas
- 2320 Vibrações - Estruturas
- 2330 Propagação de Ondas em Sólidos
- 2340 Impacto em Sólidos
- 2350 Identificação de Parâmetros
- 2420 Propagação de Ondas em Fluidos
- 2510 Interação Fluido-Estrutura
- 2610 Astronáutica - Mec. Celeste e Orbital
- 2710 Explosão e Balística
- 2810 Acústica

3000 Controle e Otimização

- 3110 Projeto e Teoria de Sistemas Mecânicos
- 3210 Sistemas de Controle Ótimo
- 3220 Sistemas de Controle Adaptativo
- 3230 Aplicações em Sistemas e Controle
- 3310 Robótica
- 3410 Otimização de Sistemas e Processos

4000 Materiais

- 4110 Biomateriais
- 4120 Materiais Metálicos
- 4130 Materiais Cerâmicos
- 4140 Materiais Poliméricos
- 4150 Materiais Conjugados
- 4210 Conformação Mecânica
- 4300 Caracterização e Controle Microestrutural
- 4410 Comp. Mecânico dos Materiais
- 4420 Comp. Mec. Mat. - Baixas Temperaturas
- 4430 Comp. Mec. Mat. - Altas Temperaturas
- 4440 Comp. Mec. Mat. - Carregamto. Variável
- 4450 Comp. Mec. Mat. - Carregamto. Dinâm.
- 4500 Mecanismos de Fratura
- 4600 Mecânica da Fratura
- 4710 Ensaios Destrutivos
- 4720 Ensaios Não-Destrutivos
- 4800 Corrosão

5000 Mecânica dos Sólidos

- 5010 Elasticidade Linear
- 5020 Elasticidade Não-Linear
- 5030 Viscoelasticidade
- 5040 Plasticidade
- 5050 Visco-Plasticidade
- 5060 Mecânica dos Materiais Conjugados
- 5070 Mecânica dos Meios Porosos
- 5110 Reologia
- 5210 Cabos, Hastas e Vigas
- 5220 Membranas, Placas e Cascas
- 5230 Estruturas - Geral
- 5240 Estruturas - Contato com o Solo
- 5250 Estruturas - Submersas/Semi-submersas
- 5260 Estruturas - Móveis
- 5270 Estruturas - Vasos e Contenção
- 5310 Mecânica dos Solos - Básico
- 5320 Mecânica dos Solos - Aplicações
- 5330 Mecânica das Rochas

5410 Efeitos Eletro-Magnéticos em M. Sólidos

- 5420 Efeitos Térmicos em M. dos Sólidos
- 5510 Estabilidade de Estruturas
- 5520 Comportamento após a Flambagem
- 5530 Estados Limites e Cargas de Colapso
- 5540 Acomodação e Acúmulo de Dano
- 5610 Mecânica de Fratura
- 5650 Tribologia
- 5655 Atrito e Desgaste
- 5710 Componentes de Máquinas
- 5720 Acoplamentos e Juntas Não-Soldadas
- 5800 Análise Experimental de Tensões
- 6000 Mecânica dos Fluidos
- 6010 Reologia
- 6110 Hidráulica
- 6210 Escoamento Incompressível
- 6220 Escoamento Compressível
- 6230 Escoamento Rarefeito
- 6240 Escoamento em Meios Porosos
- 6250 Magneto-Hidrodinâmica e Plasma
- 6270 Escoamento Multifásico
- 6310 Camada Limite - Contorno Sólido
- 6320 Camada Limite - Contorno Livre
- 6410 Escoamento Interno - Dutos, Canais, etc.
- 6430 Escoamento com Superfície Livre
- 6510 Estabilidade de Escoamento
- 6520 Turbulência
- 6530 Hidrodinâmica - Veículo de Estrut. Naval
- 6540 Aerodinâmica
- 6610 Mec. Fluidos - Aplicações em Máquinas
- 6650 Lubrificação
- 6710 Transientes em Fluidos
- 6810 Téc. Experta, e Visualização Escoamento
- 7000 Termociências
- 7010 Termodinâmica
- 7110 Transp. de Calor - Convec. Monofásica
- 7120 Transp. de Calor - Convec. Bifásica
- 7130 Transp. de Calor - Condução
- 7140 Transp. de Calor - Radiação/Mod. Comb
- 7150 Transp. de Calor - Dispositivos/Sistemas
- 7210 Termodinâmica dos Sólidos
- 7310 Transporte de Massa
- 7410 Combustão
- 7420 Combustão em Leito Fluidizado
- 7510 Acionadores e Dispositivos de Propulsão

8000 Geociências

- 8010 Micromeritos
- 8110 Meios Porosos
- 8210 Geomecânica
- 8310 Mecânica dos Abalos Sísmicos
- 8410 Hidrologia, Oceanografia, Meteorologia

9000 Energia e Meio Ambiente

- 9110 Combustíveis Fósseis
- 9120 Sistemas Nucleares - Fissão
- 9125 Sistemas Nucleares - Fusão
- 9130 Sistemas Geotérmicos
- 9140 Sistemas Solares
- 9150 Sistemas Eólicos
- 9160 Sistemas de Energia Oceânica
- 9210 Armazenamento de Energia
- 9220 Distribuição de Energia
- 9310 Mecânica dos Fluidos Ambiental
- 9410 Mecânica de Dispositivos de Armazenamento de Resíduo
- 10000 Biociências
- 10110 Biomecânica
- 10210 Ergonomia
- 10310 Reabilitação
- 10410 Mecânica nos Esportes

Áreas de Aplicação

Exemplo: um especialista em Mecânica dos Flúidos (família 6000) atuando na área de Turbulência (6520), deverá escolher a Área de Aplicação 350, se estiver trabalhando em Propulsão.

- 010 Acústica e Controle de Ruído
- 020 Aplicações em Biociências
- 030 CAD
- 040 CAM
- 050 Componentes de Máquinas
- 060 Controle Ambiental
- 070 Controle de Qualidade
- 080 Criogenia
- 090 Engenharia e Física de Reatores

- 100 Engenharia de Petróleo
- 110 Engenharia Oceanográfica
- 120 Equipamentos de Processos
- 130 Equipamentos Industriais
- 140 Fontes Alternativas de Energia
- 150 Forjamento
- 160 Fundição
- 170 Garantia de Qualidade
- 180 Indústria Têxtil e Tecnologia Correlata
- 190 Inspeção e Certificação

- 200 Instalações Industriais
- 210 Instrumentação
- 220 Lubrificação Industrial
- 230 Mancais e Rolamentos
- 240 Máquinas Ferramentas
- 250 Máquinas de Fluxo
- 260 Máquinas Motrizes
- 270 Mecânica Fina
- 280 Metalurgia Geral e Beneficiamento de Minério
- 290 Metrologia

- 300 Mineração e Metalurgia Extrativa
- 310 Óptica
- 320 Pontes e Barragens
- 330 Processos de Fabricação
- 340 Projeto de Estruturas
- 350 Propulsão
- 360 Prospeção e Propulsão
- 370 Servo Mecanismos e Controle
- 380 Siderurgia
- 390 Sistemas Hidráulicos

- 400 Sistemas Pneumáticos
- 410 Soldagem
- 420 Solicitações Acidentais - Efeitos de Vento, Sismo, Explosão, Fogo e Inundação
- 430 Tecnologia de Alimentos
- 440 Tecnologia Mineral
- 450 Transporte (excluído veículos)
- 460 Transmissão de Energia
- 470 Tratamento Térmico e Termoquímico
- 480 Tubulações Industriais e Nucleares
- 490 Usinas Hidrelétricas

- 500 Usinas Termoelétricas
- 510 Vácuo
- 520 Vasos de Pressão, Trocadores de Calor e Equipamentos Pesados
- 530 Veículos - Terrestres, Espaciais e Marítimos

SCOPE AND POLICY

- The purpose of the Journal of the Brazilian Society of Mechanical Sciences is to publish papers of permanent interest dealing with research, development and design related to science and technology in Mechanical Engineering, encompassing interfaces with Civil, Electrical, Chemical, Naval, Nuclear, Agricultural, Materials, Petroleum, Aerospace, Food, System Engineering, etc., as well as with Physics and Applied Mathematics.
- The Journal publishes Full-Length Papers, Review Papers and Letters to the Editor. Authors must agree not publish elsewhere a paper submitted to and accepted by the Journal. Exception can be made for papers previously published in proceedings of conferences. In this case it should be cited as a footnote on the title page. Copies of the conference referees reviews should be also included. Review articles should constitute a critical appraisal of published information.
- The decision of acceptance for publication lies with the Editors and is based on the recommendations of at least two ad hoc reviewers, and of the Editorial Board, if necessary.

SUBMISSION

- Manuscripts and all the correspondence should be sent to the Editor or, alternatively, to the appropriate Associate Editor.
- Four (4) copies of the manuscript are required. The author should submit the original figures, which will be returned if the paper is not accepted after the review process.
- Manuscripts should be submitted in English or Portuguese. Spanish will also be considered.
- A manuscript submitted for publication should be accompanied by a cover letter containing the full name(s) of author(s), mailing addresses, the author for contact, including phone and fax number, and, if the authors so wish, the names of up to five persons who could act as referees.

FORMAT

- Manuscripts should begin with the title, including the English title, the abstract and up to five key words. If the paper's language is not English, an extended summary of about 500 words should be included. The manuscript should not contain the author(s) name(s).
- In research papers, sufficient information should be provided in the text or by referring to papers in generally available Journals to permit the work to be repeated.
- Manuscripts should be typed double-spaced, on one side of the page, using A-4 sized paper, with 2 cm margins. The pages should be numbered and not to exceed 24 pages, including tables and figures. The lead author of a RBCM paper which exceeds the standard length of pages will be assessed a excess page charge.
- All symbols should be defined in the text. A separate nomenclature section should list, in alphabetical order, the symbols used in the text and their definitions. The greek symbols follow the English symbols, and are followed by the subscripts and superscripts. Each dimensional symbol must have SI (Metric) units mentioned at the end. In addition, English units may be included parenthetically. Dimensionless groups and coefficients must be so indicated as dimensionless after their definition.
- Uncertainties should be specified for experimental and numerical results.
- Figures and Tables should be referred in consecutive arabic numerals. They should have a caption and be placed as close as possible to the text first reference.
- Line drawings should be prepared on tracing paper or vellum, using India ink; line work must be even and black. Laser print output is acceptable. The drawings with technical data/results should have a boundary on all four sides with scale indicators (tick marks) on all four sides. The legend for the data symbols should be put in the figure as well as labels for each curve wherever possible.
- Illustrations should not be larger than 12 x 17 cm. Lettering should be large enough to be clearly legible (1.5-2.0 mm).

ILLUSTRATIONS AND TABLES

REFERENCES

- Photographs must be glossy prints.
- References should be cited in the text by giving the last name of the author(s) and the year of publication of the reference: either "Recent work (Smith and Jones, 1985) ..." or "Recently Smith and Jones (1985) ...". With four or more names, use the form "Smith et al. (1985)" in the text. When two or more references would have the same text identification, distinguish them by appending "a", "b", etc., to the year of publication.
- Acceptable references include: journal articles, dissertations, published conference proceedings, numbered paper preprints from conferences, books, submitted articles if the journal is identified, and private communications.
- References should be listed in alphabetical order, according to the last name of the first author, at the end of paper. Some sample references follow:
 - Bordalo, S.N., Ferziger, J.H. and Kline, S.J., 1989, "The Development of Zonal Models for Turbulence", Proceedings, 10th ABCM - Mechanical Engineering Conference, Vol. I, Rio de Janeiro, Brazil, pp. 41-44.
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 - Kandlikar, S.G. and Shah, R.K., 1989, "Asymptotic Effectiveness - NTU Formulas for Multiphase Plate Heat Exchangers", ASME Journal of Heat Transfer, Vol. 111, pp. 314-321.
 - McCormack, R.W., 1988, "On the Development of Efficient Algorithms for Three Dimensional Fluid Flow", Journal of the Brazilian Society of Mechanical Sciences, Vol. 10, pp. 323-346.
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 - Sparrow, E.M., 1980a, "Forced-Convection Heat Transfer in a Duct Having Spanwise-Periodic Rectangular Protuberances", Numerical Heat Transfer, Vol. 3, pp. 149-167.
 - Sparrow, E.M., 1980b, "Fluid-to-Fluid Conjugate Heat Transfer for a Vertical Pipe-Internal Forced Convection and External Natural Convection", ASME Journal of Heat Transfer, Vol. 102, pp. 402-407.

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