IDEAS OF FOUR-FERMION OPERATORS IN HADRON PHYSICS*

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(Received January 15, 2013)

Four-fermion operators have been utilized in the past to link the quark-exchange processes in the interaction of hadrons with the effective meson-exchange amplitudes. In this paper, we apply the similar idea of Fierz rearrangement to the electromagnetic processes and focus on the electromagnetic form factors of nucleon and electron. We explain the motivation of using four-fermion operators and discuss the advantage of this method in computing electromagnetic processes.

DOI:10.5506/APhysPolBSupp.6.117
PACS numbers: 12.39.Ki, 13.40.Gp, 13.40.Hq, 14.40.–n

1. Introduction

Although the calculation of the nucleon form factors based on a quark–diquark model certainly differs from the calculation of the electron form factors using quantum electrodynamics (QED), one may still discern commonalities between the two apparently different calculations. For example, both calculations in the one-loop level share essentially the same shape of triangle diagram as shown in Fig. 1 for the computation of amplitudes. While the contents of the lines drawn in the two triangle diagrams are certainly different, both calculations share the same type of one-loop integration for the amplitudes given by three vertices connected by three propagators. In

* Presented at the Light Cone 2012 Conference, Kraków, Poland, July 8–13, 2012.

particular, the structure of the two fermion lines intermediated by a boson exchange is common in the two triangle diagrams and may be generically identified as the four-fermion operator (FFO) that we discuss in this paper. Due to the commonality of the FFO, it may be conceivable to compute the two apparently different triangle amplitudes in a unified way. As we describe in this work, such a unified way of computation is possible since the FFO can be Fierz-rearranged.

Fig. 1. Triangle diagrams for (a) nucleon form factors in quark–diquark model, (b) electron form factors in QED.

A similar idea of Fierz-rearranged FFOs has been developed in a rather different context of applications in early 1980s. The basic idea of early developments was to provide a basis of the one-boson-exchange interactions of baryons at low energy from the gluon exchange which mediates quark-exchange scattering in conjunction with quark interchange in a non-perturbative bag model framework [1–5]. In the elastic nucleon–nucleon (NN) scattering, the FFO appears from the gluon-exchange mediating quark-exchange scattering and becomes bilocal when it is dressed with long-range quark–gluon correlations by means of bag-model wavefunctions [1]. As this FFO is Fierz-rearranged, the quark-interchange amplitude takes on the usual local form for each nucleon that is expected from the wealth of empirical knowledge at low energy [1]. The similar idea was applied to πN and $\pi\pi$ scattering as well as the scattering involving hyperons [2]. A partial-wave helicity-state analysis of elastic NN scattering was carried out in momentum space [3] and a mesonic NN potential from an effective quark interchange mechanism for non-overlapping nucleons was obtained from the constituent quark model [5]. Also, meson exchanges were introduced into the harmonic oscillator quark model along with a lower quark wave function [4].

In this paper, we apply the Fierz-rearranged FFO in the form factors shown in Fig. 1 and present a global formula to cover most (if not all) of the triangle diagrams for the form factor calculations. In Sec. 2, we present the basic idea and a simple illustration of an application using self-energy calculations. In Sec. 3, we apply it to the form factor calculations involving triangle diagrams and present a corresponding global formula. Conclusion and outlook follow in Sec. 4.

2. Basic idea

The basic idea of our FFO in electromagnetic processes is depicted in Fig. 2, where a two-photon process for a target nucleon is drawn as an illustration. The left and right portions of Fig. 2 correspond to the amplitude intended for computation and the equivalent amplitude after the FFO is Fierz-rearranged, respectively. In the left portion, two photons are attached

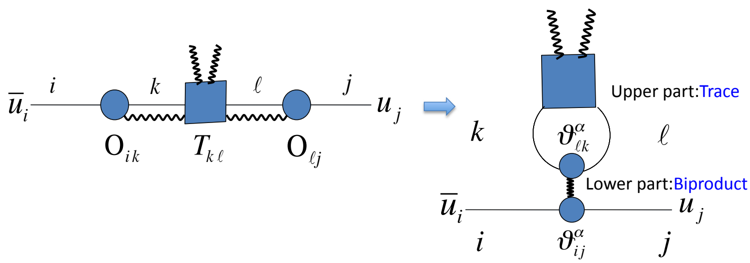

Fig. 2. Basic idea of Fierz-rearranged four-fermion operator in electromagnetic processes.

to the hadronic part which has the two intermediated quarks denoted by the spinor indices k and ℓ that inherit the fermion number from the external nucleons, $\bar{u}_i$ and u_j, with the corresponding spinor indices i and j, respectively. Here, the two vertices, O_{ik} and $O_{\ell j}$, connecting the nucleon and the corresponding quark are linked to the scattering amplitude $T_{k\ell}$, where the two photons interact with the constituents from the target nucleon: the rest of the constituents beside the quark is denoted by a wiggly line below the corresponding quark and the loop integration over the internal momentum is understood. From this configuration of the integrand in the amplitude, we may identify the FFO as the multiplication of two vertices $O_{ik}O_{\ell j}$ and rearrange it as

$$O_{ik}O_{\ell j} = C_{\rm S}\delta_{ij}\delta_{\ell k} + C_{\rm P}(\gamma_5)_{ij}(\gamma_5)_{\ell k} + C_{\rm V}(\gamma_\mu)_{ij}(\gamma^\mu)_{\ell k} + C_{\rm A}(\gamma_\mu\gamma_5)_{ij}(\gamma^\mu\gamma_5)_{\ell k} + C_{\rm T}(\sigma_{\mu\nu})_{ij}(\sigma^{\mu\nu})_{\ell k} = \Sigma_\alpha C_\alpha \mathcal{O}^\alpha_{ij}\mathcal{O}^\alpha_{\ell k}\,, \quad (1)$$

where $\sigma^{\mu\nu} = \frac{i}{2}[\gamma^\mu, \gamma^\nu]$ and the Fierz coefficients $C_\alpha(\alpha = $ S, P, V, A, T) depend on the nature of the vertices O_{ik} and $O_{\ell j}$. With this Fierz-rearrangement, we may write the integrand of the amplitude as follows

$$\bar{u}_i O_{ik} T_{k\ell} O_{\ell j} u_j = \Sigma_\alpha C_\alpha \left(\bar{u}_i \mathcal{O}^\alpha_{ij} u_j\right)\left(\mathcal{O}^\alpha_{\ell k} T_{k\ell}\right) = \Sigma_\alpha \left(\bar{u}\mathcal{O}^\alpha u\right) C_\alpha\, {\rm Tr}[\mathcal{O}^\alpha T]\,, \quad (2)$$

where the external nucleon current (or biproduct $\bar{u}\mathcal{O}^\alpha u$) part is now factorized from the internal scattering part given by the trace of the quark loop (${\rm Tr}[\mathcal{O}^\alpha T]$) as depicted in the right portion of Fig. 2. With this rearrangement of the same amplitude, one may get the general structure of the target hadron's current more immediately and factorize the details of the internal probing mechanism just due to the relevant constituents for the current of the target hadron. It provides an efficient and unified way to analyze the general structure of the amplitudes sharing the commonality of the same type of diagram for the process. For an illustration of the basic idea, we start from a simple example which does not have any external photons but just have one loop as shown in Fig. 3. Such a process may occur in chiral perturbation theory (ChPT) to yield the self-energy of the nucleon due to the surrounding pion cloud. For simplicity, we consider here either scalar or pseudoscalar coupling (rather than the pseudovector coupling in ChPT) and write the self-energy amplitude for a nucleon of four-momentum p and spin s in a unified formula for both scalar and pseudoscalar couplings

$$\Sigma(p,s) = \bar{u}_i(p,s)\hat{\Sigma}_{ij}u_j(p,s) = \Sigma_{\rm S}\bar{u}(p,s)u(p,s) + \Sigma^{\mu}_{\rm V}\bar{u}(p,s)\gamma_\mu u(p,s)\,, \quad (3)$$

where modulo corresponding normalization the self-energy operator $\hat{\Sigma}_{ij}$ is

$$\hat{\Sigma}_{ij} = \int \frac{d^4k}{(2\pi)^4}\,\frac{O_{ik}(\not{p}-\not{k}+M)_{k\ell}O_{\ell j}}{D_\pi D_N} \quad (4)$$

with $D_\pi = k^2 - m_\pi^2 + i\epsilon$ and $D_N = (p-k)^2 - M^2 + i\epsilon$. Using the Fierz-rearrangement given by Eq. (1) and computing the trace of the quark loop $\mathrm{Tr}[\mathcal{O}^\alpha(\not{p}-\not{k}+M)]$ (see Eq. (2)) for α =S, P, V, A, T, we get

$$\Sigma_{\rm S} = 4C_{\rm S}M\int\frac{d^4k}{(2\pi)^4}\,\frac{1}{D_\pi D_N}\,, \qquad \Sigma^\mu_{\rm V} = 4C_{\rm V}\int\frac{d^4k}{(2\pi)^4}\,\frac{(p-k)^\mu}{D_\pi D_N}\,, \quad (5)$$

where $C_{\rm S} = 1/4(1/4)$ and $C_{\rm V} = 1/4(-1/4)$ for the scalar (pseudoscalar) coupling case, *i.e.* $(\Sigma_{\rm S})^{\rm S} = (\Sigma_{\rm S})^{\rm PS}$ and $(\Sigma^\mu_{\rm V})^{\rm S} = -(\Sigma^\mu_{\rm V})^{\rm PS}$.

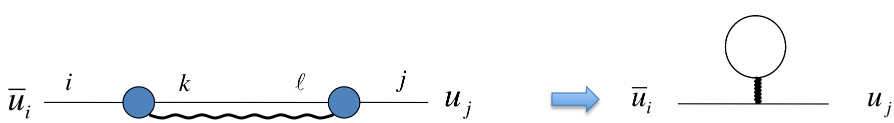

Fig. 3. Self-energy amplitude and the corresponding Fierz-rearrangement.

3. Application to form factors

We now apply the idea of FFO and Fierz rearrangement for the form factors shown in Fig. 1 and present the result that covers both the nucleon form factors in quark–diquark model and the electron form factors in QED. Using the four-fermion method illustrated in Sec. 2 and the usual Feynman parametrization for the loop integration, the current operator J_μ in the amplitude $\bar{u}(p')J_\mu u(p)$ from the triangle diagram with the external fermion mass M, the internal fermion mass m and the intermediate boson mass m_X can be given by (modulo normalization)

$$J_\mu = \int_0^1 dx \int_0^{1-x} dy \int d^4k \frac{N_\mu}{(k^2 - \Delta^2)^3} \,, \tag{6}$$

where $\Delta^2 = s_+ m^2 + (1-s_+)m_X^2 - xyq^2 - s_+(1-s_+)M^2$ and

$$\begin{aligned} N_\mu &= C_\mathrm{S} m(1-s_+)(p+p')_\mu + C_\mathrm{V}\Bigg[2k_\mu \not{k} + \frac{(1-s_+)^2}{2}\left(p+p'\right)_\mu \left(\not{p}+\not{p}'\right) \\ &\quad + \left\{m^2 - k^2 - (1-s_+)^2 M^2 + (1-s_+ + 2xy)\frac{q^2}{2}\right\}\gamma_\mu - \frac{1-s_-^2}{2} q_\mu \not{q}\Bigg] \\ &\quad + iC_\mathrm{A}\epsilon_{\mu\nu\alpha\beta}\gamma_5\gamma^\nu(1-s_+)p^\alpha p'^\beta + 2iC_\mathrm{T} m\sigma_{\mu\nu}q^\nu \end{aligned} \tag{7}$$

with $q = p' - p$ and $s_\pm = x \pm y$. Although one expects to get $J_\mu = \gamma^\mu F_1(q^2) + i\frac{\sigma_{\mu\nu}q^\nu}{2M}F_2(q^2)$, our result of J_μ appears to exhibit not only the vector and tensor currents but also the scalar and axial vector currents. This can be resolved by the Gordon decomposition and the similar extension, namely $(p+p')_\mu \to 2M\gamma_\mu - i\sigma_\mu\nu q^\nu$ and $2i\epsilon_{\mu\nu\alpha\beta}\gamma_5\gamma^\nu p^\alpha p'^\beta \to q^2\gamma_\mu - 2iM\sigma_{\mu\nu}q^\nu$, and we get the expected decomposition of J_μ in terms of just vector and tensor currents and find the form factors $(i = 1, 2)$

$$F_i\left(q^2\right) = \int_0^1 dx \int_0^{1-x} dy \int d^4k \frac{N_i}{(k^2 - \Delta^2)^3} \,, \tag{8}$$

where

$$\begin{aligned} N_1 &= 2Mm(1-s_+)C_\mathrm{S} + (1-s_+)\frac{q^2}{2}C_\mathrm{A} \\ &\quad + \left\{m^2 + (1-s_+)^2M^2 + \tfrac{1}{2}(1-s_+ + 2xy)q^2 - \tfrac{1}{2}k^2\right\}C_\mathrm{V}\,, \\ N_2 &= 4MmC_\mathrm{T} - 2Mm(1-s_+)C_\mathrm{S} - 2(1-s_+)^2M^2C_\mathrm{V} - 2(1-s_+)M^2C_\mathrm{A}\,. \end{aligned}$$

The results in Eq. (8) can cover both the nucleon form factors in a quark–diquark model and the electron form factors in QED taking the corresponding Fierz coefficients and masses. For example, $C_\mathrm{S} = 1/4$, $C_\mathrm{V} = 1/4$, $C_\mathrm{A} = -1/4$, $C_\mathrm{T} = 1/8$ if the scalar diquark is taken in the nucleon form factors, while $C_\mathrm{S} = 1$, $C_\mathrm{V} = -1/2$, $C_\mathrm{A} = 1/2$, $C_\mathrm{T} = 0$ for the electron form factors in QED. Also, M, m and m_X are the nucleon, quark and diquark masses for the nucleon form factors, while $M = m$ is the electron mass and $m_X = 0$ is the intermediate photon mass in QED calculation of the electron form factors. It is interesting to note that the Fierz coefficient $C_{\rm P}$ does not appear neither in nucleon form factors nor in electron form factors reflecting the parity conservation both in strong and electromagnetic interactions. In addition, we note that the usual decomposition of J_μ in terms of vector and tensor currents with the Dirac (F_1) and Pauli (F_2) form factors is just one of the six possible decompositions

$$\begin{aligned} J^\mu &= \gamma^\mu F_1 + i\frac{\sigma^{\mu\nu}q_\nu}{2M}F_2 = \gamma^\mu(F_1+F_2) + \frac{(p+p')^\mu}{2M}F_2 \\ &= \frac{(p+p')^\mu}{2M}\frac{4M^2F_1+q^2F_2}{4M^2-q^2} - i\epsilon^{\mu\nu\alpha\beta}\gamma_5\gamma_\nu p_\alpha p'_\beta\frac{2(F_1+F_2)}{4M^2-q^2} \\ &= \frac{(p+p')^\mu}{2M}F_1 + i\frac{\sigma^{\mu\nu}q_\nu}{2M}(F_1+F_2) \\ &= \gamma^\mu\left(F_1+\frac{q^2}{4M^2}F_2\right) - i\epsilon^{\mu\nu\alpha\beta}\gamma_5\gamma_\nu p_\alpha p'_\beta\frac{F_2}{2M^2} \\ &= i\frac{\sigma^{\mu\nu}q_\nu}{2M}\left(\frac{4M^2}{q^2}F_1+F_2\right) + i\epsilon^{\mu\nu\alpha\beta}\gamma_5\gamma_\nu p_\alpha p'_\beta\frac{2F_1}{q^2}\,. \end{aligned} \tag{9}$$

These six different decompositions in Eq. (9) are all equivalent. Any particular choice of decomposition may depend on the matter of convenience and/or effectiveness in the given situation of computation. More details of our form factor analysis using the four-fermion rearrangement idea will be presented somewhere else [6].

4. Conclusion and outlook

The idea of rearranging FFOs provides an effective way to analyze hadronic processes. It factorizes the details of the internal probing mechanism from the external global structure due to the target hadrons. Processes sharing a certain commonality (*e.g.* same type of diagrams) may be described in a unified way as we have shown in the example of electromagnetic form factors. The application to the two-photon processes would be interesting since the generalized hadronic tensor structure of DVCS still needs further investigation in view of forthcoming experiments with the 12 GeV upgrade at JLab.

This work is supported by the U.S. Department of Energy (No. DE-FG02-03ER41260). H.-M.C. and A.T.S. acknowledge partial supports from KRF Grant (KRF-2010-0009019) and CNPq (Proc. No. 201.902/2010-9), respectively.

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