

# An implementation of modified scatter search algorithm to transmission expansion planning

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## Abstract

*Transmission network expansion planning (TNEP) is one of the most important tasks in the field of power systems, especially in deregulated power system environments. TNEP is a nonlinear mixed integer programming problem that can be solved via hybrid heuristic algorithms. This paper presents a modified scatter search algorithm (MSSA) to reinforce the ordinary scatter search algorithm (SSA) to be equipped for handling large scale transmission expansion planning (TEP) problems. The proposed methodology is based on the SSA, incorporating some improved strategies so as to decrease the number of linear programming problems required to be solved iteratively. In this study, it is shown that the MSSA can handle TEP problems faster than the ordinary SSA, as well as other implemented algorithms. Case studies and simulation results show the significant performance of the proposed method in comparison with some studies addressed in common literature.*

**Key Words:** *Transmission network expansion planning, scatter search algorithm, VGS algorithm, meta-heuristic algorithm*

## 1. Introduction

In recent years, due to the ever increasing growth of electric power consumption, the need to add new circuits to the existing power transmission networks is well evident. Transmission network expansion planning (TNEP) facilitates finding the optimal plan that must specify the number and the locations of transmission lines or transformers where the power system can operate in a reliable and secure manner [1]. Since transmission network investment is very high, TNEP aims to minimize the total investment costs for a predefined time horizon [2]. In the optimization process of a complicated problem, however, one may need to implement a specific heuristic. Such heuristic techniques may use expert knowledge to achieve a superior performance [3].

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Therefore, a TNEP is inherently a mixed-integer nonlinear optimization problem that desires the application of hybrid heuristic optimization techniques [2,4]. Such a complex problem needs more challenges, especially in large-scale interconnected transmission systems. In the current literature, the initial information available for TNEP includes base-year topology, candidate circuits, forecasted demand, and scheduled generation for a predefined planning horizon as well as the investment budget [5,6]. In single-stage transmission network expansion planning (STNEP), the investment must be stated at the beginning of the planning horizon [7]. Since TNEP, as a nonconvex optimization problem, may have a great number of feasible solutions, some hybrid heuristic optimization techniques might be offered to tackle such a complicated problem [8]. In the literature, several evolutionary algorithms have been proposed to achieve the optimal solution of TNEP problems. These algorithms can be categorized into 3 groups that have been widely used in recent years: a) classical optimization algorithms such as the Benders' decomposition (BD) and branch and bound (BB) algorithms, b) heuristic algorithms, and c) metaheuristics such as simulated annealing (SA), the genetic algorithm (GA), tabu search (TS), and the greedy randomized adaptive search procedure (GRASP) [9-17].

In this paper, a modified scatter search algorithm (MSSA), as a new metaheuristic algorithm, is deployed for solving STNEP problems. The ordinary scatter search algorithm (SSA) employs the following 5 stages: diversification generation stage, improvement stage, reference set updating stage, subset generating stage, and solution combination stage. To enhance the performance of the ordinary SSA, a modification is proposed in this study (the so-called MSSA) via the following changes to the aforementioned 5 stages. In the diversification stage, unlike in the ordinary SSA where the initial population is generated randomly, an intelligent method is deployed to produce the initial population. In the improvement stage, a constructive heuristic algorithm (CHA) associated with the Villasana-Garver-Salon (VGS) algorithm is employed in order to facilitate TNEP more efficiently. In the ordinary SSA, the reference set plays a key role in reaching the optimal solution; hence, the reference set updating stage is improved to achieve better performance. In this regard, the MSSA will also handle the subset generating and the solution combination stages using GA operators. It can be argued that by making such proposed modifications, the optimal solution may be achieved by solving fewer linear programming (LP) subproblems with less computational effort. The subsequent sections of this paper are organized as follows: a mathematical model of TNEP is presented in Section 2. In Section 3, the VGS algorithm, as a constructive heuristic algorithm, is described. Section 4 discusses the ordinary SSA, while in Section 5, the MSSA is presented. Simulation studies and result analysis are provided in Section 6. Finally, concluding remarks are presented in Section 7.

## 2. Problem description and formulation

Usually, long-term TNEP is modeled by a mathematical formulation, which is the so-called DC model, as a mixed integer nonlinear problem (MINLP) that is difficult to solve, especially for large-scale systems [8,18,19]. The DC model for TNEP is formulated as follows.

$$\min \nu = \sum_{(i,j) \in \Omega} c_{ij} n_{ij} + \alpha \sum_s r_s \quad (1)$$

S.t.

$$sf + g = d \quad (2)$$

$$f_{ij} - \gamma_{ij}(n_{ij}^0 + n_{ij})(\theta_i - \theta_j) = 0 \quad (3)$$

$$|f_{ij}| \leq (n_{ij}^0 + n_{ij})\overline{f_{ij}} \tag{4}$$

$$0 \leq g \leq \overline{g} \tag{5}$$

$$0 \leq n_{ij} \leq \overline{n_{ij}} \tag{6}$$

$n_{ij}$  integer,  $f_{ij}$  and  $\theta_{ij}$  unbounded  $(i, j) \in \Omega$

Here,  $v, c_{ij}, n_{ij}, n_{ij}^0, g_{ij}, \theta_{ij}, r, \alpha, s, f, f_{ij}, \overline{f_{ij}}, g, \overline{n_{ij}}$ , and  $\Omega$  are, respectively, the value of the expansion investment costs for a predefined horizon, costs of candidate circuits added to the right-of-way  $i - j$ , number of circuits added to the right-of-way  $i - j$ , number of circuits in the initial topology, susceptance of line  $i - j$ , phase angle at bus  $i$ , vector with artificial generators added in each load bus, dummy generation penalty factor, branch node incidence matrix, vector of the power flows through added circuits with elements  $f_{ij}$ , power flow, maximum active power flow limit of line  $i - j$ , a vector with elements  $g_k$  (generation at bus  $k$ ), maximum number of circuits added to the right-of-way  $i - j$ , and a set of all right-of-way  $i - j$ .

Eq. (1) is the objective function of the DC model containing the sum of the investment costs of the newly added transmission lines, as well as the penalty load curtailment. In Eq. (2), Kirchhoff's current law in the equivalent DC network is modeled. Eq. (3) is an expression of Ohm's law for the equivalent DC network, while Kirchhoff's voltage law (KVL) is implicitly taken into consideration. Eqs. (4), (5), and (6) are based on line power flow, generator capacity, and line number limitations, respectively. The proposed MSSA algorithm is considered for solving the DC model, which is an MINLP problem; however, since there is no efficient method for solving these kinds of problems directly, we use some other relaxed linear models, extracted from the DC model, in the SSA. Although we use relaxed models in scatter search, in an intelligent way the proposed algorithm creates solutions that are also valid for DC models. These possible models are the DC operation model, hybrid linear model (HLM), and transportation model (TM). In the following subsection, we only present their mathematical models. The application and further description will be presented in Sections 3 and 5.

### 2.1. DC operation model

Considering a given proposal for the transmission lines, which means that  $n_{ij}$  is given, we have a linear model of the DC model explained by the following model [20,21].

Model 1:

$$\min \nu = \sum_s r_s$$

S.t.

$$sf + g = d$$

$$f_{ij} - \gamma_{ij}n_{ij}^1(\theta_i - \theta_j) = 0 \tag{7}$$

$$|f_{ij}| \leq (n_{ij}^1)\overline{f_{ij}}$$

where  $n_{ij}^1$  is given and defined by:

$$n_{ij}^1 = n_{ij}^0 + n_{ij}. \tag{8}$$

## 2.2. Hybrid linear model

If we assume that Eq. (3) of the DC model, i.e. the KVL, is satisfied only by the existing circuits and also assume that  $n_{ij}$  is not an integer, a HLM is obtained [22]. The HLM is employed in the proposed algorithm to calculate the sensitivity index used to determine the circuit to be added to the electrical system at each step of the CHA. It can be stated by Model 2, as follows.

Model 2:

$$\min \nu = \sum_{(i,j) \in \Omega} c_{ij} n_{ij} + \alpha \sum_s r_s$$

S.t.

$$\begin{aligned} sf + s^{01} f^{01} + g &= d \\ f_{ij}^{01} - \gamma_{ij} (n_{ij}^1 + n_{ij}^0) (\theta_i - \theta_j) &= 0 \quad \forall (i, j) \in \Omega_1 \\ |f_{ij}^{01}| &\leq (n_{ij}^1 + n_{ij}^0) \overline{f_{ij}} \quad \forall (i, j) \in \Omega_1 \\ |f_{ij}| &\leq n_{ij} \overline{f_{ij}} \quad \forall (i, j) \in \Omega \\ 0 &\leq g \leq \overline{g} \\ n_{ij} &\geq 0 \\ \theta_{ij} &\text{ unbounded} \end{aligned} \tag{9}$$

Here,  $n_{ij}^1$ ,  $\Omega_1$ ,  $S^{01}$ , and  $f_{ij}^{01}$  are, respectively, the circuits that are added during the iterative process to the base case, a set of all of the added circuits during the iterative process and all of the prime circuits of the base case, the transpose incidence branch-node matrix of the base topology and the added topology in previous iterations of the algorithm, and the power flow on path  $(i, j) \in \Omega$ .

## 2.3. Transportation model

The TM was originally proposed by Garver, where it relaxes the DC model by eliminating constraint Eq. (3), thus giving rise to a more easily manipulated linear model. Since the TM is a linear model, it is easier to solve than the original DC model. It may lead to feasible solutions but not necessarily an optimal solution, while some of these feasible solutions of the TM may not satisfy the DC model anticipations [23]. The formulation of Model 3 is as follows.

Model 3:

$$\min \nu = \sum_{(i,j) \in \Omega} c_{ij} n_{ij} + \alpha \sum_s r_s$$

S.t.

$$\begin{aligned} sf + g &= d \\ |f_{ij}| &\leq (n_{ij}^0 + n_{ij}) \overline{f_{ij}} \\ 0 &\leq g \leq \overline{g} \\ 0 &\leq n_{ij} \leq \overline{n_{ij}} \end{aligned} \tag{10}$$

$n_{ij}$  integer;  $f_{ij}$  and  $\theta_{ij}$  unbounded  $(i, j) \in \Omega$

### 3. Villasana-Garver-Salon algorithm

In this section, the VGS algorithm is described. This algorithm is used in the improvement phase of the MSSA. The VGS is a CHA that iteratively finds a solution with good quality through a step-by-step procedure [24]. In a TNEP problem, a transmission line will be added at each step of the CHA using a sensitivity index that plays a key role in the CHA. The iterative process continues until a feasible and high-quality solution (based on topology) is achieved, i.e. there is no need for new circuit additions. It can be said that the CHA is significantly robust and converges rapidly. However, for large and complex systems, the derived solutions are only of good quality; they may be far from the expected optimal topology [24]. The VGS algorithm employs the HLM (Model 2) for sensitivity index calculation in order to determine which circuit should be added to the network at each step of the CHA.

The sensitivity index is defined as follows:

$$IS = \max \{ IS_{ij} = n_{ij} \overline{f_{ij}}; n_{ij} \neq 0 \} \quad (11)$$

Here,  $n_{ij}$  is the solution of the LP problem at each step of the CHA. By using the sensitivity index via Eq. (11), a circuit will be selected and added to the current topology.

Thus, the VGS can be summarized by the following steps:

**Step 1.** Assume a base topology as a current topology and use the HLM. All of the circuits of the current topology must follow both of Kirchhoff's laws, i.e. should be in  $\Omega_0$ .

**Step 2.** Solve LP (Model 2) for the HLM using the current topology. If the LP solution indicates that the system is adequately operating with the new additions and  $v = 0$ , then stop. A new solution for the DC model has been found, so proceed to step 4.

**Step 3.** Use the sensitivity index in Eq. (11) to identify the most attractive circuit. Update the current topology with the chosen circuit, update  $n_{ij}^0$  and  $\Omega_0$ , and go to step 2.

**Step 4.** Sort the added circuits in cost-decreasing order. Remove the circuit having the maximum cost and calculate the dummy generation using the operation model of the HLM. If such removal keeps the system in the adequate operation condition (i.e. dummy generation is 0), remove that circuit; otherwise, keep the circuit. Repeat the process of simulating circuit removal until all of the added circuits have been tested. All of the added circuits that were not removed represent the solution of the CHA. It can be noted that although the VGS uses a hybrid linear model to identify the best circuit for addition in an iterative process, it complies with both of Kirchhoff's laws after adding a new circuit; thus, the final solution is also feasible in DC Model 2.

### 4. Scatter search algorithm

In this section, the SSA will be briefly described and we will discuss how it can be considered as a technique for optimizing the combination of large-scale and complex problems, where there is a high probability of finding the global optimum among many local optimum solutions. The SSA is one of the most efficient and flexible metaheuristic algorithms, since each of its elements can be implemented in a variety of ways and degrees of complexity. The SSA is a population-based metaheuristic technique that produces new solutions by combining former solutions from the reference set. The reference set plays a key role in storing better and diversified

solutions for a local search in order to achieve the optimal solution. A basic design to implement scatter search is given on the well-known 5-stage template:

1. Diversification generation stage: in this stage, a set of diverse initial trial solutions will be created; therefore, the same solutions are not included.
2. Improvement stage: the improvement stage will enhance the quality of nonqualified solutions.
3. Reference set update stage: the reference set update stage keeps the specific number of solutions with higher quality and more diversification, where these solutions will establish a reference set. Therefore, the reference set is composed of both high-quality solutions and diverse ones.
4. Subset generation stage: the subset generation stage creates subsets of 2 or more solutions, while none of the generated subsets are similar.
5. Solution combination stage: the solution combination stage creates one or more combined solutions through the subsets driven from the subset generation stage.

The flow diagram of the SSA is shown in Figure 1, in which after the diversification generation stage the solutions are improved and a set of solutions with size  $P$  is selected with higher qualities. The number of candidate solutions in the initial set is  $P$ , which is generally calculated using Eq. (12).

$$P = \max\{5b, 100\} \tag{12}$$

Here,  $b$  denotes the size of the reference set (*RefSet*). The loop that is shown in Figure 1 is repeated until the termination condition is satisfied. The above 5 stages of the SSA can vary for different applications; thus, some changes may affect the quality of our final solution. In this paper, by introducing some strategies, the SSA is improved to find a specified optimal solution for medium-scale systems, and it may find a better solution for large-scale systems that do not yet have a global optimal solutions.

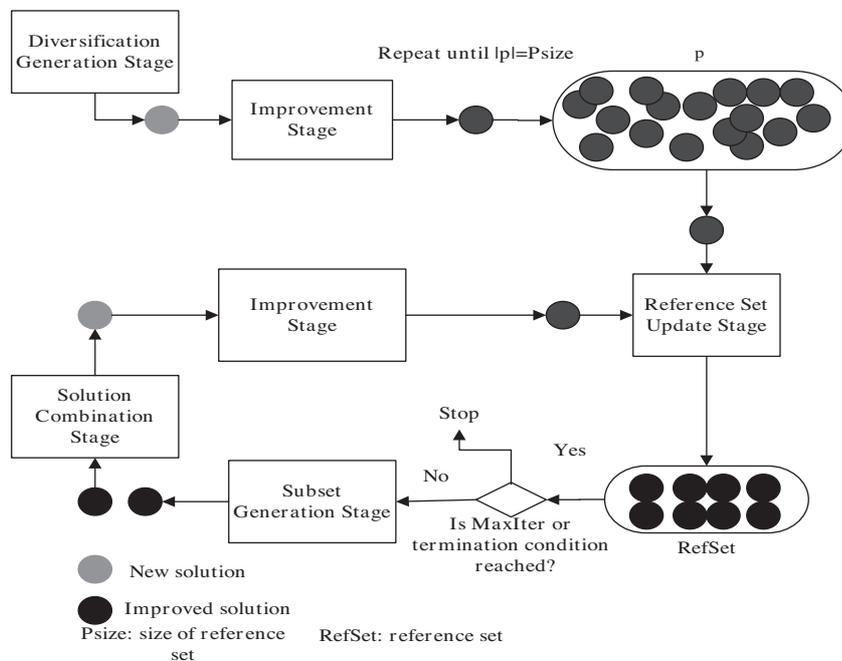


Figure 1. SSA flow diagram.

## 5. Modified scatter search algorithm

In this section, the MSSA, considering the CHA for TNEP, is described. The steps of the MSSA that are employed in TNEP are as follows:

**Step 1.** Create the initial trial solutions.

Using the diversification generation and improvement stages, an initial solution is created, where in this study, a solution is a set of bits that represents candidate paths  $(i, j)$ . Any bit of each solution is limited between the minimum and maximum number of candidate lines. Several methods can be addressed to generate the initial trial solutions, in which they may not offer high quality solutions; therefore, such initial solutions may lead to a time-consuming process. In order to generate the optimal solution, the quality of the initial solution is very important. In this paper, unlike the ordinary SSA that randomly generates the initial trial solutions, the TM (Model 3) with cost perturbation is used. In order to create diverse, high-quality initial trial solutions, the objective function of the TM (Model 3) is changed as follows:

$$\min \nu = \sum_{(i,j) \in \Omega} (w_1 c_r + w_2 c_{ij}) n_{ij} + \alpha \sum_s r_s, \tag{13}$$

where  $c_r$  is a noise vector applied to the costs of the candidate lines,  $c_{ij}$  is the costs of the candidate lines, and  $w_1$  and  $w_2$  are the cost function coefficients. By regulating these coefficients, the diversification of the generated solutions is determined, i.e. if  $w_1$  is greater than  $w_2$ , the diversification of the initial solutions is high and vice versa. To generate each initial solution, the TM must be solved individually, and this process continues until the initial set is complete. Five samples of the initial trial solutions for the Garver system that are generated by TM are shown in Table 1. The improvement stage is applied to each initial trial solution, in which new lines derived from each solution are added to the base-year topology in order to achieve the updated topology. The updated topology is then evaluated by calculating dummy generations. The solutions are categorized into 2 groups: feasible solutions (dummy generation is zero) and infeasible solution (dummy generation is nonzero). In order to improve feasible solutions, their investment costs must be decreased (via removing unnecessary circuits) while the dummy generation must be kept at zero; therefore, for feasible solutions, the costs of the added circuits is important. In order to improve the infeasible solutions, the VGS algorithm is applied in such a way that the added circuits to the primary topology can be achieved. As the value of the dummy generation is important for infeasible solutions, the improvement stage should be materialized. By improving the infeasible solution, the dummy generation of the updated topology becomes zero and this infeasible solution will be changed to a feasible counterpart.

**Step 2.** Generate the reference set.

In this stage, a reference set is generated via reference set updating methods using the high-quality solutions and diversified ones. It is worth noting that inexpensive solutions are identified as high-quality solutions. Half of the reference set is selected from the solutions with the highest quality and the rest of our trial solutions are sorted using the proposed Eq. (14), which is the combining of quality and distance.

$$\text{fitness value} = (1 - k) \text{quality} - k(\text{distance}), \quad 0 \leq k \leq 1 \tag{14}$$

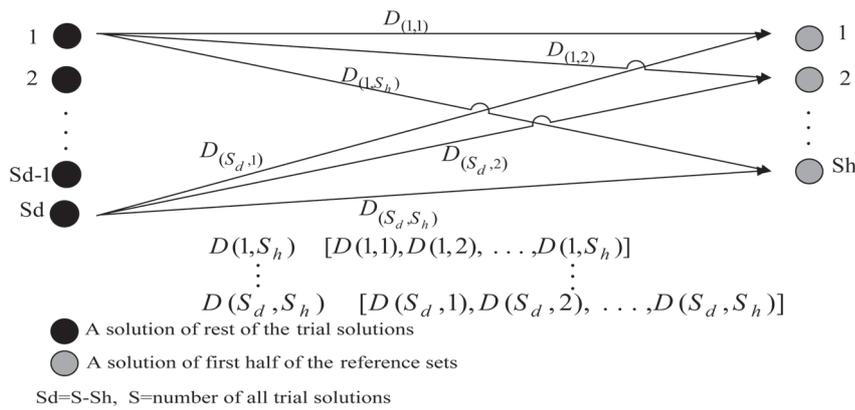
The *distance* is defined by Eq. (15).

$$\text{distance} = \max[\min(D(s_d, s_h))], \tag{15}$$

where  $S_h$  is the first half of the reference set (of high-quality solutions),  $S_d$  is the set of other trial solutions, and  $D(s_d, s_h)$  is the distance between any solution of  $S_d$  and  $S_h$  that is shown in Figure 2.

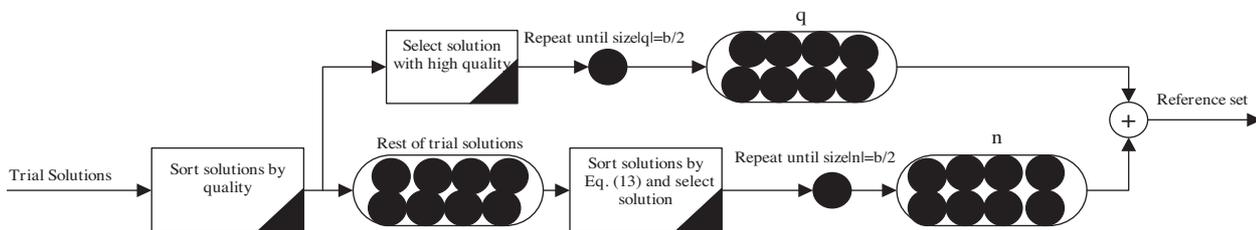
**Table 1.** A sample of the initial trial solutions.

Paths	Solutions				
	Sample 1	Sample 2	Sample 3	Sample 4	Sample 5
1-2	0	0	0	1	0
1-3	1	0	0	1	1
1-4	0	2	1	0	1
1-5	2	0	0	1	0
1-6	0	0	0	1	0
2-3	1	1	2	1	2
2-4	0	0	0	0	0
2-5	1	0	0	1	0
2-6	2	2	3	3	1
3-4	0	0	0	1	0
3-5	2	1	2	2	2
3-6	0	0	0	1	1
4-5	0	0	0	1	0
4-6	2	2	3	2	2
5-6	0	0	0	1	1



**Figure 2.** Process of computing distance.

Therefore, another half of the reference set will be produced by selecting these sorted trial solutions from top to bottom. The process of generating the reference set is shown in Figure 3.



b = size of refset

**Figure 3.** Generation of the reference set diagram.

**Step 3.** Generate the solutions subset.

From the reference set, a group of unique subsets comprising 2 solutions should be selected randomly.

**Step 4.** Create the trial solutions.

Using the solution combination stage via GA operators (crossover and mutation), a trial solution is created in which a combination is produced from subsets arranged in the previous step.

**Step 5.** Improve the trial solutions.

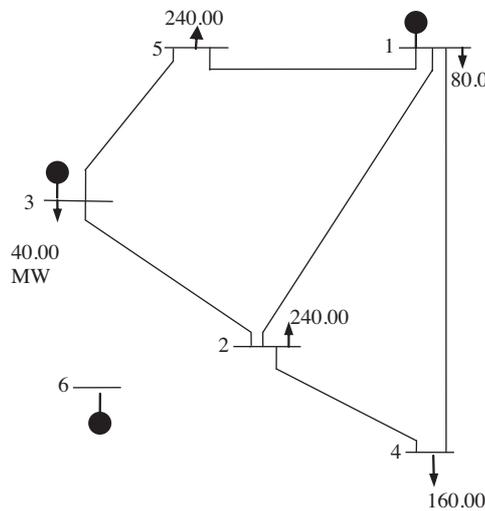
In this step, an improvement stage is applied to the current trial solutions and then the algorithm returns to step 2.

## 6. Simulation studies

In this section, the proposed MSSA is applied to the Garver, IEEE 24-bus, and 46-bus Brazilian systems. The obtained results from implementing the proposed method for the Garver and IEEE 24-bus systems are compared with different reported methods, such as SA, the GA, TS, and the GA with local search. The simulation studies of the MSSA are considered with and without generation rescheduling. To demonstrate the effectiveness of the proposed MSSA, the following studies are carried out.

### 6.1. Garver’s system

The Garver system includes 6 transmission lines and 6 buses with a 760-MW demand for base topology, which is shown in Figure 4. The number of candidate lines is 15 circuits. The maximum number of permitted parallel lines is 3 for each branch. The system data can be found in [20,25]. By applying the proposed method to the Garver system with generation rescheduling, the obtained results are:



**Figure 4.** Garver system.

*Total investment is  $v = US\$110,000,000$ ; added circuits are  $n_{3-5} = 1$  and  $n_{4-6} = 3$ .*

The results of the proposed method for Garver’s system without generation rescheduling are:

Total investment is  $v = \$200,000,000$ ; added circuits are  $n_{2-6} = 4, n_{3-5} = 1$ , and  $n_{4-6} = 2$ .

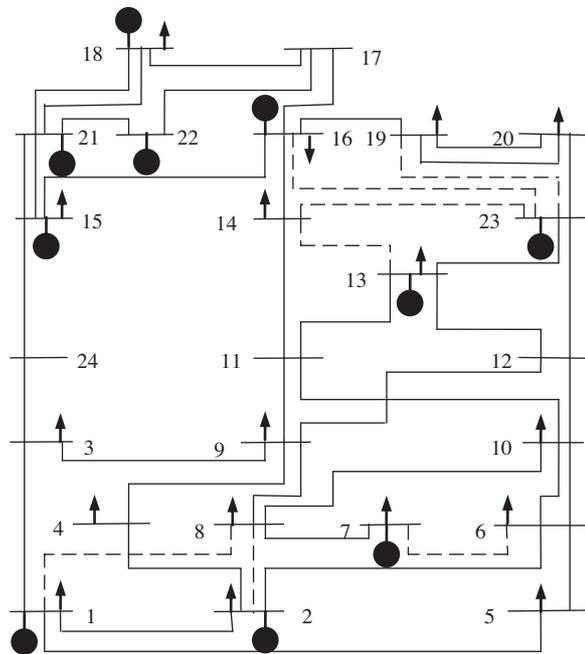
The number of LPs solved to obtain the optimal solution for all of the methods that are brought from [26] is shown in Table 2. It can be noticed that the proposed MSSA shows better performance than the other metaheuristics tested in the transmission expansion planning (TEP) problem because it executes fewer LPs to find the optimal solution for the tested system.

**Table 2.** Number of solved LPs in different methods.

Method	Number of LPs	
	With generation rescheduling	Without generation rescheduling
EGA [21]	500-750	700-1000
SA [21]	800-1000	1000-1300
TS [21]	400-500	600-700
TS-SA [21]	360-470	600-700
TS-EGA [21]	300-500	500-620
TS-SA-EGA [21]	330-460	550-700
MSSA	50-140	60-90

### 6.2. IEEE 24-bus

The IEEE 24-bus system consists of 24 buses, 41 right-of-ways for the addition of new circuits, and 8550-MW demand for base topology, as shown in Figure 5. The data are available in [27]. By applying the proposed method to the IEEE 24-bus system considering generation rescheduling, the obtained results are:



**Figure 5.** IEEE 24-bus system.

Total investment is  $\$152,000,000$ ; added circuits are  $n_{7-8} = 2, n_{6-10} = 1, n_{14-16} = 1$ , and  $n_{10-12} = 1$ .

Table 3 shows the results of the proposed method without generation rescheduling for plans G1, G2, G3, and G4 and the results obtained in [25]. Based on Table 3, the derived topology for plan G1 presents less investment using the proposed method than the obtained results from [22]. It should be noted that for plans G2 and G4, there are no results from the literature where the proposed method is applied to both, and these results are also shown in Table 3. For the IEEE 24-bus, there are no results for the number of solved LPs from the current literature, both with and without generation rescheduling.

**Table 3.** Results of the MSSA for 4 plans.

Plan	Added line to system		Cost (M\$)	
	MSSA	Result in [20]	MSSA	Result in [20]
G1	$n_{1-5} = 1, n_{3-24} = 1, n_{6-10} = 1, n_{7-8} = 2$ $n_{14-16} = 1, n_{15-24} = 1, n_{16-17} = 2$ $n_{16-19} = 1, n_{17-18} = 1$	$n_{1-5} = 1, n_{3-4} = 1, n_{6-10} = 1, n_{7-8} = 2$ $n_{14-16} = 1, n_{15-21} = 1, n_{15-24} = 1$ $n_{16-17} = 2, n_{16-19} = 1, n_{17-18} = 1$	390	438 G1
G2	$n_{1-5} = 1, n_{3-24} = 1, n_{6-10} = 1, n_{7-8} = 2$ $n_{10-12} = 1, n_{14-16} = 1, n_{15-24} = 1$ $n_{16-17} = 2, n_{17-18} = 2$	-	392	-
G3	$n_{6-10} = 1, n_{7-8} = 2, n_{10-12} = 1, n_{14-16} = 1$ $n_{16-17} = 1, n_{20-23} = 1$	$n_{6-10} = 1, n_{7-8} = 2, n_{10-12} = 1,$ $n_{14-16} = 1, n_{16-17} = 1, n_{20-23} = 1$	218	218
G4	$n_{3-24} = 1, n_{6-10} = 1, n_{7-8} = 2, n_{9-11} = 2$ $n_{10-12} = 1, n_{14-16} = 2, n_{16-17} = 1$	-	342	-

### 6.3. 46-bus southern Brazilian system

This system has 46 buses, 79 right-of-ways for the addition of new circuits, and 6880 MW of demand; the system data are available in [12,28]. There is no limit for circuit additions in each right-of-way. As in the previous case, there are 2 options, depending on whether rescheduling is considered or not.

The proposed MSSA offers the following results:

1. With generation rescheduling: *Total investment is \$70,289,000,000; added circuits are  $n_{2-5} = 1, n_{5-6} = 2, n_{13-20} = 1, n_{20-21} = 2, n_{20-23} = 1, n_{42-43} = 1,$  and  $n_{46-6} = 1.$*

The number of LPs solved to obtain the optimal solution for all of the methods that are taken from [26] is shown in Table 4. As it can be seen, the proposed method has better performance than the other methods in terms of LP numbers.

**Table 4.** Number of solved LPs in different methods.

Method	Number of LPs
	With generation rescheduling
EGA [21]	3500-4500
SA [21]	4000-5000
TS [21]	4100-6900
TS-SA [21]	1700-2500
TS-EGA [21]	1400-1900
TS-SA-EGA [21]	1450-2000
MSSA	500-1500

2. Without generation rescheduling: *Total investment is \$154,420,000,000; added circuits are  $n_{20-21} = 1, n_{42-43} = 2, n_{46-6} = 1, n_{19-25} = 1, n_{31-32} = 1, n_{28-30} = 1, n_{26-29} = 3, n_{24-25} = 2, n_{29-30} = 2,$  and  $n_{5-6} = 2.$*

For the 46-bus southern Brazilian system, without generation rescheduling, there are no results for the number of solved LPs in the literature.

In Table 5, the parameters of the MSSA (number of the initial set, number of the reference set, and number of iteration to reach the optimal solution) for solving TNEP problems are presented. In summary, the proposed MSSA has the following characteristics: a) it is effortless and reduces the number of LPs; b) it is extendable, such that it can be extended to multistage planning even considering dispersed generation; and c) it offers high-quality initial solutions and it uses the transportation model to generate high-quality initial populations, causing TNEP to converge faster.

**Table 5.** Parameters of the MSSA.

	With generation rescheduling			Without generation rescheduling		
	No. of initial solution (P)	No. of RefSet (b)	No. of iterations	No. of initial solution (P)	No. of RefSet (b)	No. of iterations
Garver system	20	6	3	20	6	4
IEEE 24-bus	40	10	5	40	10	7
46-bus	100	20	9	100	20	10

## 7. Concluding remarks

In this paper, a combinatorial approach consisting of the SSA and CHA methods was proposed for TEP as an efficient method for achieving optimal solutions. The VGS algorithm was also implemented for the MSSA in order to improve the quality of the obtained solutions. The proposed MSSA was applied to some cases in order to solve for static TNEP. The obtained results for medium- and large-scale systems showed a significant performance boost with regard to previous plausible techniques. A considerable amount of saving from the investment point of view can be achieved via implementing the MSSA to TNEP problems. The fact that the MSSA required the solving of fewer LP problems to find the optimal solutions shows that the proposed special SSA presents better performance and higher efficiency than the other metaheuristic techniques to solve static TNEP problems. It can be emphasized that unlike the other algorithms, the proposed MSSA can be applied to TNEP for large-scale systems, as well. Less computational effort shows the effectiveness of the proposed MSSA in comparison with other methods addressed in the literature. The MSSA has the capability of being applied to multistage TNEP, which is recommended for future studies.

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