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**Strategic Models for Producers and
Consumers in Electricity Markets**

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Strategic Models for Producers and Consumers in Electricity Markets

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Aos 20 dias do mês de maio do ano de 2025, às 8h, por meio de Videoconferência, realizou-se a defesa de TESE DE DOUTORADO de JOÃO AUGUSTO SILVA LÊDO, intitulada **STRATEGIC MODELS FOR PRODUCERS AND CONSUMERS IN ELECTRICITY MARKETS**. A Comissão Examinadora foi constituída pelos seguintes membros: Prof. Dr. LEONARDO NEPOMUCENO (Orientador(a) - Participação Virtual) do(a) Departamento de Engenharia Elétrica / Faculdade de Engenharia de Bauru, Prof. Dr. ANTONIO J CONEJO (Participação Virtual) do(a) Integrated System Engineering / The Ohio State University, Prof^ª. Dr^ª. CLAUDIA ALEJANDRA SAGASTIZABAL (Participação Virtual) do(a) Instituto de Matemática, Estatística e Computação Científica - IMECC / Universidade Estadual de Campinas - UNICAMP, Dr. ANDRÉ LUIZ DINIZ SOUTO LIMA (Participação Virtual) do(a) Modelos Eletroenergético / Cepel, Profa. Dra. EDILAINE MARTINS SOLER (Participação Virtual) do(a) Departamento de Matemática / UNESP/ Câmpus de Bauru. Após a exposição pelo doutorando e arguição pelos membros da Comissão Examinadora que participaram do ato, de forma presencial e/ou virtual, o discente recebeu o conceito final: aprovado . Nada mais havendo, foi lavrada a presente ata, que após lida e aprovada, foi assinada pelo(a) Presidente(a) da Comissão Examinadora.

Prof. Dr. LEONARDO NEPOMUCENO 

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POTENTIAL IMPACT OF THIS RESEARCH

This research aims to analyze equilibrium in competitive electricity markets, where both producers and consumers behave strategically. Additionally, it seeks to capture the strategic behavior of a producer operating in a hydrothermal-dominated system. In terms of the CNPq classification of knowledge fields, this research falls under the broad domain of Engineering, specifically within the field of Electrical Power Systems, encompassing the subfields of Electrical Energy Generation and Transmission. Moreover, this work has a strong interdisciplinary integration with the broader domain of Exact and Earth Sciences, particularly in the field of Mathematics (including subfields such as Applied Mathematics, Computational Mathematics, and Discrete and Combinatorial Mathematics) and Computation. It also maintains a conceptual interface with the broad field of Applied Social Sciences, particularly in Economics, covering subfields such as Quantitative Economic Methods, Mathematical Economics, and Social Welfare Economics. Furthermore, beyond its primary focus on electrical power systems, this research is applicable to various research agendas in Brazil and internationally, given its broad interdisciplinary relevance.

One of the key factors enhancing the impact of this research is its collaborative development with The Ohio State University under the guidance of Prof. Dr. Antonio J. Conejo. This collaboration began during the Sandwich Doctorate Program Abroad (PDSE) and has been instrumental in refining the methodological approach adopted in this study, as well as facilitating the publication process in high-impact scientific journals. Publishing this work in internationally recognized journals within the fields of Engineering and Earth Sciences will significantly enhance the national and global visibility of the research, thereby amplifying its academic and practical contributions.

Additionally, the recent regulatory changes in the Brazilian electricity market underscore the urgent need for analytical models and tools specifically tailored to hydrothermal systems, particularly those that incorporate strategic behavior among market participants. Given the unique characteristics of the Brazilian power system, it is essential for the national scientific community to develop specialized methodologies that not only address these unique challenges but also integrate insights from electricity markets worldwide. This effort aims to adapt and implement best practices to the Brazilian context, considering its distinctive energy mix.

Thus, this research contributes to the scientific community by offering a mathematical optimization model that captures the strategic behavior of a producer in a hydrothermal electricity market. Furthermore, it provides a single optimization model capable of identifying equilibrium conditions between strategic producers and consumers, thereby assisting system operators, regulators, and market participants in detecting and assessing market power.

Finally, through the combined efforts of international scientific collaboration—leveraging the expertise of Prof. Dr. Antonio J. Conejo in energy market equilibrium at The Ohio State University and the research initiatives of the Electric Energy Market Research Laboratory at UNESP in Bauru, Brazil—this work has contributed to scientific achievements published in prestigious journals, academic conferences, and research forums in Brazil and internationally. These contributions further highlight the significant impact of this research on the Brazilian electricity market and its broader global implications.

POTENCIAL IMPACTO DESSA PESQUISA

Esta pesquisa tem como objetivo analisar o equilíbrio em mercados de eletricidade competitivos, nos quais tanto produtores quanto consumidores atuam estrategicamente. Além disso, busca capturar o comportamento estratégico de um produtor que opera em um sistema hidrotérmico dominante. De acordo com a classificação do CNPq para áreas do conhecimento, esta pesquisa se insere na grande área da Engenharia, especificamente no campo de Sistemas Elétricos de Potência, abrangendo os subcampos de Geração e Transmissão de Energia Elétrica. Além disso, este trabalho possui uma forte integração interdisciplinar com a grande área de Ciências Exatas e da Terra, especialmente na área de Matemática (incluindo subáreas como Matemática Aplicada, Matemática Computacional e Matemática Discreta e Combinatória) e Computação. Também apresenta uma interface conceitual com a grande área das Ciências Sociais Aplicadas, particularmente na Economia, cobrindo subáreas como Métodos Quantitativos em Economia, Economia Matemática e Economia do Bem-Estar Social. Ademais, além de seu foco principal nos sistemas elétricos de potência, esta pesquisa possui aplicabilidade em diversas agendas de pesquisa no Brasil e no exterior, devido à sua ampla relevância interdisciplinar.

Um dos principais fatores que ampliam o impacto desta pesquisa é seu desenvolvimento colaborativo com a The Ohio State University, sob a supervisão do Prof. Dr. Antonio J. Conejo. Essa colaboração teve início durante o Programa de Doutorado Sanduíche no Exterior (PDSE) e tem sido fundamental para o refinamento da abordagem metodológica adotada neste estudo, além de facilitar o processo de publicação em periódicos científicos de alto impacto. A publicação deste trabalho em revistas de reconhecimento internacional nas áreas de Engenharia e Ciências da Terra contribuirá significativamente para aumentar sua visibilidade nacional e global, ampliando assim suas contribuições acadêmicas e práticas.

Além disso, as recentes mudanças regulatórias no mercado de eletricidade brasileiro ressaltam a necessidade urgente de modelos analíticos e ferramentas específicas para sistemas hidrotérmicos, especialmente aquelas que consideram o comportamento estratégico dos agentes de mercado. Dadas as características únicas do sistema elétrico brasileiro, torna-se essencial que a comunidade científica nacional desenvolva metodologias especializadas que não apenas abordem esses desafios específicos, mas também integrem conhecimentos de mercados de eletricidade ao redor do mundo. Esse esforço busca adaptar e implementar as melhores práticas ao contexto brasileiro, considerando sua matriz energética particular.

Dessa forma, esta pesquisa contribui para a comunidade científica ao oferecer um modelo de otimização matemática que captura o comportamento estratégico de um produtor em um mercado de eletricidade hidrotérmico. Além disso, fornece uma estrutura de otimização unificada capaz de identificar condições de equilíbrio entre produtores e consumidores estratégicos, auxiliando operadores do sistema, reguladores e participantes do mercado na detecção e avaliação do poder de mercado.

Por fim, por meio da colaboração científica internacional — combinando a expertise do Prof. Dr. Antonio J. Conejo em equilíbrio de mercados de energia na The Ohio State University e as iniciativas de pesquisa do Laboratório de Pesquisa em Mercado de Energia Elétrica da UNESP, em Bauru, Brasil — este trabalho tem contribuído com a pesquisa científica da área por meio de publicações em periódicos de prestígio, conferências acadêmicas e fóruns de pesquisa no Brasil e no exterior. Essas contribuições reforçam ainda mais o impacto significativo desta pesquisa no mercado de eletricidade brasileiro e suas implicações globais.

ABSTRACT

This work presents a comprehensive approach to modeling and solving strategic behavior in hydro-dominated day-ahead electricity markets. First, a bilevel strategic offering (SO) model is proposed for a strategic company (SC) that operates in such a market. The upper-level model maximizes the SC's profits, while the lower-level model represents a hydrothermal market clearing (MC) that accounts for detailed hydro, thermal, and network constraints for both the SC and non-strategic companies (NSCs). To solve this problem, the lower-level model is replaced by its primal and dual constraints along with a strong duality equality, resulting in a non-convex mathematical programming with equilibrium constraints (MPEC) formulation. This MPEC is then reformulated to include only a single bilinear term, which is iteratively convexified and solved until convergence is achieved.

Extending this research, the work also addresses the challenge of finding Nash equilibria in a wholesale electricity market with strategic producers and strategic consumers. This problem is formulated as an Equilibrium Problem with Equilibrium Constraints (EPEC), where each participant's decision-making is modeled as an MPEC. Unlike previous approaches, which rely on Karush-Kuhn-Tucker (KKT) conditions and face numerical instability due to complementarity constraints, this work employs Primal-Dual Optimality Conditions (PDOCs) to reformulate MPECs in a convex manner. The proposed method transforms MPECs into convex rotated quadratic conic problems, enabling the derivation of their exact dual counterparts and ensuring a stable solution process. The methodology is validated using a realistic case study, comparing scenarios where producers and consumers exert market power against those where perfect competition is assumed. The results demonstrate the effectiveness of the proposed approach in capturing strategic interactions and improving market modeling accuracy.

Key-words: Bilevel Optimization, Mathematical Problem with Equilibrium Constraints – MPEC, Equilibrium Problem with Equilibrium Constraints – EPEC, Complementary Problems, Strategic Offering and Strategic Bidding, Day-Ahead Electricity Markets, Hydrothermal Systems.

RESUMO

Este trabalho apresenta uma abordagem abrangente para modelar e resolver interações estratégicas em mercados de eletricidade de curto prazo dominados por usinas hidrelétricas. Primeiramente, é proposto um modelo de oferta estratégica (SO) bi-nível para uma empresa estratégica (SC) que opera nesse tipo de mercado. O modelo de nível-superior maximiza os lucros da SC, enquanto o modelo de nível-inferior representa o despacho de mercado hidrotérmico (MC), considerando restrições detalhadas de geração hidráulica, térmica e de rede para a SC e as empresas não estratégicas (NSCs). Para resolver esse problema, o modelo de nível inferior é substituído por suas restrições primais e duais, juntamente com uma equação de dualidade forte, resultando em uma formulação de programação matemática com restrições de equilíbrio (MPEC) não convexa. Esse MPEC é reformulado para incluir apenas um único termo bilinear, sendo iterativamente convexificado e resolvido até a convergência.

Ampliando essa pesquisa, o trabalho também aborda o desafio de encontrar equilíbrios de Nash em um mercado de eletricidade atacadista com produtores e consumidores estratégicos. Esse problema é formulado como um Problema de Equilíbrio com Restrições de Equilíbrio (EPEC), no qual a tomada de decisão de cada participante é modelada como um MPEC. Diferentemente de abordagens anteriores, que dependem das condições de Karush-Kuhn-Tucker (KKT) e enfrentam instabilidades numéricas devido às restrições de complementaridade, este trabalho utiliza as Condições de Otimalidade Primal-Dual (PDOCs) para reformular os MPECs de maneira convexa. O método proposto transforma os MPECs em problemas cônicos quadráticos rotacionados convexos, permitindo a derivação exata de seus respectivos duais e garantindo um processo de solução estável. A metodologia é validada por meio de um estudo de caso realista, comparando cenários em que produtores e consumidores exercem poder de mercado com aqueles em concorrência perfeita. Os resultados demonstram a eficácia da abordagem proposta na captura de interações estratégicas e na melhoria da precisão da modelagem de mercado.

Palavras chave: Otimização bi-bível, Problemas Matemáticos com Restrições de Equilíbrio – MPEC, Problemas de Equilíbrio com Restrições de Equilíbrio – EPEC, Problemas de Complementaridade, Ofertas Estratégicas e Lances Estratégicos, Mercados de Eletricidade do dia seguinte, Sistemas Hidrotérmicos.

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I would also like to express my deepest gratitude to Professor Dr. Antonio J. Conejo, whose research has had a tremendous worldwide impact in the fields of energy markets and optimization, influencing many researchers—myself included. Not only has his work been transformative, but he also kindly welcomed me as a visiting researcher in his department at The Ohio State University. His mentorship has profoundly shaped the trajectory of my research, and I remain deeply grateful for his insightful guidance.

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PUBLICATIONS

Research Publications

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LÊDO, J. A.; NEPOMUCENO, L.; CONEJO, A. J. "Equilibria among strategic producers and strategic consumers: A solution approach relying on a single optimization problem". *IIE Transactions – Operations Engineering & Analytics*. Submitted.

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LIST OF ACRONYMS

- B-CP** – Bilevel of Consumer Problem.
- B-PP** – Bilevel of Producer Problem.
- DC** – Dual Constraint.
- DSO** – Distribution System Operator.
- EPEC** – Equilibrium Problems with Equilibrium Constraints.
- ISO** – Independent System Operator.
- KKT** – Karush-Kuhn-Tucker.
- MC** – Market Clearing.
- MC-PC** – Market Clearing with Perfect Competition.
- MCP** – Mixed Complementarity Problem.
- MO** – Market Operator.
- MPEC** – Mathematical Program with Equilibrium Constraints.
- MPEC-CP** – MPEC - Consumer Problem.
- MPEC-PP** – MPEC - Producer Problem.
- NSC** – Non-Strategic Company.
- PC** – Primal Constraint.
- PDOC** – Primal-Dual Optimality Conditions.
- PF** – Producer Profit.
- SC** – Strategic Company.
- SDE** – Strong Duality Equality.
- SO** – Strategic Offering.
- SW** – Social Welfare.
- UT** – Consumer Utility.

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1 INTRODUCTION

1.1 Motivation

This work introduces and analyzes the strategic behavior of producers and consumers in day-ahead wholesale electricity markets. In this sense, we propose two different studies to examine various characteristics related to the strategic approaches employed by market agents.

The first study focuses on the application of a Stackelberg game structure, formulated as a bilevel optimization problem, to develop a strategic offering model for a producer operating in a market heavily dominated by hydrothermal generation units. To achieve this, we employ several mathematical techniques to model the decision-making strategy and analyze the impact of the strategic operation of a single large producer within a hydrothermal-driven market. Since the proposed model in the first study is capable of manipulating the market-clearing price to the benefit of the large strategic producer — while also allowing this producer to save water resources — it becomes a powerful tool both for the strategic producer itself and for any entity concerned with spot market manipulation and seeking to mitigate such behavior in hydrothermal-dominated systems.

Furthermore, the second study focuses on analyzing strategic behavior in a market where all producers and consumers act strategically at the same time, each employing its own Stackelberg game structure to guide its decision-making process regarding strategic offers and bids. In this context, we propose a novel solution approach that encompasses several mathematical procedures to formulate an equilibrium model represented by a single optimization problem. As this second model is capable of identifying shifts in market power among strategic agents depending on market setups and determining strategic offers and bids according to the implemented objective functions, it proves to be a valuable tool for market operators, regulators, and participating agents.

It is worth mentioning that strategic generation companies face several key challenges when formulating strategic offering (SO) models to maximize profits in day-ahead wholesale electricity markets (TORRE; CONTRERAS; CONEJO, 2004; ALHARBI; BHAT-TACHARYA, 2020). One of the main challenges is accurately representing the interaction between the strategic company (SC) and the market. This is typically achieved through two modeling approaches: bilevel optimization and residual demand curve modeling. The bilevel approach, which this work focuses on, involves an upper-level optimization of the

SC's profits and a lower-level market clearing (MC) procedure (HOBBS; METZLER; PANG, 2000; RUIZ; CONEJO, 2009; KAZEMPOUR; CONEJO; RUIZ, 2012; DAI; QIAO, 2017).

Another challenge arises from the market design itself, as MC procedures differ based on the bidding structure. Markets with simple bidding structures, such as most European markets, rely solely on price and quantity offers, neglecting detailed generation and network constraints in their MC procedures (KOLTSAKLIS; ZENGINIS; DAGOUMAS, 2020). This MC approach often necessitates ex-post corrective actions, which can degrade the optimality of market outcomes. Conversely, complex bidding structures, like those adopted by the California ISO, integrate detailed constraints directly into the MC procedure, reducing the need for such corrective actions. In hydrothermal markets where hydro generation is dominant, such as the Brazilian power system, additional hydro-specific constraints must also be incorporated, including water balance equations, hydro power production functions, and water release ramp rates (BREGADIOLI et al., 2016; PEREIRA et al., 2017; SANTOS et al., 2020). These constraints add complexity to the SO model but are essential for ensuring realistic and feasible strategic decisions.

Ultimately, the challenges in SO modeling stem from the need to balance strategic profit maximization with market rules, generation constraints, and the intricacies of market clearing mechanisms. The choice of market structure significantly impacts how these constraints are handled, influencing both the feasibility and optimality of strategic offers.

Also, solving equilibrium problems with equilibrium constraints (EPECs) in electricity markets presents several key challenges. First, market equilibrium depends on the strategic interactions between strategic producers and strategic consumers, where no agent has an incentive to deviate from its chosen strategy given the strategies of others. Capturing these interactions requires modeling the market as an EPEC, which results from the joint consideration of multiple mathematical programs with equilibrium constraints (MPECs) associated with different market participants. This formulation allows system operators, regulators, and market players to assess market power but significantly increases the complexity of the problem.

A major computational difficulty arises when reformulating bilevel models into MPECs. The conventional approach replaces the lower-level market-clearing problem with its Karush-Kuhn-Tucker (KKT) conditions. However, this introduces numerous complementarity constraints, which complicate the solution process and reduce numerical stability. Several methods have been proposed to address this issue, including the big-M method (JIANG et al., 2022), convex approximations (CONSTANTE-FLORES; CONEJO;

CONSTANTE-FLORES, 2022), relaxation-based techniques (DOLÁNYI; BRUNINX; DELARUE, 2024), penalty methods (KLEINERT; SCHMIDT, 2021), and metaheuristic approaches (DEB et al., 2020). However, these methods come with trade-offs in terms of computational efficiency and solution accuracy.

A particular instance of the more general KKT conditions for convex optimization problems named Primal-Dual Optimality Conditions (PDOCs) found in (CONEJO; RUIZ, 2020; TSIMOPOULOS; GEORGIADIS, 2021), can be an alternative approach on replacing the lower-level problem with its primal constraints, dual constraints, and strong duality equality. This eliminates complementarity constraints and improves numerical stability. However, a major challenge remains in solving EPECs, as existing methods do not fully represent MPECs using their exact dual counterparts due to the presence of bilinear terms and other nonlinearities. This limitation prevents the direct application of PDOCs, making it difficult to develop stable and computationally efficient solution methodologies. Overcoming these challenges is important for accurately modeling equilibria in electricity markets and ensuring a strong market analysis.

1.2 Problems and employed mathematical techniques

The main problems addressed in this thesis are described bellow along with the overall proposed solution strategies:

1. *Market clearing model for non-competitive agents under perfect competition conditions interpreted as an equilibrium problem:* We reinterpret the general market clearing (MC) optimization model under perfect competition—considering the comprehensive aspects of electricity markets—as an equilibrium model. To address this problem, we formulate a general market clearing optimization model under perfect competition conditions (where non-strategic agents offer/bid at marginal cost or maximum utility) and demonstrate its equivalence to an equilibrium model comprising all participating agents. This equivalence is proven by showing that the Karush-Kuhn-Tucker (KKT) optimality conditions of the MC model are identical to the combined KKT conditions of the individual agents' optimization problems. The modeling framework, solution approach, and broader market considerations—including the distinction between competitive and non-competitive agent behavior—are detailed in Chapter 2.
2. *Strategic behavior of a producer in a hydrothermal electricity market:* This problem is related to a strategic behavior of a producer in an electricity market under a hydrothermal system circumstances. Usually, the strategic behavior can be formulated by means of bilevel formulation, which in many cases, its solution approach involves

the use of KKT optimality conditions which generally brings many bilinear terms to the formulation, even when the original bilevel model is linear. Also, in a context of hydrothermal systems, additional hydrothermal constraints must be considered, which brings an extra layer of complexities to such markets, specially to a producer attempting to employ strategic modeling approaches. To address this problem, we replace the lower-level model of the bilevel framework with an alternative formulation based on optimality conditions (CONEJO; RUIZ, 2020). Rather than using KKT conditions, we employ a set of primal constraints, dual constraints, and a strong duality equality constraint. This approach eliminates the cumbersome complementarity conditions inherent to KKT formulations, thereby significantly reducing the number of bilinear terms. Through mathematical reformulation, we ultimately obtain a single bilinear term in the resulting single-level MPEC model. Furthermore, we convexify the remaining bilinear term using an iterative convexification approach. All developments are implemented within a hydrothermal constraint framework. The complete model and its solution methodology are presented in Chapter 3.

3. *Equilibrium of strategic agents in an electricity market:* This problem addresses the Nash equilibrium among strategic producers and strategic consumers in an electricity market, where competitive agents employ a Stackelberg game structure through bilevel formulations as their strategic modeling approach. In other words, we examine a single-level reformulation of strategic Stackelberg game bilevel models within a multi-leader-single-follower equilibrium framework. To solve this problem, we replace the lower-level model of each bilevel formulation with an alternative set of optimality conditions comprising primal constraints, dual constraints, and strong duality equality constraints instead of KKT conditions. This approach eliminates the problematic complementarity conditions inherent in KKT formulations, significantly reducing the number of bilinear terms. Through mathematical reformulation, we ultimately obtain just one bilinear term within each of these single-level MPEC models. We then convexify the remaining bilinear term in each respective MPEC model using an iterative convexification approach. Furthermore, we implement a rotated quadratic conic relaxation to enable the reformulation of these MPEC models using PDOCs as complete set of optimality conditions (primal constraints, dual constraints, and strong duality equality). This process ultimately yields a single equilibrium problem formulation encompassing both strategic producers and strategic consumers (i.e., all competitive agents). The complete model and its solution methodology are presented in Chapter 4.

In this work, the employed mathematical techniques to address the aforementioned problems are:

1. *Set of optimality conditions:* The set of optimality conditions can serve as an alternative approach to replacing an entire optimization model with its corresponding optimality conditions—effectively transforming the optimization model into a Mixed Complementarity Problem (MCP). While the literature typically employs KKT optimality conditions for this purpose, they introduce a set of complementarity constraints that can convert even a linear convex model into a non-convex MCP. This transformation may render the MCP computationally intractable and significantly increase the solving burden. To address this issue, we instead use Primal-Dual Optimality Conditions (PDOCs) as an alternative to KKT conditions. The PDOC approach retains the original primal constraints, incorporates dual constraints, and replaces the problematic complementary constraints with a strong duality equality constraint. This method is extensively applied in Chapter 3 and Chapter 4. In Chapter 3, PDOCs replace the lower-level model of the proposed bilevel formulation for strategic producer offerings, enabling its reformulation into a single-level MPEC model within a hydrothermal wholesale electricity market. In Chapter 4, PDOCs are employed in two key applications: (i) Replacing the lower-level model in bilevel formulations of strategic producers and consumers under a multi-leader-single-follower equilibrium structure, facilitating an equilibrium MPEC formulation; (ii) After deriving the individual MPEC models for strategic producers and consumers, substituting each with PDOCs to construct a single-level EPEC model that entirely avoids complementary constraints.
2. *Finding a single set of bilinear term:* Bilinear terms typically introduce additional complexity to optimization problems by creating non-convex or non-regular formulations. To mitigate this issue, we employ mathematical reformulations involving algebraic manipulations of the objective function and/or strategic complementarity constraints, ultimately reducing the problem’s bilinear terms to a single set. This approach is first presented in Chapter 3 for a strategic producer’s offering problem modeled through a Stackelberg game framework, specifically applied within hydrothermal systems subject to transmission network constraints. The methodology is then extended in Chapter 4 to a broader context, where the bilinear term reduction technique is implemented for bilevel models of both competitive strategic producers and strategic consumers operating within a multi-leader-single-follower equilibrium structure.
3. *Bilinear term relaxation/convexification:* Bilinear terms introduce non-convexities and/or non-regularity in optimization models, increasing their computational complexity. To address these challenges, relaxation and convexification methods prove particularly valuable. Our approach employs a quadratic relaxation and convexification technique that reformulates each bilinear term as a difference of quadratic

terms. Through iterative Taylor series expansion around an initial operating point, we linearize the second (negative) quadratic term, thereby converting the original non-convex bilinear expression into a convex difference between a quadratic and linear function. Chapter 3 presents this methodology applied to a strategic producer's offering problem in a Stackelberg game framework, specifically within a hydrothermal system incorporating transmission network constraints. Chapter 4 extends this approach to a more general setting, where, after identifying the relevant bilinear terms, we apply the relaxation and convexification technique to bilevel models of both strategic producers and consumers operating in a multi-leader-single-follower equilibrium structure.

4. *Rotated quadratic conic relaxation:* Recasting an optimization problem as a rotated quadratic cone problem (MOSEK ApS, 2024) (specifically, a rotated second-order cone program - SOCP) offers several computational and theoretical advantages. A key advantage explored in this work is that quadratic cone problems admit exact dual counterparts. This property enables the derivation of an exact dual formulation for any primal quadratic constraint that can be reformulated as a quadratic cone constraint. In Chapter 4, we employ a rotated quadratic cone relaxation for each convexified term involving the difference between a quadratic function and a linear function. This reformulation expresses each convexified function as a rotated quadratic cone constraint. Consequently, we obtain constraints that not only convexify the original bilinear terms but also provide their exact dual counterparts. This duality enables the application of Primal-Dual Optimality Conditions (PDOCs) to the MPEC model developed in Chapter 4.

1.3 State-of-the-art of Stackelberg game and Nash equilibrium in electricity markets

In this section, we evidence the overall state-of-the-art for the problems described in section 1.2. Further, in the corresponding chapters, a detailed literature review of each one of these problems is provided.

Strategic behavior of a producer in a hydrothermal electricity market

Most strategic offering (SO) models for day-ahead markets are designed for thermal-dominated systems (DAI; QIAO, 2015; DAI; QIAO, 2017; RUIZ; CONEJO, 2009) and do not account for hydro constraints in the lower-level market-clearing process. This makes them unsuitable for hydro-dominated markets, though some studies have attempted to incorporate hydro constraints (POUSINHO; CATALÃO; CONTRERAS, 2012; POUSINHO et al., 2013; CRUZ et al., 2016; MOISEEVA; HESAMZADEH, 2018). However, these

efforts often neglect key aspects such as hydro inter-dependencies, some specific thermal constraints, or network representation.

Bilevel SO models are typically reformulated as single-level models using the Karush-Kuhn-Tucker (KKT) conditions of the lower-level problem, resulting in mathematical programs with equilibrium constraints (MPECs) (LUO; PANG; RALPH, 1996; GABRIEL et al., 2013). However, solving MPECs is challenging due to complementarity constraints, which are commonly handled using the big-M approach (FORTUNY-AMAT; MCCARL, 1981b). This method has drawbacks, including the difficulty of selecting an appropriate big-M value without excluding feasible solutions (PINEDA; MORALES, 2019; KLEINERT et al., 2020; KLEINERT; SCHMIDT, 2023). To address these issues, heuristic techniques (PINEDA; BYLLING; MORALES, 2018), branch-and-bound methods (KLEINERT et al., 2021), and linearization techniques for bilinear terms (LAWS; HANASUSANTO, 2023) have been proposed to improve computational efficiency.

Equilibrium of strategic agents in an electricity market

The Equilibrium Problem with Equilibrium Constraints (EPEC) arising from the joint solution of Mathematical Programs with Equilibrium Constraints (MPECs) in wholesale electricity markets is often complex and computationally challenging. A common solution method is diagonalization (HOBBS; METZLER; PANG, 2000; KARDAKOS; SIMOGLOU; BAKIRTZIS, 2013; NAEBI et al., 2020), which iteratively solves MPECs for each agent while holding others' decisions fixed, though this approach may present convergence and feasibility issues. Several methods have been proposed to address these challenges. In (RUIZ; CONEJO; SMEERS, 2012), an EPEC model is used to study producer equilibria in network-constrained markets by formulating bilevel models, where strategic decisions are transformed into MPECs and solved using linearized KKT conditions. The approach in (DOLÁNYI; BRUNINX; DELARUE, 2024) explores multiple Nash equilibria by applying Scholtes' regularization to handle bilinear terms, enabling the identification of different equilibrium points. In hydro-dominated power systems, (MOISEEVA; HESAMZADEH, 2018) formulates an EPEC model using a discrete strategy set instead of continuous strategies, leading to a stochastic mixed-integer formulation solved via Benders decomposition. The interaction between conventional and wind power producers is modeled in (DAI; QIAO, 2017) and (TSIMOPOULOS; GEORGIADIS, 2021) using bilevel formulations that consider both day-ahead and real-time markets, with solutions obtained through diagonalization or by jointly solving KKT conditions. An alternative single-level approach is proposed in (LIU; CONEJO, 2021), which models the strategic behavior of producers, consumers, and the market operator without requiring an EPEC formulation. Reinforcement learning techniques are introduced in (YU et al., 2023) to represent market interactions using hierarchical Nash distributed learning, while

(PRAT; CHATZIVASILEIADIS, 2023) applies machine learning methods to predict active constraints and simplify bilevel formulations. Finally, (CHEN et al., 2021) studies the interactions between distribution and transmission markets using an EPEC model that jointly considers multiple DSOs and the wholesale market, reformulating the problem as a mixed-integer second-order cone program solved with MOSEK (MOSEK ApS, 2024).

1.4 Research goals

The main research goals involving the problems addressed in this thesis are discussed below:

- I Solving a bilevel strategic offering model for a generation company in a hydrothermal energy market: A new convexification approach:
 - i To develop a Strategic Offering (SO) model specifically designed for a generation company operating in a hydrothermal electricity market. The SO model must incorporate water balance constraints and other critical hydro constraints for both strategic and non-strategic producers, while capturing the spatial and temporal inter-dependencies of hydro cascade systems. The lower-level model must account for competitive market participants. Additionally, DC network constraints should be included to accurately represent the transmission system.
 - ii To propose a novel approach for solving the bilevel strategic offering optimization problem by employing an alternative set of constraints to represent the optimality conditions of its lower-level problem. The proposed approach reduces the number of bilinear terms in the resulting single-level MPEC problem to a single set of terms - independent of the number of inequality constraints. The single-level problem is then convexified around an initial operating point to address the remaining bilinear term.
- II Equilibria among strategic producers and strategic consumers: A solution approach relying on a single optimization problem:
 - i To propose a systematic approach for solving a general EPEC model by reformulating the associated MPEC problems for both producers and consumers as convex rotated quadratic cone problems. These reformulated problems possess analytical dual counterparts and can consequently be represented exactly using their respective Primal-Dual Optimality Conditions (PDOCs). Furthermore, this approach employs the PDOCs derived from the MPEC problems to solve the nonlinear single optimization EPEC problem involving both strategic producers and strategic consumers.

- ii To demonstrate the proposed methodology through both a simple illustrative example and a realistic case study. For simplicity, this analysis assumes the network does not constrain equilibria and is therefore not explicitly modeled. The solution approach is further applied to examine four distinct market configurations: (i) a perfectly competitive market with no expected market power; (ii) a market where a single producer may exercise market power; (iii) a market where a single consumer may exercise market power; and (iv) a market where both a producer and consumer may exercise market power.

1.5 Thesis structure

The remaining parts of this thesis is organized as follows:

Chapter 2

This chapter presents the key concepts underlying this research. First, it outlines fundamental aspects of electricity markets, including market types, structural frameworks, participating agents, and their respective roles. Next, it offers a mathematical reinterpretation of general market clearing procedures as equilibrium problems involving non-competitive agents under perfect competition conditions. The chapter then establishes a conceptual distinction between strategic and non-strategic behaviors, providing formal definitions and corresponding formulation approaches. Finally, it examines critical considerations for hourly pricing in transmission systems and essential features of hydrothermal systems that must be accounted for in market operations.

Chapter 3

This chapter presents a bilevel strategic offering model for a Strategic Company (SC) operating in a hydrothermal wholesale electricity market. The upper-level model maximizes the SC's profit, while the lower-level model represents a hydrothermal market clearing procedure incorporating both hydrothermal constraints and DC network constraints. The chapter further proposes a solution approach for this bilevel model, where the lower-level problem is replaced by its associated primal and dual constraints along with the strong duality equality. This reformulation yields a single-level MPEC model that avoids the complementary constraints typically arising from KKT optimality conditions, which are commonly used in such cases. The remaining bilinear terms in the MPEC are then reformulated as a single set of bilinear terms and iteratively convexified around an initial operating point. Numerical results evaluate two fundamental scenarios: (i) A strategic scenario where the SC exercises market power and the bilinear term in the strategic offering model is approximated using the proposed convexification approach; and (ii) A

competitive scenario where all companies behave non-strategically (perfect competition). Both scenarios are analyzed for cases with and without network congestion.

Chapter 4

This chapter presents a mathematical formulation for modeling equilibrium among strategic producers and consumers through a single-level problem, aiming to capture strategic market behavior and identify market power in competitive wholesale electricity markets, thereby providing valuable insights for market operators, regulators, and participants. The proposed model integrates two sets of Stackelberg game bilevel models within a multi-leader-single-follower equilibrium framework—one for strategic producers and another for strategic consumers—with three key methodological contributions: (i) replacing the lower-level models with Primal-Dual Optimality Conditions (PDOCs) to formulate equilibrium-constrained mathematical programs (MPECs) that eliminate complementarity conditions (typically handled via KKT conditions) while reducing bilinear terms to a single set of remaining terms; (ii) convexifying the residual bilinear terms and relaxing the resulting quadratic terms into rotated quadratic cone constraints, which admits exact dual counterparts; and (iii) reformulating each convexified/relaxed MPEC via PDOCs to yield a single-level Equilibrium Problem with Equilibrium Constraints (EPEC) that avoids KKT-based complementarity constraints entirely. The model’s applicability is demonstrated through two case studies, highlighting its flexibility for diverse stakeholders—market operators, regulators, or agents—as its utility adapts to specific objective functions and market configurations.

Chapter 5

This chapter summarizes the contributions of this thesis by presenting the main conclusions and discussing potential directions for future research.

Appendix A

This chapter presents all remaining numerical results relevant to the proposed model in Chapter 3.

Appendix B

This chapter presents all remaining numerical results relevant to the proposed model in Chapter 4.

2 ELECTRICITY MARKETS

This chapter presents the fundamental concepts of electricity markets and their participants, based on key definitions from (CONEJO; CARRIÓN; MORALES, 2010). Section 2.1 categorizes market types according to two dimensions: (1) organizational structure (organized vs. non-supervised markets, distinguished by the presence or absence of a market operator) and (2) trading timeframe (pool markets vs. futures markets). Section 2.2 identifies the primary market agents, including generators, consumers, retailers, network operators, and market/system operators. Section 2.3 develops a general mathematical model for market clearing procedures implemented by market operators, along with an analysis of agents behaviors. This section formalizes the Strategic Offering (SO) problem for producers and the Strategic Bidding (SB) problem for consumers through mathematical formulations, and examines their interaction in a Nash equilibrium framework. Section 2.4 extends the analysis by incorporating transmission network constraints into multi-period market clearing procedures. Finally, Section 2.5 presents the unique characteristics of electricity markets operating in hydrothermal systems.

2.1 Market structures

In this section, the main types of energy markets are discussed. Generally, market structures can vary, depending on the market architecture implemented. The Bertrand Equilibrium (BERTRAN, 1883), Cournot Equilibrium (COURNOT, 1838), and the Supply Function Equilibrium (SFE) (KLEMPERER; MEYER, 1989) are three fundamental theoretical models of competition in economics, each differing in how firms compete.

In Bertrand competition, a set of firms I compete by setting prices $p_i, i \in I$. Assuming homogeneous products and no capacity constraints, the firm with the lowest price captures the entire market. In Cournot competition, firms compete by choosing quantities $q_i, i \in I$, and the market price is determined by aggregate supply. In SFE, firms do not bid a single price or quantity but instead submit a supply function $S_i(p), i \in I$, which specifies how much they are willing to supply at different prices. In this work, we focus on assessing equilibria within a market structure described by an SFE competition model, similar to those commonly employed in day-ahead energy markets.

In a market structured through SFE competition, the strategic behavior of producers and consumers can vary significantly depending on their ability to influence market-clearing prices and their knowledge concerning the strategies of the other players. Two important

models for analyzing the behavior of producers and/or consumers are the Stackelberg equilibrium (STACKELBERG, 1934) and the Nash equilibrium (NASH, 1950).

In a Stackelberg equilibrium, the behavior of a market player is represented by a hierarchical leader-follower game, where the leader anticipates the optimal reaction of the follower and chooses a strategy that maximizes its own payoff. The follower, in turn, optimally responds to maximize its own payoff, given the leader's choice. The Stackelberg equilibrium is typically formulated as a bilevel optimization problem. The upper-level sub-problem represents the leader's decision (e.g., a producer maximizing profit or a consumer maximizing utility), while the lower-level sub-problem represents the follower's reaction (e.g., the market operator maximizing social welfare through a market-clearing procedure). Under conditions of lower-level convexity and certain constraint qualifications (e.g., Slater's condition), the bilevel problem can be reformulated as a single-level Mathematical Problem with Equilibrium Constraints (MPEC). This is achieved by replacing the lower-level sub-problem with its corresponding optimality conditions, which are enforced as constraints in the MPEC.

On the other hand, a Nash equilibrium is a general concept that describes the collective behavior of a group of (or possibly all) players in a market. In this type of equilibrium, each player in the group selects a strategy that maximizes their own payoff, given the strategies chosen by all other players in the group. No player can improve their payoff by unilaterally changing their strategy while the others keep theirs unchanged. A Nash equilibrium is typically formulated as a system of optimization problems, where each player's problem depends on the strategies of the other players. These problems are solved jointly, often using their respective optimality conditions (e.g., Karush-Kuhn-Tucker (KKT) conditions for convex problems). In cases involving hierarchical decision-making (e.g., multi-leader-follower games), the Nash equilibrium may involve the equilibrium of Mathematical Problems with Equilibrium Constraints (MPECs) of strategic agents, which their interaction can lead to an Equilibrium Problem with Equilibrium Constraints (EPEC).

In what concerns their organizational structure, electricity markets can be classified as either organized markets or unsupervised markets. Organized markets feature a central mediating agent, typically a Market Operator (MO), that manages energy transactions. In these markets, electricity trading occurs through an MO-operated clearinghouse that aggregates and matches selling offers from generation companies with buying bids from consumers to determine power allocations and electricity prices. In contrast, unsupervised markets function without centralized mediation, as exemplified by bilateral markets where producers and consumers establish direct contracts without MO involvement.

In what concerns the time horizon employed for electricity trading, the organized markets are basically categorized into two types: the pool markets and the future markets. Trading in pool markets involves submitting offers and bids, by generation companies and consumers, respectively, for a short-term horizon (typically in the day-ahead), whereas trading in future markets involves submitting offers and bids for medium- and long-term horizons. Therefore, futures market involve trading of electricity for longer time horizons. The futures markets may involve the forward market and the options market. Figure 2.1 illustrates a common structure for organized markets, highlighting the pool and future markets as well as the market participants, such as intermittent and non-intermittent generators, consumers, wholesale market retailers and their clients, and the MO mediator. The organization (mediation) of an electricity market by the MO involves a procedure named as market clearing, which consists in calculating the accepted offers and bids, and also the energy prices for each period of the market. Such market structure can be found in most of the US markets (e.g., MISO, PJM Interconnection, ISO New England) and Europe (e.g., Nordpool, OMEL), considering some differences among them (CONEJO; CARRIÓN; MORALES, 2010).

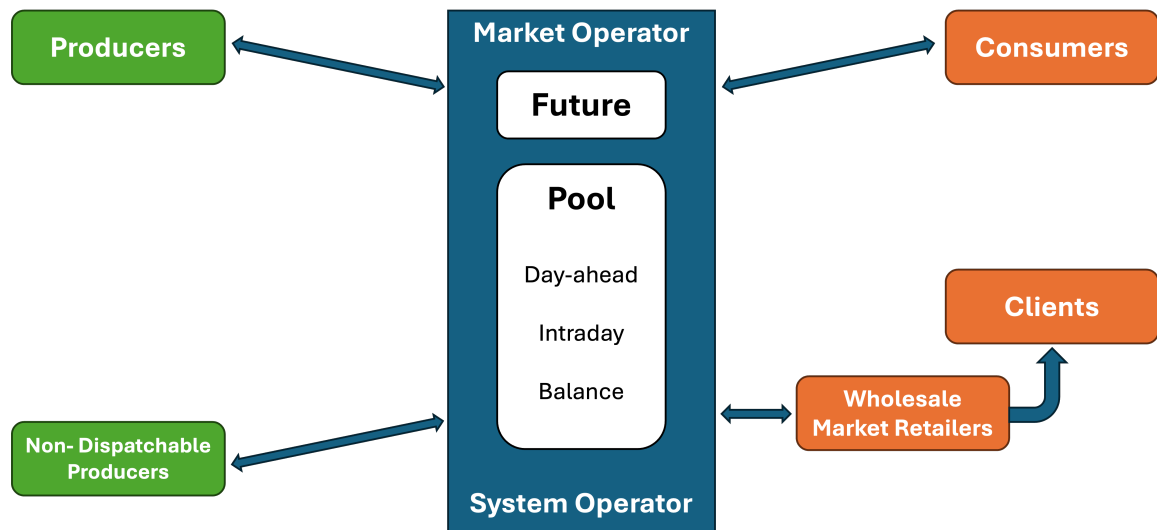


Figure 2.1 – Organized Markets Structure.

As shown in Figure 2.1, the pool market generally involves three sub-market structures: day-ahead market, intraday market and the balancing or real-time market. The description of the main aspects of the pool market is provided in Section 2.1.1. Different types of market clearing procedures employed in the day-ahead and intraday markets are described in Section 2.1.2. A general description of futures markets is provided in Section 2.1.3. Finally, a general description of the markets that are outside the organized markets is provided in Section 2.1.4.

2.1.1 Pool market

In the pool market, electricity trading occurs in the short-term horizon. Generally, trading in this type of market is conducted for the next day, with hourly discretization. As previously mentioned, the pool market consists of three different sub-markets, as depicted in Figure 2.1. These sub-markets are briefly described in Sections 2.1.1.1 through 2.1.1.3, as follows.

2.1.1.1 Day-ahead market

The day-ahead market represents the most important segment of the electricity pool market. In this market, generation companies submit hourly energy supply offers in discrete blocks, while consumers and retailers provide corresponding demand bids. Each offer and bid specifies both the desired energy quantity and its associated price. The Market Operator then clears the market through a Market Clearing (MC) procedure, which establishes: (i) hourly equilibrium prices (spot prices) and (ii) scheduled energy quantities to be traded between participants in discrete blocks for each hour of the following day.

Due to inherent price and quantity uncertainties, the day-ahead market exhibits significant volatility (CONEJO; CARRIÓN; MORALES, 2010). This characteristic enables dynamic modeling opportunities from both generator and large consumer perspectives, creating potential for strategic behavior in their market participation.

2.1.1.2 Intraday market

The intraday market operates similarly to the day-ahead market but is cleared closer to actual energy delivery and covers shorter trading periods. Intraday markets are used for adjusting energy trading to meet constantly variable system demands, and are cleared using MC procedures which are similar to those used in day-ahead markets. Given its proximity to real-time operation, intraday markets must account for voltage security to ensure system reliability. Intermittent generators (e.g., wind and photovoltaic systems) benefit most from these markets due to their need for short-term generation adjustments.

2.1.1.3 Balancing market

The balancing market (also called the real-time market) performs final power system adjustments to maintain real-time load-generation balance. This market mechanism seeks for power balancing, trying to prevent both generation surpluses and deficits. As the ultimate mechanism for maintaining supply-demand equilibrium, it operates under strict real-time conditions that require: (i) voltage security monitoring (similar to the Intraday Market requirements) and (ii) voltage stability maintenance - both critical for ensuring system reliability and security.

The market clearing procedure in balancing electricity markets serves as a real-time corrective mechanism to maintain grid stability by reconciling supply-demand imbalances. When deviations occur from scheduled power flow, due to factors like renewable generation volatility or unexpected demand shift, the Transmission System Operator (TSO) activates a dynamic optimization process. This process evaluates incremental (increase generation) and decremental (decrease generation) bids from generators and flexible loads, prioritizing the most cost-effective solutions while respecting technical constraints like ramp rates, transmission limits, and voltage requirements. Prices are typically set via marginal pricing, where the highest accepted bid determines the clearing price. Selected bids are dispatched within minutes or even seconds, with financial settlements imposing imbalance charges on deviating participants. By operating on short time granularity (5 to 15 minute intervals or continuously) and integrating real-time grid security protocols, balancing markets not only prevent blackouts but also incentivize flexibility resources, from fast-ramping gas plants to battery storage, to compensate for renewable forecast errors.

2.1.2 Simple and complex market clearing

Given the offers provided by generation companies and the bids provided by the consumers in an energy auction, the market operator must employ a market clearing procedure to clear the market. Based on offers and bids, an MC procedure must calculate the power effectively sold (accepted) by each generation company and the power effectively bought (accepted) by each consumer, as well as the energy prices for each period of the auction. Therefore, the MC procedure effectively allocates power in the system. Such task used to be performed by the traditional unit commitment problems, in the context of energy regulation, that existed previous to the energy markets.

The structure employed for MC procedures may differ depending on the market design. Some energy markets adopt purely economic bidding information (simple bidding structure), involving only price and quantity blocks of offers and bids. An example of such a market structure is the one employed in most European markets, whose MC procedure is detailed in (KOLTSAKLIS; ZENGİNİS; DAGOUMAS, 2020). In the MC models employed for such a market structure, intertemporal generation constraints are generally not considered, as well as transmission system representation. These constraints must be considered *a posteriori* by the system operator. On the other hand, some markets adopt a complex bidding structure, which integrates data regarding intertemporal generation constraints into the MC procedure. An example of such a market structure is the California ISO in the USA.

When a simple bidding structure is employed, ex-post actions may be necessary to account for feasibility. These ex-post actions are calculated using simple heuristic rules,

possibly degrading the optimality of the MC solution. On the other hand, when complex bidding information is used, these heuristic rules are avoided, since intertemporal technical constraints are intrinsically considered in the MC procedure. In this work, we assume that the market operator adopts a complex bidding structure, so that the MC model intrinsically handle these intertemporal constraints, avoiding ex-post corrective actions. In such a market structure, start-up costs and intertemporal constraints are employed for thermal units such as ramp rates, minimum up and down times, as well as network constraints (ARROYO; CONEJO, 2002; MOTTO et al., 2002). Also, if such a market structure is employed for hydrothermal systems where hydro generation is dominant (such as the Brazilian power system), hydro constraints are also considered in the MC process, such as: bounds on hydro variables, water release ramp rates, water balance constraints, hydro power production functions, etc. (BREGADIOLI et al., 2016; PEREIRA et al., 2017; SANTOS et al., 2020).

2.1.3 Futures markets

The futures market facilitates energy trading with delivery scheduled for predetermined future dates at currently agreed prices. This market functions as a hedging mechanism against price volatility in the spot market, enabling participants to mitigate financial risk (CONEJO et al., 2008). Transactions are cleared through a standardized mechanism where participants trade either physical commodities or financial instruments for future delivery. These contracts, commonly referred to as derivatives (HULL, 2016), account for the temporal discrepancy between trade execution and actual energy delivery. Notably, physical fulfillment of the contracted energy volume is not always obligatory.

2.1.4 Markets outside the organized environment (unsupervised markets)

Markets operating outside organized environments function without an intermediary agent (such as a market operator) to coordinate transactions between participants. However, they remain subject to operational constraints enforced by the Independent System Operator (ISO). In these markets, trades follow specific rules established through bilateral agreements between agents. These contracts—formed between generation companies and consumers—guarantee the delivery of specified energy quantities at predetermined prices. Bilateral agreements can be established for short-, medium-, or long-term periods. Such contracts serve as an alternative risk-hedging mechanism, protecting the interests of both parties by ensuring demand fulfillment stability for consumers while providing generators with predictable marginal profits.

2.2 Agents (Players)

This section describes the primary participants in electricity markets, detailing their respective functions, classifications, and market behaviors:

2.2.1 Generating Companies

Generating companies are the entities responsible for supplying electrical power to the market, by means of their generating units (generators). These participants can sell their energy across all market segments, including both organized and unsupervised markets. For market dispatch purposes, generators are classified as either dispatchable or non-dispatchable sources. Dispatchable generators can reliably supply power on demand, with thermal and hydroelectric plants being prime examples. In contrast, non-dispatchable generators rely on intermittent energy sources and consequently cannot guarantee consistent power availability, wind and solar photovoltaic units fall into this category.

Regarding market participation, generation companies can adopt either strategic or non-strategic behaviors. Based on their capacity to influence prices, these companies are classified as either price-makers or price-takers. Price-maker companies possess sufficient dispatchable capacity to manipulate market prices, while price-taker companies lack this ability due to their limited installed capacity. Consequently, price-makers can effectively engage in strategic market behavior, whereas price-takers typically cannot exert strategic influence since their actions generally do not affect price formation.

2.2.2 Consumers

Consumers play a fundamental role in electricity markets as the primary drivers of demand, shaping load profiles and influencing price dynamics. They range from passive residential users with fixed consumption patterns to active participants who adjust usage in response to price signals or contractual incentives. Large industrial consumers often engage directly in wholesale markets through bilateral agreements or corporate power purchase agreements (PPAs), while smaller customers typically procure electricity via retailers. A growing segment, prosumer, not only consumes power but also generates it through distributed energy resources like solar energy, blurring traditional market roles.

The degree of consumer flexibility varies significantly: critical loads (e.g., hospitals) are inflexible, whereas commercial and industrial users may offer demand response by shifting or curtailing consumption. Larger consumers can even act strategically, adjusting their demand to negotiate favorable terms or participating in demand-side auctions. Emerging trends, such as peer-to-peer energy trading and aggregation of flexible loads into virtual power plants, further expand consumer roles in modern markets. Regulatory

frameworks and network pricing mechanisms, like locational marginal pricing, also shape how consumers interact with the grid, balancing system needs with economic efficiency.

2.2.3 Market retailers

Retailers serve as intermediaries in electricity markets, bridging the gap between wholesale power suppliers and end consumers. They purchase electricity from wholesale markets, bilateral contracts, or power purchase agreements (PPAs), then resell it to customers while managing associated risks and regulatory requirements. Their core functions involve securing reliable supply at competitive prices, often employing hedging strategies to mitigate price volatility, and tailoring services to diverse customer needs, from fixed-rate residential plans to flexible industrial contracts.

Retailers vary significantly in structure and strategy. Some operate as full-service providers, handling everything from energy procurement to billing, while others specialize as green retailers focusing on renewable energy or aggregators pooling small consumers for collective bargaining power. In competitive markets, they compete for customers through innovative pricing, value-added services, or integration with distributed energy resources (DERs) like solar energy. In regulated environments, they often work within fixed tariffs and stricter service obligations.

2.2.4 Market operator and system operator

The Market Operator (MO) typically works as a non-profit entity responsible for the efficient economic management of the market environment. Therefore, the MO watches over the formulation and the enforcement of market rules and facilitates the determination of prices and quantities traded within the market. Also, one of their responsibilities is to clear both the day-ahead and adjustment energy markets. Additionally, in markets where the MO and Independent System Operator (ISO) are functionally integrated, the MO also facilitates reserve and balancing market clearing in coordination with the SO's grid-security mandates.

The Independent System Operator (ISO), which is also typically a non-profit entity, is responsible for the technical management of the power system, a critical function within the electricity market framework. Therefore, the main responsibility of the ISO is to maintain system security and reliability. Its duties also include ensuring non-discriminatory access to the transmission network for all market participants (generators, consumers, and wholesale retailers). Additionally, the ISO enables efficient trading indirectly by maintaining grid reliability and providing price signals, which the MO uses to clear transactions.

2.3 Market clearing model and agents' behavior

A market clearing procedure is a vital tool for the market operator (MOTTO et al., 2002; ARROYO; CONEJO, 2002). In the day-ahead market (as well as in intraday markets), generating units from all generation companies must submit their energy selling offers, while all participating consumers must submit their energy purchase bids. These offers and bids are processed by the market operator through a market clearing procedure, which allocates power supply and demand for each unit and consumer, and calculates the market clearing prices. To accomplish this, the market operator employs a mathematical optimization model to clear the market.

This section aims to provide a mathematical formulation of the market clearing model and to introduce the concept of perfect competition among market agents. This section also reinterprets the market clearing procedure, under perfect competition, as an equilibrium problem involving all market participants. The concept of perfect competition is discussed in 2.3.1. A typical MC model is described in Section 2.3.2. Details of strategic behavior under imperfect competition are provided in Section 2.3.3. Basic concepts regarding Nash equilibrium are described in Section 2.3.4.

2.3.1 Perfect competition in electricity markets

In an electricity market, perfect competition occurs when all agents behave non-strategically. In this case, producers submit offers at their marginal costs, and consumers submit bids at their maximum utility. On the other hand, imperfect competition arises when some or all agents behave strategically.

2.3.2 Market clearing procedure model

In this section, we formally describe a market clearing (MC) procedure model (CONEJO; CARRIÓN; MORALES, 2010). The nomenclature used in the mathematical formulation is presented in terms of sets, indices, parameters, decision variables, and dual variables. For the sake of simplicity, we assume that the network does not constrain the market clearing procedure, and the model considers only a single time period.

Notation

For expressing the cardinal of a general set S we use the following notation: $|S|$.

Sets:

I Generating companies.

J Consumer companies.

U_i Generating units of producer i .

C_j Consumer units of consumer j .

Indices:

i Index for all generating companies, $i \in I$.

k Index for all generating companies, $k \in I$, the k index is used in a context of strategic operation to differentiate from index i .

j Index for all consumer companies, $j \in J$.

l Index for all consumer companies, $l \in J$, the l index is used in a context of strategic operation to differentiate from index j .

u Index for all units belonging to generating company i , $u \in U_i$.

c Index for all units belonging to consumer company j , $c \in C_j$.

Parameters:

c_{iu} Marginal cost of generating unit u belonging to company i .

o_{iu} Offering price from generating unit u from generating company i . Under perfect competition, we have $o_{iu} = c_{iu}$.

u_{jc} Maximum utility of consumer unit c from a consumer company j .

b_{jc} Bidding price from consumer unit c from consumer company j . Under perfect competition, we have $b_{jc} = u_{jc}$.

$\bar{p}_{iu}$ Upper bound on power generated by all units u from generating company i .

$\bar{d}_{jc}$ Upper bound on power demanded from consumer unit c from consumer company j .

Decision variables:

p_{iu} Power produced by unit u from generating company i .

d_{jc} Power demanded by consumer unit c from consumer company j .

Dual variables:

The dual variables itemized below are associated with the following constraints:

- λ Associated with the power balance constraint, which represents the cleared price.
- $\bar{\mu}_{iu}^p, \underline{\mu}_{iu}^p$ Associated with upper and lower bounds, respectively, on power supply of unit u from producer i .
- $\bar{\mu}_{jc}^d, \underline{\mu}_{jc}^d$ Associated with upper and lower bounds, respectively, on power demand of consumer unit c from consumer company j .

A basic market clearing procedure model is described in (2.1) for a single period, disregarding both intertemporal constraints and the transmission system representation. In this model, (2.1a) represents the social welfare function, (2.1b) ensures the balance between power supply and demand, (2.1c) defines the upper and lower bounds for each unit of every producer company, and (2.1d) defines the upper and lower bounds for each unit of every consumer company. We highlight the dual variables for each set of constraints on the right-hand side of the constraint, enclosed in parentheses:

$$\begin{aligned} \text{Min}_{p_{iu}, d_{jc}} \quad & \sum_{i \in I} \sum_{u \in U_i} o_{iu} p_{iu} - \sum_{j \in J} \sum_{c \in C_j} b_{jc} d_{jc} \\ \text{s.t. :} \end{aligned} \quad (2.1a)$$

$$\sum_{i \in I} \sum_{u \in U_i} p_{iu} - \sum_{j \in J} \sum_{c \in C_j} d_{jc} = 0 : (\lambda) \quad (2.1b)$$

$$0 \leq p_{iu} \leq \bar{p}_{iu} : (\underline{\mu}_{iu}^p, \bar{\mu}_{iu}^p), \forall i \in I, \forall u \in U_i \quad (2.1c)$$

$$0 \leq d_{jc} \leq \bar{d}_{jc} : (\underline{\mu}_{jc}^d, \bar{\mu}_{jc}^d), \forall j \in J, \forall c \in C_j. \quad (2.1d)$$

One of the main contributions of this section is to demonstrate that the market clearing procedure model presented in (2.1) can be reinterpreted as an equilibrium model involving all producers, consumers, and the system operator, provided that the agents operate under perfect competition conditions. To this end, we describe the models associated with each of these agents below.

Model (2.2) is associated with producer company i , which seeks to maximize its profit by using the power outputs of its units $u \in U_i$ as the only optimization variables. The lower and upper bounds on the power outputs are enforced by the inequality constraints in (2.2).

$$\begin{aligned} \text{Max}_{p_{iu}} \quad & \sum_{u \in U_i} \lambda p_{iu} - o_{iu} p_{iu} \\ \text{s.t. :} \end{aligned} \quad (2.2)$$

$$0 \leq p_{iu} \leq \bar{p}_{iu} : (\underline{\mu}_{iu}^p, \bar{\mu}_{iu}^p), \forall i \in I, \forall u \in U_i.$$

Model (2.3) is associated with the consumer company j , which seeks to maximize its utility by using the power demands of its units $c \in C_j$ as the only optimization variables. The lower and upper bounds on the power demands must be enforced by the inequality constraints in (2.3):

$$\begin{aligned} & \underset{d_{jc}}{\text{Max}} \quad \sum_{c \in C_j} b_{jc} d_{jc} - \lambda d_{jc} \\ & \text{s.t. :} \\ & 0 \leq d_{jc} \leq \bar{d}_{jc} : \left(\underline{\mu}_{jc}^d, \bar{\mu}_{jc}^d \right), \forall j \in J, \forall c \in C_j. \end{aligned} \quad (2.3)$$

Finally, the feasibility problem (2.4) is associated with the system operator, where no specific objective function is employed, as the sole responsibility of the operator is to ensure the power balance between supply and demand. The dual variable associated with the power balance constraint provides the market clearing price λ . If the feasibility problem (2.4) incorporates a network formulation, therefore including a set of decision variables, it could be called an optimization model, where this agent would also be responsible for maintaining the power balance across the network and ensuring that the capacities of transmission lines are not exceeded.

$$\sum_{i \in I} \sum_{u \in U_i} p_{iu} - \sum_{j \in J} \sum_{c \in C_j} d_{jc} = 0 : (\lambda). \quad (2.4)$$

We observe that the producer problem (2.2) and the consumer problem (2.3) are intrinsically related to the system operator model (2.4), since the equilibrium price λ , used to calculate the profit in (2.2) and the consumer utility in (2.3), corresponds to the dual variable associated with the power balance constraint in (2.4).

By definition, the joint solution of problems (2.2), (2.3), and (2.4) represents an equilibrium involving all producers, consumers, and the system operator. An important aspect of this equilibrium is that, under perfect competition, producer agents offer at their marginal costs, i.e., $o_{iu} = c_{iu}$, while consumer agents bid at their maximum utility, i.e., $b_{jc} = u_{jc}$. Given these conditions, the cost term c_{iu} in the profit maximization problem (2.2) can be replaced by o_{iu} , and the utility term u_{jc} in the utility maximization objective function of (2.3) can be replaced by b_{jc} .

Now it is possible to prove that the aforementioned equilibrium model is equivalent to the model for the market clearing procedure in (2.1). This is accomplished by deriving the KKT optimality conditions for each model individually (2.2, 2.3, 2.4) and combining them to obtain (2.5):

$$0 \leq p_{iu} \leq \bar{p}_{iu} : \forall i \in I, \forall u \in U_i \quad (2.5a)$$

$$o_{iu} - \lambda - \underline{\mu}_{iu}^p + \bar{\mu}_{iu}^p = 0 : \forall i \in I, \forall u \in U_i \quad (2.5b)$$

$$0 \leq p_{iu} \perp \underline{\mu}_{iu}^p \geq 0 : \forall i \in I, \forall u \in U_i \quad (2.5c)$$

$$0 \leq (\bar{p}_{iu} - p_{iu}) \perp \bar{\mu}_{iu}^p \geq 0 : \forall i \in I, \forall u \in U_i \quad (2.5d)$$

$$0 \leq d_{jc} \leq \bar{d}_{jc} \forall j \in J, \forall c \in C_j \quad (2.5e)$$

$$-b_{jc} + \lambda - \underline{\mu}_{jc}^d + \bar{\mu}_{jc}^d = 0 : \forall j \in J, \forall c \in C_j \quad (2.5f)$$

$$0 \leq d_{jc} \perp \underline{\mu}_{jc}^d \geq 0 : \forall j \in J, \forall c \in C_j \quad (2.5g)$$

$$0 \leq (\bar{d}_{jc} - d_{jc}) \perp \bar{\mu}_{jc}^d \geq 0 : \forall j \in J, \forall c \in C_j \quad (2.5h)$$

$$\sum_{i \in I} \sum_{u \in U_i} p_{iu} - \sum_{j \in J} \sum_{c \in C_j} d_{jc} = 0. \quad (2.5i)$$

The proof is complete upon observing that the KKT optimality conditions obtained for the equilibrium model, given in (2.5), are identical to those derived from the market clearing model in (2.1), thereby demonstrating that the market clearing model (2.1) can be reinterpreted as an equilibrium of non-strategic agents in the market.

2.3.3 Strategic behavior of the agents in the market - Stackelberg Game

The aim of this section is to describe how strategic behavior can be mathematically modeled for agents that seek to act strategically in the market. For instance, a producer agent submits offers to the market; therefore, a central concern for the producer is determining the optimal values for these offers. Likewise, a consumer agent submits bids to the market, making the search for optimal bidding values a crucial concern for the consumer.

In this context, a Strategic Offering (SO) model is defined as a framework for determining optimal generation offers that maximize a producer's profit while maintaining its market position. Similarly, a Strategic Bidding (SB) model refers to a consumer-focused framework that calculates optimal purchase bids to maximize utility while preserving market share. These models enable strategic market participants to optimize their positions by balancing profit/utility objectives with competitive considerations. The design of SO and SB models must account for market-specific characteristics by incorporating multiple critical factors. These include demand patterns, competitor behavior, regulatory frameworks, and operational constraints to ensure comprehensive strategic decision-making.

A wide range of modeling approaches has been proposed in the literature to develop SO models. A fundamental aspect of formulating such a model lies in representing the behavior of a large strategic producer. If this prominent company adopts a strategic approach—meaning it intentionally manipulates the market clearing prices in the wholesale energy market to maximize its profits—it is referred to as a strategic company

(SC) or a price-maker. Conversely, if the producer company adopts a non-strategic approach—meaning it does not seek to influence market clearing prices to its advantage—it is categorized as a non-strategic company (NSC) or a price-taker.

A similar behavioral analysis applies to SB models used by consumer companies. However, this type of model is typically employed only when the electricity market features elastic demand, making consumers active participants in the market and enabling strategic behavior from the consumer agent's perspective.

The SO modeling approaches for Non-Strategic Producer Companies (NSCs) generally yield simpler models to solve compared to the modeling approaches for Strategic Producer Companies (SCs) (STEEGER; BARROSO; REBENNACK, 2014; MAZZI; KAZEMPOUR; PINSON, 2018). Two primary modeling approaches have been proposed in the literature for formulating the SO model for SCs: residual demand curves (GONZÁLEZ et al., 2017; CABANA et al., 2022) and bi-level optimization models (LIU; CONEJO, 2021; PRAT; CHATZIVASILEIADIS, 2023; LÊDO; NEPOMUCENO; CONEJO, 2024).

In SO approaches based on residual demand, a price-quota curve is first calculated. This curve provides the market clearing price (on the y -axis) corresponding to each power quota obtained by the strategic company (SC) (on the x -axis) in the market. The SC then uses this curve to determine its optimal offering strategies through a self-scheduling model (RENANI; EHSAN; SHAHIDEHPOUR, 2017; KHALOIE et al., 2020; CABANA et al., 2022). However, a key challenge with this approach lies in potential errors inherent in the residual demand curve, which can lead to inaccuracies in the calculation of optimal offers and profits. In contrast, the approach proposed in (CABANA et al., 2022) presents an improved method for mitigating such errors.

Another way to model strategic behavior is through a bilevel formulation. In this approach, SO and SB models are expressed as bilevel optimization models. Bilevel optimization allows an entire optimization problem to be embedded as a constraint within another optimization problem. The outer problem is referred to as the upper-level optimization model, while the embedded problem is known as the lower-level optimization model. In the context of strategic modeling, the upper-level model typically represents the strategic intentions of an agent aiming to maximize its own profit or utility in the market. The lower-level model, in turn, generally represents the day-ahead market clearing model, which, in most cases, aims to maximize social welfare.

In bilevel formulations, there is an inherent conflict of interest between the upper and lower levels (GABRIEL et al., 2013). The upper-level model aims to maximize the profit or utility of the strategic agent, while the lower-level model, which represents the

market clearing mechanism, seeks to maximize social welfare and remain impartial toward all market participants. Capturing this conflict of interest is a key aspect of bilevel modeling. This interaction can be interpreted through the lens of a Stackelberg game, where one player (the leader—the strategic agent in the upper level) moves first by announcing its strategy, and the other player (the follower—the market clearing mechanism in the lower level) responds accordingly. The leader in the upper level formulates its strategic move while anticipating the reaction of the follower in the lower level.

2.3.3.1 Stackelberg game and bilevel formulation

The aim of this section is to provide a clearer explanation of what a Stackelberg game is and how strategic agents can benefit from incorporating this approach into their own strategies. A Stackelberg game is characterized by a structure in which there is a leader and a follower, where the actions of one player directly influence the decisions of the other. The interaction between players in a Stackelberg game is illustrated below:

1. The leader moves first by choosing a strategy or action. This decision is made with full knowledge that the follower will observe and respond to it, such that the leader anticipates the follower's reaction when selecting his own strategy;
2. The follower observes the leader's decision and optimizes its own objective in response;
3. This process occurs until the Stackelberg equilibrium is reached and each problem is not solved independently from each other. In this state of equilibrium, both the leader and the follower have chosen their best strategies given the other's decision, respectively.

One way to capture the structure of a Stackelberg game and mathematically formulate the leader's strategy is through a bilevel optimization model. In these models, the upper-level problem represents the leader's decision-making process, including its variables, objective function, and relevant constraints. The lower-level problem represents the follower's optimization model, also including its objective function and constraints. From a broader perspective, the lower-level problem, representing the follower, acts merely as a constraint within the upper-level problem. In other words, the leader's model incorporates the follower's entire optimization model as part of its own constraint set.

Stackelberg games are mathematically described in the following sections for producers and consumers, resulting in SO models for producers and SB models for consumers.

2.3.3.2 Strategic producer offering problem

The aim of this section is to describe how a strategic producer can formulate a Strategic Offering (SO) model with the objective of maximizing its profit in an electricity market. Various approaches have been proposed in the literature to achieve this, but in this section, we focus on the Stackelberg game framework, formulated as a bilevel model, to describe an SO model for a strategic producer.

First, it is important to identify the leader and the follower in the Stackelberg game. From the strategic producer's standpoint, the producer acts as the leader. Therefore, its optimization problem constitutes the upper-level model in the bilevel formulation. On the other hand, the follower is the entire electricity market, which reacts to the leader's decisions. In this context, the Market Clearing (MC) model represents the lower-level model. The agents involved in the lower-level MC model include the strategic producer itself as well as the remaining non-strategic agents, i.e., non-strategic producers and consumers. The bilevel model presented in (2.6) captures the Stackelberg game from the perspective of a strategic producer, where (2.6a) defines the producer's profit maximization objective function.

$$\begin{aligned} \underset{o_{iu} \cup \{p_{ku}, d_{jc}\}}{Max} \quad & \sum_{u \in U_i} (\lambda p_{iu} - c_{iu} p_{iu}) \\ \text{s.t. :} \end{aligned} \quad (2.6a)$$

$$o_{iu} \geq c_{iu}, \quad \forall u \in U_i \quad (2.6b)$$

$$\begin{aligned} \underset{p_{ku}, d_{jc}}{Min} \quad & \sum_{k \in I} \sum_{u \in U_k} o_{ku} p_{ku} - \sum_{j \in J} \sum_{c \in C_j} b_{jc} d_{jc} \\ \text{s.t. :} \end{aligned} \quad (2.6c)$$

$$\sum_{k \in I} \sum_{u \in U_k} p_{ku} - \sum_{j \in J} \sum_{c \in C_j} d_{jc} = 0 : \quad (\lambda) \quad (2.6d)$$

$$0 \leq p_{ku} \leq \bar{p}_{ku} : \quad (\underline{\mu}_{ku}^p, \bar{\mu}_{ku}^p), \forall k \in I, \forall u \in U_k \quad (2.6e)$$

$$0 \leq d_{jc} \leq \bar{d}_{jc} : \quad (\underline{\mu}_{jc}^d, \bar{\mu}_{jc}^d), \forall j \in J, \forall c \in C_j \}. \quad (2.6f)$$

It is worth mentioning that the objective function in (2.6a) differs from the one presented in (2.2), which was formulated under a perfect competition context. The profit described in (2.6a) aligns with a strategic offering approach, where the producer's cost (c_{iu}) is explicitly included, and the offer (o_{iu}) becomes a decision variable, subject to constraint (2.6b), which requires that the strategic offer must be greater than or equal to the producer's own cost.

The upper-level problem is also constrained by the lower-level MC model, described from (2.6c) to (2.6f). The lower-level objective function, given in (2.6c), represents the social welfare function. The lower-level constraints include the supply-demand balance

constraint in (2.6d), as well as the upper and lower bounds on power production and consumption, given in (2.6e) and (2.6f), respectively.

The bilevel model (2.6) cannot be directly solved using commercial solvers due to its hierarchical structure. Therefore, it must be reformulated into an equivalent single-level model. To achieve this reformulation, the lower-level model—if convex—is typically replaced by its corresponding set of optimality conditions, such as the Karush-Kuhn-Tucker (KKT) conditions (LUO; PANG; RALPH, 1996), resulting in a Mixed Complementarity Problem (MCP), as shown in (2.5). The resulting single-level optimization model, obtained by replacing the lower-level problem with its KKT conditions in (2.6), is known as a Mathematical Program with Equilibrium Constraints (MPEC), which is presented in (2.7).

$$\begin{aligned} & \underset{o_{iu} \cup \{p_{ku}, d_{jc}\} \cup \{\lambda, \underline{\mu}_{ku}^p, \bar{\mu}_{ku}^p, \underline{\mu}_{jc}^d, \bar{\mu}_{jc}^d\}}{Max} \sum_{u \in U_i} (\lambda p_{iu} - c_{iu} p_{iu}) \\ & s.t. : \end{aligned} \quad (2.7a)$$

$$o_{iu} \geq c_{iu}, \quad \forall u \in U_i \quad (2.7b)$$

$$0 \leq p_{ku} \leq \bar{p}_{ku} : \forall k \in I, \forall u \in U_k \quad (2.7c)$$

$$o_{iu} - \lambda - \underline{\mu}_{iu}^p + \bar{\mu}_{iu}^p = 0 : \forall u \in U_i \quad (2.7d)$$

$$o_{ku} - \lambda - \underline{\mu}_{ku}^p + \bar{\mu}_{ku}^p = 0 : \forall k \in I : k \neq i, \forall u \in U_k \quad (2.7e)$$

$$0 \leq p_{ku} - \underline{\mu}_{ku}^p \geq 0 : \forall k \in I, \forall u \in U_k \quad (2.7f)$$

$$0 \leq (\bar{p}_{ku} - p_{ku}) - \bar{\mu}_{ku}^p \geq 0 : \forall k \in I, \forall u \in U_k \quad (2.7g)$$

$$0 \leq d_{jc} \leq \bar{d}_{jc} \forall j \in J, \forall c \in C_j \quad (2.7h)$$

$$-b_{jc} + \lambda - \underline{\mu}_{jc}^d + \bar{\mu}_{jc}^d = 0 : \forall j \in J, \forall c \in C_j \quad (2.7i)$$

$$0 \leq d_{jc} - \underline{\mu}_{jc}^d \geq 0 : \forall j \in J, \forall c \in C_j \quad (2.7j)$$

$$0 \leq (\bar{d}_{jc} - d_{jc}) - \bar{\mu}_{jc}^d \geq 0 : \forall j \in J, \forall c \in C_j \quad (2.7k)$$

$$\sum_{k \in I} \sum_{u \in U_k} p_{ku} - \sum_{j \in J} \sum_{c \in C_j} d_{jc} = 0. \quad (2.7l)$$

However, replacing the lower-level problem with its KKT optimality conditions results in numerous complementarity constraints. As a consequence, many undesirable bilinear terms appear in the MPEC formulation. From an optimization standpoint, bilinear terms are non-convex and non-regular, which significantly increases the difficulty of solving the MPEC problem. Additionally, from a computational perspective, these terms increase numerical instability and the computational burden. To address this issue, later in this work, in Chapter 4, we present an alternative set of optimality conditions that incorporates the primal constraints of the lower-level problem, its dual constraints, and a strong duality condition that enforces optimality between the primal and dual problems. This leads to an alternative MPEC formulation, which contains fewer bilinear terms and is easier to solve.

2.3.3.3 Strategic consumer bidding problem

Consumers in the electricity market can also behave strategically, and can benefit from a Stackelberg game structure as well. Similarly to the approach presented in section 2.3.3.2, in this section the strategic consumer formulates its Strategic Bidding (SB) model using bilevel formulation.

Now, from the perspective of a strategic consumer, the consumer is the leader, while the wholesale energy market is the follower in the Stackelberg game. Therefore, from the consumer perspective, the upper-level problem is represented by its utility maximization, while the lower-level problem is represented by the MC procedure model, as shown by problem (2.8):

$$\begin{aligned} \underset{b_{jc} \cup \{p_{iu}, d_{lc}\}}{Max} \quad & \sum_{c \in C_j} (u_{jc} d_{jc} - \lambda d_{jc}) \\ S.t. : \end{aligned} \quad (2.8a)$$

$$b_{jc} \leq u_{jc}, \quad \forall c \in C_j \quad (2.8b)$$

$$\begin{aligned} \underset{p_i, d_l}{Min} \quad & \sum_{i \in I} \sum_{u \in U_i} o_{iu} p_{iu} - \sum_{l \in J} \sum_{c \in C_l} b_{lc} d_{lc} \\ S.t. : \end{aligned} \quad (2.8c)$$

$$\sum_{i \in I} \sum_{u \in U_i} p_{iu} - \sum_{l \in J} \sum_{c \in C_l} d_{lc} = 0 : (\lambda) \quad (2.8d)$$

$$0 \leq p_{iu} \leq \bar{p}_{iu} : (\underline{\mu}_{iu}^p, \bar{\mu}_{iu}^p), \forall i \in I, \forall u \in U_i \quad (2.8e)$$

$$0 \leq d_{lc} \leq \bar{d}_{lc} : (\underline{\mu}_{lc}^d, \bar{\mu}_{lc}^d), \forall l \in J, \forall c \in C_l \}. \quad (2.8f)$$

We observe that the objective function in (2.8a) differs from the one presented in (2.3), which was derived under perfect competition conditions. In (2.3), the consumer bidding (b_{jc}) was treated as a fixed parameter equal to its maximum utility. In contrast, under strategic conditions in (2.8a), the consumer bidding becomes a decision variable that must be constrained to be less than or equal to its corresponding maximum utility (u_{jc}), as specified in constraints (2.8b).

We also observe that, as described for the bilevel SO problem for the producer, the lower-level problem in the bilevel SB model is likewise represented by the same MC model previously used in the producer problem. It features the same social welfare objective function, as shown in (2.8c), and is subject to the same power balance and bound constraints, given in (2.8d)–(2.8f).

Once again, the bilevel model (2.8) cannot be directly solved by commercial solvers due to its bilevel structure. Therefore, it must also be reformulated into an equivalent single-level model. To achieve this equivalent single-level formulation, the lower-level model

is replaced by its corresponding optimality conditions, following the same approach used in (2.6), resulting in the single-level MPEC shown in (2.9):

$$\begin{aligned} & \underset{b_{jc} \cup \{p_{iu}, d_{lc}\} \cup \{\lambda, \underline{\mu}_{iu}^p, \bar{\mu}_{iu}^p, \underline{\mu}_{lc}^d, \bar{\mu}_{lc}^d\}}{Max} \sum_{c \in C_j} (u_{jc} d_{jc} - \lambda d_{jc}) \\ & S.t. : \end{aligned} \quad (2.9a)$$

$$b_{jc} \leq u_{jc}, \quad \forall c \in C_j \quad (2.9b)$$

$$0 \leq p_{iu} \leq \bar{p}_{iu} : \forall i \in I, \forall u \in U_i \quad (2.9c)$$

$$o_{iu} - \lambda - \underline{\mu}_{iu}^p + \bar{\mu}_{iu}^p = 0 : \forall i \in I, \forall u \in U_i \quad (2.9d)$$

$$0 \leq p_{iu} \perp \underline{\mu}_{iu}^p \geq 0 : \forall i \in I, \forall u \in U_i \quad (2.9e)$$

$$0 \leq (\bar{p}_{iu} - p_{iu}) \perp \bar{\mu}_{iu}^p \geq 0 : \forall i \in I, \forall u \in U_i \quad (2.9f)$$

$$0 \leq d_{lc} \leq \bar{d}_{lc} \forall l \in J, \forall c \in C_l \quad (2.9g)$$

$$-b_{jc} + \lambda - \underline{\mu}_{jc}^d + \bar{\mu}_{jc}^d = 0 : \forall c \in C_l \quad (2.9h)$$

$$-b_{lc} + \lambda - \underline{\mu}_{lc}^d + \bar{\mu}_{lc}^d = 0 : \forall l \in J : l \neq j, \forall c \in C_l \quad (2.9i)$$

$$0 \leq d_{lc} \perp \underline{\mu}_{lc}^d \geq 0 : \forall l \in J, \forall c \in C_l \quad (2.9j)$$

$$0 \leq (\bar{d}_{lc} - d_{lc}) \perp \bar{\mu}_{lc}^d \geq 0 : \forall l \in J, \forall c \in C_l \quad (2.9k)$$

$$\sum_{i \in I} \sum_{u \in U_i} p_{iu} - \sum_{l \in J} \sum_{c \in C_l} d_{lc} = 0. \quad (2.9l)$$

However, as already mentioned, replacing the lower-level problem with its KKT optimality conditions results in numerous complementarity constraints, introducing many undesirable bilinear terms into the MPEC formulation. As also noted, this issue is addressed later in this work, in Chapter 4, where we present an alternative set of optimality conditions. This leads to an alternative MPEC formulation that contains fewer bilinear terms and is easier to solve.

2.3.4 Equilibrium of competitive agents in the market - Nash Equilibrium

In a market context where agents behave strategically, a problem arises: how can regulators analyze the strategic behavior of agents, assuming that all agents choose to act strategically? Solving this problem allows for the identification of market power among agents, providing a tool to mitigate it. Additionally, agents that are already acting strategically may also wish to model the strategic behavior of their competitors in order to formulate a stronger (i.e., more reliable) strategy. Therefore, mathematically formulating the equilibrium among all strategic agents (i.e a Nash equilibrium) can be a valuable tool for these market participants.

A Nash equilibrium is typically formulated as a system of optimization problems, where each player's problem depends on the strategies of the other players. These problems are solved jointly, often using their respective optimality conditions (e.g., KKT conditions for convex problems). In cases involving hierarchical decision-making (e.g., multi-leader-follower games), the Nash equilibrium may involve the joint solution of MPECs, leading to an Equilibrium Problem with Equilibrium Constraints (EPEC). Different EPEC structures can be identified, depending on how the bilevel models of the strategic agents constituting the EPEC are formulated.

A multi-leader–multi-follower game structure is associated with EPECs that incorporate bilevel models of different strategic agents, where each leader formulates a distinct follower problem in its respective lower-level model. Figure 2.2 provides a graphical illustration of this structure.

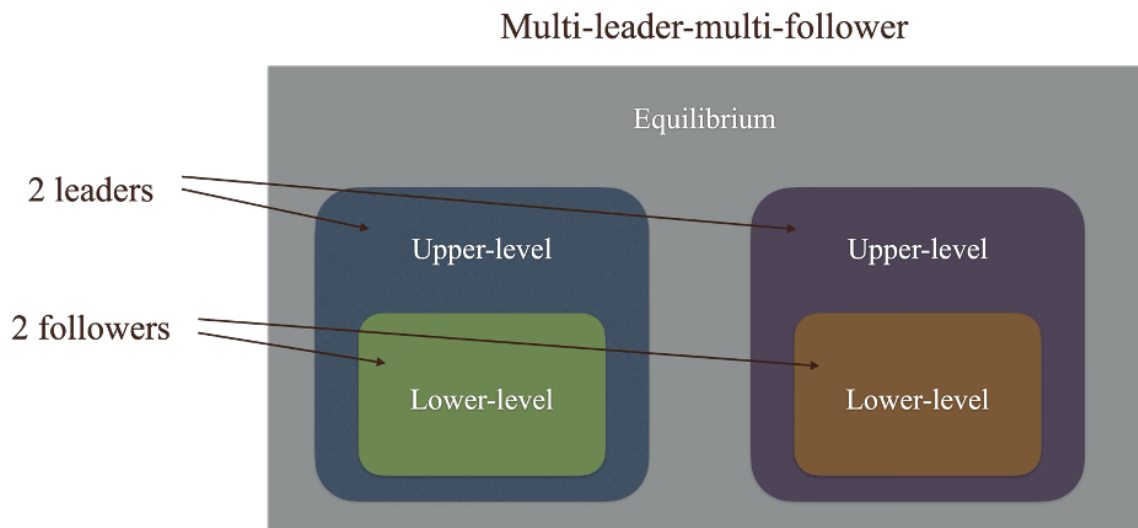


Figure 2.2 – Graphical Illustration of a multi-leader–multi-follower game structure within an Equilibrium Problem with Equilibrium Constraints (EPEC).

The multi-leader–single-follower game structure is associated with EPECs that encompass bilevel models of different strategic agents, where each leader formulates the same follower problem in their respective lower-level models. Figure 2.3 provides a graphical illustration of this structure.

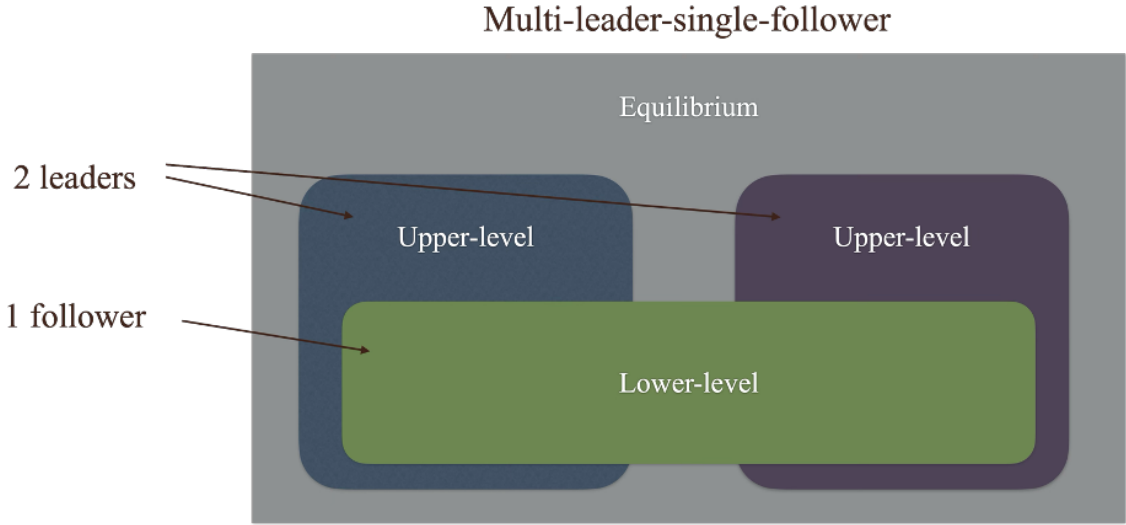


Figure 2.3 – Graphical Illustration of a multi-leader–single-follower game structure within an Equilibrium Problem with Equilibrium Constraints (EPEC).

Regardless of the specific EPEC game structure aforementioned, the EPEC resulting from the joint solution of MPECs associated with producers and consumers in a wholesale energy market may lead to an intractable or highly complex problem to solve. Numerous approaches have been proposed in the literature for finding equilibrium points and verifying whether the obtained solutions indeed correspond to a Nash equilibrium.

One of the most common methods for solving EPECs is the diagonalization approach (HOBBS; METZLER; PANG, 2000), (KARDAKOS; SIMOGLU; BAKIRTZIS, 2013), (NAEBI et al., 2020). In this method, an iterative sequence of MPECs is solved for each firm $i \in I$, while the offers/bids of the remaining players $r \in I : r \neq i$ are held fixed using the best available information from the current iteration. This procedure is repeated until no firm is willing to change its offers/bids. However, the diagonalization method has several drawbacks that have been identified in the literature: (i) the method may not converge to a stable solution (divergence); (ii) depending on the order in which the MPECs are solved, different equilibrium points may be reached; and (iii) the upper-level constraints of the remaining players are not considered when solving the MPEC of a specific player, which may lead to infeasibility in the upper-level problems of the remaining players due to their fixed decisions.

Another possible approach to solving the equilibrium involves replacing the MPECs that constitute the EPEC with their respective optimality conditions. However, to employ this approach, the equilibrium models must be convex and have regular constraints. In practice, MPECs in equilibrium are typically neither convex nor regular. As a result, replacing the MPECs with their optimality conditions may lead to solutions that do

not actually represent a true equilibrium. Therefore, it becomes necessary to verify if such solutions are effectively Nash equilibria. This is generally accomplished by using the diagonalization method, as shown in (CONEJO; RUIZ, 2020).

Later in this work, in Chapter 4, a novel method is presented for finding a Nash Equilibrium among strategic agents in a wholesale energy market. The proposed solution approach involves the use of relaxations, convexifications, and problem reformulation through rotated quadratic cones, such that each MPEC is rewritten as a convex and regular problem, also admitting an exact dual representation. The resulting single-level, single-problem EPEC can then be solved using standard nonlinear solvers.

2.4 Multi-period and network transmission system representation in electricity markets

An important aspect to consider in a market clearing model is the transmission network and the discretization of time periods. Section 2.4 aims to demonstrate how the transmission system impacts price formation in electricity supply and how hourly price formation occurs in a wholesale day-ahead electricity market.

2.4.1 Price clearing in network-constrained systems

Representing the transmission network in market clearing (MC) procedures is important, as power supply and demand are not located at a single bus but are distributed across multiple interconnected buses. The main purpose of the transmission network is to connect power supply points to load points. Incorporating the transmission system into MC models therefore involves representing constraints that are intrinsically associated with the network, such as: enforcing power balance at each bus and accounting for capacity limits on lines, transformers, and other components.

In MC procedures, modeling aspects related to voltage and reactive power are generally neglected, as the primary objective is the allocation of active power. This simplification is possible due to the decoupling between active and reactive power, which characterizes transmission systems and can be exploited in modeling.

Initially, the power balance constraint from (2.1b) should be reformulated as shown in (2.10), where the first two terms represent power generation and demand in bus n , respectively, while the third term captures the linearized active power flow from bus n to its neighboring buses:

$$\sum_{i \in I_n} \sum_{u \in U_i} p_{iu} - \sum_{j \in J_n} \sum_{c \in C_j} d_{jc} - \sum_{m \in N_n} B_{nm} (\theta_n - \theta_m) = 0 : (\lambda_n), \forall n \in N, \quad (2.10)$$

where N denotes the set of network buses and N_n is a set of buses directly connected to bus n , thus identifying its neighborhood buses. Each pair of directly connected buses $n - m$ represents a system branch, which may consist of either a transmission line or a transformer. The parameter B_{nm} is the network susceptance of the branch $n - m$. The variable θ_n denotes the voltage phase angle at bus n . Finally, the dual variables associated with constraints (2.10) are indicated in parenthesis.

It is also important to represent the capacity limits of the transmission system. Constraints (2.11) represent the flow limits on the transmission lines/transformers:

$$-\bar{P}_{nm} \leq B_{nm} (\theta_n - \theta_m) \leq \bar{P}_{nm} : (\underline{\alpha}_{nm}, \bar{\alpha}_{nm}), \forall n \in N, \forall m \in N_n, \quad (2.11)$$

where $\bar{P}_{nm}$ represents the power capacity limit of branch $n - m$. The dual variables associated with these constraints are indicated in parentheses.

Establishing a reference angle for all phasors used in the transmission system formulation is also important. Constraint (2.12) designates bus φ as the slack bus, where the voltage angle is set to zero:

$$\theta_\varphi = 0 : (\beta^0). \quad (2.12)$$

Note that the dual variable associated with the power balance constraint at bus n , denoted by λ_n , represents the cleared price at that bus. Therefore, once the power balance is reformulated to include the network representation, the cleared prices may differ across the network buses n —these are known as nodal prices. Two major factors can lead to varying nodal prices: system losses (when represented in the market-clearing model) and congestion. Congestion occurs on a system branch $n-m$ when its associated constraint (2.11), which describes the power flow limit on that branch, becomes active. In such cases, the prices at buses n and m tend to diverge. The impacts of congestion and non-congestion on system prices are numerically evaluated in Chapter 3.

Finally, the transmission system representation can be modeled for each time period of the market-clearing (MC) model to reflect the demand duration curve throughout the day. This leads to a multi-period MC model, which is discussed in the following section.

2.4.2 Multi-period price formation

During the 24 one-hour periods of the day, the demand duration curve can vary significantly. In the MC model, this variation is captured by the accepted power demand in each period. To accommodate the varying demand, power supply is also adjusted in the MC model—through the accepted power generation in the market—as well as the power flows across the transmission system. Additionally, some constraints, known

as intertemporal constraints, link decisions between consecutive periods. Examples of intertemporal constraints include ramp rate limits and minimum up and down time requirements for thermal or hydro generation units.

For all these reasons, it is important that the MC model represents decisions for each period $t \in T$. This is achieved by introducing an additional index t to the variables and parameters in the single-period MC model (2.1), resulting in a multi-period MC model, as described in (2.13), which also incorporates the transmission system representation:

$$\begin{aligned} \text{Min}_{p_{tiu}, d_{tjc}} \quad & \sum_{t \in T} \sum_{i \in I} \sum_{u \in U_i} o_{tiu} p_{tiu} - \sum_{t \in T} \sum_{j \in J} \sum_{c \in C_j} b_{tjc} d_{tjc} \\ \text{s.t. :} \quad & \end{aligned} \quad (2.13a)$$

$$\sum_{i \in I} \sum_{u \in U_{in}} p_{tiu} - \sum_{j \in J} \sum_{c \in C_{jn}} d_{tjc} - \sum_{m \in N_n} B_{nm} (\theta_{tn} - \theta_{tm}) = 0 : (\lambda_{tn}), \forall t \in T, \forall n \in N \quad (2.13b)$$

$$0 \leq p_{tiu} \leq \bar{p}_{tiu} : (\underline{\mu}_{tiu}^p, \bar{\mu}_{tiu}^p), \forall t \in T, \forall i \in I, \forall u \in U_i \quad (2.13c)$$

$$0 \leq d_{tjc} \leq \bar{d}_{tjc} : (\underline{\mu}_{tjc}^d, \bar{\mu}_{tjc}^d), \forall t \in T, \forall j \in J, \forall c \in C_j. \quad (2.13d)$$

$$-\bar{P}_{nm} \leq B_{nm} (\theta_{tn} - \theta_{tm}) \leq \bar{P}_{nm} : (\underline{\alpha}_{tnm}, \bar{\alpha}_{tnm}), \forall t \in T, \forall n \in N, \forall m \in N_n \quad (2.13e)$$

$$\theta_{t\varphi} = 0 : (\beta_t^0). \quad (2.13f)$$

In model (2.13), it is noticeable that all parameters and variables varies across time - as well as across the buses. Therefore, the dual variables λ_{tn} associated to the power supply and demand balance constraint in (2.13b) - that represents the cleared price - also varies across the time and buses, indicating that now we have hourly nodal prices.

It is also worth mentioning that representing multiple periods in the MC model is not only important for capturing the multi-period nature of electricity markets—where, in a basic setup, each period is cleared independently of the others—but also because, depending on the characteristics of the power system where the model is implemented, additional intertemporal constraints must be considered.

For instance, thermal power generation involves ramp rate constraints that define the maximum permissible change in power output that a generation unit can accommodate between consecutive periods. These time-coupled decisions inherently introduce an intertemporal aspect to the MC process, which can only be properly represented through a multi-period MC approach. Similarly, in hydro-power systems, a set of water balance constraints must be incorporated for each reservoir, introducing intertemporal coupling of reservoir volumes across successive time intervals. These and other relevant modeling aspects related to hydrothermal power systems are discussed in further detail in Section 2.5, as follows.

2.5 Electricity markets in hydrothermal power systems

In this section, we present an overview of key aspects related to hydro generating units (CICOONA, 1999a), along with their corresponding mathematical modeling features, discussed in detail in Section 2.5.1. Additionally, we introduce general characteristics of thermal generating units (CICOONA, 1999a) and describe their modeling framework in Section 2.5.2. The constraints associated with hydrothermal power systems must be properly incorporated into the MC model implemented in such systems.

2.5.1 Hydro-power generation

Hydroelectric power is one of the many abundant renewable energy sources available in certain regions of the world. This energy source harnesses the hydraulic potential of water and converts it into electricity. To utilize this potential energy, a water reservoir—such as an artificial lake or a dam—is typically constructed to concentrate the energy stored in the water. At the dam, water accumulated in the reservoir is directed under pressure through conduits leading to the powerhouse, where it flows through turbines to generate electricity.

Depending on the water storage management in a hydro plant, we may classify it as a run-of-the-river or a reservoir-based plant. Run-of-the-river plants rely on the natural, immediate flow of a river with minimal or no water storage, meaning their power output closely follows the river’s real-time discharge, making them more susceptible to seasonal and weather-related fluctuations. In contrast, reservoir plants use large artificial lakes or dams to store water, allowing for greater control over when and how much electricity is generated. This storage capability enables reservoir plants to respond more flexibly to variations in electricity demand and provides the advantage of energy generation even during dry periods. However, reservoir plants typically require more complex infrastructure and have greater environmental and social impacts due to the flooding of large areas. Figure 2.4 shows graphical representation for run-of-river reservoir-based hydro plants.

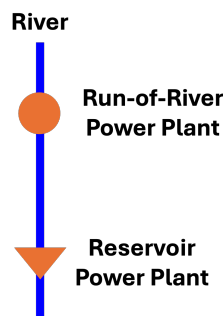


Figure 2.4 – Graphical representation of run-of-river and reservoir-based hydro plants.

Setting upper limits for the volume of a hydro reservoir is crucial to ensure its

structural safety, prevent flooding, and maintain operational flexibility. Exceeding the reservoir's maximum capacity can lead to over-topping, which poses serious risks to the integrity of the dam and downstream communities. From an operational perspective, maintaining a buffer below the upper level allows room to absorb unexpected inflows due to heavy rainfall or rapid snow-melt (which occurs in the Nord countries, for instance), thereby reducing the risk of emergency spills. Additionally, respecting maximum limits helps optimize power generation efficiency by preventing the need for frequent and uncontrolled water releases, which may not contribute to energy production and can disrupt downstream ecosystems.

On the other hand, establishing lower limits for the volume of a hydro reservoir is also essential to ensure the safe and efficient operation of the hydroelectric plant, protect environmental conditions, and support other water uses. From a technical standpoint, a minimum water level is required to maintain sufficient hydraulic head for turbine operation and to avoid issues like cavitation, which can damage equipment. Environmentally, it helps preserve aquatic ecosystems and maintain water quality by preventing stagnation. Operationally, lower limits ensure that water remains available for multiple uses such as irrigation, drinking water, recreation, and long-term drought resilience. Overall, these constraints are critical for maintaining the reliability and sustainability of hydroelectric systems.

Upper and lower limits can be imposed on the volume x_{tr}^G for each reservoir $r \in R$ and each period $t \in T$, as shown in (2.14):

$$\underline{X}_r^G \leq x_{tr}^G \leq \overline{X}_r^G, \forall t \in T, \forall r \in R \quad (2.14)$$

where $\underline{X}_r^G$ and $\overline{X}_r^G$ are the lower and upper limits, respectively.

The portion of water that flows through the turbines to generate electricity is called turbined outflow. This flow is directed through penstocks to the powerhouse, where it spins the turbine blades, converting the water's potential and kinetic energy into mechanical energy, which is then transformed into electrical energy by a generator. The turbined outflow q_{th}^G of each hydro unit $h \in H$ is subject to both upper and lower limits, as shown in (2.15):

$$\underline{Q}_r^G \leq q_{th}^G \leq \overline{Q}_r^G, \forall t \in T, \forall h \in H, \quad (2.15)$$

where $\underline{Q}_r^G$ and $\overline{Q}_r^G$ are the lower and upper limits, respectively.

It is important to highlight that in a more accurate modeling of the upper limits of the turbined outflow, the limits of both the turbine and the generator must be considered. In this approach, the upper limits for the turbined outflow are expressed as a non-differentiable

function of the net head (a concept explained below) (CICOONA, 1999b). Due to its non-differentiable nature, this type of modeling is generally not used in MC models.

The upper limit in the turbined outflow is determined by the maximum turbine capacity, which reflects the highest flow rate the turbines can handle safely and efficiently. The lower limit in the turbined outflow is a minimum operational flow requirement to maintain turbine functionality or meet environmental flow constraints downstream (e.g., to preserve aquatic habitats).

A hydro plant structure also includes a spillway, which is designed to safely release excess water from the reservoir when it reaches its maximum storage capacity. Its primary function is to prevent over-topping of the dam, which could lead to structural damage or failure, helping to maintain the integrity and safety of the plant, especially during flooding seasons. The spillway are also used to guarantee the navigability of the river downstream in a situation where the water release upstream cannot be turbined due to the lack of electricity demand to be supplied, making the spillway accountable for releasing water downstream without generating electricity. The water flow in a spillway (s_{tr}^G) for each reservoir r and period t has both upper and lower limits, as shown in (2.16):

$$\underline{S}_r^G \leq s_{tr}^G \leq \overline{S}_r^G, \forall t \in T, \forall r \in R, \quad (2.16)$$

where $\underline{S}_r^G$ and $\overline{S}_r^G$ are the lower and upper limits, respectively.

The upper limit is defined by the design capacity of the spillway, once the structure is constructed to pass a maximum flow rate, often based on extreme weather events like the probable maximum flood. Exceeding this limit could cause structural damage to the spillway or dam. The lower limit, while less critical, may be governed by environmental regulations or operational policies that require a minimum flow to sustain downstream ecosystems or meet legal water usage requirements. However, under normal conditions, if the reservoir level does not reach the spillway threshold, there may be no flow through it at all.

The total release from a hydro reservoir (u_{tr}^G) is defined as the sum of the turbined outflow and the spillage, (s_{tr}^G) for each reservoir r and period t , as shown in (2.17)

$$u_{tr}^G = \sum_{w \in W_r} q_{tw}^G + s_{tr}^G, \forall t \in T, \forall r \in R, \quad (2.17)$$

where W_r is the set of hydro units in the hydro plant r .

The total release must also be subject to upper and lower limits, as shown in (2.18). The upper limit is determined by the physical capacity of the hydraulic structures, i.e. by the maximum flow the turbines and spillway combined can safely handle, or by other

reasons such as environmental constraints concerning the water usage awareness. The lower limit is often a minimum release requirement, especially for environmental purposes, such as maintaining ecological flows in the downstream river, supporting aquatic life, and meeting legal or regulatory obligations.

$$\underline{U}_r^G \leq u_{tr}^G \leq \overline{U}_r^G : (\delta_{tr}^U, \bar{\delta}_{tr}^U), \forall t \in T, \forall r \in R \quad (2.18)$$

Water inflows into a hydro plant or reservoir can be classified as natural or incremental. Natural inflow refers to the total volume of water that naturally enters a reservoir or hydro plant from all upstream sources. Incremental inflow is the portion of the natural inflow that excludes any turbinized outflows or spillage from upstream reservoirs. Therefore, from an operational perspective, a hydro plant can control its turbinized outflows and spillage but cannot control the incremental inflow arriving at its reservoir.

An important concept in hydro power generation is the hydro production function, which defines the direct relationship between the turbinized outflow (q_{tw}^G), and the corresponding power output, denoted as p_{tw}^G . This function introduces three key components necessary to convert turbinized water flow into electrical energy. The first is the hydraulic net head, denoted by H_r^G , which represents the net distance between the upstream and downstream water levels. Depending on the level of modeling detail, H_r^G may be treated as a variable or a fixed parameter, in a more detailed modeling, it is advisable to consider H_r^G as a variable since it varies depending on the reservoir's volume which is directly correlated to the total water release. The second component is the conversion constant k_w^G , which encapsulates the product of gravitational acceleration and the specific weight of water. The third is the turbine efficiency, denoted by η_w^G , which reflects the average efficiency of the turbine in converting hydraulic energy into electrical energy. Together, these elements form the hydro production function, as expressed in equation (2.19):

$$p_{tw}^G = k_w^G \eta_w^G H_r^G q_{tw}^G, \forall t \in T, \forall r \in R, \forall w \in W_r. \quad (2.19)$$

The power output of each hydro unit calculated by (2.20) is subject to the power output limits of the units, as shown in (2.20):

$$\underline{E}_w^G \leq p_{tw}^G \leq \overline{E}_w^G, \forall t \in T, \forall w \in W. \quad (2.20)$$

Analogous to the discussion regarding the upper limit of the turbinized outflow, it is worth noting that a more accurate modeling of the power output limit exists, in which both turbine and generator limits are considered. In this approach, the upper limit of the power output is represented as a non-differentiable function of the net head (CICOONA,

1999b). However, this type of modeling is generally not used in MC models due to its complexity and non-differentiable nature.

Another important aspect of hydro power production is the water balance at the power plants in a hydro cascade, which rules the bodies of water throughout the entire watershed over time. The water balance equations are fundamental constraints that ensure the correct tracking of water volume in a hydro reservoir over time. These equations mathematically represent the conservation of mass principle, stating that the change in reservoir volume from one period to the next must equal the inflow minus the outflow. The water balance equation is shown in (2.21) for $t = 1$ and (2.22), for $t \neq 1$.

$$x_{1r}^G = X_{0r}^G + C \left(\Upsilon_{1r} + \sum_{f \in F_r^G} u_{1f}^G - u_{1r}^G \right), \forall r \in R \quad (2.21)$$

$$x_{tr}^G = x_{(t-1)r}^G + C \left(\Upsilon_{tr} + \sum_{f \in F_r^G} u_{tf}^G - u_{tr}^G \right), \forall t \in T : t \neq 1, \forall r \in R, \quad (2.22)$$

where X_{0r}^G is the initial volume of the reservoir in period $t = 0$, C is a parameter which transforms the power flow in m^3/s to water volume in hm^3 , Υ_{tr} is the incremental inflow to the plant r and F_r^G is the set of plants directly upstream plant r . Depending on how detailed those constraints are, they can also encompass the outflow of water caused by natural aspects such as water evaporation or a miscellaneous usage of water.

The total inflows to the reservoir r in (2.21) and (2.22) consist of the incremental inflow Υ_{tr} and the total water released from the plants located upstream of plant r in the cascade. The total outflow consists of the total release from the reservoir r .

Finally, it is important that the decisions regarding power dispatch and hydrothermal operation policies obtained from short-term hydrothermal scheduling models are coordinated with medium- or long-term scheduling models. The coordination of MC models with medium-term hydrothermal scheduling models is discussed in (BREGADIOLI et al., 2016), where three basic decomposition approaches are described for this purpose. The primal coordination approach, described in (BREGADIOLI et al., 2016) and adopted in the MC models in this work, involves setting minimum target values for the reservoir volumes at the last period LT of the MC model, as shown in (2.23):

$$x_{tr}^G \geq X_r^{Goal}, t = LT, \forall r \in R. \quad (2.23)$$

In the next section, a summary of thermal power generation is provided, with focus on the constraints used in the MC models in this work.

2.5.2 Thermal-power generation

A thermal power generation plant, by definition, converts thermal energy into electrical energy using a thermal generator. These plants can operate using a variety of fuels, including coal, oil, natural gas, biomass, wood, or, in the case of nuclear power plants—which are also classified as thermal power plants—enriched uranium. Moreover, the type of fuel and the method by which it is burned influence the specific design and technology of the thermal power plant. As a result, different types of generators are used in thermal power plants, including steam turbines, gas turbines, and systems based on direct combustion.

Starting with steam turbines, these systems operate by burning fuel to heat a boiler, which boils water to produce steam. The high-pressure steam is then directed to spin a turbine, thereby generating electricity. Gas turbines, on the other hand, work by first compressing air, which is then mixed with fuel in a combustion chamber. The resulting combustion produces high-temperature, high-pressure gases that expand rapidly and spin the turbine. Lastly, in direct combustion systems—such as internal combustion engines—the fuel is burned directly in a piston engine. The mechanical energy from the pistons is then converted into electrical energy via a connected generator.

With the exception of direct combustion systems, most thermal power plants—such as those using steam or gas turbines—require a certain amount of time to begin generating electricity after startup. This delay must be considered when modeling the operational constraints of thermal plants. One common way to represent this behavior is by incorporating ramp rate constraints. Ramp rates define the maximum and minimum amount by which a thermal generator's output can increase or decrease between consecutive time periods. Therefore, for a thermal unit, the difference between the power output in the next period and the current period, $p_{t+1}^G - p_t^G$ must be less than or equal to the upward ramp rate R_l^{Gup} , as shown in (2.24):

$$p_{(t+1)l}^G - p_{tl}^G \leq R_l^{Gup}, \forall t \in T : t \neq LT, \forall l \in L, \quad (2.24)$$

where L is the set of thermal units.

Also, the difference between the power output in the current period and the next period, $p_t^G - p_{t+1}^G$ must be less than or equal to the downward ramp rate R_l^{Gdw} , as shown in (2.25):

$$p_{tl}^G - p_{(t+1)l}^G \leq R_l^{Gdw} : \left(\gamma_{tl}^{Gdw} \right), \forall t \in T : t \neq LT, \forall l \in L. \quad (2.25)$$

Another important operational constraint associated to the thermal-power plants to the context of unit commitment and market clearing models are the minimum up and

down time constraints. The minimum up time constraint describes the minimum time up of a thermal unit that once started (turned on) and it must be online for a minimum number of consecutive periods before it can be turned off again. Such pattern happens because frequent startup and shutdowns are technically stressful, costly, and inefficient for thermal-power plants. On the other hand, the minimum down time describes the opposite, once a unit is shut down, it must remain offline for a minimum number of consecutive periods before it can be turned on again. This happens because restarting thermal-power plants immediately after shutdown is technically challenging and sometimes not feasible. For instance, a nuclear-power plant might require at least 12 hours of downtime before it can restart.

Typically, the minimum up and down times are formulated using Mixed-Integer Linear Programming (MILP), where the binary variables $u_{tl}, v_{tl}, w_{tl} \in \{0, 1\} : \forall i \in T, \forall l \in L$ represents the on/off status, the startup indicator and the shutdown indicator respectively. The startup and shutdown is described in 2.26 bellow:

$$v_t = u_t - u_{t-1} : \quad \forall i \in T, \forall l \in L, \quad (2.26a)$$

$$w_t = u_{t-1} - u_t : \quad \forall i \in T, \forall l \in L. \quad (2.26b)$$

Which the minimum up time is T_l^{Gup} , where if the unit starts up at time t , it must stay on for at least T_l^{Gup} periods, detailed as follows in 2.27:

$$\sum_{k=t}^{t+T_{kl}^{Gup}-1} u_{kl} \geq T_l^{Gup} v_{tl} : \quad \forall i \in T, \forall l \in L. \quad (2.27)$$

Similarly, the minimum down time is T_l^{Gdw} , where if the unit shuts down at time t it must stay off for at least T_l^{Gdw} periods, detailed as follows in 2.28:

$$\sum_{k=t}^{t+T_{kl}^{Gdw}-1} (1 - u_{kl}) \geq T_l^{Gdw} v_{tl} : \quad \forall i \in T, \forall l \in L. \quad (2.28)$$

However, since the minimum up and down time constraints require binary variables, which introduce discontinuity and non-convexity into the problem — thereby adding an additional layer of complexity to the context of a bilevel formulation — this prevents the use of KKTs and PDOCs optimality conditions as a technique to replace the lower-level of a bilevel model with its set of optimality condition constraints, in order to achieve a single-level MPEC model equivalent to the original bilevel formulation. For this reason,

we do not consider this set of operational constraints for thermal power plants in the remainder of this work.

In the next chapter, the hydrothermal generation concepts discussed in this Chapter 2 are applied to model a bilevel strategic offering framework for a generation company operating in a hydrothermal energy market.

3 SOLVING A BILEVEL STRATEGIC OFFERING MODEL FOR A GENERATION COMPANY IN A HYDROTHERMAL ENERGY MARKET: A NEW CONVEXIFICATION APPROACH

3.1 Introduction

A strategic generation company aims to maximize its profit through strategic offers in the day-ahead wholesale energy market (TORRE; CONTRERAS; CONEJO, 2004) (ALHARBI; BHATTACHARYA, 2020). To achieve this, the company employs strategic offering (SO) models, which are designed to optimize the company's profits while enforcing physical and operational constraints of the company's generating units, and accurately representing its active engagement with the market. The interaction between the strategic company (SC) and the market is typically captured by two modeling approaches: a bilevel modeling based approach (MOISEEVA; HESAMZADEH, 2018) and modeling approaches which work with residual demand curves (CABANA et al., 2022).

The bilevel modeling based approach is a classic and well-documented framework, where the upper-level model optimizes the SC's profits, while the lower-level model embodies a market clearing (MC) procedure (HOBBS; METZLER; PANG, 2000; RUIZ; CONEJO, 2009; KAZEMPOUR; CONEJO; RUIZ, 2012; DAI; QIAO, 2017). Another form of capturing the interaction between the SC and the market is by means of residual demand curves (POUSINHO; CATALÃO; CONTRERAS, 2012; POUSINHO et al., 2013; CABANA et al., 2022). This work focuses on bilevel SO models.

MC procedures may differ depending on the market design. Some energy markets adopt purely economic bidding information (simple bidding structure), involving only price and quantity block offers. An example of such a market structure is the one employed in most European markets, whose MC procedure is detailed in (KOLTSAKLIS; ZENGİNİS; DAGOUMAS, 2020). In the MC models employed for such a market structure, detailed generation constraints (i.e. intertemporal constraints as well as start-up and shut-down

offer costs) and network constraints are generally not considered. On the other hand, some markets adopt a complex bidding structure, which integrates data regarding detailed generation and network constraints. An example of such a market structure is the California ISO in the US.

If a simple bidding structure is employed, ex-post actions may be necessary to account for generation and network feasibility. These ex-post actions are derived using heuristic rules, possibly degrading the optimality of the MC solution. On the other hand, when complex bidding information is used, such heuristic rules are avoided, since detailed generation and network constraints are explicitly represented in the MC procedure. In this work, we assume that the market operator adopts a complex bidding structure, so that the MC model explicitly handle these detailed generation and network constraints, minimizing ex-post corrective actions. In such a market structure, start-up costs and intertemporal constraints are employed for thermal units such as ramp rates, minimum up and down times, as well as network constraints (ARROYO; CONEJO, 2002; MOTTO et al., 2002). Also, if such a market structure is employed for hydrothermal systems where hydro generation is dominant (such as the Brazilian power system), hydro constraints are also considered in the MC procedure, such as: bounds on hydro variables, water release ramp rates, water balance constraints, hydro power production functions, etc. (BREGADIOLI et al., 2016; PEREIRA et al., 2017; SANTOS et al., 2020).

Most SO models used by SC in day-ahead markets are specifically tailored for thermal-dominated systems (DAI; QIAO, 2015; DAI; QIAO, 2017; RUIZ; CONEJO, 2009), such that the representation of hydro constraints is neglected in their lower-level MC models. Such SO models are not suitable for a generation company which has a large share of hydro generation or a company which is operating in a hydro-dominated system. Some works attempted to represent hydro constraints within the SO model (POUSINHO; CATALÃO; CONTRERAS, 2012; POUSINHO et al., 2013; CRUZ et al., 2016; MOISEEVA; HESAMZADEH, 2018). A review of the literature on this topic, covering works up to 2014, can be found in (STEEGER; BARROSO; REBENNACK, 2014). The authors in (POUSINHO; CATALÃO; CONTRERAS, 2012; POUSINHO et al., 2013) use a residual demand curve within their SO model, which is not the focus of this work. In (CRUZ et al., 2016), the authors propose an equilibrium model for hydro-dominated systems, where hydro constraints are considered only in the upper-level of their SO model. This model also neglects thermal and hydro constraints as well as network representation in the lower-level model. In (MOISEEVA; HESAMZADEH, 2018), although hydro constraints are considered in the lower-level model, they are considered only for the SC, so that spatial and temporal hydro interrelations within hydro cascades cannot be captured.

Methods for solving bilevel SO models typically involve recasting the problem

as a single-level model, where the lower-level model is replaced by an equivalent set of constraints. Most works use KKT first order conditions of the lower-level model as such equivalent set of constraints. The equivalent single-level model that results from this recasting is termed as Mathematical Problem with Equilibrium Constraint (MPEC) (LUO; PANG; RALPH, 1996; GABRIEL et al., 2013). The solution of SO models reformulated as an MPEC is complicated by the presence of complementarity constraints, which may not meet certain constraint qualification criteria. In such context, complementarity constraints have been handled in the vast majority of the literature by the big-M approach proposed in (FORTUNY-AMAT; MCCARL, 1981b). However, some authors have pointed out deficiencies in such approach (PINEDA; MORALES, 2019; KLEINERT et al., 2020; KLEINERT; SCHMIDT, 2023), which involve finding an appropriate big-M value that does not cut-off feasible or optimal points. According to (KLEINERT et al., 2020), finding a suitable big-M can be as hard as solving the original bilevel problem. The authors in (PINEDA; BYLLING; MORALES, 2018) attempt to overcome this problem using heuristic approaches to reduce the computational burden of solving the big-M reformulation of the MPEC. Some other approaches overcome this problem using branch-and-bound methods, which are used for branching on the complementarity constraints of the lower-level problem's optimality conditions (KLEINERT et al., 2021). Finally, the authors in (LAWS; HANASUSANTO, 2023) propose a linearization approach for bilinear terms in the upper-level problem.

In the work described in this section, we propose a new solution approach for the bilevel SO problem. This approach employs an alternative set of constraints to obtain the optimality conditions of the lower-level model, instead of using the KKT conditions. In the proposed approach, the lower-level model is replaced by a set constraints involving: i) the primal constraints associated with the lower-level model; ii) the dual constraints associated with the lower-level model and iii) an equality constraint representing the strong duality theorem for the lower-level model. This alternative set of constraints are proven to assure optimality for linear lower-level models in (KLEINERT et al., 2021). The single-level MPEC that results from this recasting still presents some bilinear terms, which are associated with the strong duality equality constraint and also with the SC revenue in the objective function of the upper-level problem.

Constraints involving bilinear terms are difficult to solve since they may not satisfy strong constraint qualification, such as the linear independence. In some approaches described in (KLEINERT; SCHMIDT, 2021) (CONSTANTE-FLORES; CONEJO; CONSTANTE-FLORES, 2022), the number of bilinear terms is significantly reduced compared to approaches that utilize the KKT conditions to represent lower-level model. In the approach here proposed we reduce the bilinear terms to a single set of terms, by means of mathematical manipulations involving the complementarity constraints of the lower-level

model. We emphasize that the resulting single-level MPEC has only a single set of bilinear terms, independently of the number of inequality constraints on the lower-level problem and the system size.

The bilinear term is handled by means of a convexification technique similar to (CONSTANTE-FLORES; CONEJO; CONSTANTE-FLORES, 2022), instead of using hard-to-solve Big-M approaches (FORTUNY-AMAT; MCCARL, 1981b). The convexification is performed around an initial point, resulting in a convex MPEC problem, which is solved iteratively until a convergence criterion is achieved. Some advantages of using the proposed solution approach over the big-M approach are: i) Our solution approach solves continuous convex problems instead of mixed-integer problems; ii) The proposed approach avoids the cutting-off feasible or optimal points, which may happen in big-M approaches and iii) There are only three tuning parameters to adjust in the convexification technique, regardless of the network size, number of units, etc.

Another contribution of this work is the formulation of an SO bilevel model specifically tailored for hydro-dominated markets, where the lower-level MC model embodies the representation of hydro and thermal units for all companies, together with DC network constraints. To the best of the authors' knowledge, the only previously proposed hydrothermal bilevel SO model in the literature is described in (MOISEEVA; HESAMZADEH, 2018). However, this model has some drawbacks, which include: i) Its lower-level model represents hydro constraints associated only with the SC, such that water balance constraints are not enforced for the remaining companies; ii) Its lower-level model is an economic dispatch with inelastic demand representation. The lower-level representation by an economic dispatch would be adequate under perfect competition. However, since the SC holds market power, this representation may lead to a market design that may result in excessively high prices. In such cases, the SC exploits the fact that demand must be met at any cost (due to its inelastic nature) to leverage its market power (by submitting strategic offers to the market) and drive market clearing prices to inappropriate extreme values. iii) Its lower-level model does not take into account a nodal transmission system representation.

On the other hand, the bilevel SO model here proposed aims at enhancing these modeling aspects. In the lower-level model here proposed, hydro constraints are represented for both the SC and NSC, capturing the behavior of the hydro cascades appropriately in its lower-level model. Moreover, the lower-level model proposed is adequate for situations where the demand elasticity is represented under imperfect competition, since the social welfare function used avoids those situations where the SC leverage its market power (by submitting strategic offers to the market) and drive market clearing prices to inappropriate extreme values. A final contribution of this work is the utilization of the proposed convexification approach within a hydropower energy market. Finally, a nodal DC transmission system

representation is employed in the proposed SO bilevel model.

The main contributions of this work are twofold:

1. A Strategic Offering (SO) model specifically tailored for a hydrothermal market is proposed. The SO model is able to represent water balances and other hydro constraints for both SC and NSC companies, capturing the spatial and temporal working of hydro cascade systems. The lower-level model is adequate for situations of imperfect competition, since demand elasticity is considered. In addition, DC network constraints are represented.
2. A novel approach for solving the bilevel SO optimization problem is proposed which utilizes an alternative set of constraints to represent the optimality conditions of the lower-level problem. In the approach proposed we reduce the number of bilinear terms in the resulting single-level MPEC problem to a single set of terms, independently of the number of inequality constraints. The single-level problem is then convexified around an initial point for handling the bilinear term.

The hydrothermal bilevel SO model and the solution approach proposed are tested on an adapted version of the IEEE 24-bus system, exploring scenarios where the SC acts strategically and non-strategically (perfect competition).

The remaining parts of this chapter are organized as follows. Section 3.2 describes the proposed bilevel SO model. The reformulation of the SO model as a convex MPEC is described in Section 3.3. Numerical results and discussions are provided in Section 3.4, and final conclusions are drawn in Section 3.5.

3.2 Bilevel Strategic Offering Model

The upper-level problem of the proposed bilevel strategic offering model maximizes the profits of a strategic company (SC), while the lower-level problem represents a hydrothermal market clearing procedure, as in (PEREIRA et al., 2017) and (SANTOS et al., 2020), such that a specific formulation is employed to represent hydro constraints. The set of primal variables of the lower-level problem is defined as $\mathcal{P} = \{p_{tu}^G, p_{tk}^C, \theta_{tn}, q_{tw}^G, s_{tr}^G, u_{tr}^G, x_{tr}^G\}$. The bilevel model is described from (3.1a) to (3.1r), where the dual variable associated with each constraint of the lower-level problem is provided in parenthesis:

3.2.1 Notation

Sets:

T Periods.

J, H Thermal and hydro units from the SC, respectively.

I, O Thermal and hydro units from the NSC, respectively.

U All generating units, hence $I \cup O \cup J \cup H$.

L All thermal units, hence $I \cup J$.

W All hydro units, hence $H \cup O$.

Y Generating units of the SC, hence $H \cup J$.

Z Generating units of the NSC, hence $O \cup U$.

K Consumers.

R^{NS} Reservoirs of the NSC.

R^S Reservoirs of the SC.

R All reservoir, hence $R^{NS} \cup R^S$.

F_r Upstream reservoirs from r .

$\mathcal{T}_r$ Downstream reservoirs from r .

N Network buses.

N_n Buses directly connected to n (neighborhood).

I_n Thermal units from the SC connected to bus n .

J_n Thermal units from the NSC connected to bus n .

O_n Hydro units from the SC connected to bus n .

H_n Hydro units from the NSC connected to bus n .

U_n All generating units connected to bus n .

L_n All thermal units connected to bus n .

W_n All hydro units connected to bus n .

Y_n All units from the SC connected to bus n .

Z_n All units from the NSC connected to bus n .

K_n Consumers connected to bus n .

$\mathcal{P}$ Set of all primal variables of the lower-level problem: $\mathcal{P} = \{p_{tu}^G, p_{tk}^C, \theta_{tn}, q_{tw}^G, s_{tr}^G, u_{tr}^G, x_{tr}^G\}$.

$\mathcal{D}$ Set of all dual variables of the lower-level problem: $\mathcal{D} = \{\lambda_{tn}, \pi_{tu}^G, \bar{\pi}_{tu}^G, \pi_{tk}^C, \bar{\pi}_{tk}^C, \alpha_{tnm}, \bar{\alpha}_{tnm}, \beta_t^0, \alpha_{tw}^G, \delta_{tw}^Q, \bar{\delta}_{tw}^Q, \delta_{tr}^S, \bar{\delta}_{tr}^S, \delta_{tr}^U, \bar{\delta}_{tr}^U, \delta_{tr}^U, \bar{\delta}_{tr}^U, v_{tr}^G, \delta_{tr}^X, \bar{\delta}_{tr}^X, \delta_r^{Goal}, \gamma_{tl}^{Gup}, \gamma_{tl}^{Gdw}\}$.

Indices:

t Index for periods, $t \in T$.

j Index for thermal units from the SC, $j \in J$.

h Index for hydro units from the SC, $h \in H$.

i Index for thermal units from the NSC, $i \in I$.

o Index for hydro units from the NSC, $o \in O$.

u Index for all units, $u \in U$.

l Index for all thermal units, $l \in L$.

w Index for all hydro units, $w \in W$.

y Index for all thermal and hydro units belonging to the SC, $y \in Y$.

z Index for all thermal and hydro units belonging to the NSC, $z \in Z$.

k Index for all consumers, $k \in K$.

r Index for all reservoirs, $r \in R$.

f Index for upstream reservoir from r , $f \in F_r$.

v Index for tailrace (downstream reservoir) from r , $v \in \mathcal{T}_r$.

φ Index value indicating slack bus.

n, m Indices for buses n where $n \in N$, and m where $m \in N_n$, respectively.

Parameters:

C	Conversion constant with the value of 0.0036.
FT	First period.
LT	Last period.
ζ_y^S	Marginal cost of unit y of the SC.
λ_{tz}^{NS}	Offer price of unit z , in period t from the NSC.
λ_{tk}^C	Bid price from consumer k in a period t .
$\bar{P}_{nm}$	Power capacity of line $n - m$.
B_{nm}	Network susceptance of line $n - m$.
$\bar{E}_u^G, \underline{E}_u^G$	Upper and lower bounds, respectively, on the power offered by unit u .
$\bar{E}_{tk}^C, \underline{E}_{tk}^C$	Upper and lower bounds, respectively, on the power bid by consumer k in period t .
R_l^{up}, R_l^{dw}	Ramp-up and down limits, respectively, for thermal unit l .
$\bar{Q}_w^G, \underline{Q}_w^G$	Upper and lower bounds on the turbined outflow of unit w .
$\bar{S}_r^G, \underline{S}_r^G$	Upper and lower bounds on spillage from reservoir r .
$\bar{U}_r^G, \underline{U}_r^G$	Upper and lower bounds, respectively, on the total outflow from reservoir r .
$\bar{X}_r^G, \underline{X}_r^G$	Upper and lower bounds, respectively, on the volume of reservoir r .
H_r^G	Effective water head of reservoir r .
X_{0r}^G	Initial volume at reservoir r .
X_r^{Goal}	Goal on the final volume at reservoir r .
Υ_{tr}	Water inflow in period t at reservoir r .
U_{tf}^Y	Total water outflow in period t from the upstream reservoir f .
k_w^G	Conversion constant of gravity acceleration times the specific weight of water for hydro unit w .
η_w	Average hydro performance of hydro unit w .
$\rho^{(v)}$	Penalty term in the convexification approach.
$\bar{\rho}$	Penalty upper bound in the convexification approach.
μ	Penalty update rate in the convexification approach.

Primal decision variables:

λ_{ty}^S Offer price of unit y from the SC in period t .

p_{tu}^G Power produced by unit u in period t .

p_{tk}^C Power consumed by consumer k in period t .

θ_{tn} Voltage angle of bus n in period t .

q_{tw}^G Turbined outflow of hydro unit w in period t .

s_{tr}^G Spillage from reservoir r in period t .

u_{tr}^G Total outflow of reservoir r in period t .

x_{tr}^G Volume of reservoir r in period t .

Dual decision variables:

λ_{tn} Power balance in period t at bus n (nodal market clearing price).

$\bar{\pi}_{tu}^G, \underline{\pi}_{tu}^G$ Upper and lower bounds, respectively, on power output of unit u in period t .

$\bar{\pi}_{tk}^C, \underline{\pi}_{tk}^C$ Upper and lower bounds, respectively, on power demand of consumer k in period t .

$\bar{\alpha}_{tnm}, \underline{\alpha}_{tnm}$ Upper and lower bounds on the flow of line $n - m$ in period t .

β_t^0 Voltage angle in period t at bus $n = 1$.

α_{tw}^G Hydro production of unit w in period t .

$\bar{\delta}_{tw}^Q, \underline{\delta}_{tw}^Q$ Upper and lower bounds, respectively, on the turbined outflow of unit w in period t .

$\bar{\delta}_{tr}^S, \underline{\delta}_{tr}^S$ Upper and lower bounds, respectively, on the spillage from reservoir r in period t .

$\bar{\delta}_{tr}^U, \underline{\delta}_{tr}^U$ Upper and lower bounds, respectively, on the total outflow of reservoir r in period t .

δ_{tr}^U Total outflow of reservoir r in period t .

$\bar{\delta}_{tr}^X, \underline{\delta}_{tr}^X$ Upper and lower bounds, respectively, on the volume of reservoir r in period t .

δ_{tr}^{Goal} Goal on the volume of reservoir r of the SC in period t .

v_{tr}^G	Water balance for reservoir r in period t .
$\gamma_{tj}^{Gup}, \gamma_{tj}^{Gdw}$	Upward and downward ramp rate limits, respectively, on the power output of thermal unit j in period t .

The objective function of the upper-level problem in (3.1a) describes the profits of the SC, while the objective function of the market in lower-level in (3.1b) describes the social welfare function, where terms associated with the SC and NSC are written separately. The constraints associated with the lower-level problem are given from (3.1c) to (3.1r). The set of constraints (3.1c) represents power balances. Constraints (3.1d) and (3.1e) represent upper and lower bounds for power output dispatched to the generating units and consumers, respectively by the market clearing. Limits in active power flows are enforced in (3.1f), while the reference for voltage angles is given in (3.1g).

Hydro constraints for the lower-level problem are described from (3.1h) to (3.1p). Constraints (3.1h) represent the linear hydro production function, where the net head is considered constant. Upper and lower bounds in the water discharges, spillages and total water releases are represented in (3.1i)–(3.1k), respectively, where the total water release is calculated in (3.1l). Constraints (3.1m)–(3.1n) represent the continuity equations for reservoirs between two successive time periods, for $t \neq 1$ and $t = 1$, respectively. Upper and lower bounds in the reservoir volumes are set in (3.1o). The goal for the reservoirs volumes at the end of the horizon is given in (3.1p). Finally, ramp-up and down limits for thermal units are provided in (3.1q) and (3.1r), respectively.

$$\text{Max}_{\{\lambda_{ty}^S\} \cup \mathcal{P}} \sum_{tn(y \in Y_n)} \lambda_{tn} p_{ty}^G - \zeta_y^S p_{ty}^G \quad (3.1a)$$

subject to:

$$\text{Min}_{\mathcal{P}} \sum_{ty} \lambda_{ty}^S p_{ty}^G + \sum_{tz} \lambda_{tz}^{NS} p_{tz}^G - \sum_{tk} \lambda_{tk}^C p_{tk}^C \quad (3.1b)$$

subject to:

$$\sum_{u \in U_n} p_{tu}^G - \sum_{k \in K_n} p_{tk}^C - \sum_{m \in N_n} B_{nm} (\theta_{tn} - \theta_{tm}) = 0 : (\lambda_{tn}), \quad (3.1c)$$

$$\forall t \in T, \forall n \in N$$

$$\underline{E}_u^G \leq p_{tu}^G \leq \overline{E}_u^G : (\underline{\pi}_{tu}^G, \overline{\pi}_{tu}^G), \forall t \in T, \forall u \in U \quad (3.1d)$$

$$\underline{E}_k^C \leq p_{tk}^C \leq \overline{E}_k^C : (\underline{\pi}_{tk}^C, \overline{\pi}_{tk}^C), \forall t \in T, \forall k \in K \quad (3.1e)$$

$$-\overline{P}_{nm} \leq B_{nm} (\theta_{tn} - \theta_{tm}) \leq \overline{P}_{nm} : (\underline{\alpha}_{tnm}, \overline{\alpha}_{tnm}), \quad (3.1f)$$

$$\forall t \in T, \forall n \in N, \forall m \in N_n$$

$$\theta_{t\varphi} = 0 : (\beta_t^0), \forall t \in T \quad (3.1g)$$

$$p_{tw}^G = k_w^G \eta_w H_r^G q_{tw}^G : (\alpha_{tw}^G), \forall t \in T, \forall r \in R, \forall w \in W_r \quad (3.1h)$$

$$\underline{Q}_w^G \leq q_{tw}^G \leq \overline{Q}_w^G : (\delta_{tw}^Q, \bar{\delta}_{tw}^Q), \forall t \in T, \forall w \in W \quad (3.1i)$$

$$\underline{S}_r^G \leq s_{tr}^G \leq \overline{S}_r^G : (\delta_{tr}^S, \bar{\delta}_{tr}^S), \forall t \in T, \forall r \in R \quad (3.1j)$$

$$\underline{U}_r^G \leq u_{tr}^G \leq \overline{U}_r^G : (\delta_{tr}^U, \bar{\delta}_{tr}^U), \forall t \in T, \forall r \in R \quad (3.1k)$$

$$u_{tr}^G = \sum_{w \in W_r} q_{tw}^G + s_{tr}^G : (\delta_{tr}^U), \forall t \in T, \forall r \in R \quad (3.1l)$$

$$x_{tr}^G = x_{(t-1)r}^G + C \left(\Upsilon_{tr} + \sum_{f \in F_r^G} u_{tf}^G - u_{tr}^G \right) : (v_{tr}^G), \quad (3.1m)$$

$\forall t \in T : t \neq 1, \forall r \in R;$

$$x_{1r}^G = X_{0r}^G + C \left(\Upsilon_{1r} + \sum_{f \in F_r^G} u_{1f}^G - u_{1r}^G \right) : (v_{1r}^G), \quad (3.1n)$$

$\forall r \in R;$

$$\underline{X}_r^G \leq x_{tr}^G \leq \overline{X}_r^G : (\delta_{tr}^X, \bar{\delta}_{tr}^X), \forall t \in T, \forall r \in R \quad (3.1o)$$

$$x_{tr}^G \geq X_r^{Goal} : (\delta_r^{Goal}), \forall t \in T : t = LT, \forall r \in R \quad (3.1p)$$

$$p_{(t+1)l}^G - p_{tl}^G \leq R_l^{Gup} : (\gamma_{tl}^{Gup}), \forall t \in T : t \neq LT, \forall l \in L \quad (3.1q)$$

$$p_{tl}^G - p_{(t+1)l}^G \leq R_l^{Gdw} : (\gamma_{tl}^{Gdw}), \forall t \in T : t \neq LT, \forall l \in L \quad (3.1r)$$

The mathematical structure of the SO model is such that the decision variables of the upper-level model are given by set of variables $\{\lambda_{ty}^S\} \cup \mathcal{P}$, i.e. for the set $\{\lambda_{ty}^S, p_{tu}^G, p_{tk}^C, \theta_{tn}, q_{tw}^G, s_{tr}^G, u_{tr}^G, x_{tr}^G\}$. Therefore, strategic actions of the SC in the market involve choosing optimal values of offer prices (λ_{ty}^S) for all its units $y \in Y$. We emphasize that although the final decisions concerning the power allocations of all units owned by SC and NSC, given by (p_{tu}^G) , and the consumers' demand, given (p_{tk}^C) , are effectively calculated by the market operator in the lower-level, the offered prices (λ_{ty}^S) are calculated to maximize the profits of the SC (in the upper-level). Such a structure reflects a Stackelberg game (HOBBS; METZLER; PANG, 2000; GABRIEL et al., 2013) which can be summarized as follows: the SC calculates a strategic move given by $(\lambda_{ty}^S, \forall y \in Y)$, anticipating the reaction of the lower-level optimal decisions derived from the clearing of the market, given by $(p_{tu}^G, \forall u \in U)$ and $(p_{tk}^C, \forall k \in K)$, which are then calculated in response to the strategic moves of the SC.

It is also important to notice that, once problem (3.1) is solved, the SC will use the offering prices ("decided" by the upper-level problem) and the quantities ("decided" by the lower-level problem) to go to the market and offer these prices and quantities. However, we do not explicitly represent this step in the proposed model.

3.3 Recasting the bilevel problem as a convex MPEC

In this work, we adopt an alternative approach to reformulate the original bilevel problem by replacing the lower-level problem with its primal constraints, dual constraints, and an additional equality constraint that enforces the strong duality theorem.

To this end, we first describe the dual problem associated with the lower-level problem in Section 3.3.1. In Section 3.3.2, we construct the strong duality equality and manipulate certain complementarity conditions to obtain a single set of bilinear terms in the resulting MPEC. Finally, in Section 3.3.3, we propose a convexification method for the resulting MPEC model.

3.3.1 Building the dual lower-level problem

The equivalent dual optimization problem associated with the lower-level problem described in (3.1b)–(3.1r) is provided in (3.2)–(3.18):

$$\begin{aligned}
 \text{Max}_{\mathcal{D}} \quad & \sum_{tu} \pi_{tu}^G E_{tu}^G - \sum_{tu} \bar{\pi}_{tu}^G \bar{E}_{tu}^G + \sum_{tk} \pi_{tk}^C E_{tk}^C - \sum_{tk} \bar{\pi}_{tk}^C \bar{E}_{tk}^C \\
 & - \sum_{tn(m \in N_n)} \alpha_{tnm} \bar{P}_{nm} - \sum_{tn(m \in N_n)} \bar{\alpha}_{tnm} \bar{P}_{nm} + \sum_{tw} \delta_{tw}^Q Q_{tw}^G \\
 & - \sum_{tw} \delta_{tw}^Q \bar{Q}_{tw}^G + \sum_{tr} \delta_{tr}^S S_r^G - \sum_{tr} \bar{\delta}_{tr}^S \bar{S}_r^G + \sum_{tr} \delta_{tr}^U U_r^G \\
 & - \sum_{tr} \bar{\delta}_{tr}^U \bar{U}_r^G + \sum_{tr} \delta_{tr}^X X_r^G - \sum_{tr} \bar{\delta}_{tr}^X \bar{X}_r^G + \sum_{tr} \delta_{tr}^{Goal} X_r^{goal} \\
 & + \sum_{tr} v_{tr}^G C \Upsilon_{tr} + \sum_r v_r^G X_r^{0G} \\
 & - \sum_{t:t \neq LT, l} \gamma_{tl}^{Gup} R_l^{Gup} - \sum_{t:t \neq LT, l} \gamma_{tl}^{Gdw} R_l^{Gdw}
 \end{aligned} \tag{3.2}$$

s. t.:

$$\begin{aligned}
 & -\lambda_{tn} + \lambda_{tj}^S + \bar{\pi}_{tj}^G - \pi_{tj}^G - \gamma_{tj}^{Gup} + \gamma_{tj}^{Gdw} = 0 : (p_{tj}^G) \\
 & \forall t : t = 1, \forall n, \forall j \in J_n
 \end{aligned} \tag{3.3}$$

$$\begin{aligned}
 & -\lambda_{tn} + \lambda_{tj}^S + \bar{\pi}_{tj}^G - \pi_{tj}^G + \gamma_{(t-1)j}^{Gup} - \gamma_{tj}^{Gup} + \gamma_{tj}^{Gdw} \\
 & - \gamma_{(t-1)j}^{Gdw} = 0 : (p_{tj}^G); \forall t : t \neq 1, t \neq |T|, \forall n, \forall j \in J_n
 \end{aligned} \tag{3.4}$$

$$\begin{aligned}
 & -\lambda_{tn} + \lambda_{tj}^S + \bar{\pi}_{tj}^G - \pi_{tj}^G + \gamma_{(t-1)j}^{Gup} - \gamma_{(t-1)j}^{Gdw} = 0 : (p_{tj}^G) \\
 & \forall t : t = LT, \forall n, \forall j \in J_n
 \end{aligned} \tag{3.5}$$

$$\begin{aligned}
 & -\lambda_{tn} + \lambda_{ti}^{NS} + \bar{\pi}_{ti}^G - \pi_{ti}^G - \gamma_{ti}^{Gup} + \gamma_{ti}^{Gdw} = 0 : (p_{ti}^G) \\
 & \forall t : t = 1, \forall n, \forall i \in I_n
 \end{aligned} \tag{3.6}$$

$$\begin{aligned}
 & -\lambda_{tn} + \lambda_{ti}^{NS} + \bar{\pi}_{ti}^G - \pi_{ti}^G + \gamma_{(t-1)i}^{Gup} - \gamma_{ti}^{Gup} + \gamma_{ti}^{Gdw} \\
 & - \gamma_{(t-1)i}^{Gdw} = 0 : (p_{ti}^G); \forall t : t \neq 1, \forall n, \forall i \in I_n
 \end{aligned} \tag{3.7}$$

$$\begin{aligned}
 & -\lambda_{tn} + \lambda_{ti}^{NS} + \bar{\pi}_{ti}^G - \pi_{ti}^G + \gamma_{(t-1)i}^{Gup} - \gamma_{(t-1)i}^{Gdw} = 0 : (p_{ti}^G) \\
 & \forall t : t = NT, \forall n, \forall i \in I_n
 \end{aligned} \tag{3.8}$$

$$-\lambda_{tn} + \lambda_{th}^S + \bar{\pi}_{th}^G - \pi_{th}^G - \alpha_{th}^G = 0 : (p_{th}^G); \forall t, \forall n, \forall h \in H_n \tag{3.9}$$

$$-\lambda_{tn} + \lambda_{to}^{NS} + \bar{\pi}_{to}^G - \underline{\pi}_{to}^G - \alpha_{to}^G = 0 : (p_{to}^G); \forall t, \forall n, \forall o \in O_n \quad (3.10)$$

$$\lambda_{tn} - \lambda_{tk}^C + \bar{\pi}_{tk}^C - \underline{\pi}_{tk}^C = 0 : (p_{tk}^C); \forall t, \forall n, \forall k \in K_n \quad (3.11)$$

$$\begin{aligned} & \sum_{m \in N_n} B_{nm} (\lambda_{tn} - \lambda_{tm}) - \sum_{m \in N_n} B_{nm} (\underline{\alpha}_{tnm} - \underline{\alpha}_{tmn}) \\ & + \sum_{m \in N_n} B_{nm} (\bar{\alpha}_{tnm} - \bar{\alpha}_{tmn}) + \beta_t^0 = 0 : (\theta_{tn}); \forall t, n = 3 \end{aligned} \quad (3.12)$$

$$\begin{aligned} & \sum_{m \in N_n} B_{nm} (\lambda_{tn} - \lambda_{tm}) - \sum_{m \in N_n} B_{nm} (\underline{\alpha}_{tnm} - \underline{\alpha}_{tmn}) \\ & + \sum_{m \in N_n} B_{nm} (\bar{\alpha}_{tnm} - \bar{\alpha}_{tmn}) = 0 : (\theta_{tn}); \forall t, \forall n \neq 3 \end{aligned} \quad (3.13)$$

$$\begin{aligned} & k_w^G \eta_w^G H_r^G (\alpha_{tw}^G) + \bar{\delta}_{tw}^Q - \underline{\delta}_{tw}^Q + \delta_{tr}^U = 0 : (q_{tw}^G) \\ & \forall t \in T, \forall r \in R, \forall w \in W \end{aligned} \quad (3.14)$$

$$\bar{\delta}_{tr}^S - \underline{\delta}_{tr}^S + \delta_{tr}^U = 0 : (s_{tr}^G); \forall t, \forall r \quad (3.15)$$

$$C \left(\sum_{l \in T_r: t} v_{tl}^G - v_{tr}^G \right) - \delta_{tr}^U + \bar{\delta}_{tr}^U - \underline{\delta}_{tr}^U = 0 : (u_{tr}^G) \forall t, \forall r \quad (3.16)$$

$$\bar{\delta}_{tr}^X - \underline{\delta}_{tr}^X - v_{tr}^G + v_{(t+1)r}^G = 0 : (x_{tr}^G); \forall t : t \neq |T|, \forall r \quad (3.17)$$

$$\bar{\delta}_{tr}^X - \underline{\delta}_{tr}^X - v_{tr}^G - \delta_r^{Goal} = 0 : (x_{tr}^G); \forall t : t = |T|, \forall r. \quad (3.18)$$

3.3.2 Building the strong duality equality constraint and finding a single bilinear term

In this section, we first construct the strong duality equality, which is then mathematically manipulated using complementarity constraints derived from the KKT conditions, in order to recast it in terms of a single set of bilinear terms. It is important to note that such manipulation is not feasible when more than one generation company owns hydro plants within the same cascade. The algebraic steps involved in this process are detailed below.

Initially, it is important to split the offer costs of the strategic company in terms of its hydro and thermal components, as shown in (3.19):

$$\sum_{ty} \lambda_{ty}^S p_{ty}^G = \sum_{th} \lambda_{th}^S p_{th}^G + \sum_{tj} \lambda_{tj}^S p_{tj}^G. \quad (3.19)$$

The total revenue for the hydro units of the strategic company is obtained by multiplying the dual constraints associated to (3.9) (which happens to be the hydro power generated by the SC) to the constraint itself, summing all terms $\forall t \in T, \forall n \in N, \forall h \in H_n$, and accounting to the identity $\sum_{tnh \in H_n} (\cdot) \equiv \sum_{th} (\cdot)$, which results in (3.20):

$$\sum_{tnh \in H_n} \lambda_{tn} p_{th}^G = \sum_{th} \lambda_{th}^S p_{th}^G + \sum_{th} \bar{\pi}_{th}^G p_{th}^G - \sum_{th} \underline{\pi}_{th}^G p_{th}^G - \sum_{th} \alpha_{th}^G p_{th}^G. \quad (3.20)$$

On the other hand, the total revenue for the thermal units of the strategic company is obtained by multiplying the dual constraints (3.3), (3.4) and (3.5), respectively, by p_{tj}^G and summing all terms $\forall t \in T, \forall n \in N, \forall j \in J_n$, also considering that $\sum_{tnj \in J_n} (\cdot) \equiv \sum_{tj} (\cdot)$, which results in (3.21):

$$\begin{aligned} \sum_{tnj \in J_n} \lambda_{tn} p_{tj}^G &= \sum_{tj} \lambda_{tj}^S p_{tj}^G + \sum_{tj} \bar{\pi}_{tj}^G p_{tj}^G - \sum_{tj} \pi_{tj}^G p_{tj}^G - \sum_{tj: t \neq LT} \gamma_{tj}^{Gup} p_{tj}^G + \sum_{tj: t \neq LT} \gamma_{tj}^{Gdw} p_{tj}^G \\ &+ \sum_{tj: t \neq 1} \gamma_{(t-1)j}^{Gup} p_{tj}^G - \sum_{tj: t \neq 1} \gamma_{(t-1)j}^{Gdw} p_{tj}^G. \end{aligned} \quad (3.21)$$

Keeping in mind that $Y = H \cup J$, it is possible to obtain the overall offering costs for all hydro and thermal units of the strategic company, by merging (3.20) and (3.21), which results in (3.22):

$$\begin{aligned} \sum_{tny \in Y_n} \lambda_{tn} p_{ty}^G &= \sum_{ty} \lambda_{ty}^S p_{ty}^G + \sum_{ty} \bar{\pi}_{ty}^G p_{ty}^G - \sum_{ty} \pi_{ty}^G p_{ty}^G - \sum_{tj: t \neq LT} \gamma_{tj}^{Gup} p_{tj}^G + \sum_{tj: t \neq LT} \gamma_{tj}^{Gdw} p_{tj}^G \\ &+ \sum_{tj: t \neq 1} \gamma_{(t-1)j}^{Gup} p_{tj}^G - \sum_{tj: t \neq 1} \gamma_{(t-1)j}^{Gdw} p_{tj}^G - \sum_{th} \alpha_{th}^G p_{th}^G. \end{aligned} \quad (3.22)$$

To eliminate most of the bilinear terms in (3.22), we use some algebra involving the complementary constraints of the KKT conditions associated with constraints (3.1d), together with constraints (3.1q) and (3.1r), and the mathematical identities in (3.23), to obtain (3.24):

$$\begin{aligned} \sum_{t: t \neq LT, j} \gamma_{tj}^{Gup} p_{(t+1)j}^G &\equiv \sum_{t: t \neq 1, j} \gamma_{(t-1)j}^{Gup} p_{tj}^G \\ \sum_{t: t \neq LT, j} \gamma_{tj}^{Gdw} p_{(t+1)j}^G &\equiv \sum_{t: t \neq 1, j} \gamma_{(t-1)j}^{Gdw} p_{tj}^G \end{aligned} \quad (3.23)$$

$$\begin{aligned} \sum_{tny \in Y_n} \lambda_{tn} p_{ty}^G &= \sum_{ty} \lambda_{ty}^S p_{ty}^G - \sum_{ty} \pi_{ty}^G E_{ty}^G + \sum_{ty} \bar{\pi}_{ty}^G E_{ty}^G + \sum_{t: t \neq LT, j} \gamma_{tj}^{Gup} R_j^{Gup} \\ &+ \sum_{t: t \neq LT, j} \gamma_{tj}^{Gdw} R_j^{Gdw} - \sum_{th} \alpha_{th}^G p_{th}^G. \end{aligned} \quad (3.24)$$

We observe that only the terms $\lambda_{tn} p_{ty}^G$, $\lambda_{ty}^S p_{ty}^G$ and $\alpha_{th}^G p_{th}^G$ remain nonlinear in (3.24). Initially, to perform the linearization of the bilinear terms $\alpha_{th}^G p_{th}^G$, the production functions in (3.1h) are multiplied by their respective dual variables α_{th}^G , but only for the units of the strategic company, obtaining (3.25):

$$\alpha_{th}^G p_{th}^G = \alpha_{th}^G k_h^G \eta_h^G H_r q_{th}^G, \forall t \in T, \forall r \in R^S, \forall h \in H_r. \quad (3.25)$$

Multiplying the dual constraints (3.14) only in terms of the strategic company units by q_{th}^G , isolating the term $\alpha_{th}^G k_h^G \eta_h^G H_r q_{th}^G$, replacing it in (3.25), summing the associated terms $\forall t \in T, \forall r \in R^S, \forall h \in H_r$ and finally using the complementary constraints related to the KKT conditions of constraints (3.1i), we obtain (3.26).

$$\sum_{tr \in R^S h \in H_r} \alpha_{th}^G p_{th}^G = - \sum_{th} \bar{\delta}_{th}^Q \bar{Q}_t^G + \sum_{th} \delta_{th}^Q Q_t^G - \sum_{tr \in R^S h \in H_r} \delta_{tr}^U q_{th}^G. \quad (3.26)$$

The last bilinear term in (3.26) can be linearized by multiplying constraints (3.11) by its respectively dual variables, but only for the strategic company units ($\forall r \in R^S, \forall h \in H_r$), which results in (3.27):

$$\sum_{h \in H_r} \delta_{tr}^U q_{th}^G = \delta_{tr}^U u_{tr}^G - \delta_{tr}^U s_{tr}^G, \forall t \in T, \forall r \in R^S. \quad (3.27)$$

The two terms in the right hand side of (3.27) are still bilinear. To linearize them, for the reservoirs of the strategic company $r \in R^S$, multiply the dual constraints (3.15) and (3.16) by s_{tr}^G and u_{tr}^G , respectively, isolate the terms of $\delta_{tr}^U u_{tr}^G$ and $\delta_{tr}^U s_{tr}^G$ and replace them back in (3.27) and after in (3.26). Sum the terms of the resulting expression $\forall t \in T, \forall r \in R^S$, and finally use the complementarity constraints of the KKT conditions associated with constraints (3.1j) and (3.1k), to obtain (3.28):

$$\begin{aligned} \sum_{tr \in R^S, h \in H_r} \alpha_{th}^G p_{th}^G = & - \sum_{th} \bar{\delta}_{th}^Q \bar{Q}_t^G + \sum_{th} \underline{\delta}_{th}^Q \underline{Q}_t^G - \sum_{tr \in R^S} \bar{\delta}_{tr}^U \bar{U}_r^G + \sum_{tr \in R^S} \underline{\delta}_{tr}^G \underline{U}_r^G - \sum_{tr \in R^S} \bar{\delta}_{tr}^S \bar{S}_r^G \\ & + \sum_{tr \in R^S} \underline{\delta}_{tr}^S \underline{S}_r^G - C \left(\sum_{t, r \in R^S, l \in \mathcal{T}_r^S} v_{tl}^G u_{tr}^G - \sum_{tr \in R^S} v_{tr}^G u_{tr}^G \right). \end{aligned} \quad (3.28)$$

Two bilinear terms still remain in (3.28). To linearize them, for the reservoirs of the strategic company $r \in R^S$, multiply constraints (3.1m) and (3.1n) by their dual variables v_{tr}^G , sum the associated terms $\forall t \in T, \forall r \in R^S$, and then isolate the terms associated with $v_{tr}^G u_{tr}^G$, obtaining (3.29):

$$\begin{aligned} C \left(\sum_{t, r \in R^S, f \in F_r^S} v_{tr}^G u_{tf}^G - \sum_{tr \in R^S} v_{tr}^G u_{tr}^G \right) = & - \sum_{r \in R^S} v_{1r}^G X_r^{0G} - \sum_{tr \in R^S} v_{tr}^G C \Upsilon_{tr} \\ & + \sum_{tr \in R^S} v_{tr}^G x_{tr}^G - \sum_{t: t \neq 1, r \in R^S} v_{tr}^G x_{(t-1)r}^G. \end{aligned} \quad (3.29)$$

Using the mathematical identity (3.30) in (3.29) and replacing the resulting term in (3.28) leads to (3.31). It is important to highlight that the mathematical identity (3.30) is valid only when a single company owns all the units of a hydro cascade:

$$\sum_{t, r \in R^S, f \in F_r^S} v_{tr}^G u_{tf}^G \equiv \sum_{t, r \in R^S, l \in \mathcal{T}_r^S} v_{tl}^G u_{tr}^G \quad (3.30)$$

$$\begin{aligned} \sum_{tr \in R^S, h \in H_r} \alpha_{th}^G p_{th}^G = & - \sum_{th} \bar{\delta}_{th}^Q \bar{Q}_t^G + \sum_{th} \underline{\delta}_{th}^Q \underline{Q}_t^G - \sum_{tr \in R^S} \bar{\delta}_{tr}^U \bar{U}_r^G + \sum_{tr \in R^S} \underline{\delta}_{tr}^G \underline{U}_r^G - \sum_{tr \in R^S} \bar{\delta}_{tr}^S \bar{S}_r^G \\ & + \sum_{tr \in R^S} \underline{\delta}_{tr}^S \underline{S}_r^G + \sum_{r \in R^S} v_{1r}^G X_r^{0G} + \sum_{tr \in R^S} v_{tr}^G C \Upsilon_{tr} - \sum_{tr \in R^S} v_{tr}^G x_{tr}^G + \sum_{t: t \neq 1, r \in R^S} v_{tr}^G x_{(t-1)r}^G. \end{aligned} \quad (3.31)$$

Two bilinear terms still persist in (3.31). To linearize them, for the reservoir of the strategic company $r \in R^S$, multiply the dual constraints (3.17) and (3.18) by their respective dual variables x_{tr}^G , isolate the terms $v_{tr}^G x_{tr}^G$ and $v_{(t+1)r}^G x_{tr}^G$, and using the

complementary constraints from the KKT conditions (3.1o) and (3.1p), sum the associated terms $\forall t \in T, \forall r \in R^S$, which results in (3.32):

$$\sum_{tr \in R^S} v_{tr}^G x_{tr}^G - \sum_{t:t \neq LT, r \in R^S} v_{(t+1)r}^G x_{tr}^G = \sum_{tr \in R^S} -\delta_{tr}^X \underline{X}_r^G + \sum_{tr \in R^S} \bar{\delta}_{tr}^X \bar{X}_r^G - \sum_{r \in R^S} \delta_r^{Goal} X_r^{Goal}. \quad (3.32)$$

Using the mathematical identity (3.33) in (3.32), results in (3.34):

$$\sum_{t:t \neq LT, r \in R^S} v_{(t+1)r}^G x_{tr}^G \equiv \sum_{t:t \neq 1, r \in R^S} v_{tr}^G x_{(t-1)r}^G \quad (3.33)$$

$$\sum_{tr \in R^S} v_{tr}^G x_{tr}^G - \sum_{t:t \neq 1, r \in R^S} v_{tr}^G x_{(t-1)r}^G = \sum_{tr \in R^S} -\delta_{tr}^X \underline{X}_r^G + \sum_{tr \in R^S} \bar{\delta}_{tr}^X \bar{X}_r^G - \sum_{r \in R^S} \delta_r^{Goal} X_r^{Goal}. \quad (3.34)$$

Finally, replacing (3.34) in (3.31) and taking into account the identity $\sum_{tr \in R^S, h \in H_r} \alpha_{th}^G p_{th}^G = \sum_{th} \alpha_{th}^G p_{th}^G$ results in (3.35), which is the linear expression we were looking for:

$$\begin{aligned} \sum_{th} \alpha_{th}^G p_{th}^G = & - \sum_{th} \bar{\delta}_{th}^Q \bar{Q}_t^G + \sum_{th} \delta_{th}^Q Q_t^G - \sum_{tr \in R^S} \bar{\delta}_{tr}^U \bar{U}_r^G + \sum_{tr \in R^S} \delta_{tr}^U U_r^G - \sum_{tr \in R^S} \bar{\delta}_{tr}^S \bar{S}_r^G \\ & + \sum_{tr \in R^S} \delta_{tr}^S S_r^G + \sum_{r \in R^S} v_{1r}^G X_r^{0G} + \sum_{tr \in R^S} v_{tr}^G C \Upsilon_{tr} + \sum_{tr \in R^S} \delta_{tr}^X \underline{X}_r^G - \sum_{tr \in R^S} \bar{\delta}_{tr}^X \bar{X}_r^G + \sum_{r \in R^S} \delta_r^{Goal} X_r^{Goal}. \end{aligned} \quad (3.35)$$

By replacing (3.35) in (3.24) we obtain (3.36):

$$\begin{aligned} \sum_{tny \in Y_n} \lambda_{tn} p_{ty}^G = & \sum_{ty} \lambda_{ty}^S p_{ty}^G - \sum_{ty} \pi_{ty}^G \underline{E}_{ty}^G + \sum_{ty} \bar{\pi}_{ty}^G \bar{E}_{ty}^G + \sum_{t:t \neq LT, j} \gamma_{tj}^{Gup} R_j^{Gup} + \sum_{t:t \neq LT, j} \gamma_{tj}^{Gdw} R_j^{Gdw} \\ & + \sum_{th} \bar{\delta}_{th}^Q \bar{Q}_t^G - \sum_{th} \delta_{th}^Q Q_t^G + \sum_{tr \in R^S} \bar{\delta}_{tr}^U \bar{U}_r^G - \sum_{tr \in R^S} \delta_{tr}^U U_r^G + \sum_{tr \in R^S} \bar{\delta}_{tr}^S \bar{S}_r^G - \sum_{tr \in R^S} \delta_{tr}^S S_r^G - \sum_{r \in R^S} v_{1r}^G X_r^{0G} \\ & - \sum_{tr \in R^S} v_{tr}^G C \Upsilon_{tr} - \sum_{r \in R^S} v_{1r}^G X_r^{0G} + \sum_{tr \in R^S} \bar{\delta}_{tr}^X \bar{X}_r^G - \sum_{tr \in R^S} \delta_{tr}^X \underline{X}_r^G - \sum_{r \in R^S} \delta_r^{Goal} X_r^{Goal}. \end{aligned} \quad (3.36)$$

Isolating the term $\sum_{ty} \lambda_{ty}^S p_{ty}^G$ in (3.36), and replacing it in the strong duality equality, which is attained by equating (3.1b) and (3.2), and keeping in mind the relations among the sets involved, we obtain (3.37), where we adopt $\Psi_{tny} = \lambda_{tn} p_{ty}^G$ for future convexification purposes:

$$\begin{aligned} \sum_{tny \in Y_n} \Psi_{tny} = & - \sum_{tz} \lambda_{tz}^{NS} p_{tz}^G + \sum_{tk} \lambda_{tk}^C p_{tk}^C + \sum_{tz} \pi_{tz}^G \underline{E}_{tz}^G - \sum_{tz} \bar{\pi}_{tz}^G \bar{E}_{tz}^G + \sum_{tk} \pi_{tk}^C \underline{E}_{tk}^C - \sum_{tk} \bar{\pi}_{tk}^C \bar{E}_{tk}^C \\ & - \sum_{tn(m \in N_n)} \alpha_{nm} \bar{P}_{nm} - \sum_{tn(m \in N_n)} \bar{\alpha}_{nm} \bar{P}_{nm} + \sum_{to} \delta_{to}^Q Q_o^G - \sum_{to} \bar{\delta}_{to}^Q \bar{Q}_o^G + \sum_{tr \in R^{NS}} \delta_{tr}^S S_r^G - \sum_{tr \in R^{NS}} \bar{\delta}_{tr}^S \bar{S}_r^G \\ & + \sum_{tr \in R^{NS}} \delta_{tr}^U U_r^G - \sum_{tr \in R^{NS}} \bar{\delta}_{tr}^U \bar{U}_r^G + \sum_{t,r \in R^{NS}} \delta_{tr}^X \underline{X}_r^G - \sum_{t,r \in R^{NS}} \bar{\delta}_{tr}^X \bar{X}_r^G + \sum_{r \in R^{NS}} \delta_r^{Goal} X_r^{Goal} \\ & + \sum_{t,r \in R^{NS}} v_{tr}^G C \Upsilon_{tr} + \sum_{r \in R^{NS}} v_{1r}^G X_r^{0G} - \sum_{t:t \neq LT, i} \gamma_{ti}^{Gup} R_i^{Gup} - \sum_{t:t \neq LT, i} \gamma_{ti}^{Gdw} R_i^{Gdw}. \end{aligned} \quad (3.37)$$

Finally, the associated MPEC is given in (3.38), where the lower-level problem is replaced by the set of primal constraints, dual constraints, and the strong duality equality:

$$\begin{aligned}
 & \underset{\{\lambda_{ty}^S\} \cup \mathcal{P} \cup \mathcal{D}}{\text{Max}} & (3.1a) \\
 & \text{s.t. :} & \\
 & & (3.1d) - (3.1r) \quad (a) \\
 & & (3.3) - (3.18) \quad (b) \\
 & & (3.37) \quad (c).
 \end{aligned} \tag{3.38}$$

3.3.3 Applying the convexification approach to the MPEC

Problem (3.38) is nonconvex due to the single set of bilinear terms which appear both in the objective function and in the strong duality equality. To avoid the problems associated with hard-to-solve big-M techniques, in this work, we adopt a convexification strategy to handle the bilinear term. The resulting convex MPEC is solved through an iterative procedure. This procedure involves solving a sequence of convex MPECs that iteratively approximate the original nonconvex MPEC until a previously specified tolerance is satisfied. The convexification approach proposed is discussed below.

The bilinear term $\lambda_{tn}p_{ty}^G$ can be written as in (3.39):

$$\Psi_{tny} = \lambda_{tn}p_{ty}^G, \forall t \in T, \forall n \in N, \forall y \in Y_n. \tag{3.39}$$

From simple algebra we obtain the equality (3.40):

$$\left(\lambda_{tn} + p_{ty}^G\right)^2 - \left(\lambda_{tn} - p_{ty}^G\right)^2 = 4\lambda_{tn}p_{ty}^G, \forall t \in T, \forall n \in N, \forall y \in Y_n. \tag{3.40}$$

Therefore, by using (3.40), (3.39) can be rewritten by means of the difference of two quadratic functions, as in (3.41):

$$\Psi_{tny} = \frac{1}{4}\left(\lambda_{tn} + p_{ty}^G\right)^2 - \frac{1}{4}\left(\lambda_{tn} - p_{ty}^G\right)^2, \forall t \in T, \forall n \in N, \forall y \in Y_n, \tag{3.41}$$

which can be rewritten in terms of two sets of inequality constraints, as in (3.42):

$$\begin{aligned}
 & \frac{1}{4}\left(\lambda_{tn} - p_{ty}^G\right)^2 + \Psi_{tny} - \frac{1}{4}\left(\lambda_{tn} + p_{ty}^G\right)^2 \leq 0, \forall t \in T, \forall n \in N, \forall y \in Y_n \\
 & \frac{1}{4}\left(\lambda_{tn} + p_{ty}^G\right)^2 - \Psi_{tny} - \frac{1}{4}\left(\lambda_{tn} - p_{ty}^G\right)^2 \leq 0, \forall t \in T, \forall n \in N, \forall y \in Y_n.
 \end{aligned} \tag{3.42}$$

However, the difference of two quadratic convex functions is still non-convex. Nevertheless, we take advantage of the fact that the difference of a convex function and a linear one is convex. Therefore, the second quadratic terms in (3.42) are iteratively

linearized around an initial point $\lambda_{tn}^{(v)}, p_{ty}^{G(v)}$ using a Taylor series expansion, as in (3.43) and (3.44):

$$\begin{aligned} \frac{1}{4}(\lambda_{tn} + p_{ty}^G)^2 &\approx (a_{tny}^{(v)})^2 + a_{tny}^{(v)} (\lambda_{tn} - \lambda_{tn}^{(v)}) \\ &+ a_{tny}^{(v)} (p_{ty}^G - p_{ty}^{G(v)}), \forall t \in T, \forall n \in N, \forall y \in Y_n \end{aligned} \quad (3.43)$$

$$\begin{aligned} \frac{1}{4}(\lambda_{tn} - p_{ty}^G)^2 &\approx (b_{tny}^{(v)})^2 + b_{tny}^{(v)} (\lambda_{tn} - \lambda_{tn}^{(v)}) \\ &- b_{tny}^{(v)} (p_{ty}^G - p_{ty}^{G(v)}), \forall t \in T, \forall n \in N, \forall y \in Y_n, \end{aligned} \quad (3.44)$$

where:

$$a_{tny}^{(v)} = \frac{1}{2} (\lambda_{tn}^{(v)} + p_{ty}^{G(v)}), b_{tny}^{(v)} = \frac{1}{2} (\lambda_{tn}^{(v)} - p_{ty}^{G(v)}). \quad (3.45)$$

Replacing the second terms in the two inequalities in (3.42) by (3.43) and (3.44), respectively, finally leads to a convex difference between a quadratic term and a linearized term, which results into the set of convexified inequalities in (3.46) and (3.47), where the inequality constraints are relaxed by a variable tolerance ϵ_{tny} :

$$\begin{aligned} \frac{1}{4}(\lambda_{tn} - p_{ty}^G)^2 + \Psi_{tny} - (a_{tny}^{(v)})^2 - a_{tny}^{(v)} (\lambda_{tn} - \lambda_{tn}^{(v)}) \\ - a_{tny}^{(v)} (p_{ty}^G - p_{ty}^{G(v)}) \leq \epsilon_{tny}, \forall t \in T, \forall n \in N, \forall y \in Y_n \end{aligned} \quad (3.46)$$

$$\begin{aligned} \frac{1}{4}(\lambda_{tn} + p_{ty}^G)^2 - \Psi_{tny} - (b_{tny}^{(v)})^2 - b_{tny}^{(v)} (\lambda_{tn} - \lambda_{tn}^{(v)}) \\ + b_{tny}^{(v)} (p_{ty}^G - p_{ty}^{G(v)}) \leq \epsilon_{tny}, \forall t \in T, \forall n \in N, \forall y \in Y_n \end{aligned} \quad (3.47)$$

To keep the tolerance within acceptable bounds, penalty terms $\rho^{(v)} \sum_{tny} \epsilon_{tny}$ are introduced in the objective function, where $\rho^{(v)}$ is the penalty parameter, wherein its interactively updated rate is $\min\{\bar{\rho}, \mu * \rho^{(v)}\}$. This results in the objective function given in (3.48). We note that this procedure allows the convexification method to start from infeasible points.

$$\begin{aligned} \text{Max}_{\{\lambda_{ty}^S\} \cup \mathcal{P} \cup \mathcal{D} \cup \{\Psi_{tny}, \epsilon_{tny}\}} \quad & \sum_{tny} \Psi_{tny} - \sum_{ty} \zeta_y^S p_{ty}^G - \rho^{(v)} \sum_{tny} \epsilon_{tny}. \end{aligned} \quad (3.48)$$

Therefore, the final convex MPEC is summarized in (3.49):

$$\begin{aligned} \text{Max}_{\{\lambda_{ty}^S\} \cup \mathcal{P} \cup \mathcal{D} \cup \{\Psi_{tny}, \epsilon_{tny}\}} \quad & (3.48) \\ \text{s.t. :} \quad & (3.1d) - (3.1r) \\ & (3.3) - (3.18) \\ & (3.46) - (3.47). \end{aligned} \quad (3.49)$$

In summary, the proposed approach obtains the solution to the original nonconvex problem (3.38) by solving a series of convexified versions of (3.38) around an initial point,

using the quadratic convex problem (3.49), which results in a trajectory that iteratively approximates the solution of the original problem. Since (3.49) is convex, each problem along the trajectory has a guaranteed global solution. However, the final solution of this series of convexified problems may not correspond to the global solution of the original problem (3.38). Nevertheless, solving the set of convexified problems (3.49) always yields a local optimal solution. On the other hand, the proposed approach may produce different local solutions (trajectories) depending on the values chosen for the initial point $(\lambda_{tn}^{(v)}, p_{ty}^{G(v)})$ and for $\bar{\rho}$ and μ , which update the penalty parameter $\rho^{(v)}$. Our numerical experience with the method has shown that calibrating such parameters is generally a relatively simple task. Additionally, there are other methods that can approximate bilinear terms, such as the Outer Approximation (OA) approach. The only difference between our convexification and the OA is that the proposed convexification approach uses a sequence of convexified quadratic functions, while an OA approach works with a sequence of linearized functions (hyperplanes). Since the proposed nonlinear problem is closer to a quadratic problem rather than a linear one, our convexified approach is expected to work more efficiently than the OA approach.

3.4 Numerical results

In this Chapter, we evaluate the proposed bilevel strategic offering model for price-maker companies located in a hydropower system. For such a purpose, a modified version of the IEEE 24-bus system is used where several thermal plants are replaced by hydro plants, so as to represent a hydropower system.

3.4.1 Inputs

The data for the hydro plants are taken from Brazilian hydroelectric cascades (CICOGNA, 2008). The strategic company (SC) owns 10 hydro plants at the Paranapanema River depicted in Figure 3.1 and one thermal plant, with a total capacity of 2543.1 MW. The non-strategy company (NSC) owns 4 reservoirs at the Tietê River depicted in Figure 3.2 and three thermal units, with a total capacity of 1194.4 MW. Data for hydro and thermal plants for the SC and NSC are shown in Table 3.1, 3.2 and 3.3, where the marginal costs per unit for hydro units are taken from (OLADOSU; SASTHAV, 2022). Equation (3.50) is used for the purpose of setting a value for the initial volume X_{0r}^G . Additionally, for both SC and NSC the initial volume at the first period X_{0r}^G and the goal volume at the last period X_r^{Goal} of all reservoirs are set to be the same, therefore establishing a minimum volume at the last period to be the same volume as the one in the first period.

$$X_{0r}^G = \underline{X}_r + 0.5 * (\bar{X}_r^G - \underline{X}_r), \forall r \in R. \quad (3.50)$$

Table 3.1 – Data for hydro plants of strategic and non-strategic companies.

ID	Plant	# Units	Capacity [MW]	Bus	$\underline{U}_r^G, \bar{U}_r^G$ [m ³ /s]	$\underline{S}_r^G, \bar{S}_r^G$ [m ³ /s]	$\underline{X}_r^G, \bar{X}_r^G$ [hm ³]	Υ_{tr} [m ³ /s]	H_r^G [m]
Strategic Company									
1	Jurumirim	2	101	1	0, 1092	0, 734.2	3842.90, 7007.51	459	31.00
2	Piraju	2	80	1	0, 1086	0, 729.4	102.30, 105.58	18	25.00
3	Chavantes	4	414	18	0, 1932	0, 1323.5	5754.00, 8795.10	384	73.60
4	Ourinhos	3	44.1	2	0, 1458	0, 972.3	18.680, 20.820	10	10.40
5	Salto Grande	4	73.8	2	0, 1740	0, 1171.0	15.142, 44.506	238	15.30
6	Canoas II	3	72	2	0, 1683	0, 1136.8	123.35, 147.97	20	14.50
7	Canoas I	3	82.5	23	0, 1701	0, 1141.5	179.57, 209.90	41	16.30
8	Capivara	4	635	23	0, 4554	0, 3177.2	4816.83, 10541.35	2117	47.70
9	Taquaruçu	5	525	21	0, 7650	0, 5161.1	538.70, 676.83	136	22.50
10	Rosana	4	354	7	0, 7404	0, 4993.6	1504.70, 1912.26	293	15.70
Non-Strategic Company									
11	Barra Bonita	4	140	13	0, 2268	0, 1559.2	568.59, 3135.53	522	21.40
12	Bariri	3	144	13	0, 2313	0, 1800.5	485.29, 544.20	75	22.10
13	Ibitinga	3	131	13	0, 2106	0, 1409.4	926.91, 983.16	149	21.20
14	Promissão	3	264	13	0, 3879	0, 2604.0	5279.75, 7407.05	250	23.10

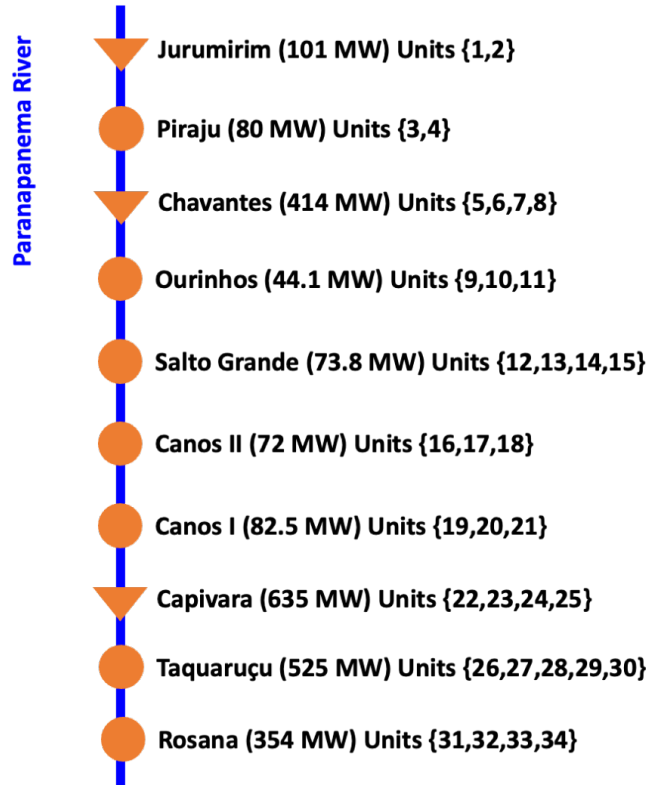


Figure 3.1 – Strategic Company cascade of its hydro power plants.

The system has 17 large consumers, whose connections to the grid are detailed in (CONEJO; CARRI6N; MORALES, 2010), with a total daily power consumption of 52192.30 MWh. Daily demand bids for each consumer are shown in Figure 3.3, while Figure 3.4 describes the hourly bid prices submitted by the consumers.

Table 3.4 shows the location of generating units and consumers in the IEEE 24-node

Table 3.2 – Data for hydro units of strategic and non-strategic companies.

Hydro Unit	Units' Cost [\$/MWh]	$\underline{E}_w^G, \overline{E}_w^G$ [MW]	η_w	k_w^G	$\underline{Q}_w^G, \overline{Q}_w^G$
Strategic Company					
1	2.66	0, 45.25	0.98	0.008862	0, 182.0
2	2.66	0, 45.25	0.98	0.008862	0, 182.0
3	3.94	0, 45.25	0.98	0.009024	0, 181.0
4	3.94	0, 45.25	0.98	0.009024	0, 181.0
5	2.66	0, 103.5	0.98	0.008749	0, 161.0
6	2.66	0, 103.5	0.98	0.008749	0, 161.0
7	2.66	0, 103.5	0.98	0.008749	0, 161.0
8	2.66	0, 103.5	0.98	0.008749	0, 161.0
9	3.94	0, 19.71	0.98	0.008694	0, 162.0
10	3.94	0, 19.71	0.98	0.008694	0, 162.0
11	3.94	0, 19.71	0.98	0.008694	0, 162.0
12	3.94	0, 19.71	0.98	0.008606	0, 145.0
13	3.94	0, 19.71	0.98	0.008606	0, 145.0
14	3.94	0, 19.71	0.98	0.008606	0, 145.0
15	3.94	0, 19.71	0.98	0.008606	0, 145.0
16	3.94	0, 19.71	0.98	0.009104	0, 187.0
17	3.94	0, 19.71	0.98	0.009104	0, 187.0
18	3.94	0, 19.71	0.98	0.009104	0, 187.0
19	3.94	0, 102.4286	0.98	0.009138	0, 189.0
20	3.94	0, 102.4286	0.98	0.009138	0, 189.0
21	3.94	0, 102.4286	0.98	0.009138	0, 189.0
22	2.66	0, 102.4286	0.98	0.008468	0, 362.0
23	2.66	0, 102.4286	0.98	0.008468	0, 388.0
24	2.66	0, 102.4286	0.98	0.008468	0, 384.0
25	2.66	0, 102.4286	0.98	0.008468	0, 384.0
26	2.66	0, 105	0.98	0.009051	0, 510.0
27	2.66	0, 105	0.98	0.009051	0, 510.0
28	2.66	0, 105	0.98	0.009051	0, 510.0
29	2.66	0, 105	0.98	0.009051	0, 510.0
30	2.66	0, 105	0.98	0.009051	0, 510.0
31	2.66	0, 88.5	0.98	0.009100	0, 617.0
32	2.66	0, 88.5	0.98	0.009100	0, 617.0
33	2.66	0, 88.5	0.98	0.009100	0, 617.0
34	2.66	0, 88.5	0.98	0.009100	0, 617.0
Non-Strategic Company					
36	2.66	0, 52.231	0.98	0.008860	0, 189.0
37	2.66	0, 52.231	0.98	0.008860	0, 189.0
38	2.66	0, 52.231	0.98	0.008860	0, 189.0
39	2.66	0, 52.231	0.98	0.008860	0, 189.0
40	2.66	0, 52.231	0.98	0.008437	0, 257.0
41	2.66	0, 52.231	0.98	0.008437	0, 257.0
42	2.66	0, 52.231	0.98	0.008437	0, 257.0
43	2.66	0, 52.231	0.98	0.008543	0, 234.0
44	2.66	0, 52.231	0.98	0.008543	0, 234.0
45	2.66	0, 52.231	0.98	0.008543	0, 234.0
46	2.66	0, 52.231	0.98	0.008788	0, 431.0
47	2.66	0, 52.231	0.98	0.008788	0, 431.0
48	2.66	0, 52.231	0.98	0.008788	0, 431.0

system. Additionally, Table 3.5 shows the information regarding the IEEE 24-node network, regarding the reactance and capacity of each line in the system. Finally, Figure 3.5 depicts

Table 3.3 – Data for thermal plants of strategic and non-strategic companies.

Thermal Unit	$\underline{E}_l^G, \bar{E}_l^G$ [MW]	Units' Cost [\$/MWh]	Bus	Ramp-up [MW]	Ramp-down [MW]
Strategic Company					
35	0, 155	45	16	152	152
Non-Strategic Company					
49	0, 107.5	45	15	55	55
50	0, 207.5	45	15	55	55
51	0, 300	45	22	295	295

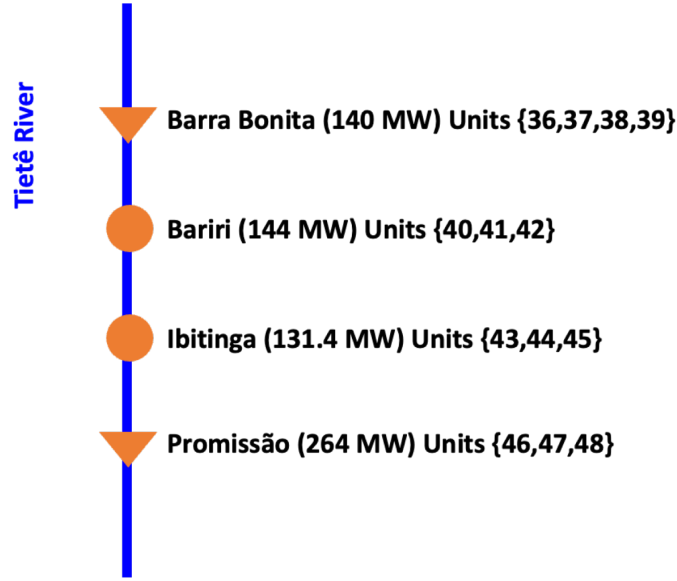


Figure 3.2 – Non-Strategic Company cascade of its hydro power plants.

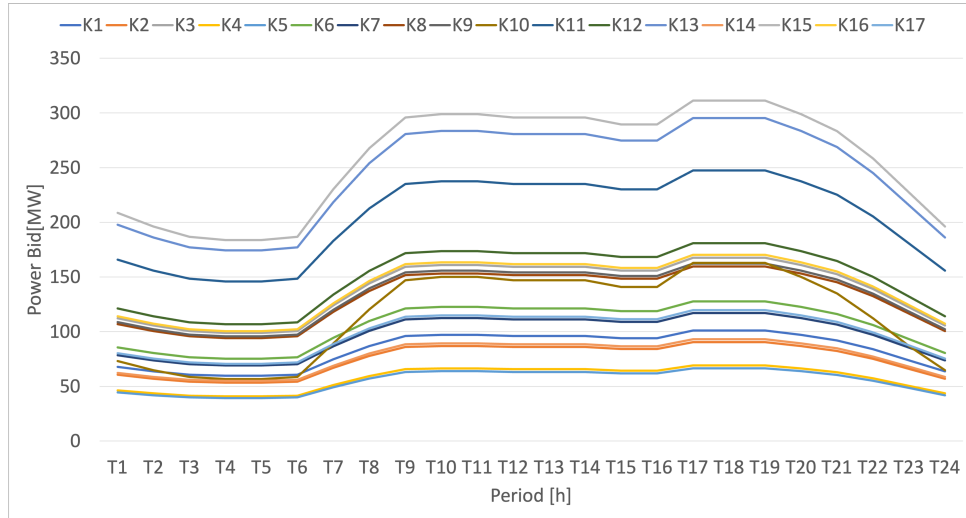


Figure 3.3 – Consumer Power Bids.

the topology of the IEEE 24-node system. Also, the slack bus is set to be bus $\varphi = 3$.

In the proposed convexification approach, the initial penalty value is set to $\rho^{(v)} = 1.3$. The step rate is also set to $\mu = 1.3$, and the penalty upper bound value is set to $\bar{\rho} = 10^5$. These values follow the advisable range proposed in (CONSTANTE-FLORES; CONEJO;

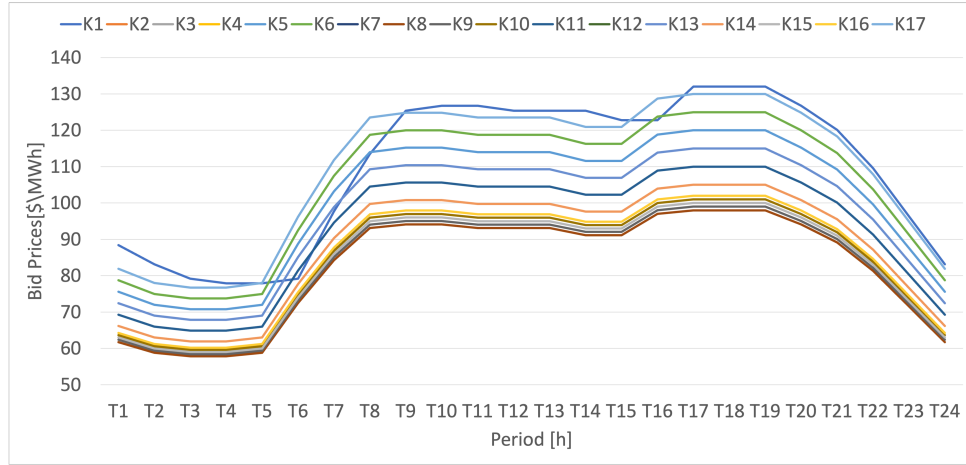


Figure 3.4 – Consumer Bid Prices.

Table 3.4 – IEEE 24-node system: node location of generating units and consumers.

Bus	Generation Units	Consumers
1	1, 2, 3, 4	1
2	9, 10, 11, 12, 13, 14, 15, 16, 17, 18	2
3	-	3
4	-	4
5	-	5
6	-	6
7	31, 32, 33, 34	7
8	-	8
9	-	9
10	-	10
11	-	-
12	-	-
13	36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48	11
14	-	12
15	49, 50	13
16	35	14
17	-	-
18	5, 6, 7, 8	15
19	-	16
20	-	17
21	26, 27, 28, 29, 30	-
22	51	-
23	19, 20, 21, 22, 23, 24, 25	-
24	-	-

CONSTANTE-FLORES, 2022). Tolerances used in the convexification processes are of 10^{-4} . Additionally, the initial values for $\lambda_{tn}^{(v)}$ are set to 1 and for $p_{ty}^{G(v)}$ are set to 5, and they are updated each iteration where $\lambda_{tn}^{(v+1)}$ is equal to the optimal solution λ_{tn}^* that results from the previous iteration. Similarly, $p_{ty}^{G(v+1)}$ is equal to the optimal solution p_{ty}^{G*} that results from the previous iteration.

The simulation analysis focus on comparing two basic scenarios:

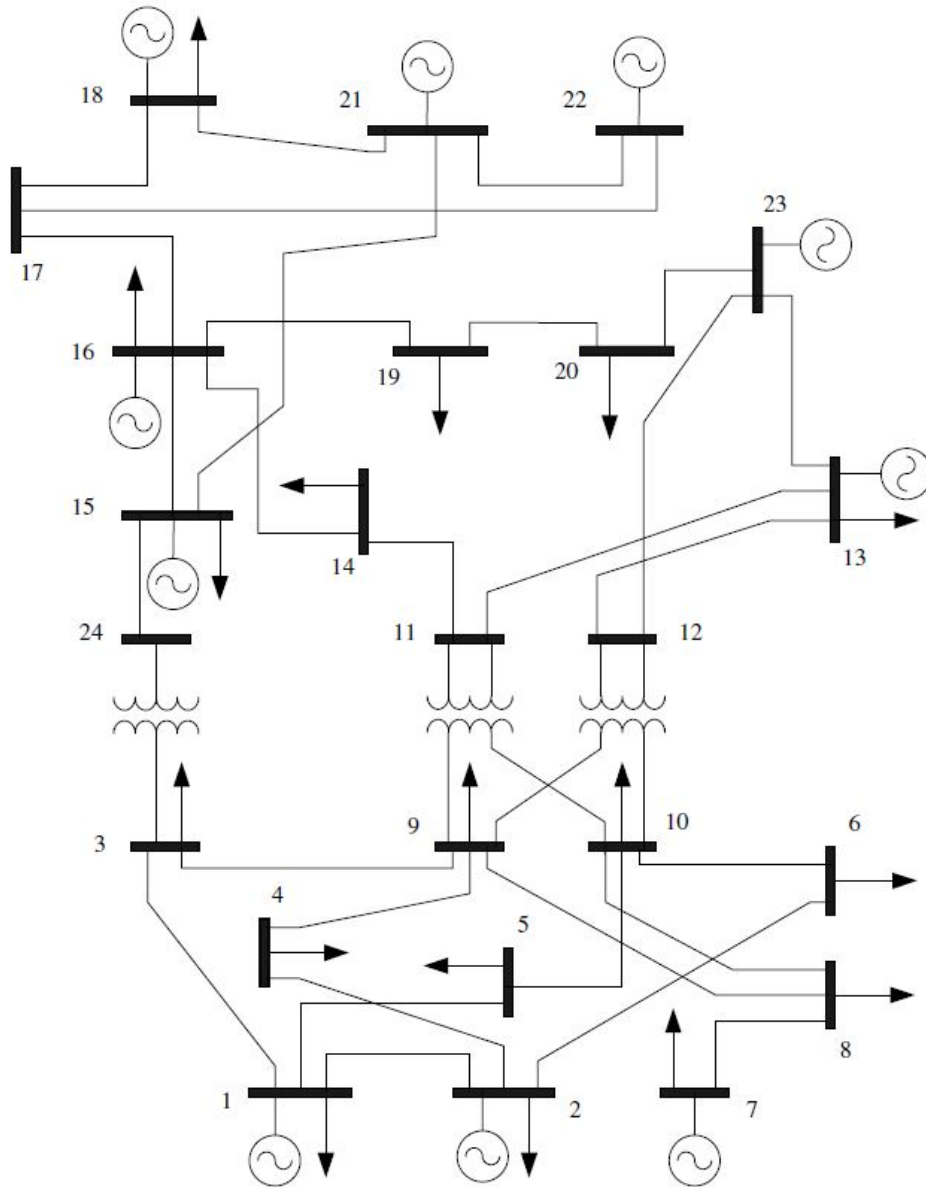


Figure 3.5 – IEEE 24-node system.
From: (CONEJO; CARRIÓN; MORALES, 2010).

- **Scenario 1 (Strategic Operation - Convexification):** The SC acts strategically and its decisions (i.e. offered prices) are calculated by using the proposed MPEC approach in (3.49), while the NSC acts non-strategically by offering at its marginal prices. In this scenario, the convexification approach is employed for iteratively approximating the second quadratic term in (3.41);
- **Scenario 2 (Perfect Competition):** Both the SC and NSC act non-strategically, such that their decision making are based on submitting offers at their respective marginal prices. Therefore, this scenario emulates a perfect competition market environment.

Table 3.5 – 24-node system: reactance and capacity of transmission lines

From node	To node	Reactance [p.u.]	Capacity [MVA]
1	2	0.0146	175
1	3	0.2253	175
1	5	0.0907	175
2	4	0.1356	175
2	6	0.205	175
3	9	0.1271	175
3	24	0.084	400
4	9	0.111	175
5	10	0.094	175
6	10	0.0642	175
7	8	0.0652	175
8	9	0.1762	175
8	10	0.1762	175
9	11	0.084	400
9	12	0.084	400
10	11	0.084	400
10	12	0.084	400
11	13	0.0488	500
11	14	0.0426	500
12	13	0.0488	500
12	23	0.0985	500
13	23	0.0884	500
14	16	0.0594	500
15	16	0.0172	500
15	21	0.0249	1000
15	24	0.0529	500
16	17	0.0263	500
16	19	0.0234	500
17	18	0.0143	500
17	22	0.1069	500
18	21	0.0132	1000
19	20	0.0203	1000
20	23	0.0112	1000
21	22	0.0692	500

Two cases are analyzed: In Case 1, the comparison of scenarios 1 and 2 is performed for a non-congested transmission network, while in Case 2 the network is artificially congested to assess the models' behavior. In both cases, market behavior is evaluated in terms of its social welfare, agents profits, hydro and thermal generation scheduling and system prices. All case studies were implemented using GAMS, while CPLEX has been used as the optimization solver to run the proposed convexified quadratic model.

3.4.2 Case 1: Non-congested network

In this case, we compare the market behavior for a non-congested network for scenarios where the SC acts strategically and non-strategically (perfect competition), respectively. Table 3.6 provides the social welfare values, profits for SC and NSC companies, the consumer utility, the total generation, and also the computation times, number of

iterations and the solver used for all scenarios. From the results, one observes that both companies experience a considerable increase in their profits when the SC acts strategically (i.e. for Scenario 1), as compared to perfect competition (Scenario 2). On the other hand, consumers experience a considerable decrease on their utilities, when the SC acts strategically (i.e. for Scenario 1), as compared to perfect competition (Scenario 2).

Table 3.6 – Social welfare, companies profits, consumer utilities and consumption: Case 1

Non-Congested Network	Strategic Operation	Perfect Competition
Strategic Company Profit [\$]	2,393,500.00	29,304.30
Non-Strategic Company Profit [\$]	1,274,924.00	9,362.464
Consumer Utility [\$]	768,196.80	4,738,977.00
Social Welfare [\$]	4,436,594.00	4,777,644.00
Total Generation/Consumption [MW]	52,192.30	52,192.30
Time [s]	21.75	0.05
Iterations	13	1
Solver	QCP: CPLEX	LP: CPLEX

The strategic behavior of the SC is better explained when one observes the market clearing prices obtained for all scenarios, which are shown in Figure 3.6. Prices are considerably higher in the scenarios of strategic operation, when compared to the scenario of perfect competition. Therefore, the increase in market clearing prices (for both Scenarios 1 and 2) results in higher profits for both SC and NSC, and also lower utility values for consumers. By analyzing figures 3.6 and 3.4, one observes that, for Scenarios 1 and 2, the SC manipulates its offered prices to the point where it matches to the cheapest consumer bid price (consumer K9 generally submits the cheapest bid prices) making the entire market clearing prices up to those value. This price manipulation is clearly better explored in the proposed Scenario 1, resulting in higher profits for the SC. On the other hand, in Scenario 2 (perfect competition) market clearing prices are formed based on the most dominant power source marginal costs, which corresponds to hydro units marginal costs, mostly.

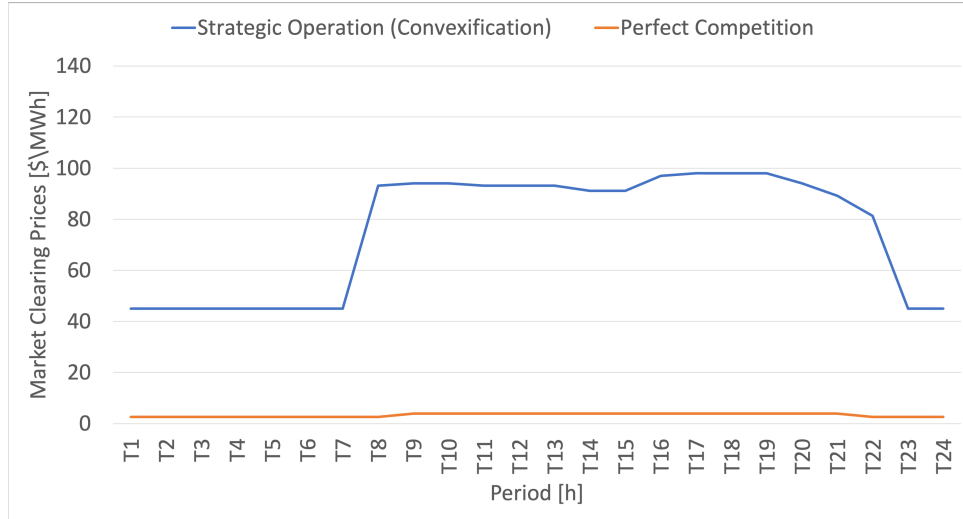


Figure 3.6 – Market Clearing Prices: Case 1.

Figures 3.7 and 3.8 show the hourly total power outputs dispatched for hydro (blue lines) and thermal (red lines) units by the strategic (solid lines) and non-strategic (dotted lines) companies. Figure 3.7 depicts the scenarios where the SC acts strategically (Scenario 1), while Figure 3.8 describes the scenario of perfect competition (Scenario 2).

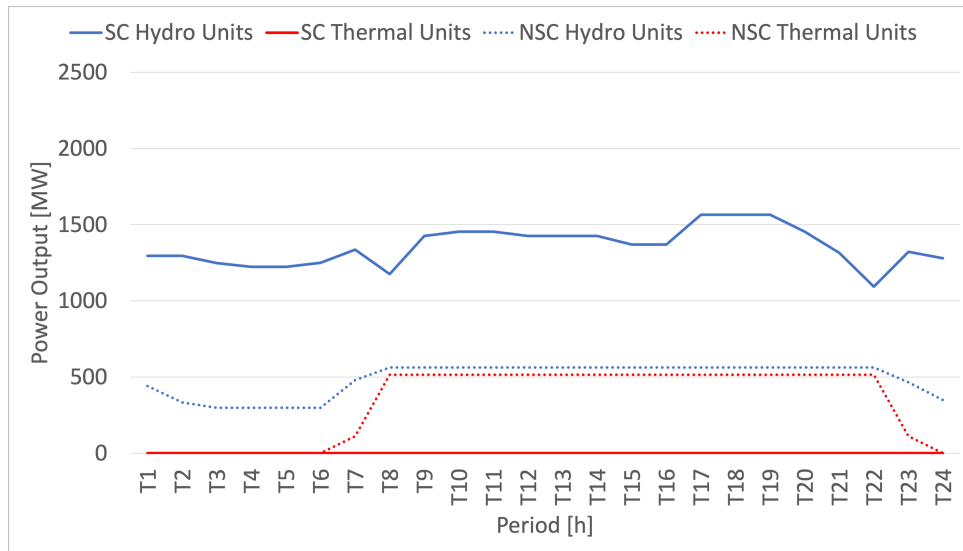


Figure 3.7 – Hydrothermal dispatch in the strategic operation scenario (Scenario 1 - convexification approach): Case 1.

In the strategic operation depicted in Figure 3.7, the SC chooses to offer high prices in order to raise market clearing prices, thereby increasing its own profit. As a result of the high offered prices submitted by the SC, the market clearing in the lower-level model allocates a smaller share of power output to the units belonging to the SC, thus granting a larger market share to its non-strategic competitors. The fact that the SC acts

to increase market clearing prices is confirmed by Figure 3.6. The perfect competition scenario depicted in Figure 3.8 (Scenario 2) reveals a markedly different dispatch schedule when compared to Scenario 1. Since there is no market power or price manipulation in Scenario 2, the plants of the SC supplies larger portions (i.e., close to the company's total capacity) of the market demand. This is because the market clearing procedure prioritizes dispatching generation with lower prices. As a result, only hydro plants are dispatched, and the power outputs of both SC and NSC thermal plants are set to zero. The choice for scheduling only hydro plants, which present lower offer prices, leads to lower market clearing prices, as shown by the orange curve in Figure 3.6.

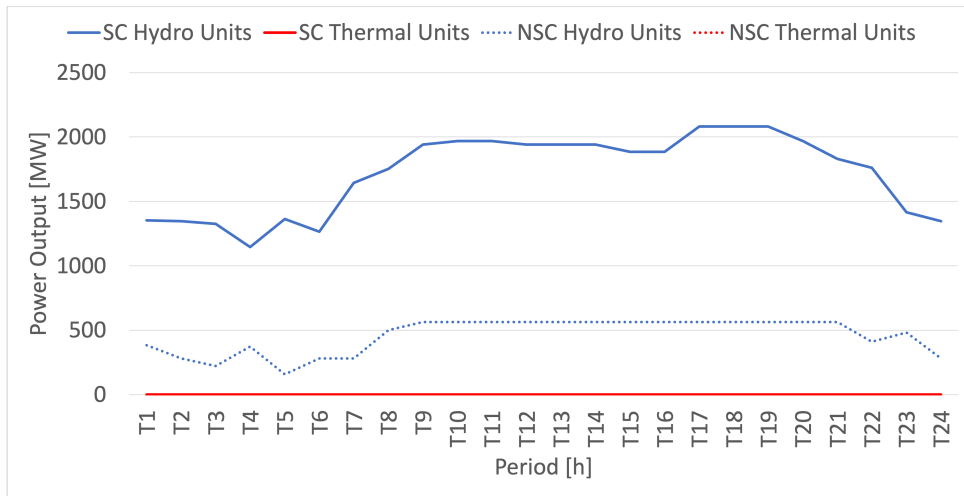


Figure 3.8 – Hydrothermal dispatch in the perfect competition scenario (Scenario 2): Case 1.

This strategic behavior of the SC is noteworthy from the short-term operational standpoint of its hydro plants, since it leads the market operator to allocate a smaller share of power output to all units belonging to the SC. For hydro units belonging to the SC, this behavior results in water savings in their reservoirs, when compared to the perfect competition operation (Scenario 2). This effect is depicted in Figure 3.9, which shows the daily evolution of total reservoir volumes for the hydro units of the SC. In this figure, we observe that in the scenario of perfect competition, the initial total reservoir volume of the SC units is equal to the final total reservoir volume (due to the final target volume constraints established for the company). Conversely, in the strategic operation the final total reservoir volume of the SC units is higher than the target value employed (note that the constraints on final target volumes for the reservoirs are inequality constraints, which are not necessarily active in the optimal solution). Therefore, Figure 3.9 shows that in relation to the perfect competition operation, the strategic operation saves water in the reservoirs of the SC units in a single day. This represents a relevant outcome from the SC's perspective, since the water in its reservoirs is stored energy that can be traded in the present or in the future. Consequently, this strategic approach yields two relevant outcomes

for the SC in the day ahead market: profit maximization and water saving. However, from the perspective of the system operator, water saving may be seen as a positive or negative outcome, depending on future water values.

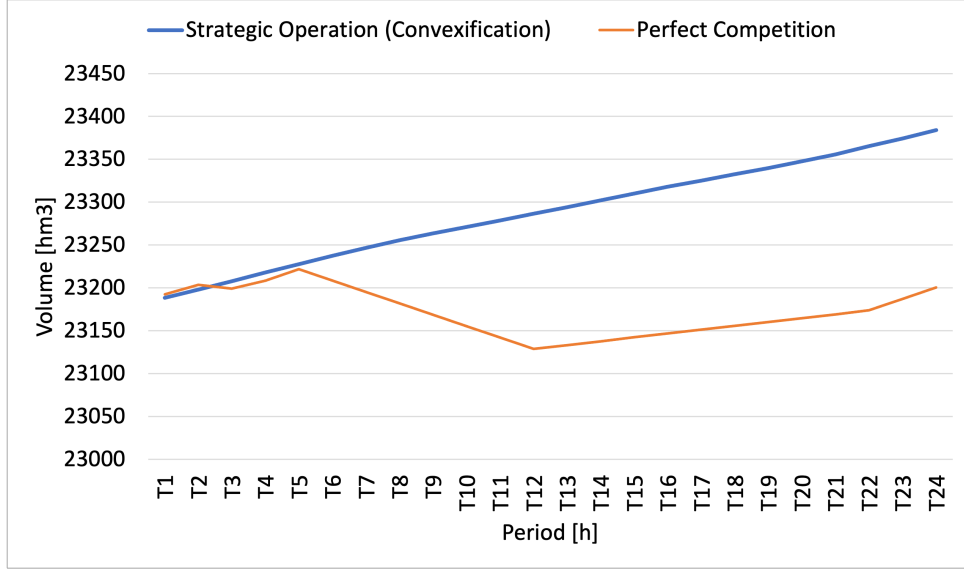


Figure 3.9 – Daily evolution of total reservoir volumes for the hydro units of the SC: Case 1.

On the other hand, from the perspective of the market/system operator, it is important to emphasize that market power is always an issue that must be mitigated. Therefore, from the perspective of the market/system operator, the strategic behavior of the SC results in a reduction of social welfare, as shown in Table 3.6.

3.4.3 Case 2: Congested network

As in Case 1, in this case we compare the market behavior for scenarios where the SC acts strategically and non-strategically, respectively. However, in Case 2 the network is artificially congested by altering the capacity of lines 3 – 24, 9 – 11, 9 – 12, 10 – 11 and 10 – 12. These areas of congestion split the system in two practically independent subsystems.

From Table 3.7, we verify that, as in Case 1, when the SC acts strategically (i.e. in Scenario 1), the profits increase for both SC and NSC, and the consumer utility decreases, when compared to the perfect competition (Scenario 2). However, by comparing Tables 3.6 and 3.7, it is clear that the demand met in Case 2 is lower than that met in Case 1, for all scenarios. Due to network congestion, the social welfare obtained in Case 2 (Table 3.7) also reduces for all scenarios, when compared to those obtained in Case 1 (Table 3.6). Consequently, network congestion poses difficulties to meet the demand and reduces social welfare in all scenarios. By analyzing the strategic operation given by Scenario 1 of Cases

1 and 2 (in Tables 3.6 and 3.7), we observe that the network congestion reduces the profits for both companies, with a deeper reduction for the NSC, while the consumer utility is considerably increased. A reverse reasoning applies in the perfect competition scenario for Cases 1 and 2, where one observes that the network congestion increases the profits for both companies in Case 2, while the consumer utility is considerably reduced in Case 2.

Table 3.7 – Social welfare, companies profits, consumer utilities and consumption: Case 2

Congested Network	Strategic Operation	Perfect Competition
Strategic Company Profit [\$]	1,481,400.00	1,185,878.00
Non-Strategic Company Profit [\$]	399,439.90	39,327.98
Consumer Utility [\$]	1,614,566.00	2,854,032.00
Social Welfare [\$]	3,496,395.00	4,079,239.00
Total Generation/Consumption [MW]	41,678.26	47,786.45
Time [s]	125.7	0.04
Iterations	27	1
Solver	QCP: CPLEX	LP: CPLEX

Figures 3.10 and 3.11 show the hydrothermal dispatches for SC and NSC for all scenarios of Case 2. As already observed in Case 1, in Scenario 1 of Case 2 (Figure 3.10) the SC also adopts the behavior of raising its offer prices so as to increase market clearing prices and its own profits. Therefore, hydro power generated by the SC in Case 2 of Scenario 1 is lower than that generated in the perfect competition scenario (Scenario 2). Once more, this market behavior leaves a larger market share to the NSC, which increases its share in the strategic operation scenario, when compared to the perfect competition scenario. Similar to the behavior described in Case 1, in Case 2 the SC experiences a decrease in its hydropower generation, which implies water savings in its own reservoirs. This, once again, represents a relevant outcome from the SC's perspective, since the water in its reservoirs is stored energy that can be traded in the present or in the future. Consequently, this strategic approach yields two relevant outcomes for the SC in the day ahead market: profit maximization and water saving. However, once again, from the perspective of the system operator, water saving may be seen as a positive or negative outcome, depending on future water values.

As a final observation, when compared to Case 1, both companies experience a reduction in their hydro power generation for both scenarios, since the elastic demand met in Case 2 is also lower than that met in Case 1.

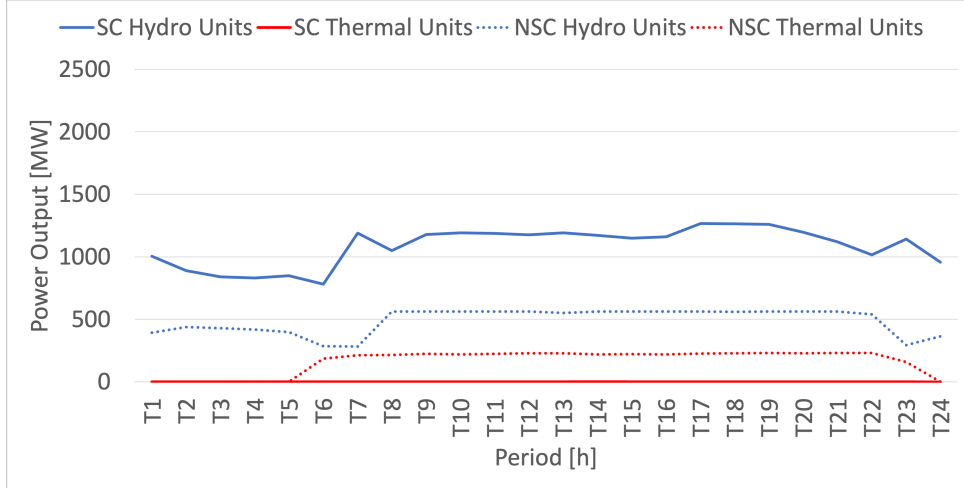


Figure 3.10 – Hydrothermal dispatch in the strategic operation scenario (Scenario 1 - convexification approach): Case 2.

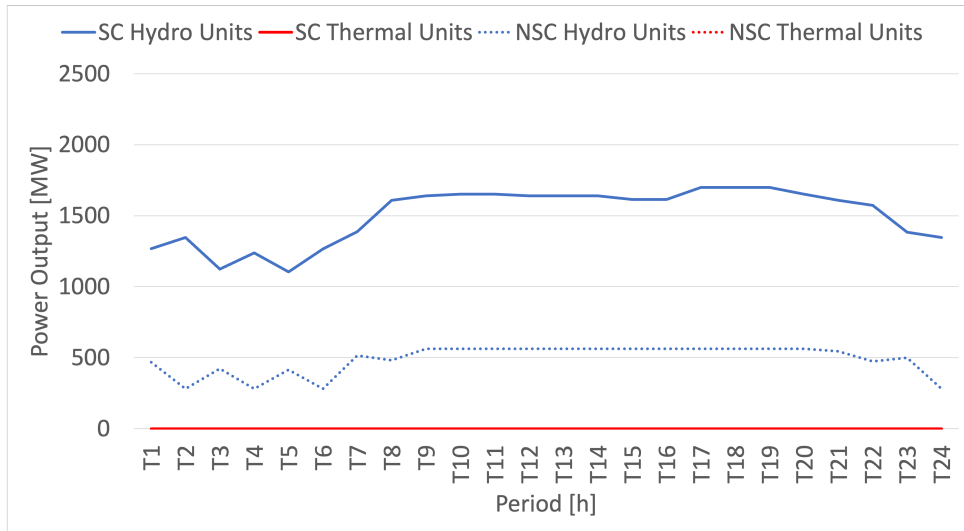


Figure 3.11 – Hydrothermal dispatch in the perfect competition scenario (Scenario 2): Case 2.

3.5 Conclusion

This work proposes a bilevel strategic offering (SO) model for a strategic company (SC) operating in a hydro-dominated day-ahead energy market, whose upper-level model maximizes its profit, while the lower-level model embodies a hydrothermal market clearing (MC) procedure. The MC considers hydrothermal constraints in addition to DC network constraints. In the proposed solution approach the lower-level model is replaced by its associate primal and dual constraints along with the strong duality equality constraint. The resulting MPEC model is further reformulated in terms of a single bilinear term, and iteratively convexified around an initial point. In traditional approaches where the lower-level problem is replaced with KKT conditions, each inequality constraint results

in a bilinear term (associated with the KKT complementarity constraint), which lead to a large number of such terms. Since bilinear terms are known to complicate the overall numerical solution, reducing the number of bilinear terms to a single term is an important feature of the proposed approach.

Two basic scenarios are evaluated in the numerical results: i) A scenario where the SC acts strategically and the bilinear term in the SO model is iteratively approximated using the proposed convexified approach, and ii) A scenario where all companies acts non-strategically (perfect competition). Also, both scenarios are studied for congested and non-congested networks cases. In strategic operation regarding scenario i), the results outline a SC market behavior, where this company decides to submit higher offered prices to the market. This behavior forces the market clearing prices to higher levels, increasing profit levels for both SC and NSC, and reducing consumer's utility. The results show that this behavior also leads to water savings in the reservoirs of the SC, due to reduced levels of power outputs from its hydro plants. This represents a relevant outcome from the SC's perspective, since the water in its reservoirs is stored energy that can be traded in the present or in the future. Consequently, this strategic behavior yields two relevant outcomes for the SC: profit maximization and water saving. The same behavior is observed in the strategic operation for scenario i) of the congested network case. On the other hand, such behavior is not observed in the perfect competition scenario. However, from the perspective of the system operator, water saving may be seen as a positive or negative outcome, depending on future water values.

A possible avenue for future research would involve proposing MPEC and Equilibrium Problem with Equilibrium Constraint (EPEC) formulations including medium-term stochastic hydrothermal scheduling problems in the lower-level of the SO model, where uncertainties may occur in water inflows, demand, etc.

In the next chapter, is presented another natural extension of our work, which involves the development of an equilibrium model suitable for evaluating the overall Nash equilibrium of the energy market by a system/market operator, resulting into an EPEC formulation, assuming the coexistence of more than one strategic company in the market.

4 EQUILIBRIA AMONG STRATEGIC PRODUCERS AND STRATEGIC CONSUMERS: A SOLUTION APPROACH RELYING ON A SINGLE OPTIMIZATION PROBLEM

4.1 Introduction

4.1.1 Description and motivation

In the context of electricity markets, an equilibrium is a stable outcome if no agent has an incentive to unilaterally deviate from its chosen strategy, given the strategies chosen by other agents. This stable outcome depends on the market rules and the way in which agents carry out their offering/bidding strategies. The Bertrand Equilibrium (BERTRAN, 1883), Cournot Equilibrium (COURNOT, 1838), and the Supply Function Equilibrium (SFE) (KLEMPERER; MEYER, 1989) are three fundamental models of competition in markets, each differing in how agents compete.

In this work, we are concerned with obtaining Nash equilibria regarding an EPEC resulting from the joint consideration of MPECs associated with producers and consumers in a wholesale electricity market. Such an EPEC is relevant to system/market operators, regulators, and market players since it characterizes market power. For this purpose, we consider an electricity market with a number of strategic producers and a number of strategic consumers. The profit maximization problem of each producer and the utility maximization problem of each consumer are formulated as bilevel models. Each upper-level problem represents the profit/utility maximization of the producer/consumer, while each lower-level problem represents the clearing of the market. Each bilevel model is then recast as an MPEC. The purpose of this work is to provide a systematic solution approach for solving the EPEC resulting from jointly considering these MPECs.

Solution approaches for solving an MPEC generally involve replacing its lower-level problem with its optimality conditions. In most approaches, the KKT conditions are employed. However, KKT conditions introduce a large number of complementarity constraints which complicate the stability of the solution approaches. Some methods for

handling complementarity constraints include the big-M method (JIANG et al., 2022), convex approximation (CONSTANTE-FLORES; CONEJO; CONSTANTE-FLORES, 2022), relaxation-based methods (DOLÁNYI; BRUNINX; DELARUE, 2024), penalty methods (KLEINERT; SCHMIDT, 2021) and metaheuristic approaches (DEB et al., 2020). An alternative to adopting KKT conditions as the optimality conditions for the lower-level problem is using Primal-Dual Optimality Conditions (PDOCs). The PDOCs consist of primal constraints, dual constraints, and the strong duality equality associated with the optimization problem being replaced. The adoption of PDOCs to represent the lower-level problem eliminate complementarity constraints, enhancing the numerical stability of solution approaches.

Some solution approaches have also been proposed for solving EPECs. In all approaches previously proposed in the literature, MPECs are not represented by their exact dual counterparts due to the presence of complementarity constraints and/or other sources of nonlinearities. This feature hinders the utilization of PDOCs for representing these MPECs. In this work, we propose a systematic solution approach for solving EPECs, where the MPECs are initially relaxed and recast as convex rotated quadratic conic problems, eliminating complementarity constraints. The resulting rotated quadratic conic MPECs have exact analytic dual counterparts. Therefore, PDOCs can be used to represent these MPECs.

The methodology proposed is summarized below:

1. Each profit/utility-maximization bilevel model, whose upper- and lower-level problems are both linear, is converted into a single-level optimization problem by replacing the lower-level linear problem with its PDOCs.
2. We note that the resulting single-level problem is nonconvex due to bilinear terms in the strong duality equality. However, such terms can be approximately convexified (CONSTANTE-FLORES; CONEJO; CONSTANTE-FLORES, 2022) (LêDO; NEPO-MUCENO; CONEJO, 2024), allowing this single-level problem to be formulated as a convex rotated quadratic conic problem (MOSEK ApS, 2024).
3. This convexified single-level problem, being a convex rotated quadratic conic problem, can be exactly represented by its PDOCs. We note that a convex rotated quadratic conic problem has an exact analytic dual problem, and strong duality holds.
4. The optimality conditions of the problems of all producers/consumers, as derived in step 3 above, are jointly considered, and an auxiliary optimization problem is used to ensure that all these constraints (equalities and inequalities) are jointly enforced. This optimization problem is nonlinear and nonconvex, and thus a global solver,

such as BARON (TAWARMALANI; SAHINIDIS, 2005), needs to be used to solve it.

5. Due to the convexification in step 2 above, the solution obtained in step 4 is approximate. Thus, an iterative procedure is needed to refine this approximate solution. Extensive computational experiments demonstrate the adequate behavior of this refinement process.

4.1.2 Literature review and contributions

The MPEC problems resulting from bilevel models of producers and consumers in an electricity market include complementarity constraints (CC), which do not satisfy certain constraint qualifications and constitute a challenge for nonlinear solvers. A variety of optimization methods have been proposed for addressing this problem, including the big-M reformulation of the KKT conditions (JIANG et al., 2022), convex approximation (CONSTANTE-FLORES; CONEJO; CONSTANTE-FLORES, 2022), relaxation methods (DOLÁNYI; BRUNINX; DELARUE, 2024), penalty methods (KLEINERT; SCHMIDT, 2021), and metaheuristic approaches (DEB et al., 2020).

The big-M method, which converts the CC associated with the KKT conditions into a mixed-integer linear constraints, was first described in (FORTUNY-AMAT; MCCARL, 1981a) and common in the literature. However, choosing inappropriate values for M can yield inaccurate solutions (KLEINERT; SCHMIDT, 2023), and checking the validity of the chosen big-Ms is shown to be as hard as solving the original bilevel problem (KLEINERT et al., 2020). Convexification approaches seek to approximate CC through an iterative approach, in an outer-approximation manner. Convex relaxation methods relax the CC by using a positive relaxation parameter ϵ . The relaxed NLP satisfies the Mangasarian-Fromovitz constraint qualification and is iteratively solved for values of $\epsilon \rightarrow 0$. Penalty methods solve alternating sub-problems, where one of the variables in the bilinear terms of the CC is fixed while the sub-problem is solved for the remaining variable, and vice versa. This procedure is repeated until convergence. A proof of convergence for this algorithm is described in (KLEINERT; SCHMIDT, 2021) for (mixed-integer) linear or quadratic upper-level and linear or quadratic lower-level problems. Finally, metaheuristic approaches are explored to solve the problem, although these methods generally cannot guarantee optimality or obtain the dual variables. We observe that all the MPEC approaches proposed in the literature involve solving a reformulated problem that cannot be dualized due to the presence of integer and/or nonlinear constraints.

The EPEC resulting from the joint solution of MPECs associated with producers and consumers in a wholesale energy market may lead to intractable or very difficult problems to solve. One of the most common solution approaches is diagonalization (HOBBS;

METZLER; PANG, 2000), (KARDAKOS; SIMOGLU; BAKIRTZIS, 2013), (NAEBI et al., 2020). In this method, a sequence of MPECs is solved for each market agent $i \in I$, where the offers/bids of the remaining players $r \in I : r \neq i$ are fixed using the best available information. This procedure is iteratively carried out until no agent changes its offers/bids. Although simple, this method has important drawbacks: it may not converge to a solution, its solution may depend on the order in which the MPECs are solved, and the upper-level constraints of the remaining agents are not considered during the solution of problem i , potentially leading to infeasibility of the upper-level problem given the decisions of the remaining players. Next, we describe some EPEC solution approaches involving only producers, as well as a few approaches involving both producers and consumers.

The EPEC model described in (RUIZ; CONEJO; SMEERS, 2012) is used to study the equilibria of producers in a network-constrained market. A bilevel model is proposed to represent each producer's strategic behavior, where the upper-level problem represents the producer's profit maximization, while the lower-level represents a market-clearing procedure. The bilevel models are recast as MPECs by replacing the lower-level problems with their associated PDOCs. In the EPEC model, the KKT conditions associated with the MPECs are linearized using the big-M approach and jointly solved to obtain the equilibrium.

The authors in (DOLÁNYI; BRUNINX; DELARUE, 2024) propose a solution approach for EPEC models aimed at exploring the multiple Nash equilibria associated with a set of producers. This method is based on the simultaneous solution of agents' bilevel problems, which are reformulated as MPECs by replacing the lower-level problems with their associated PDOCs. In this approach, Scholtes' regularization is applied before concatenating the optimality conditions associated with the strategic agents' problems. By using Scholtes' regularization, the constraints involving bilinear terms, which are difficult to handle, are relaxed. A sequence of relaxed problems is then solved by gradually reducing the violation of the relaxed constraints, enabling the exploration of multiple Nash equilibria.

In (MOISEEVA; HESAMZADEH, 2018), an EPEC problem is formulated to represent the strategic interaction of multiple producers in hydro-dominated power systems under uncertainty. The EPEC is formulated using game theory, where uncertainty in demand, wind power, and reservoir inflows is handled using two types of Nash equilibria: Bayesian and robust Nash equilibria. In their solution approach, the EPEC model is formulated using a discrete set of strategies, rather than working with a continuous one, as is typically done. This discretization leads to a mixed-integer formulation of the bilinear terms, resulting in a stochastic mixed-integer linear programming problem, which is solved using a modified Benders decomposition.

The EPEC proposed in (DAI; QIAO, 2017) considers conventional and wind power producers interacting in the market. The behavior of each strategic producer is formulated as a stochastic bilevel model, where the upper-level maximizes profits and the lower-level clears both the day-ahead and real-time markets. Uncertainty in wind power production and demand is represented by a set of plausible scenarios in the lower-level. The bilevel models are recast as MPECs by replacing the lower-level problems with their respective KKT conditions. The final EPEC model, obtained by jointly considering all MPECs. A diagonalization algorithm is used to solve this EPEC.

The authors in (TSIMOPOULOS; GEORGIADIS, 2021) propose an EPEC to model the interaction of conventional and wind power producers in a network-constrained pool market. The upper levels of the producers' stochastic bilevel models focus on expected profit maximization, while the lower-level clears both the energy and reserve markets, where uncertainty in wind power production is represented using a set of plausible scenarios. The bilevel problems are then recast as MPECs by replacing their lower-level problems with their respective equivalent PDOCs. These problems are finally jointly solved using their respective KKT conditions, where complementarity constraints are linearized using the big-M approach.

An alternative approach for solving a single-level market equilibrium model involving both producers and consumers is proposed in (LIU; CONEJO, 2021). This approach avoids the construction of bilevel models associated with the agents and, consequently, the need to solve an EPEC. In this approach, the strategic behavior of producers, consumers, and the market operator is modeled using single-level problems. These single-level problems are replaced with their respective KKT conditions and then solved jointly using some previously specified objectives.

In (YU et al., 2023), the interaction of producers and consumers in the market is represented using a hierarchical Nash distributed reinforcement learning framework. In this framework, the offering and bidding strategies provided by the agents are modeled using reinforcement learning, and a distributed dynamic-average consensus market-clearing procedure is used to compute the offers and bids of all agents.

Machine learning techniques are introduced in (PRAT; CHATZIVASILEIADIS, 2023) to solve individual bilevel problems, such as the strategic offering problem. In this case, the authors use an active-set strategy, where the lower-level problem is replaced with its associated active constraints, avoiding the use of binary variables. Machine learning techniques are used to identify the lower-level active constraints.

The authors in (CHEN et al., 2021) propose an EPEC model to explore the

interactions between various distribution markets (each operated by a distribution system operator - DSO) and the transmission wholesale market (operated by the independent system operator - ISO). In the bilevel model, the upper-level problems represents the DSOs, while the lower-level problem represents the wholesale market clearing performed by the ISO. The bilevel models are recast as single-level MPEC problems using PDOCs to represent the lower-level problem. The EPEC is used to find equilibria of multiple DSOs and the wholesale market by jointly solving the KKT conditions associated with all MPECs. The EPEC problem is then transformed into a single-level mixed-integer second-order cone problem and solved using MOSEK (MOSEK ApS, 2024) with the CVX toolbox.

From the literature review, we first observe that an MPEC may be constructed by replacing the lower-level problem in a bilevel problem with either its KKT conditions or its PDOCs. As already discussed, PDOCs are preferable since they do not introduce complementarity constraints into the MPEC formulation. We also observe the all MPECs previously proposed cannot be dualized due to the presence of integer and/or nonlinear constraints. Finally, we observe that in all previously proposed EPEC formulations, the MPECs constituting the EPEC are represented by their KKT conditions. However, since the MPECs proposed are nonlinear, PDOCs cannot be applied. In this work, we propose a systematic approach for solving an EPEC, using PDOCs both to represent the lower-level problems in the bilevel model and to represent the MPECs in the final EPEC formulation. The main contributions of the work are twofold:

1. We propose a systematic approach for solving a general EPEC model, where the MPEC problems associated with both producers and consumers are recast as convex rotated quadratic conic problems, which have analytic dual counterparts and can, therefore, be exactly represented using their respective PDOCs. We use the PDOCs associated with the MPEC problems to solve the EPEC nonlinear optimization problem involving producers and consumers.
2. We illustrate the proposed methodology using a simple example and a realistic case study. For the sake of simplicity, we assume that the network does not restrict the equilibria and, thus, do not model it. The proposed solution approach is also applied to investigate different market setups, exploring scenarios where: (i) no market power is expected; (ii) a single producer has potential market power; (iii) a single consumer has potential market power; and (iv) both a producer and a consumer have potential market power.

The remaining of this chapter is organized as follows. The bilevel models for producers and consumers are described in Section 4.2. The reformulation of these bilevel

models as MPECs is detailed in Section 4.3. The exploration of equilibria using the proposed EPEC model, and the methodology for solving it are described in Section 4.4. Numerical results are presented in Section 4.5, while the main conclusions are drawn in Section 4.6.

4.2 Bilevel models for maximization of producers' profit and consumers' utility

In this section, we introduce two bilevel models, one involving the profit maximization for strategic producer companies and another involving the utility maximization of strategic consumer companies.

Notation

In our notation bellow, the cardinality of a set S is expressed as $|S|$.

Sets:

I Producer companies.

J Consumer companies.

U_i Generating units of producer company i .

C_j Consumer units of consumer company j .

Ξ_i^{pp} Primal variables from the producers' problems: $\Xi_i^{pp} = \{o_{iu}, p_{ku}, d_{jc}, \lambda, \bar{\mu}_{ku}^p, \bar{\mu}_{jc}^d, \mu_{ku}^p, \mu_{jc}^d\} : \forall i \in I, \forall u \in U_i, \forall k \in I, \forall u \in U_k, \forall j \in J, \forall c \in C_j$.

Ξ_j^{cp} Primal variables from the consumers' problems: $\Xi_j^{cp} = \{b_{jc}, p_{iu}, d_{lc}, \lambda, \bar{\mu}_{iu}^p, \bar{\mu}_{lc}^d, \mu_{iu}^p, \mu_{lc}^d\} : \forall j \in J, \forall c \in C_j, \forall l \in J, \forall c \in C_l, \forall i \in I, \forall u \in U_i$.

Ξ_i^{pd} Dual variables from the producers' problems: $\Xi_i^{pd} = \{\eta_{iu}^p, \delta_i^p, \beta_{iku}^{pp}, \bar{\beta}_{iku}^{pp}, \beta_{ijc}^{pd}, \bar{\beta}_{ijc}^{pd}, \alpha_{iku}^{pp}, \alpha_{iuu}^{pp}, \alpha_{ijc}^{pd}, \bar{\theta}_{iku}^{pp}, \theta_{iku}^{pp}, \bar{\theta}_{ijc}^{pd}, \theta_{ijc}^{pd}, \gamma_i^p, \gamma_{iu}^{\varepsilon p}, s_{iu}^{ap}, s_{iu}^{bp}, s_{iu}^{cp}\} : \forall i \in I, \forall u \in U_i, \forall k \in I, \forall u \in U_k, \forall j \in J, \forall c \in C_j$.

Ξ_j^{cd} Dual variables from the consumers' problems: $\Xi_j^{cd} = \{\eta_{jc}^d, \delta_j^d, \beta_{jiu}^{dp}, \bar{\beta}_{jiu}^{dp}, \beta_{jlc}^{dd}, \bar{\beta}_{jlc}^{dd}, \alpha_{jiu}^{dp}, \alpha_{jlc}^{dd}, \bar{\theta}_{jiu}^{dp}, \theta_{jiu}^{dp}, \bar{\theta}_{jlc}^{dd}, \theta_{jlc}^{dd}, \gamma_j^d, \gamma_{jc}^{\varepsilon d}, s_{jc}^{ad}, s_{jc}^{bd}, s_{jc}^{cd}\} : \forall j \in J, \forall c \in C_j, \forall l \in J, \forall c \in C_l, \forall i \in I, \forall u \in U_i$.

Indices:

i, k Indices, for producer companies in set I .

j, l Indices, for consumer companies in set J .

u Index for producer units in set U_i belonging to producer company i .

c Index for consumer units in set C_j belonging to consumer company j .

Parameters:

c_{iu} Marginal cost of unit u from producer company i .

$\bar{p}_{ku}$ Upper bound on power output of unit u from producer company k .

u_{jc} Maximum utility of consumer c from consumer company j .

$\bar{d}_{lc}$ Upper bound on power demand of consumer c from consumer company l .

$\rho^{(v)}, \sigma^{(v)}$ Penalty terms associated with the convexification approach.

$\bar{\rho}, \bar{\sigma}$ Upper bounds on the penalty terms associated with the convexification approach.

μ, ω Updating rates for the penalty terms associated with the convexification approach.

Decision Variables:

o_{iu} Offering price of unit u from producer company i .

b_{jc} Bidding price of consumer c from consumer company j .

p_{ku} Power output of unit u from producer company k .

d_{lc} Power consumption of consumer c from consumer company l .

ε_{iu}^p Tolerance on the convexification of unit u from producer company i .

ε_{jc}^d Tolerance on the convexification of consumer c from consumer company j .

Bilevel's Lower-level Dual Variables:

The dual variables below are associated with constraints from the lower-level of both strategic producers and strategic consumers bilevel models:

λ Dual variable associated with the power balance constraint, representing the clearing price.

$\bar{\mu}_{ku}^p, \underline{\mu}_{ku}^p$ Upper and lower bounds, respectively, on the power supply of unit u from producer k .

$\bar{\mu}_{lc}^d, \underline{\mu}_{lc}^d$ Upper and lower bounds, respectively, on the power demand of consumer c from consumer company l .

MPEC Dual Variables:

The dual variables below are associated with constraints from both strategic producers and strategic consumers MPEC models:

η_{iu}^p, η_{jc}^d	Dual variables associated with the upper bound on the strategic offer of unit u from generating company i and the strategic bid of consumer c from the consumer company j , respectively.
δ_i^p, δ_j^d	Dual variables associated with the power balance constraint from the strategic model of the generating company i and the consumer company j , respectively.
$\bar{\beta}_{iku}^{pp}, \underline{\beta}_{iku}^{pp}$	Dual variables from the strategic model of generating company i associated with the upper and lower bounds on the power output of unit u from generating company k .
$\bar{\beta}_{jiu}^{dp}, \underline{\beta}_{jiu}^{dp}$	Dual variables from the strategic model of consumer company j associated with upper and lower bounds on the power output of unit u from generating company i .
$\bar{\beta}_{ijc}^{pd}, \underline{\beta}_{ijc}^{pd}$	Dual variables from the strategic model of generating company i associated with upper and lower bounds on the power consumed by unit c from the consumer company j .
$\bar{\beta}_{jlc}^{dd}, \underline{\beta}_{jlc}^{dd}$	Dual variables from strategic model of consumer company j associated with upper and lower bounds on the power consumed by unit c from the consumer company l .
$\alpha_{iku}^{pp}, \alpha_{jiu}^{dp}$	Dual variables from strategic models of generating company i or consumer company j , respectively, associated with the partial derivatives of the Lagrangian function with respect to the power output of unit u from generating company i or k .
$\alpha_{ijc}^{pd}, \alpha_{jlc}^{dd}$	Dual variables from strategic models of generating company i or consumer company j , respectively, associated with the partial derivatives of the Lagrangian function with respect to the power demand of unit c from consumer company j or l .
$\bar{\theta}_{iku}^{pp}, \bar{\theta}_{jiu}^{dp}$	Dual variables from strategic models of generating company i or consumer company j , respectively, associated with the non-negativity constraints associ-

ated with the bilevel's dual upper bounds variables of unit u from generating company i or k .

$\bar{\theta}_{ijc}^{pd}, \bar{\theta}_{jlc}^{dd}$ Dual variables associated to the positively constraints associated to the bilevel's dual upper boundaries variables of unit c from consuming company j or l from strategic models of producer i and consumer j respectively.

$\underline{\theta}_{iku}^{pp}, \underline{\theta}_{jiu}^{dp}$ Dual variables associated to the positively constraints associated to the bilevel's dual lower boundaries variables of unit u from generating company i or k from strategic models of producer i and consumer j respectively.

$\underline{\theta}_{ijc}^{pd}, \underline{\theta}_{jlc}^{dd}$ Dual variables associated to the positively constraints associated to the bilevel's dual upper boundaries variables of unit c from consuming company j or l from strategic models of producer i and consumer j respectively.

γ_i^p, γ_j^d Dual variables associated to the Strong Duality Constraint from strategic models of producer i and consumer j respectively.

$s_{iu}^{ap}, s_{iu}^{bp}, s_{iu}^{cp}$ Dual rotated conic variables of unit u from strategic model of producer i .

$s_{jc}^{ad}, s_{jc}^{bd}, s_{jc}^{cd}$ Dual rotated conic variables of unit c from strategic model of consumer j .

In the proposed bilevel model for profit maximization of producer company i or, more concisely, the bilevel producer problem i (B-PP $_i$), the upper-level aims to maximize the producer profits, while the lower-level sub-problem represents a market-clearing procedure, as detailed in (PEREIRA et al., 2017; CONEJO; RUIZ, 2020; LÊDO; NEPOMUCENO; CONEJO, 2024). The B-PP $_i$ model is described by (4.1), where the dual variables associated with the constraints of the lower-level sub-problem are provided in parentheses:

$$\begin{aligned} & \underset{o_{iu} \cup \{p_{ku}, d_{jc}\}}{Max} \quad \sum_{u \in U_i} (\lambda p_{iu} - c_{iu} p_{iu}) \\ & s.t. : \end{aligned} \tag{4.1a}$$

$$o_{iu} \geq c_{iu}, \quad \forall u \in U_i \tag{4.1b}$$

$$\begin{aligned} & \left\{ \underset{p_{ku}, d_{jc}}{Min} \quad \sum_{k \in I} \sum_{u \in U_k} o_{ku} p_{ku} - \sum_{j \in J} \sum_{c \in C_j} b_{jc} d_{jc} \right. \\ & s.t. : \end{aligned} \tag{4.1c}$$

$$\sum_{k \in I} \sum_{u \in U_k} p_{ku} - \sum_{j \in J} \sum_{c \in C_j} d_{jc} = 0 : \quad (\lambda) \tag{4.1d}$$

$$0 \leq p_{ku} \leq \bar{p}_{ku} : \quad (\underline{\mu}_{ku}^p, \bar{\mu}_{ku}^p), \forall k \in I, \forall u \in U_k \tag{4.1e}$$

$$0 \leq d_{jc} \leq \bar{d}_{jc} : \quad (\underline{\mu}_{jc}^d, \bar{\mu}_{jc}^d), \forall j \in J, \forall c \in C_j \}. \tag{4.1f}$$

On the other hand, in the proposed bilevel model for utility maximization of consumer company j or, more concisely, the bilevel consumer problem j (B-CP $_j$), the upper-level seeks to maximize the consumer utility, while the lower-level sub-problem represents a market-clearing procedure, as detailed in (PEREIRA et al., 2017; CONEJO; RUIZ, 2020; LêDO; NEPOMUCENO; CONEJO, 2024). The B-CP $_j$ model is described by (4.2), where the dual variables associated with the constraints of the lower-level sub-problem are also provided in parentheses:

$$\begin{aligned} & \underset{b_{jc} \cup \{p_{iu}, d_{lc}\}}{Max} \quad \sum_{c \in C_j} (u_{jc} d_{jc} - \lambda d_{jc}) \\ & s.t. : \end{aligned} \tag{4.2a}$$

$$b_{jc} \leq u_{jc}, \quad \forall c \in C_j \tag{4.2b}$$

$$\begin{aligned} & \underset{p_{iu}, d_{lc}}{Min} \quad \sum_{i \in I} \sum_{u \in U_i} o_{iu} p_{iu} - \sum_{l \in J} \sum_{c \in C_l} b_{lc} d_{lc} \\ & s.t. : \end{aligned} \tag{4.2c}$$

$$\sum_{i \in I} \sum_{u \in U_i} p_{iu} - \sum_{l \in J} \sum_{c \in C_l} d_{lc} = 0 : (\lambda) \tag{4.2d}$$

$$0 \leq p_{iu} \leq \bar{p}_{iu} : (\underline{\mu}_{iu}^p, \bar{\mu}_{iu}^p), \forall i \in I, \forall u \in U_i \tag{4.2e}$$

$$0 \leq d_{lc} \leq \bar{d}_{lc} : (\underline{\mu}_{lc}^d, \bar{\mu}_{lc}^d), \forall l \in J, \forall c \in C_l \}. \tag{4.2f}$$

Assuming that all players in the market are strategic—that is, all producers and consumers behave strategically—the joint solution of (4.1) and (4.2) for all players can be interpreted as an equilibrium among these strategic market participants. Since both bilevel models share the same lower-level formulation (the market-clearing procedure), this equilibrium involves multiple leaders and a single follower. Solving the bilevel problems (4.1) and (4.2) involves reformulating these problems by replacing their lower-level sub-problems with their associated optimality conditions, as discussed in the next section.

4.3 Recasting the considered bilevel problems as MPECs

In this work, as an alternative to using the KKT optimality conditions to replace the lower-level sub-problems of each bilevel model in equilibrium ((4.1), (4.2)), their respective Primal Dual Optimality Conditions (PDOCs) are employed, following the methodology in (CONEJO; RUIZ, 2020; LêDO; NEPOMUCENO; CONEJO, 2024). This avoids the complementarity constraints inherent in the KKT conditions. The resulting single-level MPEC models are detailed in (4.3), for the strategic producer i and in (4.4), for the strategic consumer j . These problems are shortly denoted as the MPEC for the producer problem i (MPEC-PP $_i$) and the MPEC for the consumer problem j (MPEC-CP $_j$), respectively.

MPEC for the producer problem i (MPEC-PP $_i$)

$$\begin{aligned} & \underset{\Xi_i^{pp}}{Max} \quad \sum_{u \in U_i} (\lambda p_{iu} - c_{iu} p_{iu}) \\ & s.t. : \end{aligned} \quad (4.3a)$$

$$o_{iu} \geq c_{iu} : (\eta_{iu}^p), \forall u \in U_i \quad (4.3b)$$

$$\sum_{k \in I} \sum_{u \in U_k} p_{ku} - \sum_{j \in J} \sum_{c \in C_j} d_{jc} = 0 : (\delta_i^p) \quad (4.3c)$$

$$0 \leq p_{ku} \leq \bar{p}_{ku} : (\underline{\beta}_{iku}^{pp}, \bar{\beta}_{iku}^{pp}), \forall k \in I, \forall u \in U_k \quad (4.3d)$$

$$0 \leq d_{jc} \leq \bar{d}_{jc} : (\underline{\beta}_{ijc}^{pd}, \bar{\beta}_{ijc}^{pd}), \forall j \in J, \forall c \in C_j \quad (4.3e)$$

$$o_{ku} - \lambda - \underline{\mu}_{ku}^p + \bar{\mu}_{ku}^p = 0 : (\alpha_{iku}^{pp}), \forall k \in I : k \neq i, \forall u \in U_k \quad (4.3f)$$

$$o_{iu} - \lambda - \underline{\mu}_{iu}^p + \bar{\mu}_{iu}^p = 0 : (\alpha_{iiu}^{pp}), \forall u \in U_i \quad (4.3g)$$

$$-b_{jc} + \lambda - \underline{\mu}_{jc}^d + \bar{\mu}_{jc}^d = 0 : (\alpha_{ijc}^{pd}), \forall j \in J, \forall c \in C_j \quad (4.3h)$$

$$\bar{\mu}_{ku}^p \geq 0 : (\bar{\theta}_{iku}^{pp}), \forall k \in I, \forall u \in U_k \quad (4.3i)$$

$$\bar{\mu}_{jc}^d \geq 0 : (\bar{\theta}_{ijc}^{pd}), \forall j \in J, \forall c \in C_j \quad (4.3j)$$

$$\underline{\mu}_{ku}^p \geq 0 : (\underline{\theta}_{iku}^{pp}), \forall k \in I, \forall u \in U_k \quad (4.3k)$$

$$\underline{\mu}_{jc}^d \geq 0 : (\underline{\theta}_{ijc}^{pd}), \forall j \in J, \forall c \in C_j \quad (4.3l)$$

$$\sum_{u \in U_i} o_{iu} p_{iu} + \sum_{k \in I : k \neq i} \sum_{u \in U_k} o_{ku} p_{ku} - \sum_{j \in J} \sum_{c \in C_j} b_{jc} d_{jc} + \sum_{k \in I} \sum_{u \in U_k} \bar{p}_{ku} \bar{\mu}_{ku}^p + \sum_{j \in J} \sum_{c \in C_j} \bar{d}_{jc} \bar{\mu}_{jc}^d = 0 : (\gamma_i^p). \quad (4.3m)$$

MPEC for the consumer problem j (MPEC-CP $_j$)

$$\begin{aligned} & \underset{\Xi_j^{cp}}{Max} \quad \sum_{c \in C_j} (u_{jc} d_{jc} - \lambda d_{jc}) \\ & s.t. : \end{aligned} \quad (4.4a)$$

$$b_{jc} \leq u_{jc} : (\eta_{jc}^d), \forall c \in C_j \quad (4.4b)$$

$$\sum_{i \in I} \sum_{u \in U_i} p_{iu} - \sum_{l \in J} \sum_{c \in C_j} d_{lc} = 0 : (\delta_j^d) \quad (4.4c)$$

$$0 \leq p_{iu} \leq \bar{p}_{iu} : (\underline{\beta}_{jiu}^{dp}, \bar{\beta}_{jiu}^{dp}), \forall i \in I, \forall u \in U_i \quad (4.4d)$$

$$0 \leq d_{lc} \leq \bar{d}_{lc} : (\underline{\beta}_{jlc}^{dd}, \bar{\beta}_{jlc}^{dd}), \forall l \in J, \forall c \in C_l \quad (4.4e)$$

$$o_{iu} - \lambda + \bar{\mu}_{iu}^p - \underline{\mu}_{iu}^p = 0 : (\alpha_{jiu}^{dp}), \forall i \in I, \forall u \in U_i \quad (4.4f)$$

$$-b_{lc} + \lambda + \bar{\mu}_{lc}^d - \underline{\mu}_{lc}^d = 0 : (\alpha_{jlc}^{dd}), \forall l \in J : l \neq j, \forall c \in C_l \quad (4.4g)$$

$$-b_{jc} + \lambda + \bar{\mu}_{jc}^d - \underline{\mu}_{jc}^d = 0 : (\alpha_{jjc}^{dd}), \forall c \in C_j \quad (4.4h)$$

$$\bar{\mu}_{iu}^p \geq 0 : \left(\bar{\theta}_{jiu}^{dp} \right), \forall i \in I, \forall u \in U_i \quad (4.4i)$$

$$\bar{\mu}_{lc}^d \geq 0 : \left(\bar{\theta}_{jlc}^{dd} \right), \forall l \in J, \forall c \in C_l \quad (4.4j)$$

$$\underline{\mu}_{iu}^p \geq 0 : \left(\underline{\theta}_{jiu}^{dp} \right), \forall i \in I, \forall u \in U_i \quad (4.4k)$$

$$\underline{\mu}_{lc}^d \geq 0 : \left(\underline{\theta}_{jlc}^{dd} \right), \forall l \in J, \forall c \in C_l \quad (4.4l)$$

$$\sum_{i \in I} \sum_{u \in U_i} o_i p_i - \sum_{c \in C_j} b_j d_j - \sum_{l \in J: l \neq j} \sum_{c \in C_l} b_l d_l + \sum_{i \in I} \sum_{u \in U_i} \bar{p}_i \bar{\mu}_i^p + \sum_{l \in J} \sum_{c \in C_l} \bar{d}_l \bar{\mu}_l^d = 0 : \left(\gamma_j^d \right). \quad (4.4m)$$

4.4 Market equilibrium involving convex conic MPECs

The market equilibrium is determined by jointly solving all MPECs (4.3) and (4.4), which together form an EPEC. In the literature, the joint solution of MPEC problems is typically addressed by replacing these problems with their respective optimality conditions, which are then solved as a unified set of constraints. Most solution techniques in the literature rely on the KKT conditions associated with each MPEC as the optimality conditions, as outlined in (CONEJO; RUIZ, 2020). However, this approach often leads to a large number of nonlinear constraints, which are challenging to solve due to the KKT complementarity constraints. An alternative methodology for solving the proposed EPEC problem is presented next.

4.4.1 EPEC solution methodology

In this work, we propose a novel approach in which the set of optimality conditions associated with the MPECs is represented by PDOCs. However, for this approach to be feasible, all MPECs must have explicit dual representations. This is generally not possible for most MPECs, particularly those whose optimality conditions are defined by the KKT conditions. It is also not possible for the MPEC problems described in (4.3) and (4.4) due to the presence of bilinear terms.

To overcome this challenge, we propose reformulating approximately these problems as rotated quadratic conic optimization problems, which have explicit and exact dual counterparts. Specifically, we address this issue in two steps: (i) the MPECs are reformulated approximately as convex optimization models, as discussed in Section 4.4.1.1, and (ii) we use cones to further recast the MPECs into convex rotated quadratic conic models, as described in Section 4.4.1.2, enabling explicit and exact dualization. The final EPEC reformulation as a single optimization problem is detailed in Section 4.4.1.3, and its solution is carried out using an iterative procedure outlined in Section 4.4.1.4. The iterative procedure is needed due to the approximations required to derive rotated quadratic cone formulations.

4.4.1.1 Convexification of the MPEC problems

The proposed methodology begins by reorganizing the bilinear terms to obtain a single set of bilinear terms for each MPEC (i.e., (4.3) and (4.4)). To identify this unified set of bilinear terms for each MPEC, the strong duality equality is mathematically manipulated using the complementarity constraints derived from the KKT conditions associated with each MPEC, as detailed in 4.4.1.1. This step reduces the nonlinearities to a single set of bilinear terms in each MPEC, thereby simplifying the convexification process.

Single bilinear term

In this section, the strong duality equality constraints associated with the lower-level problems in (4.1) and (4.2) are rewritten in terms of a single set of bilinear terms, which involves some algebraic manipulation, as detailed below. In the model for strategic producer i , the single set of bilinear terms corresponds to its revenue, $\lambda p_{iu}, \forall u \in U_i$, while in the model for strategic consumer j , the single set of bilinear terms corresponds to its utility, $\lambda d_{jc}, \forall c \in C_j$.

Initially, multiplying (4.3g) by p_{iu} yields (4.5), and multiplying (4.4h) by d_{jc} yields (4.6), for the producer's and consumer's problems, respectively.

$$o_{iu}p_{iu} - \lambda p_{iu} - \underline{\mu}_{iu}^p p_{iu} + \bar{\mu}_{iu}^p p_{iu} = 0, \quad \forall u \in U_i \quad (4.5)$$

$$-b_{jc}d_{jc} + \lambda d_{jc} + \bar{\mu}_{jc}^d d_{jc} - \underline{\mu}_{jc}^d d_{jc} = 0, \quad \forall c \in C_j. \quad (4.6)$$

From the complementarity conditions (4.3d), associated with the upper and lower bounds on p_{iu} , we have $\bar{\mu}_{iu}^p p_{iu} = \bar{\mu}_{iu}^p \bar{p}_{iu}$ and $\underline{\mu}_{iu}^p p_{iu} = 0$. Substituting these into (4.5), we obtain (4.7). Similarly, from the complementarity conditions (4.4e), associated with the upper and lower bounds on d_{jc} , we have $\bar{\mu}_{jc}^d d_{jc} = \bar{\mu}_{jc}^d \bar{d}_{jc}$ and $\underline{\mu}_{jc}^d d_{jc} = 0$. Substituting these into (4.6), we obtain (4.8).

$$o_{iu}p_{iu} = \lambda p_{iu} - \bar{\mu}_{iu}^p \bar{p}_{iu}, \quad \forall u \in U_i \quad (4.7)$$

$$b_{jc}d_{jc} = \bar{\mu}_{jc}^d \bar{d}_{jc} + \lambda d_{jc}, \quad \forall c \in C_j. \quad (4.8)$$

Finally, by substituting (4.7) into (4.3m) and (4.8) into (4.4m), we obtain (4.9) and (4.10), respectively. In this case, (4.9) replaces (4.3m), and (4.10) replaces (4.4m). Through this algebraic manipulation, we derive a single set of bilinear terms present in the producer's and consumer's problems, as given in (4.3) and (4.4):

$$\sum_{u \in U_i} \lambda p_{iu} + \sum_{k \in I: k \neq i} \sum_{u \in U_k} o_{ku} p_{ku} - \sum_{j \in J} \sum_{c \in C_j} b_{jc} d_{jc} + \sum_{k \in I: k \neq i} \sum_{u \in U_k} \bar{p}_{ku} \bar{\mu}_{ku}^p + \sum_{j \in J} \sum_{c \in C_j} \bar{d}_{jc} \bar{\mu}_{jc}^d = 0 \quad (4.9)$$

$$\sum_{i \in I} \sum_{u \in U_i} o_{iu} p_{iu} - \sum_{c \in C_j} \lambda d_{jc} - \sum_{l \in J: l \neq j} \sum_{c \in C_l} b_{lc} d_{lc} + \sum_{i \in I} \sum_{u \in U_i} \bar{p}_{iu} \bar{\mu}_{iu}^p + \sum_{l \in J: l \neq j} \sum_{c \in C_l} \bar{d}_{lc} \bar{\mu}_{lc}^d = 0. \quad (4.10)$$

After recasting MPEC problems (4.3) or (4.4) to contain a single set of bilinear terms, the next step is to convexify these bilinear terms in each MPEC model, as described below.

Convexification

The convexification of the bilinear terms in the producer's and consumer's problems (4.3) and (4.4) begins by associating each set of bilinear terms with newly introduced variables, ψ_{iu}^p and ψ_{jc}^d , as described in (4.11) and (4.12), respectively:

$$\psi_{iu}^p = \lambda p_{iu}, \quad \forall u \in U_i \quad (4.11)$$

$$\psi_{jc}^d = \lambda d_{jc}, \quad \forall c \in C_j. \quad (4.12)$$

Note that (4.11) and (4.12) can be rewritten as differences of squares in (4.13) and (4.14), respectively.

$$\psi_{iu}^p = \frac{1}{4}(\lambda + p_{iu})^2 - \frac{1}{4}(\lambda - p_{iu})^2, \quad \forall u \in U_i \quad (4.13)$$

$$\psi_{jc}^d = \frac{1}{4}(\lambda + d_{jc})^2 - \frac{1}{4}(\lambda - d_{jc})^2, \quad \forall c \in C_j. \quad (4.14)$$

To rewrite problems (4.3) and (4.4) as rotated quadratic conic constraints, the constraints (4.13) and (4.14) are relaxed to (4.15) and (4.16), respectively:

$$\frac{1}{4}(\lambda + p_{iu})^2 - \psi_{iu}^p - \frac{1}{4}(\lambda - p_{iu})^2 \leq 0, \quad \forall u \in U_i \quad (4.15)$$

$$\frac{1}{4}(\lambda + d_{jc})^2 - \psi_{jc}^d - \frac{1}{4}(\lambda - d_{jc})^2 \leq 0, \quad \forall c \in C_j. \quad (4.16)$$

The terms $\frac{1}{4}(\lambda - p_{iu})^2$ and $\frac{1}{4}(\lambda - d_{jc})^2$ from (4.15) and (4.16), respectively, are linearized, using a first order Taylor series expansion around an initial given point $(\lambda^{(v)}, p_{iu}^{(v)})$ in (4.17) and around an initial given point $(\lambda^{(v)}, d_{jc}^{(v)})$ in (4.18), yielding (4.19) and (4.20), respectively. These convexified constraints are used in problems (4.3) and (4.4), respectively:

$$\frac{1}{4}(\lambda - p_{iu})^2 \approx \left(b_{iu}^{(v)}\right)^2 + b_{iu}^{(v)} (\lambda - \lambda^{(v)}) - b_{iu}^{(v)} (p_{iu} - p_{iu}^{(v)}), \quad \forall u \in U_i \quad (4.17)$$

$$\frac{1}{4}(\lambda - d_{jc})^2 \approx \left(y_{jc}^{(v)}\right)^2 + y_{jc}^{(v)} (\lambda - \lambda^{(v)}) - y_{jc}^{(v)} (d_{jc} - d_{jc}^{(v)}), \quad \forall c \in C_j \quad (4.18)$$

$$\frac{1}{4}(\lambda + p_{iu})^2 - \psi_{iu}^p - \left(\left(b_{iu}^{(v)}\right)^2 + b_{iu}^{(v)} (\lambda - \lambda^{(v)}) - b_{iu}^{(v)} (p_{iu} - p_{iu}^{(v)})\right) \leq \varepsilon_{iu}^p, \quad \forall u \in U_i \quad (4.19)$$

$$\frac{1}{4}(\lambda + d_{jc})^2 - \psi_{jc}^d - \left((y_{jc}^{(v)})^2 + y_{jc}^{(v)} (\lambda - \lambda^{(v)}) - y_{jc}^{(v)} (d_{jc} - d_{jc}^{(v)}) \right) \leq \varepsilon_{jc}^d, \quad \forall c \in C_j, \quad (4.20)$$

where:

$$b_{iu}^{(v)} = \frac{1}{2} (\lambda^{(v)} - p_{iu}^{(v)}), \quad \forall u \in U_i \quad (4.21)$$

$$y_{jc}^{(v)} = \frac{1}{2} (\lambda^{(v)} - d_{jc}^{(v)}), \quad \forall c \in C_j. \quad (4.22)$$

Moreover, non-negative tolerance variables $\varepsilon_{iu}^p \geq 0, \forall u \in U_i$ and $\varepsilon_{jc}^d \geq 0, \forall c \in C_j$ are introduced to relax (4.19) and (4.20), respectively. This procedure enables the convexification method to start from infeasible points. Additionally, the penalty term $\rho^{(v)} \varepsilon_{iu}^p$ is added to the objective function of the producer's problem (4.3a), and the penalty term $\sigma^{(v)} \varepsilon_{jc}^d$ is added to the objective function of the consumer's problem (4.4a), with the goal of driving both ε_{iu}^p and ε_{jc}^d to values close to 0.

Finally, the bilinear term λp_{iu} is replaced by ψ_{iu}^p in (4.9) and in the objective function (4.3a), yielding (4.23) and (4.25). Similarly, the bilinear term λd_{jc} is replaced by ψ_{jc}^d in (4.10) and in the objective function (4.4a), yielding (4.24) and (4.26):

$$\sum_{u \in U_i} \psi_{iu}^p + \sum_{k \in I: k \neq i} \sum_{u \in U_k} o_{ku} p_{ku} - \sum_{j \in J} \sum_{c \in C_j} b_{jc} d_{jc} + \sum_{k \in I: k \neq i} \sum_{u \in U_k} \bar{p}_{ku} \bar{\mu}_{ku}^p + \sum_{j \in J} \sum_{c \in C_j} \bar{d}_{jc} \bar{\mu}_{jc}^d = 0 \quad (4.23)$$

$$\sum_{i \in I} \sum_{u \in U_i} o_{iu} p_{iu} - \sum_{c \in C_j} \psi_{jc}^d - \sum_{l \in J: l \neq j} \sum_{c \in C_l} b_{lc} d_{lc} + \sum_{i \in I} \sum_{u \in U_i} \bar{p}_{iu} \bar{\mu}_{iu}^p + \sum_{l \in J: l \neq j} \sum_{c \in C_l} \bar{d}_{lc} \bar{\mu}_{lc}^d = 0 \quad (4.24)$$

$$\underset{\Xi_i^{pp}}{\text{Min}} \sum_{u \in U_i} \left(-\psi_{iu}^p + c_{iu} p_{iu} + \rho^{(v)} \varepsilon_{iu}^p \right) \quad (4.25)$$

$$\underset{\Xi_j^{cp}}{\text{Min}} \sum_{c \in C_j} \left(-\psi_{jc}^d + u_{jc} d_{jc} + \sigma^{(v)} \varepsilon_{jc}^d \right), \quad (4.26)$$

where $\rho^{(v)}$ and $\sigma^{(v)}$ are updated at each iteration v , as shown in (4.27) and (4.28), respectively:

$$\rho^{(v+1)} = \min \left\{ \bar{\rho}, \mu \rho^{(v)} \right\} \quad (4.27)$$

$$\sigma^{(v+1)} = \min \left\{ \bar{\sigma}, \omega \sigma^{(v)} \right\}, \quad (4.28)$$

where $\bar{\rho}$ and $\bar{\sigma}$ are the maximum admissible value for $\rho^{(v+1)}$ and $\sigma^{(v+1)}$, respectively; and μ and ω are the updating step rates in (4.27) and (4.28), respectively.

Once the convexification process is applied, problems (4.3) and (4.4) are reformu-

lated as (4.29) and (4.30), respectively.

$$\begin{aligned}
 & \underset{\Xi_i^{pp}}{\text{Min}} \quad \sum_{u \in U_i} \left(-\psi_{iu}^p + c_{iu} p_{iu} + \rho^{(v)} \varepsilon_{iu}^p \right) \\
 & \text{s.t. :} \\
 & \quad (4.3b) - (4.3l) \quad (a) \\
 & \quad \varepsilon_{iu}^p \geq 0, \forall u \in U_i \quad (b) \\
 & \quad (4.23) \quad (c) \\
 & \quad (4.19) \quad (d).
 \end{aligned} \tag{4.29}$$

$$\begin{aligned}
 & \underset{\Xi_j^{cp}}{\text{Min}} \quad \sum_{c \in C_j} \left(-u_{jc} d_{jc} + \psi_{jc}^d + \sigma^{(v)} \varepsilon_{jc}^d \right) \\
 & \text{s.t. :} \\
 & \quad (4.4b) - (4.4l) \quad (a) \\
 & \quad \varepsilon_{jc}^d \geq 0, \forall c \in C_j \quad (b) \\
 & \quad (4.24) \quad (c) \\
 & \quad (4.20) \quad (d).
 \end{aligned} \tag{4.30}$$

However, problems (4.29) and (4.30) do not have an explicit and exact dual formulation as expected, due to constraints (4.29)(d) and (4.30)(d), respectively. This limitation prevents the derivation of PDOCs for these problems, rendering the proposed methodology for obtaining a single EPEC formulation for market equilibrium inapplicable. To address this issue, we propose an additional reformulation of (4.29) and (4.30) as rotated quadratic conic models, as described in the following section.

4.4.1.2 Rotated quadratic conic reformulation

The reformulation of problems (4.29) and (4.30) into rotated quadratic conic optimization problems involves rewriting constraints (4.19) and (4.20), as rotated quadratic cones. To achieve this, the constants and linear components of these constraints are moved to the right-hand side of the inequalities, multiplied by 4, and defined as auxiliary variables t_{iu}^p and t_{jc}^d , respectively. This reformulation yields constraints (4.31) and (4.32):

$$t_{iu}^p = 4\psi_{iu}^p + 4 \left(\left(b_{iu}^{(v)} \right)^2 + b_{iu}^{(v)} \left(\lambda - \lambda^{(v)} \right) - b_{iu}^{(v)} \left(p_{iu} - p_{iu}^{(v)} \right) \right) + 4\varepsilon_{iu}^p, \quad \forall u \in U_i \tag{4.31a}$$

$$t_{iu}^p \geq (\lambda + p_{iu})^2, \quad \forall u \in U_i \tag{4.31b}$$

$$t_{jc}^d = 4\psi_{jc}^d + 4 \left(\left(y_{jc}^{(v)} \right)^2 + y_{jc}^{(v)} \left(\lambda - \lambda^{(v)} \right) - y_{jc}^{(v)} \left(d_{jc} - d_{jc}^{(v)} \right) \right) + 4\varepsilon_{jc}^d, \quad \forall c \in C_j \tag{4.32a}$$

$$t_{jc}^d \geq (\lambda + d_{jc})^2, \quad \forall c \in C_j. \tag{4.32b}$$

The quadratic terms in (4.31b) and (4.32b) are expanded and then multiplied by 2, respectively, resulting in constraints (4.33) and (4.34), respectively:

$$2t_{iu}^p \geq 2\lambda^2 + 2p_{iu}^2 + 4\lambda p_{iu}, \quad \forall u \in U_i \quad (4.33)$$

$$2t_{jc}^d \geq 2\lambda^2 + 2d_{jc}^2 + 4\lambda d_{jc}, \quad \forall c \in C_j. \quad (4.34)$$

The inequalities (4.33) and (4.34) can be rewritten in vector form as in (4.35) and (4.36), respectively.

$$2t_{iu}^p \geq \begin{bmatrix} \lambda & p_{iu} \end{bmatrix} \begin{bmatrix} 2 & 2 \\ 2 & 2 \end{bmatrix} \begin{bmatrix} \lambda \\ p_{iu} \end{bmatrix}, \quad \forall u \in U_i \quad (4.35)$$

$$2t_{jc}^d \geq \begin{bmatrix} \lambda & d_{jc} \end{bmatrix} \begin{bmatrix} 2 & 2 \\ 2 & 2 \end{bmatrix} \begin{bmatrix} \lambda \\ d_{jc} \end{bmatrix}, \quad \forall c \in C_j. \quad (4.36)$$

Defining matrix Q in (4.35) and (4.36), as (4.37):

$$Q = \begin{bmatrix} 2 & 2 \\ 2 & 2 \end{bmatrix}, \quad (4.37)$$

and noticing that Q is a symmetric positive semi-definite matrix, we may factorize it, such that $Q = F^T F$, where the factorization matrix F is given by (4.38):

$$F = \begin{bmatrix} \sqrt{2} & \sqrt{2} \\ 0 & 0 \end{bmatrix}. \quad (4.38)$$

According to (MOSEK ApS, 2024), an n -dimensional rotated quadratic cone Q_r^n is defined as:

$$Q_r^n = \left\{ x \in \mathbb{R}^n : 2x_1x_2 \geq x_3^2 + \dots + x_n^2, x_1, x_2 \geq 0 \right\}. \quad (4.39)$$

It can be proven that when Q is a symmetric positive semi-definite matrix with a factorization matrix $F \in \mathbb{R}^{k \times n}$, the convex set defined by $2t \geq x^T Q x$ is a rotated quadratic cone spanned by the vector $(1, t, Fx)$, as shown in (4.40):

$$2t \geq x^T F^T F x \quad \Leftrightarrow \quad 2t \geq (Fx)^T Fx \quad \Leftrightarrow \quad 2t \geq \|Fx\|_2^2 \quad \Leftrightarrow \quad (1, t, Fx) \in Q_r^{k+2}. \quad (4.40)$$

We observe that constraints (4.35) and (4.36) share the same structure as the convex set in (4.40). Therefore, these constraints can also be expressed as rotated quadratic cones, using matrix Q from (4.37) and its factorization matrix F from (4.38), resulting in constraints (4.41) and (4.42), respectively:

$$2t_{iu}^p \geq \left\| \begin{bmatrix} \sqrt{2} & \sqrt{2} \\ 0 & 0 \end{bmatrix} \begin{bmatrix} \lambda \\ p_{iu} \end{bmatrix} \right\|_2^2, \quad \forall u \in U_i \quad (4.41a)$$

$$2t_{iu}^p \geq \left\| \begin{bmatrix} \sqrt{2}\lambda + \sqrt{2}p_{iu} \\ 0 \end{bmatrix} \right\|_2^2, \quad \forall u \in U_i \quad (4.41b)$$

$$2t_{jc}^d \geq \left\| \begin{bmatrix} \sqrt{2} & \sqrt{2} \\ 0 & 0 \end{bmatrix} \begin{bmatrix} \lambda \\ d_{jc} \end{bmatrix} \right\|_2^2, \quad \forall c \in C_j \quad (4.42a)$$

$$2t_{jc}^d \geq \left\| \begin{bmatrix} \sqrt{2}\lambda + \sqrt{2}d_{jc} \\ 0 \end{bmatrix} \right\|_2^2, \quad \forall c \in C_j. \quad (4.42b)$$

If the vectors $\alpha_{iu}^p = (\sqrt{2}\lambda + \sqrt{2}p_{iu}, 0)^T$ and $\alpha_{jc}^d = (\sqrt{2}\lambda + \sqrt{2}d_{jc}, 0)^T$ are defined, we observe, by using the last identity in (4.40), that the vectors $(1, t_{iu}^p, \alpha_{iu}^p)^T$ and $(1, t_{jc}^d, \alpha_{jc}^d)^T$ span the rotated quadratic cones described in (4.41) and (4.42), respectively. Finally, by combining (4.31a) with (4.41b), and (4.32a) with (4.42b), the associated auxiliary variables t_{iu}^p and t_{jc}^d can be eliminated, resulting in the final conic convex term (4.43) for the strategic producer problem, and (4.44) for the strategic consumer problem:

$$2 \left(4\psi_{iu}^p + 4b_{iu}^{(v)} (\lambda - p_{iu}) + 4\varepsilon_{iu}^p + 4 \left((b_{iu}^{(v)})^2 - b_{iu}^{(v)} \lambda^{(v)} + b_{iu}^{(v)} p_{iu}^{(v)} \right) \right) \geq \|p_{iu}\sqrt{2} + \lambda\sqrt{2}\|_2^2, \quad \forall u \in U_i \quad (4.43)$$

$$2 \left(4\psi_{jc}^d + 4y_{jc}^{(v)} (\lambda - d_{jc}) + 4\varepsilon_{jc}^d + 4 \left((y_{jc}^{(v)})^2 - y_{jc}^{(v)} \lambda^{(v)} + y_{jc}^{(v)} d_{jc}^{(v)} \right) \right) \geq \|d_{jc}\sqrt{2} + \lambda\sqrt{2}\|_2^2, \quad \forall c \in C_j. \quad (4.44)$$

After deriving the associated conic formulations for problems (4.29) and (4.30), given in (4.43) and (4.44), we obtain the rotated conic versions of (MPEC-PP_i) for the producer problem *i* and (MPEC-CP_j) for the consumer problem *j*, as shown in (4.45) and (4.46), respectively. These problems are not only convex but also written in a form that allows for explicit and exact dualization. This feature enables the use of PDOCs for each problem, which can then be combined to reformulate the EPEC problem as a single optimization problem, as discussed in the next section.

$$\begin{aligned} \underset{\Xi_i^{pp}}{\text{Min}} \quad & \sum_{u \in U_i} \left(-\psi_{iu}^p + c_{iu} p_{iu} + \rho^{(v)} \varepsilon_{iu}^p \right) \\ \text{s.t. :} \quad & \\ & (4.3b) - (4.3l) \quad (a) \\ & \varepsilon_{iu}^p \geq 0, \forall u \in U_i \quad (b) \\ & (4.23) \quad (c) \\ & (4.43) \quad (d). \end{aligned} \quad (4.45)$$

$$\begin{aligned}
 & \underset{\Xi_j^{cp}}{\text{Min}} \quad \sum_{c \in C_j} \left(-u_{jc} d_{jc} + \psi_{jc}^d + \sigma^{(v)} \varepsilon_{jc}^d \right) \\
 & \text{s.t. :} \\
 & \quad (4.4b) - (4.4l) \quad (a) \\
 & \quad \varepsilon_{jc}^d \geq 0, \forall c \in C_j \quad (b) \\
 & \quad (4.24) \quad (c) \\
 & \quad (4.44) \quad (d).
 \end{aligned} \tag{4.46}$$

4.4.1.3 Finding a single EPEC problem reformulation

In this section, the EPEC problem is formulated as a single optimization problem, following the proposed methodology. For this purpose, the rotated quadratic conic formulations obtained in (4.45) and (4.46) are replaced with their respective PDOCs. To achieve this, their corresponding dual problems are detailed bellow.

Dual Problem for the Producer MPEC

The dual problem associated with the rotated conic version of MPEC-PP_i for the producer problem *i*, given in (4.45), is shown in (4.47):

$$\begin{aligned}
 & \underset{\Xi_i^{pd}}{\text{Max}} \quad \sum_{u \in U_i} c_i \eta_i^p - \sum_{k \in I} \sum_{u \in U_k} \bar{p}_k \bar{\beta}_{ik}^{pp} - \sum_{j \in J} \sum_{c \in C_j} \bar{d}_j \bar{\beta}_{ij}^{pd} - \sum_{k \in I: k \neq i} \sum_{u \in U_k} o_k \alpha_{ik}^{pp} + \sum_{j \in J} \sum_{c \in C_j} b_j \alpha_{ij}^{pd} \\
 & \text{s.t. :}
 \end{aligned} \tag{4.47a}$$

$$-1 - \gamma_i^p - 4s_{iu}^{bp} = 0, \quad \forall u \in U_i \tag{4.47b}$$

$$-\eta_{iu}^p - \alpha_{iiu}^{pp} = 0, \quad \forall u \in U_i \tag{4.47c}$$

$$c_{iu} - \delta_i^p - \underline{\beta}_{iiu}^{pp} + \bar{\beta}_{iiu}^{pp} + 4b_{iu}^{(v)} s_{iu}^{bp} - \sqrt{2} s_{iu}^{cp} = 0, \quad \forall u \in U_i \tag{4.47d}$$

$$-\delta_i^p - \underline{\beta}_{iku}^{pp} + \bar{\beta}_{iku}^{pp} - o_{ku} \gamma_i^p = 0, \quad \forall k \in I : k \neq i, \forall u \in U_k \tag{4.47e}$$

$$\delta_i^p - \underline{\beta}_{ijc}^{pd} + \bar{\beta}_{ijc}^{pd} + b_{jc} \gamma_i^p = 0, \quad \forall j \in J, \forall c \in C_j \tag{4.47f}$$

$$\sum_{k \in I} \sum_{u \in U_k} \alpha_{iku}^{pp} - \sum_{j \in J} \sum_{c \in C_j} \alpha_{ijc}^{pd} - \sum_{u \in U_i} 4b_{iu}^{(v)} s_{iu}^{bp} - \sum_{u \in U_i} \sqrt{2} s_{iu}^{cp} = 0 \tag{4.47g}$$

$$-\alpha_{iku}^{pp} - \bar{\theta}_{iku}^{pp} - \bar{p}_{ku} \gamma_i^p = 0, \quad \forall k \in I : k \neq i, u \in U_k \tag{4.47h}$$

$$-\alpha_{iiu}^{pp} - \bar{\theta}_{iiu}^{pp} = 0, \quad u \in U_i \tag{4.47i}$$

$$\alpha_{iku}^{pp} - \bar{\theta}_{iku}^{pp} = 0, \quad \forall k \in I, u \in U_k \tag{4.47j}$$

$$-\alpha_{ijc}^{pd} - \bar{\theta}_{ijc}^{pd} - \bar{d}_{jc} \gamma_i^p = 0, \quad \forall j \in J, c \in C_j \tag{4.47k}$$

$$\alpha_{ijc}^{pd} - \bar{\theta}_{ijc}^{pd} = 0, \quad \forall j \in J, c \in C_j \tag{4.47l}$$

$$\rho^{(v)} - \gamma_{iu}^{ep} - 4s_{iu}^{bp} = 0, \quad u \in U_i \tag{4.47m}$$

$$2s_{iu}^{ap} s_{iu}^{bp} \geq \|s_{iu}^{cp}\|_2^2, \quad u \in U_i \tag{4.47n}$$

$$\eta_{iu}^p, \gamma_{iu}^{\varepsilon p}, s_{iu}^{ap}, s_{iu}^{bp} \geq 0, \quad \forall u \in U_i \quad (4.47o)$$

$$\underline{\beta}_{iku}^{pp}, \bar{\beta}_{iku}^{pp}, \underline{\theta}_{iku}^{pp}, \bar{\theta}_{iku}^{pp} \geq 0, \quad \forall k \in I, \forall u \in U_k \quad (4.47p)$$

$$\underline{\beta}_{ijc}^{pd}, \bar{\beta}_{ijc}^{pd}, \underline{\theta}_{ijc}^{pd}, \bar{\theta}_{ijc}^{pd} \geq 0, \quad \forall j \in J, \forall c \in C_j, \quad (4.47q)$$

where:

$$b_{iu}^{(v)} = \frac{1}{2} \left(\lambda^{(v)} - p_{iu}^{(v)} \right), \quad \forall u \in U_i. \quad (4.48)$$

Dual problem for the consumer MPEC

The dual problem associated with the rotated conic version of MPEC-CP_j for the consumer problem j , given in (4.46), is shown in (4.49):

$$\begin{aligned} \underset{\Xi_j^{cd}}{Max} \quad & - \sum_{c \in C_j} u_{jc} \eta_{jc}^d - \sum_{i \in I} \sum_{u \in U_i} \bar{p}_{iu} \bar{\beta}_{jiu}^{dp} - \sum_{l \in J} \sum_{c \in C_l} \bar{d}_{lc} \bar{\beta}_{jlc}^{dd} - \sum_{i \in I} \sum_{u \in U_i} o_{iu} \alpha_{jiu}^{dp} + \sum_{l \in J: l \neq j} \sum_{c \in C_l} b_{lc} \alpha_{jlc}^{dd} \\ s.t. : \end{aligned} \quad (4.49a)$$

$$1 + \gamma_j^d - 4s_{jc}^{bd} = 0, \quad \forall c \in C_j \quad (4.49b)$$

$$\eta_{jc}^d + \alpha_{jjc}^{dd} = 0, \quad \forall c \in C_j \quad (4.49c)$$

$$-u_{jc} + \delta_j^d - \underline{\beta}_{jjc}^{dd} + \bar{\beta}_{jjc}^{dd} + 4s_{jc}^{bd} y_{jc}^{(v)} - s_{jc}^{cd} \sqrt{2} = 0, \quad \forall c \in C_j \quad (4.49d)$$

$$-\delta_j^d - \underline{\beta}_{jiu}^{dp} + \bar{\beta}_{jiu}^{dp} - o_{iu} \gamma_j^d = 0, \quad \forall i \in I, \forall u \in U_i \quad (4.49e)$$

$$\delta_j^d - \underline{\beta}_{jlc}^{dd} + \bar{\beta}_{jlc}^{dd} - b_{lc} \gamma_j^d = 0, \quad \forall l \in J : l \neq j, \forall c \in C_l \quad (4.49f)$$

$$\sum_{i \in I} \sum_{u \in U_i} \alpha_{jiu}^{dp} - \sum_{l \in J} \sum_{c \in C_l} \alpha_{jlc}^{dd} - \sum_{c \in C_j} 4s_{jc}^{bd} y_{jc}^{(v)} - \sum_{c \in C_j} s_{jc}^{cd} \sqrt{2} = 0 \quad (4.49g)$$

$$-\alpha_{jiu}^{dp} - \bar{\theta}_{jiu}^{dp} - \bar{p}_{iu} \gamma_j^d = 0, \quad \forall i \in I, \forall u \in U_i \quad (4.49h)$$

$$-\alpha_{jlc}^{dd} - \bar{\theta}_{jlc}^{dd} - \bar{d}_{lc} \gamma_j^d = 0, \quad \forall l \in J : l \neq j, \forall c \in C_l \quad (4.49i)$$

$$-\alpha_{jjc}^{dd} - \bar{\theta}_{jjc}^{dd} = 0, \quad \forall c \in C_j \quad (4.49j)$$

$$\alpha_{jiu}^{dp} - \underline{\theta}_{jiu}^{dp} = 0, \quad \forall i \in I, \forall u \in U_i \quad (4.49k)$$

$$\alpha_{jlc}^{dd} - \underline{\theta}_{jlc}^{dd} = 0, \quad \forall l \in J, \forall c \in C_j \quad (4.49l)$$

$$\sigma^{(v)} - \gamma_{jc}^{\varepsilon d} - 4s_{jc}^{bd} = 0, \quad \forall c \in C_j \quad (4.49m)$$

$$2s_{jc}^{ad} s_{jc}^{bd} \geq \|s_{jc}^{cd}\|_2^2, \quad \forall c \in C_j \quad (4.49n)$$

$$\eta_{jc}^d, \gamma_{jc}^{\varepsilon d}, s_{jc}^{ad}, s_{jc}^{bd} \geq 0, \quad \forall c \in C_j \quad (4.49o)$$

$$\underline{\beta}_{jiu}^{dp}, \bar{\beta}_{jiu}^{dp}, \underline{\theta}_{jiu}^{dp}, \bar{\theta}_{jiu}^{dp} \geq 0, \quad \forall i \in I, \forall u \in U_i \quad (4.49p)$$

$$\underline{\beta}_{jlc}^{dd}, \bar{\beta}_{jlc}^{dd}, \underline{\theta}_{jlc}^{dd}, \bar{\theta}_{jlc}^{dd} \geq 0, \quad \forall l \in J, \forall c \in C_l, \quad (4.49q)$$

where:

$$y_{jc}^{(v)} = \frac{1}{2} \left(\lambda^{(v)} - d_{jc}^{(v)} \right), \quad c \in C_j. \quad (4.50)$$

Single equilibrium problem reformulation

The strong duality equality constraints for problem (4.45) are found by equating (4.25) with (4.47a), yielding (4.51). Similarly, the strong duality equality constraints for problem (4.46) are found by equating (4.26) with (4.49a), yielding (4.52).

$$\begin{aligned} & \sum_{u \in U_i} \left(\psi_i^p - c_i p_i - \rho^{(v)} \varepsilon_i^p \right) + \sum_{u \in U_i} c_i \eta_i^p - \sum_{k \in I} \sum_{u \in U_k} \bar{p}_k \bar{\beta}_{ik}^{pp} \\ & - \sum_{j \in J} \sum_{c \in C_j} \bar{d}_j \bar{\beta}_{ij}^{pd} - \sum_{k \in I: k \neq i} \sum_{u \in U_k} o_k \alpha_{ik}^{pp} + \sum_{j \in J} \sum_{c \in C_j} b_j \alpha_{ij}^{pd} = 0 \end{aligned} \quad (4.51)$$

$$\begin{aligned} & \sum_{c \in C_j} \left(u_{jc} d_{jc} - \psi_{jc}^d - \sigma^{(v)} \varepsilon_{jc}^d \right) - \sum_{c \in C_j} u_{jc} \eta_{jc}^d - \sum_{i \in I} \sum_{u \in U_i} \bar{p}_{iu} \bar{\beta}_{jiu}^{dp} \\ & - \sum_{l \in J} \sum_{c \in C_l} \bar{d}_{lc} \bar{\beta}_{jl}^{dd} - \sum_{i \in I} \sum_{u \in U_i} o_{iu} \alpha_{jiu}^{dp} + \sum_{l \in J: l \neq j} \sum_{c \in C_l} b_{lc} \alpha_{jl}^{dd} = 0. \end{aligned} \quad (4.52)$$

Finally, by replacing problems (4.45) and (4.46) with their respective PDOCs, the EPEC is written as a single optimization problem, as shown in (3.49), which can be solved using a global nonlinear solver (TAWARMALANI; SAHINIDIS, 2005):

$$\begin{aligned} & \text{Max :} \quad - \sum_{j \in J} \sum_{c \in C_j} \left(\sigma^{(v)} \varepsilon_{jc}^d \right) - \sum_{i \in I} \sum_{u \in U_i} \left(\rho^{(v)} \varepsilon_{iu}^p \right) \\ & \Xi_i^{pp} \cup \Xi_i^{pd} \cup \Xi_j^{cp} \cup \Xi_j^{cd} \\ & \text{s.t. :} \quad \begin{aligned} & (4.3b) - (4.3l) : \forall i \in I & (a) \\ & \varepsilon_{iu}^p \geq 0 : \forall i \in I, \forall u \in U_i & (b) \\ & (4.23) : \forall i \in I & (c) \\ & (4.43) : \forall i \in I & (d) \\ & (4.47b) - (4.47q) : \forall i \in I & (e) \\ & (4.51) : \forall i \in I & (f) \\ & (4.4b) - (4.4l) : \forall j \in J & (g) \\ & \varepsilon_{jc}^d \geq 0 : \forall j \in J, \forall c \in C_j & (h) \\ & (4.24) : \forall j \in J & (i) \\ & (4.44) : \forall j \in J & (j) \\ & (4.49b) - (4.49q) : \forall j \in J & (k) \\ & (4.52) : \forall j \in J. & (l) \end{aligned} \end{aligned} \quad (4.53)$$

Note that the objective function in (4.53) represents the penalty terms associated with the convexification method described in Section 4.4.1.1, which is an iterative procedure that approximates the convexified bilinear terms by an outer-approximation approach. Therefore, the final EPEC in (4.53) is solved as a sequence of optimization problems, and the algorithm for solving it is detailed in section 4.4.1.4.

4.4.1.4 Convexification iterative procedure

The final EPEC formulation in (4.53) includes convexified terms that must be handled by an iterative procedure. In the proposed methodology, this convexification

is carried out in the vicinity of the operational point calculated by a Market Clearing Procedure under Perfect Competition (MC-PC), where players do not behave strategically. The model used for the MC-PC is detailed in Section 2.3.

The convexification method begins with the parameters $p_{iu}^{(v)}, d_{jc}^{(v)}, \lambda^{(v)}$ calculated by the MC-PC. The iterative convexification procedure then proceeds as described in Algorithm 1:

Algorithm 1 EPEC Solution – Convexification procedure.

- 1: **Initialization** $v = 0$; set $\rho^{(v)}$ and $\sigma^{(v)}$ with values between 0 and 1; μ and ω with values slightly bigger than 1; $\bar{\rho}$ and $\bar{\omega}$ with large enough values to set an upper bound avoiding numerical instability. Set $p_{i,u}^{(v)}, d_{j,c}^{(v)}$ and $\lambda^{(v)}$ with the optimal values calculated by the MC-PC model in Section 2.3. Set ε_{iu}^p and ε_{jc}^d to infinity, and a tolerance τ for the errors in these variables.
 - 2: **while** $\varepsilon_{iu}^p \geq \tau$ or $\varepsilon_{jc}^d \geq \tau$ **do**
 - 3: Update $b_{iu}^{(v)}$ and $y_{jc}^{(v)}$ using (4.21) and (4.22), respectively.
 - 4: Solve the EPEC in (4.53) obtaining the optimal values $p_{iu}^*, d_{j,c}^*, \lambda^*, \varepsilon_{iu}^p$ and ε_{jc}^d .
 - 5: Update $p_{i,u}^{(v+1)} = p_{iu}^*$; $d_{j,c}^{(v+1)} = d_{j,c}^*$ and $\lambda^{(v+1)} = \lambda^*$.
 - 6: Update $\rho^{(v+1)}$ and $\sigma^{(v+1)}$ using (4.27) and (4.28), respectively.
 - 7: $v = v + 1$.
 - 8: **end while**
 - 9: Return the optimal solution for (4.53).
-

4.5 Numerical results

In this section, we discuss the numerical results regarding the proposed EPEC approach, where market players behave strategically. The numerical results are structured into two case studies, each with a different focus. The first case study compares the proposed EPEC solution approach with a previous one, and explores different objective functions for the proposed EPEC approach. The second case study analyzes various market setups, exploring scenarios where market power is likely to occur for a large producer only, a large consumer only, both a large producer and a large consumer simultaneously, and a case where market power is unlikely to occur.

In Case Study 1, the results obtained using the proposed EPEC solution approach are analyzed for a small test system consisting of two production units and one consumer, as described in (CONEJO; RUIZ, 2020). Similar to (CONEJO; RUIZ, 2020), we explore the adoption of different objective functions for the equivalent problem pertaining to the proposed EPEC model. It is important to note that the EPEC is formulated by simultaneously solving the optimality conditions of the MPECs associated with the agents, which means that no explicit objective function is present. Additionally, the objective

function in (4.53) serves solely to convexify the bilinear terms of the MPECs associated with producer and consumer agents. This case study enables a comparison of the results obtained using the proposed EPEC approach, which adopts PDOCs as the optimality conditions for the agents' MPECs, with those obtained using a previous approach that formulates the EPEC using KKT conditions. In Case Study 1, we observe that different equilibrium points are achieved by considering different objective functions in model (4.53).

In Case Study 2, the results obtained using the proposed EPEC solution approach are analyzed for a larger system involving 12 producer units and 17 consumer demands. In this case study, no objective function is employed in (4.53) other than the penalty terms related to the approach used to convexify the bilinear terms in the agents' MPEC problems. To explore different market setups, this case study is divided into four sub-cases. In Case 2.1, all producers and consumers are treated as individual agents. In Case 2.2, the number of producers is reduced, resulting into a large producer company among the other producers, while all consumers from Case 2.1 are retained. In Case 2.3, the number of consumers is reduced, resulting into a large consumer, while retaining the 12 producers from Case 2.1. Finally, in Case 2.4, the large producer agent from Case 2.2 and the large consumer company from Case 2.3 compete simultaneously in the market, while the remaining producers and consumers remain as in Case 2.1. The results of all sub-cases are compared to those obtained using a Market Clearing Procedure under Perfect Competition (MC-PC), whose model is described in Section 2.3.

4.5.1 Case study 1

In this case study, a market is analyzed for a small test system consisting of two producer units and one consumer unit, as described in (CONEJO; RUIZ, 2020). Details of the system's input data are provided in Section 4.5.1.1, while the numerical results are analyzed in Section 4.5.1.2.

4.5.1.1 Input data

Table 3.1 presents the input data for Case Study 1, including the following: the costs associated with each producer unit and the utility of each consumer; the capacity of each producer unit and each consumer; and the initial values for power production, $p^{(v)}$, for each producer unit and power consumption, $d^{(v)}$, for each consumer.

Table 4.1 – Case 1 - Input Data for Case Study 1.

Producers/ Consumer	Cost [\$/ <i>MW</i>]	Utility [\$/ <i>MW</i>]	Capacity [<i>MW</i>]	$p^{(v)}$ [<i>MW</i>]	$d^{(v)}$ [<i>MW</i>]
Producer 1	1	-	6	6	-
Producer 2	2	-	6	4	-
Consumer 1	-	3	10	-	10

Additionally, the convexification parameter is set to $\lambda^{(v)} = 2$, and the parameters related to the penalty terms are $\rho^{(v)} = 0.4$ and $\sigma^{(v)} = 0.4$. The update step rates are $\mu = 1.01$ and $\omega = 1.01$, and the upper bounds for the penalty terms are $\bar{\rho} = 10,000$ and $\bar{\sigma} = 10,000$.

4.5.1.2 Numerical results

The purpose of Case Study 1 is to compare the results obtained using the proposed EPEC approach in (4.53) with those obtained using the EPEC approach in (CONEJO; RUIZ, 2020). Unlike the proposed approach, which uses PDOCs for the agents' MPEC problems, the EPEC approach in (CONEJO; RUIZ, 2020) employs KKT optimality conditions to construct its final single EPEC formulation. Additionally, as in (CONEJO; RUIZ, 2020), different equilibrium points are explored in Case Study 1 by adopting various objective functions in the EPEC formulation presented in (4.53). For this purpose, the following objective functions are considered: (i) the Profit (PF_i) for each producer unit i , as described in (4.1a); (ii) the Utility (UT_j) for each consumer j , as described in (4.2a); and (iii) the Social Welfare (SW) function, as described in (2.1a).

Table 4.2 summarizes the results obtained using the proposed EPEC approach presented in (4.53), with KNITRO employed as the optimization solver, while Table 4.3 summarizes the results presented in (CONEJO; RUIZ, 2020). These tables present the following values: the employed objective functions (OF), which include either the profits for producers, the utility for the consumer, or social welfare; optimal offers (o_i^*) and bids (b_j^*); cleared prices (λ^*); optimal power outputs (p_i^*) and demand (d_j^*); as well as the optimal values for producer profits (PF_i), consumer utility (UT_j), and social welfare (SW) for each sub-case, and the number of iteration for solving each sub-case. Note that the symbol “ ε ” in Table 4.3 represents a small decimal number, indicating that the value was not rounded but is very close to the one shown in the table, as explained by the authors in (CONEJO; RUIZ, 2020).

Table 4.2 – Case 1 - Results obtained using the proposed EPEC approach, given in (4.53).

OF	o_1^* [\$/MW]	o_2^* [\$/MW]	b_1^* [\$/MW]	λ^* [\$/MW]	p_1^* [MW]	p_2^* [MW]	d_1^* [MW]	PF_1 [\$]	PF_2 [\$]	UT_1 [\$]	SW [\$]	(v)
PF_1	1.801	3	3	3	6	4	10	12	4	0	16	3
PF_2	3	2.881	3	3	4	6	10	8	6	0	14	4
UT_1	1	2	1	1	6	0	6	0	0	12	12	14
SW	2	2	2	2	6	4	10	6	0	10	16	1

Table 4.3 – Case 1 - Results presented by (CONEJO; RUIZ, 2020).

OF	o_1^* [\$/MW]	o_2^* [\$/MW]	b_1^* [\$/MW]	λ^* [\$/MW]	p_1^* [MW]	p_2^* [MW]	d_1^* [MW]	PF_1 [\$]	PF_2 [\$]	UT_1 [\$]	SW [\$]
PF_1	1	3	3	3	6	4	10	12	4	0	16
PF_2	3	3- ε	3- ε	3	0	6	6	0	6	0	6
UT_1	1	2	1	1	6	0	6	0	0	12	12
SW	2- ε	2	2	2	6	4	10	6	0	10	16

Comparing Tables 4.2 and 4.3, it is clear that the results are practically identical across all four sub-cases, reaffirming the ability of the proposed EPEC solution approach to find reliable equilibrium points. The only exception occurs in the sub-case represented by PF_2 , where the profit maximization of company 2 is employed as the objective function in (4.53). In this case, the proposed EPEC approach achieved a better equilibrium among the agents, displaying a higher social welfare (SW) value compared to the results presented in (CONEJO; RUIZ, 2020). Therefore, these results demonstrate that the approach proposed in this work not only calculates reliable equilibrium points as intended but can also find better equilibrium outcomes.

4.5.2 Case study 2

In this case study, a market is analyzed for a system comprising 12 producer units and 17 consumer demands. No objective function is employed in (3.49) apart from the penalty terms associated with the convexification approach used to linearize the bilinear terms in the agents' MPEC problems. The details of the system's input data are provided in Section 4.5.2.1, while the numerical results are analyzed in Section 4.5.2.2. To investigate different market setups, this case study is divided into four sub-cases, which are detailed below.

In Case 2.1, all 12 producer units are assigned to 12 individual strategic producer companies, and all 17 consumer demands are assigned to 17 individual strategic consumer companies. This market setup represents an almost ideal market scenario, resembling perfect competition, where no producer or consumer company has the ability to significantly influence the market clearing prices. This is because no single company has sufficient capacity to do so. Consequently, in this market setup, the likelihood of market power occurring is negligible.

In Case 2.2, producer units 3, 4, 5, 8, and 9 are aggregated into strategic producer company 3, reducing the number of producer companies from 12 to 7, while all 17 consumer demands remain as strategic consumers, as in Case 2.1. It is important to note that the aggregated producer units are the lowest-cost units, making company 3 a large, low-cost, and highly competitive producer. Consequently, this market setup represents a scenario where producer company 3 can exert market power by manipulating market clearing prices to its advantage.

In Case 2.3, consumer demands 10, 11, 12, 13, 15, and 16 are aggregated into strategic consumer company 10, reducing the number of strategic consumers from 17 to 11. However, the producer side remains unchanged, with all 12 producer companies as described in Case 2.1. Notably, the aggregated consumer demands in this case are those with higher

utility, meaning they are more willing to pay higher prices for energy, making strategic consumer company 10 highly competitive in the market from a consumer perspective. Consequently, this market setup represents a scenario where consumer company 10 can exert market power by manipulating market clearing prices to its advantage.

Finally, in Case 2.4, the same large strategic producer (company 3) from Case 2.2 and the same large strategic consumer (company 10) from Case 2.3 are employed, resulting in 7 strategic generating companies and 11 strategic consumers. The remaining producer and consumer companies remain unchanged from those described in Case 2.1. This market setup is designed to explore a scenario in which two potentially price-making companies, one from the producer side and one from the consumer side, coexist in the market.

4.5.2.1 Input data

Table 4.4 presents the input data for producers and consumers, for all four sub-cases of Case 2, including the following: the costs associated with each producer unit and the utility of each consumer demand; the capacity of each producer unit and each consumer demand; and the initial values for power production, $p^{(v)}$, for each producer unit and power consumption, $d^{(v)}$, for each consumer demand, as well as the maximum demand.

Table 4.4 – Case 2 - Input Data for Case Study 2.

Producer Units	Cost [\$/MW]	Capacity [MW]	$p^{(v)}$ [MW]	Consumer Units	Utility [\$/MW]	Max. Demand [MW]	$d^{(v)}$ [MW]
1	85.57	152	0	1	100.52	100.719	100.719
2	100	152	0	2	100.36	90.117	90.117
3	11.26	300	300	3	101.52	166.9815	166.982
4	11.26	591	591	4	100.04	68.913	68.913
5	30.4	60	60	5	100	66.2625	66.263
6	80.86	155	155	6	100.92	127.224	127.224
7	80.86	155	134.5	7	100.76	116.622	116.622
8	22.72	400	400	8	101.4	159.03	159.03
9	22.13	400	400	9	101.44	161.6805	161.68
10	69.6	300	300	10	157.04	180.234	180.234
11	80.86	310	310	11	158.04	246.4965	246.496
12	81.32	350	0	12	157.04	180.234	180.234
-	-	-	-	13	158.76	294.2055	294.205
-	-	-	-	14	100.4	92.7675	92.767
-	-	-	-	15	179	310.1085	310.108
-	-	-	-	16	156.88	169.632	169.632
-	-	-	-	17	100.8	119.2725	119.272

Additionally, the convexification parameters for all sub-cases of Case 2 are: $\lambda^{(v)} = 100$, the update step rates for the penalty parameters are $\mu = 1.01$ and $\omega = 1.01$, and the upper bounds of the penalty terms are $\bar{\rho} = 10,000$ and $\bar{\sigma} = 10,000$. Furthermore, due to the differences among the sub-cases of Case 2, each of the four sub-cases has distinct parameters for the initial penalty terms, which are: In Case 2.1, $\rho^{(v)} = 0.8$ and $\sigma^{(v)} = 0.8$;

In Case 2.2, $\rho^{(v)} = 0.75$ and $\sigma^{(v)} = 0.75$; In Case 2.3, $\rho^{(v)} = 0.78$ and $\sigma^{(v)} = 0.78$ and finally, in Case 2.4, $\rho^{(v)} = 0.9$ and $\sigma^{(v)} = 0.9$.

4.5.2.2 Numerical results

The purpose of Case Study 2 is to evaluate the different equilibrium points calculated by the proposed EPEC approach for the various market setups represented by sub-cases 2.1 to 2.4, as previously described. The corresponding EPEC formulations for all sub-cases are solved using the IPOPT solver, and are shown in Table 4.5. The results are compared to those obtained through the solution of a Market Clearing Procedure under Perfect Competition (MC-PC), whose model is detailed in Section 2.3. In this model, it is assumed that all producer and consumer companies submit offers and bids, respectively, based on their true marginal costs and marginal utilities, thereby simulating a perfect competition environment.

To assess the effectiveness of the proposed EPEC approach in obtaining equilibrium points that are genuine Nash equilibrium points, we compare the solutions obtained by the proposed EPEC approach with those derived from the diagonalization method. In the diagonalization method, a series of bilevel problems defined for producers and consumers—denoted as B-PP_{*i*} which is given in (4.1) and B-CP_{*j*} which is given in (4.2), respectively—are solved sequentially in an iterative procedure. The algorithm begins by solving the bilevel problem for the first producer unit ($i = 1$), i.e., by solving B-PP₁. In this problem, the offers and bids of the other producers ($i \in I, i \neq 1$) and consumers ($j \in J$) are treated as fixed at their initial values, and the offer of the producer unit $i = 1$ is calculated. The algorithm then proceeds to solve B-PP₂ for the next producer unit ($i = 2$), using the previously calculated offer value for producer unit $i = 1$ and the initial values for the remaining agents. This process is repeated for all producers and consumers until the offers and bids converge to values with sufficiently small deviations from the previous iteration. A detailed algorithm for the diagonalization method is provided in (CONEJO; RUIZ, 2020).

The sequential B-PP_{*i*} ($i \in I$) and B-CP_{*j*} ($j \in J$) problems are solved by replacing their lower-level sub-problems with their respective optimality conditions. Two sets of optimality conditions are considered: the KKT conditions and the PDOCs. The use of the latter results in the MPEC problems (4.3) and (4.4) for producers and consumers, respectively. Table 4.6 presents the results for all sub-cases using the diagonalization procedure, where PDOCs are employed for replacing the lower-level sub-problem in the bilevel problems. Similarly, Table 4.7 presents the results for all sub-cases using a diagonalization procedure with KKT conditions as the optimality conditions for replacing the lower-level sub-problem in the bilevel problems. The solver used for both diagonalization

procedures was CONOPT.

Additionally, it is important to note that for both diagonalization techniques, the offers o_{ku} ($k \neq i$) and bids b_{lc} ($l \neq j$) are treated as parameters for the non-strategic producers and consumers in each iteration. In contrast, these are treated as variables in the single-problem reformulation proposed in this work, as (3.49) is a unified model where all agents are considered simultaneously strategic.

Comparing Table 4.5, which presents the results of the proposed model, with Tables 4.6 and 4.7, which show the equilibrium results obtained through the diagonalization of the bilevel models—where the respective lower levels are replaced by PDOCs and KKT conditions, respectively—and further comparing each sub-case to the results presented by the MC-PC, several conclusions can be drawn.

Table 4.5 – Case 2 - Proposed EPEC Single-Model Reformulation

	MC-PC	Case 2.1	Case 2.2	Case 2.3	Case 2.4
Iterations	-	3	1	2	3
Cleared price	80.86	80.86	100.00	22.72	80.86
Total profit	115,167	109,996	165,896	8,604	109,079
Total utility	138,168	121,549	87,438	165,332	121,418
Social welfare	253,335	231,545	253,335	173,936	230,498
Profit for large producer 3	111,789	106,801	145,302	8,604	105,875
Utility for large consumer 10	112,733	101,304	86,303	129,753	101,752
Total power [MW]	2,650	2,247	2,650	1,377	2,223

Table 4.6 – Case 2 - EPEC through diagonalization using MPECs built from PDOCs.

	MC-PC	Case 2.1	Case 2.2	Case 2.3	Case 2.4
Iterations	-	1	1	1	1
Cleared price	80.86	80.86	100.00	22.72	80.86
Total profit	115,167	115,167	164,704	10,446	65,393
Total utility	138,168	123,611	87,437	203,637	121,156
Social welfare	253,335	238,778	252,142	214,084	186,549
Profit for large producer 3	111,789	111,789	143,717	10,446	62,014
Utility for large consumer 10	112,733	100,935	86,302	166,007	112,731
Total power [MW]	2,650	2,361	2,650	1,691	1,811

Table 4.7 – Case 2 - EPEC through diagonalization using MPECs built from KKT.

	MC-PC	Case 2.1	Case 2.2	Case 2.3	Case 2.4
Iterations	-	1	1	1	1
Cleared price	80.86	80.86	100.00	22.72	80.86
Total profit	115,167	114,034	164,705	10,446	65,393
Total utility	138,168	112,907	87,437	205,746	117,479
Social welfare	253,335	226,942	252,143	216,193	182,872
Profit for large producer 3	111,789	110,656	143,718	10,446	62,014
Utility for large consumer 10	112,733	87,472	86,302	168,460	108,159
Total power [MW]	2,650	2,344	2,650	1,691	1,811

For Case 2.1, where each producer/consumer unit operates as a single strategic company, it is clear that the results presented in Tables 4.5, 4.6, and 4.7 are very similar, demonstrating the reliability of the proposed model compared to the diagonalization approaches. From a market analysis perspective, the results for this case, when compared to the MC-PC, show that the cleared price is identical, and the remaining results are also closely aligned. This allows us to conclude that having an equilibrium with small strategic agents on both sides (production and consumption) is nearly equivalent to a market setup with perfect competition, where no agent behaves strategically.

For Case 2.2, where the large producer company 3 clearly dominates the other strategic agents in the market, a comparison of Tables 4.5, 4.6, and 4.7 shows that the results from the proposed model are validated by both diagonalization techniques, with only minor differences among them. From a market analysis perspective, the equilibrium among strategic agents, when producer company 3 is present, leads to an increase in the market clearing price and the total profit for all producers, while reducing the total utility for all consumers. Notably, this also results in a significant increase in the profit of producer company 3 for this Case Study. These findings clearly show that the equilibrium point reached tends to favor the large producer company 3.

For Case 2.3, where dominance is exerted by a large consumer relative to the other strategic agents in the market, a comparison of Tables 4.5, 4.6, and 4.7 once again demonstrates that the proposed EPEC single-problem reformulation is viable for solving the equilibrium of the strategic agents, yielding relatively similar results across all approaches. From a market analysis perspective, when the results of this sub-case are compared to those provided by the MC-PC, a decrease in the market clearing price and total profit is observed, while the total utility for all consumers increases. Furthermore, the utility of the large consumer shows a significant increase. These findings indicate that the equilibrium tends to favor the larger strategic consumer in this scenario.

Finally, for Case 2.4, where there is dominance by both a large producer company and a large consumer, a comparison of Tables 4.5, 4.6, and 4.7 shows that the proposed model, apart from the cleared price result, identifies a different equilibrium than the diagonalization approaches. However, the results for Case 2.4, obtained using the proposed model, reveal that from a market analysis perspective, when a large strategic producer exists on one side and a large strategic consumer on the other, the equilibrium results tend to align more with those calculated by the MC-PC and with the results from Case 2.1. This demonstrates that the equilibrium points calculated maintain a fair balance when large strategic agents are present on both sides. While this is a logical outcome, such a conclusion cannot be drawn from the results obtained through the diagonalization approaches.

4.6 Conclusion

The calculation of general Nash equilibria, considering the strategic behavior of both producers and consumers in a wholesale electricity market, may result in intractable or highly complex problems. However, obtaining such equilibria is crucial for market operators, regulators, and participants, as they provide valuable insights into market power and strategic behavior. In this work, we formulate this problem as an Equilibrium Problem with Equilibrium Constraints (EPEC), which involves solving a set of Mathematical Programs with Equilibrium Constraints (MPECs) jointly. Unlike previous EPEC approaches that solve the problem by replacing MPECs with their Karush-Kuhn-Tucker (KKT) conditions or by diagonalization, our proposed solution replaces these problems with the Primal-Dual Optimality Conditions (PDOCs). To achieve this, we reformulate the proposed MPECs as convex rotated quadratic conic problems, enabling both the computation of their dual counterparts and the application of PDOCs to represent them in the final single-problem EPEC. In this resulting EPEC model, cumbersome complementarity constraints are replaced by their respectively strong duality equalities, enhancing numerical stability and enabling the solutions within practical computation times.

The overall systematic approach for solving EPEC problems is tested in two case studies, each with a distinct analytical purpose. The first case study, based on a small test system, focuses on validating the proposed approach and comparing it with a previously established EPEC method from the literature, which represents MPECs using KKT conditions instead of PDOCs. Both approaches explore different equilibria by adopting different objective functions in their EPEC formulations, resulting in four distinct cases. The results show that in three cases, the proposed EPEC approach identified nearly the same equilibrium points as the validation method, while in one case, it achieved a more favorable equilibrium outcome.

In the second case study, a more realistic system is analyzed, involving 12 producer units and 17 consumer demands. The primary objective of this study is to utilize the proposed single-problem formulation to explore different market setups, where market power is likely to emerge.

To achieve this, producer and consumer demands are assigned to generation and consumer companies in four different ways, resulting in four distinct market setups: (i) Each unit is assigned to a separate company, representing an almost ideal market scenario where no market power is expected; (ii) A large producer company is created with the potential to exert market power, while the remaining companies remain the same as in (i); (iii) A large consumer company is created with the potential to exert market power, while the remaining companies remain the same as in (i); (iv) Both the large producer

company from (ii) and the large consumer company from (iii) coexist in the market, while the remaining companies remain as in (i).

The results obtained using the proposed single-problem formulation in Case (i), where no market power is expected, closely resemble those of a perfectly competitive market. In Case (ii), the large producer company increases its profits by driving up market-clearing prices, as anticipated. Similarly, in Case (iii), the large consumer company enhances its utility by lowering market-clearing prices, also as expected. Finally, in Case (iv), where both large companies coexist, the results tend to align more closely with a perfectly competitive market scenario. These findings demonstrate the economic coherence of the proposed EPEC approach. Similar results were obtained using the diagonalization method for the first three cases. However, in the fourth case, the diagonalization method failed to find a meaningful solution. This highlights the proposed model's effectiveness in identifying market power while also demonstrating strong reliability compared to the diagonalization method for solving equilibrium problems.

A possible avenue for future research would involve the expansion of the proposed single optimization problem of the equilibria among strategic producers and strategic consumers for hydrothermal systems, therefore including pertinent hydrothermal constraints in the lower-level model of each strategic agent in equilibrium.

5 FINAL CONCLUDING REMARKS

5.1 Summary

In Chapter 2, we presented an overview of electricity markets, covering key introductory aspects that are examined in greater detail throughout this work, including:

- I A description of market types and their organizational structures;
- II A description of market participants and their respective roles;
- III A description of a market clearing optimization model and its application for evaluating different market behaviors, including perfect competition and strategic competition among participants;
- IV Mathematical strategies that can be employed by the strategic agent including Stackelberg Game theory and Nash equilibrium;
- V A price formation in a multi-period network-constrained by market clearing procedure;
- VI Description of main aspects to be considered in a market clearing procedure model tailored for hydrothermal power systems.

In Chapter 3, we propose a bilevel Strategic Offering (SO) model for a Strategic Company (SC) operating in a hydro-dominated day-ahead electricity market. In this model, the upper-level problem aims to maximize the SC's profit, while the lower-level problem represents a hydrothermal Market Clearing (MC) procedure. The employed MC model accounts for both hydrothermal and DC network constraints.

In the proposed solution approach, the lower-level model is replaced by its associated Primal and Dual Optimality Conditions (PDOCs). The resulting Mathematical Program with Equilibrium Constraints (MPEC) is then reformulated to contain only a single bilinear term, which is iteratively convexified around an initial point.

In traditional approaches where the lower-level problem is replaced by its Karush-Kuhn-Tucker (KKT) conditions, each inequality constraint introduces a bilinear term due to the associated complementarity condition. This typically leads to a large number

of bilinear terms. Since bilinear terms are known to complicate the numerical solution, reducing them to a single term is a key advantage of the proposed approach.

To the aforementioned solution approach, in the numerical results two case studies are evaluated, as described below, each one evaluating two distinct scenarios:

I *Case Study 1* – Evaluating a non-congested network:

- i A scenario where the SC acts strategically and the bilinear term in the SO model is iteratively approximated using the proposed convexification approach;
- ii A scenario where all companies behaves non-strategically (perfect competition).

II *Case Study 2* – Evaluating a congested network:

- i A scenario where the SC acts strategically and the bilinear term in the SO model is iteratively approximated using the proposed convexification approach;
- ii A scenario where all companies behaves non-strategically (perfect competition).

The results from scenario (i) in both case studies outline the Strategic Company's (SC's) strategic market behavior. In this scenario, the SC chooses to submit higher offer prices to the market. This behavior drives market clearing prices to higher levels, thereby increasing profit for both the SC and the Non-Strategic Company (NSC), while reducing consumers' utility.

The results also indicate that this behavior leads to water savings in the SC's reservoirs, as its hydro plants generate less power. From the SC's standpoint, this is a significant outcome, since the water in its reservoirs represents stored energy that can be traded either in the present or in the future. Thus, this strategic behavior yields two key components that benefits the SC: profit maximization and water conservation.

However, such behavior is not observed in scenario (ii), which represents perfect competition in both case studies. Additionally, from the system operator's standpoint, water conservation may be viewed as either a positive or negative outcome, depending on the future value of water.

Extending this equilibrium framework, in Chapter 4, we also propose a systematic approach for calculating general Nash equilibria among strategic producers and strategic consumers. The proposed solution approach is formulated as an Equilibrium Problem with Equilibrium Constraints (EPEC), represented by a single optimization problem that incorporates a set of jointly solvable Mathematical Programs with Equilibrium Constraints

(MPECs) within the context of a wholesale electricity market. This is particularly important for market operators, regulators, and participants, as it provides valuable insights into market power and strategic behavior.

The overall systematic approach for solving EPEC problems is tested in two case studies, each with a distinct analytical purpose:

I *Case Study 1*: This case study is based on a small test system and focuses on validating the proposed approach by comparing it with a previously established EPEC method from the literature, which represents MPECs using KKT conditions instead of PDOCs. Both approaches explore different equilibria by adopting different objective functions in their EPEC formulations, resulting in four distinct cases:

- i Using a profit maximization as objective function for strategic producer 1;
- ii Using a profit maximization as objective function for strategic producer 2;
- iii Using a utility maximization as objective function for strategic consumer 1;
- iv Using the social welfare as objective function.

II *Case Study 2*: This case study is conducted on a more realistic system, involving 12 producer units and 17 consumer demands. The primary objective is to use the proposed single-problem formulation to explore different market configurations where market power is likely to emerge. To this end, generation units and consumer demands are allocated to producer and consumer companies in four different ways, resulting in four distinct market setups:

- i Each unit is assigned to a separate company, representing an almost ideal market scenario where no market power is expected;
- ii A large producer company is created with the potential to exert market power, while the remaining companies remain the same as in (i);
- iii A large consumer company is created with the potential to exert market power, while the remaining companies remain the same as in (i);
- iv Both the large producer company from (ii) and the large consumer company from (iii) coexist in the market, while the remaining companies remain as in (i).

The results from Case Study 1 show that, in three cases, the proposed EPEC approach identified equilibrium points nearly identical to those found by the validation method, while in one case, it achieved a more favorable equilibrium outcome.

The results obtained using the proposed single-problem formulation for Case Study 2 with market setup (i), where no market power is expected, closely resemble those

conditions of a market under perfect competition. In market setup (ii), the large producer company increases its profits by driving up market-clearing prices, as anticipated. Similarly, in market setup (iii), the large consumer company enhances its utility by lowering market-clearing prices, also as expected. Finally, in market setup (iv), where both large companies coexist, the results tend to align more closely to a market under perfect competition conditions.

These findings demonstrate the economic coherence of the proposed EPEC approach. Similar results were obtained using the diagonalization method for the first three cases. However, in the fourth case, the diagonalization method failed to find a meaningful solution. This highlights the effectiveness of the proposed model in identifying market power, while also demonstrating its strong reliability compared to the diagonalization method for solving equilibrium problems.

5.2 Contributions

In Chapter 3, we propose a bilevel Strategic Offering (SO) model specifically tailored for a hydrothermal market. The SO model is able to represent water balances and other hydro constraints for both SC and NSC companies, capturing the spatial and temporal working of hydro cascade systems. The lower-level model is adequate for situations of imperfect competition, since demand elasticity is considered. In addition, DC network constraints are represented. Also, a novel approach for solving the bilevel SO optimization problem is proposed which employs an alternative set of constraints to represent the optimality conditions of the lower-level problem. In the approach proposed we reduce the number of bilinear terms in the resulting single-level MPEC problem to a single term, independently of the number of inequality constraints. The single-level problem is then convexified around an initial point for handling the bilinear term. The hydro-thermal bilevel SO model and the solution approach proposed are tested on an adapted version of the IEEE 24-bus system, exploring scenarios where the SC acts strategically and non-strategically (perfect competition).

In Chapter 4, we propose a systematic approach for solving a general EPEC model, where the MPEC problems associated with both strategic producers and strategic consumers are recast as convex rotated quadratic conic problems, which have analytic dual counterparts and can, therefore, be exactly represented using their respective PDOCs. We use the PDOCs associated with the MPEC problems to solve the EPEC nonlinear optimization problem involving strategic producers and strategic consumers. Also, we illustrate the proposed methodology using a simple example and a realistic case study. For the sake of simplicity, we assume that the network does not restrict the equilibria and, thus, do not model it. The proposed solution approach is also applied to investigate

different market setups, exploring scenarios where: (i) no market power is expected; (ii) a single producer has potential market power; (iii) a single consumer has potential market power; and (iv) both a producer and a consumer have potential market power.

5.3 Possibilities for future research

We suggest below some possible avenues for future research:

1. *Medium-term stochastic hydrothermal scheduling problems:* A natural extension of the work presented in Chapter 3 would involve proposing MPEC formulation for strategic producer including medium-term stochastic hydrothermal scheduling problems in the lower-level of the strategic offering model, where uncertainties may happen in water inflows, demand, etc.
2. *Equilibrium of strategic agents in hydrothermal systems:* The work presented in Chapter 4 could be extend, where the proposed single optimization problem of the equilibria among strategic producers and strategic consumers is formulated for hydrothermal systems, therefore including pertinent hydrothermal constraints in the lower-level model of each strategic agent in equilibrium.

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Appendix

APPENDIX A – CHAPTER 3

NUMERICAL RESULTS

In this appendix, all content regarding the results obtained from Case Studies 3.4.2 and 3.4.3 in Chapter 3.4 is presented in table format rather than as charts, to facilitate better numerical visualization of the outcomes.

Table A.1 – Case 1 - Convexification: Data on power generation from hydro and thermal plants of strategic and non-strategic companies.

Units	Type	T1	T2	T3	T4	T5	T6
Strategic Company							
U1	Hydro	44.721	44.721	44.718	44.717	44.717	44.718
U2	Hydro	44.721	44.721	44.718	44.717	44.717	44.718
U3	Hydro	39.439	39.438	39.423	38.809	38.809	39.423
U4	Hydro	39.439	39.438	39.423	38.809	38.809	39.423
U5	Hydro	60.555	60.52	58.133	56.938	56.937	58.132
U6	Hydro	60.555	60.52	58.133	56.937	56.937	58.132
U7	Hydro	60.555	60.52	58.133	56.938	56.937	58.132
U8	Hydro	60.555	60.52	58.133	56.938	56.937	58.132
U9	Hydro	14.044	14.046	12.249	11.138	11.14	12.265
U10	Hydro	14.044	14.046	12.249	11.138	11.14	12.265
U11	Hydro	14.044	14.046	12.249	11.138	11.14	12.265
U12	Hydro	18.493	18.492	18.489	18.489	18.488	18.489
U13	Hydro	18.493	18.492	18.489	18.489	18.488	18.489
U14	Hydro	18.493	18.492	18.489	18.489	18.488	18.489
U15	Hydro	18.493	18.492	18.489	18.489	18.488	18.489
U16	Hydro	19.479	19.478	19.475	19.475	19.475	19.475
U17	Hydro	19.479	19.478	19.475	19.475	19.475	19.475
U18	Hydro	19.479	19.478	19.475	19.475	19.475	19.475
U19	Hydro	25.7	25.701	25.692	25.365	25.363	25.688
U20	Hydro	25.7	25.701	25.691	25.363	25.362	25.688
U21	Hydro	25.699	25.703	25.693	25.36	25.363	25.688
U22	Hydro	80.606	80.564	78.189	76.997	76.998	78.189
U23	Hydro	80.605	80.561	78.187	76.993	76.998	78.19
U24	Hydro	80.605	80.564	78.19	76.994	76.995	78.188
U25	Hydro	80.607	80.56	78.188	76.993	76.995	78.188
U26	Hydro	32.279	32.236	29.846	28.65	28.651	29.845
U27	Hydro	32.278	32.236	29.845	28.651	28.651	29.846
U28	Hydro	32.279	32.236	29.845	28.65	28.651	29.846
U29	Hydro	32.279	32.236	29.846	28.651	28.65	29.845
U30	Hydro	32.279	32.236	29.845	28.65	28.65	29.845
U31	Hydro	37.279	37.247	34.878	33.687	33.687	34.878
U32	Hydro	37.279	37.247	34.878	33.687	33.687	34.878
U33	Hydro	37.279	37.247	34.878	33.687	33.687	34.878
U34	Hydro	37.279	37.247	34.878	33.687	33.687	34.878
U35	Thermal	0.001	0.001	0.001	0.001	0.001	0.001
Non-Strategic Company							
U36	Hydro	0.675	.	.	.	.	.
U37	Hydro	.	14.48	.	.	.	.
U38	Hydro	35.118	.	.	6.694	6.691	.
U39	Hydro	.	.	6.993	.	.	6.989
U40	Hydro	46.961	36.962	.	9.387	.	.
U41	Hydro	28.756	.	9.982	.	.	.
U42	Hydro	46.961	.	.	.	9.375	9.953
U43	Hydro	41.532	41.533	41.533	41.533	41.533	41.533
U44	Hydro	41.533	41.532	41.532	41.533	41.532	41.532
U45	Hydro	41.533	41.533	41.533	41.532	41.533	41.533
U46	Hydro	52.231	52.231	52.231	52.231	52.23	52.231
U47	Hydro	52.231	52.23	52.231	52.23	52.231	52.23
U48	Hydro	52.23	52.231	52.23	52.231	52.231	52.231
U49	Thermal	2.71E-04	2.71E-04	2.71E-04	2.71E-04	2.71E-04	2.72E-04
U50	Thermal	2.71E-04	2.71E-04	2.71E-04	2.71E-04	2.71E-04	2.72E-04
U51	Thermal	2.72E-04	2.71E-04	2.71E-04	2.71E-04	2.71E-04	2.71E-04

Table A.2 – Case 1 - Convexification: Data on power generation from hydro and thermal plants of strategic and non-strategic companies.

Units	Type	T7	T8	T9	T10	T11	T12
Strategic Company							
U1	Hydro	44.683	30.724	33.588	34.562	34.676	33.702
U2	Hydro	44.683	30.724	33.588	34.561	34.677	33.702
U3	Hydro	38.425	23.142	26.001	26.976	27.089	26.115
U4	Hydro	38.425	23.143	26.001	26.976	27.09	26.115
U5	Hydro	63.529	58.958	70.635	71.3	71.193	70.525
U6	Hydro	63.529	58.959	70.635	71.3	71.193	70.525
U7	Hydro	63.529	58.959	70.635	71.3	71.193	70.525
U8	Hydro	63.529	58.959	70.635	71.3	71.193	70.525
U9	Hydro	13.371	9.318	12.524	13.481	13.592	12.641
U10	Hydro	13.371	9.318	12.524	13.481	13.592	12.641
U11	Hydro	13.371	9.318	12.524	13.481	13.592	12.641
U12	Hydro	18.034	10.652	13.499	14.472	14.584	13.612
U13	Hydro	18.034	10.652	13.499	14.472	14.584	13.612
U14	Hydro	18.034	10.652	13.499	14.472	14.584	13.612
U15	Hydro	18.034	10.652	13.499	14.472	14.584	13.612
U16	Hydro	19.084	9.968	12.818	13.792	13.904	12.931
U17	Hydro	19.084	9.968	12.818	13.792	13.904	12.931
U18	Hydro	19.084	9.968	12.818	13.792	13.904	12.931
U19	Hydro	24.636	9.299	12.164	13.138	13.254	12.288
U20	Hydro	24.633	9.296	12.17	13.153	13.254	12.285
U21	Hydro	24.633	9.298	12.167	13.129	13.256	12.277
U22	Hydro	83.545	86.127	97.945	98.62	98.501	97.825
U23	Hydro	83.544	86.127	97.945	98.62	98.501	97.825
U24	Hydro	83.545	86.128	97.945	98.62	98.501	97.825
U25	Hydro	83.545	86.126	97.945	98.62	98.501	97.825
U26	Hydro	35.246	37.713	49.423	50.089	49.98	49.313
U27	Hydro	35.246	37.713	49.423	50.089	49.98	49.313
U28	Hydro	35.246	37.714	49.423	50.089	49.98	49.313
U29	Hydro	35.246	37.713	49.423	50.089	49.98	49.313
U30	Hydro	35.245	37.713	49.423	50.089	49.98	49.313
U31	Hydro	40.207	42.71	54.617	55.299	55.181	54.497
U32	Hydro	40.207	42.71	54.617	55.299	55.181	54.497
U33	Hydro	40.207	42.71	54.617	55.299	55.181	54.497
U34	Hydro	40.207	42.71	54.617	55.299	55.181	54.497
U35	Thermal	0.001	0.001	0.001	0.001	0.001	0.001
Non-Strategic Company							
U36	Hydro	.	35.118	35.118	35.118	35.118	35.118
U37	Hydro	.	35.118	35.118	35.118	35.118	35.118
U38	Hydro	34.099	35.118	35.118	35.118	35.118	35.118
U39	Hydro	35.118	35.118	35.118	35.118	35.118	35.118
U40	Hydro	46.961	46.961	46.961	46.96	46.96	46.961
U41	Hydro	35.173	46.96	46.96	46.961	46.961	46.961
U42	Hydro	46.961	46.961	46.961	46.961	46.961	46.96
U43	Hydro	41.533	41.533	41.533	41.533	41.533	41.533
U44	Hydro	41.532	41.532	41.533	41.533	41.532	41.533
U45	Hydro	41.533	41.533	41.532	41.532	41.533	41.532
U46	Hydro	52.23	52.231	52.23	52.231	52.231	52.231
U47	Hydro	52.231	52.231	52.231	52.23	52.23	52.231
U48	Hydro	52.231	52.23	52.231	52.231	52.231	52.23
U49	Thermal	52.5	107.499	107.499	107.499	107.499	107.499
U50	Thermal	52.5	107.499	107.499	107.499	107.499	107.499
U51	Thermal	5	299.998	299.998	299.998	299.998	299.998

Table A.3 – Case 1 - Convexification: Data on power generation from hydro and thermal plants of strategic and non-strategic companies.

Units	Type	T13	T14	T15	T16	T17	T18
Strategic Company							
U1	Hydro	33.702	33.929	31.987	31.304	38.3	38.302
U2	Hydro	33.702	33.929	31.987	31.304	38.3	38.302
U3	Hydro	26.115	26.341	24.397	23.714	30.705	30.705
U4	Hydro	26.115	26.341	24.397	23.714	30.705	30.705
U5	Hydro	70.525	70.305	68.972	69.631	74.685	74.685
U6	Hydro	70.525	70.305	68.972	69.631	74.685	74.685
U7	Hydro	70.525	70.305	68.972	69.631	74.685	74.685
U8	Hydro	70.525	70.305	68.972	69.631	74.685	74.685
U9	Hydro	12.643	12.866	10.949	10.267	14.113	14.114
U10	Hydro	12.643	12.866	10.949	10.267	14.113	14.114
U11	Hydro	12.643	12.866	10.949	10.267	14.113	14.114
U12	Hydro	13.613	13.838	11.9	11.219	18.163	18.162
U13	Hydro	13.613	13.838	11.9	11.219	18.163	18.162
U14	Hydro	13.613	13.838	11.9	11.219	18.163	18.162
U15	Hydro	13.613	13.838	11.9	11.219	18.163	18.162
U16	Hydro	12.932	13.157	11.218	10.537	17.521	17.522
U17	Hydro	12.932	13.157	11.218	10.537	17.521	17.522
U18	Hydro	12.932	13.157	11.218	10.537	17.521	17.522
U19	Hydro	12.275	12.523	10.56	9.885	16.881	16.859
U20	Hydro	12.277	12.521	10.574	9.875	16.87	16.87
U21	Hydro	12.281	12.518	10.563	9.882	16.86	16.862
U22	Hydro	97.826	97.59	96.252	96.947	101.993	101.995
U23	Hydro	97.826	97.59	96.252	96.947	101.993	101.995
U24	Hydro	97.826	97.59	96.252	96.947	101.993	101.995
U25	Hydro	97.826	97.59	96.252	96.947	101.993	101.995
U26	Hydro	49.314	49.094	47.766	48.429	53.483	53.484
U27	Hydro	49.313	49.093	47.767	48.429	53.483	53.484
U28	Hydro	49.313	49.094	47.766	48.429	53.483	53.484
U29	Hydro	49.313	49.094	47.767	48.429	53.483	53.484
U30	Hydro	49.313	49.094	47.766	48.429	53.484	53.484
U31	Hydro	54.498	54.258	52.896	53.612	58.799	58.8
U32	Hydro	54.498	54.258	52.896	53.612	58.799	58.8
U33	Hydro	54.498	54.258	52.896	53.612	58.799	58.8
U34	Hydro	54.498	54.258	52.896	53.612	58.8	58.8
U35	Thermal	0.001	0.001	0.001	0.001	0.001	0.001
Non-Strategic Company							
U36	Hydro	35.118	35.118	35.118	35.118	35.118	35.118
U37	Hydro	35.118	35.118	35.118	35.118	35.118	35.118
U38	Hydro	35.118	35.118	35.118	35.118	35.118	35.118
U39	Hydro	35.118	35.118	35.118	35.118	35.118	35.118
U40	Hydro	46.961	46.96	46.96	46.96	46.961	46.961
U41	Hydro	46.96	46.961	46.961	46.961	46.96	46.96
U42	Hydro	46.961	46.961	46.961	46.961	46.961	46.961
U43	Hydro	41.533	41.533	41.532	41.533	41.533	41.533
U44	Hydro	41.533	41.533	41.533	41.533	41.532	41.533
U45	Hydro	41.532	41.532	41.533	41.532	41.533	41.532
U46	Hydro	52.231	52.231	52.23	52.231	52.231	52.231
U47	Hydro	52.231	52.23	52.231	52.23	52.231	52.23
U48	Hydro	52.23	52.231	52.231	52.231	52.23	52.231
U49	Thermal	107.499	107.499	107.499	107.499	107.499	107.499
U50	Thermal	107.499	107.499	107.499	107.499	107.499	107.499
U51	Thermal	299.998	299.998	299.998	299.998	299.998	299.998

Table A.4 – Case 1 - Convexification: Data on power generation from hydro and thermal plants of strategic and non-strategic companies.

Units	Type	T19	T20	T21	T22	T23	T24
Strategic Company							
U1	Hydro	38.303	34.566	30.706	30.769	44.725	44.926
U2	Hydro	38.303	34.565	30.707	30.77	44.725	44.926
U3	Hydro	30.703	26.97	23.106	23.163	38.392	38.743
U4	Hydro	30.704	26.969	23.106	23.163	38.392	38.743
U5	Hydro	74.686	71.299	67.005	56.168	61.767	58.956
U6	Hydro	74.686	71.299	67.005	56.168	61.767	58.956
U7	Hydro	74.686	71.299	67.005	56.168	61.767	58.956
U8	Hydro	74.686	71.299	67.005	56.168	61.767	58.956
U9	Hydro	14.114	13.485	9.656	8.807	11.315	11.243
U10	Hydro	14.114	13.485	9.656	8.807	11.315	11.243
U11	Hydro	14.114	13.485	9.656	8.807	11.315	11.243
U12	Hydro	18.165	14.47	10.612	10.674	18.033	18.489
U13	Hydro	18.165	14.47	10.612	10.674	18.033	18.489
U14	Hydro	18.165	14.47	10.612	10.674	18.033	18.489
U15	Hydro	18.165	14.47	10.612	10.674	18.033	18.489
U16	Hydro	17.522	13.789	9.928	9.991	19.081	19.475
U17	Hydro	17.522	13.789	9.928	9.991	19.081	19.475
U18	Hydro	17.522	13.789	9.928	9.991	19.081	19.475
U19	Hydro	16.865	13.136	9.28	9.337	24.636	25.702
U20	Hydro	16.863	13.158	9.282	9.34	24.635	25.7
U21	Hydro	16.865	13.157	9.283	9.341	24.638	25.701
U22	Hydro	101.993	98.619	94.26	80.741	83.546	80.568
U23	Hydro	101.993	98.619	94.26	80.741	83.546	80.568
U24	Hydro	101.993	98.619	94.26	80.741	83.547	80.569
U25	Hydro	101.993	98.619	94.26	80.741	83.544	80.566
U26	Hydro	53.484	50.087	45.807	32.356	35.255	32.246
U27	Hydro	53.484	50.088	45.807	32.356	35.255	32.246
U28	Hydro	53.484	50.088	45.807	32.356	35.255	32.246
U29	Hydro	53.484	50.087	45.807	32.356	35.255	32.246
U30	Hydro	53.483	50.087	45.807	32.356	35.255	32.246
U31	Hydro	58.8	55.297	50.878	37.196	40.205	37.243
U32	Hydro	58.8	55.297	50.878	37.196	40.205	37.243
U33	Hydro	58.8	55.297	50.878	37.196	40.205	37.243
U34	Hydro	58.8	55.297	50.878	37.196	40.205	37.243
U35	Thermal	0.001	0.001	0.001	0.001	0.001	0.001
Non-Strategic Company							
U36	Hydro	35.118	35.118	35.118	35.118	.	.
U37	Hydro	35.118	35.118	35.118	35.118	21.913	.
U38	Hydro	35.118	35.118	35.118	35.118	35.118	.
U39	Hydro	35.118	35.118	35.118	35.118	.	16.853
U40	Hydro	46.961	46.96	46.96	46.96	33.318	46.961
U41	Hydro	46.96	46.961	46.961	46.961	46.961	.
U42	Hydro	46.961	46.961	46.961	46.961	46.961	3.242
U43	Hydro	41.532	41.533	41.533	41.532	41.533	41.533
U44	Hydro	41.533	41.533	41.533	41.533	41.533	41.533
U45	Hydro	41.533	41.532	41.532	41.533	41.532	41.532
U46	Hydro	52.23	52.231	52.23	52.23	52.23	52.231
U47	Hydro	52.231	52.231	52.231	52.231	52.231	52.23
U48	Hydro	52.231	52.23	52.231	52.231	52.231	52.231
U49	Thermal	107.499	107.499	107.499	107.499	52.5	2.72E-04
U50	Thermal	107.499	107.499	107.499	107.499	52.5	2.72E-04
U51	Thermal	299.998	299.998	299.998	299.998	5	2.71E-04

Table A.5 – Case 1 - Convexification: Data of power consumed from the consumer agents.

Consumers	T1	T2	T3	T4	T5	T6
K1	67.76	63.714	60.68	59.669	59.669	60.68
K2	60.629	57.01	54.295	53.39	53.39	54.295
K3	112.325	105.619	100.59	98.913	98.913	100.59
K4	46.362	43.594	41.518	40.826	40.826	41.518
K5	44.579	41.918	39.921	39.256	39.256	39.921
K6	85.579	80.47	76.638	75.361	75.361	76.638
K7	78.449	73.765	70.253	69.082	69.082	70.253
K8	106.971	100.584	95.795	94.198	94.198	95.795
k9	108.753	102.261	97.391	95.768	95.768	97.391
k10	73.157	64.683	58.669	56.73	56.73	58.669
k11	165.811	155.911	148.487	146.012	146.012	148.487
k12	121.245	114.007	108.578	106.768	106.768	108.578
k13	197.904	186.089	177.228	174.274	174.274	177.228
k14	62.412	58.686	55.891	54.96	54.96	55.891
k15	208.607	196.153	186.812	183.699	183.699	186.812
k16	114.101	107.289	102.18	100.477	100.477	102.18
k17	80.231	75.441	71.849	70.651	70.651	71.849

Table A.6 – Case 1 - Convexification: Data of power consumed from the consumer agents.

Consumers	T7	T8	T9	T10	T11	T12
K1	74.839	86.975	96.077	97.089	97.089	96.077
K2	66.964	77.823	85.967	86.872	86.872	85.967
K3	124.061	144.179	159.267	160.944	160.944	159.267
K4	51.205	59.509	65.737	66.428	66.428	65.737
K5	49.236	57.221	63.209	63.874	63.874	63.209
K6	94.52	109.848	121.344	122.621	122.621	121.344
K7	86.645	100.695	111.233	112.404	112.404	111.233
K8	118.147	137.306	151.675	153.271	153.271	151.675
k9	120.116	139.594	154.202	155.826	155.826	154.202
k10	89.242	120.533	147.081	150.193	150.193	147.081
k11	183.134	212.831	235.105	237.579	237.579	235.105
k12	133.912	155.628	171.915	173.724	173.724	171.915
k13	218.581	254.027	280.611	283.565	283.565	280.611
k14	68.933	80.111	88.494	89.426	89.426	88.494
k15	230.402	267.764	295.786	298.899	298.899	295.786
k16	126.022	146.458	161.785	163.488	163.488	161.785
k17	88.614	102.984	113.761	114.958	114.958	113.761

Table A.7 – Case 1 - Convexification: Data of power consumed from the consumer agents.

Consumers	T13	T14	T15	T16	T17	T18
K1	96.077	96.077	94.055	94.055	101.134	101.134
K2	85.967	85.967	84.157	84.157	90.491	90.491
K3	159.267	159.267	155.914	155.914	167.65	167.65
K4	65.737	65.737	64.353	64.353	69.196	69.196
K5	63.209	63.209	61.878	61.878	66.536	66.536
K6	121.344	121.344	118.789	118.789	127.73	127.73
K7	111.233	111.233	108.891	108.891	117.088	117.088
K8	151.675	151.675	148.482	148.482	159.658	159.658
k9	154.202	154.202	150.956	150.956	162.318	162.318
k10	147.081	147.081	140.953	140.953	162.97	162.97
k11	235.105	235.105	230.155	230.155	247.478	247.478
k12	171.915	171.915	168.295	168.295	180.963	180.963
k13	280.611	280.611	274.703	274.703	295.38	295.38
k14	88.494	88.494	86.631	86.631	93.152	93.152
k15	295.786	295.786	289.559	289.559	311.354	311.354
k16	161.785	161.785	158.379	158.379	170.3	170.3
k17	113.761	113.761	111.366	111.366	119.748	119.748

Table A.8 – Case 1 - Convexification: Data of power consumed from the consumer agents.

Consumers	T19	T20	T21	T22	T23	T24
K1	101.134	97.089	92.032	83.941	73.828	63.714
K2	90.491	86.872	82.347	75.108	66.059	57.01
K3	167.65	160.944	152.561	139.149	122.384	105.619
K4	69.196	66.428	62.969	57.433	50.513	43.594
K5	66.536	63.874	60.548	55.225	48.571	41.918
K6	127.73	122.621	116.234	106.016	93.243	80.47
K7	117.088	112.404	106.55	97.183	85.474	73.765
K8	159.658	153.271	145.288	132.516	116.55	100.584
k9	162.318	155.826	147.71	134.724	118.492	102.261
k10	162.97	150.193	134.956	112.27	86.847	64.683
k11	247.478	237.579	225.205	205.407	180.659	155.911
k12	180.963	173.724	164.676	150.199	132.103	114.007
k13	295.38	283.565	268.796	245.165	215.627	186.089
k14	93.152	89.426	84.768	77.316	68.001	58.686
k15	311.354	298.899	283.332	258.423	227.288	196.153
k16	170.3	163.488	154.973	141.349	124.319	107.289
k17	119.748	114.958	108.971	99.391	87.416	75.441

Table A.9 – Case 1 - Convexification: Data of strategic offering prices from hydro and thermal plants of strategic company.

Units	Type	T1	T2	T3	T4	T5	T6
Strategic Company							
U1	Hydro	45	45	45	4.50E+01	4.50E+01	4.50E+01
U2	Hydro	45	45	45	4.50E+01	4.50E+01	4.50E+01
U3	Hydro	45	45	45	4.50E+01	4.50E+01	4.50E+01
U4	Hydro	45	45	45	4.50E+01	4.50E+01	4.50E+01
U5	Hydro	45	45	45	4.50E+01	4.50E+01	4.50E+01
U6	Hydro	45	45	45	4.50E+01	4.50E+01	4.50E+01
U7	Hydro	45	45	45	4.50E+01	4.50E+01	4.50E+01
U8	Hydro	45	45	45	4.50E+01	4.50E+01	4.50E+01
U9	Hydro	45	45	45	4.50E+01	4.50E+01	4.50E+01
U10	Hydro	45	45	45	4.50E+01	4.50E+01	4.50E+01
U11	Hydro	45	45	45	4.50E+01	4.50E+01	4.50E+01
U12	Hydro	45	45	45	4.50E+01	4.50E+01	4.50E+01
U13	Hydro	45	45	45	4.50E+01	4.50E+01	4.50E+01
U14	Hydro	45	45	45	4.50E+01	4.50E+01	4.50E+01
U15	Hydro	45	45	45	4.50E+01	4.50E+01	4.50E+01
U16	Hydro	45	45	45	4.50E+01	4.50E+01	4.50E+01
U17	Hydro	45	45	45	4.50E+01	4.50E+01	4.50E+01
U18	Hydro	45	45	45	4.50E+01	4.50E+01	4.50E+01
U19	Hydro	45	45	45	4.50E+01	4.50E+01	4.50E+01
U20	Hydro	45	45	45	4.50E+01	4.50E+01	4.50E+01
U21	Hydro	45	45	45	4.50E+01	4.50E+01	4.50E+01
U22	Hydro	45	45	45	4.50E+01	4.50E+01	4.50E+01
U23	Hydro	45	45	45	4.50E+01	4.50E+01	4.50E+01
U24	Hydro	45	45	45	4.50E+01	4.50E+01	4.50E+01
U25	Hydro	45	45	45	4.50E+01	4.50E+01	4.50E+01
U26	Hydro	45	45	45	4.50E+01	4.50E+01	4.50E+01
U27	Hydro	45	45	45	4.50E+01	4.50E+01	4.50E+01
U28	Hydro	45	45	45	4.50E+01	4.50E+01	4.50E+01
U29	Hydro	45	45	45	4.50E+01	4.50E+01	4.50E+01
U30	Hydro	45	45	45	4.50E+01	4.50E+01	4.50E+01
U31	Hydro	45	45	45	4.50E+01	4.50E+01	4.50E+01
U32	Hydro	45	45	45	4.50E+01	4.50E+01	4.50E+01
U33	Hydro	45	45	45	4.50E+01	4.50E+01	4.50E+01
U34	Hydro	45	45	45	4.50E+01	4.50E+01	4.50E+01
U35	Thermal	45	45	45	4.50E+01	4.50E+01	4.50E+01

Table A.10 – Case 1 - Convexification: Data of strategic offering prices from hydro and thermal plants of strategic company.

Units	Type	T7	T8	T9	T10	T11	T12
Strategic Company							
U1	Hydro	4.50E+01	9.31E+01	9.41E+01	94.084	93.103	93.102
U2	Hydro	4.50E+01	9.31E+01	9.41E+01	94.084	93.103	93.102
U3	Hydro	4.50E+01	9.31E+01	9.41E+01	94.084	93.103	93.102
U4	Hydro	4.50E+01	9.31E+01	9.41E+01	94.084	93.103	93.102
U5	Hydro	4.50E+01	9.31E+01	9.41E+01	94.084	93.103	93.102
U6	Hydro	4.50E+01	9.31E+01	9.41E+01	94.084	93.103	93.102
U7	Hydro	4.50E+01	9.31E+01	9.41E+01	94.084	93.103	93.102
U8	Hydro	4.50E+01	9.31E+01	9.41E+01	94.084	93.103	93.102
U9	Hydro	4.50E+01	9.31E+01	9.41E+01	94.084	93.103	93.102
U10	Hydro	4.50E+01	9.31E+01	9.41E+01	94.084	93.103	93.102
U11	Hydro	4.50E+01	9.31E+01	9.41E+01	94.084	93.103	93.102
U12	Hydro	4.50E+01	9.31E+01	9.41E+01	94.084	93.103	93.102
U13	Hydro	4.50E+01	9.31E+01	9.41E+01	94.084	93.103	93.102
U14	Hydro	4.50E+01	9.31E+01	9.41E+01	94.084	93.103	93.102
U15	Hydro	4.50E+01	9.31E+01	9.41E+01	94.084	93.103	93.102
U16	Hydro	4.50E+01	9.31E+01	9.41E+01	94.084	93.103	93.102
U17	Hydro	4.50E+01	9.31E+01	9.41E+01	94.084	93.103	93.102
U18	Hydro	4.50E+01	9.31E+01	9.41E+01	94.084	93.103	93.102
U19	Hydro	4.50E+01	9.31E+01	9.41E+01	94.084	93.103	93.102
U20	Hydro	4.50E+01	9.31E+01	9.41E+01	94.084	93.103	93.102
U21	Hydro	4.50E+01	9.31E+01	9.41E+01	94.084	93.103	93.102
U22	Hydro	4.50E+01	9.31E+01	9.41E+01	94.084	93.103	93.102
U23	Hydro	4.50E+01	9.31E+01	9.41E+01	94.084	93.103	93.102
U24	Hydro	4.50E+01	9.31E+01	9.41E+01	94.084	93.103	93.102
U25	Hydro	4.50E+01	9.31E+01	9.41E+01	94.084	93.103	93.102
U26	Hydro	4.50E+01	9.31E+01	9.41E+01	94.084	93.103	93.102
U27	Hydro	4.50E+01	9.31E+01	9.41E+01	94.084	93.103	93.102
U28	Hydro	4.50E+01	9.31E+01	9.41E+01	94.084	93.103	93.102
U29	Hydro	4.50E+01	9.31E+01	9.41E+01	94.084	93.103	93.102
U30	Hydro	4.50E+01	9.31E+01	9.41E+01	94.084	93.103	93.102
U31	Hydro	4.50E+01	9.31E+01	9.41E+01	94.084	93.103	93.102
U32	Hydro	4.50E+01	9.31E+01	9.41E+01	94.084	93.103	93.102
U33	Hydro	4.50E+01	9.31E+01	9.41E+01	94.084	93.103	93.102
U34	Hydro	4.50E+01	9.31E+01	9.41E+01	94.084	93.103	93.102
U35	Thermal	4.50E+01	9.31E+01	9.41E+01	94.084	93.103	93.102

Table A.11 – Case 1 - Convexification: Data of strategic offering prices from hydro and thermal plants of strategic company.

Units	Type	T13	T14	T15	T16	T17	T18
Strategic Company							
U1	Hydro	93.102	91.142	91.14	97.02	98.01	98.009
U2	Hydro	93.102	91.142	91.14	97.02	98.01	98.009
U3	Hydro	93.102	91.142	91.14	97.02	98.01	98.009
U4	Hydro	93.102	91.142	91.14	97.02	98.01	98.009
U5	Hydro	93.102	91.141	91.14	97.02	98.01	98.009
U6	Hydro	93.102	91.141	91.14	97.02	98.01	98.009
U7	Hydro	93.102	91.141	91.14	97.02	98.01	98.009
U8	Hydro	93.102	91.141	91.14	97.02	98.01	98.009
U9	Hydro	93.102	91.142	91.14	97.02	98.01	98.009
U10	Hydro	93.102	91.142	91.14	97.02	98.01	98.009
U11	Hydro	93.102	91.142	91.14	97.02	98.01	98.009
U12	Hydro	93.102	91.142	91.14	97.02	98.01	98.009
U13	Hydro	93.102	91.142	91.14	97.02	98.01	98.009
U14	Hydro	93.102	91.142	91.14	97.02	98.01	98.009
U15	Hydro	93.102	91.142	91.14	97.02	98.01	98.009
U16	Hydro	93.102	91.142	91.14	97.02	98.01	98.009
U17	Hydro	93.102	91.142	91.14	97.02	98.01	98.009
U18	Hydro	93.102	91.142	91.14	97.02	98.01	98.009
U19	Hydro	93.102	91.141	91.14	97.02	98.01	98.009
U20	Hydro	93.102	91.141	91.14	97.02	98.01	98.009
U21	Hydro	93.102	91.141	91.14	97.02	98.01	98.009
U22	Hydro	93.102	91.141	91.14	97.02	98.01	98.009
U23	Hydro	93.102	91.141	91.14	97.02	98.01	98.009
U24	Hydro	93.102	91.141	91.14	97.02	98.01	98.009
U25	Hydro	93.102	91.141	91.14	97.02	98.01	98.009
U26	Hydro	93.102	91.141	91.14	97.02	98.01	98.009
U27	Hydro	93.102	91.141	91.14	97.02	98.01	98.009
U28	Hydro	93.102	91.141	91.14	97.02	98.01	98.009
U29	Hydro	93.102	91.141	91.14	97.02	98.01	98.009
U30	Hydro	93.102	91.141	91.14	97.02	98.01	98.009
U31	Hydro	93.102	91.142	91.14	97.02	98.01	98.009
U32	Hydro	93.102	91.142	91.14	97.02	98.01	98.009
U33	Hydro	93.102	91.142	91.14	97.02	98.01	98.009
U34	Hydro	93.102	91.142	91.14	97.02	98.01	98.009
U35	Thermal	93.102	91.141	91.14	97.02	98.01	98.009

Table A.12 – Case 1 - Convexification: Data of strategic offering prices from hydro and thermal plants of strategic company.

Units	Type	T19	T20	T21	T22	T23	T24
Strategic Company							
U1	Hydro	98.009	94.084	89.18	81.34	45	45
U2	Hydro	98.009	94.084	89.18	81.34	45	45
U3	Hydro	98.009	94.084	89.18	81.34	45	45
U4	Hydro	98.009	94.084	89.18	81.34	45	45
U5	Hydro	98.009	94.084	89.18	81.34	45	45
U6	Hydro	98.009	94.084	89.18	81.34	45	45
U7	Hydro	98.009	94.084	89.18	81.34	45	45
U8	Hydro	98.009	94.084	89.18	81.34	45	45
U9	Hydro	98.009	94.084	89.18	81.34	45	45
U10	Hydro	98.009	94.084	89.18	81.34	45	45
U11	Hydro	98.009	94.084	89.18	81.34	45	45
U12	Hydro	98.009	94.084	89.18	81.34	45	45
U13	Hydro	98.009	94.084	89.18	81.34	45	45
U14	Hydro	98.009	94.084	89.18	81.34	45	45
U15	Hydro	98.009	94.084	89.18	81.34	45	45
U16	Hydro	98.009	94.084	89.18	81.34	45	45
U17	Hydro	98.009	94.084	89.18	81.34	45	45
U18	Hydro	98.009	94.084	89.18	81.34	45	45
U19	Hydro	98.009	94.084	89.18	81.34	45	45
U20	Hydro	98.009	94.084	89.18	81.34	45	45
U21	Hydro	98.009	94.084	89.18	81.34	45	45
U22	Hydro	98.009	94.084	89.18	81.34	45	45
U23	Hydro	98.009	94.084	89.18	81.34	45	45
U24	Hydro	98.009	94.084	89.18	81.34	45	45
U25	Hydro	98.009	94.084	89.18	81.34	45	45
U26	Hydro	98.009	94.084	89.18	81.34	45	45
U27	Hydro	98.009	94.084	89.18	81.34	45	45
U28	Hydro	98.009	94.084	89.18	81.34	45	45
U29	Hydro	98.009	94.084	89.18	81.34	45	45
U30	Hydro	98.009	94.084	89.18	81.34	45	45
U31	Hydro	98.009	94.084	89.18	81.34	45	45
U32	Hydro	98.009	94.084	89.18	81.34	45	45
U33	Hydro	98.009	94.084	89.18	81.34	45	45
U34	Hydro	98.009	94.084	89.18	81.34	45	45
U35	Thermal	98.009	94.084	89.18	81.34	45	45

Table A.13 – Case 1 - Convexification: Data of market cleared prices per node.

Node	T1	T2	T3	T4	T5	T6
N1	45	45	45	45	45	45
N2	45	45	45	44.999	45	44.999
N3	45	45	45	45	45	45
N4	45	45	45	45	45	45
N5	45	45	45	45	45	45
N6	45	45	45	45	45	45
N7	45	45	45	45	45	45
N8	45	45	45	45	45	45
N9	45	45	45	45	45	45
N10	45	45	45	45	45	45
N11	45	45	45	45	45	45
N12	45	45	45	45	45	45
N13	45	45	45	45	45	45
N14	45	45	45	45	45	45
N15	45	45	45	45	45	45
N16	45	45	45	45	45	45
N17	44.999	44.999	44.999	45	45	45
N18	45	45	45	45	45	45
N19	45	45	45	45	45	45
N20	45	45	45	45	45	45
N21	45	45	45	45	45	45
N22	45	45	45	45	45	45
N23	45	45	45	45	45	45
N24	45	45	45	45	45	45

Table A.14 – Case 1 - Convexification: Data of market cleared prices per node.

Node	T7	T8	T9	T10	T11	T12
N1	45	93.1	94.082	94.084	93.103	93.102
N2	44.999	93.1	94.082	94.084	93.103	93.102
N3	45	93.1	94.082	94.084	93.104	93.102
N4	45	93.1	94.082	94.085	93.104	93.102
N5	45	93.1	94.082	94.084	93.103	93.102
N6	45	93.1	94.082	94.084	93.103	93.102
N7	45	93.1	94.082	94.084	93.103	93.102
N8	45	93.1	94.082	94.084	93.103	93.102
N9	45	93.1	94.082	94.084	93.103	93.102
N10	45	93.1	94.082	94.084	93.103	93.102
N11	45	93.1	94.082	94.084	93.103	93.102
N12	45	93.1	94.082	94.084	93.103	93.102
N13	45	93.1	94.082	94.084	93.103	93.102
N14	45	93.1	94.082	94.084	93.103	93.102
N15	45	93.1	94.082	94.084	93.103	93.102
N16	45	93.1	94.082	94.084	93.103	93.102
N17	45	93.1	94.082	94.084	93.103	93.102
N18	45	93.1	94.082	94.084	93.103	93.102
N19	45	93.1	94.082	94.084	93.103	93.102
N20	45	93.1	94.082	94.084	93.103	93.102
N21	45	93.1	94.082	94.084	93.103	93.102
N22	45	93.1	94.082	94.084	93.103	93.102
N23	45	93.1	94.082	94.084	93.103	93.102
N24	45	93.1	94.082	94.084	93.103	93.102

Table A.15 – Case 1 - Convexification: Data of market cleared prices per node.

Node	T13	T14	T15	T16	N17	T18
N1	93.102	91.142	91.14	97.02	98.01	98.009
N2	93.102	91.142	91.14	97.02	98.01	98.009
N3	93.102	91.142	91.141	97.021	98.01	98.009
N4	93.102	91.142	91.141	97.021	98.01	98.009
N5	93.102	91.142	91.14	97.02	98.01	98.009
N6	93.102	91.142	91.14	97.02	98.01	98.009
N7	93.102	91.142	91.14	97.02	98.01	98.009
N8	93.102	91.142	91.14	97.02	98.01	98.009
N9	93.102	91.142	91.14	97.02	98.01	98.009
N10	93.102	91.142	91.14	97.02	98.01	98.009
N11	93.102	91.142	91.14	97.02	98.01	98.009
N12	93.102	91.142	91.14	97.02	98.01	98.009
N13	93.102	91.141	91.14	97.02	98.01	98.009
N14	93.102	91.141	91.14	97.02	98.01	98.009
N15	93.102	91.141	91.14	97.02	98.01	98.009
N16	93.102	91.141	91.14	97.02	98.01	98.009
N17	93.102	91.141	91.14	97.02	98.01	98.009
N18	93.102	91.141	91.14	97.02	98.01	98.009
N19	93.102	91.141	91.14	97.02	98.01	98.009
N20	93.102	91.141	91.14	97.02	98.01	98.009
N21	93.102	91.141	91.14	97.02	98.01	98.009
N22	93.102	91.141	91.14	97.02	98.01	98.009
N23	93.102	91.141	91.14	97.02	98.01	98.009
N24	93.102	91.142	91.14	97.02	98.01	98.009

Table A.16 – Case 1 - Convexification: Data of market cleared prices per node.

Node	T19	T20	T21	T22	N23	T24
N1	98.009	94.084	89.18	81.34	45	45
N2	98.009	94.084	89.18	81.34	45	45
N3	98.009	94.084	89.18	81.34	45	45
N4	98.009	94.084	89.181	81.34	45	45
N5	98.009	94.084	89.18	81.34	45	45
N6	98.009	94.084	89.18	81.34	45	45
N7	98.009	94.084	89.18	81.34	45	45
N8	98.009	94.084	89.18	81.34	45	45
N9	98.009	94.084	89.18	81.34	45	45
N10	98.009	94.084	89.18	81.34	45	45
N11	98.009	94.084	89.18	81.34	45	45
N12	98.009	94.084	89.18	81.34	45	45
N13	98.009	94.084	89.18	81.34	45	45
N14	98.009	94.084	89.18	81.34	45	45
N15	98.009	94.084	89.18	81.34	45	45
N16	98.009	94.084	89.18	81.34	45	45
N17	98.009	94.084	89.18	81.34	45	45
N18	98.009	94.084	89.18	81.34	45	45
N19	98.009	94.084	89.18	81.34	45	45
N20	98.009	94.084	89.18	81.34	45	45
N21	98.009	94.084	89.18	81.34	45	45
N22	98.009	94.084	89.18	81.34	45	45
N23	98.009	94.084	89.18	81.34	45	45
N24	98.009	94.084	89.18	81.34	45	45

Table A.17 – Case 1 – Perfect Competition: Power generation data from hydro and thermal plants, categorized by strategic and non-strategic companies.

Units	Type	T1	T2	T3	T4	T5	T6
Strategic Company							
U1	Hydro	27.693	45.25	.	45.25	45.25	45.25
U2	Hydro	.	45.25	.	45.25	45.25	45.25
U3	Hydro	.	.	.	.	.	.
U4	Hydro	.	.	.	.	.	.
U5	Hydro	101.599	101.599	101.599	101.599	101.599	101.599
U6	Hydro	101.599	101.599	101.599	101.599	101.599	101.599
U7	Hydro	101.599	101.599	101.599	101.599	101.599	101.599
U8	Hydro	101.599	101.599	101.599	101.599	101.599	101.599
U9	Hydro	.	.	.	.	.	.
U10	Hydro	.	.	.	.	.	.
U11	Hydro	.	.	.	.	.	.
U12	Hydro	.	.	.	.	.	.
U13	Hydro	.	.	.	.	.	.
U14	Hydro	.	.	.	.	.	.
U15	Hydro	.	.	.	.	.	.
U16	Hydro	.	.	.	.	.	.
U17	Hydro	.	.	.	.	.	.
U18	Hydro	.	.	.	.	.	.
U19	Hydro	.	.	.	.	.	.
U20	Hydro	.	.	.	.	.	.
U21	Hydro	.	.	.	.	.	.
U22	Hydro	102.429	102.429	102.429	102.429	102.429	102.429
U23	Hydro	102.429	102.429	102.429	102.429	102.429	102.429
U24	Hydro	102.429	102.429	102.429	102.429	102.429	102.429
U25	Hydro	102.429	102.429	102.429	102.429	102.429	102.429
U26	Hydro	101.783	101.783	101.783	101.783	101.783	.
U27	Hydro	101.783	101.783	101.783	36.296	101.783	.
U28	Hydro	101.783	101.783	101.783	.	49.608	.
U29	Hydro	101.783	57.112	101.783	.	101.783	.
U30	Hydro	101.783	.	101.783	101.783	101.783	13.333
U31	Hydro	.	.	.	.	.	86.388
U32	Hydro	.	.	.	.	.	86.388
U33	Hydro	.	.	.	.	.	86.388
U34	Hydro	.	76.842	.	.	.	86.388
U35	Thermal	.	.	.	.	.	.
Non-Strategic Company							
U36	Hydro	.	.	.	.	.	.
U37	Hydro	.	.	.	35.118	.	.
U38	Hydro	.	.	.	35.118	.	.
U39	Hydro	.	.	.	22.044	.	.
U40	Hydro	46.961	.	46.961	.	.	.
U41	Hydro	46.961	.	.	.	.	.
U42	Hydro	6.955	.	18.106	.	.	.
U43	Hydro	41.533	41.533	.	41.533	.	41.533
U44	Hydro	41.533	41.533	.	41.533	.	41.533
U45	Hydro	41.533	41.533	.	41.533	.	41.533
U46	Hydro	52.231	52.231	52.231	52.231	52.231	52.231
U47	Hydro	52.231	52.231	52.231	52.231	52.231	52.231
U48	Hydro	52.231	52.231	52.231	52.231	52.231	52.231
U49	Thermal	.	.	.	.	.	.
U50	Thermal	.	.	.	.	.	.
U51	Thermal	.	.	.	.	.	.

Table A.18 – Case 1 – Perfect Competition: Power generation data from hydro and thermal plants, categorized by strategic and non-strategic companies.

Units	Type	T7	T8	T9	T10	T11	T12
Strategic Company							
U1	Hydro	45.25	45.25	45.25	45.25	45.25	45.25
U2	Hydro	45.25	45.25	45.25	45.25	45.25	45.25
U3	Hydro	.	.	23.027	.	.	.
U4	Hydro	.	.	.	.	.	.
U5	Hydro	101.599	101.599	101.599	101.599	101.599	101.599
U6	Hydro	101.599	101.599	101.599	101.599	101.599	101.599
U7	Hydro	101.599	101.599	101.599	101.599	101.599	101.599
U8	Hydro	101.599	101.599	101.599	101.599	101.599	101.599
U9	Hydro	.	.	.	.	14.355	14.355
U10	Hydro	.	.	.	.	14.355	10.091
U11	Hydro	.	.	.	.	14.355	.
U12	Hydro	.	.	18.711	18.711	18.711	18.711
U13	Hydro	.	.	18.711	18.711	18.711	18.711
U14	Hydro	.	.	18.711	18.711	18.711	18.711
U15	Hydro	.	.	18.711	18.711	18.711	17.291
U16	Hydro	.	.	.	19.71	19.71	.
U17	Hydro	.	.	.	19.71	19.71	.
U18	Hydro	.	.	.	11.521	19.71	.
U19	Hydro	.	.	27.588	27.588	27.588	27.588
U20	Hydro	.	.	27.588	27.588	2.829	27.588
U21	Hydro	.	.	26.494	26.494	.	26.494
U22	Hydro	102.429	102.429	102.429	102.429	102.429	102.429
U23	Hydro	102.429	102.429	102.429	102.429	102.429	102.429
U24	Hydro	102.429	102.429	102.429	102.429	102.429	102.429
U25	Hydro	102.429	102.429	102.429	102.429	102.429	102.429
U26	Hydro	.	101.783	101.783	101.783	101.783	101.783
U27	Hydro	101.783	101.783	101.783	101.783	101.783	101.783
U28	Hydro	101.783	101.783	101.783	101.783	101.783	101.783
U29	Hydro	85.784	101.783	101.783	101.783	101.783	101.783
U30	Hydro	101.783	101.783	101.783	101.783	101.783	101.783
U31	Hydro	86.388	86.388	86.388	86.388	86.388	86.388
U32	Hydro	86.388	86.388	86.388	86.388	86.388	86.388
U33	Hydro	86.388	86.388	86.388	86.388	86.388	86.388
U34	Hydro	86.388	78.323	86.388	86.388	86.388	86.388
U35	Thermal	.	.	.	.	.	.
Non-Strategic Company							
U36	Hydro	.	35.118	35.118	35.118	35.118	35.118
U37	Hydro	.	35.118	35.118	35.118	35.118	35.118
U38	Hydro	.	35.118	35.118	35.118	35.118	35.118
U39	Hydro	.	35.118	35.118	35.118	35.118	35.118
U40	Hydro	.	.	46.961	46.961	46.961	46.961
U41	Hydro	.	46.961	46.961	46.961	46.961	46.961
U42	Hydro	.	31.761	46.961	46.961	46.961	46.961
U43	Hydro	41.533	41.533	41.533	41.533	41.533	41.533
U44	Hydro	41.533	41.533	41.533	41.533	41.533	41.533
U45	Hydro	41.533	41.533	41.533	41.533	41.533	41.533
U46	Hydro	52.231	52.231	52.231	52.231	52.231	52.231
U47	Hydro	52.231	52.231	52.231	52.231	52.231	52.231
U48	Hydro	52.231	52.231	52.231	52.231	52.231	52.231
U49	Thermal	.	.	.	.	.	.
U50	Thermal	.	.	.	.	.	.
U51	Thermal	.	.	.	.	.	.

Table A.19 – Case 1 – Perfect Competition: Power generation data from hydro and thermal plants, categorized by strategic and non-strategic companies.

Units	Type	T13	T14	T15	T16	T17	T18
Strategic Company							
U1	Hydro	45.25	45.25	45.25	45.25	45.25	45.25
U2	Hydro	45.25	45.25	45.25	45.25	45.25	45.25
U3	Hydro	40.017	40.017	24.496	.	40.017	40.017
U4	Hydro	40.017	38.823	.	.	40.017	19.62
U5	Hydro	101.599	101.599	101.599	101.599	101.599	101.599
U6	Hydro	101.599	101.599	101.599	101.599	101.599	101.599
U7	Hydro	101.599	101.599	101.599	101.599	101.599	101.599
U8	Hydro	101.599	101.599	101.599	101.599	101.599	101.599
U9	Hydro	.	5.277	14.355	5.277	14.355	14.355
U10	Hydro	.	.	10.114	.	8.312	14.355
U11	Hydro	.	.	.	.	.	14.355
U12	Hydro	18.711	18.711	18.711	18.711	18.711	18.711
U13	Hydro	18.711	17.581	18.711	18.711	18.711	18.711
U14	Hydro	18.711	.	18.711	18.711	18.711	18.711
U15	Hydro	18.711	.	18.711	18.711	18.711	18.711
U16	Hydro	4.953	19.71	.	4.268	19.71	19.71
U17	Hydro	.	19.71	.	19.71	19.71	19.71
U18	Hydro	19.71	19.71	.	19.71	19.71	19.71
U19	Hydro	.	.	.	.	27.588	27.588
U20	Hydro	.	.	.	.	27.588	27.588
U21	Hydro	.	.	.	.	27.588	27.588
U22	Hydro	102.429	102.429	102.429	102.429	102.429	102.429
U23	Hydro	102.429	102.429	102.429	102.429	102.429	102.429
U24	Hydro	102.429	102.429	102.429	102.429	102.429	102.429
U25	Hydro	102.429	102.429	102.429	102.429	102.429	102.429
U26	Hydro	101.783	101.783	101.783	101.783	101.783	101.783
U27	Hydro	101.783	101.783	101.783	101.783	101.783	101.783
U28	Hydro	101.783	101.783	101.783	101.783	101.783	101.783
U29	Hydro	101.783	101.783	101.783	101.783	101.783	101.783
U30	Hydro	101.783	101.783	101.783	101.783	101.783	101.783
U31	Hydro	86.388	86.388	86.388	86.388	86.388	86.388
U32	Hydro	86.388	86.388	86.388	86.388	86.388	86.388
U33	Hydro	86.388	86.388	86.388	86.388	86.388	86.388
U34	Hydro	86.388	86.388	86.388	86.388	86.388	86.388
U35	Thermal	.	.	.	.	.	.
Non-Strategic Company							
U36	Hydro	35.118	35.118	35.118	35.118	35.118	35.118
U37	Hydro	35.118	35.118	35.118	35.118	35.118	35.118
U38	Hydro	35.118	35.118	35.118	35.118	35.118	35.118
U39	Hydro	35.118	35.118	35.118	35.118	35.118	35.118
U40	Hydro	46.961	46.961	46.961	46.961	46.961	46.961
U41	Hydro	46.961	46.961	46.961	46.961	46.961	46.961
U42	Hydro	46.961	46.961	46.961	46.961	46.961	46.961
U43	Hydro	41.533	41.533	41.533	41.533	41.533	41.533
U44	Hydro	41.533	41.533	41.533	41.533	41.533	41.533
U45	Hydro	41.533	41.533	41.533	41.533	41.533	41.533
U46	Hydro	52.231	52.231	52.231	52.231	52.231	52.231
U47	Hydro	52.231	52.231	52.231	52.231	52.231	52.231
U48	Hydro	52.231	52.231	52.231	52.231	52.231	52.231
U49	Thermal	.	.	.	.	.	.
U50	Thermal	.	.	.	.	.	.
U51	Thermal	.	.	.	.	.	.

Table A.20 – Case 1 – Perfect Competition: Power generation data from hydro and thermal plants, categorized by strategic and non-strategic companies.

Units	Type	T19	T20	T21	T22	T23	T24
Strategic Company							
U1	Hydro	45.25	45.25	45.25	45.25	45.25	20.889
U2	Hydro	45.25	45.25	45.25	45.25	45.25	.
U3	Hydro	40.017	.	.	.	.	.
U4	Hydro	40.017	.	.	.	.	101.599
U5	Hydro	101.599	101.599	101.599	101.599	101.599	101.599
U6	Hydro	101.599	101.599	101.599	101.599	101.599	101.599
U7	Hydro	101.599	101.599	101.599	101.599	101.599	101.599
U8	Hydro	101.599	101.599	101.599	101.599	101.599	.
U9	Hydro	14.355	.	14.355	.	.	.
U10	Hydro	8.312	.	14.355	.	.	.
U11	Hydro	.	.	14.355	.	.	.
U12	Hydro	18.711	18.711	.	.	.	.
U13	Hydro	18.711	18.711	.	.	.	.
U14	Hydro	18.711	18.711	.	.	.	.
U15	Hydro	18.711	18.711	.	.	.	.
U16	Hydro	19.71	11.521	5.432	.	.	.
U17	Hydro	19.71	19.71	.	.	.	.
U18	Hydro	19.71	19.71	19.71	.	.	.
U19	Hydro	27.588	27.588	.	.	.	.
U20	Hydro	27.588	27.588	.	.	.	.
U21	Hydro	27.588	26.494	.	.	.	102.429
U22	Hydro	102.429	102.429	102.429	102.429	102.429	102.429
U23	Hydro	102.429	102.429	102.429	102.429	102.429	102.429
U24	Hydro	102.429	102.429	102.429	102.429	102.429	102.429
U25	Hydro	102.429	102.429	102.429	102.429	102.429	101.783
U26	Hydro	101.783	101.783	101.783	101.783	101.783	101.783
U27	Hydro	101.783	101.783	101.783	101.783	101.783	101.783
U28	Hydro	101.783	101.783	101.783	101.783	101.783	101.783
U29	Hydro	101.783	101.783	101.783	101.783	101.783	101.783
U30	Hydro	101.783	101.783	101.783	101.783	101.783	.
U31	Hydro	86.388	86.388	86.388	86.388	.	.
U32	Hydro	86.388	86.388	86.388	86.388	.	.
U33	Hydro	86.388	86.388	86.388	86.388	.	.
U34	Hydro	86.388	86.388	86.388	86.388	.	.
U35	Thermal	.	.	.	.	.	.
Non-Strategic Company							
U36	Hydro	35.118	35.118	35.118	35.118	35.118	.
U37	Hydro	35.118	35.118	35.118	23.108	35.118	.
U38	Hydro	35.118	35.118	35.118	35.118	35.118	.
U39	Hydro	35.118	35.118	35.118	35.118	35.118	.
U40	Hydro	46.961	46.961	46.961	.	46.961	.
U41	Hydro	46.961	46.961	46.961	.	13.137	.
U42	Hydro	46.961	46.961	46.961	.	.	41.533
U43	Hydro	41.533	41.533	41.533	41.533	41.533	41.533
U44	Hydro	41.533	41.533	41.533	41.533	41.533	41.533
U45	Hydro	41.533	41.533	41.533	41.533	41.533	52.231
U46	Hydro	52.231	52.231	52.231	52.231	52.231	52.231
U47	Hydro	52.231	52.231	52.231	52.231	52.231	52.231
U48	Hydro	52.231	52.231	52.231	52.231	52.231	.
U49	Thermal	.	.	.	.	.	.
U50	Thermal	.	.	.	.	.	.
U51	Thermal	.	.	.	.	.	.

Table A.21 – Case 1 - Perfect Competition: Data of power consumed from consumers agents.

Consumers	T1	T2	T3	T4	T5	T6
K1	67.76	63.715	60.681	59.669	59.669	60.681
K2	60.63	57.01	54.295	53.39	53.39	54.295
K3	112.326	105.62	100.59	98.914	98.914	100.59
K4	46.362	43.594	41.518	40.826	40.826	41.518
K5	44.579	41.918	39.922	39.256	39.256	39.922
K6	85.58	80.47	76.639	75.361	75.361	76.639
K7	78.449	73.766	70.253	69.082	69.082	70.253
K8	106.971	100.585	95.795	94.199	94.199	95.795
K9	108.754	102.261	97.392	95.768	95.768	97.392
K10	73.158	64.683	58.67	56.73	56.73	58.67
K11	165.812	155.912	148.488	146.013	146.013	148.488
K12	121.246	114.007	108.578	106.769	106.769	108.578
K13	197.906	186.09	177.229	174.275	174.275	177.229
K14	62.412	58.686	55.892	54.96	54.96	55.892
K15	208.608	196.154	186.813	183.7	183.7	186.813
K16	114.102	107.29	102.181	100.478	100.478	102.181
K17	80.232	75.442	71.849	70.652	70.652	71.849

Table A.22 – Case 1 - Perfect Competition: Data of power consumed from consumers agents.

Consumers	T7	T8	T9	T10	T11	T12
K1	74.84	86.976	96.078	97.089	97.089	96.078
K2	66.964	77.823	85.967	86.872	86.872	85.967
K3	124.061	144.18	159.268	160.945	160.945	159.268
K4	51.206	59.509	65.737	66.429	66.429	65.737
K5	49.237	57.221	63.209	63.875	63.875	63.209
K6	94.521	109.849	121.344	122.622	122.622	121.344
K7	86.645	100.696	111.234	112.405	112.405	111.234
K8	118.147	137.306	151.676	153.272	153.272	151.676
K9	120.116	139.595	154.203	155.827	155.827	154.203
K10	89.243	120.533	147.081	150.194	150.194	147.081
K11	183.135	212.833	235.106	237.581	237.581	235.106
K12	133.913	155.629	171.916	173.725	173.725	171.916
K13	218.582	254.028	280.612	283.566	283.566	280.612
K14	68.933	80.111	88.495	89.427	89.427	88.495
K15	230.403	267.766	295.788	298.901	298.901	295.788
K16	126.023	146.459	161.786	163.489	163.489	161.786
K17	88.614	102.984	113.762	114.959	114.959	113.762

Table A.23 – Case 1 - Perfect Competition: Data of power consumed from consumers agents.

Consumers	T13	T14	T15	T16	T17	T18
K1	96.078	96.078	94.055	94.055	101.135	101.135
K2	85.967	85.967	84.158	84.158	90.492	90.492
K3	159.268	159.268	155.915	155.915	167.651	167.651
K4	65.737	65.737	64.353	64.353	69.197	69.197
K5	63.209	63.209	61.879	61.879	66.536	66.536
K6	121.344	121.344	118.79	118.79	127.731	127.731
K7	111.234	111.234	108.892	108.892	117.088	117.088
K8	151.676	151.676	148.483	148.483	159.659	159.659
K9	154.203	154.203	150.957	150.957	162.319	162.319
K10	147.081	147.081	140.954	140.954	162.971	162.971
K11	235.106	235.106	230.156	230.156	247.48	247.48
K12	171.916	171.916	168.296	168.296	180.964	180.964
K13	280.612	280.612	274.705	274.705	295.382	295.382
K14	88.495	88.495	86.632	86.632	93.153	93.153
K15	295.788	295.788	289.561	289.561	311.355	311.355
K16	161.786	161.786	158.38	158.38	170.301	170.301
K17	113.762	113.762	111.367	111.367	119.749	119.749

Table A.24 – Case 1 - Perfect Competition: Data of power consumed from consumers agents.

Consumers	T19	T20	T21	T22	T23	T24
K1	101.135	97.089	92.032	83.942	73.828	63.715
K2	90.492	86.872	82.348	75.108	66.059	57.01
K3	167.651	160.945	152.562	139.15	122.385	105.62
K4	69.197	66.429	62.969	57.433	50.514	43.594
K5	66.536	63.875	60.548	55.225	48.571	41.918
K6	127.731	122.622	116.235	106.017	93.244	80.47
K7	117.088	112.405	106.55	97.183	85.474	73.766
K8	159.659	153.272	145.289	132.517	116.551	100.585
K9	162.319	155.827	147.711	134.725	118.493	102.261
K10	162.971	150.194	134.956	112.271	86.847	64.683
K11	247.48	237.581	225.207	205.408	180.66	155.912
K12	180.964	173.725	164.677	150.2	132.104	114.007
K13	295.382	283.566	268.797	245.167	215.629	186.09
K14	93.153	89.427	84.769	77.317	68.001	58.686
K15	311.355	298.901	283.333	258.425	227.289	196.154
K16	170.301	163.489	154.974	141.35	124.32	107.29
K17	119.749	114.959	108.972	99.392	87.417	75.442

Table A.25 – Case 1 - Perfect Competition: Data of market cleared prices per node.

Node	T1	T2	T3	T4	T5	T6
N1	2.66	2.66	2.66	2.66	2.66	2.66
N2	2.66	2.66	2.66	2.66	2.66	2.66
N3	2.66	2.66	2.66	2.66	2.66	2.66
N4	2.66	2.66	2.66	2.66	2.66	2.66
N5	2.66	2.66	2.66	2.66	2.66	2.66
N6	2.66	2.66	2.66	2.66	2.66	2.66
N7	2.66	2.66	2.66	2.66	2.66	2.66
N8	2.66	2.66	2.66	2.66	2.66	2.66
N9	2.66	2.66	2.66	2.66	2.66	2.66
N10	2.66	2.66	2.66	2.66	2.66	2.66
N11	2.66	2.66	2.66	2.66	2.66	2.66
N12	2.66	2.66	2.66	2.66	2.66	2.66
N13	2.66	2.66	2.66	2.66	2.66	2.66
N14	2.66	2.66	2.66	2.66	2.66	2.66
N15	2.66	2.66	2.66	2.66	2.66	2.66
N16	2.66	2.66	2.66	2.66	2.66	2.66
N17	2.66	2.66	2.66	2.66	2.66	2.66
N18	2.66	2.66	2.66	2.66	2.66	2.66
N19	2.66	2.66	2.66	2.66	2.66	2.66
N20	2.66	2.66	2.66	2.66	2.66	2.66
N21	2.66	2.66	2.66	2.66	2.66	2.66
N22	2.66	2.66	2.66	2.66	2.66	2.66
N23	2.66	2.66	2.66	2.66	2.66	2.66
N24	2.66	2.66	2.66	2.66	2.66	2.66

Table A.26 – Case 1 - Perfect Competition: Data of market cleared prices per node.

Node	T7	T8	T9	T10	T11	T12
N1	2.66	2.66	3.94	3.94	3.94	3.94
N2	2.66	2.66	3.94	3.94	3.94	3.94
N3	2.66	2.66	3.94	3.94	3.94	3.94
N4	2.66	2.66	3.94	3.94	3.94	3.94
N5	2.66	2.66	3.94	3.94	3.94	3.94
N6	2.66	2.66	3.94	3.94	3.94	3.94
N7	2.66	2.66	3.94	3.94	3.94	3.94
N8	2.66	2.66	3.94	3.94	3.94	3.94
N9	2.66	2.66	3.94	3.94	3.94	3.94
N10	2.66	2.66	3.94	3.94	3.94	3.94
N11	2.66	2.66	3.94	3.94	3.94	3.94
N12	2.66	2.66	3.94	3.94	3.94	3.94
N13	2.66	2.66	3.94	3.94	3.94	3.94
N14	2.66	2.66	3.94	3.94	3.94	3.94
N15	2.66	2.66	3.94	3.94	3.94	3.94
N16	2.66	2.66	3.94	3.94	3.94	3.94
N17	2.66	2.66	3.94	3.94	3.94	3.94
N18	2.66	2.66	3.94	3.94	3.94	3.94
N19	2.66	2.66	3.94	3.94	3.94	3.94
N20	2.66	2.66	3.94	3.94	3.94	3.94
N21	2.66	2.66	3.94	3.94	3.94	3.94
N22	2.66	2.66	3.94	3.94	3.94	3.94
N23	2.66	2.66	3.94	3.94	3.94	3.94
N24	2.66	2.66	3.94	3.94	3.94	3.94

Table A.27 – Case 1 - Perfect Competition: Data of market cleared prices per node.

Node	T13	T14	T15	T16	T17	T18
N1	3.94	3.94	3.94	3.94	3.94	3.94
N2	3.94	3.94	3.94	3.94	3.94	3.94
N3	3.94	3.94	3.94	3.94	3.94	3.94
N4	3.94	3.94	3.94	3.94	3.94	3.94
N5	3.94	3.94	3.94	3.94	3.94	3.94
N6	3.94	3.94	3.94	3.94	3.94	3.94
N7	3.94	3.94	3.94	3.94	3.94	3.94
N8	3.94	3.94	3.94	3.94	3.94	3.94
N9	3.94	3.94	3.94	3.94	3.94	3.94
N10	3.94	3.94	3.94	3.94	3.94	3.94
N11	3.94	3.94	3.94	3.94	3.94	3.94
N12	3.94	3.94	3.94	3.94	3.94	3.94
N13	3.94	3.94	3.94	3.94	3.94	3.94
N14	3.94	3.94	3.94	3.94	3.94	3.94
N15	3.94	3.94	3.94	3.94	3.94	3.94
N16	3.94	3.94	3.94	3.94	3.94	3.94
N17	3.94	3.94	3.94	3.94	3.94	3.94
N18	3.94	3.94	3.94	3.94	3.94	3.94
N19	3.94	3.94	3.94	3.94	3.94	3.94
N20	3.94	3.94	3.94	3.94	3.94	3.94
N21	3.94	3.94	3.94	3.94	3.94	3.94
N22	3.94	3.94	3.94	3.94	3.94	3.94
N23	3.94	3.94	3.94	3.94	3.94	3.94
N24	3.94	3.94	3.94	3.94	3.94	3.94

Table A.28 – Case 1 - Perfect Competition: Data of market cleared prices per node.

Node	T19	T20	T21	T22	T23	T24
N1	3.94	3.94	3.94	2.66	2.66	2.66
N2	3.94	3.94	3.94	2.66	2.66	2.66
N3	3.94	3.94	3.94	2.66	2.66	2.66
N4	3.94	3.94	3.94	2.66	2.66	2.66
N5	3.94	3.94	3.94	2.66	2.66	2.66
N6	3.94	3.94	3.94	2.66	2.66	2.66
N7	3.94	3.94	3.94	2.66	2.66	2.66
N8	3.94	3.94	3.94	2.66	2.66	2.66
N9	3.94	3.94	3.94	2.66	2.66	2.66
N10	3.94	3.94	3.94	2.66	2.66	2.66
N11	3.94	3.94	3.94	2.66	2.66	2.66
N12	3.94	3.94	3.94	2.66	2.66	2.66
N13	3.94	3.94	3.94	2.66	2.66	2.66
N14	3.94	3.94	3.94	2.66	2.66	2.66
N15	3.94	3.94	3.94	2.66	2.66	2.66
N16	3.94	3.94	3.94	2.66	2.66	2.66
N17	3.94	3.94	3.94	2.66	2.66	2.66
N18	3.94	3.94	3.94	2.66	2.66	2.66
N19	3.94	3.94	3.94	2.66	2.66	2.66
N20	3.94	3.94	3.94	2.66	2.66	2.66
N21	3.94	3.94	3.94	2.66	2.66	2.66
N22	3.94	3.94	3.94	2.66	2.66	2.66
N23	3.94	3.94	3.94	2.66	2.66	2.66
N24	3.94	3.94	3.94	2.66	2.66	2.66

Table A.29 – Case 2 – Perfect Competition: Power generation data from hydro and thermal plants, categorized by strategic and non-strategic companies.

Units	Type	T1	T2	T3	T4	T5	T6
Strategic Company							
U1	Hydro	45.242	45.242	45.242	45.242	45.242	41.85
U2	Hydro	45.242	45.242	45.242	45.242	45.242	41.874
U3	Hydro	40.009	40.009	40.009	40.009	40.009	26.061
U4	Hydro	40.009	40.009	40.009	40.009	40.009	26.012
U5	Hydro	73.833	55.919	50.357	50.042	52.805	73.05
U6	Hydro	73.888	56.042	50.434	49.942	52.967	73
U7	Hydro	73.838	55.923	50.382	50.119	53.085	72.904
U8	Hydro	73.805	56.036	50.455	50.254	53.101	72.957
U9	Hydro	7.121	14.049	14.202	14.011	13.914	11.351
U10	Hydro	7.111	14.049	14.202	14.011	13.914	11.351
U11	Hydro	7.124	14.049	14.202	14.011	13.914	11.351
U12	Hydro	18.7	18.7	18.701	18.7	18.7	11.901
U13	Hydro	18.7	18.7	18.701	18.7	18.7	11.901
U14	Hydro	18.7	18.7	18.701	18.7	18.7	11.901
U15	Hydro	18.7	18.7	18.701	18.7	18.7	11.901
U16	Hydro	19.7	19.7	19.7	19.7	19.7	11.832
U17	Hydro	19.7	19.7	19.7	19.7	19.7	11.832
U18	Hydro	19.7	19.7	19.7	19.7	19.7	11.832
U19	Hydro	25.644	21.47	19.851	20.44	21.83	14.451
U20	Hydro	25.603	21.5	19.885	20.382	21.864	14.468
U21	Hydro	25.629	21.575	19.909	20.453	21.95	14.411
U22	Hydro	62.586	54.888	53.598	53.378	55.206	47.817
U23	Hydro	62.626	54.887	53.551	53.56	55.131	47.852
U24	Hydro	62.596	54.731	53.416	53.366	55.177	47.85
U25	Hydro	62.687	54.719	53.286	53.37	55.04	47.785
U26	Hydro	0.019	0.02	0.021	0.021	0.021	0.204
U27	Hydro	0.019	0.02	0.021	0.021	0.021	0.2
U28	Hydro	0.02	0.02	0.021	0.021	0.021	0.202
U29	Hydro	0.019	0.02	0.021	0.021	0.021	0.204
U30	Hydro	0.019	0.02	0.021	0.021	0.021	0.207
U31	Hydro	14.008	8.713	4.187	2.119	0.853	0.057
U32	Hydro	13.96	8.684	4.15	2.104	0.845	0.057
U33	Hydro	13.892	8.706	4.177	2.106	0.85	0.057
U34	Hydro	13.986	8.707	4.167	2.112	0.841	0.058
U35	Thermal	0.017	0.018	0.018	0.018	0.018	0.094
Non-Strategic Company							
U36	Hydro	.	.	.	.	.	.
U37	Hydro	.	35.118	11.432	.	0.64	.
U38	Hydro	34.515	15.782	.	35.118	35.118	.
U39	Hydro	.	.	35.118	6.572	.	0.186
U40	Hydro	29.707	46.961	46.961	46.961	33.675	.
U41	Hydro	.	11.948	46.961	0.024	.	.
U42	Hydro	46.961	46.961	7.356	46.961	46.961	2.128
U43	Hydro	41.526	41.533	41.533	41.533	41.533	41.533
U44	Hydro	41.533	41.526	41.526	41.533	41.526	41.526
U45	Hydro	41.533	41.533	41.533	41.526	41.533	41.533
U46	Hydro	52.231	52.231	52.231	52.231	52.223	52.231
U47	Hydro	52.231	52.223	52.231	52.223	52.231	52.223
U48	Hydro	52.223	52.231	52.223	52.231	52.231	52.231
U49	Thermal	0.01	0.007	0.008	0.007	0.011	54.8
U50	Thermal	0.01	0.007	0.008	0.007	0.011	54.8
U51	Thermal	0.007	0.006	0.006	0.005	0.005	75.332

Table A.30 – Case 2 – Perfect Competition: Power generation data from hydro and thermal plants, categorized by strategic and non-strategic companies.

Units	Type	T7	T8	T9	T10	T11	T12
Strategic Company							
U1	Hydro	44.186	41.469	41.721	42.029	42.032	41.908
U2	Hydro	44.117	41.414	41.668	42.038	41.978	41.825
U3	Hydro	38.77	35.484	35.784	36.336	36.226	36.044
U4	Hydro	38.764	35.516	35.7	36.281	36.141	36.097
U5	Hydro	70.806	75.736	79.078	79.855	79.302	78.875
U6	Hydro	70.662	75.585	78.821	79.741	79.217	78.829
U7	Hydro	70.661	75.502	78.992	79.606	79.299	78.761
U8	Hydro	70.798	75.72	79.058	79.642	79.287	78.703
U9	Hydro	14.116	13.712	14.097	14.122	14.149	14.142
U10	Hydro	14.116	13.712	14.097	14.122	14.149	14.142
U11	Hydro	14.116	13.712	14.097	14.122	14.149	14.142
U12	Hydro	17.874	14.179	16.77	15.813	16.07	17.059
U13	Hydro	17.874	14.179	16.77	15.813	16.07	17.059
U14	Hydro	17.874	14.179	16.77	15.813	16.07	17.059
U15	Hydro	17.874	14.179	16.77	15.813	16.07	17.059
U16	Hydro	18.711	14.163	16.765	15.807	16.062	17.08
U17	Hydro	18.711	14.158	16.765	15.807	16.062	17.08
U18	Hydro	18.711	14.158	16.765	15.807	16.062	17.08
U19	Hydro	27.381	0.548	7.874	9.751	9.584	8.04
U20	Hydro	27.387	0.554	7.89	9.744	9.539	8.006
U21	Hydro	27.387	0.579	7.839	9.742	9.526	8.043
U22	Hydro	62.822	26.974	41.049	42.602	42.608	40.593
U23	Hydro	62.804	27.027	41.206	42.582	42.641	40.642
U24	Hydro	62.856	27.054	41.146	42.584	42.637	40.743
U25	Hydro	62.703	26.991	41.146	42.644	42.524	40.611
U26	Hydro	14.308	34.332	37.086	37.648	37.273	36.657
U27	Hydro	14.277	34.288	37.091	37.679	37.285	36.865
U28	Hydro	14.321	34.144	37.101	37.671	37.279	36.771
U29	Hydro	14.254	34.215	37.107	37.69	37.267	36.966
U30	Hydro	14.337	34.266	37.11	37.691	37.265	37.029
U31	Hydro	41.443	43.046	43.556	43.636	43.109	42.997
U32	Hydro	41.426	43.094	43.631	43.541	43.502	43.123
U33	Hydro	41.436	43.057	43.669	43.635	43.163	43.147
U34	Hydro	41.404	43.106	43.77	43.663	43.153	43.163
U35	Thermal	0.037	0.062	0.069	0.092	0.083	0.077
Non-Strategic Company							
U36	Hydro	.	35.118	35.009	35.118	35.118	35.118
U37	Hydro	.	35.001	35.118	35.041	35.118	35.118
U38	Hydro	.	35.118	35.118	35.118	35.02	35.035
U39	Hydro	0.854	35.118	35.118	35.118	35.118	35.118
U40	Hydro	.	46.961	46.961	46.891	46.873	46.961
U41	Hydro	.	46.855	46.861	46.961	46.961	46.961
U42	Hydro	0.366	46.961	46.961	46.961	46.961	46.884
U43	Hydro	41.533	41.533	41.533	41.533	41.533	41.533
U44	Hydro	41.526	41.526	41.533	41.533	41.526	41.533
U45	Hydro	41.533	41.533	41.526	41.526	41.533	41.526
U46	Hydro	52.223	52.231	52.223	52.231	52.231	52.231
U47	Hydro	52.231	52.231	52.231	52.223	52.223	52.231
U48	Hydro	52.231	52.223	52.231	52.231	52.231	52.223
U49	Thermal	106.092	107.437	107.452	107.432	107.448	107.449
U50	Thermal	106.092	107.437	107.452	107.432	107.448	107.449
U51	Thermal	0.029	0.876	8.787	3.51	8.032	12.298

Table A.31 – Case 2 – Perfect Competition: Power generation data from hydro and thermal plants, categorized by strategic and non-strategic companies.

Units	Type	T13	T14	T15	T16	T17	T18
Strategic Company							
U1	Hydro	43.005	41.602	41.72	42.096	44.833	42.577
U2	Hydro	42.827	41.454	41.705	42.018	44.94	42.58
U3	Hydro	36.914	35.492	35.767	36.285	39.105	36.986
U4	Hydro	37.169	35.483	35.784	36.385	39.082	37.017
U5	Hydro	78.493	79.228	78.335	78.954	81.286	81.091
U6	Hydro	78.37	79.28	78.345	78.875	81.233	81.234
U7	Hydro	78.354	79.199	78.31	78.872	81.27	81.102
U8	Hydro	78.438	79.085	78.251	78.87	81.345	81.146
U9	Hydro	14.281	14.149	14.138	14.286	14.206	14.188
U10	Hydro	14.281	14.149	14.138	14.286	14.206	14.188
U11	Hydro	14.281	14.149	14.138	14.286	14.206	14.188
U12	Hydro	17.001	15.957	16.567	16.214	16.412	18.294
U13	Hydro	17.001	15.957	16.567	16.214	16.412	18.294
U14	Hydro	17.003	15.957	16.567	16.214	16.412	18.294
U15	Hydro	17.004	15.957	16.567	16.214	16.412	18.294
U16	Hydro	17.412	15.946	16.562	16.215	16.424	18.485
U17	Hydro	17.409	15.946	16.562	16.215	16.424	18.485
U18	Hydro	17.411	15.946	16.56	16.215	16.424	18.485
U19	Hydro	9.675	8.573	5.394	5.357	14.258	14.083
U20	Hydro	9.612	8.557	5.371	5.376	14.244	14.064
U21	Hydro	9.698	8.554	5.366	5.377	14.221	14.084
U22	Hydro	42.899	41.239	38.278	38.282	47.314	47.437
U23	Hydro	42.843	41.243	38.285	38.327	47.346	47.518
U24	Hydro	42.869	41.273	38.286	38.312	47.322	47.526
U25	Hydro	42.971	41.249	38.326	38.302	47.344	47.519
U26	Hydro	36.451	37.225	36.655	36.893	39.014	38.675
U27	Hydro	36.569	37.234	36.665	36.894	39	38.71
U28	Hydro	36.439	37.237	36.671	36.907	39.012	38.697
U29	Hydro	36.551	37.222	36.668	36.917	39.011	38.697
U30	Hydro	36.556	37.226	36.669	36.912	39.002	38.699
U31	Hydro	43.065	42.343	42.356	44.272	47.114	45.547
U32	Hydro	43.204	42.348	42.094	44.618	46.999	45.822
U33	Hydro	43.217	42.248	42.225	44.542	47.028	45.649
U34	Hydro	43.117	42.223	42.425	44.578	46.776	45.678
U35	Thermal	0.084	0.095	0.075	0.072	0.068	0.067
Non-Strategic Company							
U36	Hydro	35.118	35.118	35.118	34.992	35.118	34.008
U37	Hydro	27.953	35.118	35.118	35.118	35.118	35.118
U38	Hydro	35.118	35.068	35.026	35.118	35.118	35.118
U39	Hydro	35.118	35.118	35.118	35.118	34.779	35.118
U40	Hydro	46.961	46.915	46.878	46.846	46.961	46.961
U41	Hydro	41.348	46.961	46.961	46.961	46.638	45.904
U42	Hydro	46.961	46.961	46.961	46.961	46.961	46.961
U43	Hydro	41.533	41.533	41.526	41.533	41.533	41.533
U44	Hydro	41.533	41.533	41.533	41.533	41.526	41.533
U45	Hydro	41.526	41.526	41.533	41.526	41.533	41.526
U46	Hydro	52.231	52.231	52.223	52.231	52.231	52.231
U47	Hydro	52.231	52.223	52.231	52.223	52.231	52.223
U48	Hydro	52.223	52.231	52.231	52.231	52.223	52.231
U49	Thermal	107.461	107.433	107.451	107.446	107.451	107.456
U50	Thermal	107.461	107.433	107.451	107.446	107.451	107.456
U51	Thermal	13.228	3.978	7.225	3.78	10.053	13.571

Table A.32 – Case 2 – Perfect Competition: Power generation data from hydro and thermal plants, categorized by strategic and non-strategic companies.

Units	Type	T19	T20	T21	T22	T23	T24
Strategic Company							
U1	Hydro	42.501	41.558	41.566	41.564	42.389	45.241
U2	Hydro	42.444	41.561	41.458	41.554	42.377	45.241
U3	Hydro	36.802	35.795	35.354	33.658	37.141	40.008
U4	Hydro	36.848	35.664	35.497	33.793	37.061	40.008
U5	Hydro	81.175	79.197	77.201	72.747	74.142	66.22
U6	Hydro	80.97	79.25	77.278	73.022	74.086	65.765
U7	Hydro	81.04	79.217	77.11	72.747	74.077	66.161
U8	Hydro	81.162	79.168	77.171	72.778	74.11	66.23
U9	Hydro	14.193	14.22	14.261	13.768	12.214	13.14
U10	Hydro	14.193	14.22	14.261	13.768	12.215	13.14
U11	Hydro	14.193	14.22	14.261	13.768	12.213	13.14
U12	Hydro	18.446	18.141	17.6	17.441	14.865	18.698
U13	Hydro	18.449	18.141	17.6	17.441	14.865	18.698
U14	Hydro	18.443	18.141	17.6	17.441	14.87	18.698
U15	Hydro	18.443	18.141	17.6	17.441	14.867	18.698
U16	Hydro	18.78	18.29	17.623	18.215	17.148	19.7
U17	Hydro	18.78	18.29	17.623	18.215	17.148	19.7
U18	Hydro	18.78	18.29	17.623	18.215	17.152	19.7
U19	Hydro	13.655	8.815	2.648	1.582	24.506	20.958
U20	Hydro	13.824	8.788	2.709	1.568	24.544	20.956
U21	Hydro	13.664	8.771	2.648	1.574	24.431	21.27
U22	Hydro	46.983	41.87	34.829	23.986	65.392	67.062
U23	Hydro	47.11	41.86	34.801	23.974	65.443	66.96
U24	Hydro	47.123	41.739	34.833	23.971	65.367	66.934
U25	Hydro	47.125	41.897	34.782	23.979	65.46	67.146
U26	Hydro	38.791	37.363	35.529	32.077	15.284	0.017
U27	Hydro	38.795	37.352	35.564	32.153	15.309	0.017
U28	Hydro	38.801	37.344	35.624	32.088	15.329	0.017
U29	Hydro	38.791	37.354	35.587	32.156	15.308	0.017
U30	Hydro	38.791	37.357	35.596	32.092	15.286	0.017
U31	Hydro	45.395	43.501	41.438	36.862	31.903	4.545
U32	Hydro	45.071	43.62	41.106	37.011	31.886	4.509
U33	Hydro	45.471	43.542	41.144	37.144	32.027	4.598
U34	Hydro	45.392	43.505	41.256	36.993	32.026	4.522
U35	Thermal	0.073	0.071	0.091	0.085	0.075	0.015
Non-Strategic Company							
U36	Hydro	35.118	35.014	35.118	35.118	.	.
U37	Hydro	35.118	35.118	35.118	35.118	.	.
U38	Hydro	34.954	35.118	35.079	22.215	5.707	.
U39	Hydro	35.118	35.118	35.118	35.118	.	27.038
U40	Hydro	46.961	46.865	46.924	37.304	.	46.961
U41	Hydro	46.811	46.961	46.961	46.961	.	.
U42	Hydro	46.961	46.961	46.961	46.961	7.127	7.377
U43	Hydro	41.526	41.533	41.533	41.526	41.533	41.533
U44	Hydro	41.533	41.533	41.533	41.533	41.533	41.533
U45	Hydro	41.533	41.526	41.526	41.533	41.526	41.526
U46	Hydro	52.223	52.231	52.223	52.223	52.223	52.231
U47	Hydro	52.231	52.231	52.231	52.231	52.231	52.223
U48	Hydro	52.231	52.223	52.231	52.231	52.231	52.231
U49	Thermal	107.45	107.453	107.442	107.374	54.942	0.01
U50	Thermal	107.45	107.453	107.442	107.374	54.942	0.01
U51	Thermal	14.514	14.465	14.875	15.971	47.769	0.005

Table A.33 – Case 2 - Convexification: Data of power consumed from consumers agents.

Consumer	T1	T2	T3	T4	T5	T6
K1	67.756	63.711	60.677	59.666	59.666	60.675
K2	60.624	57.004	54.289	53.384	53.385	54.291
K3	112.319	105.612	100.582	98.906	98.907	100.584
K4	46.357	43.589	41.511	40.821	40.821	41.514
K5	44.576	41.914	39.918	39.253	39.253	39.919
K6	85.574	80.465	76.634	75.356	75.356	76.634
K7	0.007	0.005	0.003	0.004	0.005	6.207
K8	0.004	0.003	0.003	0.003	0.004	0.01
K9	0.003	0.002	0.002	0.002	0.003	12.201
K10	29.427	41.495	43.475	43.112	41.305	6.309
K11	165.803	155.904	148.48	146.005	146.005	148.48
K12	121.239	114	108.572	106.762	106.762	108.572
K13	197.895	186.08	177.219	174.266	174.266	177.22
K14	62.409	58.683	55.888	54.957	54.957	55.888
K15	208.597	196.143	186.803	183.69	183.69	186.803
K16	114.096	107.284	102.175	100.472	100.472	102.175
K17	80.228	75.438	71.845	70.648	70.648	71.846

Table A.34 – Case 2 - Convexification: Data of power consumed from consumers agents.

Consumer	T7	T8	T9	T10	T11	T12
K1	74.835	86.971	96.073	97.084	97.084	96.073
K2	66.959	77.818	85.962	86.866	86.867	85.962
K3	124.054	144.171	159.259	160.935	160.935	159.259
K4	51.202	59.505	65.733	66.425	66.425	65.733
K5	49.233	57.217	63.205	63.871	63.871	63.205
K6	94.515	109.842	121.338	122.615	122.615	121.338
K7	55.648	25.73	6.622	0.108	0.271	7.352
K8	0.009	0.008	0.008	0.007	0.008	0.008
K9	59.766	29.125	15.408	12.505	12.397	15.497
K10	58.272	17.96	3.889	0.058	0.143	4.412
K11	183.126	212.822	235.094	237.569	237.569	235.094
K12	133.906	155.621	171.907	173.716	173.716	171.907
K13	218.571	254.015	280.598	283.552	283.552	280.598
K14	68.929	80.107	88.49	89.422	89.422	88.49
K15	230.391	267.752	295.772	298.886	298.886	295.772
K16	126.016	146.451	161.778	163.481	163.481	161.778
K17	88.61	102.979	113.756	114.953	114.953	113.756

Table A.35 – Case 2 - Data of power consumed from consumers agents.

Consumer	T13	T14	T15	T16	T17	T18
K1	96.073	96.073	94.05	94.05	101.129	101.129
K2	85.962	85.962	84.152	84.152	90.486	90.486
K3	159.259	159.258	155.906	155.906	167.641	167.641
K4	65.733	65.733	64.349	64.349	69.192	69.192
K5	63.205	63.205	61.875	61.875	66.532	66.532
K6	121.338	121.338	118.783	118.783	127.724	127.724
K7	9.459	0.573	10.575	14.265	0.022	0.147
K8	0.008	0.008	0.008	0.008	0.005	0.007
K9	16.268	12.666	17.673	20.193	11.533	11.147
K10	5.935	0.298	6.671	9.188	0.012	0.08
K11	235.094	235.094	230.144	230.145	247.467	247.467
K12	171.907	171.907	168.288	168.288	180.954	180.954
K13	280.598	280.598	274.691	274.691	295.366	295.366
K14	88.49	88.49	86.627	86.627	93.148	93.148
K15	295.772	295.772	289.546	289.546	311.339	311.339
K16	161.778	161.778	158.372	158.372	170.292	170.292
K17	113.756	113.756	111.361	111.361	119.743	119.743

Table A.36 – Case 2 - Data of power consumed from consumers agents.

Consumer	T19	T20	T21	T22	T23	T24
K1	101.129	97.084	92.027	83.937	73.824	63.711
K2	90.486	86.867	82.342	75.103	66.054	57.005
K3	167.641	160.935	152.553	139.142	122.378	105.613
K4	69.192	66.425	62.965	57.429	50.51	43.59
K5	66.532	63.871	60.544	55.221	48.568	41.915
K6	127.724	122.615	116.229	106.011	93.238	80.465
K7	0.141	8.092	18.448	32.571	35.509	0.013
K8	0.007	0.008	0.009	0.009	0.011	0.006
K9	11.035	15.492	22.408	34.149	35.221	0.007
K10	0.076	4.95	12.917	26.798	33.633	34.57
K11	247.467	237.569	225.195	205.398	180.651	155.904
K12	180.954	173.716	164.668	150.192	132.096	114.001
K13	295.366	283.552	268.783	245.154	215.617	186.08
K14	93.148	89.422	84.764	77.312	67.998	58.683
K15	311.339	298.886	283.319	258.412	227.277	196.143
K16	170.292	163.481	154.966	141.343	124.313	107.284
K17	119.743	114.953	108.966	99.386	87.412	75.438

Table A.37 – Case 2 - Convexification: Data on strategic offering prices from hydro and thermal plants of the strategic company.

Units	Type	T1	T2	T3	T4	T5	T6
Strategic Company							
U1	Hydro	61.925	59.678	59.646	5.90E+01	5.94E+01	7.20E+01
U2	Hydro	61.925	59.678	5.96E+01	5.90E+01	5.94E+01	7.20E+01
U3	Hydro	61.925	59.678	5.96E+01	5.90E+01	59.435	7.20E+01
U4	Hydro	61.925	59.678	59.646	59.014	5.94E+01	7.20E+01
U5	Hydro	44.422	43.833	44.045	4.37E+01	4.37E+01	4.50E+01
U6	Hydro	44.422	43.833	44.045	4.37E+01	43.697	44.963
U7	Hydro	44.422	43.833	44.045	43.682	43.697	4.50E+01
U8	Hydro	44.422	43.833	4.40E+01	4.37E+01	43.697	4.50E+01
U9	Hydro	62.136	59.892	59.892	59.232	59.639	72.184
U10	Hydro	62.136	59.892	5.99E+01	5.92E+01	5.96E+01	7.22E+01
U11	Hydro	62.136	59.892	5.99E+01	5.92E+01	59.639	72.184
U12	Hydro	62.136	59.892	5.99E+01	5.92E+01	5.96E+01	7.22E+01
U13	Hydro	62.136	59.892	5.99E+01	5.92E+01	5.96E+01	7.22E+01
U14	Hydro	62.136	59.892	5.99E+01	59.232	5.96E+01	7.22E+01
U15	Hydro	62.136	59.892	59.892	5.92E+01	5.96E+01	7.22E+01
U16	Hydro	62.136	59.892	5.99E+01	5.92E+01	5.96E+01	72.184
U17	Hydro	62.136	59.892	5.99E+01	59.232	59.639	7.22E+01
U18	Hydro	62.136	59.892	59.892	59.232	5.96E+01	7.22E+01
U19	Hydro	35.993	36.017	3.61E+01	36.027	3.60E+01	34.311
U20	Hydro	35.993	36.017	36.062	36.027	36.003	34.311
U21	Hydro	35.993	36.017	3.61E+01	3.60E+01	3.60E+01	34.311
U22	Hydro	35.993	36.017	3.61E+01	36.027	36.003	34.311
U23	Hydro	35.993	36.017	3.61E+01	3.60E+01	3.60E+01	3.43E+01
U24	Hydro	35.993	36.017	3.61E+01	3.60E+01	3.60E+01	34.311
U25	Hydro	35.993	36.017	36.062	3.60E+01	3.60E+01	3.43E+01
U26	Hydro	44.602	44.002	4.42E+01	4.38E+01	4.39E+01	4.52E+01
U27	Hydro	44.602	44.002	4.42E+01	4.38E+01	43.862	4.52E+01
U28	Hydro	44.602	44.002	44.221	43.849	4.39E+01	4.52E+01
U29	Hydro	44.602	44.002	44.221	4.38E+01	4.39E+01	4.52E+01
U30	Hydro	44.602	44.002	44.221	4.38E+01	43.862	45.154
U31	Hydro	63.985	61.731	61.959	61.093	61.404	7.40E+01
U32	Hydro	63.985	61.731	6.20E+01	6.11E+01	61.404	7.40E+01
U33	Hydro	63.985	61.731	61.959	61.093	61.404	73.999
U34	Hydro	63.985	61.731	6.20E+01	6.11E+01	6.14E+01	7.40E+01
U35	Thermal	43.81	43.259	4.34E+01	4.31E+01	43.134	44.315

Table A.38 – Case 2 - Convexification: Data on strategic offering prices from hydro and thermal plants of the strategic company.

Units	Type	T7	T8	T9	T10	T11	T12
Strategic Company							
U1	Hydro	8.51E+01	95.139	96.154	97.021	9.55E+01	95.155
U2	Hydro	8.51E+01	95.139	96.154	97.021	95.524	95.155
U3	Hydro	8.51E+01	95.139	96.154	9.70E+01	9.55E+01	95.155
U4	Hydro	8.51E+01	95.139	96.154	97.021	95.524	95.155
U5	Hydro	44.922	44.998	44.998	4.50E+01	4.50E+01	4.50E+01
U6	Hydro	44.922	44.998	44.998	4.50E+01	4.50E+01	44.998
U7	Hydro	44.922	44.998	44.998	4.50E+01	4.50E+01	4.50E+01
U8	Hydro	44.922	44.998	44.998	4.50E+01	4.50E+01	4.50E+01
U9	Hydro	85.151	95.12	96.135	9.70E+01	95.51	9.51E+01
U10	Hydro	85.151	95.12	96.135	96.988	9.55E+01	9.51E+01
U11	Hydro	85.151	95.12	96.135	9.70E+01	9.55E+01	9.51E+01
U12	Hydro	85.151	95.12	96.135	9.70E+01	95.51	95.135
U13	Hydro	85.151	95.12	96.135	96.988	95.51	9.51E+01
U14	Hydro	85.151	95.12	96.135	9.70E+01	95.51	9.51E+01
U15	Hydro	85.151	95.12	96.135	96.988	95.51	95.135
U16	Hydro	85.151	95.12	96.135	9.70E+01	9.55E+01	9.51E+01
U17	Hydro	85.151	95.12	96.135	9.70E+01	95.51	95.135
U18	Hydro	85.151	95.12	96.135	9.70E+01	9.55E+01	9.51E+01
U19	Hydro	34.096	34.649	34.437	3.43E+01	3.43E+01	3.47E+01
U20	Hydro	34.096	34.649	34.437	34.288	3.43E+01	3.47E+01
U21	Hydro	34.096	34.649	34.437	3.43E+01	3.43E+01	3.47E+01
U22	Hydro	34.096	34.649	34.437	3.43E+01	3.43E+01	34.654
U23	Hydro	34.096	34.649	34.437	34.288	34.288	3.47E+01
U24	Hydro	34.096	34.649	34.437	34.288	3.43E+01	3.47E+01
U25	Hydro	34.096	34.649	34.437	34.288	3.43E+01	34.654
U26	Hydro	45.017	44.998	44.998	44.998	45.006	44.998
U27	Hydro	45.017	44.998	44.998	4.50E+01	4.50E+01	44.998
U28	Hydro	45.017	44.998	44.998	44.998	45.006	44.998
U29	Hydro	45.017	44.998	44.998	4.50E+01	4.50E+01	4.50E+01
U30	Hydro	45.017	44.998	44.998	4.50E+01	4.50E+01	44.998
U31	Hydro	85.996	94.997	96.013	9.68E+01	9.54E+01	9.50E+01
U32	Hydro	85.996	94.997	96.013	9.68E+01	9.54E+01	9.50E+01
U33	Hydro	85.996	94.997	96.013	9.68E+01	95.442	9.50E+01
U34	Hydro	85.996	94.997	96.013	96.761	9.54E+01	9.50E+01
U35	Thermal	44.597	44.997	44.997	4.50E+01	4.50E+01	4.50E+01

Table A.39 – Case 2 - Convexification: Data on strategic offering prices from hydro and thermal plants of the strategic company.

Units	Type	T13	T14	T15	T16	T17	T18
Strategic Company							
U1	Hydro	94.535	9.37E+01	9.30E+01	9.92E+01	1.02E+02	1.01E+02
U2	Hydro	9.45E+01	9.37E+01	9.30E+01	99.153	1.02E+02	1.01E+02
U3	Hydro	9.45E+01	93.676	92.972	9.92E+01	1.02E+02	1.01E+02
U4	Hydro	94.535	93.676	9.30E+01	9.92E+01	1.02E+02	100.779
U5	Hydro	4.50E+01	44.998	4.50E+01	44.998	44.998	4.50E+01
U6	Hydro	44.987	44.998	44.995	44.998	4.50E+01	4.50E+01
U7	Hydro	4.50E+01	4.50E+01	4.50E+01	44.998	4.50E+01	44.996
U8	Hydro	4.50E+01	44.998	44.995	44.998	44.998	44.996
U9	Hydro	9.46E+01	9.36E+01	9.30E+01	9.91E+01	1.02E+02	100.755
U10	Hydro	9.46E+01	9.36E+01	9.30E+01	99.133	102.175	100.755
U11	Hydro	94.578	9.36E+01	9.30E+01	9.91E+01	1.02E+02	1.01E+02
U12	Hydro	9.46E+01	9.36E+01	9.30E+01	9.91E+01	1.02E+02	100.755
U13	Hydro	9.46E+01	9.36E+01	92.97	9.91E+01	1.02E+02	1.01E+02
U14	Hydro	94.578	93.648	9.30E+01	9.91E+01	1.02E+02	1.01E+02
U15	Hydro	94.578	9.36E+01	9.30E+01	9.91E+01	102.175	1.01E+02
U16	Hydro	94.578	9.36E+01	92.97	99.133	1.02E+02	1.01E+02
U17	Hydro	94.578	93.648	92.97	9.91E+01	1.02E+02	1.01E+02
U18	Hydro	9.46E+01	9.36E+01	92.97	9.91E+01	102.175	100.755
U19	Hydro	33.302	34.97	34.696	33.836	33.232	3.33E+01
U20	Hydro	3.33E+01	3.50E+01	3.47E+01	3.38E+01	33.232	3.33E+01
U21	Hydro	3.33E+01	3.50E+01	34.696	33.836	33.232	33.327
U22	Hydro	3.33E+01	3.50E+01	3.47E+01	3.38E+01	3.32E+01	3.33E+01
U23	Hydro	3.33E+01	3.50E+01	3.47E+01	3.38E+01	33.232	33.327
U24	Hydro	3.33E+01	34.97	3.47E+01	3.38E+01	3.32E+01	3.33E+01
U25	Hydro	33.302	3.50E+01	3.47E+01	3.38E+01	3.32E+01	3.33E+01
U26	Hydro	4.50E+01	4.50E+01	4.50E+01	44.998	4.50E+01	4.50E+01
U27	Hydro	4.50E+01	44.998	45.01	4.50E+01	4.50E+01	4.50E+01
U28	Hydro	45.043	44.998	4.50E+01	4.50E+01	4.50E+01	45.003
U29	Hydro	4.50E+01	44.998	4.50E+01	44.998	44.998	4.50E+01
U30	Hydro	45.043	44.998	45.01	44.998	4.50E+01	4.50E+01
U31	Hydro	9.50E+01	9.35E+01	9.30E+01	98.998	1.02E+02	100.611
U32	Hydro	9.50E+01	93.466	93	98.998	101.816	100.611
U33	Hydro	9.50E+01	9.35E+01	9.30E+01	9.90E+01	1.02E+02	100.611
U34	Hydro	9.50E+01	9.35E+01	9.30E+01	98.998	101.816	100.611
U35	Thermal	44.801	4.50E+01	4.49E+01	4.50E+01	4.50E+01	4.50E+01

Table A.40 – Case 2 - Convexification: Data on strategic offering prices from hydro and thermal plants of the strategic company.

Units	Type	T19	T20	T21	T22	T23	T24
Strategic Company							
U1	Hydro	100.88	96.149	91.136	81.883	71.35	61.198
U2	Hydro	1.01E+02	96.149	91.136	81.883	71.35	61.198
U3	Hydro	1.01E+02	96.149	91.136	81.883	71.35	61.198
U4	Hydro	1.01E+02	96.149	91.136	81.883	71.35	61.198
U5	Hydro	4.50E+01	44.998	44.998	44.977	44.947	44.008
U6	Hydro	4.50E+01	44.998	44.998	44.977	44.947	44.008
U7	Hydro	44.998	44.998	44.998	44.977	44.947	44.008
U8	Hydro	4.50E+01	44.998	44.998	44.977	44.947	44.008
U9	Hydro	1.01E+02	96.13	91.118	81.992	71.512	61.381
U10	Hydro	100.848	96.13	91.118	81.992	71.512	61.381
U11	Hydro	1.01E+02	96.13	91.118	81.992	71.512	61.381
U12	Hydro	100.848	96.13	91.118	81.992	71.512	61.381
U13	Hydro	1.01E+02	96.13	91.118	81.992	71.512	61.381
U14	Hydro	1.01E+02	96.13	91.118	81.992	71.512	61.381
U15	Hydro	1.01E+02	96.13	91.118	81.992	71.512	61.381
U16	Hydro	1.01E+02	96.13	91.118	81.992	71.512	61.381
U17	Hydro	100.848	96.13	91.118	81.992	71.512	61.381
U18	Hydro	100.848	96.13	91.118	81.992	71.512	61.381
U19	Hydro	33.483	34.441	35.486	34.408	35.309	35.952
U20	Hydro	33.483	34.441	35.486	34.408	35.309	35.952
U21	Hydro	33.483	34.441	35.486	34.408	35.309	35.952
U22	Hydro	33.483	34.441	35.486	34.408	35.309	35.952
U23	Hydro	33.483	34.441	35.486	34.408	35.309	35.952
U24	Hydro	33.483	34.441	35.486	34.408	35.309	35.952
U25	Hydro	33.483	34.441	35.486	34.408	35.309	35.952
U26	Hydro	44.998	44.998	44.998	45.089	45.105	44.178
U27	Hydro	44.998	44.998	44.998	45.089	45.105	44.178
U28	Hydro	44.998	44.998	44.998	45.089	45.105	44.178
U29	Hydro	44.998	44.998	44.998	45.089	45.105	44.178
U30	Hydro	44.998	44.998	44.998	45.089	45.105	44.178
U31	Hydro	100.637	96.007	90.997	82.997	72.997	62.999
U32	Hydro	100.637	96.007	90.997	82.997	72.997	62.999
U33	Hydro	100.637	96.007	90.997	82.997	72.997	62.999
U34	Hydro	100.637	96.007	90.997	82.997	72.997	62.999
U35	Thermal	44.997	44.997	44.999	44.6	44.412	43.432

Table A.41 – Case 2 - Convexification: Data of market cleared prices per node.

Node	T1	T2	T3	T4	T5	T6
N1	61.925	59.678	59.646	59.014	59.435	71.982
N2	62.136	59.892	59.892	59.232	59.639	72.184
N3	56.592	55.258	55.926	54.942	54.885	65.503
N4	63.353	61.531	62.336	61.087	61.056	72.778
N5	62.761	60.129	59.617	59.295	60.005	73.336
N6	63.272	60.429	59.66	59.502	60.368	74.13
N7	63.985	61.731	61.959	61.093	61.404	73.999
N8	63.985	61.731	61.959	61.093	61.404	73.999
N9	64.342	62.866	64.33	62.599	62.21	73.258
N10	63.627	60.597	59.587	59.587	60.597	74.74
N11	49.694	48.603	48.737	48.296	48.435	53.741
N12	22.066	23.147	22.987	23.447	23.318	15.911
N13	35.904	35.905	35.905	35.905	35.904	34.715
N14	47.237	46.371	46.528	46.133	46.221	49.805
N15	44.995	44.371	44.605	44.213	44.224	45.572
N16	43.81	43.259	43.448	43.117	43.134	44.315
N17	44.227	43.65	43.854	43.502	43.517	44.756
N18	44.422	43.833	44.045	43.682	43.697	44.963
N19	40.478	40.172	40.3	40.096	40.094	40.051
N20	37.587	37.494	37.569	37.474	37.458	36.352
N21	44.602	44.002	44.221	43.849	43.862	45.154
N22	44.454	43.864	44.077	43.713	43.727	44.998
N23	35.993	36.017	36.062	36.027	36.003	34.311
N24	49.476	48.578	48.98	48.359	48.344	50.325

Table A.42 – Case 2 - Convexification: Data of market cleared prices per node.

Node	T7	T8	T9	T10	T11	T12
N1	85.059	95.139	96.154	97.021	95.524	95.155
N2	85.151	95.12	96.135	9.70E+01	9.55E+01	9.51E+01
N3	81.458	94.439	95.439	9.58E+01	9.42E+01	9.45E+01
N4	85.147	94.535	95.536	95.92	94.71	94.542
N5	85.942	95.536	96.564	97.74	96.168	95.556
N6	86.449	95.75	96.786	98.128	96.52	95.772
N7	85.996	94.997	96.013	96.761	95.442	95.009
N8	85.996	94.997	96.013	9.68E+01	9.54E+01	9.50E+01
N9	85.136	94.046	95.037	95.037	94.047	94.047
N10	86.856	95.947	96.989	9.85E+01	9.68E+01	9.60E+01
N11	59.518	64.217	64.607	6.49E+01	6.43E+01	6.42E+01
N12	13.257	12.741	12.079	1.16E+01	1.19E+01	1.27E+01
N13	35.892	37.65	37.498	3.74E+01	3.72E+01	3.77E+01
N14	53.286	56.19	56.417	5.66E+01	5.62E+01	5.62E+01
N15	45.226	44.998	44.998	44.998	45.03	44.997
N16	44.597	44.997	44.997	4.50E+01	4.50E+01	4.50E+01
N17	44.818	44.998	44.998	4.50E+01	4.50E+01	4.50E+01
N18	44.922	44.998	44.998	44.997	44.996	44.998
N19	40.121	40.587	40.496	4.04E+01	4.04E+01	4.06E+01
N20	36.239	36.76	36.591	36.472	36.465	36.765
N21	45.017	44.998	44.998	4.50E+01	4.50E+01	4.50E+01
N22	44.939	44.998	44.998	4.50E+01	4.50E+01	4.50E+01
N23	34.096	34.649	34.437	3.43E+01	3.43E+01	3.47E+01
N24	47.607	45.004	45.003	4.50E+01	4.53E+01	4.50E+01

Table A.43 – Case 2 - Convexification: Data of market cleared prices per node.

Node	T13	T14	T15	T16	T17	T18
N1	94.535	93.676	92.972	99.153	102.229	100.779
N2	94.578	93.648	92.97	99.133	102.175	100.755
N3	92.126	92.653	91.822	98.443	100.155	99.386
N4	94.291	92.784	92.478	98.519	100.432	99.794
N5	95.233	94.26	93.444	99.564	103.41	101.489
N6	95.628	94.576	93.704	99.786	104.048	101.875
N7	95.002	93.466	93	98.998	101.816	100.611
N8	95.002	93.467	93	98.998	101.816	100.611
N9	94.047	92.067	92.067	98.006	98.997	98.997
N10	95.957	94.866	93.933	99.99	104.634	102.225
N11	63.529	63.631	63.262	65.763	66.835	66.293
N12	9.757	13.736	13.216	10.192	8.326	8.762
N13	35.92	37.881	37.473	37.082	36.64	36.619
N14	55.707	55.849	55.611	57.091	57.715	57.389
N15	45.163	44.998	45.043	44.997	44.999	45.018
N16	44.801	44.998	44.944	45	44.997	44.974
N17	44.927	44.998	44.978	44.998	44.997	44.988
N18	44.987	44.998	44.995	44.998	44.998	44.996
N19	39.9	40.724	40.576	40.241	39.983	40.01
N20	35.648	37.016	36.787	36.113	35.633	35.703
N21	45.043	44.998	45.01	44.998	44.998	45.003
N22	44.997	44.998	44.997	44.998	44.998	44.998
N23	33.302	34.97	34.696	33.836	33.232	33.327
N24	46.533	44.999	45.419	44.987	45.009	45.189

Table A.44 – Case 2 - Convexification: Data of market cleared prices per node.

Node	T19	T20	T21	T22	T23	T24
N1	100.88	96.149	91.136	81.883	71.35	61.198
N2	100.848	96.13	91.118	81.992	71.512	61.381
N3	99.678	95.439	90.478	77.822	65.914	55.47
N4	99.835	95.534	90.556	82.093	71.932	61.927
N5	101.566	96.555	91.516	82.838	72.517	62.391
N6	101.937	96.775	91.72	83.389	73.199	63.092
N7	100.637	96.007	90.997	82.997	72.997	62.999
N8	100.637	96.007	90.998	82.997	72.997	62.999
N9	98.997	95.037	90.087	82.167	72.267	62.37
N10	102.278	96.977	91.909	83.827	73.727	63.627
N11	66.386	64.606	62.685	58.206	53.759	49.19
N12	9.102	12.088	15.339	14.447	18.336	22.585
N13	36.822	37.502	38.249	35.911	35.887	35.902
N14	57.453	56.416	55.298	52.524	49.855	46.785
N15	44.998	44.998	44.998	45.331	45.449	44.548
N16	44.997	44.997	44.999	44.6	44.412	43.432
N17	44.998	44.998	44.998	44.856	44.776	43.824
N18	44.998	44.998	44.998	44.977	44.947	44.008
N19	40.09	40.498	40.944	40.256	40.532	40.244
N20	35.832	36.594	37.427	36.487	37.166	37.478
N21	44.998	44.998	44.998	45.089	45.105	44.178
N22	44.998	44.998	44.998	44.997	44.975	44.039
N23	33.483	34.441	35.486	34.408	35.309	35.952
N24	45.002	45.002	44.994	48.097	49.374	48.768

Table A.45 – Case 2 - Perfect Competition: Power generation data from hydro and thermal plants, differentiated between strategic and non-strategic companies.

Units	Type	T1	T2	T3	T4	T5	T6
Strategic Company							
U1	Hydro	45.25	45.25	45.25	45.25	45.25	45.25
U2	Hydro	45.25	45.25	45.25	45.25	45.25	45.25
U3	Hydro	.	.	.	.	.	.
U4	Hydro	19.826	.	.	.	.	.
U5	Hydro	101.599	101.599	101.599	101.599	101.599	101.599
U6	Hydro	101.599	101.599	101.599	101.599	101.599	101.599
U7	Hydro	101.599	101.599	101.599	101.599	101.599	101.599
U8	Hydro	101.599	101.599	101.599	101.599	101.599	101.599
U9	Hydro	14.355	14.355	1.522	.	.	.
U10	Hydro	14.355	14.355	14.355	.	4.792	.
U11	Hydro	14.355	14.355	.	.	14.355	.
U12	Hydro	18.711	18.711	11.817	18.711	18.711	18.711
U13	Hydro	18.711	18.711	.	11.428	17.291	18.711
U14	Hydro	18.711	4.676	18.711	.	18.711	6.45
U15	Hydro	18.711	18.711	.	18.711	18.711	18.711
U16	Hydro	19.71	19.71	19.71	19.71	.	19.71
U17	Hydro	19.71	19.71	19.71	19.71	.	19.71
U18	Hydro	19.71	19.71	19.71	19.71	.	19.71
U19	Hydro	.	.	.	.	.	.
U20	Hydro	.	.	.	.	.	.
U21	Hydro	.	.	.	.	.	.
U22	Hydro	.	102.429	.	.	.	.
U23	Hydro	.	9.321	33.442	98.675	.	.
U24	Hydro	.	.	.	.	38.316	101.909
U25	Hydro	.	102.429	.	102.429	.	102.429
U26	Hydro	101.783	101.783	41.405	.	.	.
U27	Hydro	24.113	24.504	.	.	29.438	.
U28	Hydro	.	.	.	.	.	.
U29	Hydro	.	.	101.783	.	.	.
U30	Hydro	101.783	.	.	87.223	101.783	96.999
U31	Hydro	86.388	86.388	86.388	86.388	86.388	86.388
U32	Hydro	86.388	86.388	86.388	86.388	86.388	86.388
U33	Hydro	86.388	86.388	86.388	86.388	86.388	86.388
U34	Hydro	86.388	86.388	86.388	86.388	86.388	86.388
U35	Thermal	.	.	.	.	.	.
Non-Strategic Company							
U36	Hydro	35.118	.	.	.	.	.
U37	Hydro	35.118	.	.	.	.	.
U38	Hydro	35.118	.	.	.	.	.
U39	Hydro	23.578	.	.	.	.	.
U40	Hydro	46.961	.	46.961	.	40.277	.
U41	Hydro	10.713	.	46.961	.	46.961	.
U42	Hydro	.	.	46.961	.	46.961	.
U43	Hydro	41.533	41.533	41.533	41.533	41.533	41.533
U44	Hydro	41.533	41.533	41.533	41.533	41.533	41.533
U45	Hydro	41.533	41.533	41.533	41.533	41.533	41.533
U46	Hydro	52.231	52.231	52.231	52.231	52.231	52.231
U47	Hydro	52.231	52.231	52.231	52.231	52.231	52.231
U48	Hydro	52.231	52.231	52.231	52.231	52.231	52.231
U49	Thermal	.	.	.	.	.	.
U50	Thermal	.	.	.	.	.	.
U51	Thermal	.	.	.	.	.	.

Table A.46 – Case 2 - Perfect Competition: Power generation data from hydro and thermal plants, differentiated between strategic and non-strategic companies.

Units	Type	T7	T8	T9	T10	T11	T12
Strategic Company							
U1	Hydro	45.25	45.25	45.25	45.25	45.25	45.25
U2	Hydro	45.25	45.25	45.25	45.25	45.25	45.25
U3	Hydro	40.017	40.017	40.017	40.017	40.017	40.017
U4	Hydro	40.017	40.017	40.017	40.017	40.017	40.017
U5	Hydro	101.599	101.599	101.599	101.599	101.599	101.599
U6	Hydro	101.599	101.599	101.599	101.599	101.599	101.599
U7	Hydro	101.599	101.599	101.599	101.599	101.599	101.599
U8	Hydro	101.599	101.599	101.599	101.599	101.599	101.599
U9	Hydro	14.355	14.355	14.355	14.355	14.355	14.355
U10	Hydro	14.355	14.355	14.355	14.355	14.355	14.355
U11	Hydro	14.355	14.355	14.355	14.355	14.355	14.355
U12	Hydro	18.711	18.711	18.711	18.711	18.711	18.711
U13	Hydro	18.711	18.711	18.711	18.711	18.711	18.711
U14	Hydro	18.711	18.711	18.711	18.711	18.711	18.711
U15	Hydro	18.711	18.711	18.711	18.711	18.711	18.711
U16	Hydro	19.71	19.71	19.71	19.71	19.71	19.71
U17	Hydro	19.71	19.71	19.71	19.71	19.71	19.71
U18	Hydro	19.71	19.71	19.71	19.71	19.71	19.71
U19	Hydro	.	.	.	.	.	.
U20	Hydro	.	.	.	.	.	.
U21	Hydro	.	.	.	.	.	.
U22	Hydro	.	.	30.461	42.467	42.467	30.461
U23	Hydro	.	.	.	.	.	.
U24	Hydro	.	.	.	.	.	.
U25	Hydro	.	.	.	.	.	.
U26	Hydro	.	101.783	101.783	101.783	101.783	101.783
U27	Hydro	101.783	101.783	101.783	101.783	101.783	101.783
U28	Hydro	85.223	101.783	101.783	101.783	101.783	101.783
U29	Hydro	.	101.783	101.783	101.783	101.783	101.783
U30	Hydro	101.783	101.783	101.783	101.783	101.783	101.783
U31	Hydro	86.388	86.388	86.388	86.388	86.388	86.388
U32	Hydro	86.388	86.388	86.388	86.388	86.388	86.388
U33	Hydro	86.388	86.388	86.388	86.388	86.388	86.388
U34	Hydro	86.388	86.388	86.388	86.388	86.388	86.388
U35	Thermal	.	.	.	.	.	.
Non-Strategic Company							
U36	Hydro	35.118	35.118	35.118	35.118	35.118	35.118
U37	Hydro	35.118	.	35.118	35.118	35.118	35.118
U38	Hydro	35.118	.	35.118	35.118	35.118	35.118
U39	Hydro	35.118	23.706	35.118	35.118	35.118	35.118
U40	Hydro	46.417	46.961	46.961	46.961	46.961	46.961
U41	Hydro	.	46.961	46.961	46.961	46.961	46.961
U42	Hydro	46.961	46.961	46.961	46.961	46.961	46.961
U43	Hydro	41.533	41.533	41.533	41.533	41.533	41.533
U44	Hydro	41.533	41.533	41.533	41.533	41.533	41.533
U45	Hydro	41.533	41.533	41.533	41.533	41.533	41.533
U46	Hydro	52.231	52.231	52.231	52.231	52.231	52.231
U47	Hydro	52.231	52.231	52.231	52.231	52.231	52.231
U48	Hydro	52.231	52.231	52.231	52.231	52.231	52.231
U49	Thermal	.	.	.	.	.	.
U50	Thermal	.	.	.	.	.	.
U51	Thermal	.	.	.	.	.	.

Table A.47 – Case 2 - Perfect Competition: Power generation data from hydro and thermal plants, differentiated between strategic and non-strategic companies.

Units	Type	T13	T14	T15	T16	T17	T18
Strategic Company							
U1	Hydro	45.25	45.25	45.25	45.25	45.25	45.25
U2	Hydro	45.25	45.25	45.25	45.25	45.25	45.25
U3	Hydro	40.017	40.017	40.017	40.017	40.017	40.017
U4	Hydro	40.017	40.017	40.017	40.017	40.017	40.017
U5	Hydro	101.599	101.599	101.599	101.599	101.599	101.599
U6	Hydro	101.599	101.599	101.599	101.599	101.599	101.599
U7	Hydro	101.599	101.599	101.599	101.599	101.599	101.599
U8	Hydro	101.599	101.599	101.599	101.599	101.599	101.599
U9	Hydro	14.355	14.355	14.355	14.355	14.355	14.355
U10	Hydro	14.355	14.355	14.355	14.355	14.355	14.355
U11	Hydro	14.355	14.355	14.355	14.355	14.355	14.355
U12	Hydro	18.711	18.711	18.711	18.711	18.711	18.711
U13	Hydro	18.711	18.711	18.711	18.711	18.711	18.711
U14	Hydro	18.711	18.711	18.711	18.711	18.711	18.711
U15	Hydro	18.711	18.711	18.711	18.711	18.711	18.711
U16	Hydro	19.71	19.71	19.71	19.71	19.71	19.71
U17	Hydro	19.71	19.71	19.71	19.71	19.71	19.71
U18	Hydro	19.71	19.71	19.71	19.71	19.71	19.71
U19	Hydro	.	.	.	.	.	.
U20	Hydro	.	.	.	.	.	.
U21	Hydro	.	.	.	.	.	.
U22	Hydro	30.461	.	.	.	.	.
U23	Hydro	.	.	.	.	.	.
U24	Hydro	.	30.461	6.45	6.45	90.489	90.489
U25	Hydro	.	.	.	.	.	.
U26	Hydro	101.783	101.783	101.783	101.783	101.783	101.783
U27	Hydro	101.783	101.783	101.783	101.783	101.783	101.783
U28	Hydro	101.783	101.783	101.783	101.783	101.783	101.783
U29	Hydro	101.783	101.783	101.783	101.783	101.783	101.783
U30	Hydro	101.783	101.783	101.783	101.783	101.783	101.783
U31	Hydro	86.388	86.388	86.388	86.388	86.388	86.388
U32	Hydro	86.388	86.388	86.388	86.388	86.388	86.388
U33	Hydro	86.388	86.388	86.388	86.388	86.388	86.388
U34	Hydro	86.388	86.388	86.388	86.388	86.388	86.388
U35	Thermal	.	.	.	.	.	.
Non-Strategic Company							
U36	Hydro	35.118	35.118	35.118	35.118	35.118	35.118
U37	Hydro	35.118	35.118	35.118	35.118	35.118	35.118
U38	Hydro	35.118	35.118	35.118	35.118	35.118	35.118
U39	Hydro	35.118	35.118	35.118	35.118	35.118	35.118
U40	Hydro	46.961	46.961	46.961	46.961	46.961	46.961
U41	Hydro	46.961	46.961	46.961	46.961	46.961	46.961
U42	Hydro	46.961	46.961	46.961	46.961	46.961	46.961
U43	Hydro	41.533	41.533	41.533	41.533	41.533	41.533
U44	Hydro	41.533	41.533	41.533	41.533	41.533	41.533
U45	Hydro	41.533	41.533	41.533	41.533	41.533	41.533
U46	Hydro	52.231	52.231	52.231	52.231	52.231	52.231
U47	Hydro	52.231	52.231	52.231	52.231	52.231	52.231
U48	Hydro	52.231	52.231	52.231	52.231	52.231	52.231
U49	Thermal	.	.	.	.	.	.
U50	Thermal	.	.	.	.	.	.
U51	Thermal	.	.	.	.	.	.

Table A.48 – Case 2 - Perfect Competition: Power generation data from hydro and thermal plants, differentiated between strategic and non-strategic companies.

Units	Type	T19	T20	T21	T22	T23	T24
Strategic Company							
U1	Hydro	45.25	45.25	45.25	45.25	45.25	45.25
U2	Hydro	45.25	45.25	45.25	45.25	45.25	45.25
U3	Hydro	40.017	40.017	40.017	40.017	40.017	.
U4	Hydro	40.017	40.017	40.017	40.017	40.017	.
U5	Hydro	101.599	101.599	101.599	101.599	101.599	101.599
U6	Hydro	101.599	101.599	101.599	101.599	101.599	101.599
U7	Hydro	101.599	101.599	101.599	101.599	101.599	101.599
U8	Hydro	101.599	101.599	101.599	101.599	101.599	101.599
U9	Hydro	14.355	14.355	14.355	14.355	14.355	14.355
U10	Hydro	14.355	14.355	14.355	14.355	14.355	14.355
U11	Hydro	14.355	14.355	14.355	14.355	14.355	14.355
U12	Hydro	18.711	18.711	18.711	18.711	18.711	18.711
U13	Hydro	18.711	18.711	18.711	18.711	18.711	18.711
U14	Hydro	18.711	18.711	18.711	18.711	18.711	18.711
U15	Hydro	18.711	18.711	18.711	18.711	18.711	4.676
U16	Hydro	19.71	19.71	19.71	19.71	19.71	19.71
U17	Hydro	19.71	19.71	19.71	19.71	19.71	19.71
U18	Hydro	19.71	19.71	19.71	19.71	19.71	19.71
U19	Hydro	.	.	.	.	.	.
U20	Hydro	.	.	.	.	.	.
U21	Hydro	.	.	.	.	.	.
U22	Hydro	.	.	.	.	.	.
U23	Hydro	.	.	.	.	.	102.429
U24	Hydro	90.489	42.467	.	.	.	102.429
U25	Hydro	.	.	.	.	.	9.321
U26	Hydro	101.783	101.783	101.783	66.752	.	.
U27	Hydro	101.783	101.783	101.783	101.783	.	.
U28	Hydro	101.783	101.783	101.783	101.783	101.783	24.504
U29	Hydro	101.783	101.783	101.783	101.783	80.828	.
U30	Hydro	101.783	101.783	101.783	101.783	101.783	101.783
U31	Hydro	86.388	86.388	86.388	86.388	86.388	86.388
U32	Hydro	86.388	86.388	86.388	86.388	86.388	86.388
U33	Hydro	86.388	86.388	86.388	86.388	86.388	86.388
U34	Hydro	86.388	86.388	86.388	86.388	86.388	86.388
U35	Thermal	.	.	.	.	.	.
Non-Strategic Company							
U36	Hydro	35.118	35.118	35.118	.	35.118	.
U37	Hydro	35.118	35.118	35.118	.	35.118	.
U38	Hydro	35.118	35.118	16.639	35.118	35.118	.
U39	Hydro	35.118	35.118	35.118	16.349	35.118	.
U40	Hydro	46.961	46.961	46.961	46.961	46.961	.
U41	Hydro	46.961	46.961	46.961	46.961	31.782	.
U42	Hydro	46.961	46.961	46.961	46.961	.	.
U43	Hydro	41.533	41.533	41.533	41.533	41.533	41.533
U44	Hydro	41.533	41.533	41.533	41.533	41.533	41.533
U45	Hydro	41.533	41.533	41.533	41.533	41.533	41.533
U46	Hydro	52.231	52.231	52.231	52.231	52.231	52.231
U47	Hydro	52.231	52.231	52.231	52.231	52.231	52.231
U48	Hydro	52.231	52.231	52.231	52.231	52.231	52.231
U49	Thermal	.	.	.	.	.	.
U50	Thermal	.	.	.	.	.	.
U51	Thermal	.	.	.	.	.	.

Table A.49 – Case 2 - Perfect Competition: Data of power consumed from consumers agents.

Consumers	T1	T2	T3	T4	T5	T6
K1	67.76	63.715	60.681	59.669	59.669	60.681
K2	60.63	57.01	54.295	53.39	53.39	54.295
K3	112.326	105.62	100.59	98.914	98.914	100.59
K4	46.362	43.594	41.518	40.826	40.826	41.518
K5	44.579	41.918	39.922	39.256	39.256	39.922
K6	85.58	80.47	76.639	75.361	75.361	76.639
K7	78.449	73.766	70.253	69.082	69.082	70.253
K8	106.971	100.585	95.795	94.199	94.199	95.795
K9	108.754	102.261	97.392	95.768	95.768	97.392
K10	73.158	64.683	58.67	56.73	56.73	58.67
K11	165.812	155.912	148.488	146.013	146.013	148.488
K12	121.246	114.007	108.578	106.769	106.769	108.578
K13	197.906	186.09	177.229	174.275	174.275	177.229
K14	62.412	58.686	55.892	54.96	54.96	55.892
K15	208.608	196.154	186.813	183.7	183.7	186.813
K16	114.102	107.29	102.181	100.478	100.478	102.181
K17	80.232	75.442	71.849	70.652	70.652	71.849

Table A.50 – Case 2 - Perfect Competition: Data of power consumed from consumers agents.

Consumers	T7	T8	T9	T10	T11	T12
K1	74.84	86.976	96.078	97.089	97.089	96.078
K2	66.964	77.823	85.967	86.872	86.872	85.967
K3	124.061	41.859	85.404	88.476	88.476	85.404
K4	51.206	59.509	65.737	66.429	66.429	65.737
K5	49.237	57.221	63.209	63.875	63.875	63.209
K6	94.521	109.849	121.344	122.622	122.622	121.344
K7	86.645	100.696	111.234	112.405	64.595	111.234
K8	118.147	120.336	.	.	.	.
K9	98.979	139.595	137.649	131.922	155.827	137.649
K10	89.243	75.756	87.454	82.209	106.113	87.454
K11	183.135	212.833	235.106	237.581	237.581	235.106
K12	133.913	155.629	171.916	173.725	173.725	171.916
K13	218.582	254.028	280.612	283.566	283.566	280.612
K14	68.933	80.111	88.495	89.427	89.427	88.495
K15	230.403	267.766	295.788	298.901	298.901	295.788
K16	126.023	146.459	161.786	163.489	163.489	161.786
K17	88.614	102.984	113.762	114.959	114.959	113.762

Table A.51 – Case 2 - Perfect Competition: Data of power consumed from consumers agents.

Consumers	T13	T14	T15	T16	T17	T18
K1	96.078	96.078	94.055	94.055	101.135	101.135
K2	85.967	85.967	84.158	84.158	90.492	90.492
K3	85.404	85.404	79.26	79.26	100.765	100.765
K4	65.737	65.737	64.353	64.353	69.197	69.197
K5	63.209	63.209	61.879	61.879	66.536	66.536
K6	121.344	121.344	118.79	118.79	127.731	127.731
K7	111.234	78.125	108.892	108.892	117.088	117.088
K8	.	.	.	.	.	.
K9	137.649	154.203	149.103	149.103	109.014	109.014
K10	87.454	104.008	97.944	97.944	61.228	61.228
K11	235.106	235.106	230.156	230.156	247.48	247.48
K12	171.916	171.916	168.296	168.296	180.964	180.964
K13	280.612	280.612	274.705	274.705	295.382	295.382
K14	88.495	88.495	86.632	86.632	93.153	93.153
K15	295.788	295.788	289.561	289.561	311.355	311.355
K16	161.786	161.786	158.38	158.38	170.301	170.301
K17	113.762	113.762	111.367	111.367	119.749	119.749

Table A.52 – Case 2 - Perfect Competition: Data of power consumed from consumers agents.

Consumers	T19	T20	T21	T22	T23	T24
K1	101.135	97.089	92.032	83.942	73.828	63.715
K2	90.492	86.872	82.348	75.108	66.059	57.01
K3	100.765	88.476	69.518	48.757	122.385	105.62
K4	69.197	66.429	62.969	57.433	50.514	43.594
K5	66.536	63.875	60.548	55.225	48.571	41.918
K6	127.731	122.622	116.235	106.017	93.244	80.47
K7	117.088	64.595	106.55	97.183	85.474	73.766
K8	.	.	30.477	132.517	116.551	100.585
K9	109.014	155.827	147.711	134.725	105.525	102.261
K10	61.228	106.113	93.482	78.876	86.847	64.683
K11	247.48	237.581	225.207	205.408	180.66	155.912
K12	180.964	173.725	164.677	150.2	132.104	114.007
K13	295.382	283.566	268.797	245.167	215.629	186.09
K14	93.153	89.427	84.769	77.317	68.001	58.686
K15	311.355	298.901	283.333	258.425	227.289	196.154
K16	170.301	163.489	154.974	141.35	124.32	107.29
K17	119.749	114.959	108.972	99.392	87.417	75.442

Table A.53 – Case 2 - Perfect Competition: Data of market cleared prices per node.

Node	T1	T2	T3	T4	T5	T6
N1	3.94	3.93865756	3.93865756	3.93865756	3.93865756	3.93865756
N2	3.94135203	3.94	3.94	3.94	3.94	3.94
N3	4.74343862	4.74127242	4.74127242	4.74127242	4.74127242	4.74127242
N4	4.29498042	4.29313312	4.29313312	4.29313312	4.29313312	4.29313312
N5	3.60815696	3.60720576	3.60720576	3.60720576	3.60720576	3.60720576
N6	3.42572085	3.42498282	3.42498282	3.42498282	3.42498282	3.42498282
N7	3.92434762	3.9229484	3.9229484	3.9229484	3.9229484	3.9229484
N8	3.92434762	3.9229484	3.9229484	3.9229484	3.9229484	3.9229484
N9	4.58445498	4.58220227	4.58220227	4.58220227	4.58220227	4.58220227
N10	3.26424025	3.26369452	3.26369452	3.26369452	3.26369452	3.26369452
N11	3.28021259	3.27959181	3.27959181	3.27959181	3.27959181	3.27959181
N12	2.41496143	2.41551156	2.41551156	2.41551156	2.41551156	2.41551156
N13	2.80698199	2.80698199	2.80698199	2.80698199	2.80698199	2.80698199
N14	3.03998266	3.03960954	3.03960954	3.03960954	3.03960954	3.03960954
N15	2.63762125	2.63763508	2.63763508	2.63763508	2.63763508	2.63763508
N16	2.70501418	2.70498637	2.70498637	2.70498637	2.70498637	2.70498637
N17	2.68137719	2.68136399	2.68136399	2.68136399	2.68136399	2.68136399
N18	2.67026105	2.67025471	2.67025471	2.67025471	2.67025471	2.67025471
N19	2.68577326	2.68581185	2.68581185	2.68581185	2.68581185	2.68581185
N20	2.66908134	2.66917755	2.66917755	2.66917755	2.66917755	2.66917755
N21	2.66	2.66	2.66	2.66	2.66	2.66
N22	2.66840035	2.66839516	2.66839516	2.66839516	2.66839516	2.66839516
N23	2.65987201	2.255177975	2.66	2.66	2.66	2.66
N24	2.38280522	2.164496357	2.38297645	2.38297645	2.38297645	2.38297645

Table A.54 – Case 2 - Perfect Competition: Data of market cleared prices per node.

Node	T7	T8	T9	T10	T11	T12
N1	51.94474908	95.71869956	97.55719105	97.55719105	96.54097031	96.54097031
N2	52.28922378	95.43917764	97.39425519	97.39425519	96.3797317	96.3797317
N3	82.20541617	99.75	1.01E+02	1.01E+02	99.75	99.75
N4	70.35315425	92.58576123	96.09970124	96.09970124	95.09866269	95.09866269
N5	37.62259078	95.83228348	97.26393047	97.26393047	96.25076453	96.25076453
N6	29.81698658	95.82817684	97.06356309	97.06356309	96.05248431	96.05248431
N7	53.95966981	93.1	96	96	95	95
N8	53.95966981	93.1	96	96	95	95
N9	85.14	90.25	95.04	95.04	94.05	94.05
N10	22.77933961	95.95	96.96	96.96	95.95	95.95
N11	25.47890801	43.78262142	48.40307439	48.40307439	47.91300794	47.91300794
N12	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00
N13	2.80698199	2.80698199	8.34687269	8.34687269	8.28625615	8.28625615
N14	16.38275459	29.5304269	35.09328681	35.09328681	34.74555004	34.74555004
N15	2.14327064	8.3615762	15.69914978	15.69914978	15.55857497	15.55857497
N16	3.69938573	9.65764862	16.53456892	16.53456892	16.38529182	16.38529182
N17	3.15360332	9.203072	16.24155905	16.24155905	16.09533414	16.09533414
N18	2.89692959	8.98929107	16.10376067	16.10376067	15.95897116	15.95897116
N19	1.30554848	3.99669189	10.62081823	10.62081823	10.53516744	10.53516744
N20	0	0	5.49051315	5.49051315	5.46005953	5.46005953
N21	2.66	8.79195482	15.97656216	15.97656216	15.83309764	15.83309764
N22	2.85396564	8.95350683	16.08069495	16.08069495	15.93614571	15.93614571
N23	0	0	2.66	2.66	2.66	2.66
N24	0	3.46106148	12.54038815	12.54038815	12.43271711	12.43271711

Table A.55 – Case 2 - Perfect Competition: Data of market cleared prices per node.

Node	T13	T14	T15	T16	T17	T18
N1	96.54097031	94.50852883	94.50852883	1.01E+02	1.02E+02	1.02E+02
N2	96.3797317	94.35068472	94.35068472	1.00E+02	1.01E+02	1.01E+02
N3	99.75	97.65	97.65	1.04E+02	1.05E+02	1.05E+02
N4	95.09866269	93.09658558	93.09658558	99.10281691	1.00E+02	1.00E+02
N5	96.25076453	94.22443265	94.22443265	1.00E+02	1.01E+02	1.01E+02
N6	96.05248431	94.03032674	94.03032674	1.00E+02	1.01E+02	1.01E+02
N7	95	93	93	99	1.00E+02	1.00E+02
N8	95	93	93	99	1.00E+02	1.00E+02
N9	94.05	92.07	92.07	98.01	99	99
N10	95.95	93.93	93.93	99.99	1.01E+02	1.01E+02
N11	47.91300794	46.93287505	46.93287505	49.87327373	50.36334017	50.36334017
N12	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00
N13	8.28625615	8.16502307	8.16502307	8.5287223	8.58933884	8.58933884
N14	34.74555004	34.05007649	34.05007649	36.13649713	36.48423391	36.48423391
N15	15.55857497	15.27742535	15.27742535	16.1208742	16.26144901	16.26144901
N16	16.38529182	16.08673764	16.08673764	16.98240019	17.13167728	17.13167728
N17	16.09533414	15.80288433	15.80288433	16.68023377	16.82645867	16.82645867
N18	15.95897116	15.66939215	15.66939215	16.53812918	16.68291869	16.68291869
N19	10.53516744	10.36386586	10.36386586	10.8777706	10.96342139	10.96342139
N20	5.46005953	5.39915231	5.39915231	5.58187399	5.61232761	5.61232761
N21	15.83309764	15.54616859	15.54616859	16.40695573	16.55042025	16.55042025
N22	15.93614571	15.64704724	15.64704724	16.51434267	16.65889191	16.65889191
N23	2.66	2.66	2.66	2.66	2.66	2.66
N24	12.43271711	12.21737503	12.21737503	12.86340128	12.97107232	12.97107232

Table A.56 – Case 2 - Perfect Competition: Data of market cleared prices per node.

Node	T19	T20	T21	T22	T23	T24
N1	1.02E+02	97.55719105	91.68843853	81.69154446	44.27524451	3.93865756
N2	1.01E+02	97.39425519	91.42068595	81.16878426	44.56490003	3.94
N3	1.05E+02	1.01E+02	95.55	87.15	69.82966164	4.74127242
N4	1.00E+02	96.09970124	88.68741338	74.97306585	59.79933456	4.29313312
N5	1.01E+02	97.26393047	91.79723996	82.74166854	32.18825645	3.60720576
N6	1.01E+02	97.06356309	91.79330623	83.19534157	25.60057181	3.42498282
N7	1.00E+02	96	89.18	76.86567473	45.96574963	3.9229484
N8	1.00E+02	96	89.18	76.86567473	45.96574963	3.9229484
N9	99	95.04	86.45	69.90134946	72.27	4.58220227
N10	1.01E+02	96.96	91.91	83.83	19.66149926	3.26369452
N11	50.36334017	48.40307439	42.00257858	35.64139731	21.93232149	3.27959181
N12	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	2.41551156
N13	8.58933884	8.34687269	2.80698199	2.80698199	2.80698199	2.80698199
N14	36.48423391	35.09328681	28.36702217	22.49103142	14.25103945	3.03960954
N15	16.26144901	15.69914978	8.11256229	1.91695856	2.22225083	2.63763508
N16	17.13167728	16.53456892	9.35406324	4.15460574	3.54051943	2.70498637
N17	16.82645867	16.24155905	8.91862669	3.36978687	3.07815785	2.68136399
N18	16.68291869	16.10376067	8.71384706	3.0006977	2.86071577	2.67025471
N19	10.96342139	10.62081823	3.94055108	0.67368614	1.52606083	2.68581185
N20	5.61232761	5.49051315	0	0	0	2.66917755
N21	16.55042025	15.97656216	8.52481971	2.66	2.66	2.66
N22	16.65889191	16.08069495	8.67956953	2.93891682	2.8243187	2.66839516
N23	2.66	2.66	0	0	0	2.66
N24	12.97107232	12.54038815	3.41838504	0	0	2.38297645

APPENDIX B – CHAPTER 4

NUMERICAL RESULTS

In this appendix, we present the complete results from the case study in Section 4.5.2 of Chapter 4.

Proposed Model Results

Table B.1 – Chapter 4 Case Study 2.1 - Proposed Model: Strategic Producers

Producer Company	Unit	Power Supplied [MW]	Offering Price [\$]
1	1	0	932.184
2	2	0	872.431
3	3	283.728	80.86
4	4	574.728	80.86
5	5	43.7280	80.86
6	6	0	1678.763
7	7	0	1820.635
8	8	383.728	80.86
9	9	383.728	80.86
10	10	283.728	80.86
11	11	293.715	80.86
12	12	0	1122.882
Total	-	2247.083	-

Table B.2 – Chapter 4 Case Study 2.1 - Proposed Model: Strategic Consumers

Consumer Company	Unit	Power Demanded [MW]	Bidding Price [\$]
1	1	76.989	80.86
2	2	66.387	80.86
3	3	143.252	80.86
4	4	45.183	80.86
5	5	42.533	80.86
6	6	103.494	80.86
7	7	92.892	80.86
8	8	135.3	80.86
9	9	137.95	80.86
10	10	156.504	80.86
11	11	222.766	80.86
12	12	156.504	80.86
13	13	270.475	80.86
14	14	69.037	80.86
15	15	286.378	80.86
16	16	145.902	80.86
17	17	95.542	80.86
Total	-	2247.088	-

Diagonalization PDOCs Results

Table B.3 – Chapter 4 Case Study 2.1 - Diagonalization PDOCs: Strategic Producers

Producer Company	Unit	Power Supplied [MW]	Offering Price [\$]
1	1	0	932.184
2	2	0	872.431
3	3	300	80.86
4	4	591	80.86
5	5	60	80.86
6	6	0	1678.763
7	7	0	1820.635
8	8	400	80.86
9	9	400	80.86
10	10	300	80.86
11	11	310	80.86
12	12	0	1122.882
Total	-	2361	-

Table B.4 – Chapter 4 Case Study 2.1 - Diagonalization PDOCs: Strategic Consumers

Consumer Company	Unit	Power Demanded [MW]	Bidding Price [\$]
1	1	90.381	80.86
2	2	80.867	80.86
3	3	116.306	80.86
4	4	61.84	80.86
5	5	59.461	80.86
6	6	114.166	80.86
7	7	104.652	80.86
8	8	157.309	80.86
9	9	145.086	80.86
10	10	161.735	80.86
11	11	221.196	80.86
12	12	161.735	80.86
13	13	204.008	80.86
14	14	83.246	80.86
15	15	310.108	80.86
16	16	169.632	80.86
17	17	119.272	80.86
Total	-	2361	-

Diagonalization KKT Results

Table B.5 – Chapter 4 Case Study 2.1 - Diagonalization KKT: Strategic Producers

Producer Company	Unit	Power Supplied [MW]	Offering Price [\$]
1	1	0	932.184
2	2	0	872.431
3	3	283.728	80.86
4	4	591	80.86
5	5	60	80.86
6	6	0	1678.763
7	7	0	1820.635
8	8	400	80.86
9	9	400	80.86
10	10	300	80.86
11	11	310	80.86
12	12	0	1122.882
Total	-	2344.728	-

Table B.6 – Chapter 4 Case Study 2.1 - Diagonalization KKT: Strategic Consumers

Consumer Company	Unit	Power Demanded [MW]	Bidding Price [\$]
1	1	100.719	80.86
2	2	90.117	80.86
3	3	166.982	80.86
4	4	68.913	80.86
5	5	66.263	80.86
6	6	127.224	80.86
7	7	116.622	80.86
8	8	159.03	80.86
9	9	161.68	80.86
10	10	180.234	80.86
11	11	246.496	80.86
12	12	180.234	80.86
13	13	59.624	80.86
14	14	92.767	80.86
15	15	238.918	80.86
16	16	169.632	80.86
17	17	119.272	80.86
Total	-	2344.727	-

B.0.1 Case 2.2

Proposed Model Results

Table B.7 – Chapter 4 Case Study 2.2 - Proposed Model: Strategic Producers

Producer Company	Unit	Power Supplied [MW]	Offering Price [\$]
1	1	0	330.437
2	2	0	112.103
3	3	300	11.799
3	4	591	99.997
3	5	60	99.997
6	6	154.999	99.997
7	7	134.499	99.997
3	8	400	100
3	9	400	100
10	10	299.999	100
11	11	309.999	100
12	12	0	109.804
Total	-	2650.496	-

Table B.8 – Chapter 4 Case Study 2.2 - Proposed Model: Strategic Consumers

Consumer Company	Unit	Power Demanded [MW]	Bidding Price [\$]
1	1	100.719	100
2	2	90.117	100
3	3	166.981	100
4	4	68.913	100
5	5	66.262	100
6	6	127.224	100
7	7	116.622	100
8	8	159.03	100
9	9	161.68	100
10	10	180.234	100
11	11	246.496	100
12	12	180.234	100
13	13	294.205	100
14	14	92.767	100
15	15	310.109	178.698
16	16	169.632	100
17	17	119.272	100
Total	-	2650.497	-

Diagonalization PDOCs Results

Table B.9 – Chapter 4 Case Study 2.2 - Diagonalization PDOCs: Strategic Producers

Producer Company	Unit	Power Supplied [MW]	Offering Price [\$]
1	1	0	330.437
2	2	0	112.103
3	3	300	11.799
3	4	591	99.997
3	5	60	99.997
7	7	155	99.997
6	6	155	99.997
3	8	381.723	100
3	9	397.777	100
10	10	300	100
11	11	310	100
12	12	0	109.804
Total	-	2650.5	-

Table B.10 – Chapter 4 Case Study 2.2 - Diagonalization PDOCs: Strategic Consumers

Consumer Company	Unit	Power Demanded [MW]	Bidding Price [\$]
1	1	100.719	100
2	2	90.117	100
3	3	166.982	100
4	4	68.913	100
5	5	66.262	100
6	6	127.224	100
7	7	116.622	100
8	8	159.03	100
9	9	161.68	100
10	10	180.234	100
11	11	246.496	100
12	12	180.234	100
13	13	294.205	100
14	14	92.767	100
15	15	310.108	178.698
16	16	169.632	100
17	17	119.272	100
Total	-	2650.497	-

Diagonalization KKT Results

Table B.11 – Chapter 4 Case Study 2.2 - Diagonalization KKT: Strategic Producers

Producer Company	Unit	Power Supplied [MW]	Offering Price [\$]
1	1	0	330.437
2	2	0	112.103
3	3	300	11.799
3	4	591	99.997
3	5	60	99.997
6	6	155	99.997
7	7	155	99.997
3	8	379.498	100
3	9	400	100
10	10	300	100
11	11	310	100
12	12	0	109.804
Total	-	2650.498	-

Table B.12 – Chapter 4 Case Study 2.2 - Diagonalization KKT: Strategic Consumers

Consumer Company	Unit	Power Demanded [MW]	Bidding Price [\$]
1	1	100.719	100
2	2	90.117	100
3	3	166.982	100
4	4	68.913	100
5	5	66.262	100
6	6	127.224	100
7	7	116.622	100
8	8	159.03	100
9	9	161.68	100
10	10	180.234	100
11	11	246.496	100
12	12	180.234	100
13	13	294.204	100
14	14	92.767	100
15	15	310.108	100
16	16	169.632	100
17	17	119.272	100
Total	-	2650.496	-

B.0.2 Case 2.3

Proposed Model Results

Table B.13 – Chapter 4 Case Study 2.3 - Proposed Model: Strategic Producers

Producer Company	Unit	Power Supplied [MW]	Offering Price [\$]
1	1	0	2670.122
2	2	0	7707.332
3	3	221.623	22.72
4	4	512.623	22.72
5	5	0	479.817
6	6	0	931.368
7	7	0	2499.999
8	8	321.623	22.72
9	9	321.623	22.72
10	10	0	164.045
11	11	0	81.533
12	12	0	2902.345
Total	-	1377.492	-

Table B.14 – Chapter 4 Case Study 2.3 - Proposed Model: Strategic Consumers

Consumer Company	Unit	Power Demanded [MW]	Bidding Price [\$]
1	1	25.083	22.72
2	2	14.481	22.72
3	3	91.345	22.72
4	4	0	22.714
5	5	0	22.716
6	6	51.588	22.72
7	7	40.986	22.72
8	8	83.393	22.72
9	9	86.043	22.72
10	10	104.051	22.72
10	11	170.313	22.72
10	12	104.051	22.72
10	13	218.022	22.72
14	14	17.131	22.72
10	15	233.925	22.72
10	16	93.449	22.72
17	17	43.636	22.72
Total	-	1377.497	-

Diagonalization PDOCs Results

Table B.15 – Chapter 4 Case Study 2.3 - Diagonalization PDOCs: Strategic Producers

Producer Company	Unit	Power Supplied [MW]	Offering Price [\$]
1	1	0	2670.122
2	2	0	7707.332
3	3	300	22.72
4	4	591	22.72
5	5	0	479.817
6	6	0	931.368
7	7	0	2499.999
8	8	400	22.72
9	9	400	22.72
10	10	0	164.045
11	11	0	81.533
12	12	0	2902.345
Total	-	1691	-

Table B.16 – Chapter 4 Case Study 2.3 - Diagonalization PDOCs: Strategic Consumers

Consumer Company	Unit	Power Demanded [MW]	Bidding Price [\$]
1	1	0	22.72
2	2	0	22.72
3	3	0	22.72
4	4	68.913	22.72
5	5	0	22.72
6	6	127.224	22.72
7	7	74.756	22.72
8	8	0	22.72
9	9	0	22.72
10	10	180.234	22.72
10	11	246.496	22.72
10	12	180.234	22.72
10	13	294.205	22.72
14	14	92.767	22.72
10	15	137.265	22.72
10	16	169.632	22.72
17	17	119.272	22.72
Total	-	1690.998	-

Diagonalization KKT Results

Table B.17 – Chapter 4 Case Study 2.3 - Diagonalization KKT: Strategic Producers

Producer Company	Unit	Power Supplied [MW]	Offering Price [\$]
1	1	0	2670.122
2	2	0	7707.332
3	3	300	22.72
4	4	591	22.72
5	5	0	479.817
6	6	0	931.368
7	7	0	2499.999
8	8	400	22.72
9	9	400	22.72
10	10	0	164.045
11	11	0	81.533
12	12	0	2902.345
Total	-	1691	-

Table B.18 – Chapter 4 Case Study 2.3 - Diagonalization KKT: Strategic Consumers

Consumer Company	Unit	Power Demanded [MW]	Bidding Price [\$]
1	1	25.935	22.72
2	2	82.608	22.72
3	3	0	22.72
4	4	0	22.72
5	5	0	22.72
6	6	0	22.72
7	7	0	22.72
8	8	76.507	22.72
9	9	0	22.72
10	10	180.234	22.72
10	11	246.496	22.72
10	12	180.234	22.72
10	13	294.205	22.72
14	14	92.767	22.72
10	15	223.107	22.72
10	16	169.632	22.72
17	17	119.272	22.72
Total	-	1690.997	-

B.0.3 Case 2.4

Proposed Model Results

Table B.19 – Chapter 4 Case Study 2.4 - Proposed Model: Strategic Producers

Producer Company	Unit	Power Supplied [MW]	Offering Price [\$]
1	1	0	88.169
2	2	0	320.828
3	3	300	14.809
3	4	575.52	80.861
3	5	0	7692.919
6	6	0	641.885
7	7	0	2195.058
3	8	384.52	80.861
3	9	384.52	80.861
10	10	284.521	80.861
11	11	294.482	80.861
12	12	0	788.223
Total	-	2223.563	-

Table B.20 – Chapter 4 Case Study 2.4 - Proposed Model: Strategic Consumers

Consumer Company	Unit	Power Demanded [MW]	Bidding Price [\$]
1	1	74.343	80.861
2	2	63.741	80.861
3	3	140.605	80.861
4	4	42.536	80.861
5	5	39.886	80.861
6	6	100.848	80.861
7	7	90.246	80.861
8	8	132.653	80.861
9	9	135.303	80.861
10	10	157.435	80.861
10	11	223.697	80.861
10	12	157.435	80.861
10	13	271.406	80.861
14	14	66.391	80.861
10	15	287.309	80.861
10	16	146.833	80.861
17	17	92.896	80.861
Total	-	2223.563	-

Diagonalization PDOCs Results

Table B.21 – Chapter 4 Case Study 2.4 - Diagonalization PDOCs: Strategic Producers

Producer Company	Unit	Power Supplied [MW]	Offering Price [\$]
1	1	0	88.169
2	2	0	320.828
3	3	300	14.809
3	4	591	80.861
3	5	0	7692.919
6	6	155	80.861
7	7	155	80.861
3	8	0	641.885
3	9	0	2195.058
10	10	300	80.861
11	11	310	80.861
12	12	0	788.223
Total	-	1811	-

Table B.22 – Chapter 4 Case Study 2.4 - Diagonalization PDOCs: Strategic Consumers

Consumer Company	Unit	Power Demanded [MW]	Bidding Price [\$]
1	1	42.467	80.861
2	2	62.986	80.861
3	3	0	80.861
4	4	48.166	80.861
5	5	46.313	80.861
6	6	0	80.861
7	7	18.117	80.861
8	8	0	80.861
9	9	0	80.861
10	10	180.234	80.861
10	11	246.496	80.861
10	12	180.234	80.861
10	13	294.205	80.861
14	14	92.767	80.861
10	15	310.108	80.861
10	16	169.632	80.861
17	17	119.272	80.861
Total	-	1810.997	-

Diagonalization KKT Results

Table B.23 – Chapter 4 Case Study 2.4 - Diagonalization KKT: Strategic Producers

Producer Company	Unit	Power Supplied [MW]	Offering Price [\$]
1	1	0	88.169
2	2	0	320.828
3	3	300	14.809
3	4	591	80.861
3	5	0	7692.919
6	6	155	80.861
7	7	155	80.861
3	8	0	641.885
3	9	0	2195.058
10	10	300	80.861
11	11	310	80.861
12	12	0	788.223
Total	-	1811	-

Table B.24 – Chapter 4 Case Study 2.4 - Diagonalization KKT: Strategic Consumers

Consumer Company	Unit	Power Demanded [MW]	Bidding Price [\$]
1	1	34.095	80.861
2	2	86.927	80.861
3	3	0	80.861
4	4	66.474	80.861
5	5	63.917	80.861
6	6	0	80.861
7	7	0	80.861
8	8	0	80.861
9	9	0	80.861
10	10	180.234	80.861
10	11	246.496	80.861
10	12	180.234	80.861
10	13	294.205	80.861
14	14	92.767	80.861
10	15	276.744	80.861
10	16	169.632	80.861
17	17	119.272	80.861
Total	-	1810.997	-