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Plasma confinement in tokamaks with robust torus

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ABSTRACT

We present a non-linear symplectic map that describes the alterations of the magnetic field lines inside the tokamak plasma due to the presence of a robust torus (RT) at the plasma edge. This RT prevents the magnetic field lines from reaching the tokamak wall and reduces, in its vicinity, the islands and invariant curve destruction due to resonant perturbations. The map describes the equilibrium magnetic field lines perturbed by resonances created by ergodic magnetic limiters (EMLs). We present the results obtained for twist and non-twist mappings derived for monotonic and non-monotonic plasma current density radial profiles, respectively. Our results indicate that the RT implementation would decrease the field line transport at the tokamak plasma edge.

1. Introduction

Toroidal plasmas are confined in tokamaks by the superposition of two basic magnetic fields: a toroidal field produced by coils mounted around the tokamak vessel, and a poloidal field generated by the plasma column itself [1]. The superposition of these fields results in helical magnetic field lines. These field lines lie on nested toroidal surfaces, called magnetic surfaces, on which the pressure gradient that causes the plasma expansion is counterbalanced by the Lorentz force, in an equilibrium configuration [1]. In this static axisymmetric configuration we can parameterize the field lines by using an azimuthal angle. This parameter plays the role of canonical time, so that magnetic field line equations can be viewed as canonical equations. One of the advantages of this approach is the possibility of describing field lines by means of Hamiltonian maps, reducing the number of degrees of freedom for the system [2]. In this framework, equilibrium configurations are integrable systems, whereas symmetry-breaking magnetic field perturbations spoil their integrability. This may lead to a chaotic behavior that in a Lagrangian sense means that two initially nearby field lines diverge exponentially after many turns around the toroidal chamber. Since charged plasma particles follow the magnetic field lines, the anomalous particle transport and the plasma confinement quality depends on the magnetic field configuration [3].

Plasmas are confined in tokamaks within magnetic surfaces partially destroyed at plasma edge by resonant perturbations [4]. Under those circumstances, the plasma confinement is improved by the onset of internal transport barriers (ITB) that reduce the field line transport along the destroyed magnetic surfaces [5,6]. Such barriers are created by natural magnetic perturbations, electrical currents in external coils and wave or beam injection [7,8]. Although the origin of these barriers is still an open question, there are evidences that some of them are related to dynamical invariants (toroidal magnetic surfaces) predicted by almost integrable Hamiltonian theory, as used in this work to describe the perturbed magnetic field [9]. These barriers are expected to be relevant to accomplish the controlled fusion in tokamaks, especially in

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ITER (*International Thermonuclear Experimental Reactor*), the tokamak currently in construction in Cadarache, France, under an international collaboration, whose main objective is to verify the availability of a fusion reactor [10].

In this work, we investigate the alterations of the tokamak magnetic field lines, as those observed around an ITB, by introducing a strong barrier called robust torus (RT), an invariant torus in a specified radial position of the phase space located at the plasma edge related to an ITB [11–13]. This RT is introduced in such way so as to remain intact when perturbations are added to the Hamiltonian, in contrast with the KAM torus that, depending on their winding number, may persist only for some sufficiently small perturbations. We compare the alterations caused by the RT in two kinds of magnetic configurations, a twist and also a non-twist equilibrium magnetic field perturbed by resonances [3], which partially destroy the magnetic surfaces at the plasma edge.

The resulting Hamiltonian will have a term representing the equilibrium magnetic surfaces and another one representing a resonant perturbation. There is a pre-factor in the perturbative term, which will add a RT at the plasma edge near the resonances. The non-perturbed Hamiltonian is an integrable system described by an analytical solution of the non-linear Grad–Shafranov equation. The equilibrium analytical solution used in our work describes plasma confined in a tokamak with nested toroidal magnetic surfaces with a center displaced to the external part due to the toroidal geometry [14,15], thus, the intersections of the flux surfaces with a toroidal plane exhibits invariant circles shifted toward the exterior equatorial region [16,17]. On the other hand, the perturbations are generated by electrical currents in a set of coils, poloidally distributed in toroidal sections of the tokamak [18], which creates a region of chaotic magnetic field lines at the plasma edge. In the plasma physics literature this device is known as ergodic magnetic limiter (EML) [19]. The chaotic field lines result from interactions among magnetic islands, with progressive destruction of perturbed flux surfaces as the perturbation strength increases according to different scenarios in twist magnetic configurations, where the Kolmogorov–Arnold–Moser (KAM) theory is valid [4,20], and also in non-twist configurations [21,22]. This difference occurs around the magnetic shearless surface, since that at the plasma region far from the shearless surface the KAM theory can be applied [23].

We use the Hamiltonian considered in this work to study the local barrier transport in the plasma region most affected by the resonant perturbation. Instead of obtaining numerically the Poincaré maps, we can resort to the impulsive nature of the considered perturbation and introduce a local analytical approximation of the exact mapping [24]. The basic idea is to consider an expansion of the canonical equations in the vicinity of the main islands generated by the perturbation. The control parameters of the used maps are related to quantities that can be measured in tokamak experiments, as the current density profiles and the perturbing helical currents [25].

This article is organized as follows: In Section 2 we introduce the Hamiltonian approach with a robust torus (RT) to describe the magnetic field perturbed by resonances, in Section 3 we introduce the symplectic maps we use to show how a RT modifies the field line configuration, and in Section 4 we summarize our main conclusions.

2. Hamiltonian approach

We can write the magnetic field line equation, $B \times \mathrm{d}l = 0$, in the form of Hamilton equations. As plasmas in tokamaks are independent of the toroidal angle, φ, the role of time is played by this ignorable coordinate. The remaining coordinates give the canonical position and momentum variables. The advantage of this procedure is to use the method of Hamiltonian dynamics to analyze magnetic field line topology when a static magnetic perturbation is applied on a given equilibrium field. Since the equilibrium field is axisymmetric we can put the equations in terms of the Hamiltonian description for the field line flow with $\varphi = t$ as a time-like variable, then the field line equation in a Hamiltonian form can be casted as: $\mathrm{d}J/\mathrm{d}t = -\partial H/\partial\vartheta$ and $\mathrm{d}\vartheta/\mathrm{d}t = \partial H/\partial J$ where the action–angle variables (J, ϑ) are related to the spatial coordinates (r_t, θ_t) by $J(r_t) = \frac{1}{4}\left[1 - \left(1 - 4\left(\frac{r_t}{R}\right)^2\right)^{1/2}\right]$ and $\vartheta(r_t, \theta_t) = 2\arctan\left[\frac{1}{\Omega(r_t)}\left(\frac{\sin\theta_t}{1+\cos\theta_t}\right)\right]$, where $\Omega(r_t) = \left(1 - 2\frac{r_t}{R}\right)^{(1/2)}\left(1 + 2\frac{r_t}{R}\right)^{-(1/2)}$ and R is the magnetic axis radius [26]. One can find more details about the relation of the introduced action–angle variables with magnetic field line coordinates in Ref. [19].

The addition of the resonant perturbing magnetic field produced by currents in external coils can be regarded as a Hamiltonian perturbation: $H(J, \vartheta, t) = H_0(J) + H_1(J, \vartheta, t)$ with $|H_1/H_0| \ll 1$.

The Poincaré section is taken at a fixed value of this toroidal angle φ. So, H_0 is given in terms of the canonical action J and ϑ the poloidal angle canonically associated to J [19]. On the other hand the perturbing Hamiltonian will be a function of J, ϑ and t, and it will be represented by means of a Fourier series of delta-kicks [15,19]. Each value of J labels a toroidal magnetic surface with a displaced center due to the toroidal geometry. Another Hamiltonian method has been numerically applied to create barriers to magnetic field line diffusion in tokamaks by a small dynamical modification of the magnetic perturbation [27]. This alternative method of control was also tested on a beam of electrons produced by a long traveling wave tube that mimics beam plasma interactions [28]. For both the static and the time dependent system, their Poincaré sections present similar transport barriers created by small modification of the perturbations.

In our approach, we wish to investigate the effect that a RT, close to the tokamak wall, causes when placed near the main resonances created by the EML. To do that we consider the Hamiltonian expressed in terms of action and angle variables. As we know the analytical expression of J, we expand H_0 around a magnetic surface with action J_0 and frequency $\Omega_0 = n/m$ where Ω_0 is equal to the inverse of the safety factor q and n and m are integers. In the perturbation, we keep only the two main resonances from the Fourier expansion. The interaction between these two neighbor modes reproduce quite well the transition to chaos observed in the system. Then, the Hamiltonian, with the dominant resonant modes with poloidal wave numbers m and $(m+1)$, is given as,

$$H(\Delta J, \vartheta, t) = H_0(\Delta J) + P(\Delta J)[\beta \cos(m\vartheta) + \eta \cos((m+1)\vartheta + nt)] \sum_{k=-\infty}^{+\infty} \delta\left(t - \frac{2\pi}{N_r}k\right) \tag{1}$$

where $\Delta J = J - J_0$ and $N_r = 4$ is the quantity of EML rings equally spaced around the tokamak.

The use of delta-kicks to model the t-dependence of the perturbation is motivated by the type of magnetic field generated by the ergodic magnetic limiter, which acts in determined positions along the torus curvature. Moreover, the current ring width l is supposed to be small compared to the torus mean circumference $2\pi R$, where R is the major axis radius.

We consider in this work two equilibrium configurations distinguished by two different frequency profiles of $\Omega_0 \equiv \mathrm{dH_0}/\mathrm{d}\Delta J$ given by:

$$H_0(\Delta J) = \frac{1}{2}\Delta J^2 \tag{2}$$

for the twist case, and

$$H_0(\Delta J) = \frac{1}{2}\Delta J^2 - \frac{\alpha}{3}\Delta J^3 \tag{3}$$

for the non-twist case.

The parameter $\alpha = W/M$, where $M \equiv \left.\frac{\mathrm{d}^2 H_0}{\mathrm{d}J^2}\right|_{J=J_0}$ and $W \equiv \frac{1}{2}\left.\frac{\mathrm{d}^3 H_0}{\mathrm{d}J^3}\right|_{J=J_0}$. The parameters β and η are the perturbations' strength and they are related with the electric current applied in the EML rings.

The function $P(\Delta J)$ in Eq. (1) is a polynomial and it allows us to introduce RT. We choose $P(\Delta J) = 1$ to obtain the known Hamiltonian without RT and we also choose $P(\Delta J) = (\Delta J - a)$ in order to introduce one RT at the position $\Delta J = a$. We emphasize that, for a generic polynomial function, it is possible to have a number of RT equal to the number of real roots of this polynomial. Thus, this procedure enables us to construct a Hamiltonian to study the alterations of the topology of the magnetic field lines at the plasma edge due to the presence of an infinity barrier, a RT, near the tokamak wall.

3. Conservative map and Poincaré section

As the time dependence of the Hamilton equations is in the form of periodic delta-kicks we can define discretized variables and write a 2D map. Initially, the non-twist map without the RT is,

$$\begin{aligned}
\Delta J_{k+1} &= \Delta J_k + \beta m \sin(m\vartheta_{k+1}) + \eta(m+1)\sin[(m+1)\vartheta_{k+1} + nt_{k+1}] \\
\vartheta_{k+1} &= \vartheta_k + \frac{2\pi}{N_r}(\Delta J_k - \alpha \Delta J_k^2) \bmod (2\pi) \\
t_{k+1} &= t_k + \frac{2\pi}{N_r m}.
\end{aligned} \tag{4}$$

On the other hand, for the purpose of this paper, the non-twist map with two resonant modes and one RT is given as,

$$\begin{aligned}
\Delta J_{k+1} &= \Delta J_k + (\Delta J_{k+1} - a)\{\beta m \sin(m\vartheta_k) + \eta(m+1)\sin[(m+1)\vartheta_k + nt_{k+1}]\} \\
\vartheta_{k+1} &= \vartheta_k + \frac{2\pi}{N_r}(\Delta J_{k+1} - \alpha \Delta J_{k+1}^2) + \beta\cos(m\vartheta_k) + \eta\cos[(m+1)\vartheta_k + nt_{k+1}] \bmod (2\pi) \\
t_{k+1} &= t_k + \frac{2\pi}{N_r m}.
\end{aligned} \tag{5}$$

We emphasize that the indices of the discretized variables are adjusted in a consistent way [29] in order to keep the Jacobean equal to one and to have an area-preserving map. In Eq. (5) one could exchange the indices $(k+1)$ and (k) of the variables ΔJ and ϑ in order to keep a similarity with Eq. (4), however this would not alter the dynamics. Our choice for the actual form avoided a transcendental equation, which would appear by changing the indices.

For the numerical implementations of the maps presented in Eqs. (4) and (5) we have chosen $m = 3$ and $n = 1$ introducing the resonant modes (1:3) and (1:4). The constant α that appears in the equilibrium Hamiltonian is adjusted for both maps, for the twist field $\alpha = 0$ and for the non-twist field $\alpha = 160.15$ whose value is very close of the one used experimentally in the TCABR.

In Fig. 1(a) we present the *twist case* and we see the dominant island chain (1:3) and the main secondary resonance (1:4) embedded in chaotic magnetic field lines and several other minor resonances. In this configuration, the magnetic field lines can escape toward the tokamak wall along the chaotic sea around the broken (1:3) island chain. On the other hand, in Fig. 1(b) we can see the stabilizing alterations introduced in the magnetic field line configuration by the RT whose position, $\Delta J = a = 3$, is indicated in red color. Note that the chaotic sea near the RT, around the (1:3) island, is suppressed in Fig. 1(b)

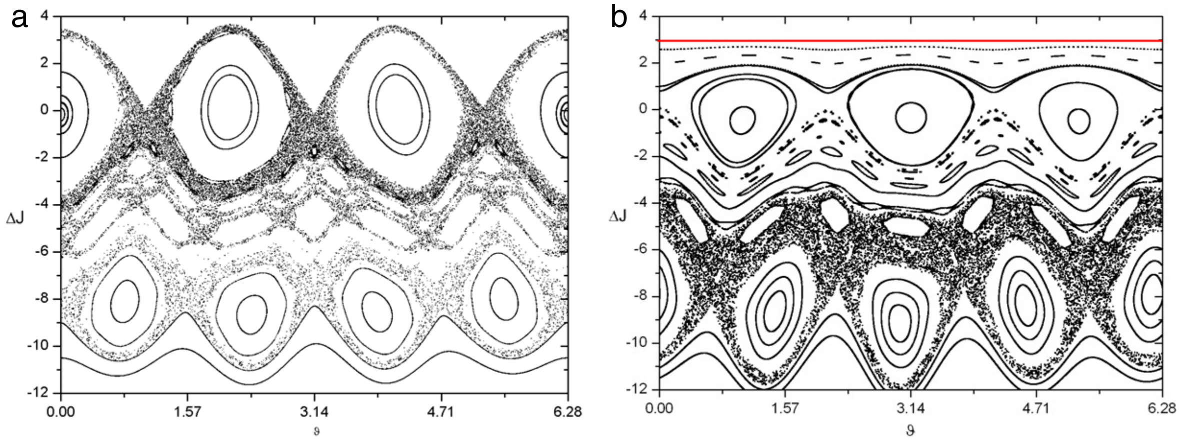

Fig. 1. Poincaré sections for the resonant modes (1:3) and (1:4) with $\alpha = 0$ for: (a) Twist map of Eq. (4) without RT with $\beta = -5.0 \times 10^{-4}$ and $\eta = 2.3 \times 10^{-4}$; (b) Twist map of Eq. (5) with RT (in red) with $\beta = -7.0 \times 10^{-3}$ and $\eta = 3.2 \times 10^{-3}$. (For interpretation of the references to colour in this figure legend, the reader is referred to the web version of this article.)

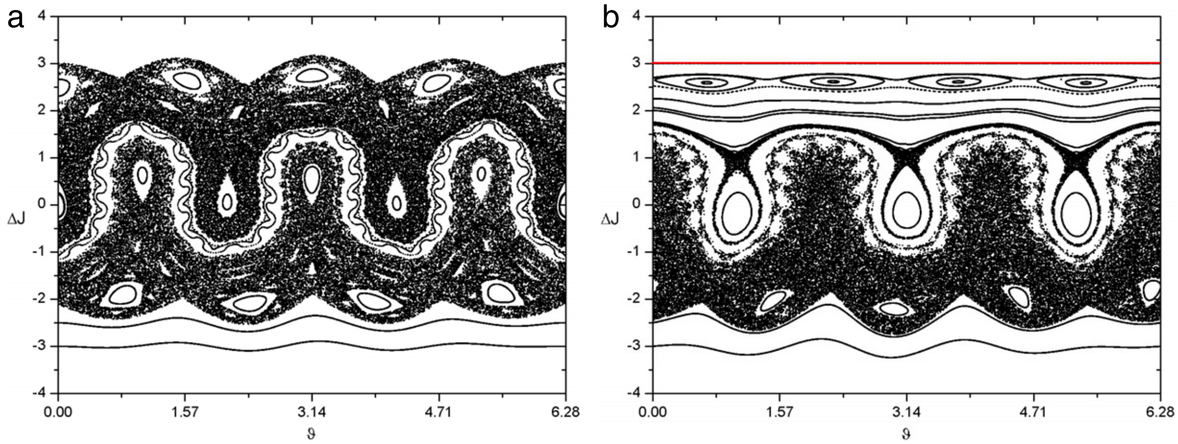

Fig. 2. Poincaré sections for two resonant modes (1:3) and (1:4) for: (a) Non-twist map of Eq. (4), without RT with $\beta = -1.5 \times 10^{-4}$ and $\eta = 6.9 \times 10^{-5}$; (b) Non-twist map of Eq. (5), with RT (in red) with $\beta = -5.0 \times 10^{-3}$ and $\eta = 2.3 \times 10^{-3}$. (For interpretation of the references to colour in this figure legend, the reader is referred to the web version of this article.)

and the neighborhood around $\Delta J = 3$ is more stable than the one of Fig. 1(a). The explanation for that is the following, for $\Delta J = a$ the perturbed Hamiltonian of Eq. (1) is algebraically null independently of β and η values. We can also see that the first equation of the map, given in Eq. (5), becomes $\Delta J_{k+1} = \Delta J_k$, defining then the RT, hence by an exigency of continuity, the perturbations are gradually weaker in the linear vicinity of the RT than far from it.

We emphasize that the presence of the RT introduces a local effect, which is seen through the regularization of the dynamics in its neighborhood, and in regions far from the RT the chaotic sea continues to exist. As we can see in Fig. 1(b) the RT traps all magnetic field lines inside the plasma edge avoiding shocks with tokamak wall unlike that of the case of Fig. 1(a).

For the non-twist equilibrium, there are isochronous resonances [30] in the phase space. In Fig. 2(a), the two isochronous resonances (1:3) have already reconnected and they are dimerized and separated by a lot of invariant meandering curves [19]. Such invariant curves encircle both sets of surviving islands (1:3) and they exist only in non-twist maps (to be more precise, inside the shearless region). This region traps the magnetic field lines for a long time and hampers the radial diffusion, and the previously termed internal transport barrier (ITB) takes place due to a strong stickiness effect.

In Fig. 2(b) the RT is indicated in red color at the position $\Delta J = 3 = a$. The alterations introduced in the magnetic configuration are noticeable. We still have the two (1:3) and (1:4) island chains, but now an interesting topological rearrangement has occurred, the chaotic sea that was close to the upper island chain (1:4), in Fig. 2(a), has been suppressed by the presence of the RT. The magnetic surfaces near the RT remain as invariant curves while on the other side of the ITB, there is a significant destruction of the magnetic surfaces around the low resonance (1:4). The rigidity of the RT and the local stabilization in its neighborhood are the main results that we address in this work.

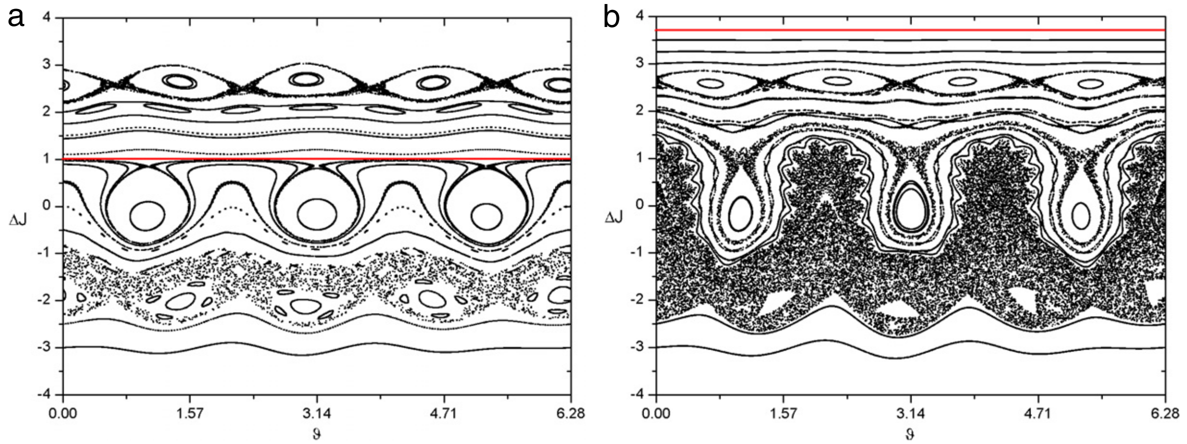

Fig. 3. Poincaré sections for the map of Eq. (5) with the modes (1:3) and (1:4), $\beta = -5.0 \times 10^{-3}$, $\eta = 2.3 \times 10^{-3}$ and a robust torus (in red) located at: (a) $\Delta J = 1$ and (b) $\Delta J = 3.7$. (For interpretation of the references to colour in this figure legend, the reader is referred to the web version of this article.)

It worthwhile to emphasize that different Hamiltonians (maps) govern the non-twist and the twist cases and so we do not keep the same parameters for both systems. For the non-twist case, we fixed the meandering curve, which appears in Fig. 2(a) without RT, as our reference. From there, we produced the plot of Fig. 2(b). For the twist case, the strategy was different since the meandering curve does not exist. We started with the plot of Fig. 1(a), without RT, which shows chaos in the neighborhood of both resonances (1:3) and (1:4) but still with some regular structures. Next, in the case with RT, we choose the parameters in order to have a similar chaotic sea around the resonance (1:4). The introduction of the RT causes the resulting difference around the resonance (1:3).

Another evidence of the effect of the position of the RT, determined by the parameter a, is shown in Fig. 3. In Fig. 3(a) we choose the position of the RT, $\Delta J = 1 = a$, closer to the dimerized island chains (1:3), and in Fig. 3(b) we choose, the position of the RT, $\Delta J = 3.7 = a$, upper than the island chain (1:4) with the same parameters of Fig. 2(b). In Fig. 3(a) the islands of the resonance (1:4) are partially destroyed in such way that the magnetic field lines can leave the plasma and reach the tokamak wall. In Fig. 3(b) the islands of the resonance (1:4) are also partially destroyed but the magnetic field lines cannot reach the tokamak wall due to the presence of the RT and also due to the stabilized curves in its vicinity. Comparing these configurations with the one shown in Fig. 2(b) we see that the position of the RT at $\Delta J = 3$ also keeps the magnetic field lines trapped in an inner region on the tokamak, near the resonance (1:4).

4. Conclusion

We presented in this paper a symplectic map that describes some alterations of the magnetic field lines inside the tokamak due to the presence of a robust torus at the plasma edge. The map describes the toroidal equilibrium magnetic field lines perturbed by resonances created by ergodic magnetic limiters. Theoretically, the robust torus has proved to be an efficient transport barrier that prevented the magnetic field lines from reaching the tokamak wall, avoiding plasma–wall interactions. The instabilities in its vicinity as well as the destruction of the equilibrium invariant curves have also reduced, which would be destroyed due to the resonant or natural perturbations.

A possible experimental implementation of a RT in tokamaks requires an active control inside the tokamak by another set of EML rings. Because a RT corresponds to a 3D surface along the toroidal direction, which remains intact in the middle of many broken magnetic surfaces, we suggest a way to create a robust magnetic surface that mimics a RT.

For instance, we may consider a set of EML rings in order to create a resonance $(n{:}m)$ at the peripheral region of the plasma from the application of a positive magnetic field. Next we consider another set of EML rings in order to create the same resonance $(n{:}m)$ at the same position of the first one, but from the application of a negative magnetic field. The superposition of the two identical resonant perturbations, but with opposite magnetic field, may create an invariant magnetic surface, similar to the RT.

The model we are using considers only the effect of a set of four EMLs, which perturbs the equilibrium magnetic surfaces. However, it is well known that many other perturbations create instabilities and anomalous transport of chaotic particles inside the plasma column, hence the action of a second EML set, with opposite magnetic field, could eliminate the effects of the main resonant mode. We emphasize that this is a local effect, which is valid for any perturbation. From the experimental point of view it is usual to have a reasonable control of the main mode of the perturbation (in this case the mode 1:3) whereas the control of the secondary modes in general is a difficult, or even an impossible job. Hence, our conjecture to apply a second perturbation, from another set of EMLs with inverse magnetic field, infers that a robust barrier will appear locally where there was a main perturbation mode. This strategy can simulate some existing barriers already observed in experiments, or propose the creation of a new barrier in the tokamak camera.

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