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Preheating in quintessential inflation

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Abstract

We perform a numerical study of the preheating mechanism of particle production in models of quintessential inflation and compare it with the usual gravitational production mechanism. We find that even for a very small coupling between the inflaton field and a massless scalar field, $g \gtrsim 10^{-6}$, preheating dominates over gravitational particle production. Reheating temperatures in the range $10^4 \lesssim T_{\rm rh} \lesssim 10^{15}$ GeV can be easily obtained.

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1. Introduction

Inflationary models of the early universe have become a paradigm that can explain the isotropy of the cosmic microwave background, the origin of the small adiabatic perturbations that seed galaxy formation and the flatness of space–time. There is also evidence that the universe has recently entered another stage of mild inflation, responsible for its accelerated expansion. It would be natural to investigate models where the two periods of inflation are related. In particular, models of quintessential inflation explore the possibility that the same scalar field is responsible for both periods of inflation [1–4].

In quintessential inflation models, the inflaton potential has no minimum in which the field could oscillate to produce the matter that would reheat the universe, as in usual chaotic inflation models. Therefore, it is assumed that all matter and energy in the universe will be created by gravitational production associated to changes in the geometry of the universe. However, it is known that this process is not very efficient [5]. In this Letter we assess the importance of another mechanism of particle production in the early universe, namely preheating [6], in which particles are produced due to the variation of the classical inflaton field. Analytical estimates of particle production in non-oscillatory models were performed in [7,8]. We find numerically that the introduction of a non-zero coupling between the inflaton and matter fields leads

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to particle production that can easily overcome the gravitational one. Furthermore, higher reheating temperatures are possible in this case as compared to pure gravitational production.

2. The model

For definiteness, we study the Peebles and Vilenkin model [1], with a potential for the inflaton field Φ of the form

$$V(\Phi) = \begin{cases} \lambda\left(\Phi^4 + M^4\right) & \text{for } \Phi < 0, \\ \dfrac{\lambda M^8}{\Phi^4 + M^4} & \text{for } \Phi \geqslant 0, \end{cases}$$

where $\lambda = 1 \times 10^{-14}$ is required from structure formation [12] and $M = 5 \times 10^8$ GeV in order to produce the present quintessential energy density [1].

Defining adimensional parameters $\phi = \Phi/M_{\text{Pl}}$, $q = M/M_{\text{Pl}}$ and $\tau = \sqrt{\lambda}\, M_{\text{Pl}} t$, the classical equation of motion for the inflaton field becomes independent of the λ parameter:

$$\phi'' + \sqrt{24\pi}\sqrt{\frac{\phi'^2}{2} + \left(\phi^4 + q^4\right)}\,\phi' + 4\phi^3 = 0 \quad \text{if } \phi < 0, \tag{1}$$

$$\phi'' + \sqrt{24\pi}\sqrt{\frac{\phi'^2}{2} + \frac{q^8}{(\phi^4 + q^4)}}\,\phi' - 4\frac{q^8}{(\phi^4 + q^4)^2}\phi^3 = 0 \quad \text{if } \phi \geqslant 0, \tag{2}$$

where the primes are the derivatives with respect to τ and we have used the Hubble parameter,

$$\tilde{H}^2(\tau) = \left(\frac{a'(\tau)}{a(\tau)}\right)^2 = \frac{8\pi}{3}\left(\frac{\phi'^2}{2} + V(\phi)\right). \tag{3}$$

In order to study preheating, we couple the classical inflaton field to a massless quantum scalar field χ through the Lagrangian:

$$\mathcal{L} = \frac{1}{2}\partial_\mu \chi \partial^\mu \chi - \frac{1}{2}\xi R \chi^2 - \frac{1}{2} g^2 \Phi^2 \chi^2, \tag{4}$$

where R is the scalar curvature, ξ is the gravitational coupling and g is a quartic coupling constant between ϕ and χ.

When such a coupling is introduced one must be careful with the possibility that radiative corrections generate an effective potential which spoils the tree-level flatness of the potential required to provide the quintessence properties observed today.[1] Given that the tree-level Lagrangian is non-renormalizable and it is difficult to make sense in quantum field theory of an inverse-power-law potential [11], we would like to take a more pragmatic point of view and regard this as an effective toy model. In order to explore the dependence with the potential, we will also consider another different non-renormalizable quintessential potential.

The second term in the Lagrangian alone gives the production of particles due to the transition of the universe from the de Sitter space–time at the end of inflation. Ford [5] calculated the energy density of such χ particles in the limit $|\xi - 1/6| \ll 1$. However, this production is not efficient since the ratio of matter and inflaton field energy densities is typically $\rho_{\text{m}}/\rho_\phi \sim 10^{-18}$ [5]. After thermalization of the products it is possible to define a reheating temperature when radiation starts to dominate and in that case it is of the order of $T_{\text{rh}} \sim 10^4$ GeV.

When preheating is operative, the picture changes drastically. Taking into account the dilution of gravitational particle production from the end of inflation, until the beginning of the particle production by the

[1] Generically, since there is no bare mass term for the inflaton field, such corrections amount to change the potential to $V(\phi,\chi) = V_0(\phi,\chi) + V_{\text{1-loop}}(\phi)$, where V_0 is the classical potential and the one-loop correction is roughly given by $V_{\text{1-loop}}(\phi) = \frac{g^2\phi^4}{256\pi^2}\log\left(\frac{\phi^2}{\Lambda^2}\right)$, and Λ is an arbitrary mass parameter defining the renormalization condition. Therefore, given the large value of the quintinflaton field, the radiative corrections can easily overcome the tree-level potential.

There are ways to circumvent this problem, but they are not fully satisfactory. One could choose the arbitrary mass parameter Λ such that $V_{\text{1-loop}} < V_0$ today or one could introduce a bare term in the tree-level potential with the correct magnitude to cancel the one-loop contribution. These alternatives obviously require some fine-tuning. It is also possible that a more complete theory, involving many heavy fields, would give rise to flat potentials even with radiative corrections taken into account. In fact, flat directions are a generic feature of supersymmetric theories, where non-renormalization theorems protect them against radiative corrections. Preheating with fields parameterizing flat directions, so-called "*flatons*", have recently being studied [9]. Protected flat directions with quartic interactions among scalar fields are also present in more involved models, as for instance in the so-called "*3–3–1*" models [10].

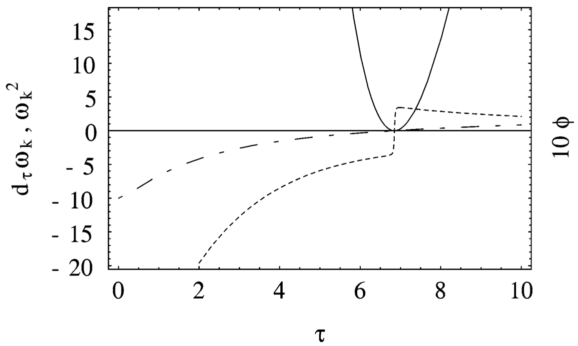

Fig. 1. ω_k^2 (solid line), ω_k' (dotted line) and $10 \times \phi$ (dot-dashed line) as a function of τ for $g = 10^{-5}$ and $k = 5\sqrt{\lambda}\, M_{\mathrm{Pl}}$. Notice that adiabaticity is violated around $\tau \simeq 7$, when the inflaton field goes through zero.

preheating mechanism, the universe is still dominated by the inflaton density energy and this is our starting point.

The equation of motion for each comoving k mode of the χ field can be written in terms of a new convenient variable $X_k = a^{3/2}\chi_k$ as:

$$X_k'' + \left\{\frac{1}{\lambda}\left(\frac{k^2}{M_{\mathrm{Pl}}^2 a^2(\tau)} + g^2\phi^2(\tau)\right) - \Delta\right\} X_k = 0. \quad (5)$$

The physical momentum is given by k/a and we have verified that $\Delta = \frac{3}{4}(\frac{a'}{a})^2 + \frac{3}{2}(\frac{a''}{a})$ is negligible during the production of χ particles.

This is an equation of motion for an harmonic oscillator with variable frequency, $\omega_k^2 = (k^2/(M_{\mathrm{Pl}}^2 a^2(\tau)) + g^2\phi^2(\tau))/\lambda$. When the adiabaticity condition is broken, that is, $\omega_k' \geqslant \omega_k^2$, particles in the k mode are produced. Fig. 1 illustrates the time interval where adiabaticity is violated, leading to particle creation. This time interval corresponds to the passage of the inflaton field through its origin.

3. Particle production

In order to compute the total amount of particles produced during preheating, we have to solve Eq. (5). To obtain $\phi(\tau)$ we choose the initial conditions as being $\phi(\tau = 0) = -1$ and $\phi'(\tau = 0) = 0$, since in $\lambda\Phi^4$ model inflation ends when $\Phi \simeq -M_{\mathrm{Pl}}$. Then we can evaluate the evolution of the scale factor $a(\tau)$ given by:

$$a(\tau) = a_0 \exp\left[\int_0^{\tau} d\tau'\, \tilde{H}(\tau')\right], \quad (6)$$

where $\tilde{H}(\tau)$ is given in Eq. (3) and $a_0 = 1$. We show in Fig. 2 the evolution of the scale factor during the period relevant for particle production.

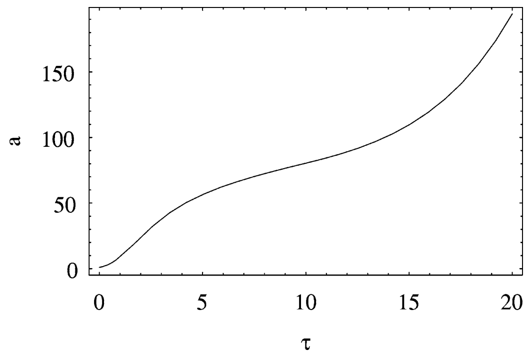

Fig. 2. Evolution of the scale factor as a function of τ.

Finally, substituting (6) and the solution of Eqs. (1) and (2) into (5) we obtain $X_k(\tau)$.

We define an adiabatic invariant n_k that can be interpreted as the comoving occupation number of particles produced with momentum k:

$$n_k = \frac{\omega_k}{2}\left(\frac{|X_k'|^2}{\omega_k^2} + |X_k|^2\right) - \frac{1}{2}. \quad (7)$$

In Fig. 3 we show the comoving occupation number of particles created during preheating obtained from the solution of Eq. (5) with vacuum initial conditions for different values of comoving momentum k, for $g = 5 \times 10^{-5}$. One can see that particle production occurs at $\tau \simeq 7$, and quickly stabilizes.

In order to illustrate the k-dependence of the density of particles, we plot in Fig. 4 the density spectrum produced during preheating for different values of λ and g.

The spectrum is well described by the expression:

$$n_k = \exp\left[-\frac{1}{52}\frac{\sqrt{\lambda}}{g}\frac{k^2}{\lambda M_{\mathrm{Pl}}^2}\right]. \quad (8)$$

This result is to be compared to the approximation for particle production in the first oscillation of a pure ϕ^4

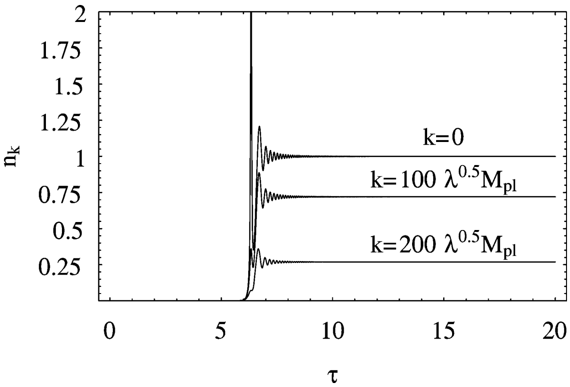

Fig. 3. Comoving occupation number of χ particles produced during preheating as a function of τ for comoving momentum $k = 0$, 100 and 200 $\sqrt{\lambda}\, M_{\mathrm{Pl}}$, for $g = 5 \times 10^{-5}$.

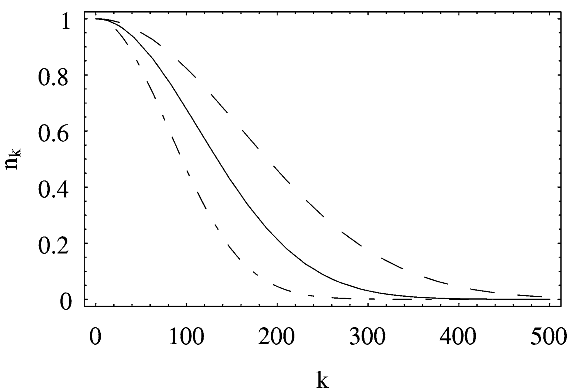

Fig. 4. Comoving occupation number of χ particles produced during preheating as a function of comoving momentum k in units of $\sqrt{\lambda}\, M_{\mathrm{p}}$. $\lambda = 1 \times 10^{-14}$ and $g = 5 \times 10^{-5}$ (solid curve); $\lambda = 1 \times 10^{-14}$ and $g = 1 \times 10^{-4}$ (dashed curve); $\lambda = 4 \times 10^{-14}$ and $g = 5 \times 10^{-5}$ (dot-dashed curve).

potential [7]:

$$n_k^{\mathrm{appr}} = \exp\left[-\pi \frac{\lambda M_{\mathrm{Pl}}^2}{g|\dot{\Phi}_{\mathrm{p}}|} \frac{k^2/a^2(\tau_{\mathrm{p}})}{\lambda M_{\mathrm{Pl}}^2}\right], \tag{9}$$

where τ_{p} is the time of particle production and $|\dot{\Phi}_{\mathrm{p}}|$ is the velocity of the inflaton field at the bottom of the potential. In our case we have $|\dot{\Phi}_{\mathrm{p}}| = 3.6 \times 10^{-2} \sqrt{\lambda}\, M_{\mathrm{Pl}}^2$ and $a(\tau_{\mathrm{p}}) = 68$, which results in:

$$n_k^{\mathrm{appr}} = \exp\left[-\frac{1}{53} \frac{\sqrt{\lambda}}{g} \frac{k^2}{\lambda M_{\mathrm{Pl}}^2}\right], \tag{10}$$

an excellent approximation indeed.

The total energy density produced at τ_{p} is given by:

$$\rho_\chi = \int \frac{d^3k}{(2\pi)^3 a^3(\tau_{\mathrm{p}})} \frac{k}{a(\tau_{\mathrm{p}})} n_k \simeq 3 \times 10^{-6} \lambda g^2 M_{\mathrm{Pl}}^4. \tag{11}$$

Since the energy density in the inflaton field around the time of particle production is

$$\rho_\phi \simeq \frac{\dot{\Phi}_{\mathrm{p}}^2}{2} \simeq 6 \times 10^{-4} \lambda M_{\mathrm{Pl}}^4, \tag{12}$$

the energy density that is transferred from the inflaton field to the χ field at the moment of production is given by:

$$\frac{\rho_\chi}{\rho_\phi} \simeq 5 \times 10^{-3} g^2. \tag{13}$$

This means that there is an upper limit in energy that can be extracted from ϕ field, at the moment of production, $\frac{\rho_\chi}{\rho_\phi} \simeq 5 \times 10^{-3}$. However, once the χ-particles are produced they rapidly become non-relativistic, since their effective masses are given by $m_\chi = g\Phi$ (see Eq. (5)). In such a case, the χ energy density is $m_\chi n_\chi$, where the χ total number density, n_χ, can be calculated integrating n_k for all modes resulting in $\rho_\chi \simeq 3 \times 10^{-5} \lambda^{3/4} g^{5/2} \Phi M_{\mathrm{Pl}}^3$. In such a case the above comparison becomes

$$\frac{\rho_\chi}{\rho_\phi} \simeq 14 g^{5/2}, \tag{14}$$

for $\Phi \simeq 10^{-1} M_{\mathrm{Pl}}$ which is a good approximation for times just after the production as shown in Fig. 1. Note that this process is very efficient since the inflaton transfer its energy to χ particles very rapidly.

In order to prove the initial statement that the diluted energy density of massless particles produced gravitationally, ρ_{m} [5] is in fact much smaller than the inflaton energy density, ρ_ϕ at $\phi \sim 0$, when the preheating starts, we take the ratio between them:[2]

$$\frac{\rho_{\mathrm{m}}(\phi = 0)}{\rho_\phi(\phi = 0)} \sim \frac{\rho_{\mathrm{m}}(\phi = -1)/a^4}{\rho_\phi(\phi = -1)/a^6} \sim 5 \times 10^{-15}. \tag{15}$$

On the other hand, we can compare ρ_{m} with our χ production (Eq. (11)):

$$\frac{\rho_\chi(\phi = 0)}{\rho_{\mathrm{m}}(\phi = 0)} \sim 3 \times 10^{12} g^2 \tag{16}$$

[2] Although ρ_ϕ is not proportional to a^{-6} through all the interval, it corresponds to a maximum dilution.

and conclude that the latter dominates over the former for $g \gtrsim 10^{-6}$.

At this point we would like to estimate the effects of back-reaction of the χ field on the inflaton equation of motion. The general equation of motion for the scalar inflaton field including back-reaction is given by:

$$\ddot{\Phi} + 3H\dot{\Phi} + V'(\Phi) = -g^2\chi^2\Phi. \tag{17}$$

In the Hartree approximation, one can substitute $\chi^2 \to \langle\chi^2\rangle$, where [13]

$$\langle\chi^2(t)\rangle = \int \frac{d^3k}{(2\pi)^3 a^3(t)} \left|\chi_k(t)\right|^2 \approx \frac{n_\chi}{g|\Phi|} \tag{18}$$

for $k/a \ll g|\Phi|$, which are the dominant modes. Therefore we can write Eq. (17) as:

$$\ddot{\Phi} + 3H\dot{\Phi} + V'(\Phi) = -gn_\chi \frac{\Phi}{|\Phi|}. \tag{19}$$

Back-reaction is important when the right-hand side is comparable to the left-hand side. We will compare the term $3H\dot{\Phi}$, which in our conventions using dimensionless variables is given by $3\lambda M_{\rm Pl}^3 \tilde{H}\phi'$ to

$$gn_\chi \approx 3 \times 10^{-5} g^{5/2}\lambda^{3/4} M_{\rm Pl}^3 \tag{20}$$

at after the time of particle creation, where we used $n_\chi = \rho_\chi/m_\chi$. In our case, this occurs at $\tau \approx 7$, when $\phi'_{\rm p} = 0.036$ and $\tilde{H}_{\rm p} = 0.0733$. Therefore, we estimate that back-reaction is not important if $g \lesssim 0.4$.

Finally, let us estimate the reheating temperature. We will make the assumption that the χ particles decay fast enough such that the transference of χ energy to relativistic particles occur just after the χ production.

As the inflaton field energy density scales as a^{-6} since it is dominated by kinetic energy whereas the relativistic particles energy density, $\rho_{\rm R}$, scales as a^{-4}, the latter will start to dominate the universe at a time $\tau_{\rm d}$ given by:

$$\frac{a^2(\tau_{\rm p})}{a^2(\tau_{\rm d})} \simeq 14g^{5/2}, \tag{21}$$

where we use the fact that the χ decays just after the its production. In fact in a time quite close to $\tau_{\rm p}$. At this time, the $\rho_{\rm R}$ will be diluted to the value:

$$\rho_{\rm R} = 3 \times 10^{-5}\lambda^{3/4} g^{5/2} M_{\rm Pl}^3 \Phi \frac{a^4(\tau_{\rm p})}{a^4(\tau_{\rm d})}$$
$$\simeq 6 \times 10^{-4}\lambda^{3/4} g^{15/2} M_{\rm Pl}^4,$$

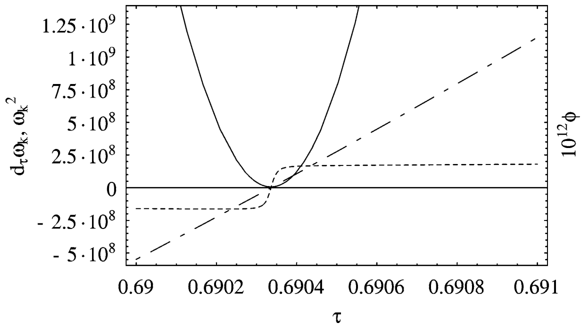

Fig. 5. ω_k^2 (solid line), ω'_k (dotted line) and $10^{12} \times \phi$ (dot-dashed line) as a function of $\tau = \kappa^2 M_{\rm Pl} t$, where $\kappa = M/M_{\rm Pl}$, for $g = 1$ and $k = 10^5\kappa^2 M_{\rm Pl}$. Notice that adiabaticity is violated around $\tau \simeq 0.69$, when the inflaton field goes through zero.

and the reheating temperature will be:

$$T_{\rm rh} \sim (\rho_{\rm R})^{1/4} \Longrightarrow T_{\rm rh} \simeq 10^{-1}\left(\lambda^{3/4} g^{15/2}\right)^{1/4} M_{\rm Pl}. \tag{22}$$

For our limits on g, $10^{-6} \lesssim g \lesssim 1$, the reheating temperature corresponds, respectively, to:

$$10^4 \lesssim T_{\rm rh} \lesssim 10^{15}\ \text{GeV}, \tag{23}$$

which is a confortable range. Further constraints in $T_{\rm rh}$ imply in new bounds on the coupling constant between the inflaton and the χ fields.

The results discussed so far are in the context of a particular model of quintessential inflation, the Peebles–Vilenkin model [1]. However, it would be interesting to see if the same behaviour is also present in other models. We present below a qualitative analysis of an alternative model, the Dimopoulos–Valle model [3]. We will adopt the potential

$$V(\Phi) = M^4\left(1 - \tanh\left(\frac{\Phi}{M_{\rm Pl}}\right)\right) \times \left(1 - \sin\left(\frac{\pi\Phi/2}{\sqrt{\Phi^2 + M^2}}\right)\right)^2, \tag{24}$$

with $M = 10^{15}$ GeV in order to obtain a successful quintessence model. Following the same steps leading to Eq. (5) we solve the corresponding equations of motion for initial conditions $\phi(0) = -0.14$ and $\phi'(0) = 0$, which are the conditions when inflation is terminated [3]. We show in Fig. 5 the region where the adiabatic condition is not obeyed and it can be seen that the behaviour is qualitatively similar to the one found in the Peebles–Vilenkin model.

We find that for an interaction not too small, $g \gtrsim 10^{-3}$, the qualitative picture does not change but for smaller values of the coupling the region of adiabaticity violation becomes large and the analytical formula seems to loose accuracy. This issue requires further investigations.

4. Conclusions

Quintessential inflation models postulate that the same field is responsible for inflation in the early universe and the accelerated expansion of the universe today. In this type of models, the potential has no minimum and the traditional reheating mechanism of the universe cannot be operative. Gravitational production of particles is usually adopted to reheat the universe.

We have demonstrated numerically in this Letter that introducing a small coupling $g \gtrsim 10^{-6}$ between the inflaton field and a massless scalar particle χ leads to particle creation in the preheating mechanism that dominates over the usual gravitational production in quintessential inflation models. Our results agree with the analytical estimates performed in Refs. [7] and [8] for the pure $\lambda\Phi^4$ potential. Reheating temperatures in the range $10^4 \lesssim T_{\rm rh} \lesssim 10^{15}$ GeV can be easily obtained for $10^{-6} \lesssim g \lesssim 1$, in contrast with the small temperature obtained for $g = 0$ of order 10^4 GeV [1]. We also present a preliminary analysis for a different quintessential inflation model [3] where the same conclusions appear to hold.

Our result is not fully complete since the gravitational production mechanism could be affected when the χ–ϕ coupling is introduced in the Lagrangian. However, the effect would be to inhibit the gravitational production, making preheating even more important.

Finally, we would like to remind that there is no realistic model for the inflaton or the χ field in the context of a well motivated particle physics model. In this sense, studies of quintessential inflation models are only exploratory and should be viewed as toy models.

Acknowledgements

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