**Andreyson Bicudo Jambersi**

# Proposal for an Angle/Time-Cyclostationary Parametric Model with Application on Internal Combustion Engine Analysis

Ilha Solteira

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UNIVERSIDADE ESTADUAL PAULISTA “JÚLIO DE MESQUITA FILHO”
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# Proposal for an Angle/Time-Cyclostationary Parametric Model with Application on Internal Combustion Engine Analysis

Thesis presented to the Faculdade de Engenharia de Ilha Solteira - UNESP as a part of the requirements for obtaining the Doctorate in Mechanical Engineering.
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Ilha Solteira, 28 de agosto de 2020

*To my beloved wife Carol,*
*for her love,*
*friendship, kindness, and support.*

*To my parents Amauri and Ana,*
*and to my sister Anna,*
*for their love and support.*

*To my friends,*
*for their humor and for the coffees.*

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*"Science may set limits to knowledge,
but should not set limits to imagination."*
Bertrand Russell (1872 - 1970)

*"C'est le temps que tu as perdu pour ta rose
qui fait ta rose si importante."*
Antoine de Saint-Exupéry (1900 - 1944)

# Abstract

The identification and model-based vibration analysis of a diesel internal combustion engine (ICE) is addressed in this thesis. An angle/time-dependent parametric model is proposed based on the existent well-known functional series time-varying autoregressive moving-average (FS-TARMA) formulation. However, instead of imposes the model coefficients' temporal evolution, this thesis suggests a model in which the coefficients are expanded and modeled as evolving according to an angular-periodic (related to the crankshaft position) basis. Due to this angle/time-dependency, we refer to this new model as an angle/time-FS-ARMA (or AT-FS-ARMA) model. The angular-periodic basis is derived from the AT-cyclostationary framework, a novel class of model extending the only time- or angle-cyclostationary framework. The proposed model considers the angle-periodicities (kinematic-related) often present in vibration-based signals from rotating and reciprocating machines, yet preserving the carrier (time-dependent dynamic). A "Relaxed" Multi-Stage Weighted-Least-Squares estimator is used to compute the models. The model structure is selected by evaluating the Bayesian information criterion through the assessment of several candidate models. A comparison between a model-based angle-frequency map and its non-parametric counterpart (obtained through smoothed-pseudo-Wigner-Ville distribution) was performed to show our proposal's potentiality. The results showed that the AT-FS-ARMA-based angle/frequency map provides a useful complementary tool for analysis; it is free from cross-terms and allows one to relate kinematics phenomena to the angular-instants in which they occur.

**Keywords**: Angle/time-cyclostationarity. Vibration analysis. Rotating and reciprocating machines. Internal combustion engines. Signal identification. Diesel engine. Parametric identification. AT-FS-ARMA model. Angle-frequency analysis.

# Resumo

Esta tese trata do problema da identificação e análise de vibrações, baseada em modelo, de sinais de vibrações de um motor de combustão interna à diesel. Um modelo paramétrico dependente do tempo/ângulo é proposto. Este modelo baseado na formulação já consolidada de modelos autorregressivos média-móveis (ARMA, do inglês autoregressive-moving-average) tempo-variantes, que utilizam espaços funcionais (FS, do inglês functional series) para a expansão dos parâmetros. Todavia, ao invés de impor uma dependência temporal nos coeficientes do modelo, esta tese sugere a expansão destes em termos de funções periódicas que são dependentes do ângulo de rotação do virabrequim do motor. Devido à esta dependência do ângulo/tempo, o novo modelo é chamado FS-ARMA ângulo/tempo-variante (ou AT-FS-ARMA). A base ângulo-periódica é proveniente da ideia de modelagem ângulo/tempo-ciclo-estacionária, que sugere uma nova classe de modelos como uma expansão das classes apenas tempo- ou ângulo-ciclo-estacionárias. O modelo proposto considera as periodicidades, frequentemente presentes em sinais de máquinas rotativas e alternativas (relacionadas à cinemática), em termos da variável angular, enquanto preserva a descrição temporal de características intrínsecas dos sinais (dinâmica dependente no tempo). Um estimador "relaxado" e de múltiplos estágios, baseado no método dos mínimos quadrados, é utilizado para estimar o modelo, cuja estrutura é selecionada utilizando a métrica do critério Bayesiano, através da avaliação de diversos modelos-candidatos. É realizada então uma comparação entre um mapa ângulo-frequência baseado no modelo estimado e um equivalente não-paramétrico deste (obtido via a computação de uma Distribuição Wigner-Ville suavizada, ou Pseudo-smoothed-Wigner-Ville Distribution), para mostrar o potencial do método proposto. Os resultados mostraram que os mapas ângulo-frequência baseados no modelo AT-FS-ARMA provêm uma ferramenta complementar útil para análise, sendo livre de termos cruzados e permitindo a associação entre fenômenos cinemáticos aos "instantes" angulares em que estes ocorrem.

**Palavras-chave**: Cicloestacionariedade ângulo/tempo. Análise de vibrações. Máquinas rotativas e alternativas. Motores de combustão interna. Identificação de sinais. Motor diesel. Identificação paramétrica. Modelo AT-FS-ARMA. Análise ângulo-frequência.

# List of Figures

# List of Acronyms

| | |
|---|---|
| 2SLS | Two-stage least squares |
| ACF | Auto-correlation function |
| AIC | Akaike's information criterion |
| APD | Aggregate parameter deviation |
| AR | AutoRegressive |
| ARMA | AutoRegressive moving-average |
| AT | Angle/time(-varying) |
| AT-CS | Angle/time-cyclostationary |
| BIC | Bayesian information criterion |
| COT | Computed order tracking |
| DFT | Discrete Fourier Transform |
| DPE | Deterministic parameter evolution |
| FS | Functional series |
| IAS | Instantaneous angular speed |
| ICE | Internal combustion engine |
| MA | Moving average |
| MIMO | Multiple-input multiple-output |
| ML | Maximum likelihood |
| MSE | Mean squared error |
| MS-WLS | Multi-stage weighted least-squares |
| NID | Normally independently distributed |
| OLS | Ordinary least squares |
| OT | Order tracking |
| RMS-WLS | "Relaxed" multi-stage weighted least squares |
| RPM | Rotations/revolutions per minute |
| P-A | Polynomial-algebraic |

| | |
|---|---|
| PSD | Power spectral density |
| RSS | Residual sum of squares |
| SISO | Single-input Single-output |
| SP | Smoothness priors |
| SPE | Stochastic parameter evolution |
| SSS | Series sum of squares |
| STFT | Short-time Fourier Transform |
| TARMA | Time-varying ARMA |
| TF | Time-frequency |
| UPE | Unconstructed parameter evolution |
| WLS | Weighted least-squares |
| WVD | Wigner-Ville distribution |
| WVS | Wigner-Ville spectrum |

# List of Symbols

**Greek Symbols**

| | |
|---|---|
| $\alpha_{i,k}$ | Coefficients of projection related to the AR-part |
| $\boldsymbol{\vartheta_{AR}}$ | Vector composed by the AR parameters |
| $\boldsymbol{\vartheta_{MA}}$ | Vector composed by the MA parameters |
| $\boldsymbol{\Theta}$ | Regression matrix |
| $\boldsymbol{\theta}$ | Vector composed by the AR/MA parameter and innovation variance vectors |
| $\boldsymbol{\Theta}_0$ | Regression matrix for estimating the initial long FS-AR model |
| $\boldsymbol{\vartheta}$ | Vector composed by the AR/MA parameter vector |
| $\boldsymbol{\vartheta}^0_{AR}$ | Vector composed by the (initial) long-AR parameters |
| $\epsilon$ | Unit-variance sequence |
| $\gamma_{i,k}$ | Coefficients of projection related to the MA-part |
| $\omega$ | Instantaneous angular speed (crankshaft position) |
| $\Phi$ | Angular period related to a reference shaft, e.g. for a four-stroke ICE $\Phi = 4\pi$ |
| $\phi$ | Angle variable (crankshaft position) |
| $\phi_{\Delta\phi}$ | Uniformly spaced angle variable (crankshaft position) |
| $\Psi$ | Angular instant of a tachometer pulse |
| $\sigma_e$ | Innovation standard deviation variance |
| $\sigma_e^2$ | Innovation variance |
| $\tau$ | Time-instant |

**Latin Symbols**

| | |
|---|---|
| $\mathbb{K}_{xx}$ | Autocovariance function of a signal $x$ |
| $\mathbb{E}\left(\cdot\right)$ | Expected value variable |
| $\hat{\mathbf{s}}$ | Initial/intermediary estimate of the innovation variance vector |
| $\hat{e}^0$ | Initial/intermediary estimate of the innovations |
| $\mathbf{g}_i$ | Basis function vector |
| $\mathbf{s}$ | Innovation variance vector |
| $\mathbf{W}$ | Weighting matrix |
| $\mathscr{F}^{\sigma^2}$ | Functional subspace related to the innovations variance |
| $\mathscr{F}^{AR}$ | Functional subspace related to the AR functional basis |
| $\mathscr{F}^{MA}$ | Functional subspace related to the MA functional basis |
| $\mathscr{L}\left(\cdot\right)$ | Model likelihood |

| | |
|---|---|
| $\mathscr{M}(\cdot)$ | Parametric model |
| AIC | Value of the Akaike's information criterion |
| BIC | Value of the Bayesian information criterion |
| $a_i$ | AR-part model coefficients |
| $b_i$ | Unknown variables of the 2-nd order equation |
| $c_i$ | MA-part model coefficients |
| $c_k$ | Fourier series' coefficients |
| $d$ | Number of independently estimated model parameters |
| $d_p$ | Number of invariant model parameters |
| $e$ | Prediction error / innovation / parametric model's input |
| $f$ | Frequency |
| $f_s$ | New sampling frequency (after downsampling) |
| $f_{sn}$ | Sampling frequency |
| $g$ | Smoothing time-window |
| $G_{b(k)}$ | Basis functions that compose the related functional subspace |
| $H$ | Smoothing frequency-window |
| $i$ | Index |
| $j$ | Imaginary unit $j = \sqrt{-1}$ |
| $K$ | Number of angular uniform samples between an angular interval / index (integer) value |
| $k$ | Index vector |
| $M$ | Number of segments in which the signal is divided |
| $m_x$ | Mean value of a signal $x$ |
| $N$ | Number of samples |
| $N_A$ | Maximum model order |
| $n_a$ | AR-part order |
| $n_c$ | MA-part order |
| $N_n$ | Number of samples of a given segment of the signal |
| $p$ | Basis dimension |
| $p_a$ | Dimension of the functional subspace related to the AR functional basis |
| $p_c$ | Dimension of the functional subspace related to the MA functional basis |
| $p_c$ | Dimension of the functional subspace related to the innovations variance functional basis |
| $p_x$ | Joint probability density function of a signal $x$ |
| $s_{i,k}$ | Coefficients of projection related to the innovations variance |
| $T$ | Time-period |
| $t$ | Time variable |
| $t_K$ | Time-instants of equally-spaced samples in angle-domain |
| $w$ | Time-window |

| | |
|---|---|
| $w_{ii}$ | Weighting matrix diagonal components |
| $W_x$ | Smoothed-pseudo-Wigner-Ville distribution computed over a signal $x$ |
| $x$ | Vibration signal / stochastic process / parametric model's output (random variable) |
| $y_k$ | Equivalent representation of AT-FS-ARMA model's input |
| $z_k$ | Equivalent representation of AT-FS-ARMA model's output |

# Contents

# 1 INTRODUCTION

This chapter presents an introduction to the subject of this thesis. First, the motivation and general context are both presented in Section 1.1. The main goal of this work is defined in Section 1.2, whereas the aimed expected contributions are summarized in Section 1.3. Finally, the organization of this thesis is presented in Section 1.4.

## 1.1 MOTIVATION AND GENERAL CONTEXT

The modeling and analysis of rotating and reciprocating machines using vibration-based signals have been important research topics for several decades. The interest in those machines is justified due to their vital position in the industry since they include pumps, turbines, compressors, internal combustion engines (ICEs), generators, fans, and gearboxes are used in the mechanical and power industries (BARDOU; SIDAHMED, 1994). For ICEs, studies are generally related to the development of condition monitoring and diagnosis tools, aiming to increase the engines' durability for longer lifecycles and reduce maintenance costs due to unexpected failure and quality control in assembly lines (DELVECCHIO; BONFIGLIO; POMPOLI, 2018). Moreover, diesel ICEs are still widely utilized since they are present in vehicles and find broad applications. These latter uses include high responsibility usage, i.e., emergency generators in nuclear power plants, used to stop the reactor in unexpected situations (MEIN; BERNARD; DEPOLLIER, 1997). Hence, those applications suggest the necessity of robust system identification methods for damage detection and fault prediction, which optionally may be used during the operation.

Generally, the methods for modeling and analyzing ICEs are classified into two approaches: 1) those based on finite element methods, and 2) those based only on experimental signals. Both methodologies for monitoring ICEs were widely reviewed in a recent paper by Delvecchio, Bonfiglio and Pompoli (2018). The vibratory characteristics of signals from ICEs, including the vibration sources, ways of propagation, and analysis methods, are also presented in references (ANTONI, 2000; DELVECCHIO; BONFIGLIO; POMPOLI, 2018). The experimental signals-based methods depend on signal processing techniques. They require feature extraction, which consists of defining specific signal characteristics to monitor and identifying the ones related to faults and damages. This thesis is focused on those signal-based methods.

The experimental signals-based approaches' fundamental principle is to establish an association between some deviation in the vibration signal statistics and some healthy-state reference(s). However, due to the complexity of the vibration and acoustic-based signals from the ICEs, analysis and feature extraction are more difficult than for other rotating machines, e.g., gearbox and bearings elements, because the signal processing tools to analyze ICEs are not well consolidated yet (MEIN; BERNARD; DEPOLLIER, 1997; ANTONI, 2000; DELVECCHIO; BONFIGLIO; POMPOLI, 2018). Different signal and system identification approaches have been used for ICEs' vibration analysis, vibratory source separation, and feature(s) extraction. Among those methods are the non-parametric techniques, which include the classical time-frequency representations based on the Short-Time Fourier Transform (STFT), Wigner-Ville Distribution (WVD), pseudo-WVD and its variations (FELDMAN; BRAUN, 1995; MEIN; BERNARD; DEPOLLIER, 1997; STASZEWSKI; WORDEN; TOMLINSON, 1997; BAYDAR; BALL, 2001), and Wavelet-based (PAN; SAS, 1996). Each of those methods has its own advantages and drawbacks. For example, the WVD-based methods may present cross-terms, and the STFT-based methods exhibit limitations due to the uncertainty principle (BOASHASH, 2015). In contrast, the Wavelet-based methods require an appropriate choice of the wavelet and are sometimes difficult to interpret by a non-specialist, and its diagnosis is hard to automatize (ANTONI, 2000).

On the other hand, parametric identification and analysis of vibration signals based on autoregressive models has been used with applications for both stationary (DOEBLING *et al.*, 1998; DA SILVA; GONSALEZ; LOPES JUNIOR, 2011) and non-stationary scenarios (GRENIER, 1983; WANG; WONG, 2002; ZHAN; JARDINE, 2005b; POULIMENOS; FASSOIS, 2006; AVENDAÑO-VALENCIA; FASSOIS, 2014), as well for time-varying structures (SPIRIDONAKOS; FASSOIS, 2009; SPIRIDONAKOS; POULIMENOS; FASSOIS, 2010). The common models are the AR/ARMA[1].

The parametric modeling (based on auto-regressions) requires the analyst's model structure, and the model order(s) must be selected. In general, this selection uses measured data and an order selection criterion using a trial-and-error procedure, which is generally based on the estimation of several candidate models and some fitness rule evaluation (STOICA *et al.*, 1986; LJUNG, 2001).

In the past, periodic time-varying parametric identification has been applied to analyze ICEs under constant speed regime of operation (KÖNIG; BÖHME, 1994; BARDOU; SIDAHMED, 1994; SHERMAN; WHITE, 1995; DELVECCHIO; D'ELIA; DALPIAZ, 2015). König, Törk and Böhme (1995) employed periodic time-varying AR filters to model and predicted the cylinder pressure signals in ICEs using vibration-based signals under constant rotation speed regime.

[1]AR: AutoRegressive. ARMA: Autoregressive Moving-Average.

The non-stationary time-dependent parametric approaches use time-varying parameters in their models. There are three most commons proposes to model the time-dependence of the parameters: the Unstructured Parameter Evolution (UPE) models, the Stochastic Parameter Evolution (SPE) models, and the Deterministic Parameter Evolution (DPE) models (SPIRIDONAKOS; POULIMENOS; FASSOIS, 2010). Choosing between those approaches is a sub-problem of the identification problem itself.

The Deterministic Parameter Evolution approach is particularly interesting for vibration analysis (POULIMENOS; FASSOIS, 2006). The method imposes the time-varying coefficients into a functional subspace (FS) constraint, expanding the time-varying parameters into constant coefficients of projection, whose evolution occurs in terms of deterministic and pre-selected time-varying functions. Through an appropriate selection of the FS, those models achieve high parsimony (their representation requires a limited number of parameters). They can track slow or fast dynamics (SPIRIDONAKOS; POULIMENOS; FASSOIS, 2010), especially when orthogonal functions are selected. For vibration and speech analysis, trigonometric functions have often been used to compose the FS (HALL; OPPENHEIM; WILLSKY, 1983; GRENIER, 1983; POULIMENOS; FASSOIS, 2006; AVENDAÑO-VALENCIA; FASSOIS, 2014). Due to the use of FS, those models are often referred to as FS-AR/ARMA. The growing interest in those models is partially due to the increase of computational capacity and the development of more efficient estimators.

Additionally, it is possible to compute parametric spectral estimation, providing spectra with high-resolution potential, avoiding cross-terms, and offering more details than obtained using traditional non-parametric approaches (LANDERS; LACOSS, 1977; ROMBERG; CASSAR; HARRIS, 1984). However, the quality of the obtained spectrum is affected by model underfitting or overfitting. Therefore, note that the quality of the results from parametric models is highly dependent on the appropriated choice of the model orders, which is not a trivial task.

More recently, parametric identification methods have been developed for vibration analysis using both vibration data and environmental and operation parameters to evaluate the effects of the later extracted features vibration response, including modeling uncertainties into the models (VAMVOUDAKIS-STEFANOU; SAKELLARIOU; FASSOIS, 2018; AVENDAÑO-VALENCIA; CHATZI; TCHERNIAK, 2020). Also, recursive DPE-based methods have been investigated with prominent applications for online identification (MA *et al.*, 2018; MA; DING, 2019).

Furthermore, the cyclostationary framework has been used to model, identify, analyze, and fault detection on rotating machines (RANDALL; ANTONI; CHOBSAARD, 2000; RANDALL; ANTONI; CHOBSAARD, 2001; ANTONI *et al.*, 2004). The cyclostationary techniques are useful in describing signals carrying (hidden-)periodicities and, therefore, are well suited for rotating machinery applications. The vibration characterization of ICEs has been performed in the past using a Wigner-Ville distribution at the angle-frequency plane, mainly using the assumption of cyclostationarity. Those results allow one to relate phenomena from the engine's cycle in terms of a signal's energy content at each angle instant (ANTONI *et al.*, 2002). Despite assuming that signals exhibit solely time or angular cyclostationarity, rotating machines' more complex phenomena in both time and angular basis are present. Thus, based on this framework, Antoni, Abboud and Baudin (2014) and Abboud *et al.* (2015) proposed the angle/time-correlated models, which is an extension of the cyclostationary modeling under some non-stationary speed conditions.

In this context, the present thesis introduces an angle/time-varying FS-ARMA model (AT-FS-ARMA) that fits the purpose of analysis of vibration-based signals of the rotating/reciprocating machine. The model is derived from the angle/time-correlated processes framework and consists of a time-varying model in which the coefficients of expansions are dictated as a function of an angle/time-periodic basis. Thus, the AT-FS-ARMA allows the description of the periodicities in terms of the angular variable, while the carrier waveform is described as a function of time. The method is illustrated by applying the identification and analysis of vibration signals from a passenger car diesel ICE operating at idle speed.

## 1.2 OBJECTIVE

This work aims to develop a new angle/time-cyclostationary parametric model, which application is illustrated for the identification and characterization of experimental signals from a diesel internal combustion engine. The analysis is performed using parametric angle-frequency mapping, which allows the analyst to identify kinematic-related events in terms of the angular position in which they occur.

## 1.3 MAIN CONTRIBUTIONS

The main contributions of the present thesis rely on:

- The proposal of a parametric model based on the angle/time-cyclostationary framework. The approach consists of a parametric model based on the FS-ARMA in which the coefficients are described as angle/time-varying processes. For that, the model parameters are expanded into functional subspaces on an angular-dependent Fourier series. The technique is applied to identify and analyze vibration-based signals measured from a diesel internal combustion engine at idle speed.

- Provide an approach for estimate a parametric angle-frequency map that is useful for identifying vibration sources in internal combustion engines and can be used as a tool for understanding signal structures from ICEs.

Part of the results and discussions of this work are also contained in the author's research paper entitled "Data-driven identification of rotating machines using ARMA deterministic parameter evolution in the angle-time domain" of Journal of the Brazilian Society of Mechanical Sciences and Engineering (JAMBERSI; SILVA; ANTONI, 2020).

## 1.4 OUTLINE

This work is organized into five chapters, as follows:

- **Chapter 1 - Introduction**: some aspects of the system/signals identifications are briefly discussed, the most common techniques are presented, and the objective is established.
- **Chapter 2 - Literature Review:** the fundamental concepts to the understanding of the proposed technique are discussed. Those concepts rely on a procedure for angular resampling, time-frequency analysis of rotating machinery signals, parametric identification, and cyclostationary theory, including the new angle/time-correlated process framework.
- **Chapter 3 - Angle-time cyclostationary parametric model**: the AT-FS-ARMA model is introduced, and some fundamental aspects of its formulation are discussed. A "Relaxed" MS-WLS estimator is presented as an efficient option for estimating the AT-FS-ARMA models. The problem of model order selection is addressed, and the BIC is formulated in terms of the angle-time model. Additionally, a simple guide for model orders selection and validation are presented. Finally, a procedure for model-based analysis is described.
- **Chapter 4 - Results and Discussion**: essential aspects of the IC engine vibration response signals' structure are discussed. The data preprocessing stage is described, and the model orders are selected. Thus, angle-frequency maps are estimated using both non-parametric and parametric methods, and their results are compared.
- **Chapter 5 - Final Remarks**: The conclusions of the thesis are drawn, and suggestions for future works are stated.

# 2 LITERATURE REVIEW

This chapter intends to provide an essential and comprehensive summary of topics covering the analysis of vibration signals from rotating and reciprocating machines. It is not a complete state-of-the-art review, but an attempt to cover tools often applied to analyze condition monitoring of internal combustion engines or topics considered as backgrounds for the model proposed in this work. For that, this chapter is divided into four main sections. First, a procedure for angular resampling, usually part of the computed order tracking methods, is presented in Section 2.1. Then, some essential background on time-frequency methods is addressed in Section 2.2. Thus, Section 2.3 is dedicated to an overview of parametric identification, and some procedure for selection of model order is presented. Basic aspects of the cyclostationary framework are addressed in Section 2.4 and its extension to angle/time-cyclostationary processes is presented in Section 2.4.1. Lastly, chapter conclusions of are outlined in Section 2.5.

## 2.1 ANGULAR RESAMPLING AND ANGLE-DOMAIN SIGNAL PROCESSING

First, we will discuss some aspects of angular resampling, a procedure often related to the order tracking (OT) techniques, that is used in this work to provide non-parametric spectra of the signal in the angular-domain. In recent decades, OT methods have gained popularity and were widely used for the vibration analysis and condition monitoring of rotating machines (FYFE; MUNCK, 1997; RANDALL, 2011). This class of methods is based on the angular domain and consists of the analysis of periodic components as a function of rotational speed instead of time (BLOUGH, 2006). Therefore, the signal is treated in the order (angle-based) domain, whereas order refers to a frequency that is a certain multiple of a reference instantaneous angular/rotational speed (BRANDT *et al.*, 2005). Those methods can detect changes attached to damage in an order-frequency distribution, distinguishing it from effects caused due to speed changes. Some order tracking techniques use the digital resampling approach to perform the conversion from time to angular constant-incremented signal (MCDONALD; GRIBLER, 1991; MUNCK; FYFE, 1991). This procedure is popularly known as angular resampling (RANDALL, 2011).

There is a lack of literature in detailing the order tracking techniques emerging since it was developed for commercial end-use. In the late ’80s, the Order Tracking techniques began to be mentioned in the scientific literature. At that time, most of the publications consisted of comparisons between the existent methods without detailing their algorithms. Those first techniques required analog tracking filters and a combination of sophisticated instrumentation and sensors, allowing them to perform angular sampling (POTTER, 1990). This means that the signals were acquired in an (assumed) equally spaced angle samples with a constant number of samples per revolution of a reference shaft, and required the measurement of both vibration and key phasor signals, and a ratio synthesizer conjointly with a low-pass filter to store a defined number of samples per revolution (FYFE; MUNCK, 1997).

At the end of the '80s, the Computed Order Tracking (COT) technique was developed, which considerably reduced the hardware requirements (POTTER, 1990). In this method, the signals are measured and sampled in uniform time-samples and then resampled to angle-domain by interpolation (a procedure called angular resampling). A tachometer (or an encoder) may be used to produce a signal with a defined number of pulses-per-revolution (PPR) and the number of resultant angular samples per revolution $K$, which is chosen by the user. The method can be used in real-time or optionally as a post-processing step. It is worth mentioning that most of those first-order tracking methods are primarily based on non-parametric approaches. More recently, a parametric method based on Vold-Kalman filters has also been developed and found useful, especially in the analysis of machines with more than one rotating part (VOLD; LEURIDAN, 1993; BRANDT *et al.*, 2005).

In the real-time COT method, the arrival time-instants $t_K$ of the tachometer pulses, in which the data will be interpolated, must be defined for each new tachometer pulse. For that, an angular interval between tachometer pulses is defined as

$$\Delta\Psi = \frac{2\pi}{\text{PPR}}, \tag{1}$$

and the interval between angular uniform spaced samples is defined as

$$\Delta\phi = \frac{2\pi}{K}. \tag{2}$$

The angular acceleration is assumed as a constant for the (short) time-period between each tachometer pulses. Thus the angular position variation $\Delta\Psi$ in this short-time period may be approximated by the quadratic function, as given by

$$\Delta\Psi = b_0 + b_1 t + b_2 t^2, \tag{3}$$

whereas the coefficients $b_{i's}$ are determined by the time instants of the three tachometer pulses $t_1$, $t_2$ and $t_3$, solving the linear system, as

$$\begin{Bmatrix} b_0 \\ b_1 \\ b_2 \end{Bmatrix} = \begin{bmatrix} 1 & t_1 & t_1^2 \\ 1 & t_2 & t_2^2 \\ 1 & t_3 & t_3^2 \end{bmatrix}^{-1} \begin{Bmatrix} 0 \\ \Delta\Psi \\ 2\Delta\Psi \end{Bmatrix}. \tag{4}$$

Once the coefficients $b_{i's}$ are determined, the time instants for each uniformly spaced angle value is calculated by

$$t_K = \frac{1}{2b_2}\left[\sqrt{4b_2\left(K\Delta\phi - b_0\right) + b_1^2} - b_1\right]. \tag{5}$$

Finally, linear interpolation (interp1) of a time-uniformly sampled signal $x(t)$ at the instants $t_K$ produces a angle-uniformly sampled signal $x(\phi)$, as

$$x(\phi) = \text{interp1}(t, x(t), t_k). \tag{6}$$

The effects of using different interpolation methods on the results are discussed by Munck (1994). In order to obtain the angle $\phi(t)$ a discrete-time integration of $\omega(t)$ can be performed. One way in achieving it would be by using a cumulative summation, which is analogous to continuous-time integral, in the form:

$$\phi(t) = \sum_{t_0}^{t_f} \omega\Delta t. \tag{7}$$

Thus, the interpolation of the angular displacement $\phi(t)$ at the instants $t_k$ gives the angle $\phi_{\Delta\phi}$ in terms of constant angular increments :

$$\phi_{\Delta\phi} = \text{interp1}(t, \phi(t), t_k). \tag{8}$$

If the procedure is performed in the post-processing phase, it is reduced to a simple interpolation procedure, where the signal may be truncated for an integer number of cycles, and the desired number of points per revolution may be defined. An equally-spaced angular displacement vector is constructed for the desired number of samples, and the interpolation is performed.

Although the procedure above is highly associated with COT methods, the use of angular resampling procedures is also appropriate jointly with other vibration analysis tools. In this work, the angular resampling procedure is used to transform the signal from time to angular-domain. After that, an angular-synchronous-average signal is obtained, and angular-frequency (non-parametric) distribution and spectrum are computed from it. Then, a qualitative comparison between their results and the ones obtained using the

AT-FS-ARMA model.

## 2.2 TIME-FREQUENCY ANALYSIS

Signals from rotating and reciprocating machines exhibit components from a periodic but also from transient nature. Therefore, it is convenient to characterize those signals in terms of some non-parametric representation. Those may be time-frequency, order-frequency (using OT methods), or angle-frequency distributions. The time-frequency approach allows one to identify the signals' energy content and simultaneously relate it to a given time-instant of the machine cycle, i.e., establishing a relation between the energy content and the involved physical phenomena (BARDOU; SIDAHMED, 1994; RANDALL, 2011). There are several different time-frequency distributions such as the Fourier-based, the Wavelet transforms (COHEN, 1995; BOASHASH, 2015), and the Wigner-Ville Distribution.

The most basic time-frequency representation is the Short-Time Fourier Transform (STFT). It is based on the assumption that the signal is stationary under the short-time window in which it is analyzed. For a continuous signal $x(t)$, the STFT is given by

$$\mathrm{STFT}(\tau, f) = \int_{-\infty}^{+\infty} x(\tau) h(t-\tau) \exp^{-j2\pi f t} dt, \tag{9}$$

where $h(\tau)$ is a moving time-window[1]. One drawback of STFT is the resolution limitations due to the uncertainty principle. There will always be a trade-off between time and frequency resolution. The STFT has been used as a tool for vibration source separation and diagnosis in ICEs (BADAWI *et al.*, 2006; TAGLIALATELA-SCAFATI; LAVORGNA; MANCARUSO, 2011).

Another commonly used time-frequency technique is the Wigner-Ville Distribution (WVD). Since the WVD is not based on the stationarity time-windows hypothesis, it does not suffer from the time-frequency resolution limitations. The WVD of a continuous signal $x(t)$ is defined as

$$\mathrm{WVD}(t, f) = \int_{-\infty}^{+\infty} x\left(t + \frac{\tau}{2}\right) x^{*}\left(t - \frac{\tau}{2}\right) \exp^{-j2\pi f \tau} d\tau, \tag{10}$$

where $(\cdot)^{*}$ stands for complex conjugation. However, the results of the WDV suffer from cross-terms interferences, which make them difficult to interpret.

Those interferences may be reduced using the smoothed-pseudo-WVD (SPWVD) at a cost on the time-frequency resolution. Thus, the smoothed version of the WVD, that

[1] $j = \sqrt{-1}$ denotes the imaginary unit.

uses smoothing kernels (low-pass windows) to smooth in time and frequency, is given by

$$\text{SPWVD}(t,f) = \int_{-\infty}^{+\infty} g(t)H(f)x\left(t+\frac{\tau}{2}\right)x^*\left(t-\frac{\tau}{2}\right)\exp^{-j2\pi f\tau}\,d\tau, \tag{11}$$

where $g(t)$ and $H(f)$ are independent smoothing windows in time and frequency.

Analogously to the previous definition presented in eq. (11), the smoothed-pseudo-WVD of an angular-synchronous averaged signal $\bar{x}(\phi)$ in its continuous form may be defined as,

$$\text{SPWVD} = W_x(\phi,f) = \int_{-\infty}^{+\infty} g(\phi)H(f)\bar{x}\left(\phi+\frac{\tau}{2}\right)\bar{x}^*\left(\phi-\frac{\tau}{2}\right)\exp^{-j2\pi f\tau}\,d\tau \tag{12}$$

where $g(\phi)$ and $H(f)$ are independent smoothing windows in angle and frequency, respectively. This equation will be used to provide non-parametric angle-frequency distributions in Chapter 4.

The WVD, the Wigner-Ville Spectrum (WVS), and their variations using smoothing kernels have been widely used to characterize vibration signals in reciprocating machines. In ICEs, the WVD has been applied for knock detection (SAMIMY; RIZZONI, 1994) and for vibration signature identification in the time-frequency domain (ALBARBAR *et al.*, 2010). Under the cyclostationary framework, a WVS was derived used for vibration signature identification and condition monitoring using an angle-frequency based approach, which required the signal to be resampled into the angular domain (ANTONI *et al.*, 2002). The WVD was also used as a complementary tool for detecting lousy adjustment in rocker arm clearances (MEIN; BERNARD; DEPOLLIER, 1997), and, in other reciprocating machines, STFT and WVD have been used for leakage detection in compressors (BARDOU; SIDAHMED, 1994).

## 2.3 PARAMETRIC IDENTIFICATION

This section presents a short overview of parametric identification. In Section 2.3.1 a brief discussion about stationary models is carried out, while Section 2.3.2 is addressed to time-dependent representations. The unstructured and stochastic parameter evolution approaches are presented only in terms of its main idea. On the other hand, the deterministic parameter evolution is discussed in more detail. An overview of model order selection is presented in Section 2.3.3.

### 2.3.1 Stationary Models

An ARMA model for a $N$-length zero-mean random signal $x(t_n)$ is given by (LJUNG, 2001):

$$x(t_n) = \underbrace{-\sum_{i=1}^{n_a} a_i x(t_{n-i})}_{\text{AR-part}} + \underbrace{\sum_{i=1}^{n_c} c_i e(t_{n-i})}_{\text{MA-part}} + e(t_n), \quad e(t_n) \sim \text{NID}(0, \sigma_e^2), \tag{13}$$

where $t_n$ denotes the discrete-time index for $n = 1, \ldots, N$ samples, while $a_i$ for $i = 1, \ldots, n_a$ and $c_i$ for $i = 1, \ldots, n_c$ are AR and MA $i$-th coefficients, $n_a$ and $n_b$ are the corresponding AR- and MA-part orders (i.e. the number of regressions), $e(t_n)$ is the error or innovations sequence, which is assumed as a Normally Independently Distributed (NID) white-Gaussian noise with unitary variance $\sigma_e^2 = 1$. In fact, the sequence $e(t)$ has multiple signify: it may be interpreted as the input signal, the prediction error or the innovation of $x(t_n)$ (GRENIER, 1986). Additionally, the signal $x(t_n)$ may be interpreted as the model output. Less general cases of the model presented in eq.(13) include only AR-part models, that are called AutoRegressive (AR) models. Conversely, models MA-part only are called Moving-Average (MA) models.

For pure-AR models, there are very efficient methods for the estimation of the coefficients $a_i$. The most common approaches are based on the Yule-Walker equation, on the Levinson-Durbin algorithm, and the Burg's method (LJUNG, 2001; BOX *et al.*, 2015). On the other hand, the development of accurate and efficient algorithms to compute the coefficients $c_i$ for MA signals stills in the literature (MCKELVEY; STOICA; MARI, 2000; STOICA; MOSES, 2005; KIZILKAYA; KAYRAN, 2006; ABDOU *et al.*, 2015).

The stationary parametric models are useful to represent processes with no time-dependency or, at least, does not exhibit this behavior during the (short-)time of observation. Additionally, they may be useful in some situations to provide a first insight into signals properties before a deeper analysis using more complex techniques. Stationary models have been applied for structural health diagnosis (SOHN; CZARNECKI; FARRAR, 2000; SOHN; FARRAR, 2001; DA SILVA; DIAS JÚNIOR; LOPES JUNIOR, 2007; DA SILVA *et al.*, 2008b) and condition monitoring in the past decades (BAILLIE; MATHEW, 1996; WANG; WONG, 2002; MECHEFSKE, 2009; AYAZ, 2014).

### 2.3.2 Time-Varying Parametric Model

A traditional time-varying ARMA (TARMA) model is given by (GRENIER, 1983):

$$x(t_n) = -\sum_{i=1}^{n_a} a_i(t_n) x(t_{n-i}) + \sum_{i=1}^{n_c} c_i(t_n) e(t_{n-i}) + e(t_n), \quad e(t_n) \sim \text{NID}(0, \sigma_e^2(t_n)), \tag{14}$$

and is presented as a generalization of the time-invariant ARMA model for the non-stationary case. Here, $x(t_n)$ is a $N$-sample scalar and time-dependent stochastic process, and $a_i(t_n)$ denotes $i = 1, \ldots, n_a$ and $c_i(t_n)$ for $i = 1, \ldots; n_c$ are the time-varying AR and MA $i$-th coefficients; $e(t_n)$ is an unobservable uncorrelated white-Gaussian non-stationary innovation process, with zero-mean and variance $\sigma_e^2(t_n)$, which is also time-varying.

The time-varying models are classified according to the method in which their dynamics are defined, i.e., approaches to model how the parameters $a_i(t_n)$ and $c_i(t_n)$ that evolve a long time. The three most common are: the Unstructured Parameter Evolution (UPE) models, the Stochastic Parameter Evolution (SPE) models, and the Deterministic Parameter Evolution (DPE) models.

The Unstructured Parameter Evolution (UPE) group uses no mathematical structure for its parameters' time evolution. This model class is estimated using short-time windows, computing a simple stationary AR/ARMA model to successive short-time segments of the signal. The estimation procedure, in general, applies recursion or adaptive algorithms (POULIMENOS; FASSOIS, 2006). While UPE models show low parsimony because their parameters are updated at each new signal sample become available, they may be adopted for online system identification. Those models are recommended to - and capable of - representing slowly-time evolution of the signal dynamics due to the same limitations found on the Short-Time Fourier Transform (STFT): short-time window deteriorates the spectral resolution (GRENIER, 1983). The time evolution sensibility depends on a forgetting factor ($\lambda$), which value must be pre-selected by the analyst. Some applications of UPE models for structural health monitoring and condition monitoring may be found in (WANG; WONG, 2002; DA SILVA; GONSALEZ; LOPES JUNIOR, 2011).

The Stochastic Parameter Evolution (SPE) methods, also named Smoothness Priors (SP) constraints, are models in which it is assumed that the coefficients emerge in time in a Markovian chain, i.e., commanding a stochastic construction on the structure of their parameters. Although this family of the method is conceptually the simplest one[2], it is computationally the more complex one between the suggested methods (POULIMENOS; FASSOIS, 2006). The SP-TAR/TARMA models are recursively estimated, generally using Kalman filter and backward smoothing for a simultaneous estimate of the Markov chain and parameters resulting in a nonlinear problem. Consequently, these models exhibit low parsimony. SPE usage for damage detection may be found in some papers, e. g., (ZHAN; JARDINE, 2005a; ZHAN; JARDINE, 2005b; AVENDAÑO-VALENCIA; FASSOIS, 2014); they use for system identification is discussed in (AVENDANO-VALENCIA; SPIRIDONAKOS; FASSOIS, 2011).

[2]It consists of estimates of successive conventional stationary ARMA models through a moving time-window.

Finally, there are the Deterministic Parameter Evolution (DPE) models. In the DPE models, a deterministic structure on the evolution of its parameters is imposed. In this approach, the model coefficients are assumed as known/pre-selected deterministic time-functions belonging to a functional subspace (FS). Due to that, often, the time-varying models with DPE structure are referred to as FS-TARMA[3]. The FS-TARMA models are particularly interesting for vibration modeling and analysis since, through an appropriated selection of the FS, they can achieve high parsimony (a representation that requires a restricted number of parameters). They can also track slow, fast, or even discontinuous dynamics. The tracking of fast time-evolution may be achieved by choosing an orthogonal basis for the FS (POULIMENOS; FASSOIS, 2006; SPIRIDONAKOS; POULIMENOS; FASSOIS, 2010). The FS-TARMA models were already used in the context of ICEs analysis for knock intensity identification (ETTEFAGH *et al.*, 2010).

The FS-TARMA models have been properly used for conditioning monitoring of diesel ICEs (BARDOU; SIDAHMED, 1994), time-varying structure identification (SPIRIDONAKOS; FASSOIS, 2009; AVENDANO-VALENCIA; SPIRIDONAKOS; FASSOIS, 2011), and fault diagnosis (SPIRIDONAKOS; POULIMENOS; FASSOIS, 2010; AVENDAÑO-VALENCIA; FASSOIS, 2014).

To help understand the DPE-models, let us first define the functional subspace $\mathscr{F}$ where variance of both AR/MA-part coefficients and the innovations are expanded in terms of the functions $G_{b(k)}(t_n)$ for $k = 1, \dots, \max(p_a, p_c, p_s)$ with dimensions $p_a$, $p_c$ and $p_s$, as

$$\begin{aligned}
\mathscr{F}^{AR} &\triangleq \Big[G_{b_a(1)}(t_n), \dots, G_{b_a(p_a)}(t_n)\Big],\\
\mathscr{F}^{MA} &\triangleq \Big[G_{b_c(1)}(t_n), \dots, G_{b_c(p_c)}(t_n)\Big],\\
\mathscr{F}^{\sigma^2} &\triangleq \Big[G_{b_s(1)}(t_n), \dots, G_{b_s(p_s)}(t_n)\Big].
\end{aligned}$$

As aforementioned, the FS-TARMA$(n_a, n_c)_{[p_a,p_c,p_s]}$ model requires the expansion of each of the $i$-th parameters $a_i(t_n)$ and $c_i(t_n)$ and innovation variance $\sigma_e^2(t_n)$ into a finite-order basis of the form:

$$a_i(t_n) \triangleq \sum_{k=1}^{p_a} \alpha_{i,k} G_{b_a(k)}(t_n), \quad c_i(t_n) \triangleq \sum_{k=1}^{p_c} \gamma_{i,k} G_{b_c(k)}(t_n), \quad \sigma_e^2(t_n) \triangleq \sum_{k=1}^{p_s} s_{i,k} G_{b_s(k)}(t_n), \tag{15}$$

where $G_{b(k)}(t_n)$ is the predefined set of functions in which the coefficients of projection $\alpha_{i,k}$, $\gamma_{i,k}$, and $s_{i,k}$ are expanded into. The coefficients of expansions are constant values, which means that the basis expansions restrict the temporal evolution of $a_i(t_n)$ and $c_i(t_n)$ by a subspace constraint. The resultant model is, therefore, a time-invariant vector problem that replaces the original non-stationary scalar one (GRENIER, 1983).

[3]Or FS-TAR for AR-only models.

As mentioned above, the method requires the pre-selection of a proper family of basis functions, e.g., polynomial basis such as Legendre, Chebyshev, trigonometric, exponential, or other functions. This choice of an appropriate FS may require some *a priori* knowledge about the dynamics of the system and previous experience of the analyst. Furthermore, the FS-ARMA model's performance in tracking the dynamics of the system is highly conditioned on this choice. For analysis of vibration-based signals, researchers have used the Fourier series, i.e., using trigonometric basis functions to perform this expansion (SPIRIDONAKOS; FASSOIS, 2009; SPIRIDONAKOS; POULIMENOS; FASSOIS, 2010). The Fourier series for the FS achieved prominence after its use in speech signal analysis, first for the TAR-part only models (HALL; OPPENHEIM; WILLSKY, 1983), and later for the complete TARMA model (GRENIER, 1983). In the broader sense, the Fourier series for basis expansions is recommended for cases where periodic or quasi-periodic time evolution of the parameters is expected.

The Fourier series expansion coefficients may be defined as:

$$\begin{cases} G_1(t_n) = 1 \\ G_{2k}(t_n) = \sin(kt_n), & k > 1 \\ G_{2k+1}(t_n) = \cos(kt_n), & k > 1 \end{cases} \tag{16}$$

Therefore, the FS-TARMA model is written as:

$$\begin{aligned} x(t_n) &= -\sum_{i=1}^{n_a}\sum_{k=1}^{p_a} \alpha_{i,k} G_{b_a(k)}(t_n) x(t_{n-i}) + \sum_{i=1}^{n_c}\sum_{k=1}^{p_c} \gamma_{i,k} G_{b_c(k)}(t_n) e(t_{n-i}) + e(t_n), \\ e(t_n) &\sim \text{NID}(0, \sigma_e^2(t_n)), \end{aligned} \tag{17}$$

The FS-TARMA from eq.(17) is described in terms of $n_a\,(p_a+1) + n_c\,(p_c+1) + p_s$ coefficients. One advantage in using the DPE method is that expanding the model coefficients in terms of the basis functions, a scalar linear non-stationary problem is described in terms of a vectorial linear time-invariant process (GRENIER, 1983). The following subsection is dedicated to the subproblem of selecting the model orders and in defining it structure.

### 2.3.3 On the Selection of Orders of Autoregressive Models

The selection of the model orders and basis dimensions is a subproblem of the identification problem. Usually, it is based on the data and an order selection rule and, generally, consists of an arbitrary choice based on trial-and-error or integer optimization. This procedure often requires estimating several candidate models and evaluating their fitness by some criterion (STOICA *et al.*, 1986). Optimally, the selected order must be the one which provides more information using the minimum number of parameter. If the chosen order is smaller than the optimum one, the model is under-fitted. In contrast, if order more significant than the theoretically optimal is chosen, the model is over-fitted. The effect of model under-fitting or over-fitting is various. For instance, for parametric-based spectral analysis, the model order should be the one that gives the most suitable mean square agreement between bias and variance in the spectral estimates. If the selected orders are too inadequate, it may cause the resonances in data to be unresolved. On the other hand, too high orders may be less noise-sensitive (MOLINARO; CASTANIÉ, 1995) while they are susceptible to produce spurious peaks (numerical instabilities), which result in significant variance in the spectral estimates (ROMBERG; CASSAR; HARRIS, 1984; ZHAN; JARDINE, 2005a). Due to those reasons, the model order selection is treated by some authors as a primary concern when defining the model order structure (POULIMENOS; FASSOIS, 2006; SPIRIDONAKOS; FASSOIS, 2013).

When dealing with time-constant models, i.e., filters with time-invariant coefficients, the model order selection problem was broadly reviewed in many papers through the second half of the last century (AKAIKE, 1974; SCHWARZ, 1978; AKAIKE, 1979; HANNAN; QUINN, 1979; HANNAN, 1980; PRIESTLEY, 1981; PUKKILA; KRISHNAIAH, 1988; HURVICH; TSAI, 1989; MARPLE JR; CAREY, 1989).

Although order selection for autoregressive models is still considered an open topic in the signal processing literature (LIU *et al.*, 2013; XIONG *et al.*, 2017), in practice, some criteria are in general well accepted. Between the methods for order selection, the information criterion proposed by Akaike (1974)[4] and the Bayesian information criterion (BIC), proposed Schwarz (1978) and Kashyap (1982) are worth to be mentioned. Both criteria evaluate the model fitness while penalizes the model size, discouraging possible over-fitting. The optimal order should be the one in which the criterion presents a minimum value. However, there are many criteria, and their usage is not consent. Stoica *et al.* (1986) presents an extensive survey on the model order selection methods, while both AIC and BIC criteria are compared and discussed in detail in terms of probability of over-fitting a research paper by Stoica and Selen (2004). Their results showed that BIC outperforms AIC in most scenarios. In the context of vibration modeling and SHM, Figueiredo *et al.* (2011) discussed the effect of the chosen criterion on the estimated models and its consequence in damage detection.

Nonetheless, for time-varying models, the order selection procedure is an even more laborious task. This is especially true for the case of FS-based ARMA models where, besides the selection of the orders, $n_a$ and $n_c$, the dimension of the functional subspaces $p_a$ and $p_c$ must be chosen as well. To select the structure of an FS-TARMA model,

[4] Originally, the criterion was named by the author as “an information criterion”. Years later, it became popularly known as the “Akaike's information criterion.”

Poulimenos and Fassois (2006) decouple the problem of selecting model orders and the FS dimensions. For that, an extended and complete functional subspace is held, and the model order is selected by minimizing the BIC criterion. Thus, several FS-TARMA($n$,$n$) are determined for many orders until a predefined theoretical maximum order, the optimal AR-part order $n_a$ is selected through the minimization of BIC. Then, the procedure is replicated to optimize the MA-part order $n_c$. Furthermore, the aggregate parameter deviation (APD) identifies the functional subspace's excessive dimensions to achieve a more economical model representation. Likewise, for the case of FS-TAR models, a similar approach is used by Spiridonakos and Fassois (2013). Again, they also decouple the problem assuming extended and complete functional subspace. However, although the model order is selected through the minimization of the BIC, they use BIC again instead of the APD to successively remove each excessive subspace dimension that produces any reduction of the BIC.

For vibration analysis, the AIC and/or BIC criteria are generally chosen to select the model order (POULIMENOS; FASSOIS, 2006). Thus, for a given $N$-samples discrete-time series $x(t_n)|_{n=1}^{N}$ and for a defined model structure $\mathscr{M}\left(\boldsymbol{\theta}^N, (\sigma_e^2)^N\right)$, the AIC value is given by:

$$\text{AIC} = -2\ln\mathscr{L}\left(\mathscr{M}\left(\boldsymbol{\theta}^N, \left(\sigma_e^2\right)^N\right)\right) + 2d, \tag{18}$$

while the BIC value is given by:

$$\text{BIC} = -\ln\mathscr{L}\left(\mathscr{M}\left(\boldsymbol{\theta}^N, \left(\sigma_e^2\right)^N\right)\right) + \frac{\ln N}{2}d, \tag{19}$$

where $d$ is the number of independently estimated model parameters $d = n_a p_a + n_c p_c$, and $\boldsymbol{\theta} \triangleq [\boldsymbol{\vartheta}^\mathsf{T}|\mathbf{s}^\mathsf{T}]^\mathsf{T}$ is a vector composed by the AR/MA parameter vectors $\boldsymbol{\vartheta} \triangleq [\boldsymbol{\vartheta}_{AR}^\mathsf{T}|\boldsymbol{\vartheta}_{MA}^\mathsf{T}]^\mathsf{T}$ and by the innovation variance vector $\mathbf{s} \triangleq [s_1, \ldots, s_{p_s}]^\mathsf{T}$. The term $\ln\mathscr{L}\left(\mathscr{M}\left(\boldsymbol{\theta}^N, (\sigma_e^2)^N\right)\right)$ is the Gaussian log-likelihood function of the model $\mathscr{M}\left(\boldsymbol{\theta}^N, (\sigma_e^2)^N\right)$, that is given (in its time-dependent form) by:

$$\ln\mathscr{L}\left(\mathscr{M}\left(\boldsymbol{\theta}^N, \left(\sigma_e^2\right)^N\right)\right) = -\frac{N}{2}\ln 2\pi - \frac{1}{2}\sum_{n=1}^{N}\left(\ln\left(\sigma_e^2(t_n)\right) + \frac{e^2(t_n)}{\sigma_e^2(t_n)}\right). \tag{20}$$

Once the model orders are selected and the model structure is defined, the chosen model must still be validated using some validation criterion.

### 2.3.4 Model Validation

The acceptance of the estimated model depends on its performance under procedures that check it for an example for Gaussianity and the innovations sequence whiteness by one or more procedures. When dealing with stationary models, this validation and if more than one dataset for a similar condition is available, the identification and validation may be performed using different time-series assuming they present similar statistical properties. Nevertheless, if only a single realization of the process is available, half of the time-series is used to estimate the model, and after, the second half is used to validate it. For stationary models, a common approach is to validate the model by evaluating the correlation between the innovations and the model output/input of a given confidence level. If the model is well estimated, this function should be of an impulse-like shape (DA SILVA; DIAS JUNIOR; LOPES JUNIOR, 2008a). A similar approach would be evaluating the Auto-Correlation Function (ACF) of the innovation process to verify if there is a correlation among the residual themselves (LJUNG, 2001).

However, for non-stationary models, the innovations variance exhibit time-varying behavior, and the simple evaluation of the ACF is not an adequate metric to attest model fitting (SPIRIDONAKOS; FASSOIS, 2014). In order to overcome this issue, the innovations sequence can be normalized into a unit-variance sequence $\epsilon(t_n)$ (FOUSKITAKIS; FASSOIS, 2002; SPIRIDONAKOS; FASSOIS, 2014) as

$$\epsilon(t_n) = \frac{e(t_n)}{\sigma_e(t_n)} \qquad \epsilon(t_n) \sim \mathrm{NID}(0, 1), \tag{21}$$

where $\epsilon(t_n)$ is the normalized variance. When the ACF is computed from the normalized sequence $\epsilon(t_n)$ it is known as normalized ACF.

Additionally, a simple method to validate non-stationary models would be using the run test for randomness or residual sign test. This test evaluates the whiteness of the residual/innovation signal through an analysis based on the number of signal changes for a given confidence level (STRAUME; JOHNSON, 1992). It consists of a hypothesis test that evaluates that group consecutive samples with a common sign and counts the number of signal changes to test it for Gaussianity. In this procedure, the null hypothesis would state that the innovation sequence is uncorrelated.

## 2.4 CYCLOSTATIONARY PROCESSES

Cyclostationarity is a property that characterizes processes whose statistical moments are periodic as a function of some generic variable, e.g., time or angle. By definition, cyclostationary processes provide a formal description for a class of non-stationarity signals with hidden-periodicities in its structure (ANTONI *et al.*, 2004).

The theory of cyclostationary processes has been explored in telecommunications engineering for decades (GARDNER; FRANKS, 1975; GARDNER, 1986; GARDNER, 1991), whereas periodicities are generally treated in terms of time-periodicity. However, more recently, this theory has gained researchers' attention in the context of signals from rotating and reciprocating machines (ANTONI *et al.*, 2002; ANTONI *et al.*, 2004; RANDALL, 2011) and rolling element bearings analysis (RANDALL; ANTONI; CHOBSAARD, 2000). Overviews on cyclostationary signal analysis and applications are addressed by Giannakis and Madisetti (1998), by Gardner, Napolitano and Paura (2006), and Napolitano (2016).

In order to introduce some concepts regarding cyclostationary processes, let us define an $N$-sample process $x(t_n)$, for $n = 1, \ldots, N$, that is cyclostationary if its joint probability density function is periodic in $t_n$ for some period $T$, as

$$p_x(x_1, \ldots, x_N; t_1, \ldots, t_N) = p_x(x_1, \ldots, x_N; t_1 + T, \ldots, t_N + T). \tag{22}$$

A first-order cyclostationary process is an $N$-sample stochastic process $x(t_n)$ whose first-order statistical moment, i.e. its expected value $m_x(t_n)$ is periodic with period $T$,

$$m_x(t_n) \triangleq \mathbb{E}\{x(t_n)\} = m_x(t_n + T), \tag{23}$$

where $\mathbb{E}\{\cdot\}$ stands for the expected value.

In the same way, an $N$-sample stochastic process $x(t_n)$ is a second-order cyclostationary process if its second-order moment, i.e., its auto-covariance function is periodic with period $T$ between instants $t_n$ and $\tau_n$, as

$$\mathbb{K}_{xx}(t_n, \tau_n) \triangleq \mathbb{E}\{x^*(t_n)x(\tau_n)\} = \mathbb{K}_{xx}(t_n + T, \tau_n + T). \tag{24}$$

Signals that are both first and second-order cyclostationary are called wide-sense cyclostationary processes. For instance, this is the case of vibration signals from internal combustion engines (ANTONI; ABBOUD; XIN, 2016).

Wide-sense cyclostationary processes are often referred to as "periodically correlated" processes, and admit the following Fourier series representation:

$$x(t_n) = \sum_{k=-\infty}^{\infty} c_k(t_n) e^{j2\pi \frac{kt_n}{T}}, \tag{25}$$

where the Fourier coefficients $c_k(t_n)$ are mutually correlated signals.

The cyclostationary framework has been proven useful for modeling and describing vibration signals and their signatures and the development of signal processing tools for condition monitoring of the rotating and reciprocating machines (ANTONI; ABBOUD; XIN, 2016). A tutorial on the application of cyclostationary analysis focusing on mechanical signals and systems is presented by Antoni (2009). When describing rotating machine signals, this formulation imposes as a limitation that the operating speed must exactly/perfectly constant. One solution to this problem would be the reformulation of all these definitions in terms of the angular variable $\phi_n$ instead of $t_n$. This would allow the analysis to proceed in terms of $f$ and the cyclic order $\alpha$ expressed in orders rather than in Hz. This change of variable improves the analysis of cyclostationary signals in the presence of slight speed fluctuations, but at the same time, introduces a distortion of the spectral content (in variable $f$)of phenomena described by time-dependent dynamics.

In the case of vibration signals from rotating and reciprocating machines, phenomena related to the kinematics of the machines are often produced as a function of the angular position(s) of one or more reference(s) shaft(s). Therefore, a natural choice to represent those signals is often assumed as angle-cyclostationary processes instead of time and describing it in terms of an angular variable. This procedure would require the signal to be angular (re-)sampled, and the equations (22) to (25) must be reformulated in terms of the angular variable $\phi_n$ Thus, the signals may be described as angle-cyclostationary with angular-period $\Phi$, instead of their time-domain counterparts. This would provide the following Fourier series representation of the angle-cyclostationary process $x(\phi_n)$ in the angle-domain, as

$$x(\phi_n) = \sum_{k=-\infty}^{\infty} c_k(\phi_n) e^{j2\pi \frac{k\phi_n}{\Phi}}. \tag{26}$$

whereas eq. (26) defines an angle-cyclostationary signal only if $c_k(\phi_n)$ is angle-stationary (ABBOUD *et al.*, 2016).

For stationary signals, the description of signals in time or angular-domain is completely equivalent since the signal statistics are constant; however, for cyclostationary signals from machines, this equivalence generally does not hold (ABBOUD, 2015). In a study regarding the situations in which time and angle-cyclostationary equivalence persist, Antoni *et al.* (2004) demonstrated that an angle-cyclostationary signal remains time-cyclostationary only if the speed profile is periodic, stationary, or cyclostationary.

As remarked by Abboud *et al.* (2015), under non-stationary speed profile, if the time-cyclostationary description is used, the angle-dependent modulations will not be adequately described. On the other hand, if an angle-cyclostationary description is adopted, the time-invariant characteristics will be angle-varying.

Finally, even in conditions where solely angle-stationarity is kept, the resulting signal is not angle-cyclostationary in general, since it contains adding noise and time-invariant (therefore, angle-varying) characteristics related to the transient waveform (e.g., natural frequencies and relaxation times). Another concern regarding rotating machinery that contributes to invalidate the angle-cyclostationary hypothesis of the resulting signal is that the transmission paths from the vibration (caused by angular-related forces) to the sensors act like structural (linear time-invariant) filters that are governed by time-dependent differential equations. Therefore, neither time nor angle-domain description solely would be enough to describe the signal content accurately (ANTONI; ABBOUD; BAUDIN, 2014).

### 2.4.1 Angle-Time Periodically Correlated Processes

The angle/time-cyclostationary (AT-CS) framework extends the cyclostationary class to provide a formal description of mechanical signals in terms of angle/time-correlated processes. The AT-cyclostationary framework was originally proposed by Antoni, Abboud and Baudin (2014) and Abboud *et al.* (2015) and has found different applications in the last few years Abboud *et al.* (2016), Abboud, Marnissi and Elbadaoui (2020), especially for analysis of signals from machines under speed fluctuations.

In this novel framework, the signals are described considering angle and time jointly instead of independently, i.e., the signals are defined by the angle-periodicity of the time correlation measure of the signal. In this context, Antoni, Abboud and Baudin (2014) presented three useful models for modeling of rotating machine signals. One model particularly interesting consists into the following angular-dependent Fourier series representation in its exponential form, as

$$x(t_n) = \sum_{k=-\infty}^{\infty} c_k(t_n) e^{j2\pi \frac{k\phi_n}{\Phi}} \tag{27}$$

with mutually correlated stationary random Fourier coefficients. Unlike the equation (26), this novel model presented in equation (27) describes the signal in both time and angle-domain conjointly, and it is useful for the modeling of AT-CS processes.

This angle-dependent Fourier series has a vital role in developing the proposed AT-FS-ARMA models for the analysis of vibrations from rotating/reciprocating machines. This role will be taken up in Section 3.1 of this thesis.

## 2.5 PARTIAL REMARKS

In this chapter, some literature review on topics related to this work's central core were presented and discussed. First, the angular resampling procedure was discussed. This procedure constitutes the heart of the computed order tracking methods and is a common approach to transforming signals from time to angle. It requires both vibration and tachometer signals and may be performed in real-time or post-processing stages. In this work, the angular resampling is used to compute the angular-domain signal to compute a non-parametric angle-frequency map from it. After that, the most popular time-frequency techniques were briefly reviewed. Those methods provide a family of powerful tools to characterize signals in terms of their transients components and allows one to obtain non-parametric time/order-frequency distribution from time/angular-domain signals. The techniques based on the WVD have been used for the analysis of vibration signals ICEs.

Additionally, some models commonly used parametric identification was presented. The TARMA models were introduced as a generalization of the stationary ARMA for the modeling of time-varying random processes. After, the three most common approaches to defining how the time-varying model coefficients will evolve with time were presented. Particular attention was given to the Deterministic Parameter Evolution representation, which provides the FS-TARMA models. A deterministic structure is imposed upon the coefficients' evolution by postulating the parameters as belonging to specific (pre-selected) functional subspaces. FS-TARMA model has been used for vibration analysis in the past, often using the Fourier-basis functions. The subproblem of selecting the model structure was addressed, and some procedures for model validation based on the analysis of the whiteness of the innovation sequence were presented.

Finally, some background on the cyclostationary theory and its extension: the angle/time-cyclostationary framework, was introduced. The AT-cyclostationary class provides a formal description of signals from mechanical systems even under some speed-varying conditions. The angular-dependent Fourier series was stated. The AT-FS-ARMA model is derived based on the AT-cyclostationary framework and on the FS-TARMA model representation. It assumes the signal as angle/time-varying describing this evolution by coefficients of projection in angle-domain while preserves the time-evolution of the signal. This model is, as far as we know, an original contribution of this work.

# 3 ANGLE-TIME CYCLOSTATIONARY PARAMETRIC MODEL

The proposed AT-FS-ARMA model is described in this chapter. First, it is derived from the FS-ARMA (time-dependent) formulation in Section 3.1. Additionally, the RMS-WLS estimator used for model computation is described in Section 3.2, while the and a simple guide for model order selection is discussed in Section 3.3 where the criteria for model validation used in this thesis are also resumed. A procedure for the computation of the (parametric) angle-frequency map is presented in Section 3.4. Finally, a summary of the content of this chapter is presented in Section 3.5.

## 3.1 THE ANGLE/TIME-VARYING FS-ARMA MODEL

In contrast to the FS-TARMA models, the proposed AT-FS-ARMA approach expands the model coefficients in an angle-function functional basis belonging to a functional subspace, where the angular dependency is directly related to the instantaneous position of a reference shaft instead of time. For the case of ICEs, this reference shaft is the crankshaft. This model can be used to identify vibration-based signals from rotating, and reciprocating machines and are derived from a new framework, where the signals from machines are modeled as angle/time-cyclostationary processes. The signal periodicity is described in terms of the angular variable, with the signal dynamics represented varying through time.

Let us define the instantaneous angular shaft displacement variable, which depends on the discrete-time instant $\phi_n = \phi(t_n)$. Hence, an AT-FS-ARMA model with parameters dependent on both time and angle is proposed as:

$$x(t_n) = -\sum_{i=1}^{n_a} a_i(\phi_n)x(t_{n-i}) + \sum_{i=1}^{n_c} c_i(\phi_n)e(t_{n-i}) + e(t_n), \tag{28}$$

where,

$$a_i(\phi_n) = a_i(\phi_n + \Phi), \quad c_i(\phi_n) = c_i(\phi_n + \Phi), \quad \sigma_e^2(\phi_n) = \sigma_e^2(\phi_n + \Phi), \tag{29}$$

are functions dependent on both time and angle that are angle-periodic with a period equal to $\Phi$. The angle-periodicity also indicates that the innovation variance $\sigma_e^2(\phi_n)$ is cyclic with a period related to a shaft/rotating component position. This period is in general equal to $\Phi = k2\pi$, where $k \in \mathbb{Z}$. Assuming the DPE-based structure for the

AT-FS-ARMA model, the parameters for the AR-part $a_i(\phi_n)$ and MA-part $c_i(\phi_n)$ as well as the innovation variance $\sigma_e^2(\phi_n)$ are expanded into angle-dependent functional basis in the form:

$$\begin{aligned} a_i(\phi_n) &\triangleq \sum_{k=1}^{p_a} \alpha_{i,k} G_{b_a(k)}(\phi_n), \quad c_i(\phi_n) \triangleq \sum_{k=1}^{p_c} \gamma_{i,k} G_{b_c(k)}(\phi_n), \\ \sigma_e^2(\phi_n) &\triangleq \sum_{k=1}^{p_s} s_{i,k} G_{b_s(k)}(\phi_n), \end{aligned} \tag{30}$$

and, substituting Eq. (30) into Eq. (28), we have a closed-form formulation of the AT-FS-ARMA model as:

$$\begin{aligned} x(t_n) &= -\sum_{i,k} \alpha_{i,k} \underbrace{G_{b_a(k)}(\phi_n) x(t_{n-i})}_{z_k(\phi_{n-i})} + \sum_{i,k} \gamma_{i,k} \underbrace{G_{b_c(k)}(\phi_n) e(t_{n-i})}_{y_k(\phi_{n-i})} + e(t_n), \\ e(t_n) &\sim \text{NID}(0, \sigma_e^2(\phi_n)), \end{aligned} \tag{31}$$

Then, the AT-FS-ARMA model can be expressed as:

$$x(t_n) = -\sum_{i,k} \alpha_{i,k} z_k(\phi_{n-i}) + \sum_{i,k} \gamma_{i,k} y_k(\phi_{n-i}) + e(t_n), \quad e(t_n) \sim \text{NID}(0, \sigma_e^2(\phi_n)). \tag{32}$$

The AT-FS-ARMA model is described in terms of $n_a(p_a + 1) + n_c(p_c + 1) + p_s$ coefficients. As previously mentioned, the DPE-based method requires the preselection of a proper family of basis functions, which in general may assume in *a priori* knowledge about the signal and previous analyst experience. As previously discussed in the sections 2.1 and 2.4, vibration signals from reciprocating/rotating machines suggest angular-periodicity of the sources of vibrations linked to a reference shaft, while the waveforms are properly described in time-domain. Therefore, an angular-periodic basis is an appropriate choice to express this dependency, yet the signal is still described varying as the time variable's function.

Thus, to express the angle/time periodicities, a functional subspace is composed of an angular-dependent Fourier series, from eq. 27, is expanded into its trigonometric form as

$$\begin{cases} G_1(\phi_n) = 1 \\ G_{2k}(\phi_n) = \sin\left(2\pi \frac{k\phi_n}{\Phi}\right), & k > 1 \\ G_{2k+1}(\phi_n) = \cos\left(2\pi \frac{k\phi_n}{\Phi}\right), & k > 1, \end{cases} \tag{33}$$

where $G_{b(k)}$ for $k = 1, \ldots, \max(p_a, p_c)$ are the angle/time-dependent functional subspaces, and $k$ are the indices of the subspace basis functions.

Most of the rotating and reciprocating machines will present signals with angular dependence in terms of a period equal to $\Phi = 2\pi$. For the 4-stroke ICEs however, this angular-period is equal to $\Phi = 4\pi$, therefore rewritten equation (33) in terms of the angular period, we have

$$\begin{cases} G_1(\phi_n) = 1 \\ G_{2k}(\phi_n) = \sin\left(\frac{k\phi_n}{2}\right), & k > 1 \\ G_{2k+1}(\phi_n) = \cos\left(\frac{k\phi_n}{2}\right), & k > 1. \end{cases} \tag{34}$$

where $k$ are the indexes.

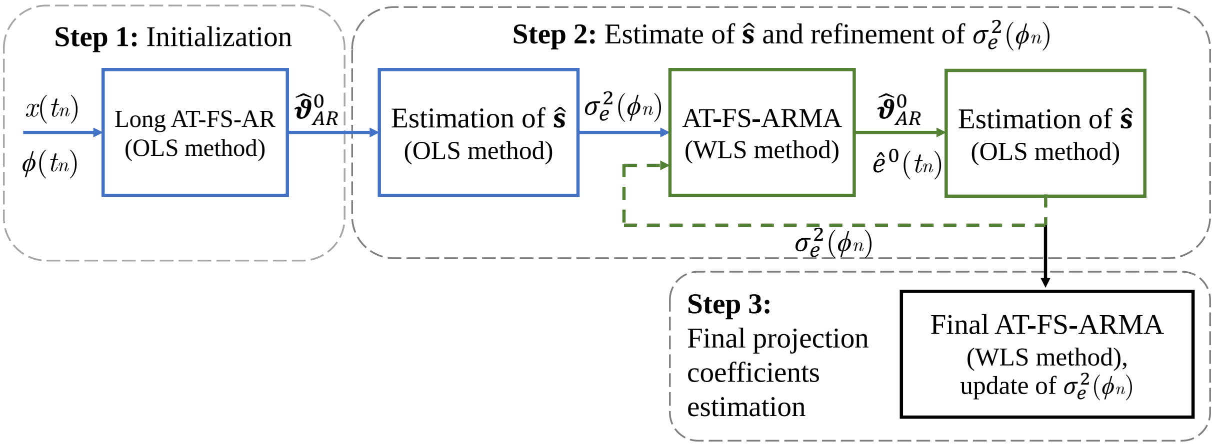
As a result, the AT-FS-ARMA model presented in equation (28) has a time-dependent signal as an output, but this time-dependent signal is represented through expansion into angular-periodic (but yet time-dependent) basis, accordingly to equation (34). Therefore the model can capture the time-dependent dynamics while preserving the angular-dependent modulations related to the system's kinematics. This model is expected to exhibit good performance under low or periodic speed variations, as well as in conditions in which its changes are considered low when compared to the total length of the dataset. The next section is dedicated to the RMS-WLS estimator used to compute the AT-FS-ARMA models.

## 3.2 THE RMS-WLS ESTIMATOR

A "Relaxed" Multi-Stage Weighted Least Squares (RMS-WLS) method is used to compute the model coefficients of projections and the innovation variance (SPIRIDONAKOS; FASSOIS, 2010; SPIRIDONAKOS; FASSOIS, 2014). The RMS-WLS estimator replaces the non-linear operations of the Weighted Least Squares (MS-WLS) estimator with linear ones. The original MS-WLS method is efficient, asymptotically equivalent to the Maximum-Likelihood (ML) estimator, consists in decoupling the AR and MA estimation procedures, and presents reduced optimization space dimensional for the FS-ARMA case (POULIMENOS; FASSOIS, 2005). The RMS-WLS replaces the non-quadratic optimization problems with suboptimal but linear counterparts, providing refinement by iterative techniques. Although the RMS-WLS method does not share the same asymptotic qualities as the MS-WLS estimator, it is not susceptible to the potential convergence problems of the non-linear techniques (POULIMENOS; FASSOIS, 2005).

Other "relaxed" estimators have been successfully used to estimate FS-ARMA models such the Two-Stage Least Squares (2SLS) method (GRENIER, 1983; POULIMENOS; FASSOIS, 2006; BERTHA; GOLINVAL, 2017) and a Polynomial-Algebraic (P-A) method (MRAD; FASSOIS; LEVITT, 1998; POULIMENOS; FASSOIS, 2006). The RMS-WLS estimator, in which schematic representation is presented in Fig. 1, is implemented as a part of the *Non-stationary modeling toolbox* (AVENDAÑO-VALENCIA *et al.*, 2016).

Figure 1: Schematic representation of the RMS-WLS method for estimation of the AT-FS-ARMA model.




Source: prepared by the author.

First, let us define the AR/MA-part projection coefficient vectors:

$$\boldsymbol{\vartheta}_{AR} = [\alpha_{1,1}, \ldots, \alpha_{1,p_a}, \alpha_{2,1}, \ldots, \alpha_{2,p_a}, \ldots, \alpha_{n_a,1}, \ldots \alpha_{n_a,p_a}]^{\mathsf{T}},$$
$$\boldsymbol{\vartheta}_{MA} = [\gamma_{1,1}, \ldots, \gamma_{1,p_c}, \gamma_{2,1}, \ldots, \gamma_{2,p_c}, \ldots, \gamma_{n_c,1}, \ldots, \gamma_{n_c,p_c}]^{\mathsf{T}},$$

The model orders $n_a$ and $n_c$, and the basis dimensions $p_a$, $p_c$ and $p_s$ are assumed as *a priori* known. Let[1]:

$$\boldsymbol{\theta} \triangleq [\boldsymbol{\vartheta}^{\mathsf{T}} | \mathbf{s}^{\mathsf{T}}]^{\mathsf{T}}_{(n_a p_a + n_c p_c + p_s)\times 1},$$
$$\boldsymbol{\vartheta} \triangleq [\boldsymbol{\vartheta}^{\mathsf{T}}_{AR} | \boldsymbol{\vartheta}^{\mathsf{T}}_{MA}]^{\mathsf{T}}_{(n_a p_a + n_c p_c)\times 1}, \qquad \mathbf{s} \triangleq [s_1, \ldots, s_{p_s}]^{\mathsf{T}}_{p_s \times 1},$$

where $\boldsymbol{\vartheta}$ and $\mathbf{s}$ are, again, the AR/MA projection coefficient and innovation variance vectors.

To properly describe the estimator, let us rewrite Eq. (31) in its regression-type form:

[1]The subscripts in the brackets designate the vector dimensions, and bold symbols indicate column vectors.

$$x(t_n) = \underbrace{\begin{bmatrix} -G_{b_a(1)}(\phi_n)x(t_{n-1}) \\ -G_{b_a(2)}(\phi_n)x(t_{n-1}) \\ \vdots \\ -G_{b_a(p_a-1)}(\phi_n)x(t_{n-n_a}) \\ -G_{b_a(p_a)}(\phi_n)x(t_{n-n_a}) \\ \hdashline G_{b_c(1)}(\phi_n)e(t_{n-1}) \\ G_{b_c(2)}(\phi_n)e(t_{n-1}) \\ \vdots \\ G_{b_c(p_c-1)}(\phi_n)e(t_{n-n_c}) \\ G_{b_c(p_c)}(\phi_n)e(t_{n-n_c}) \end{bmatrix}^{\intercal}}_{\boldsymbol{\Theta}(\phi_n)} \underbrace{\begin{bmatrix} \alpha_{1,1} \\ \alpha_{1,2} \\ \vdots \\ \alpha_{n_a,p_{a-1}} \\ \alpha_{n_a,p_a} \\ \hdashline \gamma_{1,1} \\ \gamma_{1,2} \\ \vdots \\ \gamma_{n_c,p_{c-1}} \\ \gamma_{n_c,p_c} \end{bmatrix}}_{\boldsymbol{\vartheta}} + e(t_n), \quad e(t_n) \sim \text{NID}(0, \sigma_e^2(\phi_n)), \tag{35}$$

where $\boldsymbol{\Theta} = \boldsymbol{\Theta}(\phi_n)$ is the regression matrix of the angle/time-dependent FS-ARMA model.

The estimation procedure may be summarized in the following steps:

- **Step 1:** First, a long AT-FS-AR model is computed. This initial FS-AR model is given by

$$x(t_n) = \boldsymbol{\Theta}_0 \boldsymbol{\vartheta}_{AR}^0 + e(t_n), \tag{36}$$

where $\boldsymbol{\Theta}_0 = \boldsymbol{\Theta}_0(\phi_n)$ is the regression matrix of the long FS-AR model. Since the true innovation variance $\sigma_e^2(\phi_n)$ is *a priori* unknown, the bounded weighting sequence is assumed at this step as an unitary sequence $w_{ii}(\phi_n) = 1$ to guarantee the estimation consistency. Then, the estimation of the projection coefficients $\hat{\boldsymbol{\vartheta}}_{AR}^0$ is estimated using the OLS method:

$$\hat{\boldsymbol{\vartheta}}_{AR}^0 = \left(\boldsymbol{\Theta}_0 \boldsymbol{\Theta}_0^{\intercal}\right)^{-1} \left(\boldsymbol{\Theta}_0 x(t_n)\right), \tag{37}$$

- **Step 2:** The second step consists in estimates in compute the innovations process $\hat{e}^0(t_n)$ from the $\hat{\boldsymbol{\vartheta}}_{AR}^0$ by $\hat{e}^0(t_n) = x(t_n) - \boldsymbol{\Theta}_0 \hat{\boldsymbol{\vartheta}}_{AR}^0$, and then the instantaneous innovations variance projection coefficient vector $\hat{\mathbf{s}}$ is estimated also by the OLS method:

$$\hat{\mathbf{s}} = \left(\mathbf{g}_s \mathbf{g}_s^{\intercal}\right)^{-1} \left(\mathbf{g}_s \left|\hat{e}^0(t_n)\right|\right), \tag{38}$$

where $\mathbf{g}_s = \mathbf{g}_s(\phi_n) \triangleq \left[G_{b_s(1)}(\phi_n), \ldots, G_{b_s(p_s)}(\phi_n)\right]^{\intercal}$. Then, the angle/time-dependent innovations variance $\sigma_e^2(\phi_n)$ is estimated, as

$$\sigma_e^2(\phi_n) = \left(\mathbf{g}_s \hat{\mathbf{s}}\right)^2. \tag{39}$$

- **Step 2.1:** An intermediary step involves the refinement of the innovation variance estimation. This iterative procedure consists of a recursive refinement of the estimated coefficients and residual sequence, where the initial estimates are then refined by using the WLS method to compute the projection coefficients. The weightings may be assumed as $w_{ii} = \sigma_e^2(\phi_n)$. Since the errors are expected to be uncorrelated, the weighting matrix may be defined as $\mathbf{W} = \mathrm{diag}(1/\sqrt{w_{ii}})$ (STRUTZ, 2010). Then, the WLS estimator is:

$$\hat{\boldsymbol{\vartheta}}_{AR} = \left(\mathbf{\Theta}_0 \mathbf{W} \mathbf{\Theta}_0^{\intercal}\right)^{-1} \left(\mathbf{\Theta}_0 \mathbf{W} x(t_n)\right), \tag{40}$$

  This provides updated AR-part projection coefficients $\hat{\boldsymbol{\vartheta}}_{AR}$ and the residual sequence of $\hat{e}^0(t_n)$ is again updated by $\hat{e}^0(t_n) = x(t_n) - \mathbf{\Theta}_0 \hat{\boldsymbol{\vartheta}}_{AR}^0$. This residual sequence is then employed to estimate the innovations variance, again by the OLS method. This step is performed until a) the iteration Mean-Square-Error (MSE) is lower than a objective value ($|\mathrm{MSE} - \mathrm{MSE}_0| / \mathrm{MSE}_0 \leq 1 \times 10^{-8}$); b) the change in the parameters is lower than a threshold ($\|A(\phi_n) - A(\phi_{n-1})\| \,/\, \|A(\phi_{n-1})\| \leq 1 \times 10^{-8}$); or c) a maximum number of iterations is reached.

- **Step 3:** The last step involves estimation of the AT-FS-ARMA final projection coefficients, also using the WLS estimator. The projection coefficients $\hat{\boldsymbol{\vartheta}}$ are finally approximated to replace the past values (but not the current ones) of the residual error sequence $\hat{e}(t_n)$ by the previously estimated $\hat{e}^0(t_n)$ in Eq. (35). After the estimation of $\hat{\boldsymbol{\vartheta}}$, the innovations variance $\sigma_e^2(\phi_n)$ is again updated.

Once the RMS-WLS estimator was chosen and described, the next natural step should be discussing how the model's mathematical structure is defined. This procedure will require the model orders to be selected and is the primary concern of the following section.

## 3.3 MODEL ORDERS SELECTION PROCEDURE AND MODEL VALIDATION

In this thesis, the selection of the AT-FS-ARMA model orders is performed adopting the BIC, previously discussed in Section 2.3.3. Therefore, again, for a given $N$-samples time-series $x(t_n)|_{n=1}^{N}$ and an angle/time-dependent model structure $\mathscr{M}\left(\boldsymbol{\theta}^N, (\sigma_e^2)^N\right)$, the BIC value is given by

$$\mathrm{BIC} = -\ln \mathscr{L}\left(\mathscr{M}\left(\boldsymbol{\theta}^N, \left(\sigma_e^2\right)^N\right)\right) + \frac{\ln N}{2} d, \tag{19}$$

where the angle/time-dependent model likelihood $\ln \mathscr{L}\left(\mathscr{M}\left(\boldsymbol{\theta}^N, (\sigma_e^2)^N\right)\right)$ is defined in an analogous form to the previously presented in eq. (20) by

$$\ln \mathscr{L}\left(\mathscr{M}\left(\boldsymbol{\theta}^N, \left(\sigma_e^2\right)^N\right)\right) = -\frac{N}{2}\ln 2\pi - \frac{1}{2}\sum_{n=1}^{N}\left(\ln\left(\sigma_e^2(\phi_n)\right) + \frac{e^2(t_n)}{\sigma_e^2(\phi_n)}\right). \tag{41}$$

It is worth mentioning the procedure for model order selection adopted here is not optimal and does not provide the parsimonious model. Complex schemes based on integer optimization or additional steps for excessive dimension would provide more economic representations capable of modeling the process. The adopted approach is summarized as follows:

1. Data is truncated to an integer number of engine cycles and downsampled using a low-pass anti-aliasing filter (if necessary). Then, a preprocessing step is performed by data mean-value and trend removal;

2. Arbitrarily, a maximum order ($N_A$) is chosen, and a complete subspace with dimensions $p_a = p_c = p_s$ is defined;

3. Several AT-FS-ARMA$(n, n)_{[p_a,p_c,p_s]}$ candidate models are estimated for $n = 1, \ldots, N_A$. The chosen order $n_a = n$ provides the minimum BIC value. Then, an analogous procedure is repeated for candidates AT-FS-ARMA$(n_a, n_c)_{[p_a,p_c,p_s]}$ models for $n_c = 1, \ldots, n_a$) to select MA-part order. Again, the $n_c$ order returns the minimal BIC value. If it is not clear that BIC value is the minimum value, this procedure may be repeated assuming a new maximum order $N_A$ greater than the previously defined;

4. The steps above are repeated for other values of sampling frequency and subspace dimensions;

5. The chosen model exhibits lower BIC value among all models.

After the (supposedly) adequate orders are selected, the next step consists of evaluating the model. As previously described in Section 2.3.4 in this thesis, the following criteria are used to check the model for adequacy:

- The normalized ACF, computed over the angle/time-dependent standardized unit-variance $\epsilon(\phi_n)$ is verified, as

$$\epsilon(\phi_n) = \frac{e(t_n)}{\sigma_e(\phi_n)} \qquad \epsilon(\phi_n) \sim \text{NID}(0, 1), \tag{42}$$

  where the computed ACF should present an impulse-like form for a well-estimated (and, therefore valid) model.

- A metric based on the evaluation of the Residual Sum of Squares normalized by the Series Sum of Squares (RSS/SSS) value. An adequate model would present low RSS/SSS value.
- Finally, a hypothesis test is carried to check for the whiteness of the residue sequence (run test for randomness). If the residue is uncorrelated, the hypothesis test returns a test decision for the null hypothesis at a given confidence level.

Conclusively, if the model is considered valid, its usage for analysis is adequate. Otherwise, it is unable to capture the signal dynamics. If the model is not approved during the validation stage, other model candidates must be defined, and a new model structure must be selected. The following section provides a procedure for model-based analysis.

## 3.4 MODEL-BASED ANALYSIS

Once the AT-FS-ARMA$(n_a, n_c)_{[p_a,p_c,p_s]}$ model is defined and validated, the next step consists in proceed with the model-based analysis. For that, an angle-frequency map is estimated. This spectrum allows one to relate the vibration signals' energy content to the "angular-instants" in which they occur and can be useful for identifying vibration sources and relating them to kinematic events.

Thus, a simple way to estimate the angle-frequency map is running the model for a periodically-angle-varying inputs for given frequencies $f_k \geq 0$, in the form,

$$e(t_n, f_k) = \sigma_e(\phi_n) \exp^{j2\pi f_k t_n}, \tag{43}$$

where $\sigma_e(\phi_n)$ is the innovations standard-deviation of the identified model. Thus, new model outputs are estimated for all frequencies $f_{k's}$. For that, the input signal $e(t_n, f_k)$ from eq. (43) is introduced in eq. (31) jointly to the previous estimated model coefficients $a_i(\phi_n)$ and $c_i(\phi_n)$ and innovations variance $\sigma_e^2(\phi_n)$ to estimate each new output $x(t_n(\phi_n, f_k)$. Finally, the "frozen" angle/time-varying spectrum is obtained by,

$$S(\phi_n, f_k) = |x(t_n(\phi_n), f_k)|^2. \tag{44}$$

In Chapter 4, the angle-frequency map obtained from eq. (44) is discussed conjointly to its non-parametric counterpart.

## 3.5 PARTIAL REMARKS

The AT-FS-ARMA model was proposed for the identification of signals from rotating and reciprocating machines. The model mainly consists of an FS-based model, but which its coefficients are expanded on an angle/time-dependent trigonometric basis, which is angular-periodic. This basis consists of the angular-dependent Fourier series in its trigonometric form and is derived from a model for a formal description of AT-CS signals. The approach requires the vibration signal and additional information about the angular variable, which may be estimated using the IAS signal. A "relaxed" estimator based on the MS-WLS method was described, and the subproblem of a selection of model order was formulated. A simple (but non-optimal) procedure for model orders selection was presented, and two methods for evaluating and validating the model were presented. The first consists of the residue's runs test, and the second consists of the analysis of the unit-variance sequence. Finally, a procedure for model-based analysis assuming angular-periodic input was discussed to estimate the parametric "frozen-configuration" angle-frequency map from it.

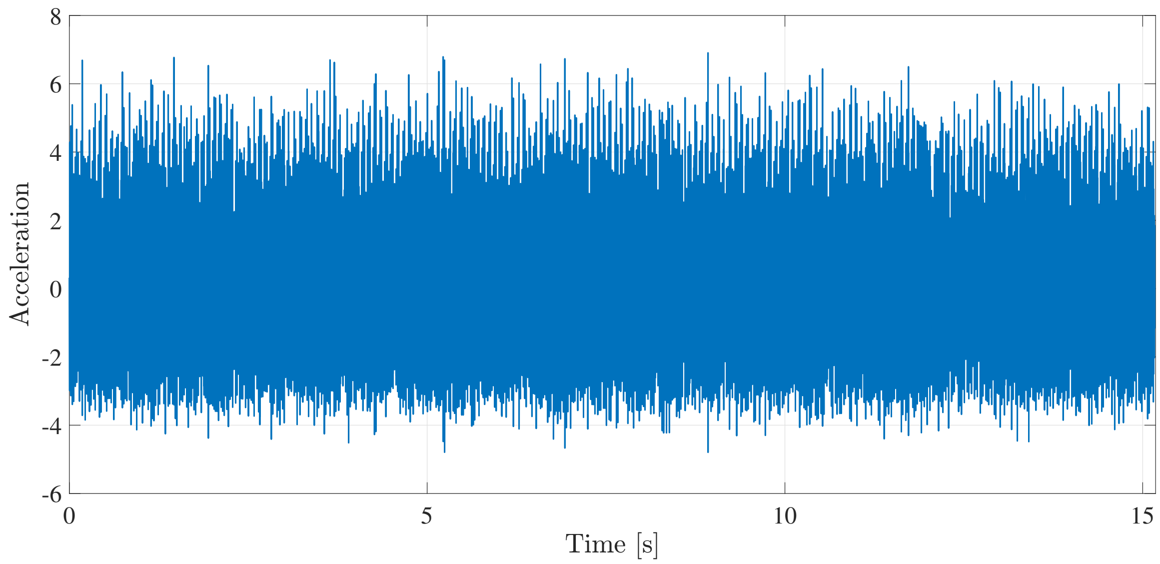
# 4 RESULTS AND DISCUSSION

The main results of this work are presented in this chapter. First, some essential characteristics of the experimental data of the diesel ICE are discussed in Section 4.1. Furthermore, the provided data is preprocessed and used for selecting the model order, and once the model structure is defined, the model is estimated and validated in Section 4.2. In Section 4.3, a parametric-based angle/time-frequency distribution is estimated and compared to its non-parametric counterpart, which is computed through smoothed-pseudo-Wigner Ville Distribution using the signal resampled to the angular-domain. Finally, the chapter's summary is presented in Section 4.4.

## 4.1 EXPERIMENTAL SIGNAL FROM A DIESEL IC ENGINE

The experimental data were measured from a four-stroke four-cylinder IC diesel engine from a passenger car. These data were provided to the author by Professor Jerome Antoni. The experiments have been carried out with the transmission at the neutral position. The time series (Figure 2) was saved at a constant operation speed regime using one accelerometer positioned on the engine block close to a cylinder. Additionally, an encoder provides a one-top-per-revolution signal. All data were acquired at the sampling frequency of $f_s = 20.48$ kHz.

Figure 2: The vibration response signal for the steady-state condition at 1500 RPM idle speed.




Source: prepared by the author.

A complete cycle of the 4-stroke engine has four stages: 1) admission, 2) compression, 3) ignition/combustion, and 4) expansion/exhaust. The dependence between the thermodynamic cycles and the crankshaft position suggests that the vibration signals are angle-periodic at every two turns of the crankshaft ($\Phi = 4\pi$), which contains four combustions. In other words, in a four-stroke engine, each stage corresponds to one stroke or half revolution of the crankshaft. One may notice this angular-period depends on the machine characteristics, e.g., for two-stroke engines, all four stages would occur during two strokes (or one crankshaft revolution) (RANDALL, 2011).

The sources of the vibrations in ICEs are mostly due to the movement of the moving parts, the tilting of pistons, the distributor and injection systems, and forces caused by the pressure variations inside the combustion chambers. The combination of all those vibratory contributions results in a complex signal to analyze. More details on IC diesel engines' vibration characteristics can be found in Chapter 3 of the thesis of Antoni (2000) or in the recent research paper Delvecchio, Bonfiglio and Pompoli (2018) in which the vibroacoustic mechanisms that produce sound and vibration in ICEs are discussed in details.

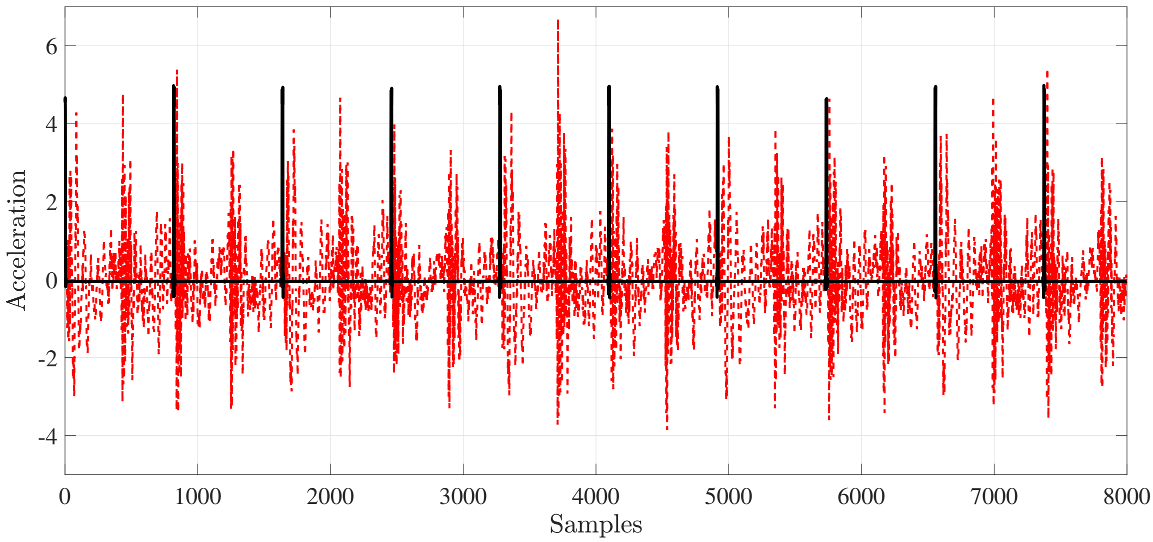
The counting of the tachometer pulses and the instantaneous angular speed were estimated based on the encoder signal. Then, the vibration signal was initially preprocessed to include an integer and even number of the thermodynamic cycles by windowing. The angular displacement of the crankshaft is approximated through a discrete-time integral of the IAS signal. The measured vibration response signal initially has a duration of about 15 seconds and is showed in Figure 2. This signal structure presents high amplitude peaks produced by the explosions, followed by sharp transients until the energy produced by the impacts dissipates through the engine block.

A zoom of a segment of the vibration signal is shown in Figure 3 where the black vertical lines illustrate the tachometer pulses to highlight the explicit dependence between the vibration signal and the engine's cycles. Additionally, Figure 4 illustrates the relationship between the crankshaft angular position and the tachometer pulses.

Figures 5a and 5b show the extracted IAS signal and the estimated angular displacement $\phi(t_n)$ estimated through discrete-time cumulative sum.

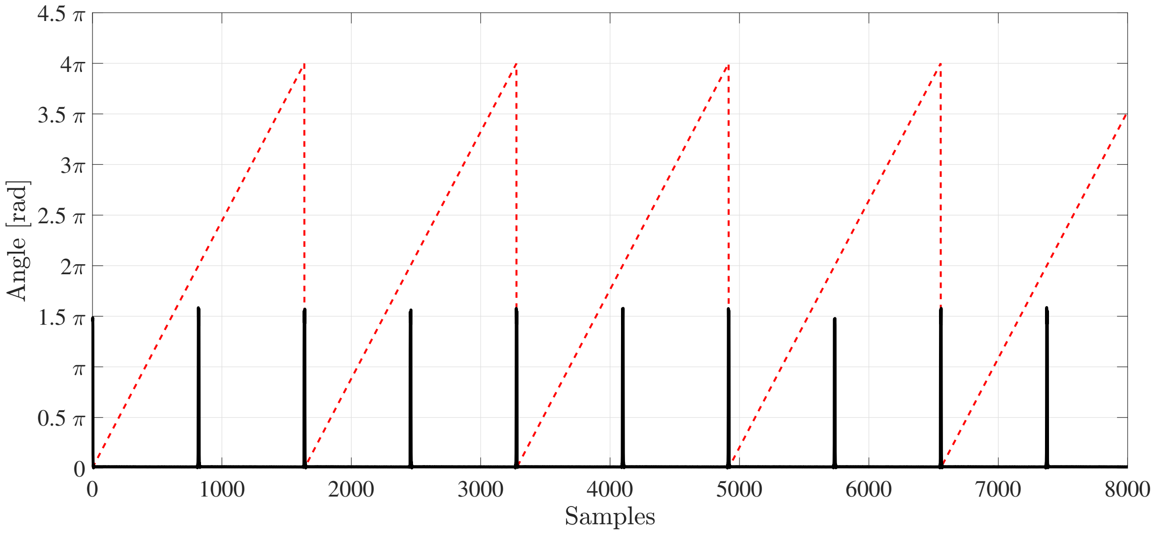
A schematic overview of the procedures involved in model-based analysis is showed in Figure 6. In this flowchart, all the stages involved in the model identification are represented, starting in the preprocessing phase, the model identification and validation procedures, until finally, the model-based analysis.

Figure 3: Segment of the vibration response signal (- -) and its relation with the tachometer signal (–) for the steady-state condition at 1500 RPM idle speed.



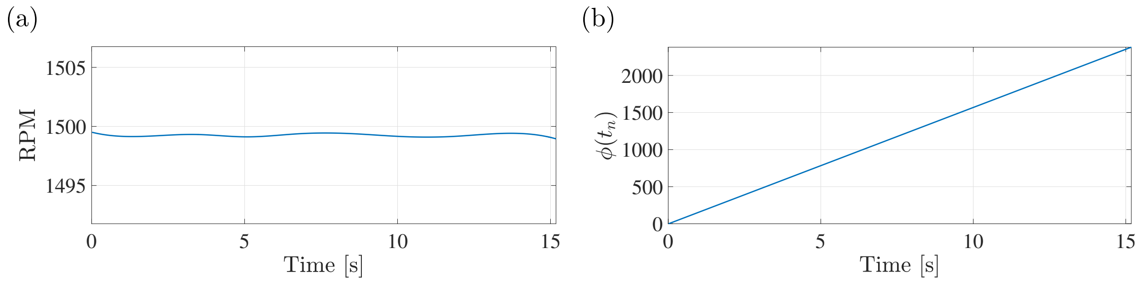

Source: prepared by the author.

Figure 4: Segment of the crankshaft position signal (mapped from 0 to $4\pi$) (- -) and its relation with the tachometer signal (–) for the steady-state condition at 1500 RPM idle speed.



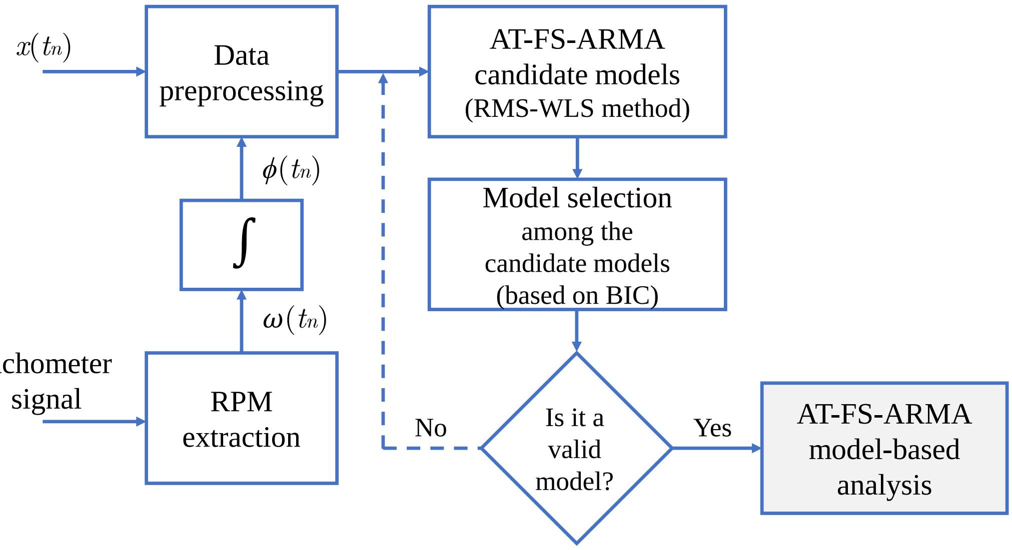

Source: prepared by the author.

Figure 5: Engine's (a) IAS extracted using the encoder signal, and (b) crankshaft angular displacement.




Source: prepared by the author.

Figure 6: Schematic overview of data preprocessing, and the AT-FS-ARMA identification and validation procedures.




Source: prepared by the author.

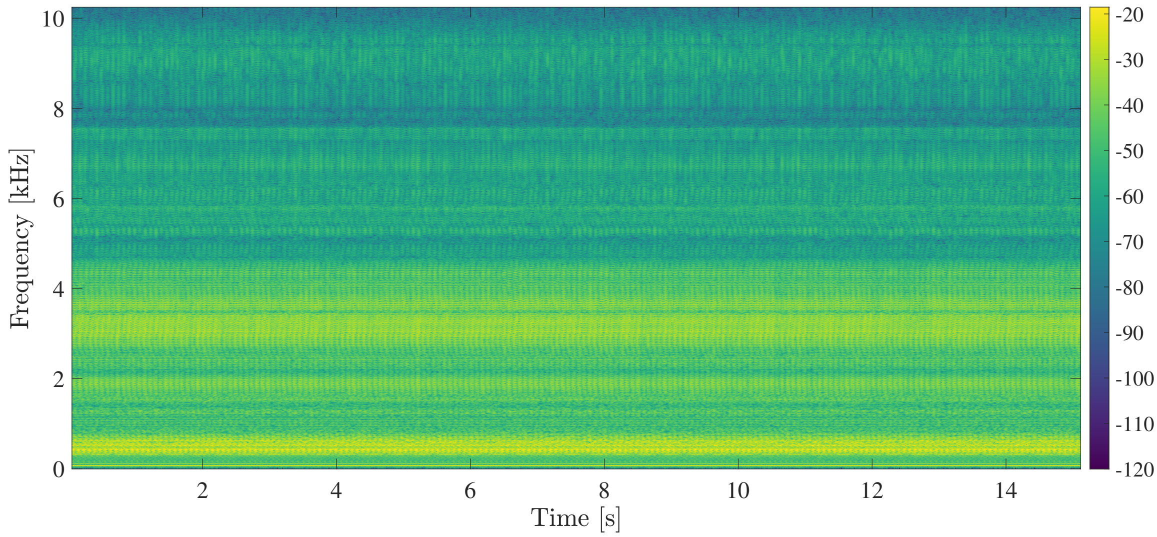
## 4.2 MODEL IDENTIFICATION AND VALIDATION

First, a non-parametric spectrogram of the vibration signal was estimated for an initial analysis. Figure 7 corresponds to the non-parametric PSD obtained through STFT using a Gaussian moving-window with 2048 data-points with 200 advance samples and 2048 DFT points. In the PSD, the darker the color, the lower the energy content at this frequency range, as indicated by the color bar. Also, from Fig. 7, the energy of the signal decreases considerably for frequencies above 4.5 kHz. This information may help determine if the signals can be down-sampled before using the parametric identification method.

After the range of frequency with high energy content is established, the next step is to select the model structure. This procedure requires the selection of the model orders and basis dimensions for the AT-FS-ARMA$(n_a, n_c)_{[p_a,p_c,p_s]}$ based on some selection rule criterion. The Bayesian criterion (using the BIC value as a metric) is used, as previously described in Section 3.3, and several conditions are then examined varying the data-length and down-sampling the signals. This is necessary because the original sampling frequency is too high, and the data length is too long. Complete and high-dimensional subspaces are defined to check the sensibility of varying those dimensions.

As aforementioned, the original signal length is too long, and taking into account that it presents periodicity, only a segment of the vibration signal is selected for analysis. This may be achieved by dividing the signal into $M$ segments. This selected $M$-th segment was then truncated to include an integer number of thermodynamic cycles. Therefore, a

Figure 7: Two-dimensional non-parametric PSD obtained via STFT, using a Gaussian moving-window with 2048 data-points with 200 advance samples and 2048 DFT points. The darker the color, the lower the energy content.




Source: prepared by the author.

new number of $N$ samples are obtained. The signal is also resampled into a new sampling frequency $f_{sn}$ using a poly-phase anti-aliasing filter. Again, Figure 7 demonstrates that the high energy content of the signal occurs below 4.5 kHz, so the new sampling frequency was established as at least $f_{sn} \geq 10$ kHz to include the information below at least 5 kHz and satisfy the Shannon sampling theorem.

The structure of the model is chosen through the selection of the orders $n_a$ and $n_c$ and the dimension of the functional subspaces $p_a$ and $p_c$. In this thesis, the selection is based on the BIC criterion (POULIMENOS; FASSOIS, 2006) following the procedure previously described in Section 3.3. Thus, several AT-FS-ARMA($n$,$n$) candidate models are estimated for $n = 1, \ldots, 30$ (extended to 50 for situations where the initial evaluation was considered inconclusive). The selected AR-part order $n_a$ is the one that minimizes the BIC value. Then, the same procedure is used to optimize the MA-part order $n_c$, but now considering $n_c = 1, \ldots, n_a$, where the selected $n_c$ order will again be the one that minimizes the BIC. The above procedure is repeated for different signal lengths $N_{n's}$ and sampling frequency $f_{sn}$. All the basis were adopted as containing the same dimension $p = p_a = p_c = p_s$.

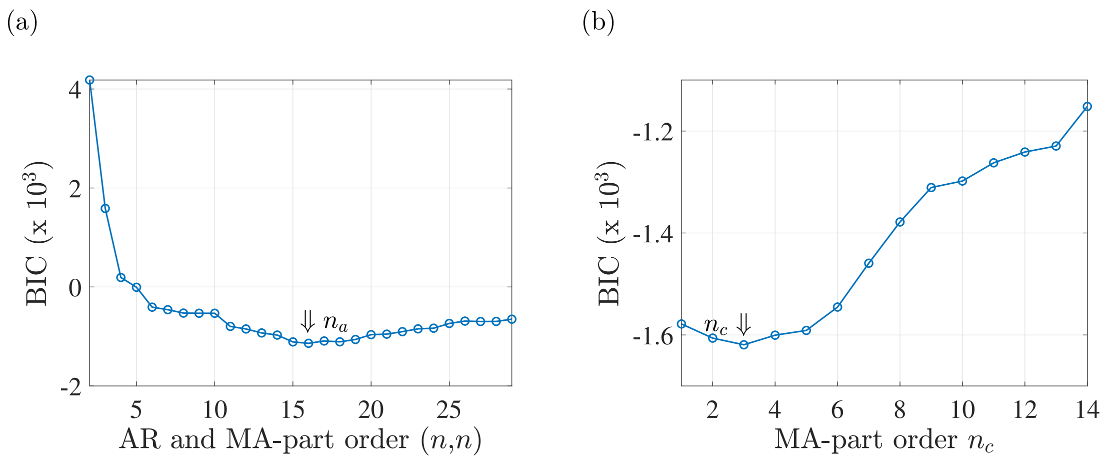
Table 1 provides the obtained results, where $d_p = (n_a p_a + n_c p_c)$ indicates the number of invariant parameters that describe the resultant model. This table shows that the combination of parameters from Test #6 provided the lowest BIC value. Therefore, the AT-FS-ARMA$(16, 3)_{[25,25,25]}$ model was selected as the one that would best represent the signal. For Test #6, the sampling frequency is reduced to 3/4 of its original value, and the resulting signal contains 14605 samples and is approximately 1/15 part of the original duration (which includes 24 complete cycles of the engine). The BIC values obtained for the AT-FS-ARMA$(n,n)_{[25,25,25]}$ candidate models from Test # 6 are presented at Figure 8.

Table 1: AT-FS-ARMA model identification: BIC minimum values obtained for each candidate model.

| Model | $f_{sn}$ [Hz] | $N_n$ [samples] | $p$ | $n_a$ | $n_c$ | $d_p$ | min(BIC value ($\times 10^3$)) |
|---|---|---|---|---|---|---|---|
| # 1 | 10240 | 9737 | 25 | 13 | 1 | 350 | 3.5231 |
| # 2 | 10240 | 14651 | 25 | 26 | 7 | 825 | 3.2290 |
| # 3 | 10240 | 9737 | 15 | 25 | 8 | 495 | 3.3879 |
| # 4 | 10240 | 9737 | 50 | 12 | 1 | 650 | 4.2239 |
| # 5 | 15360 | 14605 | 15 | 18 | 6 | 345 | -0.5253 |
| # 6 | 15360 | 14605 | 25 | 16 | 3 | 475 | -1.1384 |
| # 7 | 10240 | 9737 | 15 | 18 | 6 | 525 | 3.7280 |

Source: prepared by the author.

Figure 8: AT-FS-ARMA model identification: (a) BIC values of AT-FS-ARMA$(n,n)_{[25,25,25]}$ models ($n = 2,\ldots,30$) for AR-part order selection. (b) BIC values of AT-FS-ARMA$(16,n_c)_{[25,25,25]}$ ($n_c = 1,\ldots,14$) for MA-part order selection.

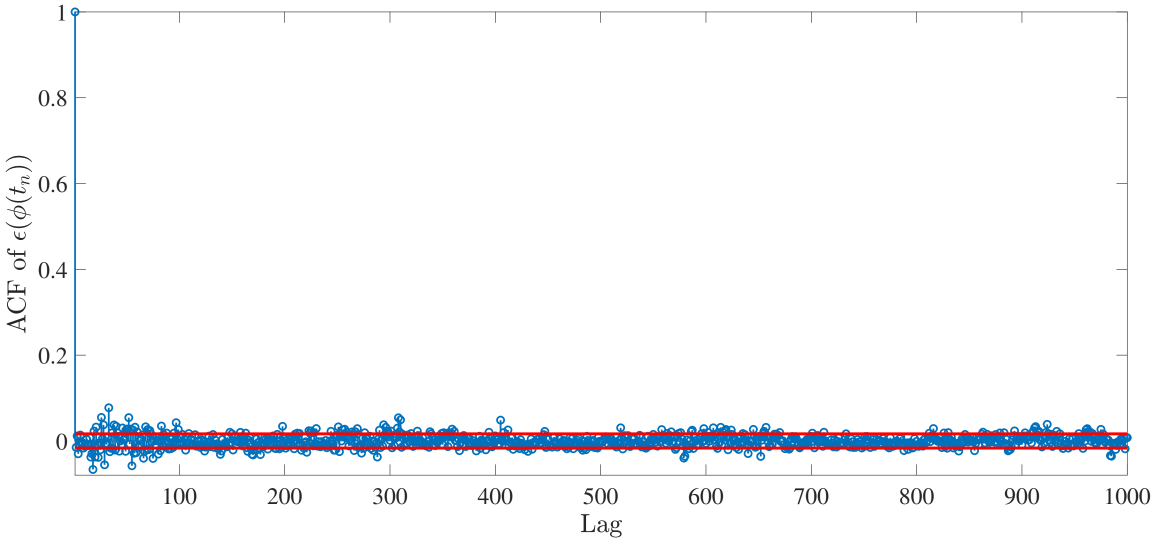



Source: prepared by the author.

Once the model is estimated, it must be validated. The model's acceptance depends on its performance under procedures that check it for an example for Gaussianity and the innovations sequence whiteness by one or more procedures. The procedures used here are also described in Section 3.3.

Thus, the normalized ACF computed over the normalized variance $\epsilon(\phi_n)$ is showed in Figure 9. The impulse-like signature for the ACF indicates the uncorrelatedness of the normalized innovations sequence and, therefore, an overall adequate model fitting. Another common procedure for is the residual sign test (run test for randomness) (STRAUME; JOHNSON, 1992; POULIMENOS; FASSOIS, 2006) in which the AT-FS-ARMA$(16,3)_{[25,25,25]}$ innovations sequence is tested for 95% confidence level and attests the hypothesis that the uncorrelated residual series is valid.

Figure 9: AT-FS-ARMA model identification: Normalized Auto-Correlation Function (ACF) of the residual error of AT-FS-ARMA$(16, 3)_{[25,25,25]}$ for 95% confidence level computed for a number of 1000 lags.

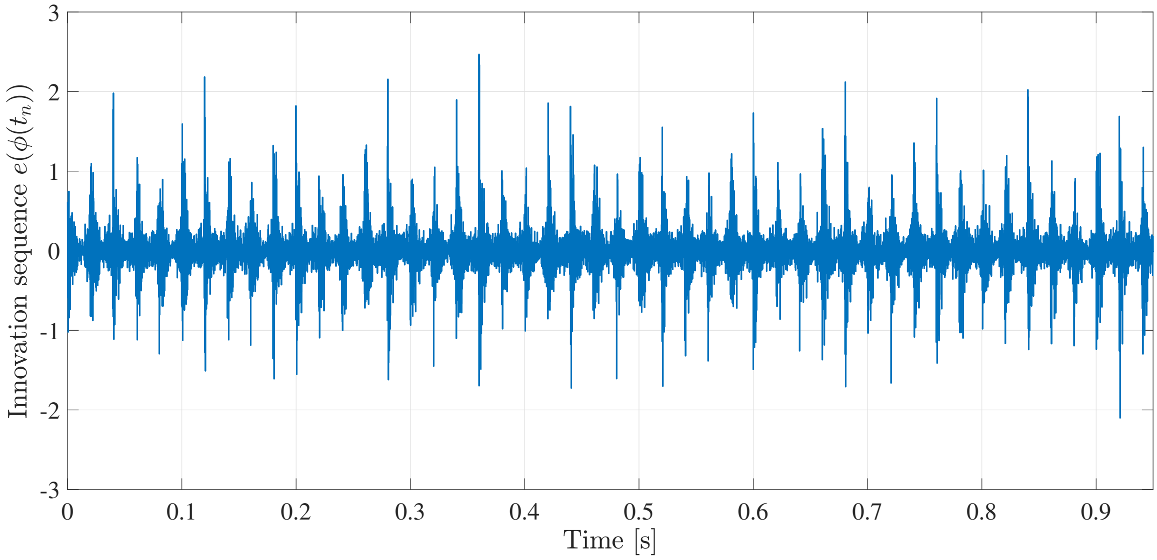



Source: prepared by the author.

The comparison between the vibration response and the AT-FS-ARMA$(16, 3)_{[25,25,25]}$ model output is presented in terms of the residual error sequence in Figure 10. The estimated angle/time-dependent innovation variance is in Figure 11 and shows periodic behavior, as expected. The vibration response and model prediction in Figure 12 were adequate in accordance.

The obtained value for the RSS/SSS is approximately 5.96%, which indicates a high modeling accuracy. For the effect of comparison, the same signal was modeled using a traditional FS-TARMA (with a time-based basis) with precisely the same orders and basic dimensions, and the RSS/SSS obtained was equal to 11.30%. This does not exclude, however, the possibility that an even more parsimonious FS-TARMA may be identified with RSS/SSS lower than that obtained here. As expected, the estimated model's success depends on the tachometer signal quality or, more generally, on the angular displacement estimation. Thus, low tachometer signal features may result in problems in selecting the orders or compromise the model's statistical characteristics. Some preliminary tests, not included in this work, were affected by a weak tachometer signal. Due to missing tachometer pulses at the beginning of the acquisition, the first 1000 samples of the raw vibration signal were dropped before the preprocessing phase.

Figure 10: AT-FS-ARMA model identification: The residual error sequence $e(\phi(t_n))$.




Source: prepared by the author.

Figure 11: AT-FS-ARMA model identification: the angle/time-dependent innovations variance $\sigma_e^2(\phi(t_n))$.

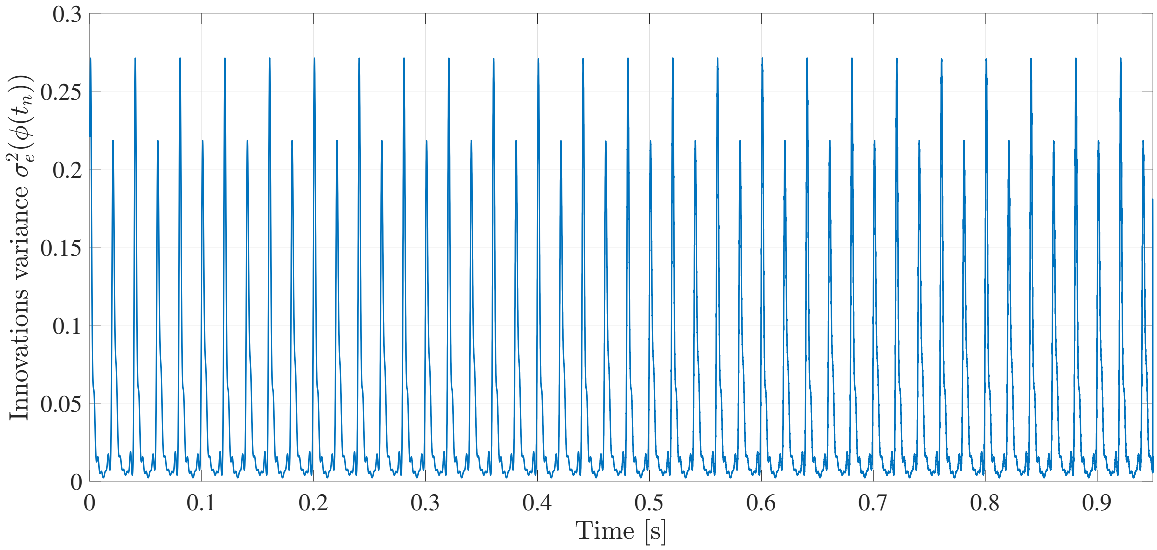



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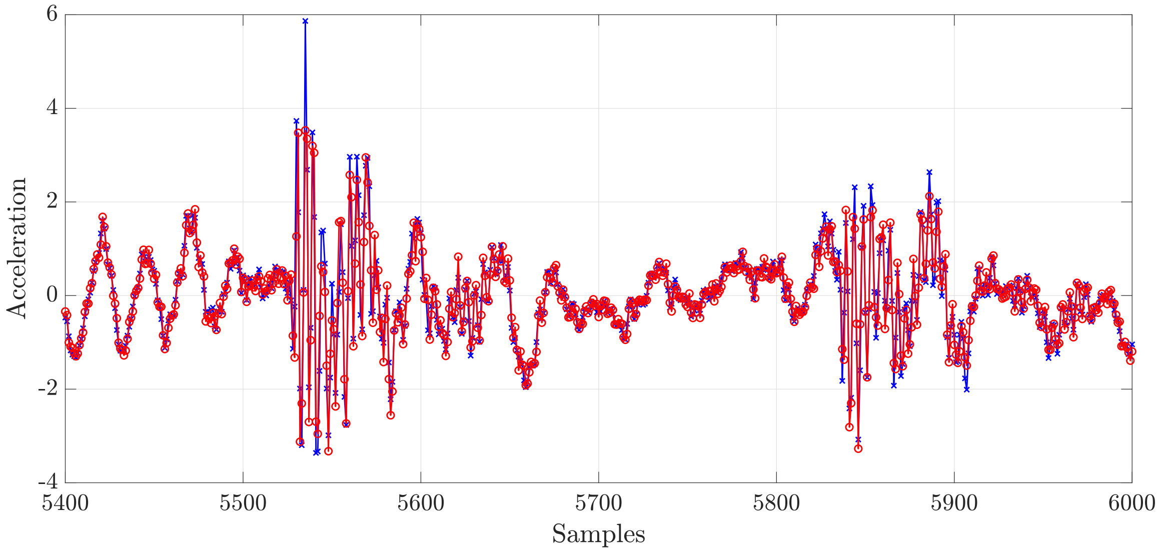
Figure 12: AT-FS-ARMA identification: segment of the vibration response (—x—) and AT-FS-ARMA$(16, 3)_{[25,25,25]}$ prediction (—o—).

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## 4.3 ICE VIBRATION ANALYSIS THROUGH ANGLE/TIME-FREQUENCY DISTRIBUTION

A qualitative comparison between parametric and non-parametric angle-frequency distribution is performed to discuss the potential of the AT-FS-ARMA for vibration analysis. It is worth to mention that the intention here is to obtain similar information from two different ways (parametric and non-parametric) and not to oppose both strategies. The smoothed-pseudo-WVD is chosen for the non-parametric time/angle-frequency representation since it is often used to analyze ICEs signals.

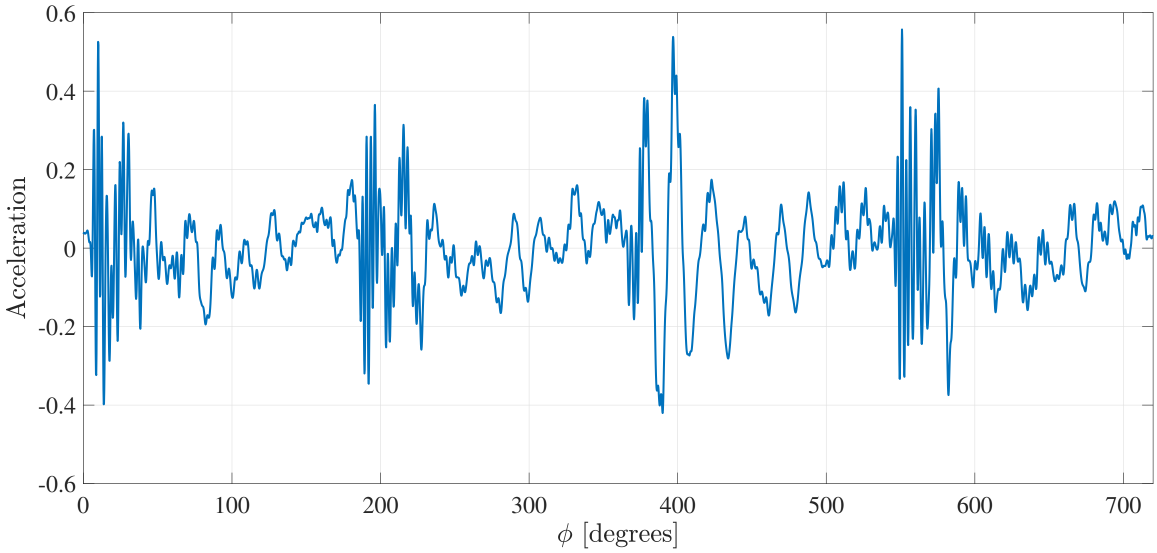
A simple way to obtain an angular-averaged SPWVD-like would be considering the operational speed as perfectly constant and assuming $\phi_n = \bar{\omega} t_n$ where $\bar{\omega}$ is the mean value of the crankshaft angular speed. Hence, the SPWVD would be computed using the times-series $x(t_n)$ and simply plotting its result as a function of the angle instead of time. Nevertheless, in this thesis a most general procedure is performed as the following: the signal $x(t_n)$ is angular resampled to obtain $x(\phi)$[1], then this signal is an angular-averaged to obtain $\bar{x}(\phi_n)$ and the SPWVD$(\phi, f)$ is computed from it. This procedure provides more reliable results even under speed fluctuations.

In this work, the angular resampling technique is used using the raw vibration signal on its original sampling frequency and length. This procedure is performed using linear interpolation accordingly to previously described in eq. (6). Moreover, the angular-synchronous average signal is computed over this angular-uniformly spaced signal. The

[1]For the sake of simplicity $x(\phi) = x(\phi_{\Delta\phi})$ is defined in terms of equally-spaced samples in angle. Again, $\phi_n = \phi(t_n)$ denotes angle in function of time.

angular-synchronous averaged signal is calculated directly from the angular-domain signal and is exhibited in Figure 13.

Figure 13: Angular-synchronous averaged signal $\bar{x}(\phi)$ computed over 189 cycles of the signal resampled in the angular-domain.



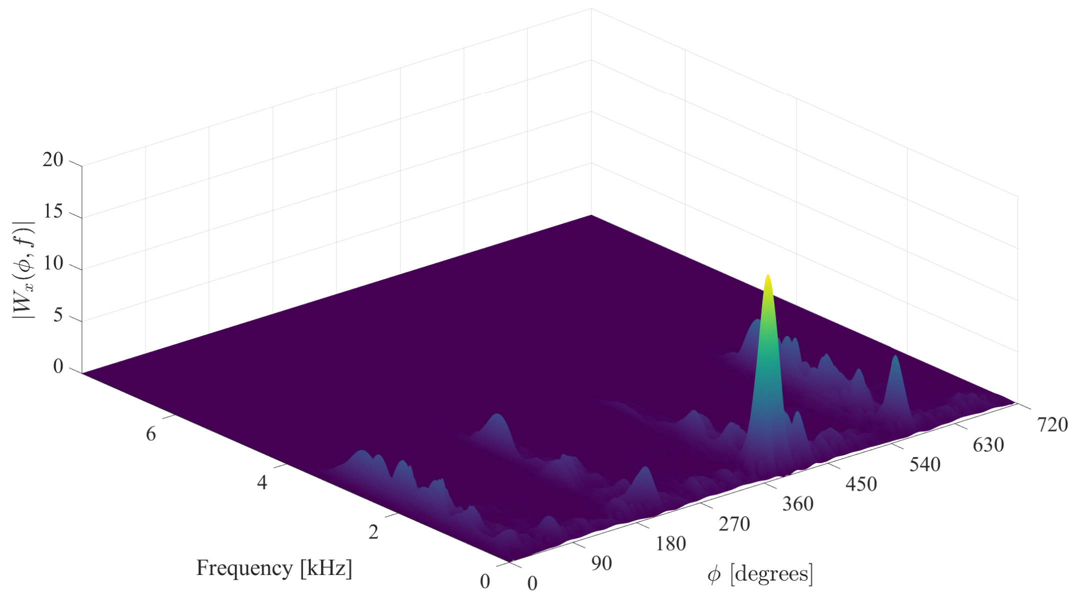

Source: prepared by the author.

Figure 14 and 15 show the obtained smoothed-pseudo-WV distribution and spectrum computed on the angular-synchronous averaged signal over 189 cycles and calculated using Eq. (12). In those figures, the frequency-axis is limited to 7680 Hz for a direct comparison with the parametric spectra that will be presented below. From the Figure 14, it is possible to notice the highest peak in amplitude related to the combustion phenomena in the cylinder, closer to the accelerometer's position. The obtained spectrum is represented on the logarithm scale in Figure 15, the succession of four combustion events that excite the system over a wide frequency range can be observed. Also, it is noticeable that a stationary component around 570 Hz may be related to the vehicle's gearbox. Finally, transients are detected around 3 kHz after each of the combustion events. Those transients may be related to the openings and closures of the inlet and exhaust valves.

The "frozen" configuration angle/time-frequency distribution is obtained running a realizations of the AT-FS-ARMA$(16,3)_{[25,25,25]}$ model for angle-periodic inputs. Hence, the previously estimate trajectories $a_i(\phi_n)$, $c_i(\phi_n)$ and innovations variance $\sigma_e^2(\phi_n)$ are used to estimate the new output assuming a sinusoidal innovations sequence, modulated by an angle-periodic amplitude $\sigma_e(\phi_n) = \sigma_e(\phi_n + \Phi)$, accordingly to eq. (43). Then the parametric spectrum is computed from eq. (44). Figure 16 shows the obtained parametric "frozen" configuration angle/time-frequency spectrum for the first four cycles of the analyzed signal.

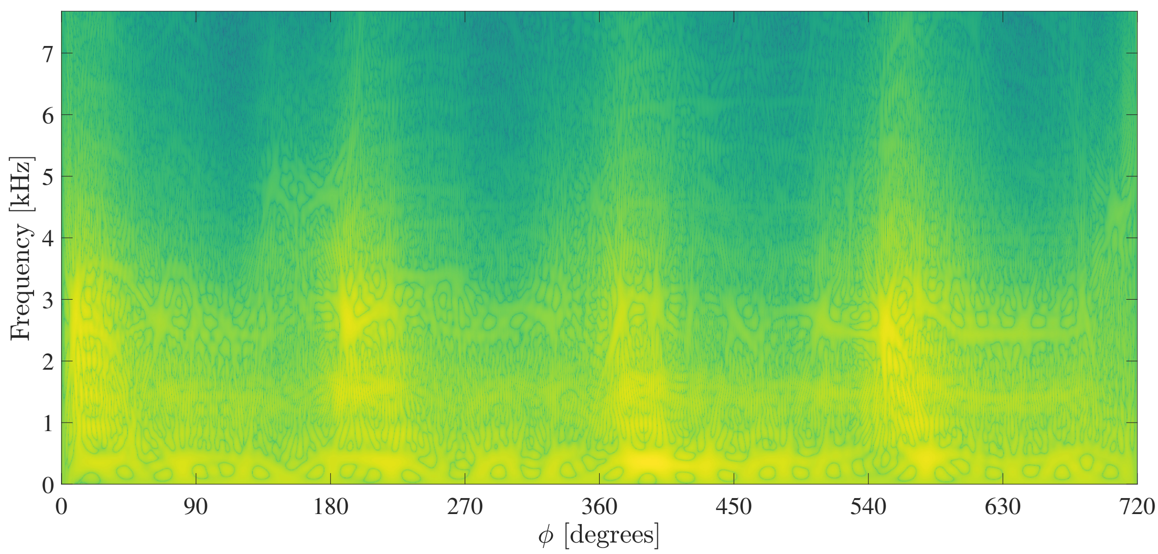
Figure 17 shows the angle-frequency distribution obtained from the parametric model.

Figure 14: Averaged smoothed-pseudo-WVD $|\text{SPWVD}(\phi, f)|$ computed over the signal resampled in the angular-domain synchronously averaged on 189 cycles.



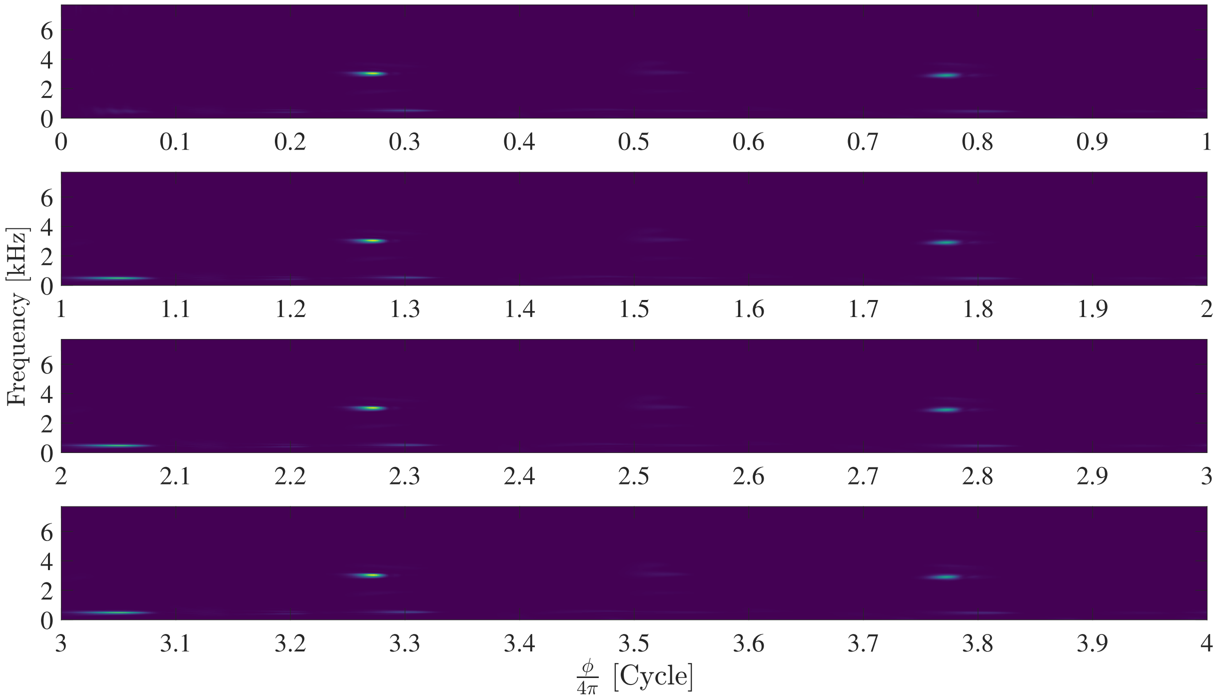

Source: prepared by the author.

Figure 15: Averaged smoothed-pseudo-WVS $\log_{10}(\text{SPWVD}(\phi, f))$ computed over the signal resampled in the angular-domain synchronously averaged on 189 cycles.




Source: prepared by the author.

Figure 16: "Frozen" angle-frequency distribution of the first 4 cycles (the horizontal axis is the angle-position normalized by the period of the engine cycle).



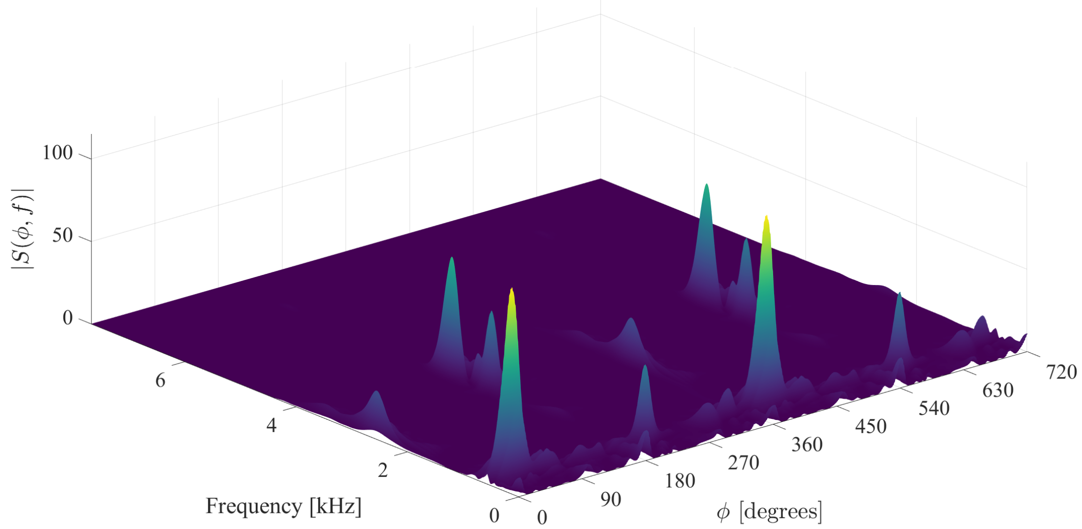

Source: prepared by the author.

It is possible to notice that this figure seems "periodic" than the one obtained by the SPWVD (Figure 14).

Figure 18 presents the "frozen" angle-frequency map. In comparison to Figure 15, the parametric spectrum in Figure 18 seems smoother along the angle-axis. In Figure 18, the events related to the engine cycle are evidenced, where the combustion events and the transients, probably due to the openings/closures of the valves are evidenced by dashed black-colored vertical lines, while the horizontal red signalizes the stationary component around 570 Hz dashed lines. Gears may produce this stationary component.

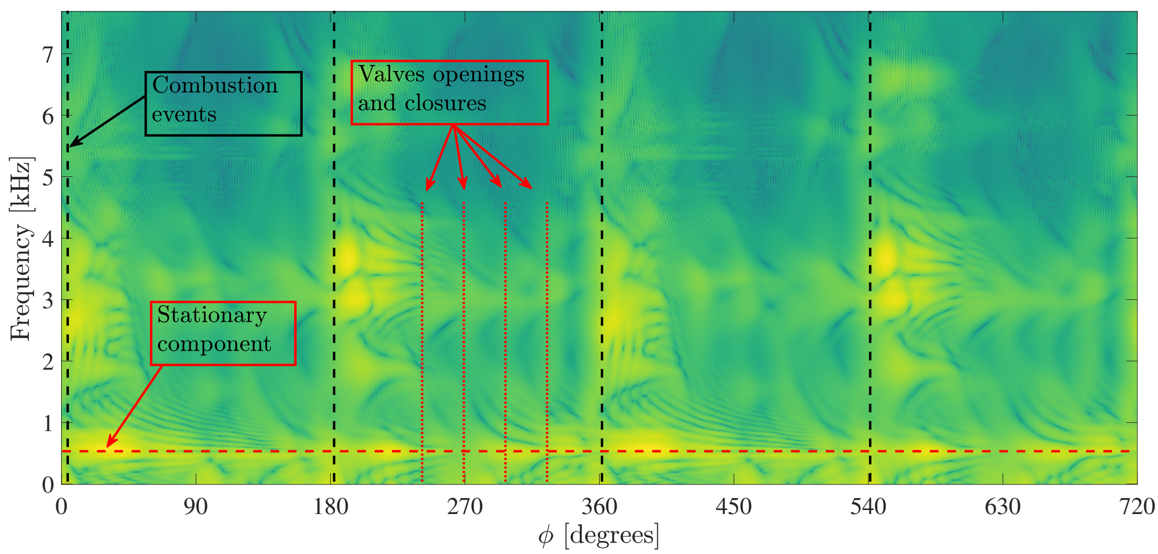
One advantage of the parametric spectra is that they are free from cross-terms. Hence, the parametric spectrum being smoother than the non-parametric one, the kinematic related events are highlighted, which suggest that the technique can be useful as a helpful complementary tool for identifying and characterizing vibrations of rotating and reciprocating machines.

Figure 17: "Frozen" angle-frequency distribution for the 5-th engine cycle. The angular scale is mapped between 0 and 720°.




Source: prepared by the author.

Figure 18: "Frozen" log-scaled angle-frequency spectrum for the 5-th engine cycle. The angular scale is mapped 0 and 720°. Events related to the engine's kinematics are indicated in the figure.




Source: prepared by the author.

## 4.4 PARTIAL REMARKS

This chapter illustrated the parametric AT-FS-ARMA model application for the analysis of experimental data from an IC diesel engine. First, in Section 4.1, some aspects of the experimental data from an IC diesel were briefly discussed. The dataset is composed of two signals: 1) the vibration signal, measured by an accelerometer mounted in the engine's block; 2) an encoder/tachometer signal provides a one-top-per-revolution indication, used to reconstruct the instantaneous angular speed. Data were acquired from a passenger car at idle speed. First, a preliminary analysis of the data was performed using a non-parametric technique to verify the corresponding frequency value's energy content. For that, a non-parametric PSD was estimated through the STFT, and it was possible to observe that the higher energy content was mainly below 4.5 kHz. This information is useful to determine if it is possible to resample the data before the parametric identification procedure, which may be desirable due to the high computational memory cost to compute the data in its original length and sampling frequency.

Thus, in Section 4.2, the data was pre-processed to reduce its sample frequency and its length, but including an integer number of cycles of the engine and the high energy content, in lower frequency (below 5 kHz) was preserved. Several candidate models were estimated, and the AR/MA part orders were selected using the Bayesian Information Criterion (BIC). It is worth mentioning that the model order selection procedure presents a high computational cost since it requires several candidate models to be estimated, and some of them present high order values. After that, the vibration-based signal was modeled using the AT-FS-ARMA parametric method. The estimated model is validated by evaluating the normalized ACF of the standardized innovations sequence and the residual sign test, which indicated an overall satisfactory model fitting.

In Section 4.3, an angle-frequency distribution was estimated simulating the model for the desired frequency range. An angle-frequency distribution was also computed using the smoothed-pseudo WVD, computed over the angular synchronous averaged signal, for the effects of comparison. An advantage of the parametric technique is that the parametric angle-frequency map does not contain cross-terms. It can also present high resolution according to the number of terms of the angular-based Fourier series expansion. This may provide some versatility in the analysis of ICEs signals.

However, the reliability of the AT-FS-TARMA model estimate is considered sensitive to the tachometer signal quality. This may restrict the applications in scenarios where the tachometer data is not sufficiently accurate. Some procedures may reduce this issue, as the IAS reconstruction directly from the vibration signals, but this was not the concern in this thesis. Reliable methods for IAS obtention from vibration signals are still in the literature as the current investigation topic. Another drawback is the procedure of selecting model structure/orders, which consists basically of a trial-by-error strategy and demands high computational cost and time. An automated and standardized procedure for model order selection would be desirable for industrial applications.

Finally, the results showed the potential of the proposed AT-FS-ARMA model to analyze rotating and reciprocating machines. The obtained angle-frequency map highlights the kinematic related events, which helps identify vibration sources in ICEs.

# 5 FINAL REMARKS

In this chapter, the final remarks of this thesis are summarized in Section 5.1. Additionally, some perspectives and suggestions for future works are carried out in Section 5.2.

## 5.1 CONCLUSIONS

Time-varying parametric models have been a topic of research in recent decades. Some of those methods are DPE-based, where the models' parameters are expanded in terms of projection coefficients that vary respectively to a deterministic functional subspace (FS), which are chosen a priori accordingly to nature of the signal. Those models are popularly known as FS-ARMA. Over the years, the FS-TARMA models have been successfully applied to vibration analysis, and the Fourier-based (or trigonometric) basis is often used to model vibration signals. In the vibration analysis field, this FS's evolution is often described in terms of the time variable. However, their applications in rotating and reciprocating machinery are still not typical. Those machines exhibit time-dependent dynamics while their signals have angular-periodic data. Additionally, an (jointly-)angle/time class of models has been proposed over recent years. Those models gave rise to the angle-time-cyclostationary approach, an extension of the cyclostationary framework. This framework has provided useful analytical models and tools for rotating machine analysis. In this thesis, a novel data-driven angle/time-varying FS-ARMA model is proposed.

The proposed method attempts to couple the advantages of parametric methods of identification with concepts from the angle/time-cyclostationary class of signals. The formulation was based on the well-known FS-TARMA models, but the expansion basis is set in terms of a given reference shaft's angular displacement. The signals are described in terms of a jointly angle-time form, in which the periodic modulations are characterized in terms of the angular variable while the carrier waveform is preserved in its temporal description. This is achieved by modeling the signal output as time-domain signals but imposing an angular-dependent evolution on the model coefficients.

The method was illustrated using experimental data from a diesel ICE operating at a constant operational speed. The method demands both vibration signal and angular position information. The angular information was provided by a tachometer from which the IAS was extracted. Then this IAS signal was used to compute the angular displacement. An RMS-WLS estimator was used to compute the models, and its structure was chosen using the BIC and a trial-and-error procedure. The model was validated through the evaluation by evaluating the normalized ACF, residual sign test, and the RSS/SSS evaluation. For industry applications, defining the model structure and selection may be viewed as a drawback since it may present a high computational cost and demand time and some experience. The development of an automated procedure for model orders selection would improve the applicability of this technique.

Once the model structure is defined, the analysis was performed computing parametric angle-frequency spectra and distributions and compared them to the obtained through smoothed-pseudo-WVD. For that, the data was preprocessed to include an integer number of the engine's cycle (using only a small segment of the original signal, to reduce computational cost) and filtered to a new sampling frequency. The angle-frequency map was obtained simulating the model over the desired frequency range.

Finally, The SPWVD was estimated using the angular-synchronous averaged signal. Both parametric and non-parametric distributions presented similar information, yet the non-parametric angle-frequency map presented a better resolution, the parametric-based was smoother and allowed ones to distinguish kinematic-related phenomena. Therefore, the proposed AT-FS-ARMA model may be used as a complementary tool for analyzing reciprocating and rotating machines. However, further investigations are required to assure the proposed model capacity for signal analysis under varying speed profiles but in which the angle/time-cyclostationarity is preserved.

## 5.2 SUGGESTIONS FOR FUTURE WORKS

In this section, some suggestions for future works are enumerated. Those proposals may be conducted to investigate the angle/time-varying models' deeper aspects and applications or provide optimized solutions for some identification sub-problems. The proposed ideas are the following:

- Reproduce the procedure for signal analysis of data from which the technical information, i.e., the theoretical angular-instants of kinematic phenomena, are known. Thus, a more reliable comparison would assure the AT-FS-ARMA model capacity concerning signal content to the angular-instants in which they occur.

- Investigate deeper the relation between the angular-dependent Fourier series coefficients and the harmonics/orders that it represents. It would be useful to establish an angular-dependent series capable of tracking non-integer orders.

- The development of an optimal model order selection procedure based on optimization algorithms. That would allow one to estimate more parsimonious models and establish a more reliable methodology for model order and structure selection.

- Apply the methodology for others with lower signal-to-noise levels, as acoustic signals from ICEs.
- Check the model performance for modeling signals from machines under other speed profiles inside the angle/time-cyclostationarity assumption, e.g.., linear increase speed profile, or operating under small-to-medium speed fluctuations.
- Provide a model benchmark for damage detection in ICEs for different types of faults (e.g.., misfire, knocking, injection advance/delay).
- Check the method's potential of the method under broader applications involving rotating and reciprocating machines from other nature. For, for instance, for condition monitoring of compressors, gearboxes, or rolling element bearings signals. In this latter, model capacity in working with high -frequency data may be explored.

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