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Áxions, Májorons e Neutrinos em Extensões do Modelo Padrão

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DEDICATION

Ao Charme da minha vida,
Flávia Sobreira Sánchez.

"I seem to have been only like a small boy playing on the sea-shore, diverting myself in now and then finding a smoother pebble or a prettier shell than the ordinary, whilst the great ocean of truth lay all undiscovered before me"

Sir Isaac Newton.

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RESUMO

Nesta tese, partículas tais como áxions, Májorons e neutrinos são consideradas em duas extensões eletrofracas do modelo padrão da física de partículas. Especificamente, os modelos considerados estão baseados nas simetrias de gauge $SU(3)_L \otimes U(1)_X$ e $SU(2)_L \otimes U(1)_{Y'} \otimes U(1)_{B-L}$. Primeiramente, no contexto do modelo 3-3-1 com um sector escalar mínimo é realizado um estudo detalhado referente à implementação da simetria de Peccei-Quinn (PQ) para resolver o problema CP forte. Para a versão original do modelo, que possui apenas dois tripletos escalares, é mostrado que a Lagrangiana total é invariante sobre uma simetria PQ. No entanto, o áxion não é produzido porque um sub-grupo permanece sem quebrar. Embora, neste caso, o problema CP forte possa ser resolvido, a solução é amplamente desfavorecida porque três quarks não têm massa em todas as ordens da teoria de perturbação. A adição de um terceiro tripleto escalar resolve o problema dos quarks sem massa, mas o áxion que aparece é visível. Para fazer o modelo realístico teremos que modificá-lo. É mostrado que a adição de um singleto escalar junto com uma simetria de gauge discreta Z_N é capaz de levar a cabo esta tarefa e proteger o áxion de efeitos da gravidade quântica. Para ter segurança que a simetria de gauge discreta que protege o áxion é livre de anomalias, é usada uma versão discreta do mecanismo de Green-Schwarz.

A seguir, é considerado um modelo eletrofraco baseado na simetria de gauge $SU(2)_L \otimes U(1)_{Y'} \otimes U(1)_{B-L}$, no qual temos neutrinos de mão direita com números quânticos exóticos e diferentes. Devido a esta particular caraterística, é possível termos de massa e de Yukawa para os neutrinos, com campos escalares que podem adquirir valores esperados do vácuo (VEVs) pertencendo a escalas de energia diferentes. É feito um estudo detalhado dos setores dos escalares e dos neutrinos para mostrar que o modelo é compatível simultaneamente com as escalas de massa e a matriz de mistura tribimaximal que são inferidas dos dados de neutrinos solares e atmosféricos. Também, é mostrado que o modelo poderia possuir candidatos à matéria escura se uma simetria Z_2 é incluída.

Finalmente, é discutido uma extensão supersimétrica $N = 1$ do modelo $B - L$ com três neutrinos de mão-direita.

Palavras Chaves: Áxions, Májorons, neutrinos, modelos 3-3-1, modelo $B - L$, o problema CP forte.

Áreas do conhecimento: Física de Partículas.

ABSTRACT

In this doctoral thesis axions, Majorons and neutrinos are considered into different electroweak extensions of the standard model of the particle physics. Specifically, the two models considered are based on the $SU(3)_L \otimes U(1)_X$ and $SU(2)_L \otimes U(1)_{Y'} \otimes U(1)_{B-L}$ gauge symmetries. Firstly, in the framework of a 3-3-1 model with a minimal scalar sector a detailed study concerning the implementation of the PQ symmetry in order to solve the strong CP problem is made. For the original version of the model, with only two scalar triplets, it is shown that the entire Lagrangian is invariant under a PQ-like symmetry but no axion is produced since a $U(1)$ subgroup remains unbroken. Although in this case the strong CP problem can still be solved, the solution is largely disfavored since three quark states are left massless to all orders in perturbation theory. The addition of a third scalar triplet removes the massless quark states but the resulting axion is visible. In order to become realistic the model must be extended to account for massive quarks and invisible axion. It is shown that the addition of a scalar singlet together with a Z_N discrete gauge symmetry can successfully accomplish these tasks and protect the axion field against quantum gravitational effects. To make sure that the protecting discrete gauge symmetry is anomaly free, a discrete version of the Green-Schwarz mechanism is used.

Secondly, an electroweak model based on the gauge symmetry $SU(2)_L \otimes U(1)_{Y'} \otimes U(1)_{B-L}$ which has right-handed neutrinos with different quantum numbers is considered. Because of this particular feature it is possible to write Yukawa terms, and neutrino mass terms, with scalar fields that can develop VEVs belonging to different energy scales. A detailed study of the scalar and the Yukawa neutrino sectors is made to show that this model is compatible with the observed solar and atmospheric neutrino mass scales and the tribimaximal mixing matrix. Also, it is shown that there are dark matter candidates if a Z_2 symmetry is included.

Finally, a $N = 1$ supersymmetric extension of the a $B - L$ model with three right-handed neutrinos is briefly discussed.

Key words: Axions, Majorons, neutrinos, 3-3-1 models, $B - L$ model, strong CP problem.

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Chapter 1

THE AXION: GENERALITIES

1.1 Introduction

Axions are fascinating hypothetical particles whose existence was proposed by S. Weinberg and F. Wilczek to give a resolution of the strong CP problem [1, 2]. From its beginning, axion physics has motivated several experimental searches and theoretical models. Since the invisible axions have extremely small coupling with the ordinary matter, their search has challenged the imagination and the experimental skills of the most of the physical community. Searches for solar, laser induced, relic and thermal axions have been performed. The majority of these experiments have been based on the Primakoff process which allows one photon to become an axion in the presence of an electromagnetic field and vice versa. The absence of any axion signal has imposed strong limits on the axion properties, such as its mass and its coupling to two photons. Currently, there is a narrow window for the decay coupling constant of the axion f_a, 10^9 GeV $\leq f_a \leq 10^{12}$ GeV, where the axion can be still found.

Although, from the effective theory point of view axion physics is relatively simple, it involves a great amount of pieces of the physical knowledge, such as non-perturbative QCD effects (instantons) only to mention one, which require a more detailed study. Keeping it in mind, the motivation of this chapter is only to give a brief review of the main topics that axion physics involves and introduce some tools and ideas that will be necessary to get a better understanding of the rest of this thesis.

Thus, this chapter is organized as follows. In Sec. 1.2 the $U(1)$ problem and its resolution are revisited. It is important because the solution to this problem gives origin to the $\overline{\theta}$ term violating the CP symmetry in the effective QCD Lagrangian. Consequently, in Sec. 1.3 the strong CP problem and its resolutions are briefly revisited. The most accepted solutions to the strong CP problem are discussed. In Secs. 1.4

and 1.5 the classical Dine-Fischler-Srednicki-Zhitnisky (DFSZ) and Kim-Shifman-Vainshtein-Zakharov (KSVZ) axion models are reviewed. Furthermore, some techniques which will be used in the following chapters are introduced. Finally, in Sec. 1.6 a concise review of some bounds on the axion properties coming from astrophysical and cosmological considerations is presented.

1.2 The $U(1)$ Problem and its Resolution

In the 1970s the strong interactions had a great puzzling problem, which became clear with the development of quantum chromodynamics (QCD). The QCD Lagrangian entails an $U(1)$ axial current, whose conservation is only broken by the quark mass terms. As consequence, the usual arguments of current algebra require a neutral pseudoscalar Nambu-Goldstone (NG) boson with a mass of the same order of magnitude as the pion mass as shown in Refs. [3, 4]. But, this strongly interacting particle does not exist. This problem was called the $U(1)$ problem by S. Weinberg.

To better understand this problem, consider the QCD theory with only three flavors of quarks, u, d, and s. This is perfectly justified if we are interested in hadron physics at energies below ~ 1 GeV. Also to begin, let us assume that these quarks are massless. This is sensible because the masses m_u, m_d, m_s are small in the following sense. The gauge coupling g of QCD becomes large at low energies. If we truncate the beta function after some number of terms, and integrate it, we find that g becomes infinite at some finite, nonzero value of the $\overline{\text{MS}}$ (modified minimal-substraction renormalization scheme) parameter μ [5, 6]; this value is commonly called Λ_{QCD}. Measurements of the strength of the gauge coupling at high energies imply $\Lambda_{\text{QCD}} \sim 380 \pm 60$ MeV [7]. Since $m_u \simeq 0.0017$, $m_d \simeq 0.0039$, $m_s \simeq 0.076$ in GeV [8] are much less than Λ_{QCD} this is a reasonable approximation to start with. With those approximations done, and ignoring the effects of the quantum anomaly which will play a key role later, the Lagrangian of QCD is written as

$$\mathcal{L}_{\text{QCD}} = i\chi^{\dagger\alpha i}\overline{\sigma}^{\mu}\left(D_{\mu}\right)_{\alpha}{}^{\beta}\chi_{\beta i} + i\xi^{\dagger}_{\bar{i}\alpha}\overline{\sigma}^{\mu}\left(\overline{D}_{\mu}\right)^{\alpha}{}_{\beta}\xi^{\beta\bar{i}} - \frac{1}{4}G^{a\mu\nu}G^{a}_{\mu\nu}, \tag{1.1}$$

where $D_{\mu} = \partial_{\mu} - ig\lambda^{a}A^{a}_{\mu}$ and $\overline{D}_{\mu} = \partial_{\mu} - ig\overline{\lambda}^{a}A^{a}_{\mu}$, with $\left(\overline{\lambda}^{a}\right)^{\alpha}{}_{\beta} = -\left(\lambda^{a}\right)_{\beta}{}^{\alpha}$ ($a = 1, \ldots, 8$), are the covariant derivatives. The λ^{a} are the Gell-Mann matrices for $SU(3)$ color group. We also have that $\chi_{\alpha i}$ are left-handed Weyl fields in the 3 representation of the $SU(3)$ color group. The α, $\beta = 1, 2, 3$ and $i = 1, 2, 3$, are color and flavor indices, respectively. The $\xi^{\alpha\bar{i}}$ are left-handed Weyl fields in the $\overline{3}$ representation of the $SU(3)$ color group, with color indices, α, $\beta = 1, 2, 3$ and flavor indices $\bar{i} = 1, 2, 3$. Notice that the spinor index carried by both χ and ξ have been omitted. Finally, the color field strengths, $G^{a}_{\mu\nu}$, are given by

$$G^{a}_{\mu\nu} = \partial_{\mu}A^{a}_{\nu} - \partial_{\nu}A^{a}_{\mu} + f^{abc}A^{b}_{\mu}A^{c}_{\nu}, \tag{1.2}$$

where A_μ^a represent the eight gluon fields, and the f^{abc} are the structure functions of the $SU(3)$ group.

In addition to the $SU(3)$ color gauge symmetry, this Lagrangian has a global $U(3) \otimes U(3)$ flavor symmetry

$$\begin{aligned} \chi_{\alpha i} &\to L_i{}^j \chi_{\alpha j}, & (1.3)\\ \xi^{\alpha \bar{i}} &\to (R^*)^{\bar{i}}{}_{\bar{j}} \xi^{\alpha \bar{j}}, & (1.4) \end{aligned}$$

where L and R^* are independent 3×3 constant unitary matrices. In terms of the Dirac field

$$\Psi_{\alpha i} = \begin{pmatrix} \chi_{\alpha i} \\ \xi^\dagger_{\alpha \bar{i}} \end{pmatrix}, \tag{1.5}$$

Eqs. (1.3) and (1.4) read

$$\begin{aligned} P_{\rm L} \Psi_{\alpha i} &\to L_i{}^j P_{\rm L} \Psi_{\alpha j}, & (1.6)\\ P_{\rm R} \Psi_{\alpha \bar{i}} &\to R_{\bar{i}}{}^{\bar{j}} P_{\rm R} \Psi_{\alpha \bar{j}}, & (1.7) \end{aligned}$$

where $P_{\rm L,R} = \frac{1}{2}(1 \mp \gamma_5)$. Thus the global flavor symmetry is often called $U(3)_{\rm L} \otimes U(3)_{\rm R}$. A symmetry that treats the left- and right-handed parts of a Dirac field differently is said to be chiral.

Reconciliation of the experimental observations with the $U(3)_{\rm L} \otimes U(3)_{\rm R}$ symmetry of the underlying Lagrangian is only possible if this symmetry is spontaneously broken. Since in QCD there are no fundamental scalar fields that could acquire a nonzero VEV, the spontaneous symmetry breaking must happen through a quark-antiquark condensate. The simplest candidate is

$$\langle 0 | \chi_{\alpha i a} \xi_b^{\beta \bar{j}} | 0 \rangle = -\frac{1}{6} \Lambda^3 \delta_\alpha{}^\beta \delta_i{}^{\bar{j}} \epsilon_{ab}, \tag{1.8}$$

where a, $b = 1$, 2 are the undotted spinor indices; ϵ_{ab} is the antisymmetric invariant symbol of $SU(2)$; and Λ is a parameter with dimensions of mass. The rest of the indices have the same meaning as in the Eq. (1.1). The condensate is unchanged only by transformations in the "vector" subgroup $U(3)_{\rm V}$ specified by $R = L$. Thus, $U(3)_{\rm L} \otimes U(3)_{\rm R}$ is spontaneously broken down to $U(3)_{\rm V}$. To see that, note that under the transformations of Eqs. (1.3) and (1.4),

$$\begin{aligned} \langle 0 | \chi_{\alpha i a} \xi_b^{\beta \bar{j}} | 0 \rangle &\to L_i{}^k (R^*)^{\bar{j}}{}_{\bar{n}} \langle 0 | \chi_{\alpha k a} \xi_b^{\beta \bar{n}} | 0 \rangle, \\ &\to -\frac{1}{6} \Lambda^3 \delta_\alpha{}^\beta \epsilon_{ab} \left(L R^\dagger \right)_i{}^{\bar{j}}. & (1.9) \end{aligned}$$

Therefore, when $L = R$, corresponding to an $U(3)_{\rm V}$ transformation, the right-hand side of Eq. (1.9) is unchanged from its value in Eq. (1.8). This means that $U(3)_{\rm V}$ is unbroken. However, for a more general transformation with $L \neq R$, the right-hand side of Eq. (1.9) does not match that of Eq. (1.8), signifying that the axial generators are broken down.

At this point it is important to say that Eq. (1.8) is non-perturbative, i.e. $\langle 0|\, \chi_{\alpha ia}\, \xi^{\beta}_{\bar{j}b}\, |0\rangle$ vanishes at tree level and perturbative corrections also vanish, because of the chiral flavor symmetry of the Lagrangian. Thus the value of Λ is not accessible in perturbation theory. It is expect that $\Lambda \sim \Lambda_{\text{QCD}}$, since Λ_{QCD} is the only mass scale in the theory when the quarks are massless.

A low-energy effective Lagrangian [9], also known as chiral Lagrangian, for the nine expected NG bosons, $18-9$, can be constructed in the following way. These NG bosons can be thought as long wavelength excitations of the condensate,

$$\langle 0|\, \chi_{\alpha ia}\xi^{\beta\bar{j}}_{b}\, |0\rangle = -\frac{1}{6}\Lambda^3 \delta_{\alpha}{}^{\beta}\delta_i{}^{\bar{j}}\epsilon_{ab}\Sigma\left(x\right)_i{}^{\bar{j}}, \tag{1.10}$$

where $\Sigma\left(x\right)$ is an unitary matrix field. Under a $U\left(3\right)_{\text{L}} \otimes U\left(3\right)_{\text{R}}$ transformation,

$$\Sigma\left(x\right)_i{}^{\bar{j}} \to L_i{}^k \left(R^*\right)^{\bar{j}}{}_{\bar{n}}\Sigma\left(x\right)_k{}^{\bar{n}}. \tag{1.11}$$

The $\Sigma\left(x\right)$ field can be written as

$$\Sigma\left(x\right) = \exp\left[-i\sum_{a=1}^{8}\frac{\lambda^a\pi^a\left(x\right)}{f_\pi} - i\frac{\pi^9\left(x\right)}{f_9}\right], \tag{1.12}$$

where the Gell-Mann λ matrices are hermitian and normalized via $\text{Tr}\lambda^a\lambda^b = 2\delta^{ab}$; the $\pi^a\left(x\right)$ ($a = 1, ..., 8$) are the hermitian NG fields; and the f_π and f_9 are parameters with dimensions of mass. f_π also is known as the pion decay constant, and it has the measured values of 92.4 MeV [10].

The terms in the effective Lagrangian for $\Sigma\left(x\right)$ can be organized by the number of derivatives that they contain. There is no allowed term with no derivatives because $\Sigma^\dagger\Sigma = 1$. As the Lagrangian must be $U\left(3\right)_{\text{L}} \otimes U\left(3\right)_{\text{R}}$ invariant, there are no terms with only one derivative. There are two terms with two derivatives,

$$\mathcal{L}_{\text{chiral}} \supset -\frac{1}{4}f_\pi^2\,\textbf{Tr}\left[\partial^\mu\Sigma^\dagger\partial_\mu\Sigma\right] - \frac{1}{4}F^2\partial^\mu\left(\det\Sigma^\dagger\right)\partial_\mu\left(\det\Sigma\right). \tag{1.13}$$

By requiring all nine NG fields to have canonical kinetic terms, the F parameter can be written in terms of f_π and f_9. To see this let us focus on the π^9 dependence,

$$\begin{aligned}\Sigma\left(x\right) &= \left(1-\frac{1}{f_9}i\pi^9\left(x\right)\right)\mathbf{I}_{3\times3} + \mathcal{O}\left(f_9^{-2}\right), &(1.14)\\ \det\Sigma\left(x\right) &= \left(1-\frac{1}{f_9}3i\pi^9\left(x\right)\right) + \mathcal{O}\left(f_9^{-2}\right), &(1.15)\end{aligned}$$

and

$$\mathcal{L}_{\text{chiral}} = -\frac{1}{4}\left[\textbf{Tr}\left[\mathbf{I}_{3\times3}\right]f_\pi^2 + 9F^2\right]f_9^{-2}\partial^\mu\pi^9\partial_\mu\pi^9 + ..., \tag{1.16}$$

thus requiring the coefficient of $\partial^\mu \pi^9 \partial_\mu \pi^9$ to be $-\frac{1}{2}$ yields

$$F^2 = \frac{2}{9}\left(f_9^2 - \frac{3}{2}f_\pi^2\right). \tag{1.17}$$

With all these done, the Lagrangian reads

$$\mathcal{L}_{\text{chiral}} = -\frac{1}{4}f_\pi^2 \mathbf{Tr}\partial^\mu \Sigma^\dagger \partial_\mu \Sigma - \frac{1}{18}\left(f_9^2 - \frac{3}{2}f_\pi^2\right)\partial^\mu \left(\det \Sigma^\dagger\right)\partial_\mu \left(\det \Sigma\right) + \dots . \tag{1.18}$$

In the real world, the three light quarks have small masses, thus the $\mathcal{L}_{\text{QCD}}$ given in Eq. (1.1) contains

$$\mathcal{L}_{\text{QCD}} \supset -M_i^{\bar{j}} \epsilon^{ab} \chi^i_{\alpha a} \xi^\alpha_{b\bar{j}} + \mathbf{H.c.}, \tag{1.19}$$

where M is an arbitrary complex matrix. M can be made diagonal with positive real entries m_u, m_d, and m_s via an $U(3)_{\mathrm{L}} \otimes U(3)_{\mathrm{R}}$ transformation. In terms of the effective Lagrangian,

$$\mathcal{L}_{\text{chiral}} \supset \Lambda^3 \mathbf{Tr}\left(M\Sigma + \mathbf{H.c.}\right). \tag{1.20}$$

Expanding in the NG fields, we obtain the following mass term

$$\mathcal{L}_{\text{mass}} = -\frac{1}{4f_\pi^2}\Lambda^3 \mathbf{Tr}\left[M\left\{\lambda^a, \lambda^b\right\}\right]\pi^a \pi^b, \tag{1.21}$$

where $a, b = 1, \dots, 9$, and $\lambda^9 \equiv (f_\pi/f_9)\,\mathbf{1}_{3\times 3}$. It is usual to define

$$\begin{aligned} \pi^a \quad &\equiv \quad \pi^a \lambda^a = \sqrt{2}\begin{pmatrix} \frac{1}{\sqrt{2}}\pi^3 + \frac{1}{\sqrt{6}}\pi^8 & \pi^+ & K^+ \\ \pi^- & -\frac{1}{\sqrt{2}}\pi^3 + \frac{1}{\sqrt{6}}\pi^8 & K^0 \\ K^- & \overline{K}^0 & -\sqrt{\frac{2}{3}}\pi^8 \end{pmatrix} \\ &+\frac{f_\pi}{f_9}\begin{pmatrix} \pi^9 & 0 & 0 \\ 0 & \pi^9 & 0 \\ 0 & 0 & \pi^9 \end{pmatrix}. \end{aligned} \tag{1.22}$$

To find the eigenvalues of the mass-squared matrix in Eq. (1.21) let us write this explicitly

$$\begin{aligned} \mathcal{L}_{\text{mass}} \quad = \quad & -2\Lambda^3 f_\pi^{-2}\Big[(m_u + m_d)\,\pi^+\pi^- + (m_u + m_s)\,K^+K^- + (m_d + m_s)\,\overline{K}^0 K^0 \\ & m_u\left(\frac{1}{\sqrt{3}}\pi^8 + \pi^3 + r\pi^9\right)^2 + m_d\left(\frac{1}{\sqrt{3}}\pi^8 - \pi^3 + r\pi^9\right)^2 \\ & + m_s\left(\frac{2}{\sqrt{3}}\pi^8 + r\pi^9\right)^2\Big], \end{aligned} \tag{1.23}$$

where $r \equiv f_\pi/f_9$. The squared masses of the charged NG bosons are easily read from Eq. (1.23)

$$m^2_{\pi^\pm} \quad = \quad 2\Lambda^3 f_\pi^{-2}\left(m_u + m_d\right), \tag{1.24}$$

$$m^2_{K^\pm} \quad = \quad 2\Lambda^3 f_\pi^{-2}\left(m_u + m_s\right), \tag{1.25}$$

$$m^2_{K^0\overline{K}^0} \quad = \quad 2\Lambda^3 f_\pi^{-2}\left(m_d + m_s\right). \tag{1.26}$$

To simplify the calculation of the squared masses of the neutral NG bosons, set $m_u = m_d = m \ll m_s$. By doing so,

$$\mathcal{L}_{\text{mass}}^{\text{neutral}} = -2\Lambda^3 f_\pi^{-2}\left[2m\left(\pi^3\right)^2 + 2m\left(\frac{1}{\sqrt{3}}\pi^8 + r\pi^9\right)^2 + \right. \tag{1.27}$$

$$\left. m_s\left(\frac{2}{\sqrt{3}}\pi^8 + r\pi^9\right)^2\right]. \tag{1.28}$$

Thus the squared masses of the neutral NG bosons are

$$m_{\pi^0}^2 \equiv m_{\pi^3}^2 = 4m\Lambda^3/f_\pi^2, \tag{1.29}$$

$$m_\eta^2 = \frac{8}{3}m_s\Lambda^3 f_\pi^{-2}\left(1+\frac{3}{4}r^2\right), \tag{1.30}$$

$$m_{\pi^9}^2 = \frac{9r^2}{4+3r^2}m_{\pi^0}^2, \tag{1.31}$$

where, as it is usual, the neutral eigenstates as π^0, η, π^9 have been defined. From the Eq. (1.31) we see that the maximum possible value of m_{π^9} is $\sqrt{3}m_{\pi^0}$, attained in the limit $f_9 \to 0$. This particle does not appear in nature. This discrepancy between theory and experiment is known as the $U(1)$ problem, as it has been mentioned before.

At first glance, one might think that the Adler-Bell-Jackiw anomaly [11–14] provides a possible solution to the $U(1)$ problem, i.e. because the divergence of the axial current J_5^μ associated with this symmetry gets quantum corrections from the triangle graph which connects it to two gluon fields, A^a, with quarks going around the loop Fig. 1.2,

$$\partial_\mu \langle 0| J_5^\mu |A^a(p) A^a(q)\rangle = -\frac{g^2 N}{16\pi^2}\left\langle 0 \left| G^{a\mu\nu}\widetilde{G}_{\mu\nu}^a \right| A^a(p) A^a(q)\right\rangle, \tag{1.32}$$

where N is the number of massless quarks and $\widetilde{G}_{\mu\nu}^a = \frac{1}{2}\epsilon_{\mu\nu\alpha\beta}G^{a\alpha\beta}$, the axial current is not conserved. Thus there would not be an $U(1)$ symmetry to worry about. In other

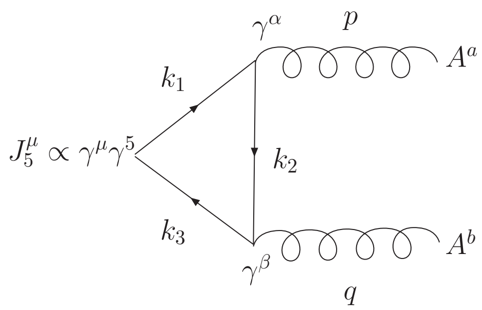

Figure 1.1: *The lowest order Feynman graph leading to the chiral anomaly.*

words, in the massless quark limit, although formally QCD is invariant under $U(1)$ axial transformations,

$$\Psi_i \to e^{i\alpha\gamma_5/2}\Psi_i, \tag{1.33}$$

the chiral anomaly affects the action,

$$\delta S=\alpha\int \mathbf{d}^4x\;\partial_\mu J_5^\mu=-\alpha\frac{g^2N}{16\pi^2}\int \mathbf{d}^4x\;\epsilon^{\mu\nu\alpha\beta}\mathbf{Tr}\left[G_{\mu\nu}G_{\alpha\beta}\right], \tag{1.34}$$

and thus the $U\left(1\right)$ axial symmetry would not be a true quantum symmetry of QCD. However, since

$$\begin{aligned}-\frac{g^2N}{16\pi^2}\epsilon^{\mu\nu\alpha\beta}\mathbf{Tr}\left[G_{\mu\nu}G_{\alpha\beta}\right] &= -\frac{g^2N}{4\pi^2}\epsilon^{\mu\nu\alpha\beta}\partial_\mu\mathbf{Tr}\left[A_\nu\partial_\alpha A_\beta-\frac{2}{3}igA_\nu A_\alpha A_\beta\right]\\ &\equiv -\frac{g^2N}{4\pi^2}\partial_\mu W^\mu, \end{aligned}\tag{1.35}$$

where

$$W^\mu=\epsilon^{\mu\nu\alpha\beta}\mathbf{Tr}\left[A_\nu\partial_\alpha A_\beta-\frac{2}{3}igA_\nu A_\alpha A_\beta\right], \tag{1.36}$$

the Eq. (1.32) can be reexpressed as

$$\partial_\mu\left\langle 0\right|J_5^\mu\left|A^a\left(p\right)A^a\left(q\right)\right\rangle=\frac{g^2N}{4\pi^2}\partial_\mu W^\mu. \tag{1.37}$$

Because of these identities δS is a pure surface integral

$$\delta S=-\alpha\frac{g^2N}{4\pi^2}\int \mathbf{d}^4x\;\partial_\mu W^\mu=-\alpha\frac{g^2N}{4\pi^2}\int \mathrm{d}\sigma_\mu\;W^\mu. \tag{1.38}$$

Hence, one might think that $\int \mathrm{d}\sigma_\mu W^\mu=0$, and thus, the $U\left(1\right)$ axial symmetry would appear as a symmetry of the QCD again. However, this it is not correct from the quantum point of view as showed by G. 't Hooft [15, 16].

To understand how it is possible that $\int \mathrm{d}\sigma_\mu W^\mu\neq 0$ and how this provides a solution to the $U\left(1\right)$ problem, consider the QCD vacuum structure. The classical QCD energy density is the sum of the square of the chromo-electric and chromo-magnetic fields. Thus the classical field configuration corresponding to the ground state is $G^a_{\mu\nu}=0$. This happens whenever the vector potential A_μ, is a gauge transformation of zero, i.e. $A_\mu\equiv A^a_\mu\lambda^a=\frac{i}{g}U\left(x\right)\partial_\mu U^\dagger\left(x\right)$, where $U\left(x\right)$ is any $SU\left(3\right)$ group transformation

$$U\left(x\right)=\exp\left[iw^a\left(x\right)\frac{\lambda^a}{2}\right], \tag{1.39}$$

with $a=1,\ldots,8$. Each configuration correspond to a particular map $w^a\left(x\right)$ of space-time into the eight-dimensional group manifold of $SU\left(3\right)$.

Now, a fundamental question arises. Are these different vacuum configurations gauge equivalent? In other words, can every $U\left(x\right)$ be smoothly deformed into every other $U\left(x\right)$? If the answer is yes, they correspond to a single quantum vacuum state. Otherwise, there must be more than one quantum vacuum state. To answer this question, let us restrict ourselves to the non-abelian $SU\left(2\right)$ group. This is not a loss of generality because the following arguments will also apply to the $SU(3)$ group, since this last contains the $SU(2)$ group.

First, note that any 2×2 special unitary matrix U can be written in the form

$$U = a_4 + i\vec{a}.\vec{\sigma}, \tag{1.40}$$

where a_4 and the three vector $\vec{a}$ are real, and

$$\vec{a}^2 + a_4^2 = 1. \tag{1.41}$$

Thus the $a_\mu \equiv (\vec{a}, a_4)$ specifies an Euclidean four-vector of unit length, $a_\mu a^\mu = 1$, and hence one point on three sphere S_3. To determine the topology of the space-time, consider the possible vacuum configurations in the temporal gauge, $\mathrm{A}_0 = 0$. This restriction entails no loss of generality and means the unitary maps $U(\vec{x})$ depend only on space. These maps, from one vacuum configuration to the next, are local in space, and thus they must satisfy the boundary conditions

$$\lim_{|\vec{x}| \to \infty} U(\vec{x}) = \mathbf{I}, \tag{1.42}$$

which do not depend on the direction at the spatial infinity. Hence, the maps are defined in a three dimensional space, with all of its directions at ∞ identified. Thus the topology of the space-time is S_3. Then, $U(\vec{x})$ provides a map from the spatial three-sphere to the vacuum three-sphere, $S_3 \to S_3$. These maps are characterized by a topological winding number n, which can be defined as the number of times that each sphere is mapped into the other. Given a smooth map $U(\vec{x})$, its winding number can be written as [17]

$$n = -\frac{1}{24\pi^2} \int \mathbf{d}^3x \; \epsilon^{ijk} \mathbf{Tr} \left[(U\partial_i U)(U\partial_j U)(U\partial_k U) \right]. \tag{1.43}$$

To verify that the equation above agrees with the previous definition of winding number, consider the identity map (with winding number 1)

$$\begin{aligned} U(\widehat{x}_\mu) &= \frac{x_4 + i\vec{x}.\vec{\sigma}}{\rho} \\ &= \begin{pmatrix} \cos\chi + i\sin\chi\cos\psi & i\sin\chi\sin\psi e^{-i\phi} \\ i\sin\chi\sin\psi e^{+i\phi} & \cos\chi - i\sin\chi\cos\psi \end{pmatrix}, \end{aligned} \tag{1.44}$$

where

$$\widehat{x}_\mu = (\sin\chi\sin\psi\cos\phi, \sin\chi\sin\psi\sin\phi, \sin\chi\cos\psi, \cos\chi), \tag{1.45}$$

defines the polar angles χ and ψ, and the azimuthal angle ϕ. Also, $\rho \equiv (x_\mu x^\mu)^{1/2}$ has been defined. Plugging Eq. (1.44) in Eq. (1.43) is straightforward to get

$$\begin{aligned} (U\partial_\chi U)(U\partial_\psi U)(U\partial_\phi U) &= -(U\partial_\psi U)(U\partial_\chi U)(U\partial_\phi U) \\ &= -\left(\sin^2\chi\sin\psi\right)\, \mathbf{I}_{2\times 2}, \end{aligned} \tag{1.46}$$

and

$$n=-\frac{1}{24\pi^2}\int_0^{\pi}\mathrm{d}\chi\int_0^{\pi}\mathrm{d}\psi\int_0^{2\pi}\mathrm{d}\phi\,6\left(\sin^2\chi\sin\psi\right)\ \mathbf{Tr}\left[\mathbf{I}_{2\times 2}\right]=1, \tag{1.47}$$

as it should be. Moreover, it is evident that if ϕ in Eq. (1.44) is replaced by $n\phi$, that map will have a winding number equal n. With this done, it has been checked that the definition of the winding number given in Eq. (1.44) agrees with the previous definition of the winding number.

Now, consider the variation of n, δn, under smooth deformations of $U\left(\overrightarrow{x}\right)$. A deformation δU induces

$$\begin{aligned}\epsilon^{ijk}\delta\mathbf{Tr}\left[\left(U\partial_i U\right)\left(U\partial_j U\right)\left(U\partial_k U\right)\right] &= 3\epsilon^{ijk}\mathbf{Tr}\left[\left(U\partial_i U\right)\left(U\partial_j U\right)\delta\left(U\partial_k U\right)\right]\\ &= -3\epsilon^{ijk}\mathbf{Tr}\left[\left(U\partial_i U\right)\left(U\partial_j U\right)U\partial_k\left(U^\dagger\delta U\right)U^\dagger\right]\\ &= -3\epsilon^{ijk}\mathbf{Tr}\left[\left(\partial_i U\right)\left(U\partial_j U\right)U\partial_k\left(U^\dagger\delta U\right)\right],\end{aligned} \tag{1.48}$$

where the cyclic property of the trace, $U^\dagger U=\mathbf{I}$ and $\delta\left(U\partial_k U\right)=-U\partial_k\left(U^\dagger\delta U\right)U^\dagger$ have been used. Continuing with the calculation

$$\begin{aligned}(1.48) &= -3\epsilon^{ijk}\mathbf{Tr}\left\{\partial_k\left[\left(\partial_i U\right)\left(U\partial_j U\right)U\left(U^\dagger\delta U\right)\right]-\partial_k\left[\left(\partial_i U\right)\left(U\partial_j U\right)U\right]\left(U^\dagger\delta U\right)\right\}\\ &= 3\epsilon^{ijk}\mathbf{Tr}\left[\left(\partial_i U\right)\left(\partial_k U\right)\left(\partial_j U\right)\delta U+\left(\partial_i U\right)\left(U\partial_j U\right)\left(\partial_k U\right)\left(U^\dagger\delta U\right)\right],\end{aligned} \tag{1.49}$$

where the surface term is zero after integrating because of $\delta U=0$ at the boundary. Terms with two derivatives acting on a single U vanish when contracted with ϵ^{ijk}. Now, using $U\partial_j U^\dagger=-\left(\partial_j U\right)U^\dagger$ and $\left(\partial_k U\right)U^\dagger=-U\partial_k U^\dagger$, followed by $U^\dagger U=\mathbf{I}$, we have that the remaining terms in Eq. (1.49) become

$$(1.49)=3\epsilon^{ijk}\mathbf{Tr}\left[\left(\partial_i U\right)\left(\partial_k U\right)\left(\partial_j U\right)\delta U+\left(\partial_i U\right)\left(\partial_j U\right)\left(\partial_k U\right)\delta U\right]. \tag{1.50}$$

The two terms are now symmetric on $j\leftrightarrow k$, and thus cancel when contracted with ϵ^{ijk}. Therefore, the winding number n is invariant under smooth transformations of $U\left(\overrightarrow{x}\right)$.

The two previous results show that $SU\left(2\right)$ gauge theory has an infinity number of classical field configurations of zero energy, distinguished by an integer n, and that these can not be smoothly deformed into each other. Furthermore, the existence of these non-equivalent classical field configuration of zero energy can be understood as the existence of different quantum vacuum states separated by energy barriers. To see why this is the case, suppose that $U\left(\overrightarrow{x}\right)$ and $\widehat{U}\left(\overrightarrow{x}\right)$ have different winding numbers so that they can not be deformed into each other. The associated vector potentials, A_μ and $\widehat{A}_\mu$ are both gauge transformations of zero, and so both $G_{\mu\nu}$ and $\widehat{G}_{\mu\nu}$ vanish. However, if we try to smoothly deform A_μ into $\widehat{A}_\mu$, we must pass through vector potentials that are not gauge transformations of zero, and whose field strengths therefore do not vanish. These nonzero field strengths imply nonzero energy, which means that there is an energy barrier between the field configurations A_μ and $\widehat{A}_\mu$.

At this point, it would seem that we have many inequivalent theories, each starting from a different winding number, living in a different Hilbert space with vacuum state $|\Omega_n\rangle$, labeled by an integer. Thus, the key questions to make here are: Are there many disconnected Hilbert spaces, one for each winding number?, and is the theory only invariant under gauge transformations that do not change the winding number? The answers depend on the existence of transition amplitudes between these different vacuum states. If no such transitions exist, unitary is realized in each Hilbert space, and we have many equivalent theories. On the other hand, if transitions between Hilbert spaces do exist, all the (sub)Hilbert spaces must be included in order to have an unitary theory.

The answer to those questions is that actually, the winding number is not conserved by QCD; there is quantum tunneling between vacua of different winding number, which are analyzed in the saddle-point approximation of the path integral [18], by expanding away of configurations of minimum action in Euclidean space. These configurations are called instantons. They mediate the tunneling. Their contributions to the tunneling amplitude are of the order of

$$e^{-\frac{8\pi^2}{g^2}}, \tag{1.51}$$

where g is the strong (QCD) coupling constant. Normally, such amplitude would lead to a negligible rate, but the QCD coupling constant can be very large in the infrared, effectively avoiding this typical exponential suppression of tunneling processes.

To better understand the appearance of those configurations let us show some of the main arguments that lead to those solutions without going into all details of the calculation.

The first task will be to construct a Bogomolny bound on the Euclidean action

$$S = \frac{1}{2}\int \mathrm{d}^4x\ \mathbf{Tr}\left[G^{\mu\nu}G_{\mu\nu}\right], \tag{1.52}$$

of a field that obeys the boundary condition

$$\lim_{\rho\to\infty} A_\mu\left(x\right) = \frac{i}{g}U\left(\widehat{x}_\mu\right)\partial_\mu U^\dagger\left(\widehat{x}_\mu\right), \tag{1.53}$$

where $\widehat{x}_\mu$ is defined in Eq. (1.45), $\rho = \left(x_\mu x^\mu\right)^{1/2}$, and $U\left(\widehat{x}_\mu\right)$ is a map with winding number n. Using the fact that Eq. (1.43) can be written as

$$n = \frac{1}{24\pi^2}\int \mathrm{d}S_\mu\ \epsilon^{\mu\nu\alpha\beta}\mathbf{Tr}\left[\left(U\partial_\nu U\right)\left(U\partial_\alpha U\right)\left(U\partial_\beta U\right)\right], \tag{1.54}$$

where $\partial_\mu = \partial/\partial x^\mu$ and $\epsilon^{1234} = +1$. Now, using Eq. (1.53) in (1.54), the winding number can be written in terms of the vector potential,

$$n = \frac{ig^3}{24\pi^2} \int \mathrm{d}S_\mu \; \epsilon^{\mu\nu\alpha\beta} \mathrm{Tr}\left[A_\nu A_\alpha A_\beta\right]. \tag{1.55}$$

Besides, using $\int \mathrm{d}S_\mu \; \epsilon^{\mu\nu\alpha\beta}\mathrm{Tr}[A_\nu G_{\alpha\beta}] = 0$, the Gauss's theorem, the Eq. (1.35), and $\epsilon^{\mu\nu\alpha\beta}\mathrm{Tr}\left[A_\nu G_{\alpha\beta} + \frac{2}{3} ig A_\nu A_\alpha A_\beta\right] = 2\epsilon^{\mu\nu\alpha\beta}\mathrm{Tr}\left[A_\nu \partial_\alpha A_\beta - \frac{2}{3} ig A_\nu A_\alpha A_\beta\right]$, the winding number becomes

$$\begin{aligned} (1.55) &= \frac{g^2}{16\pi^2} \int \mathrm{d}S_\mu \; \epsilon^{\mu\nu\alpha\beta} \mathrm{Tr}\left[A_\nu G_{\alpha\beta} + \frac{2}{3} ig A_\nu A_\alpha A_\beta\right] \\ &= \frac{g^2}{8\pi^2} \int \mathrm{d}^4x \; \partial_\mu W^\mu \\ n &= \frac{g^2}{16\pi^2} \int \mathrm{d}^4x \; \mathrm{Tr}\left[G_{\mu\nu} \widetilde{G}^{\mu\nu}\right]. \end{aligned} \tag{1.56}$$

The first equality that has been used, $\int \mathrm{d}S_\mu \; \epsilon^{\mu\nu\alpha\beta}\mathrm{Tr}[A_\nu G_{\alpha\beta}] = 0$, is due to that on the surface at infinity, the vector potential is a gauge transformation of zero, and so the field strength $G_{\alpha\beta}$ vanishes there. With this done, it easy to construct a Bogomolny bound. First, note that

$$\frac{1}{2}\mathrm{Tr}\left[\widetilde{G}_{\mu\nu} \pm G_{\mu\nu}\right]^2 = \mathrm{Tr}\left[G_{\mu\nu}G^{\mu\nu}\right] \pm \mathrm{Tr}\left[\widetilde{G}_{\mu\nu}G^{\mu\nu}\right], \tag{1.57}$$

where $\widetilde{G}_{\mu\nu}\widetilde{G}^{\mu\nu} = G_{\mu\nu}G^{\mu\nu}$ has been used. The left-handed side of Eq. (1.57) is non-negative and so

$$\int \mathrm{d}^4x \; \mathrm{Tr}\left[G_{\mu\nu}G^{\mu\nu}\right] \geq \left| \int \mathrm{d}^4x \; \mathrm{Tr}\left[\widetilde{G}_{\mu\nu}G^{\mu\nu}\right] \right|. \tag{1.58}$$

Therefore, finally, from Eq. (1.52), Eq. (1.56) and Eq. (1.58) the Bogomolny bound is given by

$$S \geq 8\pi^2 \left|n\right| / g^2. \tag{1.59}$$

This bound gives us the minimum value of the Euclidean action for a solution of the Euclidean field equations that mediates between a vacuum configuration with winding number n_- at $x_4 = -\infty$ and a vacuum configuration with winding number $n_+ = n_- + n$ at $x_4 = +\infty$.

The instanton solution can be found by resolving

$$\widetilde{G}_{\mu\nu} = (\mathrm{sign}\; n)\, G_{\mu\nu}, \tag{1.60}$$

for a map with winding number $n = 1$. To find it explicitly let us make the following ansatz [19]

$$A_\mu(x) = \frac{i}{g} f(\rho)\, U(\widehat{x}_\mu)\, \partial_\mu U^\dagger(\widehat{x}_\mu), \tag{1.61}$$

where $f(\infty) = 1$, so that the solution obeys the boundary conditions; $f(0) = 0$, since A_μ should be well defined at $\rho = 0$; and $U(\widehat{x}_\mu)$ is given by Eq. (1.44). Plugging Eq. (1.61) in Eq. (1.60), it is straightforward to find

$$f(\rho) = \frac{\rho^2}{\rho^2 + a^2}, \tag{1.62}$$

where a, the size of the instanton, is a constant of integration. The instanton solution is also parameterized by the location of its center; here it has been used the space-time origin, but the translation invariance allows us to displace it.

Now, since the vacua of different winding numbers can be reached via instantons, as showed above, we come to two important conclusions. Writing these in a short way,

- The chiral $U(1)$ symmetry is no more a symmetry of the QCD Lagrangian and thus the $U(1)$ problem does not exist anymore.
- The QCD Lagrangian has a new parameter, θ.

The first conclusion is easily reached. The existence of instantons affect the charge $Q_5 = \int \mathrm{d}^4x \, \partial_\mu J_5^\mu$ in the sense that the charge after the instanton differs from the charge before the instanton. This can be seen in the following way

$$\begin{aligned} \Delta Q_5 &= \int \mathrm{d}^4x \, \partial_\mu J_5^\mu \\ &= \frac{g^2 N}{8\pi^2} \int \mathrm{d}^4x \, \mathrm{Tr}\left[G^{\mu\nu}\widetilde{G}_{\mu\nu}\right] \\ &= \pm 2N, \end{aligned} \tag{1.63}$$

where the Eqs. (1.34) and (1.56) have been used. Note that, although the instanton solution has been computed using analytic continuation to Euclidean space-time, the tunneling process between different vacua take place in the Minkowski space-time. Furthermore, being topological equations, Eqs. (1.32), (1.56) and (1.59) hold both in Euclidean and Minkowski space-times, since they are independent of the metric.

In conclusion, in the limit of N massless quarks, although the QCD Lagrangian has the global symmetry

$$\begin{aligned} U(N)_\mathrm{L} \otimes U(N)_\mathrm{R} &\sim SU(N)_\mathrm{L} \otimes SU(N)_\mathrm{R} \otimes U(1)_\mathrm{L} \otimes U(1)_\mathrm{R} \\ &\sim SU(N)_\mathrm{L} \otimes SU(N)_\mathrm{R} \otimes U(1)_\mathrm{V} \otimes U(1)_\mathrm{A}\,, \end{aligned} \tag{1.64}$$

the instanton contribution violates the $U(1)_\mathrm{A}$ symmetry, and thus it provides a solution to the $U(1)$ problem.

The appearance of one new parameter in the QCD Lagrangian can be understood in the following way. Since all vacua of different winding number can be reached via instantons, the physical vacuum state must be a linear superposition of all states of zero energy and different winding numbers, $|\Omega_n\rangle$. Its precise form is determined by gauge invariance. Under a gauge transformation $\mathcal{U}$ which changes the winding number by one unit, we have

$$\mathcal{U}\,|\Omega_n\rangle = |\Omega_{n+1}\rangle\,. \tag{1.65}$$

None of these states are physical since they can be transformed into one another by a gauge transformation. The vacuum state is the one on which a gauge transformation can at most result in a phase transformation. This fixes the physical vacuum to be given by the Bloch superposition

$$|\Omega_\theta\rangle = \sum_n e^{-in\theta}\,|\Omega_n\rangle\,, \tag{1.66}$$

which is designed such that

$$\mathcal{U}\,|\Omega_\theta\rangle = e^{i\theta}\,|\Omega_\theta\rangle\,. \tag{1.67}$$

Therefore, the true vacuum state of the $SU(3)$ gauge theory depends on an angle θ, defined module 2π. This new parameter has for effect to alter the Lagrangian by a surface term which does not appear in the equations of motion, yielding the effective Lagrangian in Minkowski space

$$\mathcal{L}_{SU(3)} = \mathbf{Tr}\left[-\frac{1}{2}G^{\mu\nu}G_{\mu\nu} - \frac{g^2\theta}{16\pi^2}\widetilde{G}^{\mu\nu}G_{\mu\nu}\right]. \tag{1.68}$$

The appearance of the extra term in the Lagrangian is better understood by considering the Euclidean path integral formalism. To do that, suppose that we are interested in starting with a particular theta vacuum $|\Omega_\theta\rangle$, and ending with a (possibly different) theta vacuum $|\Omega_{\theta'}\rangle$. Then from Eq. (1.66)

$$\langle\Omega_{\theta'}|\,\Omega_\theta\rangle = Z_{\theta'\leftarrow\theta}\,(J) = \sum_{n_-,n_+} e^{i(n_+\theta'-n_-\theta)}Z_{n_+\leftarrow n_-}\,(J)\,, \tag{1.69}$$

where

$$\left\langle\Omega_{n_+}\middle|\,\Omega_{n_-}\right\rangle = Z_{n_+\leftarrow n_-}\,(J) = \int \mathcal{D}A_{n_+-n_-}\;e^{-S+JA}, \tag{1.70}$$

with $JA \equiv \int \mathrm{d}^4x\;\mathrm{Tr}[J^\mu A_\mu]$, and the subscript on the field differential means that we integrate only over fields with that winding number. Now, defining $n_+ = n_- + n$, and summing over n_- we have

$$Z_{\theta'\leftarrow\theta}\,(J) = \delta\left(\theta'-\theta\right)\sum_n e^{in\theta}\int \mathcal{D}A_n\;e^{-S+JA}. \tag{1.71}$$

The $\delta\left(\theta'-\theta\right)$ in the above equation means that the value of θ is time independent. Thus we can drop the delta function, and just define

$$Z_\theta\,(J) \equiv \sum_n e^{in\theta}\int \mathcal{D}A_n\;e^{-S+JA}. \tag{1.72}$$

Next, combining the sum over n and the integral over A_n into an integral over all A, and using Eq. (1.56) results

$$Z_\theta(J) = \int \mathcal{D}A \ \exp \int \mathrm{d}^4x \ \mathbf{Tr}\left[-\frac{1}{2}G^{\mu\nu}G_{\mu\nu} + \frac{ig^2\theta}{16\pi^2}\widetilde{G}^{\mu\nu}G_{\mu\nu} + J^\mu A_\mu\right], \tag{1.73}$$

thus the θ angle appears as the coefficient of an extra term in the $SU(3)$ Yang-Mills Lagrangian. When we return to the Minkowski space (by setting $x_0 = it$), we obtain the Lagrangian given in Eq. (1.68)

1.3 The Strong CP Problem and its Resolutions

As showed in the previous section, the QCD Lagrangian has a new parameter θ. This parameter can have any value between 0 and 2π, and it is hoped to be of order one, $\mathcal{O}(1)$. However, the absence of a measurable electric dipole moment for the neutron, $|d_n| < 2.9 \times 10^{-26}$ e cm [20], suggests that

$$|\overline{\theta}| \lessapprox 0.7 \times 10^{-11}, \tag{1.74}$$

where $\overline{\theta} \equiv \theta - \arg\det \mathbf{M}_q$, being $\mathbf{M}_q$ the quark mass matrix. The reason why this parameter is so small is known as the strong CP problem. Notice that in the Eq. (1.74), the parameter used was $\overline{\theta}$ instead of θ. This is so, because the physical parameter is $\overline{\theta}$ as it will be explained later.

Although there is no experimental evidence that the strong interactions violate either P and CP symmetries, the QCD is capable of breaking these symmetries both spontaneously and explicitly. The former through the quark condensate given in Eq. (1.8), the latter through interactions which violate these symmetries. Here, we concentrate in the last case.

Explicitly breaking comes from the term

$$-\frac{g^2\theta}{16\pi^2}\mathbf{Tr}\left[\widetilde{G}^{\mu\nu}G_{\mu\nu}\right]. \tag{1.75}$$

This term is clearly odd under P and CP. Note that

$$\epsilon^{\mu\nu\alpha\beta}G_{\mu\nu}G_{\alpha\beta} \sim \epsilon^{ijk}G_{0i}G_{jk}, \tag{1.76}$$

and making an analogy with the electromagnetic field, where $\epsilon^{\mu\nu\alpha\beta}F_{\mu\nu}F_{\alpha\beta} \sim \epsilon^{ijk}F_{0i}F_{jk} = \overrightarrow{E}.\overrightarrow{B}$, the transformation of Eq. (1.75) under P and CP is evident. Here, it is also important to note that the parameter in Eq. (1.75) is not a physical parameter. This can be understood if we consider the quark mass term in the QCD Lagrangian. Suppose that we have only a quark field given by Eq. (1.5) with the mass term

$$\begin{aligned} \mathcal{L}_{\text{mass}} &= -m\chi\xi - m^*\xi^\dagger\chi & (1.77)\\ &= -|m|\,\overline{\Psi}e^{-i\phi\gamma_5}\Psi, & (1.78)\end{aligned}$$

where $m = |m| e^{i\phi}$. Then, a $U(1)_{\rm A}$ transformations given by Eq. (1.33) changes ϕ to $\phi - \alpha/2$. Since θ simultaneously changes to $\theta - \alpha/2$, we see that $\theta - \phi$ is invariant. Thus the actual physical parameter is $\theta - \phi$. With more quarks fields, the mass term is $\mathcal{L}_{\rm mass} = -\left(\mathbf{M}_q\right)_{ij} \chi_i \xi_j +$ H.c., and the physical parameter is

$$\begin{aligned} \overline{\theta} &= \theta - \arg\det \mathbf{M}_q \\ &\equiv \theta_{\rm QCD} - \theta_{\rm QFD}, \end{aligned} \tag{1.79}$$

where the definitions $\theta_{\rm QCD} \equiv \theta$ and $\theta_{\rm QFD} \equiv \arg\det\mathbf{M}_q$ have been done. The acronym QFD in $\theta_{\rm QFD}$ mean Quantum Flavor Dynamics.

Having showed that the $\overline{\theta}$ in the Lagrangian violates the P and CP symmetries, consider the places where strong CP violation can manifest itself. Two places where it is possible, in principle, to see CP violation are the P- and T-violating $\eta \to 2\pi$ decay [21] and the electric dipole moment d_n [20]. As it is well known, the most stringent limit on $\overline{\theta}$ is set by the neutron electric dipole moment (NEDM).

To give one estimate for NEDM, we follow Ref. [22]. The diagrams that most contribute to the NEDM are shown in Fig. 1.3. These diagrams are enhanced by a $\ln\left(\Lambda^2/m_\pi^2\right) \sim 4.2$, where $\Lambda \sim 4\pi f_\pi$ is the ultraviolet cutoff in the effective theory. No other contributing diagrams have this enhancement. Of course, this number is not impressively large number, and thus we can not be certain that the remaining contributions are not significant. However, here we restrict ourselves to consider only these kind of diagrams as it was done in Ref. [22]. To calculate these contributions it

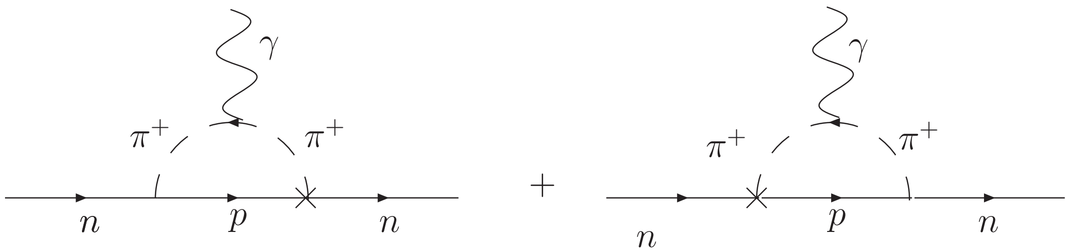

Figure 1.2: *Diagrams contributing to the electric dipole moment of the neutron. The CP violating vertex is denoted with a cross.*

is needed to know the couplings of $np\pi^+$. The Lagrangian for this interaction is given by

$$\mathcal{L}_{\pi\overline{N}N} = -i\sqrt{2}\left(g_{\rm A} m_N/f_\pi\right)\left(\pi^+\overline{p}\gamma_5 n + \pi^-\overline{n}\gamma_5 p\right), \tag{1.80}$$

$$\mathcal{L}_{\overline{\theta}\pi\overline{N}N} = -\sqrt{2}\left(\overline{\theta} c_+ \widetilde{m}/f_\pi\right)\left(\pi^+\overline{p}n + \pi^-\overline{n}p\right), \tag{1.81}$$

where m_N is the neutron mass, ≈ 939.56 MeV, $f_\pi = 92.4$ MeV, $g_{\rm A} \simeq 1.27$,

$$c_+ = \frac{m_{\Xi^0} - m_{\Sigma^0}}{m_s - \frac{1}{2}m_u - \frac{1}{2}m_d} \simeq 1.7, \tag{1.82}$$

$$\widetilde{m} = \frac{m_u m_d}{m_u + m_d} \simeq 1.2 \text{ MeV}. \tag{1.83}$$

Without going into the details, the amplitude for the diagrams in the Fig. 1.3

$$T = -4\left(e\overline{\theta} g_{\rm A} c_+ \widetilde{m}/f_\pi^2\right) \varepsilon_\mu^* \overline{u} S^{\mu\nu} q_\nu i\gamma_5 u \int_0^\Lambda \frac{\mathrm{d}^4 l}{(2\pi)^4} \frac{1}{\left(l^2 + m_\pi^2\right)^2}, \tag{1.84}$$

where q is the momentum of the photon, and $S^{\mu\nu} = \frac{i}{4}\left[\gamma^\mu, \gamma^\nu\right]$. Using standard techniques

$$d_n = \frac{e\overline{\theta} g_{\rm A} c_+ \widetilde{m}}{8\pi^2 f_\pi^2} \ln\left(\Lambda^2/m_\pi^2\right) \simeq 3.2 \times 10^{-16} \overline{\theta}\ e \text{ cm}. \tag{1.85}$$

The experimental upper limit is $|d_n| < 2.9 \times 10^{-26}\ e$ cm [20], thus $\left|\overline{\theta}\right| < 0.9 \times 10^{-10}$. Note that there are a lot of other estimates of the NEDM using different methods [23–25], but the important thing here is that in all of these methods the parameter $\overline{\theta}$ is extremely small.

Before going to the different solutions to the strong CP problem, let us rephrase it in different ways. First, it is a CP hierarchy problem. Weak CP violation in $K^0 - \overline{K}^0$ systems is characterized by ε parameter which is of order of 10^{-3}, whereas strong CP is measured by the very small parameter $\overline{\theta}$. Also, it is a problem of fine-tuning in the sense that the combinations of two non physical parameters $\theta_{\rm QCD}$ and $\theta_{\rm QFD}$ is so small irrespective of the arbitrary value of $\theta_{\rm QCD}$.

If QCD is supposed to be the correct theory of the strong interactions, a solution to the strong CP problem must be found. Several "solutions" have been proposed. These can be classified as follows

- Unconventional dynamics.
- Spontaneously broken CP.
- An additional chiral symmetry.

Solutions based on unconventional dynamics suggest that the boundary conditions that give rise to the θ vacuum are an artifact [26], however this does not provide one solution to the $U(1)$ problem. Other approaches use the periodicity of the vacuum energy to deduce that $\overline{\theta}$ vanishes [27]. This type of approach is not satisfactory because it fails to motive the minimization of the vacuum energy.

The approach of the spontaneously broken CP is equivalent to set the parameter $\overline{\theta}$ at zero at tree-level. This requires that the tree-level determinant of the quark mass matrix to be real. However, the observed weak interaction phenomena exhibit the weak CP symmetry violation in the neutral K mesons system and $B \to K^+\pi^-$ decay [8], and hence the potential is arranged to break CP spontaneously. In these models must be guaranteed that the parameter $\overline{\theta}$ is small enough to be within the experimental value after radiative corrections are done. In other words [28],

$$\overline{\theta} \sim \frac{1}{16\pi^2}\Delta f^2 \sum (\text{loop integrals}) \leq 0.7 \times 10^{-11}, \tag{1.86}$$

where Δf^2 is the product of coupling constants and the Feynmann loop integrals are of $\mathcal{O}(1)$. Therefore, Δf^2 must be small enough to satisfy the experimental bound. Along this line, many ideas were proposed [29–31]. These ideas have difficulties in satisfying the bounds of the current CP violation data, for example, flavor changing neutral currents (FCNC) and domain walls [32]. Another drawback for this type of approach is that the experimental data are in excellent agreement with the Cabibbo-Kobayashi-Maskawa model (CKM model), a model where CP is explicitly broken.

However, there is a type of weak CP violation [33, 34], known as Nelson-Barr, that mimics CKM type CP violation even though the fundamental reason for the CP violation is spontaneous. To understand how this mechanism works we follow here S. M. Barr [34]. Suppose that in a model the fermions can be classified in two sets: F consisting of fermions with the same $SU(3)\otimes SU(2)\otimes U(1)$ quantum numbers as the ordinary light families, and R consisting of a real set of representations of $SU(3)\otimes SU(2)\otimes U(1)$ (R may content complex representations as long as it contains an equal number of conjugate representations.) Then $\overline{\theta}$ will be zero at tree level if two conditions are satisfied

- The $SU(2)\otimes U(1)$ breaking vacuum expectation values (VEVs) appear only in $F - F$ Yukawa terms not in $F - R$ or $R - R$ terms.
- CP- nonconserving VEVs appear only in $F - R$ Yukawa terms, not in $F - F$ or $R - R$ terms.

To illustrate the idea, consider augment the standard model chiral quarks with vector-like weak doublet with standard model hypercharge, and two vector-like weak singlets of charges $2/3$ and $-1/3$:

$$R = \begin{pmatrix} R_1 \\ R_2 \end{pmatrix}_{1/3}, \quad \overline{R} = \begin{pmatrix} \overline{R}_1 \\ \overline{R}_2 \end{pmatrix}_{-1/3}, \quad (R'_1)_{4/3} + (\overline{R}'_1)_{-4/3}, \quad (R'_2)_{-2/3} + (\overline{R}'_2)_{2/3}. \tag{1.87}$$

Taking into account the two conditions given previously, the standard model Higgs is required to couple only to chiral quarks $d\overline{d}$, but not to $d\overline{R}'_2$, $R_2\overline{R}'_2$, $R_2\overline{d}$, $R'_2\overline{R}_2$. Mass terms $R_2\overline{R}_2$, $R'_2\overline{R}'_2$ are allowed. CP violation comes about only through the phase carried by the scalar singlet, S, that couples to dR_2 and $R'_2\overline{d}$. These rules yield the mass matrix

$$\begin{pmatrix} d, & R_2, & R_2' \end{pmatrix} \begin{pmatrix} \langle H_d \rangle & \langle S \rangle & 0 \\ 0 & M & 0 \\ \langle S' \rangle & 0 & M' \end{pmatrix} \begin{pmatrix} \overline{d} \\ \overline{R}_2 \\ \overline{R}_2' \end{pmatrix}, \tag{1.88}$$

where $\langle H_d \rangle$, M, M' are real, and $\langle S \rangle$, $\langle S' \rangle$ are complex. The determinant of mass matrix, $\langle H_d \rangle (MM')$, is real. A similar matrix obtains in the $2/3$ charged sector. There will be loop corrections to the phase of the determinant which will induce a non-zero value of $\overline{\theta}$. These will be model dependent. In the particular model of the Ref. [33] they are shown to be small.

Introducing an additional chiral symmetry is a very natural solution for the strong CP problem, as this is chiral, it rotates this $\overline{\theta}$ parameter away. There are two ways to introduce this symmetry:

- The up-quark is massless [35].
- The standard model has an additional global $U(1)$ chiral symmetry, known as $U(1)_{\mathrm{PQ}}$ [36, 37].

The first possibility works in the following way. The path integral for QCD with one quark Ψ (up-quark) massless is

$$\begin{aligned} Z_\theta(J) &= \int \mathcal{D}A\, \mathcal{D}\Psi\, \mathcal{D}\overline{\Psi} \\ &\times \exp\, i \int \mathbf{d}^4x\, \mathbf{Tr} \left[i\overline{\Psi}\gamma^\mu D_\mu \Psi - \frac{1}{2} G^{\mu\nu} G_{\mu\nu} - \frac{g^2 \overline{\theta}}{16\pi^2} \widetilde{G}^{\mu\nu} G_{\mu\nu} + J^\mu A_\mu \right]. \end{aligned} \tag{1.89}$$

Under a $U(1)_{\mathrm{A}}$ transformation

$$\Psi \to e^{-i\alpha\gamma_5}\Psi, \tag{1.90}$$

$$\overline{\Psi} \to \overline{\Psi} e^{-i\alpha\gamma_5}, \tag{1.91}$$

the integration measure picks up a phase factor

$$\mathcal{D}\Psi\, \mathcal{D}\overline{\Psi} \to \exp\left[-i \int \mathbf{d}^4x\, \frac{g^2\alpha}{16\pi^2} \widetilde{G}^{a\mu\nu} G^a_{\mu\nu} \right] \mathcal{D}\Psi\, \mathcal{D}\overline{\Psi}, \tag{1.92}$$

because the $U(1)_{\mathrm{A}}$ symmetry is anomalous as showed in the previous section. Thus

$$\begin{aligned} Z_\theta(J) &\to \int \mathcal{D}A\, \mathcal{D}\Psi\, \mathcal{D}\overline{\Psi} \\ &\times \exp\, i \int \mathbf{d}^4x\, \mathbf{Tr} \left[i\overline{\Psi}\gamma^\mu D_\mu \Psi - \frac{1}{2} G^{\mu\nu} G_{\mu\nu} - \left(\overline{\theta} + 2\alpha\right) \frac{g^2}{16\pi^2} \widetilde{G}^{\mu\nu} G_{\mu\nu} + J^\mu A_\mu \right]. \end{aligned} \tag{1.93}$$

Hence, $\overline{\theta}$ can be taken away from the QCD Lagrangian by doing a chiral transformation with $\alpha = -\overline{\theta}/2$. The question for this possibility is, "is the massless up-quark phenomenologically viable?" It is widely believed that within the context of the lowest-order chiral perturbation theory, $m_u = 0$ is inconsistent with the observed mesons and baryon masses, $\rho - \omega$ mixing, $\Sigma^0 - \Lambda$ mixing, $\eta \to 3\pi$, and lattice QCD calculations [38–42]. Those calculations show that the ratio between m_u and m_d is

$$m_u/m_d = 0.410 \pm 0.036. \tag{1.94}$$

Therefore this possibility is disfavored.

The second possibility, which seems to be the most attractive one, is when the additional global $U(1)_{\text{PQ}}$ chiral symmetry introduced into the entire Lagrangian is spontaneously broken down. Thus a new pseudo-NG boson called the axion, $a(x)$, appears in the physical scalar spectrum. Because the axion is the NG boson of the $U(1)_{\text{PQ}}$ symmetry, it shifts

$$a(x) \to a(x) + \alpha f_a, \tag{1.95}$$

when an $U(1)_{\text{PQ}}$ transformation is done. The f_a parameter in the Eq. (1.95) is associated with the breaking of the $U(1)_{\text{PQ}}$ symmetry. In other words, this solution try to mimic dynamically the shift symmetry $\overline{\theta} \to \overline{\theta} + 2\alpha$ of the previous "solution".

Under the symmetry transformation given in the Eq. (1.95), the effective Lagrangian undergoes the transformation

$$\delta\mathcal{L}_{\text{eff}} = -\frac{\alpha A}{32\pi^2}\epsilon_{\mu\nu\rho\sigma}G^{a\mu\nu}G^{a\rho\sigma}, \tag{1.96}$$

where A is a dimensionless constant of order unity, characterizing the anomaly. Then the terms in the effective Lagrangian involving $a(x)$ are

$$\mathcal{L}_a = -\frac{1}{2}\partial_\mu a\partial^\mu a - \frac{1}{32\pi^2}\frac{Aa}{f_a}\epsilon_{\mu\nu\rho\sigma}G^{a\mu\nu}G^{a\rho\sigma} + \dots\,. \tag{1.97}$$

Comparing Eqs. (1.89) and (1.97) for a $a(x)$ constant, it can be seen that all observables will be functions not of $a(x)$ and $\overline{\theta}$ separately, but only of $\overline{\theta} + Aa/f_a$. If everything in the theory apart from the theta term (1.89) and (1.97) conserves P and CP, then effective potential will be even in $\overline{\theta} + Aa/f_a$, and it will have a stationary point at $\overline{\theta} + Aa/f_a = 0$, preserving the conservations of P and CP. This is basically the philosophy behind the the PQ mechanism.

Now, it is useful to illustrate explicitly some other properties of this type of solution showing the main features of two classical models of the axion, known as DFSZ axion [43, 44] and KSVZ axion [45, 46]. Thus, let us briefly review them.

1.4 DFSZ Axion

Since this model is identical to that of Peccei and Quinn (PQ) [37] model except for the addition of a complex scalar field, Φ, which is a singlet under the $SU(2)_L \otimes U(1)_Y$ gauge group (the $SU(2)_L \otimes U(1)_Y$ group is known as the standard model (SM) model group), we consider firstly that model.

The fermionic matter content of the PQ model is the same as in the SM model, i.e. neither leptons nor quarks are added. The way as the $U(1)_{\mathrm{PQ}}$ symmetry is implemented is introducing a new Higgs field, H_u, which couples only to the quarks. To avoid naturally tree-level flavor changing neutral current effects (FCNC), each Higgs doublet is coupled to one quark charge sector. With this, the relevant Yukawa interactions read

$$\mathcal{L}_{\mathrm{Y}} = G^u_{ij}\overline{Q}_{Li}u_{Rj}H_u + G^d_{ii}\overline{Q}_{Li}d_{Ri}H_d + G^e_{ii}\overline{L}_{Li}e_{Ri}H_d + \mathrm{H.c.}, \tag{1.98}$$

where

$$Q_{Li} = (u_i, d_i)^T\,,\; L_{Li} = (\nu_i, e_i)^T\,,\; H_d = \left(H_d^+, H_d^0\right),\; H_u = \left(H_u^0, H_u^-\right), \tag{1.99}$$

and $i,\, j = 1,\, 2,\, 3$. The Yukawa Lagrangian given in Eq. (1.98) has the required global PQ symmetry. The charge assignment is given in the Table (1.1). With two

Table 1.1: Assignment of PQ charges in the DFSZ model.

	H_u	H_d	Q_{Li}	u_{Ri}	d_{Ri}	L_{Li}	e_R
Y	1	-1	$1/3$	$4/3$	$-2/3$	-1	-2
PQ	1	1	$-1/2$	$1/2$	$1/2$	$-1/2$	$1/2$

Higgs scalar fields, the most general scalar potential which is invariant under the PQ symmetry is given by

$$\begin{aligned} V_{H_{u,d}} &= \sum \left(-\mu_a^2 H_a^\dagger H_a + \lambda_{aa}\left(H_a^\dagger H_a\right)^2\right) + \lambda_{ud}H_u^\dagger H_u H_d^\dagger H_d \\ &\quad + \lambda'_{ud}H_u^\dagger H_d H_d^\dagger H_u\,. \end{aligned} \tag{1.100}$$

When both H_d and H_u acquire vacuum expectation values (VEVs), the PQ symmetry is broken at the same scale as the electroweak symmetry. Since V_{PQ} breaks the SM group it can not be larger than $G_{\mathrm{F}}^{-1/2}$, where G_{F} is the Fermi coupling constant. Thus the axion has mass and coupling strength that make it ruled out by the experiments [47].

As it was previously said, to overcome this difficulty in this model one singlet scalar Φ is introduced. Because Φ is a singlet under the SM group, it can acquire a VEV, V_Φ, which is required to be larger than $G_{\mathrm{F}}^{-1/2}$, i.e. $V_\Phi \gg G_{\mathrm{F}}^{-1/2}$. With this, the couplings of the axion to the matter ($\propto 1/V_\Phi$) become suppressed and in agreement with experiments. It is the reason why this type of axion is known as the “invisible” axion.

The singlet Φ field can gain PQ charge coupling to the fields H_u and H_d in two different ways. The former is a cubic interaction

$$\mu H_u^i \epsilon_{ij} H_d^j \Phi + \text{H.c.}, \tag{1.101}$$

where μ is a constant with mass units and ϵ_{ij} is the completely antisymmetric symbol of $SU(2)$. This type of interaction was considered in the Ref. [44]. And the second possibility, which is considered here, has a quartic term

$$\lambda_{PQ} H_u^i \epsilon_{ij} H_d^j \Phi^2 + \text{H.c.}, \tag{1.102}$$

where λ_{PQ} is a dimensionless constant. This was considered in the Ref. [43]. The cubic interaction can be set to zero naturally, by imposing the discrete symmetry $\Phi \to -\Phi$. In the case with a quartic term the scalar potential is simply

$$V_{\text{classic}} = V_{H_{u,d}} + V_\Phi + \lambda_{\text{PQ}} H_u^i \epsilon_{ij} H_d^j \Phi^2 + \text{H.c.}, \tag{1.103}$$

where V_Φ

$$V_\Phi = -\mu_\Phi^2 \Phi^\dagger \Phi + \lambda_\Phi \left(\Phi^\dagger \Phi\right)^2. \tag{1.104}$$

Now, it is the time to give an explicitly expression to the axion. To do that, let us before discussing briefly the procedure that has been used throughout this work. At the classical level, the squared masses of the scalars in the theory are the eigenvalues of the following matrix [5]

$$m_{ij}^2 = \left.\frac{\partial^2 V_{\text{classic}}}{\partial \phi_i \partial \phi_j}\right|_{\phi_i = V_i}, \tag{1.105}$$

where the ϕ_i in Eq. (1.105) makes reference to the components of the scalar fields of the model, for instance Re H_u^0, Im H_u^0, Re H_u^-, Im H_u^-, etc. The V_is are the VEVs of the scalar fields. It is important to say that

$$\left.\frac{\partial V_{\text{classic}}}{\partial \phi_i}\right|_{\phi_i = V_i} = 0, \tag{1.106}$$

because $\phi_i = V_i$ minimizes V_{classic}. After finding the eigenvalues and eigenstates of the matrix in Eq. (1.105), it is necessary isolate the physical axion field, which at the classical level is massless, from the would be NG bosons which in the unitary gauge will become the longitudinal components of the massive gauge bosons. In general, the would be NG bosons are generated by some linear combination of the rows of the matrix

$$F^a{}_i = g_a \left(T^a\right)_{ij} V_j, \tag{1.107}$$

where i runs on the total number of real fields in the model, a runs on all generators of the gauge groups, g_as are the coupling constants of the gauge groups, and the T^a matrices are given by [9]

$$T^a = \begin{pmatrix} -\text{Im }\tau^a & -\text{Re }\tau^a \\ \text{Re }\tau^a & -\text{Im }\tau^a \end{pmatrix}. \tag{1.108}$$

The τ^as are the generators of the gauge group. To put all that in a short way, the axion field, a, must satisfy

$$\sum_i a_i * F^b{}_i = 0, \ \forall\, b, \tag{1.109}$$

where a_i are the components of the axion field. These are given by

$$a = \sum_n c_n * b_n, \tag{1.110}$$

where b_n are the eigenstates that are a base to the linear vector space of the massless scalars. The c_n are constants which are found using the Eqs. (1.109).

Now, applying the above procedure to the DFSZ axion, we have

$$H_u = \begin{pmatrix} \text{Re } H_u^0 \\ \text{Re } H_u^- \\ \text{Im } H_u^0 \\ \text{Im } H_u^- \end{pmatrix}, \ H_d = \begin{pmatrix} \text{Re } H_d^- \\ \text{Re } H_d^0 \\ \text{Im } H_d^- \\ \text{Im } H_d^0 \end{pmatrix}, \ \Phi = \begin{pmatrix} \text{Re } \Phi \\ \text{Im } \Phi \end{pmatrix}. \tag{1.111}$$

Thus we have ten real scalar fields. The generators of the gauge groups, $SU(2)_L \otimes U(1)_Y$, in the real representations are

$$\begin{aligned} T^1 &= \frac{-1}{2}\begin{pmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & -1 & 0 & 0 \\ -1 & 0 & 0 & 0 \end{pmatrix}, \ T^2 = \frac{-1}{2}\begin{pmatrix} 0 & -1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 \\ 0 & 0 & 1 & 0 \end{pmatrix}, \\ T^3 &= \frac{-1}{2}\begin{pmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 \\ -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{pmatrix}, \ T^4 = -Y_s\begin{pmatrix} 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{pmatrix}. \end{aligned} \tag{1.112}$$

where the Y_s are the hypercharges of the scalar fields given in the Table 1.1. The vector of the VEVs is

$$V = (V_u, 0, 0, 0, 0, V_d, 0, 0, V_\Phi, 0)^T. \tag{1.113}$$

Finally, using the Eqs. (1.105-1.107), (1.109-1.110), and (1.112) the axion reads [43]

$$\begin{aligned} a(x) &= \left[2V_u V_d \left(V_u \text{Im } H_d^0 + V_d \text{Im } H_u^0\right) - V_{\text{EW}}^2 V_\Phi \text{Im } \Phi\right] \\ &\quad \times \left[V_{\text{EW}} \left(V_{\text{EW}}^2 V_\Phi^2 + 4V_u^2 V_d^2\right)^{1/2}\right]^{-1}, \end{aligned}$$

where $V_{\rm EW} \equiv \left(V_u^2 + V_d^2\right)^{1/2}$. Note that in the limit $V_\phi \gg V_u$, V_d,

$$a\left(x\right) \simeq -\text{Im }\Phi + \left(2V_uV_d/V_\Phi V_{\rm EW}^2\right)\left(V_u\text{Im }H_d^0 + V_d\text{Im }H_u^0\right), \tag{1.114}$$

i.e. the axion is primarily composed of the Φ field.

Although, the axion is massless at tree-level, it gains mass because the $U\left(1\right)_{\rm PQ}$ symmetry is anomalous. QCD effects (such as instantons [15]) that violate $U\left(1\right)_{\rm PQ}$ give a small mass to the axion [48] given by

$$m_a^2 = \left(f_\pi^2/f_a^2\right)m_\pi^2 N^2 Z\left(1+Z\right)^{-2}, \tag{1.115}$$

where $Z = m_u/m_d \simeq 0.56$ (m_u and m_d are the u- and d-quark masses respectively), N is the number of quark doublets, m_π and $f_\pi \approx 130$ MeV are the mass and decay constant of the π^0. f_a is the axion decay constant given in this model by [43]

$$f_a = \left(2V_{\rm EW}\right)^{-1}\left(V_{\rm EW}^2V_\Phi^2 + 4V_u^2V_d^2\right)^{1/2}. \tag{1.116}$$

Astrophysical and cosmological considerations limit f_a to be between

$$10^9 \text{ GeV} \leq f_a \leq 10^{12} \text{ GeV}. \tag{1.117}$$

1.5 KSVZ Axion

This type of model for the axion is simpler than the DFSZ axion model and was introduced by the first time in the Refs. [45, 46]. The gauge group is the same as the SM group and except for weak singlets both Q quark and σ scalar, the matter content is the same as in the SM one. To introduce naturally the PQ symmetry a Z_2 symmetry is imposed, such that

$$Z_2: \ Q_L \to -Q_L, \ Q_R \to Q_R, \ \sigma \to -\sigma; \tag{1.118}$$

and all the other fields are invariant. Notice that this symmetry guarantees the absence of the bare-mass term $m\overline{Q}Q$. The invariant Yukawa term of Q and the Higgs potential V are simply

$$\mathcal{L}_{Y_Q} = Y\overline{Q_L}Q_R\sigma + \text{H.c.}, \tag{1.119}$$

$$\begin{aligned} V &= -\mu_H^2 H_d^\dagger H_d - \mu_\sigma^2\sigma^\dagger\sigma + \lambda_H\left(H_d^\dagger H_d\right)^2 \\ &\quad +\lambda_\sigma\left(\sigma^\dagger\sigma\right)^2 + \lambda_{H\sigma}\left(H_d^\dagger H_d\right)\left(\sigma^\dagger\sigma\right), \end{aligned} \tag{1.120}$$

where H_d is the SM Higgs and it is given by the Eq. (1.99).

In this model the PQ symmetry is given by

$$Q \to e^{i\gamma_5\alpha}Q, \ \sigma \to e^{-2i\alpha}\sigma. \tag{1.121}$$

which is used to rotate away the term

$$\mathcal{L}_{\overline{\theta}} = \frac{g^2\overline{\theta}}{16\pi^2}\mathbf{Tr}\left[\widetilde{G}^{\mu\nu}G_{\mu\nu}\right]. \tag{1.122}$$

It is important to say that this is possible provided the Q quark belongs to a nontrivial representation of $SU(3)_C$.

The field σ develops a non-vanishing vacuum expectation value,

$$|\langle\sigma\rangle| \equiv V_\sigma = \mu_\sigma/\sqrt{2\lambda_\sigma}, \tag{1.123}$$

thus the model has a new scalar σ_R with mass $\mu_\sigma\sqrt{2}$ and a pseudoscalar, the axion, whose mass vanishes in the classical approximation. The new quark Q gain a mass equal to YV_σ. In order to the axion phenomenology agree with the cosmological and astrophysical data $V_\sigma \gg V_{H_d}$. More precisely $V_\sigma \gtrsim 10^9$ GeV. Thus the Q quark is not directly observable in low energy experiments. The only interesting effect in low energy is determined by the heavy quark loop which induces a term in the effective Lagrangian

$$\frac{g^2}{16\pi^2}\frac{a(x)}{V_\sigma}\mathbf{Tr}\left[\widetilde{G}^{\mu\nu}G_{\mu\nu}\right], \tag{1.124}$$

where $a(x)$ is the axion field and it is related to the original field $\sigma(x)$ in the following way

$$\sigma(x) = 2^{-1/2}(V_\sigma + \sigma_R)\,e^{ia(x)/V_\sigma}. \tag{1.125}$$

The interaction axion-gluon in the Eq. (1.124) results in the substitution of the parameter $\overline{\theta}$ by $\theta_{\text{eff}} = \overline{\theta} + a(x)/V_\sigma$ and the vacuum expectation value of $a(x)$ can be seen to cancel the original $\overline{\theta}$.

In this model, the term in the Eq. (1.124) also gives mass to the axion [46]

$$m_a^2 = \frac{f_\pi^2 m_\pi^2}{\sqrt{2}V_\sigma^2}\frac{m_u m_d}{(m_u^2 + m_d^2)}, \tag{1.126}$$

similarly to the DFSZ model.

1.6 Searches for the Axion

During the last thirty years, several experiments searching for signals of the axion existence have been performed. Among the most important ones are Cern Axion Solar Telescope (CAST), Tokyo Axion helioscope experiment, Brookhaven-Fermilab-Rutherford-Trieste (BFRT) collaboration, SOLAX, COSME, DAMA. The majority of the previous experiments are based on the (reversed) Primakoff effects, i.e. $a + \gamma_{\text{virtual}} \to \gamma$, where the axion interacts with a virtual photon provided by a transversal magnetic field and reconverts into a real photon. The absence of any axion signal has imposed bounds on the mass and the couplings of the axion.

As seen from the Earth, the most important and strongest astrophysical source for axion is the core of the Sun. Particles like the axion have the following coupling to two photons

$$\mathcal{L}_{a\gamma\gamma} = \frac{g_{a\gamma\gamma}}{4} F_{\mu\nu} \widetilde{F}^{\mu\nu} a = -g_{a\gamma\gamma} \mathrm{E}.\mathrm{B} a, \tag{1.127}$$

where $F_{\mu\nu}$ is the electromagnetic field-strength tensor, $\widetilde{F}^{\mu\nu}$ its dual, and E and B the electric and magnetic fields, respectively. The coupling constant $g_{a\gamma\gamma}$ is given by

$$g_{a\gamma\gamma} = \frac{\alpha}{2\pi f_a} \left(\frac{E}{N} - \frac{2}{3} \frac{4+z}{1+z} \right) = \frac{\alpha}{2\pi} \left(\frac{E}{N} - \frac{2}{3} \frac{4+z}{1+z} \right) \frac{1+z}{z^{1/2}} \frac{m_a}{m_\pi f_\pi}, \tag{1.128}$$

where $z = m_u/m_d \simeq 0.56$, α is the fine-structure constant; E and N, respectively, are the electromagnetic and color anomaly of the axial current associated with the axion field. E and N are given by [49]

$$N = \sum X_i T(R_i), \quad E = \sum X_i Q_i^2 D(R_i), \tag{1.129}$$

where $T(R_i)$ is the Dynkin index of the $SU(3)_C$ representation of the quark q_i, $D(R_i)$ is the dimension of the representation, Q_i and X_i, respectively, are the electric and PQ charges of the quark q_i. In general, $g_{a\gamma\gamma}$ is model-dependent.

The coupling in Eq. (1.127) allows the production of axions from thermal photons in the fluctuating electromagnetic fields of the stellar plasma. Thus if axions scape from the Sun core, these reach the Earth, and in principle, they can be detected using the reversed Primakoff effect, since the axion interacts with electromagnetic field producing X-rays. This type of experiment is known as the axion helioscope experiment and the idea was firstly proposed by P. Sikivie in 1983 [50]. The absence of the axion signals resulted in upper limits on $g_{a\gamma\gamma}$ [51]

$$g_{a\gamma\gamma} \leq 6 \times 10^{-10} \text{ GeV}^{-1} \text{ for } m_a < 0.03 \text{ eV}, \tag{1.130}$$

$$g_{a\gamma\gamma} \leq 6.8 - 10.9 \times 10^{-10} \text{ GeV}^{-1} \text{ for } m_a < 0.3 \text{ eV}, \tag{1.131}$$

for the Tokyo Axion helioscope experiment, and

$$g_{a\gamma\gamma} \leq 0.88 \times 10^{-10} \text{ GeV}^{-1} \text{ for } m_a < 0.02 \text{ eV}, \tag{1.132}$$

at the 95% confidence level for the CAST [52]. Using different experimental techniques the collaborations SOLAX, COSME and DAMA achieved similar limits [53–56]

$$g_{a\gamma\gamma} \leq 2.7 \times 10^{-9} \text{ GeV}^{-1} \text{ (SOLAX)}, \tag{1.133}$$

$$g_{a\gamma\gamma} \leq 2.8 \times 10^{-9} \text{ GeV}^{-1} \text{ (COSME)}, \tag{1.134}$$

$$g_{a\gamma\gamma} \leq 1.7 \times 10^{-9} \text{ GeV}^{-1} \text{ (DAMA)}, \tag{1.135}$$

$$g_{a\gamma\gamma} \leq 6.7 \times 10^{-7} \text{ GeV}^{-1} \text{ for } m_a < 10^{-3} \text{ eV (BFRT)}. \tag{1.136}$$

Other important and strong type of limits come from Globular-Cluster stars and the supernova (SN) 1987A. Roughly speaking, these limits are based on energy-loss argument which say, in a short way, that the existence of new particles like the axion coupling to the photons, leptons and hadrons would be a new channel for energy loss in the stars, and thus, the evolutions of these objects should significantly change. Studies considering these factors have been performed obtain strong limits on $g_{a\gamma\gamma}$,

$$g_{a\gamma\gamma} < 10^{-10}\ \text{GeV}^{-1}, \tag{1.137}$$

comes from the Globular-Cluster [57] stars and

$$f_a < 4 \times 10^8\ \text{GeV and } m_a \lesssim 16\ \text{meV}, \tag{1.138}$$

comes from the SN 1987A [58].

Finally, from cosmology it was found that a general lower limit could be placed on the axion mass. At the time of the big bang, axions would be produced in a significant amount by different mechanisms as misalignment, or the decay of axionic strings. Although there are still substantial uncertainties on the calculations of the relic axion abundance, an estimate for the total contributions to the energy density of the universe from axions created via the vacuum misalignment method can be expressed as [59, 60]

$$\Omega_a \sim \left(\frac{5\ \mu\text{eV}}{m_a}\right)^{7/6}, \tag{1.139}$$

which put a lower limit on the axion mass of $m_a \geq 10^{-6}$ eV because any lighter axion would overclose the universe, $\Omega_a \geq 1$.

Chapter 2

NATURAL PECCEI-QUINN SYMMETRY IN THE 3-3-1 MODEL WITH A MINIMAL SCALAR SECTOR

2.1 Introduction

The standard model (SM) of the elementary particles physics successfully describes almost all of the phenomenology of the strong, electromagnetic, and weak interactions. However, from the experimental point of view, the need to go to physics beyond the standard model comes from the neutrino masses and mixing, which are required to explain the solar and atmospheric neutrino data. On the other hand, from the theoretical point of view, the SM cannot be taken as the fundamental theory since some important contemporary questions, like the number of generations of quarks and leptons, do not have an answer in its context. Unfortunately we do not know what the physics beyond the SM should be. A likely scenario is that at the TeV scale physics will be described by models which, at least, give some insight into the unanswered questions of the SM.

A way of introducing new physics is to enlarge the symmetry gauge group. For example, the gauge symmetry may be $SU(3)_C \otimes SU(3)_L \otimes U(1)_X$, instead of that of the SM. Models based on this gauge group have become known as 3-3-1 models [61–63]. Although the 3-3-1 models coincide with the SM at low energies, they explain some fundamental questions. This is the case of the number of generations cited above. In the 3-3-1 model framework, the number of generations must be three, or a multiple of three, in order to cancel anomalies. This is because the model is anomaly-free only if there is an equal number of triplets and antitriplets, including the color degrees of freedom. In this case, each generation is anomalous. The anomaly cancellation only occurs for the three, or multiple of three, generations together, and not generation by generation like in the SM. This provides, at least, a first step towards the understanding of the flavor question. Other interesting features of the 3-3-1 models concern the electric charge quantization and the vectorial character of the electromagnetic interaction [64, 65]. These questions can be accommodated in the SM. However, in the 3-3-1 models these questions are related one to another and are independent of the nature of the neutrinos.

In recent literature it is found studies about the most different aspects of the 3-3-1 model phenomenology. Among others, a fundamental puzzling aspect is, Why is the CP nonconservation in the strong interactions so small [66, 67]? The last question, quantified by the $\overline{\theta}$ parameter of the effective QCD Lagrangian, is known as the strong CP problem. Several solutions based on different ideas have been proposed. According to the framework, they are based on unconventional dynamics [26], spontaneously broken CP [33], and an additional chiral symmetry. In the framework of introducing an additional chiral symmetry, two suggestions have been made. If this symmetry is not broken, the symmetry is realized in the Wigner-Weyl manner and the only possible way of relating this unbroken chiral symmetry with flavor conserving gluons is to have at least one massless quark [35]. This suggestion is disfavored by standard current algebra analysis [38, 39]. The second possibility is that the global $U(1)$ chiral symmetry, known as $U(1)_{\mathrm{PQ}}$ [36], is spontaneously broken down which implies a Nambu-Goldstone boson (NG boson), currently known as the axion [43–45].

In this chapter, which is based on Ref. [68, 69] written by J. C. Montero and I, is considered the strong CP problem in the framework of a version of the 3-3-1 model in which the scalar sector is minimal [70]. This model has become known as the "economical 3-3-1 model." The appealing feature of this 3-3-1 model is the natural existence of a Peccei-Quinn-like (PQ-like) $U(1)$ symmetry. To study the consequences of this symmetry in this model, this chapter is organized as follows: in Sec. 2.2 the model is briefly described, and in Sec. 2.3 the consequences of the natural PQ-like symmetry in the model are analyzed and it is found that the symmetry is realized in the Wigner-Weyl manner implying three massless quarks, which disagrees with the standard current algebra analysis. Thus, the introduction of two new scalar fields, η and ϕ, are proposed in order to both give a solution to the massless quarks and implement the PQ mechanism. Since this mechanism needs the $U(1)_{\mathrm{PQ}}$ to be anomalous in order to solve the strong CP problem, it does not seem *natural* to impose this symmetry on the Lagrangian. However, it could be understood as being *natural* if it is a residual symmetry of a larger one which is not anomalous and spontaneously broken. Then, it is considered a Z_N discrete gauge symmetry to be a symmetry of the Lagrangian. The discrete gauge anomalies are canceled by a discrete version of the Green-Schwarz mechanism. After this, two Z_N symmetries, Z_{10} and Z_{11}, which protect the axion against quantum gravity effects, are explicitly shown. Finally, our conclusions are given in Sec. 2.4.

2.2 A Brief Review of the Economical 3-3-1 Model

The different models based on a 3-3-1 gauge symmetry can be classified according to the electric charge operator

$$Q = T^3 - bT^8 + X, \tag{2.1}$$

where T^3 and T^8 are the diagonal Gell-Mann matrices, X refers to the quantum number of the $U(1)_X$ group, and $b = 1/\sqrt{3}$, $\sqrt{3}$. The embedding b parameter defines the model. Here, it will be considered the model with both $b = 1/\sqrt{3}$ and the simplest scalar sector, which was proposed for the first time in Ref. [71]. It has become known in the literature as "economical 3-3-1 model." This model had origin in a systematic study of all possible 3-3-1 models without exotic electric charges [72].

To give a brief review of the main features of this model, let us say that it has a fermionic matter content given by

$$\begin{aligned}
&\Psi_{aL} = \left(\nu_a, e_a, (\nu_{aR})^C\right)_L^T \sim (\mathbf{1}, \mathbf{3}, -1/3), \quad e_{aR} \sim (\mathbf{1}, \mathbf{1}, -1),\\
&Q_{\alpha L} = \left(d_\alpha, u_\alpha, d'_\alpha\right)_L^T \sim (\mathbf{3}, \mathbf{3}^*, 0), \quad Q_{3L} = \left(u_3, d_3, u'_3\right)_L^T \sim (\ \mathbf{3}, \mathbf{3}, 1/3),\\
&u_{aR} \sim (\mathbf{3}, \mathbf{1}, 2/3), \quad u'_{3R} \sim (\mathbf{3}, \mathbf{1}, 2/3),\\
&d_{aR} \sim (\mathbf{3}, \mathbf{1}, -1/3), \quad d'_{\alpha R} \sim (\mathbf{3}, \mathbf{1}, -1/3),
\end{aligned} \tag{2.2}$$

where $a = 1, 2, 3$, $\alpha = 1, 2$ (from now on Latin and Greek letters always take the values $1, 2, 3$ and $1, 2$, respectively), and the values in the parentheses denote quantum numbers based on the $(SU(3)_C, SU(3)_L, U(1)_X)$ factor, respectively. In this model the electric charges of the exotic quarks are the same as the usual ones, i.e. $Q(d'_\alpha) = -1/3$ and $Q(u'_3) = 2/3$.

In the bosonic matter content there are only two scalar triplets, χ and ρ:

$$\chi = \left(\chi^0, \chi^-, \chi_1^0\right)^T \sim (\mathbf{1}, \mathbf{3}, -1/3), \qquad \rho = \left(\rho^+,\ \rho^0,\ \rho_1^+\right) \sim (\mathbf{1}, \mathbf{3}, 2/3). \tag{2.3}$$

These two scalars spontaneously break down the $SU(3)_L \otimes U(1)_X$ gauge group. The vacuum expection values (VEVs) in this model satisfy the constraint

$$V_{\rho^0} \equiv \left\langle \mathrm{Re}\,\rho^0 \right\rangle, \qquad V_{\chi^0} \equiv \left\langle \mathrm{Re}\,\chi^0 \right\rangle \ll V_{\chi_1^0} \equiv \left\langle\ \mathrm{Re}\,\chi_1^0 \right\rangle.$$

With the quark, lepton, and scalar multiplets above the Yukawa interactions are given by

$$\mathcal{L}_{\mathrm{Y}}^l = Y_{ab}\overline{\Psi_{aL}}e_{bR}\rho + Y'_{ab}\epsilon^{ijk}\left(\overline{\Psi_{aL}}\right)_i \left(\Psi_{bL}\right)_j^C \left(\rho^*\right)_k + \text{ H.c.}, \tag{2.4}$$

for leptons. Y_{ab} and Y'_{ab} are arbitrary complex matrices and Y'_{ab} is also antisymmetric. Throughout the chapter the convention that an addition over repeated indices is implied. The lepton masses are generated by the interactions in Eq. (2.4). The first term gives a general tree level mass matrix for the charged leptons [70]. However, for the neutrino mass generation, the interactions in the second term are not able to provide a realistic mass spectrum at the tree level. At least 1-loop corrections must be considered in order to obtain neutrino masses compatible with the solar and atmospheric neutrino data [73].

For quarks we have

$$\begin{aligned}\mathcal{L}_{\mathrm{Y}}^{q} &= G^{1}\overline{Q_{3L}}u'_{3R}\chi + G^{2}_{\alpha\beta}\overline{Q_{\alpha L}}d'_{\beta R}\chi^{*} + G^{3}_{a}\overline{Q_{3L}}d_{aR}\rho \\ &+ G^{4}_{\alpha a}\overline{Q_{\alpha L}}u_{aR}\rho^{*} + G^{5}_{a}\overline{Q_{3L}}u_{aR}\chi + G^{6}_{\alpha a}\overline{Q_{\alpha L}}d_{aR}\chi^{*} \\ &+ G^{7}_{\alpha}\overline{Q_{3L}}d'_{\alpha R}\rho + G^{8}_{\alpha}\overline{Q_{\alpha L}}u'_{3R}\rho^{*} + \text{H.c.},\end{aligned} \tag{2.5}$$

where G^i are arbitrary complex matrices. Notice that the Yukawa interactions given in Eqs. (2.4) and (2.5) are the most general allowed by the gauge symmetries. Notice that this model is exactly one given in Refs. [70] and [74]; i.e., no additional symmetries are imposed, contrary to what is done in Ref. [75] where a Z_2 symmetry is imposed.

The most general scalar potential invariant under the gauge symmetry is

$$\begin{aligned}V_{\mathrm{H}} &= \mu_{\chi}^{2}\chi^{\dagger}\chi + \mu_{\rho}^{2}\rho^{\dagger}\rho + \lambda_{1}\left(\chi^{\dagger}\chi\right)^{2} + \lambda_{2}\left(\rho^{\dagger}\rho\right)^{2} \\ &+ \lambda_{3}\left(\chi^{\dagger}\chi\right)\left(\rho^{\dagger}\rho\right) + \lambda_{4}\left(\chi^{\dagger}\rho\right)\left(\rho^{\dagger}\chi\right).\end{aligned} \tag{2.6}$$

One of the main features of this model is that its scalar sector is the simplest possible. In principle, this should make the scalar potential analysis easier. A study of the stability of this scalar potential is presented in Ref. [76].

2.3 $U(1)_{\mathbf{PQ}}$ Symmetry in the Economical 3-3-1 Model

A $U(1)_{\mathrm{PQ}}$ symmetry is global and chiral [36, 37]; i.e., it treats the left- and right-handed parts of a Dirac field differently. Moreover, it must be both a symmetry of the entire Lagrangian and valid only at the classical level. In renormalizable theories, the key ingredient of the $U(1)_{\mathrm{PQ}}$ is that it must be afflicted by a color anomaly; i.e., its associated current, j_{μ}^{PQ}, must obey

$$\partial^{\mu} j_{\mu}^{\mathrm{PQ}} \supset \frac{Ng^{2}}{16\pi^{2}}G\widetilde{G}, \tag{2.7}$$

being $G\widetilde{G} = \frac{1}{2}\epsilon^{\mu\nu\sigma\tau}G^{b}_{\mu\nu}G^{b}_{\sigma\tau}$, and $G^{b}_{\mu\nu}$ is the color field strength tensor ($b = 1,\ldots,8$). N must not be zero.

Now, it is time to prove that the economical 3-3-1 model entire Lagrangian is naturally invariant under a $U(1)_{\rm PQ}$ symmetry transformation. To do so, let us search for how many $U(1)$ symmetries the model has. First of all, the relations that these symmetries must obey in order to keep the entire Lagrangian invariant are written. From Eqs. (2.4-2.6) the following relations are obtained

$$-X_{Q_3} + X_{u'_{3R}} + X_\chi = 0, \qquad -X_Q + X_{d'_R} - X_\chi = 0, \tag{2.8}$$

$$-X_{Q_3} + X_{u_R} + X_\chi = 0, \qquad -X_Q + X_{d_R} - X_\chi = 0, \tag{2.9}$$

$$-X_{Q_3} + X_{d_R} + X_\rho = 0, \qquad -X_Q + X_{u_R} - X_\rho = 0, \tag{2.10}$$

$$-X_{Q_3} + X_{d'_R} + X_\rho = 0, \qquad -X_Q + X_{u'_{3R}} - X_\rho = 0, \tag{2.11}$$

$$-X_\Psi + X_{e_R} + X_\rho = 0, \qquad -2X_\Psi - X_\rho = 0, \tag{2.12}$$

where the notation X_ψ above is to be understood as the $U(1)$ charge of the ψ field. Solving the equations above, three independent $U(1)$ symmetries are found. One of these is the $U(1)_X$ gauge symmetry. The other two are the usual baryon number symmetry, $U(1)_B$, and a chiral symmetry acting on the quarks, $U(1)_{\rm PQ}$. Thus, the model actually has a larger symmetry: $SU(3)_C \otimes SU(3)_L \otimes U(1)_X \otimes U(1)_B \otimes U(1)_{\rm PQ}$. The two last symmetries are global. This is summarized in Table 2.1. Notice that the

Table 2.1: Assignment of quantum charges in the economical 3-3-1 model.

	$Q_{\alpha L}$	Q_{3L}	(u_{aR}, u'_{3R})	$(d_{aR}, d'_{\alpha R})$	Ψ_{aL}	e_{aR}	ρ	χ
$U(1)_X$	0	1/3	2/3	$-1/3$	$-1/3$	-1	2/3	$-1/3$
$U(1)_B$	1/3	1/3	1/3	1/3	0	0	0	0
$U(1)_{\rm PQ}$	-1	1	0	0	$-1/2$	$-3/2$	1	1

$U(1)_{\rm PQ}$ chiral symmetry is afflicted by a color anomaly in the following way:

$$A_{\rm PQ} \propto -X_\rho - 2X_\chi = -3, \tag{2.13}$$

where $A_{\rm PQ}$ is the coefficient of the $[SU(3)_C]^2 U(1)_{\rm PQ}$ anomaly. Therefore, this chiral symmetry is a PQ-like symmetry. Also, notice that in this case the $U(1)_{\rm PQ}$ is an accidental symmetry; i.e., it follows from the gauge local symmetry plus renormalizability. In other words, the economical model naturally has a PQ symmetry. The naturalness of the $U(1)_{\rm PQ}$ in the economical 3-3-1 model is a key point. In my understanding, since $U(1)_{\rm PQ}$ symmetry is anomalous its imposition is not sensible in the sense that in the absence of further constraints on very high energy physics it should be expected all relevant and marginally relevant operators that are forbidden only by this symmetry to appear in the effective Lagrangian with coefficient of order one, but if this symmetry follows from some other free anomaly symmetry, in this case from the gauge symmetry, all terms which violate it are then irrelevant in the renormalization group sense.

Unfortunately, when χ and ρ acquire VEVs different from zero, a subgroup of $U(1)_X \otimes U(1)_{\mathrm{PQ}}$ remains unbroken; i.e., the symmetry-breaking pattern is

$$
\begin{aligned}
SU(3)_L \otimes U(1)_X \otimes U(1)_{\mathrm{PQ}} &\xrightarrow{\langle\chi\rangle} SU(2)_L \otimes U(1)_Y \otimes U(1)'_{\mathrm{PQ}} \\
&\xrightarrow{\langle\rho\rangle} U(1)_Q \otimes U(1)''_{\mathrm{PQ}}\,,
\end{aligned}
\tag{2.14}
$$

where $U(1)_Q$ is the electromagnetic symmetry. The $SU(3)_C$ and $U(1)_B$ groups have been omitted in the expression above because these are both unbroken and irrelevant to the current analysis. An explicit expression of the $U(1)'_{\mathrm{PQ}}$ symmetry can be easily written as

$$
U(1)'_{\mathrm{PQ}} \equiv U(1)_{\mathrm{PQ}} + 3U(1)_X\,. \tag{2.15}
$$

Also, note that $U(1)'_{\mathrm{PQ}}$ and $U(1)''_{\mathrm{PQ}}$ are PQ-like symmetries because these are chiral and afflicted by a color anomaly.

As a consequence of the unbroken $U(1)''_{\mathrm{PQ}}$ chiral symmetry [i.e., $U(1)''_{\mathrm{PQ}}$ is realized in the Wigner-Weyl manner], no axion appears in the scalar mass spectrum. Instead of that, some quarks remain massless after the spontaneous symmetry breaking, and these will remain massless to all orders of perturbation theory.

To illustrate the preceding, let us explicitly calculate the mass spectra of scalars and quarks. First, the scalar mass spectrum is given by

$$
\begin{aligned}
m^2_{H_1,H_2} &= \lambda_1 V^2_{\rho^0} + \left(V^2_{\chi^0} + V^2_{\chi^0_1}\right)\lambda_2 \\
&\pm \sqrt{\left(V^2_{\rho^0}\lambda_1 - \left(V^2_{\chi^0} + V^2_{\chi^0_1}\right)\lambda_2\right)^2 + \left(V^2_{\chi^0} + V^2_{\chi^0_1}\right)V^2_{\rho^0}\lambda_3^2},
\end{aligned}
\tag{2.16}
$$

$$
m^2_{H_3^\pm} = \frac{1}{2}\left(V^2_{\rho^0} + V^2_{\chi^0} + V^2_{\chi^0_1}\right)\lambda_4, \tag{2.17}
$$

where V_{ρ^0}, V_{χ^0}, $V_{\chi^0_1}$ are the VEVs of ρ^0, χ^0, χ^0_1, respectively. For simplicity, all the VEVs have been assumed to be real. Additionally, there are exactly 8 NG bosons that will become the longitudinal components of the 8 gauge bosons [71]. The absence of one physical massless state (or axion) in the scalar spectrum shows that the $U(1)''_{\mathrm{PQ}}$ symmetry remains unbroken after the spontaneous symmetry breaking.

On the other hand, in the quark spectra, there are three massless states, one in the up-quark sector and two in the down-quark sector. First, consider the up-quark mass matrix at the tree level which is written as

$$
\overline{\mathbf{u}_L} M_u^{(0)} \mathbf{u}_R \equiv \frac{1}{\sqrt{2}}\overline{\mathbf{u}_L}
\begin{bmatrix}
G^4_{11}V_{\rho^0} & G^4_{12}V_{\rho^0} & G^4_{13}V_{\rho^0} & G^8_1 V_{\rho^0} \\
G^4_{21}V_{\rho^0} & G^4_{22}V_{\rho^0} & G^4_{23}V_{\rho^0} & G^8_2 V_{\rho^0} \\
G^5_1 V_{\chi^0} & G^5_2 V_{\chi^0} & G^5_3 V_{\chi^0} & G^1 V_{\chi^0} \\
G^5_1 V_{\chi^0_1} & G^5_2 V_{\chi^0_1} & G^5_3 V_{\chi^0_1} & G^1 V_{\chi^0_1}
\end{bmatrix}
\mathbf{u}_R, \tag{2.18}
$$

where $\overline{\mathbf{u}_L} \equiv \left(\overline{u_{1L}}, \overline{u_{2L}}, \overline{u_{3L}}, \overline{u'_{3L}}\right)$ and $\mathbf{u}_R \equiv (u_{1R}, u_{2R}, u_{3R}, u'_{3R})^T$. The third and fourth rows of the $M_u^{(0)}$ matrix are proportional, thus there is a massless up quark (this massless up quark will be referred as u) at the tree level. An analytical expression for this massless state can be given but it is useless for our analysis. Later arguments that the u quark remain massless to all orders of perturbation theory [77] are given. Similarly, the down-quark mass matrix at the tree level, $M_d^{(0)}$, defined as $\frac{1}{\sqrt{2}}\overline{\mathbf{d}_L} M_d^{(0)} \mathbf{d}_R$, reads

$$\begin{bmatrix} G^6_{11}V_{\chi^0} & G^6_{12}V_{\chi^0} & G^6_{13}V_{\chi^0} & G^2_{11}V_{\chi^0} & G^2_{12}V_{\chi^0} \\ G^6_{21}V_{\chi^0} & G^6_{22}V_{\chi^0} & G^6_{23}V_{\chi^0} & G^2_{21}V_{\chi^0} & G^2_{22}V_{\chi^0} \\ G^3_{1}V_{\rho^0} & G^3_{2}V_{\rho^0} & G^3_{3}V_{\rho^0} & G^7_{1}V_{\rho^0} & G^7_{2}V_{\rho^0} \\ G^6_{11}V_{\chi^0_1} & G^6_{12}V_{\chi^0_1} & G^6_{13}V_{\chi^0_1} & G^2_{11}V_{\chi^0_1} & G^2_{12}V_{\chi^0_1} \\ G^6_{21}V_{\chi^0_1} & G^6_{22}V_{\chi^0_1} & G^6_{23}V_{\chi^0_1} & G^2_{21}V_{\chi^0_1} & G^2_{22}V_{\chi^0_1} \end{bmatrix}, \tag{2.19}$$

where $\overline{\mathbf{d}_L} \equiv \left(\overline{d_{1L}}, \overline{d_{2L}}, \overline{d_{3L}}, \overline{d'_{1L}}, \overline{d'_{2L}}\right)$ and $\mathbf{d}_R \equiv (d_{1R}, d_{2R}, d_{3R}, d'_{1R}, d'_{2R})^T$. Since the first and fourth rows, and the second and fifth rows, are proportional to each other, the $M_d^{(0)}$ matrix has two eigenvalues equal to zero (these massless down quarks will be referred as d and s). Thus, the economical model has three massless quark states: one in the up-quark sector and two in the down-quark sector. In other words, the economical 3-3-1 model has a remaining unbroken chiral symmetry, $U(1)''_{\text{PQ}}$, which allows us to transform $u_L \to e^{i\alpha}u_L$, $d_L \to e^{i\alpha}d_L$, $s_L \to e^{i\alpha}s_L$, leaving the Lagrangian invariant. This symmetry will protect these massless quarks to acquire mass at any level of perturbation theory [77]. At this point it is important to say that, since the $U(1)''_{\text{PQ}}$ symmetry is anomalous, these quarks will acquire mass only through QCD nonperturbative effects (for example, by instanton effects [15]). Although the quarks could acquire some mass through these nonperturbative processes, this is in conflict with both chiral QCD and lattice calculation where the ratio m_u/m_d is 0.410 ± 0.036 [28, 38, 39].

Before considering a possible solution to the problem mentioned above, for the sake of completeness, it important to say that in Ref. [70] one-loop contributions to the up-quark mass matrix were calculated, even though a subtle flaw makes these contributions incorrect. To demonstrate that, the same lines as in Ref. [70] are exactly followed. There, in Sec. IV, the authors consider, for simplicity, one-loop contributions to the submatrix

$$M^{(0)}_{u_3u'_3} \equiv \frac{1}{\sqrt{2}} \begin{bmatrix} G^5_3V_{\chi^0} & G^1V_{\chi^0} \\ G^5_3V_{\chi^0_1} & G^1V_{\chi^0_1} \end{bmatrix}, \tag{2.20}$$

where $M^{(0)}_{u_3u'_3}$ is written in the base (u_3, u'_3). The other two massive quark states, u_1 and u_2, which acquire mass at tree level [$m_1 = G^4_{11}V_{\rho^0}/\sqrt{2}$, $m_2 = G^4_{22}V_{\rho^0}/\sqrt{2}$, see Eq. (27) in Ref. [70]] are not important in the analysis. The matrix Eq. (2.20) mixes together the states u_3 and u'_3. A combination of them will be a massless quark and the orthogonal combination acquires a mass $\sim V_{\chi_1^0}$.

Now, the idea is to calculate the one-loop contributions coming from the Feynman diagrams in Fig. 2.1 to the up-quark mass submatrix defined in Eq. (2.20). Following

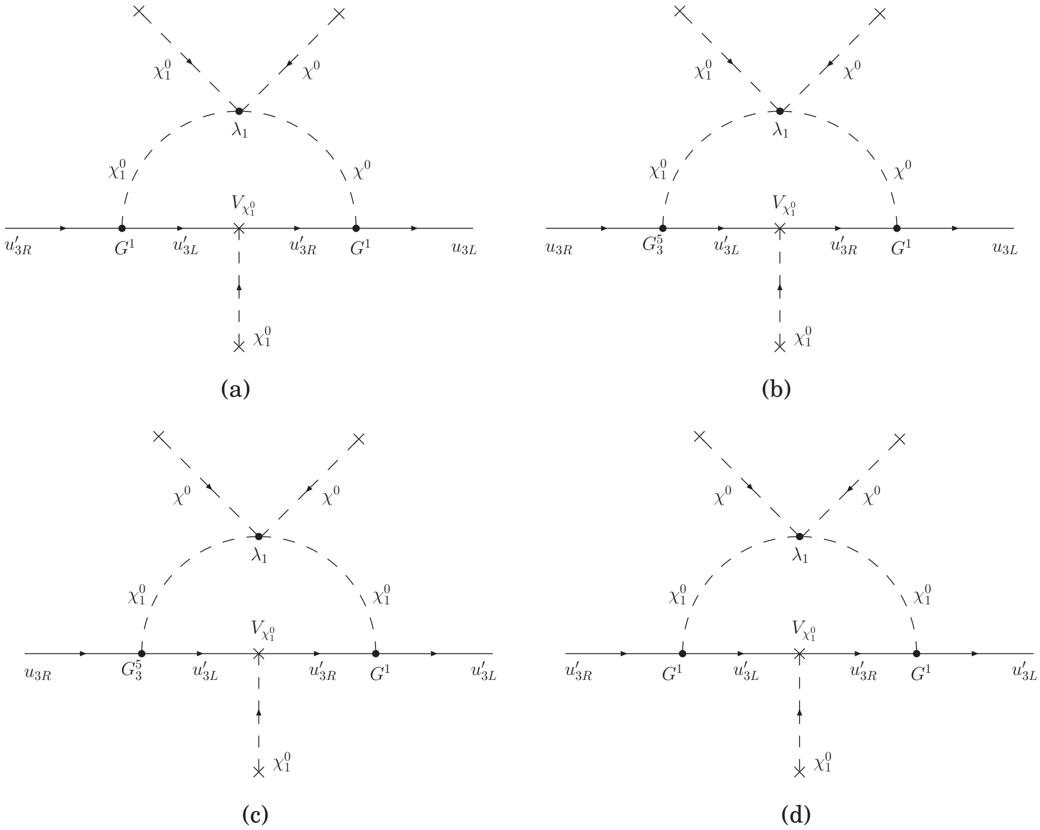

Figure 2.1: *One-loop contributions to the up-quark mass matrix.*

Ref. [70], it is obtained

$$\begin{aligned}\Delta_{u_{3L},u'_{3R}} &= -2iV_{\chi^0}V_{\chi_1^0}\lambda_1 M_{u'_3}\left(G^1\right)^2 \\ &\times \int \frac{\mathrm{d}^4p}{(2\pi)^4}\frac{p^2}{\left(p^2-M^2_{u'_3}\right)^2\left(p^2-M^2_{\chi^0}\right)\left(p^2-M^2_{\chi_1^0}\right)} \\ &\equiv 2V_{\chi^0}V_{\chi_1^0}\lambda_1 M_{u'_3}\left(G^1\right)^2 I\left(M^2_{u'_3},\, M^2_{\chi^0},\, M^2_{\chi_1^0}\right),\end{aligned} \tag{2.21}$$

where $I(M^2_{u'_3}, M^2_{\chi^0}, M^2_{\chi_1^0})$ is defined as

$$I\left(M^2_{u'_3},\, M^2_{\chi^0},\, M^2_{\chi_1^0}\right) \equiv -i\int \frac{\mathrm{d}^4p}{(2\pi)^4}\frac{p^2}{\left(p^2-M^2_{u'_3}\right)^2\left(p^2-M^2_{\chi^0}\right)\left(p^2-M^2_{\chi_1^0}\right)}, \tag{2.22}$$

and $\Delta_{u_{3L},u'_{3R}}$ is the one-loop contribution to the element $(M^{(0)}_{u_3u'_3})_{12}$ given by the Feynman diagram in Fig. 2.1(a). The value of the integral in Eq. (2.22) is not relevant in our analysis and thus it is not calculated. Now, $\Delta_{u_{3L},u_{3R}}$ is found in a similar way from the diagram in Fig. 2.1(b),

$$\begin{aligned}\Delta_{u_{3L},u_{3R}} &= -2iV_{\chi^0}V_{\chi_1^0}\lambda_1 M_{u_3'}G_3^5 G^1 \\ &\times \int \frac{\mathrm{d}^4 p}{(2\pi)^4}\frac{p^2}{\left(p^2-M_{u_3'}^2\right)^2\left(p^2-M_{\chi^0}^2\right)\left(p^2-M_{\chi_1^0}^2\right)} \\ &= \frac{G_3^5}{G^1}\Delta_{u_{3L},u'_{3R}}. \end{aligned} \tag{2.23}$$

One-loop contributions to $(M^{(0)}_{u_3u_3'})_{21}$ and $(M^{(0)}_{u_3u_3'})_{22}$, found from the Feynman diagrams in Figs. 2.1(c) and 2.1(d), respectively, are also proportional to each other, i.e.,

$$\Delta_{u'_{3L},u_{3R}} = \frac{G_3^5}{G^1}\Delta_{u'_{3L},u'_{3R}}. \tag{2.24}$$

Therefore, when considering simultaneously all the one-loop contributions above, the $M^{(0)}_{u_3u_3'}$ becomes

$$\frac{1}{\sqrt{2}}\begin{bmatrix} G_3^5\left(V_{\chi^0}+\frac{\Delta_{u_{3L},u'_{3R}}}{G^1}\right) & G^1\left(V_{\chi^0}+\frac{\Delta_{u_{3L},u'_{3R}}}{G^1}\right) \\ G_3^5\left(V_{\chi_1^0}+\frac{\Delta_{u'_{3L},u'_{3R}}}{G^1}\right) & G^1\left(V_{\chi_1^0}+\frac{\Delta_{u'_{3L},u'_{3R}}}{G^1}\right)\end{bmatrix}. \tag{2.25}$$

This matrix still has a determinant equal to zero. In other words, it has been shown that one combination of the up quarks still remains massless, as it should be. In the down-quark sector a similar analysis can be easily made. Thus, what makes the contributions to the up-quark and down-quark masses made in Ref. [70] incorrect is that those contributions were not considered simultaneously.

To conclude, the 3-3-1 economical model has three massless quarks (one up quark and two down quarks) to all order of perturbation theory, which is in conflict with both chiral QCD and lattice calculation where the ratio m_u/m_d is 0.410 ± 0.036 [39]. Therefore, the economical model is not realistic and it must be modified to overcome that difficulty. One manner of doing that is introducing a new scalar triplet, η:

$$\eta = \left(\eta^0, \eta^-, \eta_1^0\right)^T \sim (\mathbf{1}, \mathbf{3}, -1/3). \tag{2.26}$$

When the scalar triplet, η, is introduced into the model, the Yukawa Lagrangian given in Eq. (2.5) has the following extra terms:

$$\begin{aligned}\mathcal{L}^q_{\text{Y, extra}} &= G_a^9\overline{Q_{3L}}u_{aR}\eta + G^{10}_{\alpha a}\overline{Q_{\alpha L}}d_{aR}\eta^* \\ &+ G^{11}\overline{Q_{3L}}u'_{3R}\eta + G^{12}_{\alpha\beta}\overline{Q_{\alpha L}}d'_{\beta R}\eta^* + \text{H.c.}. \end{aligned} \tag{2.27}$$

As can be seen from Eq. (2.5) and Eq. (2.27), the quark fields interact with different neutral scalar fields simultaneously. Hence, flavor-changing neutral currents (FC-NCs) are, in general, induced. This characteristic is shared by most of multi-Higgs models [78]. In order to suppress the FCNC effects we must use some model dependent strategies, for instance, choosing an appropriate direction in the VEV space, resorting to heavy scalars and/or small mixing angles in the quark and the scalar sectors, and considering adequate Yukawa coupling matrix textures [63, 78–80]. In particular, in this model the exotic quarks have the same electric charge as the ordinary ones. This means that they can mix with the later ones and hence also induce FCNC. However, this kind of FCNC is suppressed when the VEV which controls the exotic quark masses is taken much larger than the electroweak mass scale [63, 80]. FCNC also occurs in models which have an extra neutral vector boson. They can be handled in a similar way. See, for example, [81]. Finally, from Eq. (2.4) we see that the lepton sector of the model is not afflicted by FCNC.

The most general scalar potential invariant under the gauge symmetry, $V = V_{\rm H} + V_{\rm NH}$, has now the following extra terms:

$$\begin{aligned} V_{\text{H, extra}} = {} & \mu_\eta^2 \eta^\dagger \eta + \lambda_5 \left(\eta^\dagger \eta\right)^2 + \eta^\dagger \eta \left[\lambda_6 \left(\rho^\dagger \rho\right) + \lambda_7 \left(\chi^\dagger \chi\right)\right] \\ & + \lambda_8 \left(\rho^\dagger \eta\right) \left(\eta^\dagger \rho\right) + \lambda_9 \left(\chi^\dagger \eta\right) \left(\eta^\dagger \chi\right), \end{aligned} \tag{2.28}$$

and

$$\begin{aligned} V_{\rm NH} = {} & \mu_4^2 \chi^\dagger \eta + f \epsilon^{ijk} \eta_i \rho_j \chi_k + \lambda_{10} \left(\chi^\dagger \eta\right)^2 + \lambda_{11} \left(\chi^\dagger \rho\right) \left(\rho^\dagger \eta\right) \\ & + \lambda_{12} \left(\chi^\dagger \eta\right) \left(\eta^\dagger \eta\right) + \lambda_{13} \left(\chi^\dagger \eta\right) \left(\rho^\dagger \rho\right) + \lambda_{14} \left(\chi^\dagger \eta\right) \left(\chi^\dagger \chi\right) + \text{H.c.}\ . \end{aligned} \tag{2.29}$$

Now, when the scalar triplets acquire VEVs, it is straightforward to see that the quark mass matrices do not have determinant equal to zero; thus all the quarks are massive. Additionally, as it will be shown below, there will be no accidental anomalous PQ–like symmetry.

Returning to the question of the PQ symmetry, note that due to these new terms in the Lagrangian the charges of the $U(1)$ symmetries must obey the following relations

$$-X_{Q_3} + X_{u_R} + X_\eta = 0, \qquad -X_{Q_3} + X_{u'_R} + X_\eta = 0, \tag{2.30}$$

$$-X_Q + X_{d'_R} - X_\eta = 0, \qquad -X_Q + X_{d_R} - X_\eta = 0, \tag{2.31}$$

$$X_\rho + X_\eta + X_\chi = 0, \qquad -X_\chi + X_\eta = 0\ , \tag{2.32}$$

besides the ones given in Eqs. (2.8-2.12). Solving Eqs. (2.8- 2.12) and Eqs. (2.30-2.32) simultaneously, it is found that there are only two $U(1)$ symmetries, $U(1)_X$ and $U(1)_B$. The assignment of quantum charges for these two $U(1)$ symmetries when η is included is shown in Table 2.2. Thus, in this case, in contrast to the previous one, the $U(1)_{\rm PQ}$ is not allowed by the gauge symmetry. But, if the Lagrangian is slightly modified by imposing a Z_2 symmetry such that $\chi \to -\chi$, $u'_{3R} \to -u'_{3R}$, $d'_{\beta R} \to -d'_{\beta R}$, and all the other fields being even under Z_2, the trilinear term of the scalar potential, $f\epsilon^{ijk}\eta_i\rho_j\chi_k$, is eliminated. Consequently, the $U(1)_{\rm PQ}$ symmetry is automatically

Table 2.2: Assignment of quantum charges when η is included.

	$Q_{\alpha L}$	Q_{3L}	(u_{aR}, u'_{3R})	$(d_{aR}, d'_{\alpha R})$	Ψ_{aL}	e_{aR}	ρ	(χ, η)
$U(1)_X$	0	1/3	2/3	−1/3	−1/3	−1	2/3	−1/3
$U(1)_B$	1/3	1/3	1/3	1/3	0	0	0	0

introduced. This can be seen by solving Eqs. (2.8-2.12) and Eqs. (2.30-2.32) without the equation

$$X_\rho + X_\eta + X_\chi = 0. \tag{2.33}$$

Note that, in addition to the assignment of quantum charges given in Table 2.1, the charge $U(1)_{\mathbf{PQ}}$ of the η triplet scalar is 1. Unfortunately, the axion that appears when the neutral components of the scalar triplets acquire VEV is visible. This is easy to see as follows. In this model the χ field is responsible for breaking the symmetry from $SU(3)_C \otimes SU(3)_L \otimes U(1)_X$ to $SU(3)_C \otimes SU(2)_L \otimes U(1)_Y$. Thus, to obtain an invisible axion, $V_{\chi_1^0}$ that breaks the PQ symmetry must be greater than 10^9 GeV. But, when χ acquires a VEV the combination $U(1)'_{\mathbf{PQ}} = U(1)_{\mathbf{PQ}} + 3U(1)_X$ is not broken. Therefore, the new PQ symmetry is truly broken when the ρ field acquires a VEV. As $V_{\rho^0} \lesssim 246$ GeV, the axion induced is visible. A visible axion was long ago ruled out by experiments [47].

One usual way to resolve that problem is to introduce an electroweak scalar singlet, ϕ [43, 45]. Its role is to break the PQ symmetry at a scale much larger than the electroweak scale. This field does not couple directly to quarks and leptons; however, it acquires a PQ charge by coupling to the scalar triplets. With the PQ charges given in Table 2.1, the ϕ scalar acquires a PQ charge by coupling to the η, ρ, χ scalar triplets through the interaction term

$$\lambda_{\mathbf{PQ}}\epsilon^{ijk}\eta_i\rho_j\chi_k\phi. \tag{2.34}$$

From this coupling, the ϕ field obtains a PQ charge of -3. Also, notice that this term is permitted provided the ϕ field is odd under the Z_2 symmetry, i.e., $Z_2(\phi) = -\phi$. However, the Z_2 and gauge symmetries do not prohibit some terms in the scalar potential violating the PQ symmetry, such as ϕ^2, ϕ^4, $\rho^\dagger\rho\phi^2$, $\eta^\dagger\eta\phi^2$, $\chi^\dagger\chi\phi^2$, from appearing. Thus, the PQ symmetry should be imposed. Since the PQ symmetry is anomalous, it is somewhat awkward to do so. However, there is a way to overcome this difficulty. Consider that the entire Lagrangian is invariant under a Z_N discrete gauge symmetry [82], with $N \geq 5$, instead of a Z_2 symmetry. The Z_N charge assignment that allows the scalar potential to be naturally free of awkward terms violating the PQ symmetry must satisfy the following minimal conditions:

$$
\begin{aligned}
Z_N(\phi) &\neq (0,\, N/2,\, N/3,\, N/4)\,, && (2.35)\\
Z_N(\eta)+Z_N(\rho)+Z_N(\chi) &\neq pN, && (2.36)\\
-Z_N(\chi)+Z_N(\eta) &= rN;\;\; p,r\in\mathbb{Z}, && (2.37)
\end{aligned}
$$

and, obviously, the other ones that leave the rest of the Lagrangian invariant under Z_N. The $-Z_N(\chi)+Z_N(\eta)=rN$ condition, with $r\in\mathbb{Z}$, is necessary to allow the terms in the scalar potential given in Eq. (2.29), except the trilinear $f\epsilon^{ijk}\eta_i\rho_j\chi_k$ term and, thus, avoid the appearance of an additional dangerous massless scalar in the physical spectrum. In other words, with the conditions imposed by Eqs. (2.35-2.37) for this Z_N discrete symmetry, none of Lagrangian terms, except the violating PQ terms, such as $f\epsilon^{ijk}\eta_i\rho_j\chi_k$, ϕ^2, ϕ^3, ϕ^4, etc., are prohibited from appearing.

Furthermore, to stabilize the axion solution from quantum gravitational effects [83, 84] the same Z_N discrete symmetry with anomaly cancellation by a discrete version of the Green-Schwarz mechanism [85–88] will be used. Quantum gravity effective operators, allowed by the gauge symmetry, of the form $\phi^N/M_{\mathrm{Pl}}^{N-4}$ can induce a nonzero $\overline{\theta}$ given by

$$
\overline{\theta}\simeq\frac{f_a^N}{\Lambda^4_{\mathrm{QCD}}M_{\mathrm{Pl}}^{N-4}}. \tag{2.38}
$$

From the neutron electric dipole moment (EDM) experimental data $\overline{\theta}\lesssim 10^{-11}$ [28], and using $f_a\sim 10^{10}$ GeV, it is found that $N\geq 10$, in order to keep the PQ solution stable. It means that effective operators with $N<10$ must be forbidden by the Z_N symmetry.

The neutron EDM will also receive contributions that do not come from the $\overline{\theta}\tilde{G}G$ term. Those which are similar to the SM contributions will pose no problems since they will have approximately the same values and will give $d_n^{\text{SM-CKM}}\sim 10^{-32}$ e cm [89], i.e., 6 to 7 orders of magnitude smaller than the experimental limit [90]. The other contributions, which are specific of the present 3-3-1 model, like the 1-loop contribution due to the exchange of a charged scalar (χ^-), can be used to constrain the still free parameters of the model, in order to be consistent with the experimental neutron EDM data [90, 91]. This point will be considered later.

Before introducing the Z_N symmetry to stabilize the PQ mechanism, let us calculate the axion state. With the introduction of the scalar singlet ϕ, the scalar potential gains the following extra terms:

$$
V_{\phi,\,\text{extra}}=-\mu_\phi^2\phi^\dagger\phi+\lambda_\phi\left(\phi^\dagger\phi\right)^2+\lambda_{15}\left(\rho^\dagger\rho\right)\left(\phi^\dagger\phi\right)+\lambda_{16}\left(\eta^\dagger\eta\right)\left(\phi^\dagger\phi\right)+\lambda_{17}\left(\chi^\dagger\chi\right)\left(\phi^\dagger\phi\right). \tag{2.39}
$$

Now, to calculate the eigenstate of the axion field, the fields are written as

$$\rho = \begin{pmatrix} \rho^{+} \\ \frac{1}{\sqrt{2}}\left(V_{\rho^0} + \mathrm{Re}\,\rho^0 + i\mathrm{Im}\,\rho^0\right) \\ \rho^{++} \end{pmatrix}, \qquad \eta = \begin{pmatrix} \frac{1}{\sqrt{2}}\left(V_{\eta^0} + \mathrm{Re}\,\eta^0 + i\mathrm{Im}\,\eta^0\right) \\ \eta^{-} \\ \frac{1}{\sqrt{2}}\left(V_{\eta_1^0} + \mathrm{Re}\,\eta_1^0 + i\mathrm{Im}\,\eta_1^0\right) \end{pmatrix},$$

$$\chi = \begin{pmatrix} \frac{1}{\sqrt{2}}\left(V_{\chi^0} + \mathrm{Re}\,\chi^0 + i\mathrm{Im}\,\chi^0\right) \\ \chi^{-} \\ \frac{1}{\sqrt{2}}\left(V_{\chi_1^0} + \mathrm{Re}\,\chi_1^0 + i\mathrm{Im}\,\chi_1^0\right) \end{pmatrix}, \qquad \phi = \frac{1}{\sqrt{2}}\left(V_{\phi} + \ \mathrm{Re}\,\phi + i\mathrm{Im}\,\phi\right). \tag{2.40}$$

The axion field must be isolated from the eight NG bosons that are absorbed by the gauge bosons in the unitary gauge. This is fundamental to do a correct phenomenological study of the axion properties. By following standard procedures, the axion field, $a\,(x)$, is determined to be

$$\begin{aligned} a\,(x) &= \frac{1}{f_a}\left[\frac{V_-^2}{V_{\rho^0}}\mathrm{Im}\,\rho^0 - V_{\chi_1^0}\mathrm{Im}\,\eta^0 + V_{\chi^0}\mathrm{Im}\,\eta_1^0 + V_{\eta_1^0}\mathrm{Im}\,\chi^0\right. \\ &\quad \left. - V_{\eta^0}\mathrm{Im}\,\chi_1^0 - \left(\frac{V_-^2}{V_{\rho^0}^2} + \frac{V_+^2}{V_-^2}\right)V_{\phi}\mathrm{Im}\,\phi\right], \end{aligned} \tag{2.41}$$

where

$$V_-^2 \equiv V_{\chi^0}V_{\eta_1^0} - V_{\chi_1^0}V_{\eta^0}, \tag{2.42}$$

$$V_+^2 \equiv V_{\chi^0}^2 + V_{\chi_1^0}^2 + V_{\eta^0}^2 + V_{\eta_1^0}^2, \tag{2.43}$$

and f_a is the normalization constant given by

$$f_a \equiv \sqrt{\left(\frac{V_-^2}{V_{\rho^0}}\right)^2 + V_+^2 + \left(\frac{V_-^2}{V_{\rho^0}^2} + \frac{V_+^2}{V_-^2}\right)^2 V_{\phi}^2}. \tag{2.44}$$

Note that in the limit $V_{\phi} \gg V_{\chi^0}, V_{\chi_1^0}, V_{\eta^0}, V_{\eta_1^0}$

$$\begin{aligned} a\,(x) &\simeq -\mathrm{Im}\,\phi + \left(\frac{V_-^2}{V_{\rho^0}^2} + \frac{V_+^2}{V_-^2}\right)^{-1} V_{\phi}^{-1}\left[\frac{V_-^2}{V_{\rho^0}}\mathrm{Im}\,\rho^0 - V_{\chi_1^0}\mathrm{Im}\,\eta^0 + V_{\chi^0}\mathrm{Im}\,\eta_1^0\right. \\ &\quad \left. + V_{\eta_1^0}\mathrm{Im}\,\chi^0 - V_{\eta^0}\ \mathrm{Im}\,\chi_1^0\right]; \end{aligned} \tag{2.45}$$

i.e., the axion is primarily composed of the $\mathrm{Im}\,\phi$ field. As is well-known, to make the invisible axion compatible with astrophysical and cosmological considerations, the axion decay constant, f_a, must be in the range 10^9 GeV $\leq f_a \leq 10^{12}$ GeV.

Now, returning to the stabilization of the axion by the Z_N symmetry, let us put that in a short way. If the Z_N symmetry that survives at low energies was part of an "anomalous" $U\,(1)_A$ gauge symmetry, the Z_N charges of the fermions in the low energy theory must satisfy nontrivial conditions: The anomaly coefficients for the full theory are given by the coefficients for the low energy sector, in our case $A_{3C} \equiv [SU(3)_C]^2 U(1)_A$ and $A_{3L} \equiv [SU(3)_L]^2 U(1)_A$, plus an integer multiple of $N/2$ [92, 93], i.e.,

$$\frac{A_{3C} + pN/2}{k_{3C}} = \frac{A_{3L} + rN/2}{k_{3L}} = \delta_{\text{GS}}, \tag{2.46}$$

with p and r being integers. The k_{3C} and k_{3L} are the levels of the Kac-Moody algebra for the $SU(3)_C$ and $SU(3)_L$, respectively. In the present case these are positive integers. Finally, the δ_{GS} is a constant that is not specified by the low energy theory alone. Other anomalies such as $[U(1)_A]^3$, $[U(1)_A]^2 U(1)_X$ do not give useful low energy constraints because these depend on some arbitrary choices concerning $U(1)_A$ [94]. This is why these do not appear in Eq. (2.46). Now, to identify that anomalous $U(1)_A$ symmetry, it is helpful to write it as a linear combination of the $U(1)_{\text{PQ}}$ and the $U(1)_B$ symmetries, i.e.,

$$U(1)_A = \alpha \left[U(1)_{\text{PQ}} + \beta U(1)_B \right], \tag{2.47}$$

where α is a normalization constant used to make the $U(1)_A$ charges integer numbers. With the charges given in Table 2.1, it is straightforward to calculate the anomaly coefficients A_{3C} and A_{3L},

$$A_{3C} = -\frac{3}{2}\alpha, \quad A_{3L} = \left[-\frac{9}{4} + \frac{3}{2}\beta \right] \alpha. \tag{2.48}$$

Thus, the β parameter that satisfies the condition given in Eq. (2.46) is

$$\beta = \frac{1}{3} \left[-3\frac{k_{3L}}{k_{3C}} + \frac{9}{2} + \frac{N}{\alpha} \left(\frac{k_{3L}}{k_{3C}} p - r \right) \right]. \tag{2.49}$$

Taking the simplest possibility for the parameters k_{3C} and k_{3L}, i.e., $k_{3C} = k_{3L}$, the parameter β becomes

$$\beta = \frac{1}{3} \left[\frac{3}{2} + \frac{N}{\alpha} (p - r) \right]. \tag{2.50}$$

Recalling that to stabilize the axion from the quantum gravity corrections it is necessary $N \geqslant 10$, two possible solutions with $N = 10$ and 11 are shown. The corresponding charge assignment of these two discrete subgroups of the $U(1)_A$ symmetry is given in Table 2.3. Also, it is important to remember that those charges are defined mod N.

Table 2.3: The charge assignment for Z_{10} and Z_{11} that stabilize the axion, for $\alpha = 6$.

	$Q_{\alpha L}$	Q_{3L}	(u_{aR}, u'_{3R})	$(d_{aR}, d'_{\alpha R})$	Ψ_{aL}	e_{aR}	ρ	(χ, η)	ϕ
Z_{10}	$+5$	$+7$	$+1$	$+1$	$+7$	$+1$	$+6$	$+6$	$+2$
Z_{11}	$+6$	$+7$	$+1$	$+1$	$+8$	$+2$	$+6$	$+6$	$+4$

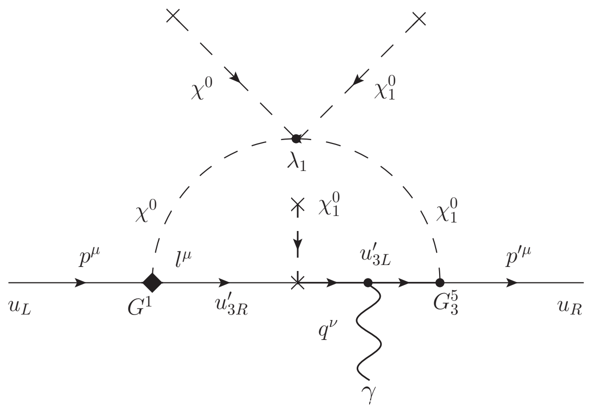

Figure 2.2: One loop diagram contributing to the electric dipole moment of the up-quark. The CP violating vertex is denoted with a diamond.

It can be explicitly verified that the charges in Table 2.3 satisfy Eq. (2.46), as it should be, since Z_{10} and Z_{11} are discrete subgroups of $U\left(1\right)_A$, which is anomaly-free by the Green-Schwarz mechanism.

At this point, an important remark is in order. In its most general form, this model possesses other CP-violating sources apart from the strong CP-violating $\overline{\theta}$ term, which can give additional contributions to the electric dipole moment of the neutron. The reason is that not all phases can be absorbed into the quark and lepton field definitions. Therefore, it is necessary to estimate if these additional contributions do not require tuning the model parameters at the same order of the $\overline{\theta}$ parameter. Then, let us compute a representative case: the up-quark electric dipole moment, d_u^e. One dominant diagram contributing to d_u^e is derived from the one given in Fig. 2.1(b), when an external photon line is attached. See Fig. 2.2. To compute the resultant diagram, it is necessary to know the mixing of the scalar fields, $\mathcal{C}_{ij}$, coming from the diagonalization of the scalar mass matrix. However, it will be considered the $\mathcal{C}_{ij} \sim \mathcal{O}(1)$, which is the worst case. Standard calculations lead to

$$d_u^e|_{m \ll m_{u'}, m_\chi} \approx \frac{eG_3^5 \left|G_1\right| \sin\phi}{48\pi^2} \frac{m_{u'}}{m_\chi^2} \mathcal{K}\left(r\right), \tag{2.51}$$

where $\mathcal{K}\left(r\right)$,

$$\mathcal{K}\left(r\right) = \frac{1}{2r} - \frac{1}{r^2} + \frac{1}{r^3} \ln\left(1+r\right), \tag{2.52}$$

with $r = \frac{m_{u'}^2}{m_\chi^2} - 1$; $m_{u'}$ and m_χ are the exotic quark and scalar masses, respectively; G_3^5 and G_1 are the Yukawa couplings given in Eq. (2.5); $\sin\phi$ is the sine of the CP-violating phase ϕ. Also, the limit $m_u \ll m_\chi$ and $m_u \ll m_{u'}$, with m_u the up-quark mass, have been taken. Furthermore, to give numerical results, it is interesting to consider $m_{u'} \approx m_\chi$ in Eq. (2.51), since these two exotic particles obtain mass from the same VEV, $V_{\chi_1^0}$. In this case, it is obtained

$$G_3^5\,|G_1|\sin\phi \times \left(\frac{1\,\text{TeV}}{m_\chi}\right) \lesssim 2.1\times 10^{-6}. \tag{2.53}$$

To obtain the bound in Eq. (2.53), $d_n^e \sim \frac{4}{3}d_d^e - \frac{1}{3}d_u^e \approx \mathcal{O}(d_u^e) < 0.29\times 10^{-25}\,e\cdot$cm has been used [8]. Now, for instance, let us assume that the CP-violating phase is such that $\sin\phi \sim 10^{-2}$ and $m_\chi \sim 1$ TeV. In this case the parameters $G_3^5 \sim 10^{-2}$ and $|G_1| \sim 10^{-2}$ satisfy the upper bound given in Eq. (2.53). In the general case, i.e., $m_\chi \neq m_{u'}$, it can be shown that when $m_\chi > m_{u'}$ the limit on the couplings is softer than the one given in Eq. (2.53). To see more details about the EDM estimate see Appendix.

Hence, since the order of the model parameters differs from $\overline{\theta} \lesssim 10^{-11}$ by several order of magnitude, a solution to the strong CP problem is required.

2.4 Conclusions

In this chapter it has been shown a detailed and comprehensive study concerning the implementation of the PQ symmetry into a 3-3-1 model in order to solve the strong CP problem. A version of the 3-3-1 model in which the scalar sector is minimal have been considered. In its original form this version has only two scalar triplets (χ, ρ) and it is found that the model presents an automatic PQ-like symmetry. However, for this scalar content, there is a $U(1)$ subgroup of $U(1)_X \otimes U(1)_{\text{PQ}}$ that remains unbroken and hence no axion field, $a(x)$, arises. Therefore, the strong CP problem cannot be solved by the dynamical properties of the axion field. However, as it has been shown in the text, the problem can be solved due to the appearance of three massless quark states. These massless quark states remain massless to all orders in perturbation theory as shown explicitly above. This solution is disfavored since results from lattice and current algebra do not point in that direction. When the model is slightly extended by the addition of a third scalar triplet η, with the same quantum numbers as χ, the massless quarks do not appear anymore but the PQ symmetry cannot be implemented in a natural way. The trilinear term in the scalar potential forbids this symmetry. A Z_2 symmetry can be imposed to remove the trilinear term. In this case, a PQ symmetry can be defined and an axion field appears in the physical scalar spectrum. Unfortunately this axion is visible since it is related to the V_{ρ^0} energy scale, which is of the order of the electroweak scale. Therefore, the model must be extended. A successful implementation of a stable PQ mechanism by introducing a ϕ scalar singlet and a Z_N discrete gauge symmetry has been achieved. The introduction of the ϕ scalar makes the axion invisible provided $V_\phi \gtrsim 10^9$ GeV, i.e., $a\,(x) \simeq \text{Im}\,\phi$. On the another hand, the Z_N protects the axion against quantum gravity effects because both it is anomaly-free, as it was shown by using a discrete version of the Green-Schwarz mechanism, and it forbids all effective operators of the form $\sim \phi^N/M_{Pl}^{N-4}$, with $N < 10$, which could destabilize the PQ mechanism.

Chapter 3

NEUTRINO MASSES AND THE SCALAR SECTOR OF A $B-L$ EXTENSION OF THE STANDARD MODEL

3.1 Introduction

The neutrino masses and mixing which are required for giving a consistent explanation for the solar and atmospheric neutrino anomalies are the most firm evidence of physics beyond the electroweak standard model (ESM). New physics can be implemented in a variety of different scenarios. There are basically two main schemes that are often followed: (i) new matter content is added to the model respecting the original ESM gauge symmetry and (ii) to consider a model with a larger gauge symmetry. Certainly both schemes can be implemented together. In this vein, extensions of the ESM having an extra $U(1)$ gauge symmetry factor are interesting for a variety of reasons. They are the simplest way of extending the ESM gauge symmetry and can be thought of as an intermediate energy scale symmetry coming from the breaking, at a higher energy scale, of a larger gauge symmetry describing some yet unknown physics. For instance, $U(1)$ gauge factors are contained in grand unified theories, supersymmetric models, and left-right models. One major feature of these models is the existence of an extra neutral vector boson, usually denoted by Z', whose mass is related to the energy scale of the extra $U(1)$ symmetry spontaneously broken. It is expected to have Z' signals at the TeV scale and its discovery is one of the goals of the LHC and future lepton colliders. Depending on the implementation of this kind of model, it can have a natural candidate for dark matter (DM) and/or furnish a mechanism for leptogenesis. Through the years much work has been done considering the features of this extra $U(1)$ gauge factor and some particular formulations of the model were made. See, for example, Refs. [95, 96]. In particular, when the charge of the extra $U(1)$ factor is identified with $B-L$ (baryon number minus lepton number), there is extensive literature concerning the most different versions of the model and a large variety of phenomenological aspects.

In this chapter is considered a $B-L$ gauge model which has the particularity of being rendered anomaly free by introducing right-handed neutrinos with exotic $B-L$ charges. The number of right-handed neutrinos and their $B-L$ exotic charges is fixed by the anomaly cancellation equations. Since in this model not all of these right-handed neutrinos have the same exotic charge, we can construct Yukawa terms with different $SU(2)_L$ scalar doublets. Appropriate $SU(2)_L$ scalar singlets are also introduced to write the most general mass terms for the right-handed neutrinos. A detailed study of the scalar potential is made, concerning the mass spectra and the physical Goldstone bosons, and take advantage of this rich scalar sector to construct a seesaw mechanism at low energies (TeV scale) to give realistic masses to the light active neutrinos. Notice that all this analysis is based on the Refs. [97, 98] written by J. C. Montero and I.

The outline of this chapter is as follows. In Sec. 3.2 the particular $B-L$ gauge model under consideration is presented. In Sec. 3.3 the analysis the scalar potential of the model is performed –the symmetries, the mass spectra and the model compatibility with experimental constraints –and introduce a Z_2 symmetry to allow the model to have DM candidates. In Sec. 3.4 the neutrino mass generation and the compatibility of the model with the observed neutrino masses and the tribimaximal mixing is explicitly shown. In the Sec. 3.5 the constraints coming from the first derivatives of the classical scalar potential are presented. In the Sec. 3.6 an analytical expression for the masses and the eigenstates of the gauge bosons are found. In the Sec. 3.7 it is explicitly shown that the model has a $O(2)$ symmetry which allows to interpret the model as one based on the $SU(2)_L \otimes U(1)_Y \otimes U(1)_Z$ gauge symmetry instead of $SU(2)_L \otimes U(1)_{Y'} \otimes U(1)_{B-L}$. Finally, the conclusions are given in Sec. 3.8.

3.2 The Model

The model of Ref. [99] will be considered here. The model is an extension of the ESM based on the gauge symmetry $SU(2)_L \otimes U(1)_{Y'} \otimes U(1)_{B-L}$ where B and L are the usual baryonic and leptonic numbers, respectively, and Y' is a new charge. The values of Y' are chosen to obtain the ESM hypercharge Y through the relation $Y = [Y' + (B-L)]$, after the first spontaneous symmetry breaking. In order to make the model anomaly free introduce right-handed neutrinos (n_R) have been introduced. Solving the anomaly equations is found that the number of n_R cannot be less than 3, if the quantum numbers are restricted to be only integer. For the minimal number (3) these equations have two solutions: the usual one, where all right-handed neutrinos are identical and have $L = 1$, and the exotic one, where two of them have $L = 4$ and the third one has $L = -5$. The model under consideration has the right-handed neutrinos having such exotic lepton numbers.

The fermionic content of the model is the same as the ESM plus the right-handed neutrinos introduced above. The respective charge assignment is shown in Table 3.1. In the framework of a gauge theory with spontaneous symmetry breaking, at least a scalar doublet, H, have to be introduced in order to give mass to the lighter massive neutral vector boson (Z) and the charged fermions, as in the ESM. However, more scalar fields are needed to give mass to the extra neutral vector boson (Z'), which is expected to be heavier than Z, and to the neutrinos of the model. Respecting gauge invariance, a general choice is to introduce two $SU(2)$ scalar doublets, $\Phi_{1,2}$, and three $SU(2)$ neutral scalar singlets, $\phi_{1,2,3}$, with the charge assignments shown in Table 3.2. With these fields, and omitting summation symbols, the most general

Table 3.1: Quantum number assignment for the fermionic fields.

	I_3	I	Q	Y'	$B-L$	Y
ν_{eL}	$1/2$	$1/2$	0	0	-1	-1
e_L	$-1/2$	$1/2$	-1	0	-1	-1
e_R	0	0	-1	-1	-1	-2
u_L	$1/2$	$1/2$	$2/3$	0	$1/3$	$1/3$
d_L	$-1/2$	$1/2$	$-1/3$	0	$1/3$	$1/3$
u_R	0	0	$2/3$	1	$1/3$	$4/3$
d_R	0	0	$-1/3$	-1	$1/3$	$-2/3$
n_{1R}	0	0	0	4	-4	0
n_{2R}	0	0	0	4	-4	0
n_{3R}	0	0	0	-5	5	0

Yukawa Lagrangian respecting the gauge invariance is given by

$$\begin{aligned} -\mathcal{L}_{\mathrm{Y}} \;=\; & Y_i^{(l)}\overline{L}_{Li}e_{Ri}H + Y_{ij}^{(d)}\overline{Q}_{Li}d_{Rj}H + Y_{ij}^{(u)}\overline{Q}_{Li}u_{Rj}\widetilde{H} + \mathcal{D}_{im}\overline{L}_{Li}n_{Rm}\Phi_1 \\ & +\mathcal{D}_{i3}\overline{L}_{Li}n_{R3}\Phi_2 + \mathcal{M}_{mn}\overline{(n_{Rm})^c}n_{Rn}\phi_1 + \mathcal{M}_{33}\overline{(n_{R3})^c}n_{R3}\phi_2 \\ & +\mathcal{M}_{m3}\overline{(n_{Rm})^c}n_{R3}\phi_3 + \text{H.c.}, \end{aligned} \tag{3.1}$$

where $i, j = 1, 2, 3$ are lepton family numbers and represent e, μ and τ, respectively,

$m, n = 1, 2$, and $\widetilde{H} = i\tau_2 H^*$. The corresponding scalar potential is

$$\begin{aligned} V_{B-L} &= -\mu_H^2 H^\dagger H + \lambda_H (H^\dagger H)^2 - \mu_{11}^2 \Phi_1^\dagger \Phi_1 + \lambda_{11} \left|\Phi_1^\dagger \Phi_1\right|^2 - \mu_{22}^2 \Phi_2^\dagger \Phi_2 + \lambda_{22} \left|\Phi_2^\dagger \Phi_2\right|^2 \\ &\quad -\mu_{s\alpha}^2 |\phi_\alpha|^2 + \lambda_{s\alpha} |\phi_\alpha^* \phi_\alpha|^2 + \lambda_{12} |\Phi_1|^2 |\Phi_2|^2 + \lambda'_{12} (\Phi_1^\dagger \Phi_2)(\Phi_2^\dagger \Phi_1) \\ &\quad +\Lambda_{H\gamma} |H|^2 |\Phi_\gamma|^2 + \Lambda'_{H\gamma} (H^\dagger \Phi_\gamma)(\Phi_\gamma^\dagger H) + \Lambda_{Hs\alpha} |H|^2 |\phi_\alpha|^2 + \Lambda'_{\gamma\alpha} |\Phi_\gamma|^2 |\phi_\alpha|^2 \\ &\quad + \left[\beta_{13} \Phi_1^\dagger \Phi_2 \phi_1 \phi_3^* + \beta_{23} \Phi_1^\dagger \Phi_2 \phi_2^* \phi_3 + \beta_{123} \phi_1 \phi_2 (\phi_3^*)^2 + \text{H.c.}\right] \\ &\quad +\Delta_{\alpha\beta} (\phi_\alpha^* \phi_\alpha)(\phi_\beta^* \phi_\beta), \end{aligned} \tag{3.2}$$

where $\gamma = 1, 2$; $\alpha, \beta = 1, 2, 3$; and $\alpha < \beta$ in the last term. In $\mathcal{L}_\text{Y}$, the motivation for introducing such scalar fields is to write the most general neutrino mass terms. Because of the fact that not all right-handed neutrinos have the same Y' and $(B - L)$ charges, the neutrino mass matrix will have entries proportional to vacuum expectation values (VEVs) which can, in principle, belong to different energy scales. The scalar potential is a consequence of the fields we have previously introduced, and the terms in Eq. (3.2) are only dictated by gauge invariance. Now, notice that when potential terms, based on general grounds, are written, although correct, this may have introduced more symmetries than necessary. Hence, a detailed study of the scalar potential must be done to avoid inconsistencies with the present phenomenology. The

Table 3.2: Quantum number assignment for the scalar fields.

	I_3	I	Q	Y'	$B-L$	Y
$H^{0,+}$	$\mp 1/2$	$1/2$	$0, 1$	1	0	1
$\Phi_1^{0,-}$	$\pm 1/2$	$1/2$	$0, -1$	-4	$+3$	-1
$\Phi_2^{0,-}$	$\pm 1/2$	$1/2$	$0, -1$	5	-6	-1
ϕ_1	0	0	0	-8	$+8$	0
ϕ_2	0	0	0	10	-10	0
ϕ_3	0	0	0	1	-1	0

scalar doublets of the model, H and $\Phi_{1,2}$, contribute to the Z boson mass, so their vacuum expectation values are bounded by the electroweak energy scale. Hence, the largest energy scale of the model comes from the $SU(2)$ scalar singlets $\phi_{1,2,3}$. In this way, the pattern of the spontaneous symmetry breaking is

$$\begin{aligned} SU(2)_L \otimes U(1)_{Y'} \otimes U(1)_{B-L} &\xrightarrow{\langle \phi_{1,2,3} \rangle} SU(2)_L \otimes U(1)_Y \\ &\xrightarrow{\langle H, \Phi_{1,2} \rangle} U(1)_\text{em}, \end{aligned} \tag{3.3}$$

3.3 The Scalar Potential Analysis

Now, it is considered the V_{B-L} scalar potential given in Eq. (3.2) when all neutral scalar fields develop nonvanishing VEVs, with the usual shifting $\varphi^0 = \frac{1}{\sqrt{2}}(V_\varphi + \mathrm{Re}\,\varphi + i\mathrm{Im}\,\varphi)$. By using standard procedures it is found the constraint equations coming from the linear terms in the scalar potential after the symmetry breaking. See the appendix. In the same way, the mass-squared matrices for the charged, real, and imaginary scalar fields can be constructed. Look at the mass-squared matrix for the charged fields. It is a complete 3×3 symmetric matrix in the basis $(H^+, \Phi_1^+, \Phi_2^+)$ that can be easily diagonalized and, after taking into account the constraint equations, gives the following mass spectrum: two charged Goldstone bosons

$$G_W^{\pm} = \frac{1}{\sqrt{1+\frac{V_H^2}{V_{\Phi_2}^2}+\frac{V_{\Phi_1}^2}{V_{\Phi_2}^2}}}\left(-\frac{V_H}{V_{\Phi_2}}\,H^{\pm} + \frac{V_{\Phi_1}}{V_{\Phi_2}}\,\Phi_1^{\pm} + \Phi_2^{\pm}\right), \tag{3.4}$$

and two massive states whose expressions are not shown by shortness. The fields $G_W^{\pm}$ will be absorbed to form the longitudinal components of the charged massive vector bosons $W^{\pm}$. The other two physical states remain in the spectrum and are a prediction of the model. Later all the mass spectra in some different situations will be numerically calculated.

The mass eigenstates of the neutral imaginary scalar sector are found from a 6×6 mass-squared matrix that, after the diagonalization procedure, is found to have two massive scalar and four massless fields. Two of them will become the longitudinal components of the Z and the Z' neutral vector bosons. The other two massless states remain in the physical spectrum. At the present analysis, it is only necessary to write the two physical NG bosons in the limit where $V_{\phi_{1,2,3}} \gg V_H, V_{\Phi_{1,2}}$ and they are given by

$$G_{F_1}^0 \approx \sqrt{\frac{V_{\Phi_1}^2+V_{\Phi_2}^2}{V_H^2+V_{\Phi_1}^2+V_{\Phi_2}^2}}\left(\mathrm{Im}\,H^0 + \frac{V_H V_{\Phi_1}}{V_{\Phi_1}^2+V_{\Phi_2}^2}\mathrm{Im}\,\Phi_1^0 + \frac{V_H V_{\Phi_2}}{V_{\Phi_1}^2+V_{\Phi_2}^2}\mathrm{Im}\,\Phi_2^0\right), \tag{3.5}$$

$$G_{F_2}^0 \approx \frac{1}{\sqrt{110}}\left(7\,\mathrm{Im}\,\phi_1 + 5\,\mathrm{Im}\,\phi_2 + 6\,\mathrm{Im}\,\phi_3\right). \tag{3.6}$$

From the expressions above it is seen that $G_{F_{1,2}}^0$ are mainly doublet and singlet, respectively. This fact will be analyzed later on.

For the neutral real scalar sector the model has a symmetric 6×6 mass-squared matrix with a nonvanishing determinant. Hence the spectrum will not contain massless states. Analytical expressions are not shown here but numerical values will be given below.

The number of NG bosons found by doing explicit calculations can be easily understood by studying the global symmetries of the scalar potential before and after the spontaneous symmetry breaking (SSB). Before the SSB, the global symmetries of the scalar potential are (a) $SU(2)$ acting on H and $\Phi_{1,2}$ doublets, (b) $U(1)$ acting on H with charge $+1$, (c) $U(1)$ acting on $\Phi_{1,2}$ with charge $+1$, and (d) two independent $U(1)_{\beta,\gamma}$ transformations acting on the fields $\Phi_1, \Phi_2, \phi_1, \phi_2, \phi_3$ with charges $(\frac{1}{2}, -\frac{1}{2}, 1, -1, 0)$ and $(-\frac{1}{2}, \frac{1}{2}, 0, +2, 1)$, respectively. After the SSB the global symmetries of the scalar potential are reduced to a single $U(1)_\alpha$ acting on the charged components of the doublets, $H^\pm$ and $\Phi^\pm_{1,2}$ with charges $(\pm 1, \pm 1)$. Following the Goldstone theorem, the number of NG bosons is equal to the number of broken symmetry generators. In this case the original symmetry has $(3 + 4 \times 1) = 7$ generators and the remaining symmetry has 1. Then there must be 6 NG bosons, which is exactly the number that has been found: two charged and four neutral imaginary fields.

Notice that the scalar potential given in Eq. (3.2) corrects the one given in Eq. (16) of Ref. [99] in which the terms proportional to $\Lambda'_{H\gamma}$ are missing. The lack of these terms alters the global symmetries under which the scalar potential is invariant and, consequently, the number of Goldstone bosons in the spectra. In that case the symmetries before the SSB are (a) $O(4)$ acting on the four components of H, (b) $SU(2)$ acting on $\Phi_{1,2}$, (c) $U(1)$ acting on $\Phi_{1,2}$, (d) the two $U(1)_{\beta,\gamma}$ defined above. After the SSB the remaining symmetries are (i) $O(3)$ acting on the components $(\mathrm{Im}\, H^0, \mathrm{Re}\, H^+, \mathrm{Im}\, H^+)$ and (ii) $U(1)$ acting on $\Phi^\pm_{1,2}$ with charge ± 1. Therefore,there will be $(6+3+3\times 1) - (3+1) = 12 - 4 = 8$ NG bosons. The same result is obtained by doing explicit calculations. From the mass-squared matrices it is found that the number of NG bosons is the expected one, and besides four massless states in the charged sector given by

$$G^\pm_W = H^\mp, \tag{3.7}$$

$$G^\pm_C = \frac{1}{\sqrt{V^2_{\Phi_1} + V^2_{\Phi_2}}} \left(V_{\Phi_1} \Phi^\pm_1 + V_{\Phi_2} \Phi^\pm_2 \right). \tag{3.8}$$

Hence, this result is in conflict with the present phenomenology since there are two extra charged massless scalars in the spectrum.

Now, return to the present analysis. Since the analysis of the scalar potential shows the existence of two physical NG bosons, it is time to care about the safety of the model. Before that, some remarks about the VEVs of the model are due. The $V_{\phi_{1,2,3}}$ are the largest energy scale of the model. The main contribution to the Z boson square mass comes from the doublets so that $(V^2_H + V^2_{\Phi_1} + V^2_{\Phi_2}) = V^2_{\mathrm{ESM}} = (246)^2$ GeV^2. The doublet H is the one that couples to quarks and to charged leptons via Yukawa interactions, and hence, V_H must be close to V_{ESM} to give the correct tree level mass to the quark top, as the ESM do, for a $\mathcal{O}(1)$ top Yukawa coupling. Then it is concluded that $V^2_{\Phi_1} + V^2_{\Phi_2} \ll V^2_H$.

The major challenge to models with physical NG bosons, also called Majorons (J), comes from the energy loss in stars through the processes $\gamma + e^- \to e^- + J$. This process is used to put limits on the $\bar{e}eJ$ coupling, and it is found that it has to be $g_{eeJ} \leq 10^{-10}$ for the Sun, and $g_{eeJ} \leq 10^{-12}$ for the red-giant stars [100]. However, the dynamics of the red giants has not the same level of confidence as that of the Sun, and this fact considerably weakens the second constraint.

The physical NG $G^0_{F_2}$ has components only in the $SU(2)$ singlets $\phi_{1,2,3}$, which couple only to right-handed neutrinos. Therefore, it is safe since there is no tree level contribution to the energy loss process. The case for $G^0_{F_1}$ is not that simple. $G^0_{F_1}$ has a component in the ESM-like doublet H, and it contributes to $\bar{e}eJ$ through $\mathrm{Im}\, H^0$. The components in $\Phi_{1,2}$, which couple only to neutrinos at the tree level, pose no problem. Since in this case symmetry eigenstates and mass eigenstates are connected by orthogonal matrices, from Eq. (3.6) we find

$$\mathrm{Im}\, H^0 \approx \sqrt{\frac{V_{\Phi_1}^2 + V_{\Phi_2}^2}{V_H^2 + V_{\Phi_1}^2 + V_{\Phi_2}^2}} G^0_{F_1} + ..., \tag{3.9}$$

and hence,

$$g_{eeJ} \approx \frac{Y_e}{\sqrt{2}} \sqrt{\frac{V_{\Phi_1}^2 + V_{\Phi_2}^2}{V_H^2 + V_{\Phi_1}^2 + V_{\Phi_2}^2}} = \left(\frac{\sqrt{2}\, m_e}{V_H}\right)\left(\frac{V_\Phi}{\sqrt{2} V_H}\right) \approx 2 \times 10^{-6} \frac{V_\Phi}{V_H} \leq 10^{-12} - 10^{-10}, \tag{3.10}$$

where Y_e is the electron Yukawa coupling to the doublet H, it has been defined $V_{\Phi_1}^2 + V_{\Phi_2}^2 \equiv V_\Phi^2$, and used the shift $H^0 \to \frac{1}{\sqrt{2}}(V_H + \mathrm{Re}\, H^0 + i\, \mathrm{Im}\, H^0)$. From the equation above it is concluded that the VEVs of the $SU(2)$ doublets $\Phi_{1,2}$ must be less than 12 MeV to satisfy the Sun constraint or less than 120 KeV to satisfy rigorously the red-giant constraint. Let us adopt for practical purposes an intermediate scale: $V_\Phi = 1$ MeV.

Once the energy scale of the VEVs of the model have been established, and verified its safety up to now, an exemplary study of the full scalar mass spectra can be made. That will be done it numerically since the excessive length of the analytical expressions make them useless.

In order to compute the masses a set of parameters is considered: the dimensional ones $V_H = 246$, $V_{\Phi_{1,2}} = 0.001$, $V_{\phi_{1,2,3}} = 1000$, in GeV, and $\lambda_H = 0.2$, $\lambda_{11,22} = \Lambda'_{13,21,22,H1,H2} = \lambda_{s\alpha} = 1$, $\Lambda_{H1,H2} = \Lambda_{Hs\alpha} = \Lambda'_{11,12,23}$, $\Delta_{12,13,23} = 0.1$, $\beta_{123} = -0.8$, which are dimensionless, for $\alpha = 1, 2, 3$. Note that the values for the μ parameters are found by solving the constraint equations for the scalar potential: $\mu_H^2 = (402)^2$, $\mu_{11}^2 = -(630)^2$, $\mu_{22}^2 = (230)^2$, $\mu_{s1}^2 = \mu_{s2}^2 = (838)^2$, $\mu_{s3}^2 = (550)^2$, in GeV^2. It is also used $g_{Y'} = g_{B-L} = 0.4885$, and $g = 0.6298$, where $g_{Y'}$, g_{B-L}, and g are the coupling constants of the $U(1)_{Y'}$, $U(1)_{B-L}$, and $SU(2)_L$ gauge factors, respectively, which are related to the electric charge through $1/e^2 = 1/g_{Y'}^2 + 1/g_{B-L}^2 + 1/g^2$ [99].

The charged scalar sector gives the masses $m_{C_j} = (1424.9,\ 173.9,\ 0)$, also in GeV, where the massless complex field is responsible for the longitudinal components of the charged vector bosons W^+ and W^-.

In the imaginary neutral scalar sector it is found a 6×6 square mass matrix and after diagonalization results, in GeV, $m_{I_i} = (1549.2,\ 1414.13,\ 0,\ 0,\ 0,\ 0)$. Notice the correct number of the massless fields: two are absorbed to form the longitudinal component of the neutral vector bosons of the model, Z and Z', and the other two are the physical NG bosons $G^0_{F_{1,2}}$, as discussed above.

In the real neutral scalar sector, in the same way, it is found $m_{R_i} = (1743.9,\ 1643.2,\ 1414.2,\ 1029.8,\ 150.0,\ 0.0014)$, in GeV. A very light scalar of about 1.4 MeV, which poses a new challenge to the model: the Z invisible decay width, appears. The presence of such a light scalar field, say R, and the $G^0_{F_1} \equiv J$ zero mass state, allows the decay $Z \to RJ \to JJJ$, which will contribute to the Z invisible decay width as half of the decay $Z \to \overline{\nu}\nu$, for a single flavor family [101]. According to the experimental data there is no room for such an extra contribution [8].

The light scalar found above is not the result of a particular choice of the input parameters, as it could be thought at the first moment. Here, a qualitative but convincing argument is provided. As was observed in Ref. [101], the reason is as follows. As mentioned above, before the SSB, the scalar potential has a $U(1)$ global symmetry acting on each of the $\Phi_{1,2}$ doublets, say, Φ. This means that a rotation in the $\mathrm{Re}\,\Phi^0$–$\mathrm{Im}\,\Phi^0$ plane can be freely done, so that as long as this $U(1)$ symmetry holds, the fields $\mathrm{Re}\,\Phi^0$ and $\mathrm{Im}\,\Phi^0$ are mass degenerate. However, this symmetry is broken when the real neutral component acquires a nonvanishing VEV and, hence, the fields are no longer mass degenerate. The square mass difference must be, then, of the order of the square of the energy scale responsible for breaking the symmetry, i.e., $m^2_{\mathrm{Re}\,\Phi^0} - m^2_{\mathrm{Im}\,\Phi^0} = \mathcal{O}(V^2_\Phi)$. When $\mathrm{Im}\,\Phi^0$ is a NG boson, $m^2_{\mathrm{Im}\,\Phi^0} = 0$, we are left with $m^2_{\mathrm{Re}\,\Phi^0} = \mathcal{O}(V^2_\Phi)$, which, in the current case, it is a very light scalar since V_Φ must be of the order of 1 MeV, in order to be consistent with the star energy loss data. Then, a way to reconcile the present model with the experimental constraints must be found.

Some attempts can be made to remove such inconsistency. Since the origin of the problem is in the breaking of the $U(1)$ symmetry acting on the doublets $\Phi_{1,2}$, consider the situation where $V_{\Phi_2} = 0$, and all other VEVs are different from zero. In this case it is found the same number of neutral Goldstone bosons (4): two would be Goldstone bosons and two physical ones $G^0_{F_{1,2}}$. $G^0_{F_2}$ is given by the same expression as in Eq. (3.6), and

$$G^0_{F_1} \approx \frac{1}{\sqrt{V^2_H + V^2_{\Phi_1}}} \left(V_{\Phi_1} \mathrm{Im}\, H^0 + V_H \mathrm{Im}\, \Phi^0_1 \right). \tag{3.11}$$

For the same input parameters, but now providing an input value for $\mu^2_{22} = (230)^2$

GeV^2, the mass spectra are practically not affected and a light real scalar whose mass is about 1 MeV$\sim$ $\mathcal{O}(V_{\Phi_1})$ is still found. The same conclusion is obtained if $V_{\Phi_1} = 0$ and $V_{\Phi_2} \neq 0$ are considered. The only that have to be done is the replacement $\Phi_1 \longleftrightarrow \Phi_2$ in the above results.

As the problem persists, now consider the case where $V_{\Phi_1} = 0$ and $V_{\Phi_2} = 0$. In this case, the number of NG bosons is reduced to 3. There is only one physical Goldstone, the $G^0_{F_2}$ given in Eq. (3.6), which is safe, as discussed above. The mass spectra are now considerably affected. For the same input parameters as above, and with $\mu^2_{11} = -(800)^2, \mu^2_{22} = (230)^2$, in GeV^2, the spectra, with all the masses in GeV, are the following. For the charged scalars it is found $m_{C_j} = (1469.4, 380.1, 0)$. For the imaginary scalars and for the real scalars $m_{I_i} = (1549.2, 1459.1, 337.9, 0, 0, 0)$, $m_{R_i} = (1743.9, 1643.2, 1459.1, 1029.8, 337.8, 150.0)$ are found, respectively. As it can be seen, there is no a light real scalar anymore. The lighter real scalar is heavier than the Z vector boson, so that the problematic decay $Z \to RJ$ is kinetically forbidden. Then, the model is safe. However, this solution is not satisfactory since the choice for the doublet VEVs ($V_{\Phi_{1,2}} = 0$) does not allow the light neutrinos to get mass. It is easy to see that in this case there is a remaining $U(1)$ quantum symmetry, say, $U(1)_\zeta$, protecting the neutrino mass generation at any level. A possible ζ-charge assignment is: $\zeta(\nu_{eL}, e_L, e_R, \Phi_{1,2}) = -1$, $\zeta(u_L, d_L, u_R, d_R) = 1/3$, and $\zeta(n_{(1,2,3)R}, \phi_{1,2,3}) = 0$. In order to make the model compatible with the experimental data and, hence, with massive neutrinos, a new kind of solution have to be looked for.

Before continuing the search for a satisfactory solution, observe that before the SSB the model has a Z_2 exact symmetry with the transformation rules $Z_2(n_{R3}) = -n_{R3}$, $Z_2(\Phi_2) = -\Phi_2$, $Z_2(\phi_3) = -\phi_3$, and all the other fields being even under Z_2. It is interesting to preserve this symmetry after the SSB if the model is supposed to have DM candidates. This is true when $V_{\Phi_2} = V_{\phi_3} = 0$. In this case the Z_2 symmetry is not spontaneously broken, and a mechanism similar to that of Ref. [102] can be implemented. The number of NG bosons is 4, and the physical ones are given by the following: $G^0_{F_1}$ is given by the same expression in Eq. (3.11), and

$$G^0_{F_2} = \frac{1}{\sqrt{16V^2_{\phi_1} + 25V^2_{\phi_2}}} \left(5V_{\phi_2} \mathrm{Im}\,\phi_1 + 4V_{\phi_1} \mathrm{Im}\,\phi_2\right). \tag{3.12}$$

However, as it is already known, there is a very light real scalar that, together with $G^0_{F_1}$, has severe implications on the Z invisible decay width. This Z_2 picture will be considered later on.

3.3.1 The Solution

With the aim of constructing a consistent model, a new $SU(2)$ neutral scalar singlet ϕ_X with the quantum numbers $Y' = -(B - L) = 3$ is introduced. The Yukawa

Lagrangian remains as in Eq. (3.1), but to the scalar potential in Eq. (3.2), besides extending the range of the indices to $\alpha, \beta = 1, 2, 3, X$, the following non-Hermitian terms,

$$V_{B-L}^{X} = -i\kappa_{H1X}\Phi_1^T\tau_2 H\phi_X - i\kappa_{H2X}(\Phi_2^T\tau_2 H)(\phi_X^*)^2 + \beta_X(\phi_X^*\phi_1)(\phi_2\phi_3) + \beta_{3X}(\phi_X^*\phi_3^3) + \text{H.c.}, \tag{3.13}$$

to have to be added in order to account for all the gauge invariant terms after the introduction of ϕ_X. The terms above reduce the number of global symmetries of the scalar potential, so that changes in the scalar spectra are expected.

Before the SSB, the global symmetries of the total scalar potential are (a) $SU(2)$ acting on H and $\Phi_{1,2}$ doublets, (b) $U(1)_\alpha$ acting on H and $\Phi_{1,2}$, and (c) $U(1)_\beta$ acting on the fields $H, \Phi_1, \Phi_2, \phi_1, \phi_2, \phi_3, \phi_X$ with charges $(\frac{3}{8}, 0, -\frac{9}{8}, 1, -\frac{5}{4}, -\frac{1}{8}, -\frac{3}{8})$, respectively. After the SSB the global symmetries of the scalar potential are reduced to a single $U(1)$ acting on the charged components of the doublets, $H^\pm$ and $\Phi_{1,2}^\pm$, with charges $(\pm 1, \pm 1)$. The total number of Goldstone bosons will be given by the number of broken generators, i.e., $5 - 1 = 4$, which is the number of massless fields needed to form the longitudinal components of the charged (W^+, W^-) and neutral vector bosons (Z, Z'). In this case, there are no physical NG bosons at all. It means that the inclusion of the $SU(2)$ scalar singlet ϕ_X has removed all physical massless states from the spectrum, and a solution for the safety of the model have been found.

Now, numerical applications require expanding the input parameters set to account for the new ones related to ϕ_X. Choosing $V_{\phi_X} = 1000$, and $\kappa_{H1X} = 0.01$ in GeV, and the dimensionless $\lambda_{sX} = \Lambda'_{1X} = 1$, $\Lambda_{HsX} = \Lambda'_{2X} = \Delta_{1X} = \Delta_{2X} = \Delta_{3X} = \beta_{3X} = 0.1$, $\beta_X = -0.6$, and $\kappa_{H2X} = 0.001$. As before the μ parameters are found by solving the constraint equations given in the appendix. With the above parameter set, plus the one that been used previously, it is found $m_{C_j} = (11\,137.3,\ 1661.7,\ 0)$ for the charged scalar sector, $m_{I_i} = (11\,135.9,\ 1652.6,\ 1467.0,\ 973.6,\ 0.002,\ 0,\ 0)$ for the neutral imaginary sector, and $m_{R_i} = (11\,135.9,\ 1927.6,\ 1816.6,\ 1652.7,\ 1508.8,\ 900.5,\ 146.2)$ for the real scalar sector, in GeV. Notice that, now a very light pseudoscalar, which has components mainly in the $SU(2)$ singlet fields $\phi_{1,2,3,X}$, has appeared. For instance, its component in Im H^0 is 7.3×10^{-12}, which implies $g_{eeJ} \approx 10^{-18}$. Hence, it is compatible with the astrophysical constraint, and poses no problem to the Z invisible decay width, since all the real scalar fields are heavier than the Z boson.

In this case the introduction of the ϕ_X scalar provides the right elements to make the model safe. Moreover, concerning the neutrino mass generation, from the Yukawa terms in Eq. (3.1) the most general neutrino mass matrix can be constructed, since now all VEVs are different from zero.

3.3.2 A Z_2 Symmetry and Dark Matter

Now let us consider the Z_2 symmetry again, after the introduction of the scalar ϕ_X. The field symmetry transformation rules are

$$Z_2(n_{R3}) = -n_{R3}, \quad Z_2(\Phi_2) = -\Phi_2, \quad Z_2(\phi_3) = -\phi_3, \tag{3.14}$$

and all the other being even. It is easy to see that all the Hermitian terms in the scalar potential involving ϕ_X are invariant under Z_2. However, the non-Hermitian terms

$$-i\kappa_{H2X}(\Phi_2^T\tau_2 H)(\phi_X^*)^2, \quad \beta_X(\phi_X^*\phi_1)(\phi_2\phi_3), \quad \text{and} \quad \beta_{3X}(\phi_X^*\phi_3^3), \tag{3.15}$$

in V_{B-L}^X, Eq. (3.13), are not invariant. The ϕ_X transformation rule could be changed to odd, in order to have some of them invariant. In this case, however, if we want to keep the Lagrangian invariant under Z_2, the $-i\kappa_{H1X}\Phi_1^T\tau_2 H\phi_X$ in the scalar potential given in Eq. (3.13) is also forbidden, and thus, an additional symmetry $U(1)$ acting only on the doublets $\Phi_{1,2}$ appears again, provoking a severe problem as discussed above.

Motivated by the possibility of having DM candidates a Z_2 symmetry to the entire Lagrangian is imposed. Then, the terms in Eq. (3.15) will be removed from the scalar potential and the only non-Hermitian terms allowed are

$$V_{B-L}^{NH} = \Phi_1^\dagger\Phi_2(\beta_{13}\phi_1\phi_3^* + \beta_{23}\phi_2^*\phi_3) + \beta_{123}\phi_1\phi_2(\phi_3^*)^2 - i\kappa_{H1X}\Phi_1^T\tau_2 H\phi_X + \text{H.c.} \tag{3.16}$$

After the SSB, the Z_2 symmetry is not broken if $V_{\Phi_2} = V_{\phi_3} = 0$, and the mass eigenstates that are also eigenstates of this symmetry. However, as it is shown from previous analysis, before introducing ϕ_X, that this vacuum configuration challenges the safety of the model due to a physical NG boson and a light real scalar. Now, after introducing ϕ_X there are four massless states in the neutral imaginary sector. However, in this case, both of the physical massless states are mainly singlets: $G_{F_2}^0$ is given by the same expression in Eq. (3.12), and

$$G_{F_1}^0 \approx \frac{1}{\sqrt{7093}}\left(12\,\mathrm{Im}\,\phi_1 - 15\,\mathrm{Im}\,\phi_2 + 82\,\mathrm{Im}\,\phi_X\right). \tag{3.17}$$

In fact, for the parameter set used above, the $G_{F_1}^0$ component in $\mathrm{Im}\,H^0$ is $\approx 2 \times 10^{-12}$ which implies $g_{eeJ} \approx 4 \times 10^{-18}$; thus it is safe with respect to the star energy loss constraint. This main feature is due to the introduction of the trilinear term $-i\kappa_{H1X}\Phi_1^T\tau_2 H\phi_X$. Qualitative arguments to explain this behavior can be given. The number of $U(1)$ symmetries is the same in both situations, with and without ϕ_X, 4. Without ϕ_X, there are two independent $U(1)$ symmetries involving only the doublets, say, $U(1)_\sigma$ acting on H, and $U(1)_\alpha$ acting on $\Phi_{1,2}$. With ϕ_X, the trilinear term relates the $U(1)$ charge of H to that of Φ_1, reducing the number of $U(1)$ symmetries involving only doublets to just one, say, $U(1)_\alpha$ acting on H and $\Phi_{1,2}$, and at the same time, it introduces a new $U(1)$ symmetry acting on H and ϕ_X, say, $U(1)_\gamma$. The number of broken generators is the same in both situations, since there are the same number of massless states; however, the origin of these physical massless states is different. The introduction of ϕ_X works in a very similar way to the singlet introduced in Refs. [44] and [43] to form the terms $H_u^T \tau_2 H_d \phi$ and $H_u^T \tau_2 H_d \phi^2$, respectively, in order to make the axion invisible.

The numerical spectra for the different scalar sectors, in GeV, are $m_{C_j} = (1489.9, 1330.3, 0)$ for the charged scalar sector; $m_{I_i} = (1479.7, 1433.9, 1318.9, 0, 0, 0, 0)$ for the neutral imaginary sector, and $m_{R_i} = (1483.5, 1479.7, 1378.4, 1378.4, 1318.9, 675.3, 152.9)$ for the real scalar sector. The point here is that all real scalar fields are now heavier than the Z boson, avoiding in this way the Z invisible decay width constraint. Therefore, the model is safe from these most severe constraints.

As the Z_2 symmetry still holds after the SSB, due to this particular vacuum configuration, the model can present some DM candidates. In general, a candidate must be the lightest particle odd under Z_2, in order to be stable. In this case, it can be the lightest odd mass eigenstate of the n_{R3} or the lightest odd imaginary mass eigenstate, or its odd real counterpart. This subject will be not considered in detail here. However, the relic abundance and the direct detection cross section for the case of the fermionic cold DM candidate n_{R3} are estimated (referred as χ from now on).

The most relevant annihilation process of χ occurs via the t-channel exchange of $\Phi_2^\pm(\Phi_2^0)$ to charged (neutral) leptons' final states. See Fig. 3.1. The thermally averaged χ annihilation cross section, $\langle\sigma v\rangle$, is given by [103]

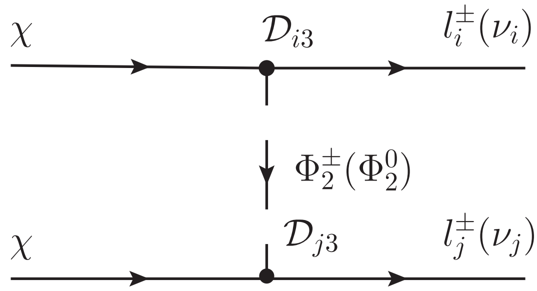

Figure 3.1: *The most relevant annihilation process of χ occurs via the t-channel exchange of $\Phi_2^\pm(\Phi_2^0)$ to charged (neutral) leptons' final states.*

$$\langle\sigma v\rangle \approx a + b\,\langle v^2\rangle \approx \frac{1}{16\pi}\sum_{ij} G^2_{\text{eff},ij}\, c_c\, M_\chi^2\, \langle v^2\rangle\,. \tag{3.18}$$

where $i, j = e, \mu, \tau$ and c_c are the color factors, equal to 1 for leptons. Also, the lepton masses have been neglected. The $G_{\text{eff},ij} = \mathcal{D}_{i3}\mathcal{D}^*_{j3}/\left(m^2_{C_1,R_2} + M_\chi^2\right)$ are the effective couplings, where $m_{C_1} \approx 1489$ GeV ($m_{R_2} \approx 1489$ GeV), the mass to be considered when charged (neutral) leptons are produced. The relic abundance of χ is approximately given by [104]

$$\Omega h^2 \approx \frac{1.04 \times 10^{-9} x_f}{M_{\mathrm{Pl}} \sqrt{g_*}(a + 3b/x_f)}, \tag{3.19}$$

where, in this model, $g_* = 107.75$ is the number of relativistic degrees of freedom available at the freeze-out temperature, T_f, and $x_f = M_\chi/T_f$ is given by

$$x_f = \ln\left[c\sqrt{\frac{45}{8}} \frac{g_\chi M_{\mathrm{Pl}} M_\chi (a + 6b/x_f)}{2\pi^3 \sqrt{x_f\, g_*}}\right], \tag{3.20}$$

with $c = 5/4$ and $g_\chi = 2$. Using the following set of parameters, $\mathcal{D}_{e3} = 0.06$, $\mathcal{D}_{\mu 3} = 0.9$, $\mathcal{D}_{e3} = 1$, and for $M_\chi = 750$ GeV, $x_f = 24.81$ and $\Omega h^2 = 0.11$ are found, which is in agreement with the experimental bounds [105]. The same interaction allowing the χ annihilation in charged leptons, which are proportional $\mathcal{D}_{i3}$, also induces lepton flavor violation (LFV) such as $\mu \to e\gamma$ and $\tau \to \mu\gamma$ (see below).

The next task is to compute the direct detection cross section. In the present case, the elastic scattering of χ with nuclei occurs via the t-channel $\chi + N \to \chi + N$ process due to the exchange of the scalar mass eigenstate R_7, which is the Higgs scalar boson in the model with mass $m_{R_7} \approx 152.9$ GeV. See Fig. 3.2. The spin-independent cross section is given by [106]

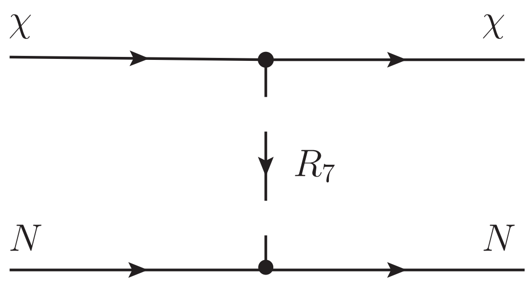

Figure 3.2: *The elastic scattering of χ with nuclei, $\chi + N \to \chi + N$, occurring via the t-channel due to the exchange of the scalar mass eigenstate R_7.*

$$\sigma_{\chi N} = \frac{4}{\pi} \frac{M_\chi^2 m_N^2}{(M_\chi + m_N)^2} \left[Z f_p + (A - Z) f_n\right]^2, \tag{3.21}$$

where the effective couplings to protons and neutrons, $f_{p,n}$, are

$$f_{p,n} = \sum_{q=u,d,s} \frac{G_{\mathrm{eff},q}}{\sqrt{2}} f_{Tq}^{(p,n)} \frac{m_{p,n}}{m_q} + \frac{2}{27} f_{TG}^{(p,n)} \sum_{q=c,b,t} \frac{G_{\mathrm{eff},q}}{\sqrt{2}} \frac{m_{p,n}}{m_q}. \tag{3.22}$$

In this case $G_{\mathrm{eff},q} = G_0 \times m_q \equiv \left[C_{\phi_2 R_7} C_{HR_7} M_\chi / \left(V_{\phi_2} V_H m_{R_7}^2\right)\right] \times m_q$, where $C_{\phi_2 R_7} \approx 0.01$ and $C_{HR_7} \approx 0.99$ are the coefficients relating the symmetry eigenstates (ϕ_2, H) to the relevant mass eigenstate R_7, respectively. By using $f_{Tq}^{(p,n)}$ and $f_{TG}^{(p,n)}$ given in Ref. [107]

$$\sigma_{\chi,p} \approx 3 \times 10^{-7}\,\mathrm{pb} \times \left[\frac{G_{\mathrm{eff},q} \times (1\,\mathrm{GeV}/\,m_q)}{10^{-7}\,\mathrm{GeV}^{-2}}\right]^2, \tag{3.23}$$

which gives $\sigma_{\chi,p} \approx 4.74 \times 10^{-11}$ pb, for $M_\chi = 750$ GeV, which is in agreement with the most recent present bounds [108–110]. The parameter set used in all the cases above is compatible with the following requirements: (i) the constraint equations are satisfied, (ii) all obtained masses are $m^2 > 0$, and (iii) results for the already known fields are consistent with those of the SM at the tree level.

Notice that the values that have been found for the lightest real scalar, the Higgs boson, are in agreement with the last combined CDF and D0 results for the ESM Higgs boson, which have excluded, at the 95% C.L., a region at high mass in $158 < m_H < 175$ GeV [111].

3.4 Neutrino Masses

The model without the Z_2 symmetry already has a satisfactory solution to the neutrino masses, since it is possible to construct a general neutrino mass matrix. However, it is going to be considered the case with this symmetry because the model becomes more attractive due to the presence of stable candidates to DM.

The Yukawa Lagrangian in Eq. (3.1) gives the following neutrino mass terms:

$$-\mathcal{L}_{m_\nu} = \mathcal{D}_{im}\overline{\nu_{Li}}n_{Rm}V_{\Phi_1} + \mathcal{M}_{mn}\overline{(n^c_m)_L}n_{Rn}V_{\phi_1} + \mathcal{M}_{33}\overline{(n^c_3)_L}n_{R3}V_{\phi_2} + \mathrm{H.c.}, \tag{3.24}$$

where $i, j = 1, 2, 3$ (or e, μ, τ, respectively, when convenient) and $m, n = 1, 2$. In matrix form Eq. (3.24) reads

$$-\mathcal{L}_{m_\nu} = \left[\begin{array}{cc} \overline{\nu_L} & \overline{(n^c)_L} \end{array}\right] \left[\begin{array}{cc} 0 & M_D \\ M_D^T & M_M \end{array}\right] \left[\begin{array}{c} (\nu^c)_R \\ n_R \end{array}\right], \tag{3.25}$$

with

$$\nu_L = [\nu_e\ \nu_\mu\ \nu_\tau]^T_L, \qquad n_R = [n_1\ n_2\ n_3]^T_R. \tag{3.26}$$

The Majorana mass matrix (M_M) and the Dirac mass matrix (M_D) are given by

$$M_M = V_{\phi_1}\begin{pmatrix} \mathcal{M}_{11} & \mathcal{M}_{12} & 0 \\ \mathcal{M}_{12} & \mathcal{M}_{22} & 0 \\ 0 & 0 & \frac{V_{\phi_2}}{V_{\phi_1}}\mathcal{M}_{33} \end{pmatrix}, \qquad M_D = V_{\Phi_1}\begin{pmatrix} \mathcal{D}_{11} & \mathcal{D}_{12} & 0 \\ \mathcal{D}_{21} & \mathcal{D}_{22} & 0 \\ \mathcal{D}_{31} & \mathcal{D}_{32} & 0 \end{pmatrix}, \tag{3.27}$$

since $V_{\Phi_2} = V_{\phi_3} = 0$, where $M_M = M_M^T$. For $V_{\Phi_1} \ll V_{\phi_1}$, the mass matrix in Eq. (3.25) can be diagonalized by an approximate scheme. The masses of the heavy neutrinos are related to the energy scale of the VEVs of the singlets ϕ_1 and ϕ_2, and are given by the eigenvalues of M_M:

$$M_{1,2} = \frac{(\mathcal{M}_{11} + \mathcal{M}_{22}) \mp \sqrt{4\mathcal{M}_{12}^2 + (\mathcal{M}_{11} - \mathcal{M}_{22})^2}}{2}V_{\phi_1}, \qquad M_3 = \mathcal{M}_{33}V_{\phi_2}.$$

The masses of the light neutrinos are given by the eigenvalues of the matrix

$$M_\nu \approx M_D M_M^{-1} M_D^T, \tag{3.28}$$

which are

$$m_{1,2} = \frac{1}{2D_M}\left[\Delta \mp \sqrt{\Delta^2 + r}\right]\frac{V_{\Phi_1}^2}{V_{\phi_1}}, \qquad m_3 = 0, \tag{3.29}$$

where the following definitions have been used:

$$\overrightarrow{C}_1 = (\mathcal{D}_{11}, \mathcal{D}_{21}, \mathcal{D}_{31}), \quad \overrightarrow{C}_2 = (\mathcal{D}_{12}, \mathcal{D}_{22}, \mathcal{D}_{32}), \tag{3.30}$$

$$r = 4D_M\left[\left(\mathcal{D}_{12}\overrightarrow{C}_1 - \mathcal{D}_{11}\overrightarrow{C}_2\right)^2 + D_D^2\right], \tag{3.31}$$

$$\Delta = \mathcal{M}_{11}(\overrightarrow{C}_2)^2 + \mathcal{M}_{22}(\overrightarrow{C}_1)^2 - 2\mathcal{M}_{12}(\overrightarrow{C}_1.\overrightarrow{C}_2), \tag{3.32}$$

$$D_M = \mathcal{M}_{12}^2 - \mathcal{M}_{11}\mathcal{M}_{22}, \quad D_D = \mathcal{D}_{21}\mathcal{D}_{32} - \mathcal{D}_{22}\mathcal{D}_{31}. \tag{3.33}$$

The parameters in M_M and M_D have to be chosen in order to have light neutrino masses consistent with the solar and atmospheric experimental data. However, since there is no a standard procedure to do that, a particular solution to show that this model can generate realistic active neutrino masses will be presented.

A Particular Solution

From the experimental neutrino data it is found that the neutrino mixing matrix is compatible with the so-called tribimaximal (TB) one [112], which is given by

$$U_{\text{TB}} = \begin{pmatrix} \sqrt{\frac{2}{3}} & \frac{1}{\sqrt{3}} & 0 \\ -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & -\frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} \end{pmatrix}, \tag{3.34}$$

and where it is assumed that the neutrino mixing angles are in a good approximation given by $\sin^2\theta_{12} = 1/3$, $\sin^2\theta_{23} = 1/2$, and $\sin^2\theta_{13} = 0$. Working in a basis where the charged lepton mass matrix is already diagonal, the U_{TB} matrix diagonalizes the light neutrino mass matrix in Eq. (3.28): $U_{\text{TB}}^T M_\nu U_{\text{TB}} = \hat{M}_\nu =$ diag(m_1, m_2, m_3). It can be shown that the most general neutrino mass matrix that can be exactly diagonalized by U_{TB} has the form

$$M_{\text{TB}} = \begin{pmatrix} x & y & y \\ y & x+\nu & y-\nu \\ y & y-\nu & x+\nu \end{pmatrix}, \tag{3.35}$$

using the same notation as in Ref. [112].

The M_{TB} mass eigenstates are

$$m_1 = x - y, \qquad m_2 = x + 2y, \qquad m_3 = 2\nu + x - y. \tag{3.36}$$

The square mass differences $\Delta m^2_{\rm sol}$ and $\Delta m^2_{\rm atm}$, needed to explain de solar and atmospheric neutrino anomalies, can be obtained by imposing conditions on x, y, and ν. The simplest way to apply this analysis to this particular case is as follows. Let us consider

$$\mathcal{M}_{11} = \mathcal{M}_{22} = \frac{V_{\phi 2}}{V_{\phi_1}} \mathcal{M}_{33}, \qquad \mathcal{M}_{12} = 0, \tag{3.37}$$

so that the Majorana and Dirac mass matrices are now given by

$$M_M = \mathcal{M}_{11} V_{\phi_1} \mathbf{1}_{3\times 3}; \qquad M_D = V_{\Phi_1} \begin{pmatrix} \mathcal{D}_{11} & \mathcal{D}_{12} & 0 \\ \mathcal{D}_{21} & \mathcal{D}_{22} & 0 \\ \mathcal{D}_{31} & \mathcal{D}_{32} & 0 \end{pmatrix}. \tag{3.38}$$

Then, the light neutrino mass matrix becomes

$$\begin{aligned} M_\nu &= M_D M_M^{-1} M_D^T = \frac{V^2_{\Phi_1}}{V_{\phi_1}} \frac{1}{\mathcal{M}_{11}} M_D M_D^T \\ &= \frac{V^2_{\Phi_1}}{\mathcal{M}_{11} V_{\phi_1}} \begin{pmatrix} \mathcal{D}^2_{11} + \mathcal{D}^2_{12} & \mathcal{D}_{11}\mathcal{D}_{21} + \mathcal{D}_{12}\mathcal{D}_{22} & \mathcal{D}_{11}\mathcal{D}_{31} + \mathcal{D}_{12}\mathcal{D}_{32} \\ \mathcal{D}_{11}\mathcal{D}_{21} + \mathcal{D}_{12}\mathcal{D}_{22} & \mathcal{D}^2_{21} + \mathcal{D}^2_{22} & \mathcal{D}_{21}\mathcal{D}_{31} + \mathcal{D}_{22}\mathcal{D}_{32} \\ \mathcal{D}_{11}\mathcal{D}_{31} + \mathcal{D}_{12}\mathcal{D}_{32} & \mathcal{D}_{21}\mathcal{D}_{31} + \mathcal{D}_{22}\mathcal{D}_{32} & \mathcal{D}^2_{31} + \mathcal{D}^2_{32} \end{pmatrix}. \end{aligned} \tag{3.39}$$

The matrix above has a null determinant and, therefore, a zero mass eigenstate. Thus, in order to make both matrices compatible, there must be a vanishing eigenvalue in Eq. (3.36). A possible solution is $m_3 = 0$, i.e., $2\nu + x - y = 0$, and, hence, $x + \nu = y - \nu$.

Comparing Eq. (3.35) with Eq. (3.39) the following equations appear:

$$\frac{x}{K} = \mathcal{D}^2_{11} + \mathcal{D}^2_{12}, \tag{3.40}$$

$$\frac{y}{K} = \mathcal{D}_{11}\mathcal{D}_{21} + \mathcal{D}_{12}\mathcal{D}_{22} = \mathcal{D}_{11}\mathcal{D}_{31} + \mathcal{D}_{12}\mathcal{D}_{32}, \tag{3.41}$$

$$\frac{x+\nu}{K} = \mathcal{D}^2_{21} + \mathcal{D}^2_{22} = \mathcal{D}^2_{31} + \mathcal{D}^2_{32}, \tag{3.42}$$

$$\frac{y-\nu}{K} = \mathcal{D}_{21}\mathcal{D}_{31} + \mathcal{D}_{22}\mathcal{D}_{32}, \tag{3.43}$$

where the dimensional constant $K = \frac{V^2_{\Phi_1}}{\mathcal{M}_{11} V_{\phi_1}}$ has been defined. A solution for the above equations is

$$\mathcal{D}_{21} = \mathcal{D}_{31}, \quad \text{and} \quad \mathcal{D}_{22} = \mathcal{D}_{32}. \tag{3.44}$$

From the above equations it is seen that the condition to have $m_3 = 0$, $x + \nu = y - \nu$, is automatically satisfied. The following equations to fit the atmospheric and solar neutrino data,

$$m_1 = x - y, \qquad m_2 = x + 2y, \qquad m_3 = 0, \tag{3.45}$$

are necessary, and, therefore,

$$\Delta m^2_{\text{sol}} = m_2^2 - m_1^2 = 3y(2x+y) > 0, \tag{3.46}$$

$$|\Delta m^2_{\text{atm}}| = |m_3^2 - m_1^2| = (x-y)^2. \tag{3.47}$$

Assuming that $x - y > 0$, the equations to be solved are

$$3y(2x+y) = 7.67 \times 10^{-5}\ (\text{eV})^2, \quad \text{and} \quad x - y = (2.4 \times 10^{-3})^{1/2}\ \text{eV}, \tag{3.48}$$

which are satisfied by $x = 0.049\,248\,7$ and $y = 0.000\,258\,887$, in eV. The corresponding mass eigenvalues are then given by $m_1 = 0.048\,989\,8$, $m_2 = 0.049\,766\,5$, and $m_3 = 0$, in eV, showing an inverse hierarchy pattern. Now the Eqs. (3.40) and (3.41) for the $\mathcal{D}_{ij}$ parameters have to be solved. In order to do that the value of the dimensional constant K is necessary. For $V_{\Phi_1} = 1$ MeV, $V_{\phi_1} = 1$ TeV, and assuming $\mathcal{M}_{11} = 1$, then $K = 1$ eV. Choosing the input values $\mathcal{D}_{22} = 0.25$ and $\mathcal{D}_{21} = 0.15$, it is obtained $\mathcal{D}_{11} = 0.190\,751$, and $\mathcal{D}_{12} = -0.113\,415$. Experiments on $0\nu\beta\beta$ can put bounds on $|m_{ee}|$, and the strongest one is $|m_{ee}| < 0.26\,(0.34)$ eV at 68% (90%) C.L. [113]. This quantity is related to the mass eigenvalues through $|m_{ee}| = |c_{13}^2(m_1 c_{12}^2 e^{i\delta_1} + m_2 s_{12}^2 e^{i\delta_2}) + m_3 e^{2i\phi_{CP}} s_{13}^2|$. In the current case, with no CP violation nor phases in the leptonic mixing matrix, it is found $|m_{ee}| \approx 0.05$ eV. Future experiments, however, expect to improve sensitivity up to ≈ 0.01 eV [114].

The procedure followed for finding a particular solution for the light neutrino masses can also be realized by using, instead of the matrices given in Eqs. (3.34) and (3.35), the ones given in Ref. [115], provided, in the notation of this reference, $c = -d/2$, and the identifications $\nu = d - (a+b)$, $y = d$, $x = a + 2b - d$ are made. It results $-a = x - y + 2\nu = m_3$, and we take a=0.

The results showed above demonstrate that the model is fully compatible with the experimental neutrino data, and that light neutrino masses can be generated neither appealing for very large energy scales nor imposing fine-tuning. Now, it is time to verify if the set of parameters used above is in agreement with the LFV constraints coming from a process like $l_i \to l_j + \gamma$, where $i = 2, 3 = \mu, \tau$ and $j = 1, 2 = e, \mu$, respectively. See Fig. 3.3. This model has one loop contributions to such a process since charged leptons couple to charged scalars and right-handed heavy neutrinos. The branching ratio is estimated as [116]

$$B\,(l_i \to l_j + \gamma) = \frac{96\pi^3\alpha}{G_F^2 m_{l_i}^4}\left(|f_{M1}|^2 + |f_{E1}|^2\right), \tag{3.49}$$

where $\alpha \simeq 1/137$ and $G_F \simeq 1.16 \times 10^{-5}$ GeV^{-2} is the Fermi constant and

$$f_{M1} = f_{E1} = \sum_{k=1}^{3} \frac{\mathcal{D}_{ik}\mathcal{D}_{jk}}{4\,(4\pi)^2}\frac{m_{l_i}^2}{m_\Phi^2} F_2\left(\frac{m_{N_k}^2}{m_\Phi^2}\right), \tag{3.50}$$

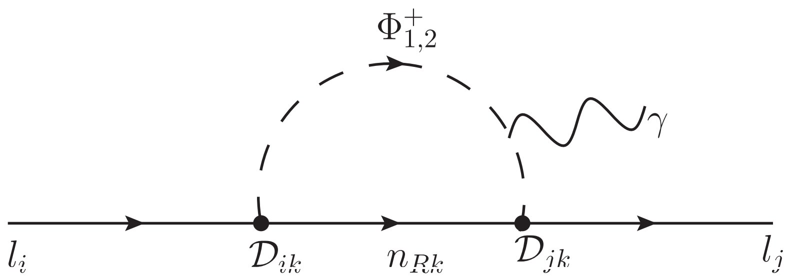

Figure 3.3: *Diagram giving rise to* $l_i \to l_j + \gamma$.

with $F_2(x)$ being

$$F_2(x) = \frac{1 - 6x + 3x^2 + 2x^3 - 6x^2 \ln x}{6(1-x)^4}. \tag{3.51}$$

Using the parameters needed to fit the neutrino masses and the ones to estimate $\Omega h^2 \simeq 0.11$ ($\mathcal{D}_{e3} \simeq 0.06$, $\mathcal{D}_{\mu 3} \simeq 0.9$, $\mathcal{D}_{\tau 3} \simeq 1$, $m_{n_{R3}} = 750$ GeV, $m_{C_1} \simeq m_{\Phi_1^\pm} = 1.33$ TeV, $m_{C_2} = m_{\Phi_2^\pm} = 1.48$ TeV), it is possible give an estimate for the branching ratio $B(\mu \to e + \gamma) \simeq 7.9 \times 10^{-12}$ and $B(\tau \to \mu + \gamma) \simeq 2.5 \times 10^{-9}$. These values are in agreement with the present upper bounds $B(\mu \to e + \gamma) < 1.2 \times 10^{-11}$ and $B(\tau \to \mu + \gamma) \simeq 6.8 \times 10^{-8}$ [117, 118].

The ratio between the VEVs used for finding the neutrino mass eigenvalues is $V_{\Phi_1}/V_{\phi_1} = 10^{-6}$. This is of the same order as the ratio $m_e/m_{\text{top}} = Y_e/Y_{\text{top}} \approx 10^{-6}$, and it is comparable with $m_u/m_{\text{top}} = Y_u/Y_{\text{top}} \approx 10^{-5}$. The values for V_{Φ_1} and V_{ϕ_1} have been chosen in order to have light neutrino masses without resorting to very tiny neutrino Yukawa coupling constants, or fine-tuning, and, at the same time, to have the Z' vector boson not extremely heavy. This is a kind of seesaw mechanism where the heaviest scale, V_{ϕ_1}, is constrained by the Z' vector boson, which should be not too heavy in order to not decouple from the spectrum. The light scale, V_{Φ_1}, is then used to fix the absolute neutrino mass scale through the ratio $V_{\Phi_1}^2/V_{\phi_1}$.

As discussed above, the absolute neutrino mass scale depends on the ratio $V_{\Phi_1}^2/V_{\phi}$, where V_{Φ_1} is a tiny value. Although this value can be affected by radiative corrections, it can be argued that, when the Z_2 symmetry is considered, setting V_{Φ_1} to a tiny value, at the tree level, is natural because if it were in fact taken to be zero this would increase the symmetry of the entire Lagrangian ('t Hooft's principle of naturalness). This can be seen considering the constraint equations with $V_{\Phi_1} \to 0$. It implies that $\kappa_{H1X} = 0$, since V_H and V_{ϕ_X} differ from zero. Then the term $-i\kappa_{H1X}\Phi_1^T \tau_2 H \phi_X$ does not appear in the scalar potential, Eq. (3.16), and the entire Z_2 invariant Lagrangian is now invariant under an additional global quantum symmetry, say, $U(1)_\zeta$. A possible ζ-charge assignment is $\zeta(\nu_{eL}, e_L, e_R, \Phi_{1,2}) = -1$, $\zeta(u_L, d_L, u_R, d_R) = 1/3$, and $\zeta(n_{(1,2,3)R}, \phi_{1,2,3}) = 0$. Thus, it is expected that the VEV hierarchy will remain stable when radiative corrections are taken into account.

3.5 The Constraint Equations

Here, the constraint equations for the scalar potential given in Eq. (3.2) plus the terms after the ϕ_X introduction and without the Z_2 symmetry are given. These equations are obtained by considering, after the spontaneous symmetry breaking, the linear terms ($t_\varphi \varphi$) in the scalar potential, and the solutions to the equations $t_\varphi = 0$ are the critical points of the scalar potential.

$$
\begin{aligned}
t_H = \;& V_H \left(2\lambda_H V_H^2 + \Lambda_{H1} V_{\Phi_1}^2 + \Lambda_{H2} V_{\Phi_2}^2 + \Lambda_{Hs1} V_{\phi_1}^2 + \Lambda_{Hs2} V_{\phi_2}^2 + \Lambda_{Hs3} V_{\phi_3}^2 + \Lambda_{HsX} V_{\phi_X}^2 \right. \\
& \left. -2\mu_H^2\right) - \sqrt{2}\kappa_{H1X} V_{\Phi_1} V_{\phi_X} - \kappa_{H2X} V_{\Phi_2} V_{\phi_X}^2,
\end{aligned} \tag{3.52}
$$

$$
\begin{aligned}
t_{\Phi_1} = \;& V_{\Phi_1} \left(\Lambda_{H1} V_H^2 + 2\lambda_{11} V_{\Phi_1}^2 + (\lambda_{12} + \lambda'_{12}) V_{\Phi_2}^2 + \Lambda'_{11} V_{\phi_1}^2 + \Lambda'_{12} V_{\phi_2}^2 + \Lambda'_{13} V_{\phi_3}^2 \right. \\
& \left. +\Lambda'_{1X} V_{\phi_X}^2 - 2\mu_{11}^2\right) - \sqrt{2}\kappa_{H1X} V_H V_{\phi_X} + V_{\Phi_2} V_{\phi_3} (\beta_{13} V_{\phi_1} + \beta_{23} V_{\phi_2}),
\end{aligned} \tag{3.53}
$$

$$
\begin{aligned}
t_{\Phi_2} = \;& V_{\Phi_2} \left(\Lambda_{H2} V_H^2 + (\lambda_{12} + \lambda'_{12}) V_{\Phi_1}^2 + 2\lambda_{22} V_{\Phi_2}^2 + \Lambda'_{21} V_{\phi_1}^2 + \Lambda'_{22} V_{\phi_2}^2 + \Lambda'_{23} V_{\phi_3}^2 \right. \\
& \left. +\Lambda'_{2X} V_{\phi_X}^2 - 2\mu_{22}^2\right) - \kappa_{H2X} V_H V_{\phi_X}^2 + V_{\Phi_1} V_{\phi_3} (\beta_{13} V_{\phi_1} + \beta_{23} V_{\phi_2}),
\end{aligned} \tag{3.54}
$$

$$
\begin{aligned}
t_{\phi_1} = \;& V_{\phi_1} \left(\Lambda_{Hs1} V_H^2 + \Lambda'_{11} V_{\Phi_1}^2 + \Lambda'_{21} V_{\Phi_2}^2 + 2\lambda_{s1} V_{\phi_1}^2 + \Delta_{12} V_{\phi_2}^2 + \Delta_{13} V_{\phi_3}^2 + \Delta_{1X} V_{\phi_X}^2 \right. \\
& \left. -2\mu_{s1}^2\right) + \beta_{13} V_{\Phi_1} V_{\Phi_2} V_{\phi_3} + V_{\phi_2} V_{\phi_3} (\beta_{123} V_{\phi_3} + \beta_X V_{\phi_X}),
\end{aligned} \tag{3.55}
$$

$$
\begin{aligned}
t_{\phi_2} = \;& V_{\phi_2} \left(\Lambda_{Hs2} V_H^2 + \Lambda'_{12} V_{\Phi_1}^2 + \Lambda'_{22} V_{\Phi_2}^2 + \Delta_{12} V_{\phi_1}^2 + 2\lambda_{s2} V_{\phi_2}^2 + \Delta_{23} V_{\phi_3}^2 + \Delta_{2X} V_{\phi_X}^2 \right. \\
& \left. -2\mu_{s2}^2\right) + \beta_{23} V_{\Phi_1} V_{\Phi_2} V_{\phi_3} + V_{\phi_1} V_{\phi_3} (\beta_{123} V_{\phi_3} + \beta_X V_{\phi_X}),
\end{aligned} \tag{3.56}
$$

$$
\begin{aligned}
t_{\phi_3} = \;& V_{\phi_3} \left(\Lambda_{Hs3} V_H^2 + \Lambda'_{13} V_{\Phi_1}^2 + \Lambda'_{23} V_{\Phi_2}^2 + \Delta_{13} V_{\phi_1}^2 + \Delta_{23} V_{\phi_2}^2 + 2\lambda_{s3} V_{\phi_3}^2 + \Delta_{3X} V_{\phi_X}^2 \right. \\
& \left. +3\beta_{3X} V_{\phi_3} V_{\phi_X} - 2\mu_{s3}^2\right) + V_{\Phi_1} V_{\Phi_2} (\beta_{13} V_{\phi_1} + \beta_{23} V_{\phi_2}) + V_{\phi_1} V_{\phi_2} 2\beta_{123} V_{\phi_3}) \\
& +V_{\phi_1} V_{\phi_2} \beta_X V_{\phi_X}
\end{aligned} \tag{3.57}
$$

$$
\begin{aligned}
t_{\phi_X} = \;& V_{\phi_X} \left(\Lambda_{HsX} V_H^2 + \Lambda'_{1X} V_{\Phi_1}^2 + \Lambda'_{2X} V_{\Phi_2}^2 + \Delta_{1X} V_{\phi_1}^2 + \Delta_{2X} V_{\phi_2}^2 + \Delta_{3X} V_{\phi_3}^2 \right. \\
& \left. +2\lambda_{sX} V_{\phi_X}^2 - 2\kappa_{H2X} V_H V_{\Phi_2} - 2\mu_{sX}^2\right) - \sqrt{2}\kappa_{H1X} V_H V_{\Phi_1} + \beta_X V_{\phi_1} V_{\phi_2} V_{\phi_3} \\
& +\beta_{3X} V_{\phi_3}^3.
\end{aligned} \tag{3.58}
$$

3.6 Gauge Bosons

Although the main objective of this chapter is not perform a study about the phenomenological properties of the gauge bosons, it is important, at least, give an analytical expression of both their masses and their eigenstates since signals of these gauge bosons can be found in current experiments (for example LHC). Here, this will be done in a general way.

Suppose that the symmetry $SU(2)_L \otimes U(1)_Y \otimes U(1)_X$, where Y and X represent the generators of any $U(1)$ gauge groups, is broken down through N scalar mutiplets Φ_i with isospin I^a, and quantum numbers Y^a and X^a respectively. Thus the N multiplets of Higgs read

$$\Phi_1 = \begin{pmatrix} \phi_{1,1} \\ \phi_{1,2} \\ \vdots \\ \phi_{1,(2\mathrm{I}^1+1)} \end{pmatrix}, \cdots, \Phi_N = \begin{pmatrix} \phi_{N,1} \\ \vdots \\ \vdots \\ \phi_{N,(2\mathrm{I}^N+1)} \end{pmatrix}. \tag{3.59}$$

Besides that, let's that the VEVs of these multiplets are written as

$$\langle\Phi_1\rangle_0 = \begin{pmatrix} 0 \\ \vdots \\ v_{1,i} \\ \vdots \\ 0 \end{pmatrix}, \cdots, \langle\Phi_N\rangle_0 = \begin{pmatrix} 0 \\ \vdots \\ v_{N,j} \\ \vdots \\ 0 \end{pmatrix}. \tag{3.60}$$

To find the masses of the gauge bosons it is necessary to calculate $\sum_a \left|D^a_\mu \phi^a_0\right|^2$, where $D^a_\mu = \partial_\mu + ig_Y Y^a A_\mu + ig_X X^a B_\mu + ig_2 T^i W^i_\mu$. The g_2, g_Y, g_X are the coupling constants of the gauge groups $SU(2)$, $U(1)_Y$ and $U(1)_X$ respectively. The A_μ and B_μ are the gauge bosons of the groups $U(1)_Y$ and $U(1)_X$ respectively. And, finally, the T^i are the three generators of $SU(2)_L$. When calculated explicitly, it is easy to see that

$$\begin{aligned}\sum_a \left(D^a_\mu \langle\Phi^a\rangle_0\right)^\dagger \left(D^{a\mu} \langle\Phi^a\rangle_0\right) &= \sum_a g_2^2 \frac{v_a^2}{4}\left[\mathrm{I}^a(\mathrm{I}^a+1) - (\mathrm{I}^a_3)^2\right] \times \left|W^1_\mu - iW^2_\mu\right|^2 \\ &+ \sum_a \left(v_a g_Y Y^a A_\mu + v_a g_X X^a B_\mu + v_a g_2 \mathrm{I}^a_3 W^3_\mu\right)^2 .\end{aligned} \tag{3.61}$$

Defining $W^\pm \equiv \frac{1}{\sqrt{2}}\left(W^1_\mu \mp iW^2_\mu\right)$, the expressions of the masses of the charged gauge boson, $M_{W^\pm}$, are

$$M^2_{W^\pm} = g_2^2 \sum_a v_a^2 \left[\mathrm{I}^a(\mathrm{I}^a+1) - (\mathrm{I}^a_3)^2\right]. \tag{3.62}$$

The masses of the neutral gauge bosons Z, Z', γ are obtained from the following matrix

$$\mathbf{M}^2 \equiv \begin{pmatrix} g_2^2 \sum_a v_a^2 (\mathrm{I}^a_3)^2 & g_2 g_Y \sum_a v_a^2 Y^a \mathrm{I}^a_3 & g_2 g_X \sum_a v_a^2 X^a \mathrm{I}^a_3 \\ g_2 g_Y \sum_a v_a^2 Y^a \mathrm{I}^a_3 & g_Y^2 \sum_a v_a^2 (Y^a)^2 & g_Y g_X \sum_a v_a^2 Y^a X^a \\ g_2 g_X \sum_a v_a^2 X^a \mathrm{I}^a_3 & g_Y g_X \sum_a v_a^2 Y^a X^a & g_X^2 \sum_a v_a^2 (X^a)^2 \end{pmatrix}. \tag{3.63}$$

The basis used to write M^2 was $\left(W^{3\mu},\, A^{\mu},\, B^{\mu}\right)$. Now, to avoid the spontaneous symmetry breaking of the electric charged the quantum numbers must satisfy

$$\mathrm{I}_3^a = -\left(Y^a + X^a\right). \tag{3.64}$$

Using the relations above

$$\mathrm{M}^2 = \begin{pmatrix} g_2^2\left(K+L+2N\right) & -g_2 g_Y\left(K+N\right) & -g_2 g_X\left(L+N\right) \\ -g_2 g_Y\left(K+N\right) & g_Y^2 K & g_Y g_X N \\ -g_2 g_X\left(L+N\right) & g_Y g_X N & g_X^2 L \end{pmatrix}, \tag{3.65}$$

where the definitions

$$K \equiv \sum_a v_a^2 (Y^a)^2, \quad L \equiv \sum_a v_a^2 (X^a)^2, \quad N \equiv \sum_a v_a^2 Y^a X^a. \tag{3.66}$$

have been done. Now, it is straightforward find the eigenvalues of M^2, which are the squared masses of the neutral gauge bosons, to be

$$M_\gamma^2 = 0, \quad M_Z^2 = \frac{1}{2}\left[P - \sqrt{R}\right], \quad M_{Z'}^2 = \frac{1}{2}\left[P + \sqrt{R}\right], \tag{3.67}$$

with $P \equiv g_Y^2 K + g_X^2 L + g_2^2\left(K+L+2N\right)$ and R is defined by

$$R \equiv P^2 - 4\left(KL - N^2\right)\left[g_X^2 g_Y^2 + g_2^2\left(g_X^2 + g_Y^2\right)\right]. \tag{3.68}$$

Finally, an analytical and exact expression of the γ, Z, Z' in the basis $\left(W^{3\mu},\, A^{\mu},\, B^{\mu}\right)$ can be given. The photon γ, Z and Z' bosons read

$$\gamma^\mu = \frac{1}{N_\gamma}\left[\left(1/g_2\right) W^{3\mu} + \left(1/g_Y\right) A^\mu + \left(1/g_X\right) B^\mu\right], \tag{3.69}$$

$$\begin{aligned} Z^\mu &= \frac{1}{N_Z}\left[g_2\left(g_X^2 L - g_Y^2 N - M_Z^2\right) W^{3\mu} - g_Y\left(g_X^2 L + g_2^2\left(L+N\right) - M_Z^2\right) A^\mu\right. \\ &\quad \left. + g_X\left(g_Y^2 N + g_2^2\left(L+N\right)\right) B^\mu\right], \end{aligned} \tag{3.70}$$

$$\begin{aligned} Z'^\mu &= \frac{1}{N_{Z'}}\left[g_2\left(g_X^2 L - g_Y^2 N - M_{Z'}^2\right) W^{3\mu} - g_Y\left(g_X^2 L + g_2^2\left(L+N\right) - M_{Z'}^2\right) A^\mu\right. \\ &\quad \left. + g_X\left(g_Y^2 N + g_2^2\left(L+N\right)\right) B^\mu\right], \end{aligned} \tag{3.71}$$

where N_γ, N_Z, $N_{Z'}$ are the usual normalization constants. For example,

$$N_\gamma = \sqrt{\frac{1}{g_2} + \frac{1}{g_Y} + \frac{1}{g_X}}, \tag{3.72}$$

for the photon eigenstate. Similar relations can be given to N_Z and $N_{Z'}$.

Applying the equations above to the present model, it is straightforward find

$$
\begin{aligned}
K &= \frac{1}{4}V_H^2 + 4V_{\Phi_1}^2 + \frac{25}{4}V_{\Phi_2}^2 + 16V_{\phi_1}^2 + 25V_{\phi_2}^2 + \frac{1}{4}V_{\phi_3}^2 + \frac{9}{4}V_{\phi_X}^2, &(3.73)\\
L &= \frac{9}{4}V_{\Phi_1}^2 + 9V_{\Phi_2}^2 + 16V_{\phi_1}^2 + 25V_{\phi_2}^2 + \frac{1}{4}V_{\phi_3}^2 + \frac{9}{4}V_{\phi_X}^2, &(3.74)\\
N &= -3V_{\Phi_1}^2 - \frac{15}{2}V_{\Phi_2}^2 - 16V_{\phi_1}^2 - 25V_{\phi_2}^2 - \frac{1}{4}V_{\phi_3}^2 - \frac{9}{4}V_{\phi_X}^2. &(3.75)
\end{aligned}
$$

Putting the numerical values used in the previous sections, the masses of the gauge bosons in GeV are

$$M_\gamma = 0,\;\; M_{W^\pm} \approx 80.3,\;\; M_Z \approx 91.1,\;\; M_{Z'} \approx 4.5 \times 10^3. \tag{3.76}$$

The Eq. (3.76) shows that the parameters chosen previously make the masses of the gauge bosons be in agreement with the ones in the SM. Moreover, the mass of the Z' gauge boson, which is a prediction of the model, is about 4 TeV. The mass of this gauge boson can be done lower if the VEVs of the ϕ_i fields are chosen lower. An analysis of the behavior of Z' mass is beyond the scope of this chapter.

3.7 A $O(2)$ Symmetry

The motivation of this short section is to show that the model $B-L$ studied in this chapter, which is based on the $SU(2)_L \otimes U(1)_{Y'} \otimes U(1)_{B-L}$ gauge symmetry (notice that the second group is not the $U(1)_Y$ of hypercharge of the SM) is equivalent to one based on $SU(2)_L \otimes U(1)_Y \otimes U(1)_Z$, i.e. SM $\otimes\, U(1)_Z$. To show that, the use of a $O(2)$ symmetry will be done.

Consider the transformation on the gauge bosons of the $U(1)_{Y'}$ and $U(1)_{B-L}$ groups, A_μ and B_μ respectively, given by

$$
\begin{aligned}
A_\mu &= \cos\theta X_\mu + \sin\theta V_\mu, &(3.77)\\
B_\mu &= -\sin\theta X_\mu + \cos\theta V_\mu, &(3.78)
\end{aligned}
$$

where V_μ and X_μ are the generators of the $U(1)_Y$ and $U(1)_Z$, respectively. The remain fields in the model being invariant under this transformation. Now, because the quantum numbers of the fields ϕ_i and n_{iR} in the studied model were chosen to make these singlets of the SM group, their kinetic terms determine what is $\cos\theta$ that makes possible write the model in $SU(2)_L \otimes U(1)_Y \otimes U(1)_Z$ form. Notice that the kinetic terms are given by

$$D_\mu\psi_i = \left(\partial_\mu + i g'_Y \frac{Y'_{\psi_i}}{2} A_\mu + i g_{BL} \frac{BL_{\psi_i}}{2} B_\mu\right)\psi_i$$
$$= \left(\partial_\mu + i \frac{Y'_{\psi_i}}{2}\left(g'_Y A_\mu - g_{BL} B_\mu\right)\right)\psi_i$$
$$= \left(\partial_\mu + i\sqrt{g'^2_Y + g^2_{BL}}\frac{Y'_{\psi_i}}{2}\frac{1}{\sqrt{g'^2_Y + g^2_{BL}}}\left(g'_Y A_\mu - g_{BL} B_\mu\right)\right)\psi_i, \tag{3.79}$$

where ψ_i above makes reference to the fields ϕ_i and n_{iR}. Moreover, the relation $Y'_{\psi_i} + BL_{\psi_i} = 0$ between the $U(1)_{Y'}$ and $U(1)_{B-L}$ charges of the fields ψ_i has been used. From Eq. (3.79) it is easy to see that $\cos\theta$ has to be equal to $g'_Y/\sqrt{g'^2_Y + g^2_{BL}}$, where g'_Y and g_{BL} are the coupling constants of the $U(1)_{Y'}$ and $U(1)_{B-L}$ groups. In other words, the new gauge bosons, are given

$$X_\mu \equiv \frac{g'_Y}{\sqrt{g'^2_Y + g^2_{BL}}} A_\mu - \frac{g_{BL}}{\sqrt{g'^2_Y + g^2_{BL}}} B_\mu, \tag{3.80}$$
$$V_\mu \equiv \frac{g_{BL}}{\sqrt{g'^2_Y + g^2_{BL}}} A_\mu + \frac{g'_Y}{\sqrt{g'^2_Y + g^2_{BL}}} B_\mu. \tag{3.81}$$

Thus, doing this transformation, the fields ϕ_i and n_{iR} will decouple of the $U(1)_Y$ group that can now be interpreted as the hypercharge group. From Eq. (3.79), also, it can be seen that the coupling constant of the $U(1)_Z$ group reads

$$g_Z \equiv \sqrt{g'^2_Y + g^2_{BL}}. \tag{3.82}$$

Now, to determine the relation between the coupling constants g_Y and g'_Y, g_{BL} it useful look at the kinetic term of the rest of fields, i.e. H, L_i, Q_i, etc. called of Ψ_i for simplicity. Doing so,

$$D_\mu\Psi_i = \left(\ldots + i g_Z \frac{1}{2}\left(Y'_{\Psi_i}\cos^2\theta - BL_{\Psi_i}\sin^2\theta\right) X_\mu \right.$$
$$\left. + i\frac{g'_Y g_{BL}}{\sqrt{g'^2_Y + g^2_{BL}}}\frac{1}{2}\left(Y'_{\Psi_i} + BL_{\Psi_i}\right) V_\mu\right)\Psi_i. \tag{3.83}$$

From Eq. (3.83) it is obvious that the hypercharge coupling constant, g_Y is written as

$$g_Y \equiv \frac{g'_Y g_{BL}}{\sqrt{g'^2_Y + g^2_{BL}}}. \tag{3.84}$$

Furthermore, the quantum numbers of the new $U(1)_Y$ and $U(1)_Z$ groups are related to the ones of the $U(1)_{Y'}$ and $U(1)_{BL}$ groups by

$$Y_{\Psi_i} \equiv Y'_{\Psi_i} + BL_{\Psi_i}, \tag{3.85}$$
$$Z_{\Psi_i} \equiv Y'_{\Psi_i}\cos^2\theta - BL_{\Psi_i}\sin^2\theta. \tag{3.86}$$

As the transformation given in the Eqs. (3.77) and (3.78) changes the field strength tensors $F_{\mu\nu}^{A}$ and $F_{\mu\nu}^{B}$ of the $U(1)_{Y'}$ and $U(1)_{BL}$ groups respectively, it is necessary, finally, to verify that the kinetic term of these is invariant under these transformation. Starting with $F_{\mu\nu}^{A}$

$$\begin{aligned} F_{\mu\nu}^{A} &= \partial_\mu A_\nu - \partial_\nu A_\mu \\ &= \partial_\mu \left(X_\nu \cos\theta + V_\nu \sin\theta\right) - \partial_\nu \left(X_\mu \cos\theta + V_\mu \sin\theta\right) \\ &= \cos\theta \left(\partial_\mu X_\nu - \partial_\nu X_\mu\right) + \sin\theta \left(\partial_\mu V_\nu - \partial_\nu V_\mu\right) \\ &\equiv X_{\mu\nu} \cos\theta + V_{\mu\nu} \sin\theta, \end{aligned} \tag{3.87}$$

where the definitions $X_{\mu\nu} \equiv \partial_\mu X_\nu - \partial_\nu X_\mu$ and $V_{\mu\nu} \equiv \partial_\mu V_\nu - \partial_\nu V_\mu$ have been used. With this the kinetic term of $F_{\mu\nu}^{A}$ become

$$\begin{aligned} -\frac{1}{4} F^{A\mu\nu} F_{A\mu\nu} &= -\frac{1}{4} \left(X^{\mu\nu} \cos\theta + V^{\mu\nu} \sin\theta\right) \left(X_{\mu\nu} \cos\theta + V_{\mu\nu} \sin\theta\right) \\ &= -\frac{1}{4} \left(X^{\mu\nu} X_{\mu\nu} \cos^2\theta\right) - \frac{1}{4} \left(V^{\mu\nu} V_{\mu\nu} \sin^2\theta\right) \\ &\quad -\frac{1}{2} X^{\mu\nu} V_{\mu\nu} \cos\theta \sin\theta. \end{aligned} \tag{3.88}$$

Similarly doing for kinetic term of $F_{\mu\nu}^{B}$

$$\begin{aligned} -\frac{1}{4} F^{B\mu\nu} F_{\mu\nu}^{B} &= -\frac{1}{4} \left(-X^{\mu\nu} \sin\theta + V^{\mu\nu} \cos\theta\right) \left(-X_{\mu\nu} \sin\theta + V_{\mu\nu} \cos\theta\right) \\ &= -\frac{1}{4} \left(X^{\mu\nu} X_{\mu\nu} \sin^2\theta\right) - \frac{1}{4} \left(V^{\mu\nu} V_{\mu\nu} \cos^2\theta\right) \\ &\quad +\frac{1}{2} X^{\mu\nu} V_{\mu\nu} \cos\theta \sin\theta. \end{aligned} \tag{3.89}$$

Thus, when both kinetic terms are added

$$-\frac{1}{4} F^{A\mu\nu} F_{A\mu\nu} - \frac{1}{4} F^{B\mu\nu} F_{B\mu\nu} = -\frac{1}{4} X^{\mu\nu} X_{\mu\nu} - \frac{1}{4} V^{\mu\nu} V_{\mu\nu}. \tag{3.90}$$

Therefore, the Lagrangian is invariant under the $O(2)$ transformation given in the Eqs. (3.77) and (3.78).

In conclusion, the model $SU(2)_L \otimes U(1)_{Y'} \otimes U(1)_{B-L}$ analyzed in this chapter has a $O(2)$ symmetry given by

$$A_\mu = \cos\theta X_\mu + \sin\theta V_\mu, \tag{3.91}$$

$$B_\mu = -\sin\theta X_\mu + \cos\theta V_\mu, \tag{3.92}$$

with $\tan\theta \equiv g_{BL}/g_Y'$, and the rest of fields being invariant. This allows to interpreted the model as being based on SM$\otimes U(1)_Z$ gauge group without loss of generality. The advantage of interpreting the model in this way is that if a study of the phenomenological properties of the model is required, the results will be easier to compare with the ones of the SM. In other words, the new phenomenological properties of the model will come from the $U(1)_Z$, instead of the $U(1)_{Y'} \otimes U(1)_{B-L}$ group.

3.8 Conclusions

In this chapter it has been studied in detail the scalar and the neutrino Yukawa sectors of an extension of the electroweak standard model which has an extra $U(1)$ gauge factor, as described in Sec. 3.2. A detailed study of the scalar spectra of the potential given in Eq. (3.2) shows that this is inconsistent with the experimental data coming from the star energy loss and the Z invisible decay width. It is important to stress that this is a general result for this scalar potential.

A suitable solution to this problem is the addition of a new $SU(2)$ scalar singlet, called ϕ_X in the text. The new terms introduced by ϕ_X are able to remove all the physical NG bosons and, at the same time, to have all the real mass eigenstates heavier than the Z boson. This solution is particularly interesting since, in this case, all VEVs can be different from zero, which allows for the construction of a general neutrino mass matrix.

In order to have a still more attractive model it is considered the possibility of having DM candidates by including a Z_2 symmetry. Before the SSB the only fields having odd transformation under Z_2 are n_{R3}, Φ_2, and ϕ_3. Z_2 will still be a symmetry if the scalar fields Φ_2 and ϕ_3 do not develop VEVs. Hence, after the SSB there will be states which are mass and Z_2 eigenstates simultaneously. It opens the possibility of having DM fields since the lighter Z_2 odd eigenstate will be stable. Moreover, a preliminary study that the fermionic field n_{R3} is a viable cold DM candidate is performed.

The neutrino mass generation in the framework of the model with the Z_2 symmetry is considered in detail. In this case an inverted hierarchy compatible with the solar and atmospheric neutrino data and the tribimaximal mixing matrix is found. Two appealing features are (i) the absolute scale of the neutrino masses is obtained by a seesaw mechanism at $\mathcal{O}\,(\text{TeV})$ energy scale, which is the scale of the first symmetry breaking, and (ii) the observed mass-squared differences are obtained without resorting to fine-tuning the neutrino Yukawa couplings.

The model has also some phenomenological implications. One of them is the existence of an extra neutral vector boson, Z', which can be in principle detected at the LHC or International Linear Collider. In fact, there are studies showing that the Z' of this particular model can be distinguished from that of other models by comparing, for instance, the forward-backward asymmetry for the process $p + p \to \mu^+ + \mu^- + X$ as a function of the dilepton invariant mass, or the muon transverse momentum distribution at the LHC [119], and the same asymmetry for the process $e^+ + e^- \to f + \overline{f}$ ($f = q,\, l$) at International Linear Collider [120]. At first glance, another interesting feature is that the model seems to indicate that the LFV and DM are closely related. It implies that when the parameters are appropriate to satisfy the DM requirements, the LFV is relatively close to the present experimental bounds. In this way, the model can be confronted by the next generation of LFV experiments.

Chapter 4

PROSPECTS: A FIRST GLANCE AT THE MINIMAL SUPERSYMMETRIC $B - L$ MODEL

4.1 Introduction

Although the Standard Model (SM) gives very good results in explaining the observed properties of the charged fermions, it is unlikely to be the ultimate theory because it maintains the neutrinos massless to all orders in perturbation theory, and even after non-perturbative effects are included. The recent groundbreaking discoveries of nonzero neutrino masses and its oscillations [121–125] have put massive neutrinos as one of evidences of physics beyond the SM.

Recently it was proposed a very interesting extension of the SM, where $B - L$ (being B and L the baryonic and leptonic number respectively) is a gauge symmetry [99]. In that reference, the authors use the constraints on the number of right-handed neutrinos coming from anomaly cancelations in order to propose extensions of the SM which may be part of a more general theory, in which $B - L$ would appear naturally as a local symmetry. The models in that reference are based on $SU(2)_L \otimes U(1)_{Y'} \otimes U(1)_{B-L}$ gauge symmetry which is broken down in a first step to $SU(2)_L \otimes U(1)_Y$ (SM gauge group) when some scalar fields, singlets of the SM gauge group, obtain VEVs. Finally, the group of the SM is spontaneously broken down to electric charge group, called $U(1)_Q$.

An important characteristic of this type of model is that the parameter Y' is chosen to obtain the hypercharge Y of the SM, given by $Y = Y' + (B - L)$. In this case, the remaining unbroken generator is given by

$$\frac{Q}{e} = I_3 + \frac{1}{2}\left(Y' + (B-L)\right). \tag{4.1}$$

Using a series of assumptions such as restrict $B-L$ assignment to have only integer numbers, the anomaly cancelation implies that with only three right-handed neutrinos added to the minimal representation of SM arise two types of model:

1. The assignment $B-L=-1$ for all sterile neutrinos.

2. Two of the sterile neutrinos have $B-L=-4$ and the third one $B-L=5$.

Recently, it was studied the capability of the future ILC in determining the parameter space of Z' and Z vector bosons in two models in which the $B-L$ symmetry is gauged [120]. A detailed study of the scalar and the Yukawa neutrino sectors for the second model was done in Chap. 3 [97, 98]. There was shown that this model is compatible with the observed solar and atmospheric neutrino mass scales and the tri-bimaximal mixing matrix if a new scalar called ϕ_X is added.

On another hand, one of the most appealing solutions to the hierarchy problem which extent not only the space-time group but also the matter content of the SM is the minimal supersymmetric standard model (MSSM) [126–128]. However, in this model the neutrinos are massless. If neutrino mass is required the R-parity must be broken [129–131]. The simplest supersymmetric model that explains the left-handed neutrino masses is known as the minimal supersymmetric standard model with three generation right-handed neutrinos (MSSMRN). The MSSMRN can be embedded in the supersymmetric $SU(5)$ grand unified theory (GUT) with right-handed neutrinos [132]. The Minimal gauged $U(1)_{B-L}$ with spontaneous R-parity violation was presented in Ref. [133].

At the same time in the MSSM the R-parity is introduced to forbid interactions that violates baryonic and leptonic number violation. The existence of this extra symmetry has far-reaching experimental consequences. Superpartners, which have odd R-parity, must be pair-produced in the laboratory, from ordinary matter which is R-even. In addition, R-parity conservation requires that they decay into an odd number of superpartners. At the end of a chain of sequential decays there will remain one odd R-parity particle, the lightest superpartner (LSP). Kinematically forbidden to further decay, this particle is absolutely stable. Therefore a good candidate for the cold dark matter (CDM) in the Universe if this LSP is neutral.

This chapter is based on a work which Ph.D. Marcos Rodriguez and I are finishing in the near time. Here, it will be presented the minimal $N=1$ supersymmetric version for the $SU(2)_L \otimes U(1)_{Y'} \otimes U(1)_{B-L}$ gauge model with three identical right-handed neutrinos. This model has some interesting facts. The main one is that the neutrinos gain masses without breaking the R-parity symmetry, and therefore, the LSP is stable. The outline of this chapter is as follows. In Sec. 4.2 the minimal matter content is presented. In Sec. 4.3 the entire Lagrangian is shown using the formalism of superfield, which is useful in $N = 1$ supersymmetric theories. In Secs. 4.4, 4.5, 4.6, the masses of the usual fermions (Leptons, Quarks, Neutrinos, Charginos and Neutralinos), gauge bosons, and the Higgs scalars are shown. Finally, in Sec. 4.7 some final commentaries on future studies of this model are done.

4.2 The Matter Content

In order to construct the minimal $N = 1$ supersymmetric extension of the $B - L$ model with three identical right-handed neutrinos given in Ref. [99], only those fields necessary to reproduce the $N = 0$ minimal $B - L$ model and to satisfy theoretical consistency are introduced. Thus, the minimal baryonic and leptonic matter content is given by

$$\begin{aligned}
\hat{L}_i &= \begin{pmatrix} \hat{\nu}_i \\ \hat{l}_i \end{pmatrix}_L \sim (\mathbf{1}, \mathbf{2}, 0, -1), \quad i = 1, 2, 3. \\
\hat{l}^c_{iL} &\sim (\mathbf{1}, \mathbf{1}, 1, 1), \quad \hat{n}^c_{iL} \sim (\mathbf{1}, \mathbf{1}, -1, 1), \\
\hat{Q}_{iL} &= \begin{pmatrix} \hat{u}_i \\ \hat{d}_i \end{pmatrix}_L \sim \left(\mathbf{3}, \mathbf{2}, 0, \frac{1}{3}\right), \\
\hat{u}^c_{iL} &\sim \left(\mathbf{3}^*, \mathbf{1}, -1, -\frac{1}{3}\right), \quad \hat{d}^c_{iL} \sim \left(\mathbf{3}^*, \mathbf{1}, 1, -\frac{1}{3}\right).
\end{aligned} \tag{4.2}$$

In parenthesis, it appears the transformation properties under the respective factors $SU(3)_C$, $SU(2)_L$, $U(1)_{Y'}$, $U(1)_{B-L}$ gauge groups. Here, it has been used the notation $\widehat{\Psi}$ to generically denote Wess-Zumino chiral superfield of the corresponding Ψ field. For instance, $\hat{L}_i$ makes reference to

$$\hat{L}_i : \begin{pmatrix} \nu_i \\ l_i \end{pmatrix}_L \quad \text{and} \quad \begin{pmatrix} \widetilde{\nu}_i \\ \widetilde{l}_i \end{pmatrix}_L , \tag{4.3}$$

where the fields ν_i, l_i make reference to the neutrino and lepton left-handed fields. And, on the other hand, $\widetilde{\nu}_i$, $\widetilde{l}_i$ make reference to left sneutrinos and left sleptons which are the superpartners of the ν_i, l_i fields respectively. The word left in left sneutrinos and left sleptons indicates they are, respectively, the superpartners of the left neutrinos and left leptons and not theirs helicity, since they have no spin.

In $N = 1$ supersymmetry, the standard model Higgs doublet must be viewed as the scalar component of a chiral superfield, which contains one weak doublet of Weyl fermions, the Higgsinos. These chiral fermions have the same electroweak quantum numbers as the Higgs, which generate contributions to two different anomalies.

The first is the usual Adler-Bell-Jackiw (ABJ) triangle anomaly associated with the $U(1)_{Y'}$ symmetry. The second is the global anomaly, according to which, any theory with an odd number of chiral fermions which transform as $SU(2)$ doublets, path-integrates to zero. The simplest way to remedy these two potential disasters is to introduce another Higgsino doublet with opposite $U(1)_{Y'}$ charge, to act as the vector-like completion of the first, and cancel both anomalies. By the reverse argument, this introduces a new Higgs spin zero doublet with opposite $U(1)_{Y'}$ charge. Thus superfields needed to generate the masses are

$$\hat{H}_1 = \begin{pmatrix} \hat{H}_1^+ \\ \hat{H}_1^0 \end{pmatrix} \sim (\mathbf{1}, \mathbf{2}, +1, 0), \quad \hat{H}_2 = \begin{pmatrix} \hat{H}_2^0 \\ \hat{H}_2^- \end{pmatrix} \sim (\mathbf{1}, \mathbf{2}, -1, 0). \tag{4.4}$$

The minimal $B - L$ non-supersymmetric model has a ϕ SU(2) singlet, thus a new superfield, $\hat{\phi}_1$, is necessary. In addition, to cancel the ABJ triangle anomaly another superfield, $\hat{\phi}_2$, is also introduced. The corresponding quantum numbers of these two superfields are

$$\hat{\phi}_1 \sim (\mathbf{1}, \mathbf{1}, -2, 2), \quad \hat{\phi}_2 \sim (\mathbf{1}, \mathbf{1}, 2, -2). \tag{4.5}$$

The neutral spin zero components of these superfields will gain VEVs different from zero, which are denoted here as

$$\begin{aligned} \langle H_1^0 \rangle &= \frac{v_1}{\sqrt{2}}, \quad \langle \phi_1 \rangle = \frac{u_1}{\sqrt{2}}, \\ \langle H_2^0 \rangle &= \frac{v_2}{\sqrt{2}}, \quad \langle \phi_2 \rangle = \frac{u_2}{\sqrt{2}}. \end{aligned} \tag{4.6}$$

The gauge bosons and their superpartners are now part of gauge multiplets, described by vector superfields $\hat{G}^a$ (with $a = 1,\ldots,8$), $\hat{W}^i$ (with $i = 1, 2, 3$), $\hat{b}_{Y'}$ and $\hat{b}_{BL}$. Specifically, the gluons will be denoted by g^b and theirs respective superpartners, the gluinos, by $\tilde{g}^b$, with $b = 1,\ldots,8$; and in the electroweak sector are the W^i, with $i = 1, 2, 3$, the gauge bosons of $SU(2)_L$, and their gauginos superpartners $\tilde{W}^i$; finally the gauge boson of $U(1)_{Y'}$, are denoted by $b_{Y'}$, and its supersymmetric partner $\tilde{b}_{Y'}$, while for the symmetry $U(1)_{B-L}$, the gauge boson is b_{BL}, and its gaugino is $\tilde{b}_{BL}$. This is the minimal particle content of the minimal supersymmetric $B - L$ model.

4.3 The Lagrangian

With the superfields introduced above, the entire Lagrangian of the model considered here has the following form

$$\mathcal{L} = \mathcal{L}_{\text{SUSY}} + \mathcal{L}_{\text{Soft}}. \tag{4.7}$$

Here $\mathcal{L}_{\text{SUSY}}$ is the supersymmetric invariant piece, while $\mathcal{L}_{\text{soft}}$ explicitly breaks SUSY. Now, it is time to write each of these Lagrangians in terms of the respective superfields. The supersymmetric term can be divided as follows

$$\mathcal{L}_{\text{SUSY}} = \mathcal{L}_{\text{Leptons}} + \mathcal{L}_{\text{Quarks}} + \mathcal{L}_{\text{Gauge}} + \mathcal{L}_{\text{Higgs}}, \tag{4.8}$$

where each term is given by

$$\begin{aligned}
\mathcal{L}_{\text{Leptons}} &= \int \mathrm{d}^4\theta \, \Big[\hat{\bar{L}} e^{2[g\hat{W}+g_{BL}(\frac{-1}{2})\hat{b}_{BL}]}\hat{L} + \hat{\bar{l}}^c_{aL} e^{2[g_{Y'}(\frac{1}{2})\hat{b}_{Y'}+g_{BL}(\frac{1}{2})\hat{b}_{BL}]}\hat{l}^c_{aL} \\
&\quad + \hat{\bar{n}}^c_{aL} e^{2[g_{Y'}(\frac{-1}{2})\hat{b}_{Y'}+g_{BL}(\frac{1}{2})\hat{b}_{BL}]}\hat{n}^c_{aL}\Big],
\end{aligned} \tag{4.9}$$

$$\begin{aligned}
\mathcal{L}_{\text{Quarks}} &= \int \mathrm{d}^4\theta \, \Big[\hat{\bar{Q}}_i e^{2[g_s\hat{G}+g\hat{W}+g_{BL}(\frac{1}{6})\hat{b}_{BL}]}\hat{Q}_i + \hat{\bar{u}}_i e^{2[g_s\hat{G}+g_{Y'}(\frac{-1}{2})\hat{b}_{Y'}+g_{BL}(\frac{-1}{6})\hat{b}_{BL}]}\hat{u}_i \\
&\quad + \hat{\bar{d}}_i e^{2[g_s\hat{G}+g\hat{W}+g_{Y'}(\frac{1}{2})\hat{b}_{Y'}+g_{BL}(\frac{-1}{6})\hat{b}_{BL}]}\hat{d}_i\Big].
\end{aligned} \tag{4.10}$$

The $\mathcal{L}_{\text{Leptons}}$ and $\mathcal{L}_{\text{Quarks}}$ Lagrangians give the kinetic terms for the leptons, sleptons, quarks and squarks, also as their interactions with the gauge boson and gauginos. Finally the kinetic terms for the gauge bosons and gauginos are given by

$$\mathcal{L}_{\text{Gauge}} = \frac{1}{4}\int \mathrm{d}^2\theta \, [\mathcal{W}^a_c\mathcal{W}^a_c + \mathcal{W}^i\mathcal{W}^i + \mathcal{W}^{Y'}\mathcal{W}^{Y'} + \mathcal{W}^{BL}\mathcal{W}^{BL}] + \text{H.c.}, \tag{4.11}$$

where the strength fields read

$$\begin{aligned}
\mathcal{W}^a_{\alpha c} &= -\frac{1}{8g_s}\bar{D}\bar{D}e^{-2g_s\hat{G}}D_\alpha e^{2g_s\hat{G}}, \quad \mathcal{W}^a_\alpha = -\frac{1}{8g}\bar{D}\bar{D}e^{-2g\hat{W}}D_\alpha e^{2g\hat{W}}, \\
\mathcal{W}^{Y'}_\alpha &= -\frac{1}{4}\bar{D}\bar{D}D_\alpha\hat{b}_{Y'}, \quad \mathcal{W}^{BL}_\alpha = -\frac{1}{4}\bar{D}\bar{D}D_\alpha\hat{b}_{BL}.
\end{aligned} \tag{4.12}$$

In the expressions above it has been used the notation that $\hat{G} = T^a\hat{G}^a$, $\hat{W} = T^i\hat{W}^i$ where $T^a = \lambda^a/2$ (with $a = 1,\cdots,8$) are the generators of $SU(3)_C$, and $T^i = \tau^i/2$ (with $i = 1, 2, 3$) are the generators of $SU(2)_L$. Also, g_s, g, $g_{Y'}$ and g_{BL} are the gauge coupling of $SU(3)_C$, $SU(2)_L$, $U(1)_{Y'}$ and $U(1)_{B-L}$, respectively. The D_α are the usual covariant derivatives.

Finally the interaction and kinetic terms for the Higgs boson and their superpartners the Higgsinos come from

$$\begin{aligned}
\mathcal{L}_{\text{Higgs}} &= \int \mathrm{d}^4\theta \, \Big[\hat{\bar{H}}_1 e^{2[g\hat{W}+g_{Y'}(\frac{1}{2})\hat{b}_{Y'}]}\hat{H}_1 + \hat{\bar{H}}_2 e^{2[g\hat{W}+g_{Y'}(\frac{-1}{2})\hat{b}_{Y'}]}\hat{H}_2 \\
&\quad + \hat{\bar{\phi}}_1 e^{2[g_{Y'}(\frac{-2}{2})\hat{b}_{Y'}+g_{BL}(\frac{2}{2})\hat{b}_{BL}]}\hat{\phi}_1 + \hat{\bar{\phi}}_2 e^{2[g_{Y'}(\frac{2}{2})\hat{b}_{Y'}+g_{BL}(\frac{-2}{2})\hat{b}_{BL}]}\hat{\phi}_2\Big] \\
&\quad + \left(\int \mathrm{d}^2\theta \, W + \text{H.c.}\right),
\end{aligned} \tag{4.13}$$

where W is the superpotential, which will be discussed below.

The most general gauge invariant superpotential of this model which gives the usual renormalizable Yukawa interactions and also contributes to give some mass terms for the different matter fields is given by

$$W = \frac{W_2}{2} + \frac{W_3}{3}, \tag{4.14}$$

where W_2 and W_3 have only two and three chiral superfields, respectively. The terms permitted by the gauge symmetry are

$$W_2 = \mu_H \hat{H}_1^\alpha \epsilon_{\alpha\beta} \hat{H}_2^\beta + \mu_\phi \hat{\phi}_1 \hat{\phi}_2. \tag{4.15}$$

As the quantum numbers of right-handed neutrino superfields are $\hat{n}^c_{iL} \sim (\mathbf{1}, \mathbf{1}, -1, 1)$, it is prohibited the term $\mu_n \hat{n}^c \hat{n}^c$ from appearing, because it has $B - L = 2$. The part of the superpotential with three chiral superfields

$$\begin{aligned} W_3 \;\; = \;\; & \epsilon_{\alpha\beta} \left[f^l_{ij} \hat{H}_2^\alpha \hat{L}_i^\beta \hat{l}^c_j + f^d_{ij} \hat{H}_2^\alpha \hat{Q}_i^\beta \hat{d}^c_j + f^u_{ij} \hat{H}_1^\alpha \hat{Q}_i^\beta \hat{u}^c_j + f^\nu_{ij} \hat{H}_1^\alpha \hat{L}_i^\beta \hat{n}^c_j \right] \\ & + f^M_{ij} \hat{\phi}_2 \hat{n}^c_i \hat{n}^c_j. \end{aligned} \tag{4.16}$$

Notice that the supersymmetry does not allow conjugates to appear in the superpotential which has to be holomorphic in the superfields. As a result, the $U(1)_{Y'}$ and $U(1)_{B-L}$ symmetries forbids the same Higgs superfield from coupling analytically to both charged quarks sectors. In addition, it is important note that the terms that are proportional to f^ν and f^M generate, respectively, the Dirac and Majorana mass terms for the neutrinos. Of course, when ϕ_2 get VEV, the last term in equation above generate a mass term as $\sim u_2 f^M_{ij} \hat{n}^c \hat{n}^c / \sqrt{2}$.

Now, it is time to write down the last part of the Lagrangian $\mathcal{L}$ given in Eq. (4.7), the $\mathcal{L}_{\text{Soft}}$ Lagrangian. It is obvious that to account for data, supersymmetry must be broken. But, a predictive mechanism for SUSY breaking is still a major puzzle that have attracted the attention of the many experts in the field. Many renormalizable models of spontaneous supersymmetry breaking have been proposed [134, 135] but none of these models are entirely satisfactory. Fortunately, the impact of the supersymmetry breaking can be described without committing ourselves to a specific theory for the breaking mechanism. Indeed, in the context of an effective theory, it is natural to parameterize the spontaneous breaking of a symmetry by adding soft terms to its Lagrangian. These are terms of dimension two and cubic interactions of scalar with dimension three. Intuitively, these terms can not affect the theory in the ultraviolet since they all have prefactors with positive mass dimensions, which vanish in the limit where all masses are taken to zero, relative to the scale of the interest. The usual strategy is to add soft terms that break supersymmetry but not gauge symmetry. There are several types of masses that explicitly break supersymmetry: mass terms for the superpartners of the massless chiral fermions; mass terms for each

Higgs scalar but not for the Higgsinos; and finally gaugino mass terms. These clearly split the mass degeneracy between particles and their superpartners, thus breaking supersymmetry. There are also terms of dimension three that describe interactions among the scalar components of chiral superfields. These must preserve the gauge symmetry. In conclusion, in this model the soft terms can be written as

$$\mathcal{L}_{\text{Soft}} = \mathcal{L}_{\text{SMT}} + \mathcal{L}_{\text{GMT}} + \mathcal{L}_{\text{INT}}, \tag{4.17}$$

where

$$\begin{aligned}
\mathcal{L}_{\text{SMT}} &= -\Big[M_L^2 \tilde{L}^\dagger \tilde{L} + M_l^2 \tilde{l^c}^\dagger \tilde{l^c} + M_n^2 \tilde{n^c}^\dagger \tilde{n^c} + M_Q^2 \tilde{Q}^\dagger \tilde{Q} \\
&\quad + M_u^2 \tilde{u^c}^\dagger \tilde{u^c} + M_d^2 \tilde{d^c}^\dagger \tilde{d^c} + M_{H_1}^2 H_1^\dagger H_1 \\
&\quad + M_{H_2}^2 H_2^\dagger H_2 + \beta_H \epsilon_{ij} \left(H_1^i H_2^j + \text{H.c.}\right) + M_{\phi_1}^2 \phi_1^\dagger \phi_1 + M_{\phi_2}^2 \phi_2^\dagger \phi_2 \\
&\quad - \beta_\phi \left(\phi_1 \phi_2 + \text{H.c.}\right)\Big], \\
\mathcal{L}_{\text{GMT}} &= -\frac{1}{2}\Bigg(M_{\tilde{g}} \sum_{a=1}^{8} \tilde{g}^a \tilde{g}^a + M_{\tilde{W}} \sum_{i=1}^{3} \tilde{W}^i \tilde{W}^i + M_{\tilde{b}_{Y'}} \tilde{b}_{Y'} \tilde{b}_{Y'} \\
&\quad + M_{\tilde{b}_{BL}} \tilde{b}_{BL} \tilde{b}_{BL}\Bigg) + \text{H.c.}\ . \\
\mathcal{L}_{\text{INT}} &= \Big(A_l H_2 \tilde{L} \tilde{l^c} + A_d H_2 \tilde{Q} \tilde{d^c} + A_u H_1 \tilde{Q} \tilde{u}^c + A_\nu H_1 \tilde{L} \tilde{n}^c \\
&\quad + A_M \phi_2 \tilde{n}^c \tilde{n}^c + \text{H.c.}\Big).
\end{aligned} \tag{4.18}$$

To close this section, let's show explicitly the pattern of the symmetry breaking of the model

$$\begin{aligned}
\mathcal{L}_{\text{SUSY}} &\xrightarrow{\mathcal{L}_{\text{Soft}}} \text{SU}(3)_C \otimes \text{SU}(2)_L \otimes \text{U}(1)_{Y'} \otimes \text{U}(1)_{B-L} \\
&\xrightarrow{\langle \phi_{1,2} \rangle} \text{SU}(3)_C \otimes \text{SU}(2)_L \otimes \text{U}(1)_Y \\
&\xrightarrow{\langle H_{1,2} \rangle} \text{SU}(3)_C \otimes \text{U}(1)_Q.
\end{aligned} \tag{4.19}$$

Therefore, the following hierarchy

$$u_1,\, u_2 \gg v_1,\, v_2, \tag{4.20}$$

between the VEVs of this model can be used.

4.4 Fermion Masses

In this section the mass spectrum from the fermion sector at tree level will be briefly commented.

4.4.1 Charged Lepton and Quark Masses

The charged lepton and quarks masses are given by the $f^l_{ij}\hat{H}^\alpha_2\hat{L}^\beta_i\hat{l}^c_j$, $f^d_{ij}\hat{H}^\alpha_2\hat{Q}^\beta_i\hat{d}^c_j$, $f^u_{ij}\hat{H}^\alpha_1\hat{Q}^\beta_i\hat{u}^c_j$ in the superpotential W_3, Eq. (4.16). These terms will produce the following mass matrix for the charged leptons when the H^0_1, H^0_2 obtain their VEVs

$$(m_l)_{ij} = \frac{v_2}{\sqrt{2}}f^l_{ij},\ (m_d)_{ij} = \frac{v_2}{\sqrt{2}}f^d_{ij},\ (m_u)_{ij} = \frac{v_1}{\sqrt{2}}f^u_{ij}. \tag{4.21}$$

The mass matrices above are the same as the ones in the minimal supersymmetric standard model (MSSM). Thus, these are not discussed here.

4.4.2 Neutrino Masses

In the case of neutrinos, the $f^\nu_{ij}\hat{H}^\alpha_1\hat{L}^\beta_i\hat{n}^c_j$ and $f^M_{ij}\hat{\phi}_2\hat{n}^c_i\hat{n}^c_j$ in the superpotential W_3, Eq. (4.16) generate mass to all the neutrinos in the model. The term proportional to f^ν_{ij} generates a Dirac mass matrix for the neutrinos while the f^M_{ij} term generates a Majorana mass matrix for the neutrinos. In the base

$$\Psi^0 = \left(\begin{array}{cccccc} \nu_1 & \nu_2 & \nu_3 & n^c_1 & n^c_2 & n^c_3 \end{array} \right)^T, \tag{4.22}$$

the following mass matrix for the neutrinos is obtained

$$\left(\begin{array}{cccccc}
0 & 0 & 0 & \frac{v_1}{\sqrt{2}}f^\nu_{11} & \frac{v_1}{\sqrt{2}}f^\nu_{12} & \frac{v_1}{\sqrt{2}}f^\nu_{13} \\
0 & 0 & 0 & \frac{v_1}{\sqrt{2}}f^\nu_{21} & \frac{v_1}{\sqrt{2}}f^\nu_{22} & \frac{v_1}{\sqrt{2}}f^\nu_{23} \\
0 & 0 & 0 & \frac{v_1}{\sqrt{2}}f^\nu_{31} & \frac{v_1}{\sqrt{2}}f^\nu_{32} & \frac{v_1}{\sqrt{2}}f^\nu_{33} \\
\frac{v_1}{\sqrt{2}}f^\nu_{11} & \frac{v_1}{\sqrt{2}}f^\nu_{21} & \frac{v_1}{\sqrt{2}}f^\nu_{31} & \frac{u_2}{\sqrt{2}}f^M_{11} & \frac{u_2}{\sqrt{2}}f^M_{12} & \frac{u_2}{\sqrt{2}}f^M_{13} \\
\frac{v_1}{\sqrt{2}}f^\nu_{12} & \frac{v_1}{\sqrt{2}}f^\nu_{22} & \frac{v_1}{\sqrt{2}}f^\nu_{32} & \frac{u_2}{\sqrt{2}}f^M_{21} & \frac{u_2}{\sqrt{2}}f^M_{22} & \frac{u_2}{\sqrt{2}}f^M_{23} \\
\frac{v_1}{\sqrt{2}}f^\nu_{13} & \frac{v_1}{\sqrt{2}}f^\nu_{23} & \frac{v_1}{\sqrt{2}}f^\nu_{33} & \frac{u_2}{\sqrt{2}}f^M_{31} & \frac{u_2}{\sqrt{2}}f^M_{32} & \frac{u_2}{\sqrt{2}}f^M_{33}
\end{array} \right). \tag{4.23}$$

This type of matrix generates the seesaw mechanism.

4.4.3 Chargino and Neutralino Masses

This model contains two new charged fermions, the Higgsinos, $\tilde{H}^+_1$ and $\tilde{H}^-_1$, and the Winos $\tilde{W}^+$ and $\tilde{W}^-$, which are defined as

$$\tilde{W}^\pm = \frac{\tilde{W}^1 \mp i\tilde{W}^2}{\sqrt{2}}. \tag{4.24}$$

These states will be mixed by the electroweak breaking, to yield two charginos, $\widetilde{C}_{1,2}$ mass eigenstates. The terms

$$g\left[\tilde{W}^-\widetilde{H}^+_1 H^0_1 + \tilde{W}^+\widetilde{H}^-_2 H^0_2\right] \tag{4.25}$$

result in mixing terms when the Higgs doublets get their VEVs. Adding the direct mass terms for the Higgsinos and Winos, the mass matrix is given by

$$\mathcal{L}_{\widetilde{C}} = -\frac{1}{2}\psi^T M_{\widetilde{C}}\psi + \text{H.c.}, \tag{4.26}$$

where the base $\psi^T = \left(\tilde{W}^+, \widetilde{H}_1^+, \tilde{W}^-, \widetilde{H}_2^-\right)$ and

$$M_{\widetilde{C}} = \begin{pmatrix} 0 & M^T \\ M & 0 \end{pmatrix}, \tag{4.27}$$

with

$$M = \begin{pmatrix} M_{\tilde{W}} & \sqrt{2}\sin\beta M_W \\ \sqrt{2}\cos\beta M_W & \mu \end{pmatrix}. \tag{4.28}$$

M_W above is the W-boson mass defined in Eq. (4.42). The β angle is defined by $\tan\beta = v_1/v_2$. The mass matrix for the charginos in this model is similar to the one in the MSSM model. The chargino mass matrix can be diagonalized by a singular value decomposition

$$L^\dagger M R^{-1} = \begin{pmatrix} m_{\widetilde{C}_1} & 0 \\ 0 & m_{\widetilde{C}_2} \end{pmatrix}, \tag{4.29}$$

with mass eigenstates given by

$$\begin{pmatrix} \widetilde{C}_1^+ \\ \widetilde{C}_2^+ \end{pmatrix} = R\begin{pmatrix} \tilde{W}^+ \\ \widetilde{H}_1^+ \end{pmatrix}, \quad \begin{pmatrix} \widetilde{C}_1^- \\ \widetilde{C}_2^- \end{pmatrix} = L\begin{pmatrix} \tilde{W}^- \\ \widetilde{H}_2^- \end{pmatrix}, \tag{4.30}$$

where L and R are unitary matrices. After diagonalization, the elements of L and R appear in the interaction vertices for chargino mass eigenstates. The masses of these eigenstates are

$$m_{\widetilde{C}_1,\widetilde{C}_2} = \frac{1}{2}\left[M_{\tilde{W}}^2 + \mu^2 + 2M_W^2 \mp \sqrt{\left(M_{\tilde{W}}^2 + \mu^2 + 2M_W^2\right)^2 - 4\left(\mu M_{\tilde{W}} - M_W^2\sin 2\beta\right)^2}\right]. \tag{4.31}$$

In the limit that $\left||\mu| \pm M_{\tilde{W}}\right| \gg M_W$ the charginos are approximately a Wino and a Higgsino with masses $\left|M_{\tilde{W}}\right|$ and $|\mu|$.

On another hand, in the neutralino sector, the mass matrix comes from the following terms

$$\begin{aligned} &- \frac{g}{2}\left[v_2\tilde{W}^3\tilde{H}_2^0 - v_1\tilde{W}^3\tilde{H}_1^0\right] + \frac{\mu_H}{2}\tilde{H}_1^0\tilde{H}_2^0 + \frac{\mu_\phi}{2}\tilde{\phi}_1\tilde{\phi}_2 - \frac{M_{\tilde{W}}}{2}\tilde{W}^3\tilde{W}^3 \\ &- \frac{M_{\tilde{b}_{Y'}}}{2}\tilde{b}_{Y'}\tilde{b}_{Y'} - \frac{M_{\tilde{b}_{BL}}}{2}\tilde{b}_{BL}\tilde{b}_{BL} - \frac{M_{\tilde{b}_{Y'}}}{2}\tilde{b}_{Y'}\tilde{b}_{Y'} - \frac{g_{BL}}{2}\left(2u_1\tilde{b}_{BL}\tilde{\phi}_1 - 2u_2\tilde{b}_{BL}\tilde{\phi}_2\right) \\ &- \frac{g_{Y'}}{2}\left(v_1\tilde{b}_{Y'}\tilde{H}_1^0 - v_2\tilde{b}_{Y'}\tilde{H}_2^0 - 2u_1\tilde{b}_{Y'}\tilde{\phi}_1 + 2u_2\tilde{b}_{Y'}\tilde{\phi}_2\right), \end{aligned} \tag{4.32}$$

using the following base

$$\tilde{\chi}^{0T} = \begin{pmatrix} \tilde{W}^3 & \tilde{b}_{Y'} & \tilde{b}_{BL} & \tilde{H}_1^0 & \tilde{H}_2^0 & \tilde{\phi}_1 & \tilde{\phi}_2 \end{pmatrix}, \tag{4.33}$$

the mass term of neutralinos can be write as $-\frac{1}{2}\tilde{\chi}^{0T}Y^0\tilde{\chi}^0$, with Y^0

$$\left(\begin{array}{ccccccc} M_{\tilde{W}} & 0 & 0 & -\frac{gv_1}{2} & \frac{gv_2}{2} & 0 & 0 \\ 0 & M_{\tilde{b}_{Y'}} & 0 & \frac{g_{Y'}v_1}{2} & -\frac{g_{Y'}v_2}{2} & -g_{Y'}u_1 & g_{Y'}u_2 \\ 0 & 0 & M_{\tilde{b}_{BL}} & 0 & 0 & g_{BL}u_1 & -g_{BL}u_2 \\ -\frac{gv_1}{2} & \frac{g_{Y'}v_1}{2} & 0 & 0 & \frac{\mu_H}{4} & 0 & 0 \\ \frac{gv_2}{2} & -\frac{g_{Y'}v_2}{2} & 0 & \frac{\mu_H}{4} & 0 & 0 & 0 \\ 0 & -g_{Y'}u_1 & g_{BL}u_1 & 0 & 0 & 0 & \frac{\mu_\phi}{4} \\ 0 & g_{Y'}u_2 & -g_{BL}u_2 & 0 & 0 & \frac{\mu_\phi}{4} & 0 \end{array}\right). \tag{4.34}$$

Due Y^0 matrix is symmetric, we need only one rotation matrix, V, such that

$$(\mathcal{M}_{\tilde{N}})_{ij} = [VY^0V^T]_{ij} = m_{\tilde{N}_i}\delta_{ij}. \tag{4.35}$$

is a diagonal matrix with real non-negative entries. Thus, the mass term of the neutralinos is

$$-\frac{1}{2}m_{\tilde{N}_i}\tilde{N}_i\tilde{N}_i \tag{4.36}$$

where $\tilde{N}_i$

$$\tilde{N}_i = V_{ij}(\tilde{\chi}^0)_j,\ i = 1, \ldots, 7.$$

The $\tilde{N}_i$ fields are defined such that their absolute masses increase with increasing i. The Lightest supersymmetric particle (LSP) of this model is the $\tilde{N}_1$.

4.5 Gauge Boson Masses

The masses of the gauge bosons come from the following terms

$$\begin{aligned}
(\mathcal{D}^\mu H_1)^\dagger(\mathcal{D}_\mu H_1) &= \frac{g^2v_1^2}{4}W^{+\mu}W^-_\mu + \frac{g^2v_1^2}{8}W^{3\mu}W^3_\mu \\
&+ \frac{g^2_{Y'}v_1^2}{8}b^\mu_{Y'}b_{\mu Y'} - \frac{gg_{Y'}v_1^2}{4}b^\mu_{Y'}W^3_\mu, \quad (4.37)\\
(\mathcal{D}^\mu H_2)^\dagger(\mathcal{D}_\mu H_2) &= \frac{g^2v_2^2}{4}W^{-\mu}W^+_\mu + \frac{g^2v_2^2}{8}W^{3\mu}W^3_\mu \\
&+ \frac{g^2_{Y'}v_2^2}{8}b^\mu_{Y'}b_{Y'\mu} - \frac{gg_{Y'}v_2^2}{4}b^\mu_{Y'}W^3_\mu, \quad (4.38)\\
(\mathcal{D}^\mu \phi_1)^\dagger(\mathcal{D}_\mu \phi_1) &= \frac{g^2_{Y'}}{2}u_1^2b^\mu_{Y'}b_{Y'\mu} + \frac{g^2_{BL}}{2}u_1^2b^\mu_{BL}b_{BL\mu} \\
&- g_{Y'}g_{BL}u_1^2b^\mu_{Y'}b_{BL\mu}, \quad (4.39)\\
(\mathcal{D}^\mu \phi_2)^\dagger(\mathcal{D}_\mu \phi_2) &= \frac{g^2_{Y'}}{2}u_2^2b^\mu_{Y'}b_{Y'\mu} + \frac{g^2_{BL}}{2}u_2^2b^\mu_{BL}b_{BL\mu} \\
&- g_{Y'}g_{BL}u_2^2b^\mu_{Y'}b_{BL\mu}. \quad (4.40)
\end{aligned}$$

The charged gauge boson mass is given by

$$M_W^2W^{+\mu}W^-_\mu, \tag{4.41}$$

where

$$M_W^2 = \frac{g^2 v_2^2}{4}\left(1+\tan^2\beta\right). \tag{4.42}$$

The mass-squared matrix for the neutral vector bosons in the $(W^3, b_{Y'}, b_{BL})$ base is

$$M^2_{\text{neutral}} = \begin{pmatrix} \frac{g^2}{4}(v_1^2+v_2^2) & -\frac{g g_{Y'}}{4}(v_1^2+v_2^2) & 0 \\ -\frac{g g_{Y'}}{4}(v_1^2+v_2^2) & g_{Y'}^2\left(\frac{v_1^2+v_2^2}{4}+u_1^2+u_2^2\right) & -g_{Y'}g_{BL}(u_1^2+u_2^2) \\ 0 & -g_{Y'}g_{BL}(u_1^2+u_2^2) & g_{BL}^2(u_1^2+u_2^2) \end{pmatrix}. \tag{4.43}$$

Det $M^2_{\text{neutral}} = 0$. The exact mass eigenvalues are: zero for the photon field, and two massive fields with masses given by

$$M_{1,2}^2 = \frac{1}{8}\left(U \pm \sqrt{U^2 - V}\right), \tag{4.44}$$

where it has been defined

$$U = 4\left(g_{Y'}^2+g_{BL}^2\right)\left(u_1^2+u_2^2\right) + \left(g^2+g_{Y'}^2\right)\left(v_1^2+v_2^2\right), \tag{4.45}$$

$$V = 16\left[g^2\left(g_{Y'}^2+g_{BL}^2\right)+g_{Y'}^2 g_{BL}^2\right]\left(u_1^2+u_2^2\right)\left(v_1^2+v_2^2\right). \tag{4.46}$$

The gauge neutral boson sector is the same as presented in Ref. [120].

4.6 The Scalar Potential

The scalar potential of this model has the following form

$$V = \sum_i F_i^\dagger F_i + \frac{1}{2}\left(D_{Y'}^2 + D_{BL}^2 + D^a D^a + D^A D^A\right) + V_{\text{Soft}}, \tag{4.47}$$

where $i = H_1$, H_2, ϕ_1, ϕ_2, L, Q, l^c, u^c, d^c, n^c; $a = 1, 2, 3$; and $A = 1, ...,8$. The F terms are obtained from the superpotential Eq. (4.14)

$$\begin{aligned}
F_{H_1}^{\dagger a} &= -\mu_H H_2^a - f_{ij}^u \tilde{Q}_i^a \tilde{u}_{jL}^c - f_{ij}^\nu \tilde{L}_i^a \tilde{n}_{jL}^c, \\
F_{H_2}^{\dagger a} &= \mu_H H_1^a - f_{ij}^l \tilde{L}_i^a \tilde{l}_{jL}^c - f_{ij}^d \tilde{Q}_i^a \tilde{d}_{jL}^c, \\
F_{\phi_1}^{\dagger} &= -\mu_\phi \phi_2, \\
F_{\phi_2}^{\dagger} &= -\mu_\phi \phi_1 - f_{ij}^M \tilde{n}_{iL}^c \tilde{n}_{jL}^c, \\
F_{L_i}^{\dagger a} &= f_{ij}^l H_2^a \tilde{l}_{jL}^c + f_{ij}^\nu H_1^a \tilde{n}_{jL}^c, \\
F_{l_i^c}^{\dagger} &= -f_{ji}^l H_2^a \epsilon_{ab} \tilde{L}_j^b, \\
F_{Q_i}^{\dagger a} &= f_{ij}^d H_2^a \tilde{d}_{jL}^c + f_{ij}^u H_1^a \tilde{u}_{jL}^c, \\
F_{u_i^c}^{\dagger} &= -f_{ji}^u H_1^a \epsilon_{ab} \tilde{Q}_j^b, \\
F_{d_i^c}^{\dagger} &= -f_{ji}^d H_2^a \epsilon_{ab} \tilde{Q}_j^b, \\
F_{n_i^c}^{\dagger} &= -f_{ji}^\nu H_1^a \epsilon_{ab} \tilde{L}_j^b - 2 f_{ij}^M \phi_2 \tilde{n}_{jL}^c.
\end{aligned} \tag{4.48}$$

There is one D-term for each of the four gauge groups

$$
\begin{aligned}
U(1)_{Y'}: \quad D_Y &= - \frac{1}{2} g_{Y'} \left[H_1^\dagger H_1 - H_2^\dagger H_2 - 2\phi_1^\dagger \phi_1 + 2\phi_2^\dagger \phi_2 - \tilde{u}_{iL}^{c\dagger} \tilde{u}_{iL}^c \right. \\
&+ \left. \tilde{d}_{iL}^{c\dagger} \tilde{d}_{iL}^c + \tilde{l}_{iL}^{c\dagger} \tilde{l}_{iL}^c - \tilde{n}_{iL}^{c\dagger} \tilde{n}_{iL}^c \right], \\
U(1)_{BL}: \quad D_{BL} &= - \frac{1}{2} g_{BL} \left[2\phi_1^\dagger \phi_1 - 2\phi_2^\dagger \phi_2 + \frac{1}{3} \tilde{Q}_{iL}^\dagger \tilde{Q}_{iL} - \frac{1}{3} \tilde{u}_{iL}^{c\dagger} \tilde{u}_{iL}^c \right. \\
&- \left. \frac{1}{3} \tilde{d}_{iL}^{c\dagger} \tilde{d}_{iL}^c - \tilde{L}_{iL}^\dagger \tilde{L}_{iL} + \tilde{l}_{iL}^\dagger \tilde{l}_{iL} + \tilde{n}_{iL}^{c\dagger} \tilde{n}_{iL}^c \right], \\
SU(2)_L: \quad D^a &= - \frac{g}{2} \left[H_1^\dagger \sigma^a H_1 + H_2^\dagger \sigma^a H_2 + \tilde{L}_{iL}^\dagger \sigma^a \tilde{L}_{iL} + \tilde{Q}_{iL}^\dagger \sigma^a \tilde{Q}_{iL} \right], \\
SU(3)_C: \quad D^A &= - \frac{g_s}{2} \left[\tilde{Q}_{iL}^\dagger \lambda^A \tilde{Q}_{iL} + \tilde{u}_{iL}^{c\dagger} \lambda^A \tilde{u}_{iL}^c + \tilde{d}_{iL}^{c\dagger} \lambda^A \tilde{d}_{iL}^c \right].
\end{aligned} \tag{4.49}
$$

Here, it is only shown the Higgs potential, V_{Higgs}. This is given by

$$
\begin{aligned}
V_{\text{Higgs}} &= V_{\text{SUSY}} + V_{\text{Soft}}, \\
V_{\text{SUSY}} &= |\mu_H|^2 H_1^\dagger H_1 + |\mu_H|^2 H_2^\dagger H_2 + |\mu_\phi|^2 \phi_1^\dagger \phi_1 + |\mu_\phi|^2 \phi_2^\dagger \phi_2 \\
&+ \frac{g^2}{8} \left[H_1^\dagger \sigma^a H_1 + H_2^\dagger \sigma^a H_2 \right]^2 + \frac{g_{BL}^2}{8} \left[2\phi_1^\dagger \phi_1 - 2\phi_2^\dagger \phi_2 \right]^2 \\
&+ \frac{g_{Y'}^2}{8} \left[H_1^\dagger H_1 - H_2^\dagger H_2 - 2\phi_1^\dagger \phi_1 + 2\phi_2^\dagger \phi_2 \right]^2, \\
V_{\text{Soft}} &= M_{H_1}^2 H_1^\dagger H_1 + M_{H_2}^2 H_2^\dagger H_2 + M_{\phi_1}^2 \phi_1^\dagger \phi_1 + M_{\phi_2}^2 \phi_2^\dagger \phi_2 + \beta_H \epsilon_{ab} H_1^a H_2^b \\
&- \beta_\phi \phi_1 \phi_2 + \text{H.c.},
\end{aligned} \tag{4.50}
$$

To calculate the mass matrices, the neutral scalars are written as

$$
\begin{aligned}
H_1 &= \begin{pmatrix} H_1^+ \\ \frac{1}{\sqrt{2}} \left(v_1 + \text{Re}\, H_1^0 + i \text{Im}\, H_1^0 \right) \end{pmatrix}, \quad \phi_1 = \frac{1}{\sqrt{2}} \left(u_1 + \text{Re}\, \phi_1 + i \text{Im}\, \phi_1 \right), \\
H_2 &= \begin{pmatrix} \frac{1}{\sqrt{2}} \left(v_2 + \text{Re}\, H_2^0 + i \text{Im}\, H_2^0 \right) \\ H_2^- \end{pmatrix}, \quad \phi_2 = \frac{1}{\sqrt{2}} \left(u_2 + \text{Re}\, \phi_2 + i \text{Im}\, \phi_2 \right).
\end{aligned} \tag{4.51}
$$

Thus, the mass matrices can be calculated, using

$$
M_{ij}^2 = \frac{\partial^2 V_{\text{Higgs}}}{\partial \phi_i \partial \phi_j} \tag{4.52}
$$

evaluated at the chosen minimum, where ϕ_i are the scalars of this model described above.

4.6.1 Pseudoscalars

The imaginary mass-squared matrix with constraints in the base (Im H_1^0, Im H_2^0, Im ϕ_1, Im ϕ_2) is given by

$$M^2_{\text{Pseudoscalars}} = \begin{pmatrix} \frac{v_2}{v_1}\beta_H^2 & \beta_H^2 & 0 & 0 \\ \beta_H^2 & \frac{v_1}{v_2}\beta_H^2 & 0 & 0 \\ 0 & 0 & \frac{u_2}{u_1}\beta_\phi^2 & \beta_\phi^2 \\ 0 & 0 & \beta_\phi^2 & \frac{u_1}{u_2}\beta_\phi^2 \end{pmatrix}. \tag{4.53}$$

The determinant of this matrix is zero. There are two eigenvalues equal zero which correspond to the NG bosons, G_1 and G_2 (they will generate the mass to Z^0 and Z'^0), and two massive eigenstates, I_1 and I_2. Their masses are given by

$$m^2_{I_1} = \frac{v_1^2 + v_2^2}{v_1 v_2}\beta_H^2, \quad m^2_{I_2} = \frac{u_1^2 + u_2^2}{u_1 u_2}\beta_\phi^2, \tag{4.54}$$

where the states have been chosen to satisfy $m_{I_2} > m_{I_1}$. The corresponding eigenvectors are given by

$$\begin{aligned} G_1 &= \frac{1}{N_{G_1}}\left(-\frac{u_1}{u_2}\text{Im}\,\phi_1 + \text{Im}\,\phi_2\right), \quad G_2 = \frac{1}{N_{G_2}}\left(-\frac{v_1}{v_2}\text{Im}\,H_1^0 + \text{Im}\,H_2^0\right), \\ I_1 &= \frac{1}{N_{I_1}}\left(\frac{v_2}{v_1}\text{Im}\,H_1^0 + \text{Im}\,H_2^0\right), \quad I_2 = \frac{1}{N_{I_2}}\left(\frac{u_2}{u_1}\text{Im}\,\phi_1 + \text{Im}\,\phi_2\right), \end{aligned} \tag{4.55}$$

where N_s are the normalization constants.

4.6.2 Scalars

The scalar mass-squared matrix with constraints in the base (Re H_1^0, Re H_2^0, Re ϕ_1, Re ϕ_2) is given by

$$M^2_{\text{Scalars}} = \begin{pmatrix} A & E & -\frac{1}{2}g_{Y'}^2 v_1 u_1 & \frac{1}{2}g_{Y'}^2 v_1 u_2 \\ E & B & \frac{1}{2}g_{Y'}^2 v_2 u_1 & -\frac{1}{2}g_{Y'}^2 v_2 u_2 \\ -\frac{1}{2}g_{Y'}^2 v_1 u_1 & \frac{1}{2}g_{Y'}^2 v_2 u_1 & C & F \\ \frac{1}{2}g_{Y'}^2 v_1 u_2 & -\frac{1}{2}g_{Y'}^2 v_2 u_2 & F & D \end{pmatrix}, \tag{4.56}$$

where

$$\begin{aligned} A &= \frac{1}{4}(g^2 + g_{Y'}^2)v_1^2 + \frac{v_2}{v_1}\beta_H^2, \quad B = \frac{1}{4}(g^2 + g_{Y'}^2)v_2^2 + \frac{v_1}{v_2}\beta_H^2, \\ C &= (g_{BL}^2 + g_{Y'}^2)u_1^2 + \frac{u_2}{u_1}\beta_\phi^2, \quad D = (g_{BL}^2 + g_{Y'}^2)u_2^2 + \frac{u_1}{u_2}\beta_\phi^2, \\ E &= -\frac{1}{4}(g^2 + g_{Y'}^2)v_1 v_2 - \beta_H^2, \quad F = -(g_{BL}^2 + g_{Y'}^2)u_1 u_2 - \beta_\phi^2. \end{aligned} \tag{4.57}$$

The determinant of this matrix is given by

$$\text{Det}\, M^2_{\text{Scalars}} = m^2_{R_1} m^2_{R_2} m^2_{R_3} m^2_{R_4} = \frac{[g_{BL}^2 g_{Y'}^2 + g^2(g_{BL}^2 + g_{Y'}^2)]}{4 v_1 v_2 u_1 u_2}(v_1^2 - v_2^2)^2 (u_1^2 - u_2^2)^2 \beta_H^2 \beta_\phi^2. \tag{4.58}$$

Therefore, $v_1 \neq v_2$ and $u_1 \neq u_2$ must be satisfied in order to have determinant different from zero. In this sector of the model there are four massive states denoted as R_1, R_2, R_3, R_4. Because the analytical expressions for their eigenstates are not very illuminating, these are not shown here.

4.6.3 Charged Scalars

The charged mass-squared matrix with constraints in the basis $(H_1^{\pm}, H_2^{\pm})$ is given by

$$\begin{pmatrix} \frac{g^2 v_2^2}{4} + \frac{v_2}{v_1}\beta_H^2 & \frac{1}{4}v_1 v_2 g^2 + \beta_H^2 \\ \frac{1}{4}v_1 v_2 g^2 + \beta_H^2 & \frac{g^2 v_1^2}{4} + \frac{v_1}{v_2}\beta_H^2 \end{pmatrix}. \tag{4.59}$$

The determinant of this matrix is zero. Thus, this matrix has one charged NB boson, $G^{\pm}$ (it gives mass to $W^{\pm}$), and one massive charged eigenstate $h^{\pm}$. Its mass is given by

$$m_{h^{\pm}}^2 = \frac{v_1^2 + v_2^2}{4 v_1^2 v_2^2}\left(g^2 v_1 v_2 + 4\beta_H^2\right). \tag{4.60}$$

The corresponding eigenvectors are given by

$$G^{\pm} = \frac{1}{N_{G^{\pm}}}\left(-\frac{v_1}{v_2}H_1^{\pm} + H_2^{\pm}\right), \quad h^{\pm} = \frac{1}{N_{h^{+}}}\left(\frac{v_2}{v_1}H_1^{\pm} + H_2^{\pm}\right), \tag{4.61}$$

where N_s are the normalization constants.

4.7 Conclusions

In this chapter, the superfield formalism is used to build a supersymmetric version of a $SU(3)_C \otimes SU(2)_L \otimes U(1)_{Y'} \otimes U(1)_{B-L}$ gauge model. This model is interesting because it generates masses for all neutrinos and at the same time preserves the R-parity, therefore the lightest supersymmetric particle (LSP) is stable and could be a candidate for the cold dark matter in the Universe. Additionally, an analysis of the mass spectrum of the fermions, gauge bosons and the scalar fields is briefly shown.

Although, an analysis about the phenomenological consequences is beyond the scope of this chapter, in my opinion, the model has nice predictions that can be explored in the near future.

Appendix A

AN ESTIMATE OF THE NEUTRON ELECTRIC DIPOLE MOMENT IN THE ECONOMICAL 3-3-1 MODEL

As it was pointed out in Chap. 2, this model, in its most general form, possess some additional CP-violating phases. The reason is that some phases cannot be absorbed into the definitions of the quark and lepton fields. Among these new CP-violating phases, one of them induces a new contribution to the electric dipole moment of the neutron, EDMN, denoted as d_n^e. However, this new source of the EDMN can be left in agreement with the experimental bounds, $d_n^e < 0.29 \times 10^{-25}\, e \cdot$ cm [8], by choosing an adequate set of parameters. In addition, as it will be shown below, this tuning of the parameters is far from being of the same order of the $\overline{\theta}$ parameter. Therefore, considering the PQ solution to the strong CP problem in this model is perfectly reasonable.

To explicitly show the tuning of the parameters that allows the model to be in agreement with EDMN experimental bounds, let us take a representative case, the up-quark electric dipole moment, d_u^e. A diagram contributing to d_u^e is given in the Fig. A.1. This diagram is derived from the one given in the Fig. 2.1(b), when an external photon line is attached. To compute the diagram in the Fig. A.1, it is necessary to know the mixing of the scalar fields, $\mathcal{C}_{ij}$, coming from the diagonalization of the scalar mass matrix. However, it will be considered $\mathcal{C}_{ij} \sim \mathcal{O}(1)$, which is the worst case. Taking into account all the previously said, it is obtained

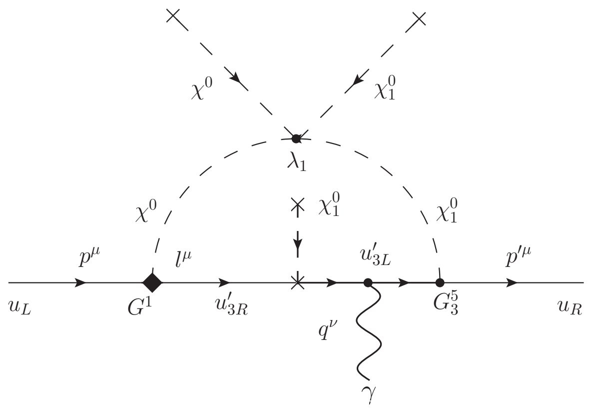

Figure A.1: One loop diagram contributing to the electric dipole moment of the up-quark. The CP violating vertex is denoted with a diamond.

$$\begin{aligned}
\rightarrow \quad & \overline{u}(p')\left[\int \frac{\mathrm{d}^4 l}{(2\pi)^4} i G_3^5 P_L \frac{i\left(\gamma^\mu\left(l_\mu+p'_\mu\right)+m_{u'}\right)}{(l+p')^2-m_{u'}^2+i\epsilon} i\frac{2}{3}e\gamma^\nu \frac{i\left(\gamma^\rho\left(l_\rho+p_\rho\right)+m_{u'}\right)}{(l+p)^2-m_{u'}^2+i\epsilon}\right. \\
& \left. i\,\mathrm{Im}\,\left(G_1\right) P_L \frac{i}{l^2-m_\chi^2+i\epsilon}\right] u\left(p\right)\varepsilon_\nu\left(q\right), \\
\equiv\; & \frac{2}{3}e\,\overline{u}(p')\left[-\frac{1}{4}i\left|G_1\right|\sin\phi G_3^5 \int \mathrm{d}F_3 \int \frac{\mathrm{d}^4 l}{(2\pi)^4}\frac{N^\nu}{\left(l^2-D\right)^3}\right] u\left(p\right)\varepsilon_\nu\left(q\right), \qquad \text{(A.1)}
\end{aligned}$$

where G_3^5 and G_1 are the Yukawa couplings given in Eq. (2.5); and Im $\left(G_1\right) \equiv i\left|G_1\right|\sin\phi$ has been used in the last line. Also, it has been defined

$$N^\nu \;\equiv\; 2m_{u'}\left[\left(\hat{l}+\hat{p}'\right)\gamma^\nu+\gamma^\nu\left(\hat{l}+\hat{p}\right)\right]\left(1-\gamma^5\right), \qquad \text{(A.2)}$$

with $\hat{r} \equiv \gamma^\mu r_\mu$. To get D in the Eq. (A.1), the Feynman parameters have been used

$$\frac{1}{A_1A_2A_3} \;=\; \int \mathrm{d}F_3\left(X_1A_1+X_2A_2+X_3A_3\right)^{-3}. \qquad \text{(A.3)}$$

where the integral over the Feynman parameters reads

$$\int \mathrm{d}F_3 \;\equiv\; 2\int_0^1 \mathrm{d}X_1\mathrm{d}X_2\mathrm{d}X_3\,\delta\left(X_1+X_2+X_3-1\right). \qquad \text{(A.4)}$$

Furthermore,

$$\begin{aligned}
X_1A_1+X_2A_2+X_3A_3 \;=\; & \left(l^2+2l\cdot\left(X_1p'+X_2p\right)+\left(X_1p'+X_2p\right)^2\right)-\left(X_1p'+X_2p\right)^2 \\
& +X_1p'^2+X_2p^2-\left(X_1+X_2\right)m_{u'}^2-X_3m_\chi^2+i\epsilon, \qquad \text{(A.5)}
\end{aligned}$$

where $X_1+X_2+X_3=1$ has been used. Now, doing $l'=l+X_1p'+X_2p$ it is obtained

$$\begin{aligned}
X_1A_1+X_2A_2+X_3A_3 \;=\; & l'^2-\left(X_1p'+X_2p\right)^2+X_1p'^2+X_2p^2-\left(X_1+X_2\right)m_{u'}^2, \\
& -X_3m_\chi^2+i\epsilon \\
=\; & l'^2-D, \qquad \text{(A.6)}
\end{aligned}$$

where D reads

$$D = X_1 (X_1 - 1) p'^2 + X_2 (X_2 - 1) p^2 + 2X_1 X_2 p' \cdot p + (X_1 + X_2) m_{u'}^2 + X_3 m_\chi^2 - i\epsilon.$$

Moreover, let us use $p^2 = p'^2 = m_u^2$ ($m_u = 1.7 - 3.1$ MeV is the up-quark mass [8]), $X_1 + X_2 + X_3 = 1$ and $p \cdot p' = m^2 - q^2/2$, in order to simplify D

$$D = X_3 (X_3 - 1) m^2 + (1 - X_3) m_{u'}^2 + X_3 m_\chi^2 - X_1 X_2 q^2 - i\epsilon. \tag{A.7}$$

Now, the numerator N^ν has to be written in terms of $l' = l + X_1 p' + X_2 p$. Doing so,

$$N^\nu = 2m_{u'} \left[\hat{l}'\gamma^\nu + \gamma^\nu \hat{l}' + \hat{p}'\gamma^\nu + \gamma^\nu \hat{p} - 2X_1 p'^\nu - 2X_2 p^\nu\right] \left(1 - \gamma^5\right). \tag{A.8}$$

To simplify the numerator, let us use

$$\int \frac{\mathrm{d}^4 l}{(2\pi)^4} \frac{l_\mu}{(l^2 - D)^3} = 0, \tag{A.9}$$

and writing l rather than l' for simplicity. In this way, N^ν reads

$$N^\nu \to 2m_{u'} \left[\hat{p}'\gamma^\nu + \gamma^\nu \hat{p} - 2X_1 p'^\nu - 2X_2 p^\nu\right] \left(1 - \gamma^5\right). \tag{A.10}$$

Putting the numerator into an useful form is just a matter of some tedious Dirac algebra. The most straightforward way to accomplish this is considering an expression of the form

$$\overline{u}(p') \left[i (p' - p)^\mu \sigma_{\nu\mu} \left(A + B\gamma^5\right) + \gamma^\nu \left(C + D\gamma^5\right) + (p' - p)_\nu \left(E + F\gamma^5\right)\right] u(p). \tag{A.11}$$

Attaining this form requires only the relations

$$\{\gamma^\nu, \gamma^\mu\} = 2g^{\nu\mu}, \quad \{\gamma^\nu, \gamma^5\} = 0, \quad \left(\gamma^5\right)^2 = 1, \quad \sigma_{\mu\nu} \equiv \frac{i}{2} [\gamma_\mu, \gamma_\nu], \tag{A.12}$$

and the Dirac equation

$$\hat{p} u(p) = m_u u(p), \quad \overline{u}(p')\hat{p}' = \overline{u}(p') m_u, \tag{A.13}$$

and because we are interested in calculating the EDM, the relevant terms are those that are multiplied by γ^5. Using the previous relations, the relevant part of the numerator is

$$\begin{aligned} &\to -2m_{u'}\overline{u}(p') \left[\hat{p}'\gamma^\nu + \gamma^\nu \hat{p} - 2X_1 p'^\nu - 2X_2 p^\nu\right] \gamma^5 u(p), \\ &= 4m_{u'}\overline{u}(p') \left[X_1 p'^\nu + X_2 p^\nu\right] \gamma^5 u(p). \end{aligned} \tag{A.14}$$

Now, let us group the p^ν and p'^ν terms into $p_+^\nu \equiv p'^\nu + p^\nu$ and $p_-^\nu \equiv p'^\nu - p^\nu$ combinations

$$X_1 p'^\nu + X_2 p^\nu = \frac{1}{2} p_+^\nu [X_1 + X_2] + \frac{1}{2} p_-^\nu [X_1 - X_2], \tag{A.15}$$

and using

$$0 = \overline{u}(p',\, s)\left[p_{+}^{\nu} + i\sigma^{\nu\rho}p_{-\rho}\right]\gamma^{5}u\,(p,\, s)\,, \tag{A.16}$$

where s is the spin projection. Now, substituting Eq. (A.15) and Eq. (A.16) into Eq. (A.14), and picking out the relevant term to the EDM calculation, $i\,\sigma^{\nu\rho}p_{-\rho}\gamma^{5}$, the following integral

$$\int \frac{\mathrm{d}^{4}l}{(2\pi)^{4}}\frac{-i2m_{u'}\left[1-X_{3}\right]\sigma^{\nu\rho}p_{-\rho}\gamma^{5}}{\left(l^{2}-D\right)^{3}}, \tag{A.17}$$

have to be evaluated, where D is given by Eq. (A.7) and $X_1 + X_2 = 1 - X_3$. To do this, let us use

$$\int \frac{\mathrm{d}^{d}l}{(2\pi)^{d}}\frac{1}{\left(l^{2}-D\right)^{n}} = \frac{(-1)^{n}\,i\,\Gamma\left(n-d/2\right)}{(4\pi)^{d/2}\quad\Gamma\left(n\right)}\left(\frac{1}{D}\right)^{n-d/2}. \tag{A.18}$$

Using this result it is obtained

$$\int \frac{\mathrm{d}^{4}l}{(2\pi)^{4}}\frac{-i2m_{u'}\left[1-X_{3}\right]\sigma^{\nu\rho}p_{-\rho}\gamma^{5}}{\left(l^{2}-D\right)^{3}} = -\frac{m_{u'}\left[1-X_{3}\right]}{16\pi^{2}D}\sigma^{\nu\rho}p_{-\rho}\gamma^{5}. \tag{A.19}$$

Thus, the general expression to calculate the B coefficient in the Eq. (A.11) is

$$B\left(q^{2}\right) = \frac{2}{3}e \times \frac{m_{u'}}{32\pi^{2}}G_{3}^{5}\left|G_{1}\right|\sin\phi \times I(m_{u}, m_{u'},\, m_{\chi},\, q^{2}), \tag{A.20}$$

where

$$I(m_{u}, m_{u'},\, m_{\chi},\, q^{2}) \equiv \frac{1}{2}\int \mathrm{d}F_{3}\frac{1-X_{3}}{X_{3}\left(X_{3}-1\right)m_{u}^{2}+\left(1-X_{3}\right)m_{u'}^{2}+X_{3}m_{\chi}^{2}-X_{1}X_{2}q^{2}}. \tag{A.21}$$

has been defined. But, we need only to calculate the EDM, i.e. $d_{u}^{e} \equiv B\left(q^{2}\rightarrow 0\right)$. Then

$$d_{u}^{e} = \frac{em_{u'}G_{3}^{5}\left|G_{1}\right|\sin\phi}{48\pi^{2}}\int_{0}^{1}\mathrm{d}X_{3}\frac{\left(1-X_{3}\right)^{2}}{X_{3}\left(X_{3}-1\right)m_{u}^{2}+\left(1-X_{3}\right)m_{u'}^{2}+X_{3}m_{\chi}^{2}}. \tag{A.22}$$

Although an exact expression for d_u^e in the Eq. (A.22) can be given, this is not illuminating. Therefore, the limit when $m_u \ll m_\chi$ and $m_u \ll m_{u'}$ will be taken. Being more precise

$$d_{u}^{e}|_{m_{u}\ll m_{u'},m_{\chi}} \approx \frac{eG_{3}^{5}\left|G_{1}\right|\sin\phi\, m_{u'}}{48\pi^{2}\qquad m_{\chi}^{2}}\mathcal{K}\left(r\right), \tag{A.23}$$

where $\mathcal{K}\left(r\right)$,

$$\mathcal{K}\left(r\right) = \frac{1}{2r}-\frac{1}{r^{2}}+\frac{1}{r^{3}}\ln\left(1+r\right), \tag{A.24}$$

with $r = \frac{m_{u'}^{2}}{m_{\chi}^{2}} - 1$.

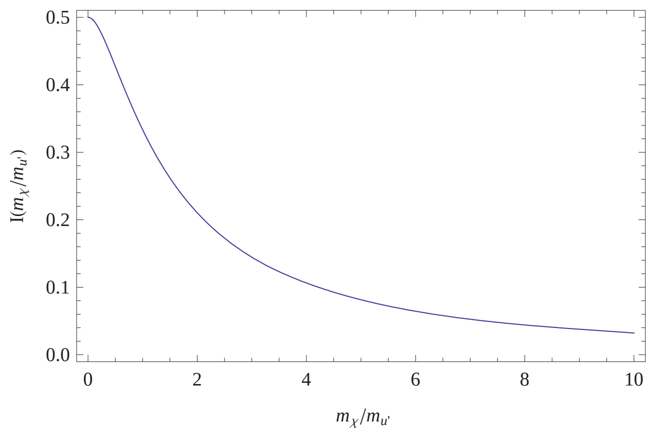

Figure A.2: Integral $I(m_u, m_{u'}, m_\chi, q^2)$ when $q^2 \to 0$ and $m_u = 0$.

Furthermore, it is interesting to consider the limit $m_{u'} \approx m_\chi$ to the Eq. (A.23), since these two exotic particles obtain mass from the same VEV, $V_{\chi_1^0}$. Considering this situation, it is obtained

$$d_u^e|_{m \ll m_{u'}, m_\chi,\, \text{and}\, m_{u'} \to m_\chi} \approx \frac{eG_3^5\,|G_1|\sin\phi}{144\pi^2 m_\chi}. \tag{A.25}$$

In this approximation, there are a bound on $G_3^5\,|G_1|\sin\phi/m_\chi$ coming from the experimental bound of the EDMN, $d_n^e \sim \frac{4}{3}d_d^e - \frac{1}{3}d_u^e \approx \mathcal{O}(d_u^e) < 0.29 \times 10^{-25}\, e\cdot\text{cm}$ [8]. Thus, we have

$$G_3^5\,|G_1|\sin\phi \times \left(\frac{1\ \text{TeV}}{m_\chi}\right) \lesssim 2.1 \times 10^{-6}, \tag{A.26}$$

where the conversion factor $(1\ \text{GeV})^{-1}\,(\hbar c) = 1.9733 \times 10^{-14}$ cm has been used. For instance, let us assume that the CP phase is such that $\sin\phi \approx 10^{-2}$ and $m_\chi \sim 1$ TeV. In this case the parameters $G_3^5 \sim 10^{-2}$ and $|G_1| \sim 10^{-2}$ satisfy the limit given in the Eq. (A.26). The general case, i.e. $m_\chi \neq m_{u'}$, is shown in the Fig. A.3. From this figure it can be seen that when $m_\chi > m_{u'}$ the limit on the couplings is slightly less severe. Notice that suppressions coming from the neutral scalar mixing angles, $\mathcal{C}_{ij}$, are not been considered.

In conclusion, the tuning of the coupling constants required to satisfy the experimental bound coming from the EDMN is not as severe as the strong CP problem, where the $\overline{\theta}$ parameter has to be smaller than $\lesssim 10^{-11}$. Therefore, a solution for the strong CP problem in this model must be found. In particular, the PQ solution was successful implemented in the Chap. 2.

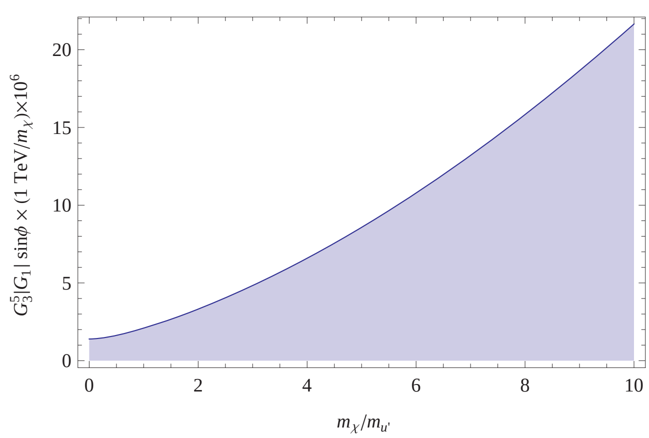

Figure A.3: The quantity $G_3^5\,|G_1|\sin\phi\times(1\,\text{TeV}/m_\chi)$ as function of the ratio $m_\chi/m_{u'}$. The shaded region is the allowed one.

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