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Zero Mode Effect Generalization for the Electromagnetic Current in the Light Front: Fermion Case

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Abstract We investigate the effect of the γ^+ term in the fermion propagator on the zero mode in a system composed of two free fermions propagating in a background field. For this purpose we use the technique of global propagators in the light front and current operator in order to obtain the electromagnetic current component J^-.

1 Introduction

In the traditional approach to restore covariance of the electromagnetic current in the light front [1] an "ad hoc" prescription of dislocating the pole is employed [2]. However, this procedure of "pole dislocation" has no physical grounds and the arrival at the correct result is just fortuitous. We demonstrate that the light front Fock space of positive quanta solutions is incomplete and that as a consequence the non triviality of the light front vacuum is a mandatory feature in the new scenario [3].

In an article Bakker et al. [4] studied how the behavior of the contour integration in the k^- complex plane could influence the results of integrations in the light front. Our work differs in the fact that we have no need to consider these "treacherous" arc contributions in order to recover covariance of the results from the light front calculations. In our work we show that these arc contributions are absent for we deal with "ladder" type diagrams and evaluating all the relevant ranges of integration for k^+ variables we get the correct terms equivalent to the results of computations obtained via covariant Minkowski calculations. This being our present case, it is well worth noting that no "treacherous" points have to be dealt with in our "ladder" diagram type calculations and also that our methodology does not involve any subtle or difficult points like in the case of those treacherous arc contributions, recognized as not only very difficult to handle but requiring careful consideration of the limiting procedure and thus of end-point singularities [3].

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We show how the pair production is necessary to the complete calculation of the current's J^- component in the Drell–Yan's reference frame ($q^+ = 0$).

2 Propagator of Fermions in a Background Field

In a recent article [3] it has been considered, in the zeroth order of perturbative coupling, the calculation of the electromagnetic current in the light front coordinates for scalar bosons in the electromagnetic background field. We refer to the notation and conventions used there.

The propagator for a spin-$\frac{1}{2}$ particle is related to the scalar propagator through the following

$$S_{\mathrm{f}}(x^\mu) = \left(i\gamma^\mu \partial_\mu + m\right) S(x^\mu). \tag{1}$$

where

$$S(x^\mu) = \int \frac{d^4k}{(2\pi)^4} \frac{e^{-ik^\mu x_\mu}}{k^2 - m^2 + i\varepsilon}, \tag{2}$$

In the light front, we have

$$S(x^+) = \int \frac{dk_1^-}{(2\pi)} \frac{i e^{-ik_1^- x^+}}{2k_1^+ \left(k_1^- - \frac{\mathbf{k}_{1\perp}^2 + m^2 - i\varepsilon}{2k_1^+}\right)} \tag{3}$$

Using Eq. (2) in Eq. (1) and differentiating, we have:

$$S_{\mathrm{f}}(x^\mu) = \int \frac{d^4k}{(2\pi)^4} \frac{i(\not{k} + m)}{k^2 - m^2 + i\varepsilon} e^{-ik^\mu x_\mu}, \tag{4}$$

where the Feynman's "slash" for any four-vector is defined as $\not{k} = k_\mu \gamma^\mu$.

If we project the integrand of Eq. (4) onto the light front coordinates we may write it (after appropriate Fourier transformation and henceforth omitting the subindex "f" for fermions) as

$$\widetilde{S}(k^-) = i \frac{\not{k}_{\mathrm{on}} + m}{2k^+ \left(k^- - k^-_{\mathrm{on}}\right)} + \frac{i\gamma^+}{2k^+} \equiv \widetilde{S}^{\mathrm{prop}}(k^-) + \frac{i\gamma^+}{2k^+}, \tag{5}$$

where $k^-_{\mathrm{on}} = \frac{(\mathbf{k}_\perp^2 + m^2)}{2k^+}$, and the propagating part is explicitly separated out into the term $\widetilde{S}^{\mathrm{prop}}(k^-)$ with $k^+ > 0$ as identified above, i.e.,

$$\widetilde{S}^{\mathrm{prop}}(k^-) = i \frac{\not{k}_{\mathrm{on}} + m}{2k^+ \left(k^- - k^-_{\mathrm{on}} + \frac{i\varepsilon}{2k^+}\right)}. \tag{6}$$

Thus, the propagator $\widetilde{S}(k^-)$ represents the covariant propagation and $\widetilde{S}^{\mathrm{prop}}(k^-)$ the propagation in the light front without the instantaneous term $\frac{\gamma^+}{2k^+}$. We also note that in the propagation in the light front time, only one state—with positive energy—propagates to the future, $x^+ > 0$.

The part of the interaction Lagrangian that contains the fermions-scalar vertex can be written as

$$\mathcal{L}_{\mathrm{Interaction}} = -ieA^\mu \left(\psi_1 D_\mu \overline{\psi}_1 - \overline{\psi}_1 D_\mu \psi_1\right) = J_\mu A^\mu. \tag{7}$$

We can indicate the propagation of two fermions S_1 and S_2 that propagate from $x^+ = 0$ to $x^+ > 0$ interacting with an external electromagnetic field $A^\mu(x^+)$ at $\bar{x}_3^+$ and with the exchange of an intermediate σ boson between $\bar{x}_1^+$ and $\bar{x}_2^+$. The propagator $S_3(\bar{x}_1^+ - \bar{x}_3^+)$ which is the propagation of a fermion after the emission of σ boson at $\bar{x}_1^+$ later interacts with the external field at $\bar{x}_3^+$. The propagator S_5 is the fermion propagation after the interaction with the external field. The propagator S_4 is the fermion propagation after the absorption of the intermediate σ boson. Therefore the correction to the free propagator of two fermions in the light front with background field in the ladder diagram is:

$$S(x^+) = (-ie)\,(ig)^2 \int d\bar{x}_1^+ d\bar{x}_2^+ d\bar{x}_3^+ dq^- A^\mu(q^-) e^{-iq^-\bar{x}_3^+} S_1(\bar{x}_1^+) S_2(\bar{x}_2^+) \times S_4(x^+ - \bar{x}_2^+) S_\sigma(\bar{x}_2^+ - \bar{x}_1^+) \left[S_3 \frac{\partial S_5}{\partial \bar{x}_3^\mu} - \frac{\partial S_3}{\partial \bar{x}_3^\mu} S_5 \right], \tag{8}$$

where $A^\mu(q^-)$ is the Fourier transform and μ indicates the components $-$, $+$, $\perp$. In Eq. (8) the presence of $S_\sigma(\bar{x}_2^+ - \bar{x}_1^+)$ indicates that there is a virtual σ boson that is emitted from fermion 1 at light-front time $\bar{x}_1^+$ and absorbed by fermion 2 at time $\bar{x}_2^+$. Thus, this equation represents the time-ordered vacuum expectation value of a system composed of two fermions exchanging an intermediate sigma boson and propagating in a background field, calculated up to the perturbative order $\mathcal{O}(g^2)$ in the coupling constant, which corresponds to the so-called "ladder" diagram approximation.

So, we observe that the operator component in the current J^μ, $\mu = +, -, \perp$ is obtained from the operator $\mathcal{O}_\mu$ which corresponds to the components of the external electromagnetic field A^μ, $\mu = -, +, \perp$ respectively such that

$$\mathcal{O}_\mu = (-ie)\,(ig)^2 \int d\bar{x}_1^+ d\bar{x}_2^+ d\bar{x}_3^+ e^{-iq^-\bar{x}_3^+} \times S_1(\bar{x}_1^+) S_2(\bar{x}_2^+) S_4(x^+ - \bar{x}_2^+) \times S_\sigma(\bar{x}_2^+ - \bar{x}_1^+) \left[S_3 \frac{\partial S_5}{\partial \bar{x}_3^\mu} - \frac{\partial S_3}{\partial \bar{x}_3^\mu} S_5 \right], \tag{9}$$

where the Greek index μ over the $\mathcal{O}_\mu$ reminds us which light-front components $+$, $-$, or $\perp$ are we dealing with.

The final propagator can therefore be written as a function of only two momenta, and in this case we choose "spectator" particles with respect to the current, those labeled as 2 and 4:

$$\widetilde{S}(k_f^-) = -\frac{ie\,(ig)^2}{2^6(2\pi)^2} \int dq^- A^\mu(q^-) \times \left\{ \int \frac{dk_2^- dk_4^- \left(k_f + k_i - 2k_4\right)_\mu}{(k_i - k_2)^+ k_2^+ (k_i - k_4)^+ k_4^+ \left(k_f - k_4\right)^+ (k_4 - k_2)^+} \right.$$
$$\times \frac{1}{\left[k_2^- - k_i^- + (k_i - k_2)_{\text{on}} - \frac{i\epsilon}{2(k_i - k_2)^+}\right]\left[k_2^- - k_{2\text{on}} + \frac{i\epsilon}{2k_2^+}\right]}$$
$$\times \frac{1}{\left[k_4^- - k_i^- + (k_i - k_4)_{\text{on}} - \frac{i\epsilon}{2(k_i - k_4)^+}\right]\left[k_4^- - k_{4\text{on}} + \frac{i\epsilon}{2k_4^+}\right]}$$
$$\left. \times \frac{1}{\left[k_2^- - k_4^- + (k_4 - k_2)_{\text{on}} - \frac{i\epsilon}{2(k_4 - k_2)^+}\right]\left[k_4^- - k_f^- + (k_f - k_4)_{\text{on}} - \frac{i\epsilon}{2(k_f - k_4)^+}\right]} \right\}, \tag{10}$$

where

$$k_{2\text{on}} = \frac{\mathbf{k}_{2\perp}^2 + m^2}{2k_2^+}; \quad (k_i - k_2)_{\text{on}} = \frac{(\mathbf{k}_i - \mathbf{k}_2)_\perp^2 + m^2}{2(k_i^+ - k_2^+)}$$
$$(k_i - k_4)_{\text{on}} = \frac{(\mathbf{k}_i - \mathbf{k}_4)_\perp^2 + m^2}{2(k_i^+ - k_4^+)}; \quad k_{4\text{on}} = \frac{\mathbf{k}_{4\perp}^2 + m^2}{2k_4^+}$$
$$(k_f - k_4)_{\text{on}} = \frac{(\mathbf{k}_f - \mathbf{k}_4)_\perp^2 + m^2}{2(k_i^+ - k_4^+)}; \quad (k_4 - k_2)_{\text{on}} = \frac{(\mathbf{k}_4 - \mathbf{k}_2)_\perp^2 + m_\sigma^2}{2(k_4^+ - k_2^+)}. \tag{11}$$

We begin our discussion with an illustrative example, where the pair term appears. To this end we use the "Z-graph", that is momentum possibilities $0 < k_2^+ < k_i^+ < k_4^+ < k_f^+$. In this example we show that the current's J^- component does not have a contribution from the pair production in the Drell–Yan reference frame [5], that is, in the limit $q^+ = q^- = 0$.

We look after the components of the current operator J^μ, which as we referred before, will be obtained from the operator $\mathcal{O}_\mu$ which is represented by the square brackets in Eq. (9), so

$$\mathcal{O}_\mu = -\frac{ie\,(ig)^2}{2^6(2\pi)^2}\int \frac{dk_2^- dk_4^- \left(k_f + k_i - 2k_4\right)_\mu}{(k_i - k_2)^+ k_2^+ (k_i - k_4)^+ k_4^+ \left(k_f - k_4\right)^+ (k_4 - k_2)^+}$$
$$\times \frac{1}{\left[k_2^- - k_i^- + (k_i - k_2)_{\text{on}} - \frac{i\epsilon}{2(k_i - k_2)^+}\right]\left[k_2^- - k_{2\text{on}} + \frac{i\epsilon}{2k_2^+}\right]}$$
$$\times \frac{1}{\left[k_4^- - k_i^- + (k_i - k_4)_{\text{on}} - \frac{i\epsilon}{2(k_i - k_4)^+}\right]\left[k_4^- - k_{4\text{on}} + \frac{i\epsilon}{2k_4^+}\right]}$$
$$\times \frac{1}{\left[k_2^- - k_4^- + (k_4 - k_2)_{\text{on}} - \frac{i\epsilon}{2(k_4 - k_2)^+}\right]\left[k_4^- - k_f^- + (k_f - k_4)_{\text{on}} - \frac{i\epsilon}{2(k_f - k_4)^+}\right]}. \tag{12}$$

where $k_{i\mu}$ and $k_{f\mu}$ are the initial and final four-momentum of the system and m is the mass of the boson. The integration in Eq. (12), using the Cauchy integral formula over k_2^- and k_4^-, have ten nonvanishing contributions for the residue calculation, but for our example, we concentrate our attention in a specific region, that is, in the range of momenta satisfying $0 < k_2^+ < k_i^+ < k_4^+ < k_f^+$, which corresponds to the "Z-graph".

3 Zero Mode Contribution at $\mathcal{O}(g^2)$

To calculate the electromagnetic current generated by the diverse configurations we must have the matrix elements $J^{-,+,\perp} = \langle \Gamma\,|\mathcal{O}_{+,-,\perp}|\,\Gamma\rangle$, where Γ is the constant vertex and $\mathcal{O}_{+,-,\perp}$ are the current operator components, which we can obtain directly from the sum of the final results in each region. With our metric convention, we have respectively $\mathcal{O}_{+,-} = \mathcal{O}^{-,+}$ and $\mathcal{O}_\perp = -\mathcal{O}^\perp$. Introducing the unit resolution into the matrix element we have:

$$\begin{aligned}
\left\langle \Gamma \left| \mathcal{O}^{-,+,\perp} \right| \Gamma \right\rangle &= \int dk_j^+ d^2\mathbf{k}_{j\perp} \left\langle \Gamma \left| k_j^+, \mathbf{k}_{j\perp} \right.\right\rangle \left\langle k_j^+, \mathbf{k}_{j\perp} \right| \mathcal{O}^{-,+,\perp} \\
&\quad \times \int dk_j'^+ d^2\mathbf{k}_{j\perp}' \left| k_j'^+, \mathbf{k}_{j\perp}' \right\rangle \left\langle k_j'^+, \mathbf{k}_{j\perp}' \right| \Gamma \,\rangle \\
&= \Gamma \int dk_j^+ d^2\mathbf{k}_{j\perp} dk_j'^+ d^2\mathbf{k}_{j\perp}' \left\langle k_j^+, \mathbf{k}_{j\perp} \left| \mathcal{O}^{-,+,\perp} \right| k_j'^+, \mathbf{k}_{j\perp}' \right\rangle \Gamma \\
&= \Gamma^2 \int dk_j^+ d^2\mathbf{k}_{j\perp} dk_j'^+ d^2\mathbf{k}_{j\perp}' \delta\left(k_j^+ - k_j'^+ - q^+\right) \delta\left(\mathbf{k}_{j\perp} - \mathbf{k}_{j\perp}' - \mathbf{q}_\perp\right) \\
&\quad \times \left\langle k_j^+, \mathbf{k}_{j\perp} \left| \mathcal{O}^{-,+,\perp} \right| k_j'^+, \mathbf{k}_{j,\perp}' \right\rangle \\
&= \Gamma^2 \int dk_2^+ d^2\mathbf{k}_{2\perp} dk_4^+ d^2\mathbf{k}_{4\perp} \mathcal{O}^{-,+,\perp},
\end{aligned} \tag{13}$$

Thus, for example the electromagnetic current $J^{-,+,\perp}$, pertinent to the region with momenta range $0 < k_2^+ < k_i^+ < k_4^+ < k_f^+$ is obtained by substituting $\mathcal{O}^{-,+,\perp}$ given in Eq. (12).

Our next step is to perform the remaining momentum integration over k_2^+ and k_4^+ and take the limit $q^+ \to 0$. To calculate the momentum integrations we make two changes of variables that will facilitate our job of integrating them

$$x = \frac{k_i^+ - k_2^+}{q^+}; \quad y = \frac{k_f^+ - k_4^+}{q^+}. \tag{14}$$

On the other hand, taking advantage of the momentum conservation relations, we get

$$\begin{aligned} k_1^+ &= xq^+; \quad k_3^+ = (y-1)\,q^+; \\ k_5^+ &= yq^+; \quad k_\sigma^+ = (x-y+1)\,q^+. \end{aligned} \tag{15}$$

Now it is just a matter of putting things together. For space constraints here we restrict ourselves to investigate the minus component of the current. Other components can be worked out in a similar manner. Thus, for

- **Current** J^-:

Substituting Eqs. (14) and (15) in Eq. (13) we get:

$$\begin{aligned} J^- &= \langle \Gamma \left| \mathcal{O}^- \right| \Gamma \rangle = \Gamma^2 \int dk_2^+ d^2\mathbf{k}_{2_\perp} dk_4^+ d^2\mathbf{k}_{4_\perp} \mathcal{O}^- \\ &= \Gamma^2 \int d^2\mathbf{k}_{2_\perp} d^2\mathbf{k}_{4_\perp} \left\{ \left(q^+\right)^2 \int dx dy \mathcal{O}^- \right\}, \end{aligned} \tag{16}$$

where the operator contribution $\mathcal{O}^-$ takes the following form

$$\begin{aligned} \mathcal{O}^- = &\frac{ie\,(ig)^2}{2^6} \frac{\theta\left(k_i^+ - k_2^+\right)\theta\left(k_2^+\right)\theta\left(k_4^+ - k_i^+\right)\theta\left(k_4^+\right)\theta\left(k_f^+ - k_4^+\right)\theta\left(k_4^+ - k_2^+\right)}{xq^+\left(k_i^+ - xq^+\right)(y-1)\,q^+\left(k_f^+ - yq^+\right)yq^+\,(x-y+1)\,q^+} \\ &\times \frac{\left[k_f^- - k_i^- - \frac{(\mathbf{k}_f - \mathbf{k}_4)_\perp^2 + m^2}{yq^+}\right]}{\left[k_i^- - \frac{(\mathbf{k}_i - \mathbf{k}_2)_\perp^2 + m^2}{2xq^+} - \frac{\mathbf{k}_{2_\perp}^2 + m^2}{2(k_i^+ - xq^+)}\right]} \frac{1}{\left[k_f^- - \frac{\mathbf{k}_{2_\perp}^2 + m^2}{2(k_i^+ - xq^+)} - \frac{(\mathbf{k}_f - \mathbf{k}_4)_\perp^2 + m^2}{2yq^+} - \frac{(\mathbf{k}_4 - \mathbf{k}_2)_\perp^2 + m_\sigma^2}{2(x-y+1)q^+}\right]} \\ &\times \frac{1}{\left[k_f^- - k_i^- + \frac{(\mathbf{k}_i - \mathbf{k}_4)_\perp^2 + m^2}{2(y-1)q^+} - \frac{(\mathbf{k}_f - \mathbf{k}_4)_\perp^2 + m^2}{2yq^+}\right]} \frac{1}{\left[k_f^- - \frac{\mathbf{k}_{4_\perp}^2 + m^2}{2\left(k_f^+ - yq^+\right)} - \frac{(\mathbf{k}_f - \mathbf{k}_4)_\perp^2 + m^2}{2yq^+}\right]}, \end{aligned} \tag{17}$$

which can be written in a more convenient form, factoring out all the relevant factors of q^+ to make more evident how this particular operator component depends on q^+:

$$\begin{aligned} \mathcal{O}^- = &\frac{ie\,(ig)^2}{2^6}\left(\frac{1}{q^+}\right) \frac{\theta\left(k_i^+ - k_2^+\right)\theta\left(k_2^+\right)\theta\left(k_4^+ - k_i^+\right)\theta\left(k_4^+\right)\theta\left(k_f^+ - k_4^+\right)\theta\left(k_4^+ - k_2^+\right)}{x\left(k_i^+ - xq^+\right)(y-1)\left(k_f^+ - yq^+\right)y\,(x-y+1)} \\ &\times \frac{\left[\left(k_f^- - k_i^-\right)q^+ - \frac{(\mathbf{k}_f - \mathbf{k}_4)_\perp^2 + m^2}{y}\right]}{\left[k_i^- q^+ - \frac{(\mathbf{k}_i - \mathbf{k}_2)_\perp^2 + m^2}{2x} - \frac{\mathbf{k}_{2_\perp}^2 + m^2}{2(k_i^+ - xq^+)}q^+\right]} \frac{1}{\left[k_f^- q^+ - \frac{\mathbf{k}_{2_\perp}^2 + m^2}{2(k_i^+ - xq^+)}q^+ - \frac{(\mathbf{k}_f - \mathbf{k}_4)_\perp^2 + m^2}{2y} - \frac{(\mathbf{k}_4 - \mathbf{k}_2)_\perp^2 + m_\sigma^2}{2(x-y+1)}\right]} \\ &\times \frac{1}{\left[\left(k_f^- - k_i^-\right)q^+ + \frac{(\mathbf{k}_i - \mathbf{k}_4)_\perp^2 + m^2}{2(y-1)} - \frac{(\mathbf{k}_f - \mathbf{k}_4)_\perp^2 + m^2}{2y}\right]\left[k_f^- q^+ - \frac{\mathbf{k}_{4_\perp}^2 + m^2}{2\left(k_f^+ - yq^+\right)}q^+ - \frac{(\mathbf{k}_f - \mathbf{k}_4)_\perp^2 + m^2}{2y}\right]}. \end{aligned} \tag{18}$$

Having achieved this result we want to generalize to J^- for this specific example, counting the terms that bear q^+, that is, performing a power counting on the factors q^+ for a quick analysis of the result in the frame $q^+ \to 0$:

$$\text{Momentum integration} \int \mathrm{dk}_2^+ \mathrm{dk}_4^+ \Rightarrow \left(q^+\right)^2 \int dxdy$$
$$\frac{1}{(k_i-k_2)^+ k_2^+ (k_i-k_4)^+ k_4^+ \left(k_f-k_4\right)^+ (k_4-k_2)^+} \Rightarrow \frac{1}{\left(q^+\right)^4}$$
$$\text{Legs of the type } \frac{1}{\left[\mathrm{a}+\frac{\mathrm{b}}{\mathrm{cq}^+}+\frac{\mathrm{d}}{\mathrm{eq}^+}+\cdots\right]^4} \Rightarrow \left(q^+\right)^4$$
$$\text{Numerator of the type } (\mathrm{a}+\mathrm{k}_{2\mathrm{on}}) \text{ or } (\mathrm{a}+\mathrm{k}_{4\mathrm{on}}) \Rightarrow \left(q^+\right)^0 = 1$$
$$\text{Numerator of the type } \left(\mathrm{a}+\mathrm{k}_{\mathrm{jon}}\right) \text{ with } \mathrm{j} \neq 2 \text{ or } 4 \Rightarrow \frac{1}{q^+}\ , \tag{19}$$

where a represents the momenta k_i, k_f, $k_f - k_i$, etc. and b, d, etc. are $k_{2\text{on}}$, $k_{4\text{on}}$ or $k_{j\text{on}}$. We note that as we multiply all these factors together it will always remain at least $\left(q^+\right)^1$, which in the limit for $q^+ \to 0$, makes the current in all regions to vanish. What we conclude here is that the introduction of a virtual boson in comparison to the configuration considered in [3], does not alter the current because the factors in the second and third line of Eq. (19) cancel each other. The important factor is the photon vertex, since this increases the power in the numerator and only with correct factors of $\frac{1}{q^+}$ we can cancel the factor coming from the change of variables.

Now finally, if we have m external sources for n interacting bosons, we obtain

$$\text{Momentum integration} \int \prod_{\mathrm{j}=1}^{\mathrm{n}+1} \mathrm{dk}_{2\mathrm{j}}^+ \Rightarrow \left(q^+\right)^{n+1} \int \prod_{j=1}^{n+1} dx_j$$
$$\frac{1}{\left(k_i^+ - k_2^+\right) \cdots \left(k_{2n+2}^+ - k_{2n}^+\right) \left(k_f^+ - k_{2n+2}^+\right) \left(k_f^+ - k_{2j+2}^+\right)} \Rightarrow \frac{1}{\left(q^+\right)^{2n+m+2}}$$
$$\text{Legs of the type } \frac{1}{\left[\mathrm{a}+\frac{\mathrm{b}}{\mathrm{cq}^+}+\frac{\mathrm{d}}{\mathrm{eq}^+}+\cdots\right]} \Rightarrow \left(q^+\right)^{2n+m+2}$$
$$\text{Numerator of the type } (\mathrm{a}+\mathrm{k}_{2\mathrm{on}}) \text{ or } (\mathrm{a}+\mathrm{k}_{4\mathrm{on}}) \Rightarrow \left(q^+\right)^0 = 1$$
$$\text{Numerator of the type } \left(\mathrm{a}+\mathrm{k}_{\mathrm{jon}}\right) \text{ with } \mathrm{j} \neq 2, 4, \ldots, 2\mathrm{n}+2 \Rightarrow \frac{1}{(q^+)^m}. \tag{20}$$

In this manner it is only possible to observe the contributions of antiparticles when we put more energy in the system of two interacting bosons. We can check in the case shown previously: In second order of coupling constant for a virtual boson it results in no observation of antiparticle contributions for $q^+ \to 0$ in a background field. However in the expression Eq. (20) we have a case of two external sources ($m = 2$) and one interacting intermediate boson ($n = 1$) in which we obtain a cancelation of the factors $\frac{(q^+)^{n+1}}{(q^+)^m} = 1$. As a consequence, in this case we will have a nonvanishing contribution from the diagrams of antiparticles. Therefore, as we increase the number of photons (more energy input to the system) on the n bosons, we will encounter nonvanishing contributions from pair production diagrams in the limit $q^+ \to 0$.

We have demonstrated that the propagator of two fermions in a background field, has a non-vanishing contribution coming from the pair creation by the photon. In particular, in an example of bound state with constant vertex, we demonstrated that the J^- current component in the Breit's reference frame ($q^+ = 0$) has a non-zero contribution from the process of pair creation by the photon. This conclusion is reached as long as we first have q^+ different from zero, integrating in k^- and then taking the limit $q^+ \to 0$. The integration in k^- and the limit $q^+ \to 0$ does not commute in general.

4 Conclusion

We have demonstrated that the propagator of two fermions in a background field, has a non-vanishing contribution coming from the pair creation by the photon. In particular, in an example of bound state with constant vertex, we demonstrated that the J^- current component in the Breit's reference frame ($q^+ = 0$) has a non-zero contribution from the process of pair creation by the photon. This conclusion is reached as long as we first have q^+ different from zero, integrating in k^- and then taking the limit $q^+ \to 0$. The integration in k^- and the limit $q^+ \to 0$ does not commute in general.

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