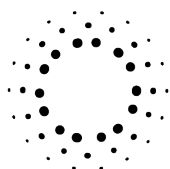


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MASTER'S DISSERTATION

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Complex Fermion Mass Poles in the Linear Sigma Model at Finite Temperature

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Advisor

Gastão Inácio Krein

Agosto de 2024

S586c	Silva, Alisson Matheus Araujo Complex fermion mass poles in the linear sigma model at finite temperature / Alisson Matheus Araujo Silva. – São Paulo, 2024 80 f.: il. (algumas color.) Dissertação (mestrado) – Universidade Estadual Paulista (Unesp), Instituto de Física Teórica (IFT), São Paulo Orientador: Gastão Inácio Krein 1. Cromodinâmica quântica. 2. Partículas (Física nuclear) - Quiralidade. 3. Teoria quântica de campos. I. Título
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Complex Fermion Mass Poles in the Linear Sigma Model at Finite Temperature

Dissertação de Mestrado apresentada
ao Instituto de Física Teórica do
Câmpus de São Paulo, da Universidade
Estadual Paulista “Júlio de Mesquita
Filho”, como parte dos requisitos para
obtenção do título Mestre em Física,
Especialidade Física Teórica.

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São Paulo, 30 de agosto de 2024.

*Dedico esta tese a
minha mãe, que sempre me apoiou.*

Agradecimentos

Agradeço minha família e meus amigos que me apoiaram durante o período da elaboração dessa dissertação. Em especial minha mãe que sempre me aconselhou da melhor forma possível, apesar de algumas preocupações excessivas características de mãe. Agradeço também meu orientador, Prof. Gastão Krein, pela paciência e disponibilidade que me permitiram crescer como pessoa e pesquisador. Por fim, agradeço ao CNPq pelo apoio financeiro, sem o qual esse trabalho não teria sido possível. Muito obrigado a todos.

*"The best way to explain it,
is to do it."*

Lewis Carroll, *Alice in Wonderland*

Resumo

Investigamos o aparecimento de polos de massa complexa não físicos no propagador dos férmions no modelo sigma linear (LSM) a temperatura finita. O LSM é uma teoria de campos efetiva para a quebra espontânea da simetria quiral (SCSB) na cromodinâmica quântica (QCD). Os graus de liberdade efetivos são descritos por campos de mésons escalares e pseudoescalares e campos de férmions de spin-1/2. Quando os férmions são núcleons, o LSM realiza SCSB na fase confinada da QCD e quando os férmions são quarks, ele realiza SCSB na fase desconfiada da QCD. A presença de pólos complexos nos propagadores leva a resultados não físicos em observáveis, como na equação de estado termodinâmica e coeficientes de transporte. Especificamente, quantificamos ao nível de um loop as contribuições térmicas para a função espectral da autoenergia do férmion e os polos complexos. Encontramos que embora a temperatura altere as propriedades espectrais dos propagadores, ela não elimina os polos, mas próximo à temperatura de restauração quiral há mudanças qualitativas significativas nessas quantidades. Concluimos com algumas considerações sobre as implicações de nossos resultados para a interpretação do aparecimento de polos de massa complexa em propagadores da QCD como sinalização de confinamento de quark-glúon.

Palavras Chaves: Cromodinâmica Quântica; Simetria Quiral; Quebra Espontânea de Simetria, Teorias Efetivas; Modelo Sigma-Linear .

Áreas do conhecimento: Física de Partículas; Física Nuclear; Teoria Quântica de Campos

Abstract

We investigate the appearance of unphysical complex-mass poles in the fermion propagator in the linear sigma model (LSM) at finite temperature. The LSM is an effective field theory for spontaneous chiral symmetry breaking (SCSB) in quantum chromodynamics (QCD). The effective degrees of freedom are described by scalar and pseudoscalar meson fields and spin-1/2 fermion fields. When the fermions are nucleons, the LSM model realizes SCSB in the confined phase of QCD and when the fermions are quarks, it realizes SCSB in the QCD deconfined phase. The presence of complex poles in the propagators leads to unphysical results in observables such as in the thermodynamic equation of state and transport coefficients. Specifically, we quantify at the one-loop level the thermal contributions to the fermion self-energy spectral function and on the complex poles. We found that although temperature changes the spectral properties of the propagators it does not eliminate the poles but close to the chiral restoration temperature there are significant qualitative changes in those quantities. We conclude with considerations about the implications of our results for the interpretation of the appearance of complex-mass poles in QCD propagators as signaling quark-gluon confinement.

Keywords: Quantum Chromodynamics; Chiral Symmetry; Spontaneous Symmetry Breaking; Effective Theories; Linear Sigma Model.

Areas of Knowledge: Particle Physics; Nuclear Physics; Quantum Field Theory.

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Chapter 1

Introduction

Quantum Chromodynamics (QCD) is the fundamental theory describing the strong interactions among quarks and gluons, the elementary constituents of hadrons such as protons and neutrons. The theory is governed by the non-Abelian gauge group $SU(3)$ and describes the dynamics of the strong force, which is responsible for binding quarks together to form protons, neutrons, and other hadrons. Unlike the electromagnetic force, which diminishes with increasing distance, the strong force exhibits a unique property known as confinement. Confinement refers to the phenomenon where quarks and gluons are perpetually bound within hadrons, preventing their isolation as free particles [1]. This aspect of QCD is not only central to our understanding of nuclear physics but also remains one of the most challenging aspects to study both theoretically and experimentally [1, 2, 3, 4].

The mathematical aspects of confinement are intricately linked to the unresolved question of proving the existence of a mass gap in Yang-Mills theory, a problem that stands at the frontier of both physics and mathematics. Specifically, it requires showing that in a non-Abelian gauge theory, such as $SU(3)$ Yang-Mills theory, there exists a finite energy gap between the vacuum state and the first excited state, which is a necessary condition for confinement. [5] The mass gap in Yang-Mills theory refers to the existence of a positive lower bound on the energy of any possible excitation above the vacuum state. In simpler terms, it implies that the lightest possible particle in the theory (in this case, a hypothetical glueball) would have a non-zero mass, creating a gap between the energy of the vacuum and the energy of this particle. This is a crucial aspect of confinement because if quarks or gluons could exist as free particles, there would be no such gap—massless or nearly massless particles would dominate, as is the case in QED with photons. The mass gap is essential for ensuring that quarks and gluons remain bound within hadrons. Despite the compelling physical evidence and numerical simulations supporting the existence of this gap, a rigorous mathematical proof within the framework of Yang-Mills theory remains one of the most significant open problems in theoretical physics [5, 6].

Another manifestation of confinement in QCD is the formation of confining flux tubes. When quarks are pulled apart, the strong force between them does not diminish as it does with other forces like electromagnetism. Instead, the field lines of the strong force, carried by gluons, become concentrated into a narrow tube or string between the quarks. This tube is known as a confining flux tube, and its tension is responsible for the increasing potential energy between quarks as they are separated [7]. If quarks are separated beyond a certain point, the energy in the flux tube becomes sufficient to create a new quark-antiquark pair, effectively preventing the isolation of individual quarks. This phenomenon is beautifully captured by lattice QCD simulations, where the flux tubes are directly observed, and the linear rise of potential with distance is clearly demonstrated [8, 9]. The study of these flux tubes is not only essential for understanding the mechanism of confinement but also provides insights into the behavior of QCD at different temperatures. As the temperature increases, the energy in the system can reach a point where these flux tubes cease to exist, leading to the deconfinement of quarks and gluons.

Exploring QCD under extreme conditions, such as at finite temperature, reveals many intriguing phenomena. At sufficiently high temperatures, quarks and gluons become deconfined, forming a quark-gluon plasma (QGP)—a state of matter that existed shortly after the Big Bang. A number of works have investigated this transition and its implications for understanding the early universe and the nature of phase transitions in the strong interaction regime. We refer the reader to the recent review [10] which provides a comprehensive overview of the current theoretical and experimental approaches to studying these effects.

Experimental efforts, particularly in heavy-ion collisions at facilities like the Large Hadron Collider (LHC) and the Relativistic Heavy Ion Collider (RHIC), have been instrumental in probing these extreme conditions. These experiments aim to recreate the environment of the early universe by colliding nuclei at near-light speeds, generating temperatures high enough to potentially produce the quark-gluon plasma (QGP). Observations from these collisions have provided valuable information on the properties of QGP, such as its near-perfect fluidity. [11, 12]. Furthermore, lattice QCD simulations and other theoretical approaches complement these experimental studies, offering predictions and a deeper understanding of the behavior of QCD under such extreme conditions and the nature of the transition between the confined and deconfined phases. Together, these experimental and theoretical investigations continue to refine our understanding

of the strong force and the early moments of our universe.

Understanding the transition from confinement to deconfinement, and the behavior of QCD at finite temperatures is therefore a key focus in high-energy physics, providing deep insights into the fundamental processes and the conditions that shaped the formation of matter as we know it today. A contemporary challenge for theorists, in addition to providing guidance to the experimental efforts, is to build on results from the first-principles lattice QCD numerical simulations to develop effective theories and models to understand how QCD works. The present dissertation builds on one piece of knowledge brought from lattice QCD on quark-gluon correlation functions, namely that quark and gluon propagators exhibit a violation of positivity of the associated spectral functions [13, 14, 15]. Such positivity violation has long been associated with the confinement of quarks and gluons [16, 17, 18]. Since the spectral function is related to the probability of measuring a determined state, the violation of positivity implies the existence of states of negative norm which lie outside the Hilbert space of the theory and are thus not associated with observable particles.

This dissertation is concerned with one aspect of the connection between positivity violation of spectral functions and confinement, namely if such violation is also manifest in effective models of QCD which lie outside of the confining regime. This was first shown, for example, in meson nuclear models [19] and pursued later on in [20, 21, 22]. In the work [22], in particular, the linear-sigma model (LSM), an effective field theory was investigated for describing pion nuclear processes (pN) and the spectral functions have been shown to exhibit complex-conjugated mass poles and violation of the positivity of spectral densities. The negativity that arises in this case is usually related to a pair of complex poles found in the propagator, which invalidate the Källén–Lehmann spectral representation, since such poles correspond to eigenstates of the system that have complex energy and probability, and are thus unphysical. Our main objective in this dissertation is to analyze whether such effects are maintained in the model at finite temperature and whether this carries some implications to the interpretation on the quark-gluon confinement phenomenon in QCD. Alternatively, it is equally important to consider whether the observed complex structure could represent an artifact of the approximation scheme employed. To address these issues, our strategy involves computing the one-loop contributions to the fermion propagator in the LSM fermion propagator. This approach, while not entirely novel, has a solid foundation, having been applied in similar contexts, such as the Walecka model [23]

and the quark-meson-coupling model [24]. These studies demonstrated the utility of employing nonperturbative effective masses within exchange self-energies, a technique often utilized in frameworks like optimized perturbation theory or the δ -expansion method [25]. However, none of these previous applications addressed the issue of complex mass poles at finite temperature, a question that remains mostly unexplored.

Our original contribution lies precisely in this area, filling a gap in the existing literature. By focusing on the fermion self-energy, both in vacuum and under finite-temperature conditions, we aim to elucidate the significance of these complex poles and their broader implications for the underlying physics of QCD. Our calculations are performed within the finite temperature field theory formalism in real time [26, 27, 28], which is the formalism required to calculate dynamical processes and transport properties of strongly interacting matter such as shear and bulk viscosities and thermal and electric conductivities [29, 30, 31]. Unlike the imaginary-time formalism, which is formulated in Euclidean space, the real-time formalism remains in Minkowski space, allowing direct access to time-dependent observables and real-time dynamics.

We carry out our study by inspecting the poles in the fermion propagator of the LSM at finite temperature. We employ two versions of the LSM, one in which the fermions are nucleons and the other in which they are quarks. By studying these two versions, we explore the model across different scales, from low to high temperatures. This will allow us to assess how temperature influences the existence and nature of the complex-mass poles and draw conclusions about, first, potential flaws in the interpretations about confinement in QCD and, second, on using the LSM to compute finite-temperature observables.

We structure the the dissertation as follows. After briefly reviewing the LSM and its origins in Chapter 2, we introduce the spectral representation of propagators in Chapter 3 and prove the important property that the physical poles of the propagator must lie in the real axis. In Chapter 4, we briefly review the real-time formalism of thermal field theory which we will be using to perform most of the calculations. In Chapter 5, we compute the one-loop corrections to nucleon and propagators first in vacuum and then at finite temperature, and present numerical results. Our conclusions and perspectives are presented in Chapter 6.

Chapter 6

Conclusions

We have investigated the effects of temperature on the pole structure of fermions within the linear sigma model, employing the real-time formalism and one-loop propagators. The main motivation for this work comes from the varied nature of pole interpretations. Although complex poles in QCD propagators are often associated with quark-gluon confinement in some theoretical contexts [18, 70, 71, 72], their appearance in models that do not relate to confinement phenomena [19, 57, 73, 21] raises questions about this interpretation. In these latter instances, the complex pole structure might be attributed as being an artifact of the approximation methods employed, rather than an indication of physical properties. Another interpretation [74] is that theories not asymptotically free, such as the LSM, are not nonperturbatively renormalizable, in the sense that quantum fluctuations at strong coupling destabilize the theory.

Given this understanding, our analysis emphasizes the need to discern the origins and implications of complex poles. In effective theories that are not asymptotically free, our results are significant because unphysical spectral properties ruin thermodynamic quantities, such as the equation of state and transport coefficients [29, 30]. By quantifying the influence of thermal corrections on the pole structure, we ensure a deeper and more accurate interpretation of the fermion dynamics within the linear sigma model at finite temperatures. Our findings reveal that the complex structure of the fermion propagator is maintained at finite temperature, reinforcing the need for further studies to settle the question whether the appearance of the complex poles is an artifact of the one-loop mean field approximation or is intrinsic to the model because it is not asymptotically free. Although the complex poles aren't present when one employs solely the one-loop Hartree approximation, they nevertheless become apparent when including one-loop exchange contributions to the fermion self-energy. There is no a priori reason to neglect exchange contributions and keep only the Hartree contribution since both are one-loop contributions. However, in some cases, such as in the large- N limit, where N refers to the number of colors, the Hartree contribution scales as

N , while the exchange contribution scales as $1/N$. This disparity in scaling could allow one to neglect the exchange diagrams in this limit, simplifying the analysis.

The appearance of complex poles become less problematic when considering the quark-level linear sigma model, as they appear at scales much higher than the validity of the model. This suggests that they might be neglected. Nevertheless, the presence of complex poles remains problematic and could potentially influence observable quantities evaluated at finite temperature, as there still remains the difficulty of how to control the ultraviolet of such nonasymptotically free theories.

More work in this investigation is needed to provide more numerical results to accurately characterize the pole structures. To address the limitations of the one-loop approximation, future research should incorporate higher-loop orders and explore self-consistent calculations using the Dyson-Schwinger equations [20]. Furthermore, from a phenomenological point of view, the inclusion of the vector meson ω in future studies involving nucleons could provide valuable information on the dynamics within the linear sigma model under thermal conditions, such as the potential violation of positivity in the spectral function. Such studies would help in refining our understanding of non-perturbative effects and provide a more robust framework for analyzing nucleon-meson dynamics at finite temperatures.

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