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CAMPUS DE ILHA SOLTEIRA

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EXTRAPOLATION OF AUTOREGRESSIVE MODEL FOR DAMAGE PROGRESSION ANALYSIS

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EXTRAPOLATION OF AUTOREGRESSIVE MODEL FOR DAMAGE PROGRESSION ANALYSIS

Dissertation presented to the Faculdade de Engenharia de Ilha Solteira - UNESP as a part of the requirements for obtaining the MSc in Mechanical Engineering.
Knowledge area: Solid Mechanics.

Advisor: Prof. Dr. Samuel da Silva

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TÍTULO DA DISSERTAÇÃO: Extrapolation of Autoregressive Model for Damage Progression Analysis

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Ilha Solteira, 16 de abril de 2019

To my parents, Neuza and Jorge, and my girlfriend, Carol.

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Resumo

O principal objetivo deste trabalho é usar métodos de extrapolação em coeficientes de modelos autorregressivos (AR), para fornecer informações futuras de condições de estruturas na existência de mecanismo de danos pré-definidos. Os modelos AR são estimados considerando a predição de um passo à frente, verificados e validados a partir de dados de vibração de uma estrutura na condição não danificada. Os erros de predição são usados para extrair um indicador para classificar a condição do sistema. Então, um novo modelo é identificado se qualquer variação de índices de dano ocorrer, e seus coeficientes são comparados com os do modelo de referência. A extrapolação dos coeficientes de AR é realizada através das splines cúbicas por partes que evitam possíveis instabilidades e alterações indesejáveis dos polinômios, obtendo aproximações adequadas através de polinômios de baixa ordem. Uma curva de tendência para o indicador capaz de predizer o comportamento futuro pode ser obtida a partir da extrapolação direta dos coeficientes. Uma estrutura de três andares com um para-choque e uma coluna de alumínio colocada no centro do último andar são analisados com diferentes cenários de dano para ilustrar a abordagem. Os resultados indicam a possibilidade de estimar a condição futura do sistema a partir dos dados de vibração nas condições de danos iniciais.

Palavras-chave: Monitoramento de integridade estrutural. Análise da progressão de dano. Modelos autorregressivos. Extrapolação dos coeficientes de modelos AR.

Abstract

The main purpose of this work is to apply extrapolation methods upon coefficients of autoregressive models (AR), to provide future condition information of structures in the existence of predefined damage mechanism. The AR models are estimated considering one-step-ahead prediction, verified and validated from vibration data of a structure in the undamaged condition. The prediction errors are used to extract an indicator to classify the system state condition. Then, a new model is identified if any variation of damage indices occurs, and its coefficients are compared to the ones from the reference model. The extrapolation of the AR coefficients is performed through the piecewise cubic splines that avoid possible instabilities and undesirable changes of the polynomials, obtaining suitable approximations through low-order polynomials. A trending curve for the indicator capable of predicting future behavior can be obtained from direct coefficient extrapolation. A benchmark of a three-story building structure with a bumper and an aluminum column placed on the center of the top floor is analyzed with different damage scenarios to illustrate the approach. The results indicate the feasibility of estimating the future system state from the vibration data in the initial damage conditions.

Keywords: Structural health monitoring. Damage progression analysis. Autoregressive models. Extrapolation of AR models coefficients.

List of Symbols

$A(q)$	-	AR model polynomial
a_{n_a}	-	AR model coefficient
$E_{[\%]}$	-	Relative percentage error of each index
$e(k)$	-	Noise signal
f_i	-	Cubic splines
H_0	-	Null hypothesis
H_1	-	Alternative hypothesis
n	-	Damage severity condition
n_a	-	AR model polynomial order
p	-	p-value
q^{-1}	-	Shift operator
x_n	-	Intervals of damage condition
$y(k)$	-	Measured output
$\hat{y}(k)$	-	System response obtained through the AR model

Greek Letters

α	-	Significance level
γ	-	Damage index
$\Gamma(.)$	-	Gamma function
λ	-	Smoothing parameter
$\sigma^2(.)$	-	Variance operator
ν	-	Degree of freedom
ψ	-	AR model coefficient extrapolated
ξ	-	Independent random error

List of Acronyms

AIC	-	Akaike Information Criterion
AR	-	Autoregressive model
FRF	-	Frequency Response Function
PDF	-	Probability Density Function
ROC	-	Receiver Operating Characteristic
RUL	-	Remaining useful life
SHM	-	Structural health monitoring

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1 Introduction

This chapter presents the introduction of the subject of the dissertation. The motivation of this research and its main objectives are presented in sections 1.1 and 1.2, respectively. Finally, section 1.3 highlights the outline of the dissertation presenting the main topics.

1.1 Motivation

Damage identification algorithms for the analysis of structural integrity are currently defined as Structural Health Monitoring (SHM) (SBARUFATTI; MANES; GIGLIO, 2014). The continuous evaluation of structural integrity under different environmental and operational conditions, and the determination of damage prior to a critical stage allow the reduction of maintenance costs and ensures structural safety (WENTAO *et al.*, 2018; TIBADUIZA; MUJICA; RODELLAR, 2013). The process of damage identification is classified into some steps defined as detection, diagnosis, and prognosis. First, the damage detection investigates the existence of damage based on the analysis of the damage-sensitive features, which are characterized by the physical properties of mass, stiffness, and damping, that are capable of describing changes in the dynamics of the system (LIU; AKIRA; JIN, 2015; HOELL; OMENZETTER, 2016). Then, the diagnosis is performed, where the localization, type of damage, and severity are determined. Finally, the prognosis is carried out, normally using physical models, in an attempt to evaluate the effects of future states, forecasting its response based on past/current condition to estimate the Remaining Useful Life (RUL) of the system (FARRAR; LIEVEN, 2007; SHANKAR; SANKARAN, 2013).

In the last decades, data-based methods received significant attention in the fields of mechanical, civil, and aeronautical engineering, and their application is extensively found in the literature (RÈBILLAT; HAJRYA; MECHBAL, 2014; SANTOS *et al.*, 2017). Data-based methods overcome the drawbacks of physics-based methods by not requiring the knowledge of the physical phenomena involved and allow the analysis of complex structures dynamics, which may not be feasible through physics-based methods in some cases (FIGUEIREDO *et al.*, 2012). The structural analysis is based on the application of pattern recognition methods to damage indices, and the employment of these methods depends on the purpose of the damage identification analysis (WORDEN *et al.*, 2015). The AR model and their variations (DIONISIO; DANIELE; MATTEO, 2012; SILVA, 2018), Volterra series (SHIKI; SILVA; TODD, 2017; VILLANI; SILVA; CUNHA, 2019), Mahalanobis distance (FIGUEIREDO *et al.*, 2010; YEAGER *et al.*, 2019), and machine learning algorithms (SANTOS *et al.*, 2017; SILVA *et al.*, 2019) are some examples of methods applied in SHM.

This dissertation proposes the application of the well-known input/output data-based mathematical model, the AR model. The structural condition evaluation is performed through the analysis of the model employed with the investigated data, where it is assumed that the structure and the identified AR model present a linear behavior in the undamaged condition, also denominated healthy condition. However, the presence of damage changes the behavior of the estimated AR model, resulting in an inadequate model fitting (BORNN *et al.*, 2016). Several works involving AR models were developed mainly on damage detection and diagnosis. The application of the AR model for damage detection was proposed by Yao and Pakzad (2012), where the vibration data of a mass-spring-damper model and laboratory experiments were investigated. The authors investigated the pattern of two damage indices, defined by the residuals correlation and the model spectrum defined by the AR coefficients. The results presented more accurate performance when compared to traditional methods.

Entezami and Shariatmadar (2018) employed the AR model to focus on localization and severity analysis of a three-story building structure and a four-story steel structure. The importance of defining an appropriate model order was emphasized, and it was determined by the correlation analysis of the AR model residuals through the Ljung-Box Q-test. The proposed damage indices, defined by the linearity coefficients of the coefficients and the statistical moments of the model residuals, presented amplitude variations according to the presence of the damage and positions in relation to the sensor arrays, allowing the damage detection and localization. Nardi *et al.* (2016) proposed the application of an AR model for damage detection in composite laminated plates, where low-velocity impacts caused delamination in the structure. The measured responses of the structure were applied for the AR model identification in the undamaged condition, and the application of Linear Discriminant Analysis with the AR model coefficients was proposed for structural condition classification, in which the AR parameters were projected to a new sub-space where the same condition groups present low dispersion.

The present work aims to go beyond the detection and diagnosis levels, performing the prediction of structural behavior based on the application of piecewise cubic splines, even though the complete prognosis is not performed. The proposed methodology is attractive as a consequence of the application of a straightforward and innovative approach, which does not require knowledge of physics-based models as proposed by Corbetta *et al.* (2017). Four different physical models were applied to composite materials and compared their performances with experimental data of run-to-failure experiments, or complex methods with high computational cost as presented by Giagopoulos *et al.* (2018), who estimates the fatigue crack in a linear steel substructure through a Finite Element model.

The cubic spline interpolation represents a simple and efficient approach used in the numerical analysis of data sets for curve and surface fitting (SARFRAZ; HUSSAIN; NISAR, 2010). The opportunity to employ low-order polynomial functions at intervals between data points to approximate a function is convenient, and avoids possible undesirable oscillations obtained by single high-order polynomials (KNOTT, 2012). Rucka and Wilde (2006) took advantage of the cubic splines stability and extrapolated the original data sets, maintaining the smoothness of the signals and avoiding possible inconsistencies in their boundaries. Noel *et al.* (2014) highlighted the efficiency of the application of piecewise cubic splines to determine the stiffness and damping factor even in a nonlinear system, proving to be superior when compared to ordinary polynomials.

Wang, Liang and Xiang (2014) investigated wind turbine blades with damage at different positions through the Finite Element method, and the cubic splines were applied to approximate the curvatures of the mode shapes. The difference between the curvatures of the mode shape under different structural conditions was applied for the detection and localization, and the severity analysis was related to the increase of the differences of these curvatures. Dilena, Limongelli and Morassi (2015) proposed a similar approach for the analysis of a concrete bridge, investigating the divergences between curves. The authors proposed the application of a non-parametric method based on the interpolation errors of the Frequency Response Functions (FRFs) obtained by the cubic splines and the vibration data. The methodology was able to perform the damage detection and localization based on the significant variation between the FRFs obtained by the sensor arrays distributed by the structure.

In the present dissertation, the vibration responses of a three-story building structure for AR model extrapolation are investigated to predict its structural behavior. At first, the damage detection is addressed considering damage indices defined by the prediction errors and, as long as the structural damage is detected, new AR models must be estimated to represent the system dynamics. The main novelty of this dissertation is to benefit from the coefficients of these models, extrapolating them to more severe conditions before the occurrence of possible future damage. The AR models are identified through the responses measured under predefined damage conditions, and their coefficients are extrapolated by the piecewise cubic splines to more severe damage conditions. A convergence criterion is defined based on the damage indices obtained by the extrapolated model to validate the methodology, comparing the performance between the cubic spline, linear, and quadratic polynomial functions.

1.2 Objectives

The main objective of this work is to propose the application of classical statistical tools for the extrapolation of AR models coefficients, to more severe predefined damage conditions before they occur and, therefore, to predict future structural conditions considering the existence of predefined damage.

1.3 Outline

This dissertation is structured into the following chapters:

- **Chapter 1 - Introduction:** The motivation of the research and the main objectives;
- **Chapter 2 - Description of Autoregressive Models for Damage Detection and Extrapolation Method:** This chapter presents a description of the proposed methodology and the major contribution of this dissertation. First, a brief review of the proposed method is described. Then, the theoretical overview of AR models for damage detection step is discussed. Finally, it is presented the novelty methodology based on extrapolation of AR coefficients;
- **Chapter 3 - Application of methodology:** The description of the three-story building structure characteristics and the data acquisition procedure are presented. Then, the application of the proposed methodology in chapter 2 for the damage detection and extrapolation, and the main results are described in this chapter;
- **Chapter 4 - Final Remarks:** Exhibition of the main conclusions related to the results obtained and discussion about the future works and the continuity of the research.

2 Description of Autoregressive Models for Damage Detection and Extrapolation Method

This chapter describes a general description of the methodology applied to extrapolate the AR parameters in section 2.1, and a flowchart explaining the applied methodology is illustrated. Section 2.2 describes the damage detection step based on the damage indices defined by the prediction errors. The classification of the structural condition is performed using a hypothesis test defined according to the probability distribution of the damage indices. Section 2.3 brings the mathematical explanation regarding the extrapolation, to predict the future behavior of the structure considering the damage progression.

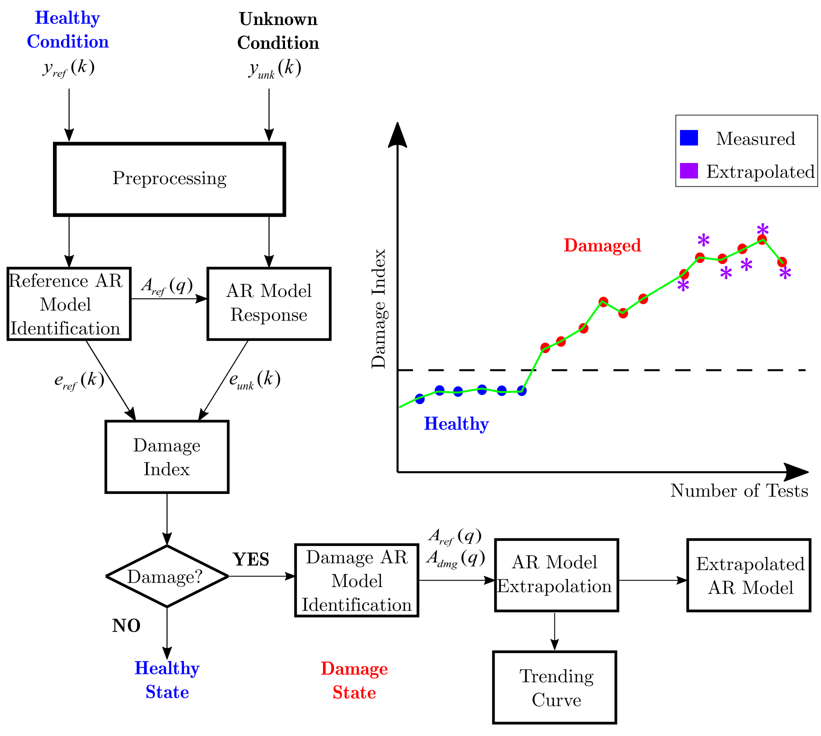
2.1 General Description of the Proposed Methodology

The methodology proposed in this dissertation is divided into two steps. First, the damage detection is performed, and then the AR model coefficients are extrapolated. The methodology proposed in this dissertation is shown in Figure 1. All data sets from each accelerometer are filtered in the preprocessing step before the analysis, avoiding interference from possible noise during laboratory tests. The data sets in the healthy condition are considered as reference data, and the data sets under unknown conditions are used as test data for the damage detection step.

The reference AR model is estimated upon the reference data for further comparison, and the adequate model order is defined according to a technique that analyzes the performance of linear models. The model is verified and validated through the non-correlation between the residuals analysis. The damage index considered in this work is defined by the relationship between the prediction errors determined based on the reference AR model estimated, assuming the undamaged and unknown conditions. A hypothesis test is applied according to the probability distribution of the damage indices for structural integrity classification, ensuring statistical reliability. This test analyzes the modification in the probability distribution of the damage indices according to a defined significance level. If the damage is determined, a new AR model must be identified as a consequence of the current model losing its representation in describing the system behavior. Otherwise, a new test data set is used again with the reference AR model until the damage is detected. It is considered that the new AR model uses the same order of the reference model, to facilitate the comparison of performance between the models, and for later extrapolation of its coefficients. However, this hypothesis may not always be correct due to the type of damage that the structure is subjected.

After the detection stage, the extrapolation is performed. The proposed methodology is based on the extrapolation of AR model coefficients to more severe conditions through cubic splines method. It is relevant to affirm that it is considered a single source of damage in the structure, which enables the proper application of the method, and the average coefficient on each condition analyzed for the extrapolation. First, the AR models of the same order are estimated at initial predefined damage conditions. The piecewise cubic splines are defined in each AR coefficients investigated, and the curves are extrapolated to more severe scenarios. The proposed methodology is attractive, since more information on the structural states can be applied for future extrapolations, and a better trending curve fitting can be achieved. Finally, the damage detection performance of the AR models obtained through the measured signals in the most severe state, and by different extrapolation methods are compared through a convergence criterion, to validate the proposed methodology.

Figure 1 – Flowchart of the proposed methodology.



Source: Prepared by the author.

2.2 Damage Detection

The AR models are a powerful tool for system identification methodology to describe structural behavior through its vibrational responses. These linear parametric models associate the measured output $y(k)$ with an n-th order polynomial and noise $e(k)$, to describe the system dynamics. The AR models can be written as (LJUNG, 1999):

$$A(q)y(k) = e(k) \tag{1}$$

where the polynomial $A(q)$, with order n_a and coefficients $a_1, \ldots, a_{n_a}$, is described as a function of the shift operator q^{-1}, in such a way that $y(k)q^{-1} = y(k-1)$. In this work, the Least Squares Method is applied for the determination of the model coefficients. In a more formal approach, the polynomial $A(q)$ can be defined as:

$$A(q) = \sum_{i=0}^{n_a} a_i q^{-i} \tag{2}$$

The AR models are identified, verified, and validated through the measured vibration data. The adequate order definition is important to represent the structural dynamics as described by Figueiredo *et al.* (2011). The classic Akaike Information Criterion (AIC) is employed, and it analyzes the fitting accuracy of the identified statistical model in relation to the number of parameters used. It is important to highlight the significance of the correct order definition. If a lower order than the required one is selected, the model may not be able to represent the system dynamics. Otherwise, the overfitting is achieved, resulting in the particularization of the model application.

After the AR model identification, the damage detection is performed. There are two different approaches for structural analysis through AR models regarding the AR parameters and residual errors (SHIN *et al.*, 2012). The first approach uses AR coefficients as damage-sensitive features. Lu and Gao (2005) considered that the coefficients are directly related to the dynamic properties of the structure, resulting in parameters that are sensitive to the presence of damage. Meanwhile, the second approach assumes that the damage results in the modification of the structural responses, and impairs the performance of the reference model in an accurate representation of the system. In this dissertation, the prediction errors approach is employed considering the one-step-ahead prediction, where the observations related to the structure responses are considered to be known (BILLINGS, 2013). The prediction errors can be described as:

$$e_{ref}(k) = y_{ref}(k) - \hat{y}(k) \tag{3}$$

$$e_{unk}(k) = y_{unk}(k) - \hat{y}(k) \tag{4}$$

where y_{ref} is the measured system response when the structure is operating in its reference condition, y_{unk} is the system response measured when the structure is in an unknown condition and $\hat{y}$ is the system response obtained through the AR model identified with the structure in the healthy condition.

The damage index employed in this dissertation was proposed by Sohn and Farrar (2001), considering the ratio of the variance between the prediction errors in the reference and unknown conditions. The authors applied a hypothesis test defined according to the $\mathcal{F}$-distribution achieved by the damage index, ensuring statistical reliability in the structural analysis of an eight-degrees of freedom mass-spring system. The damage index can be determined as:

$$\gamma = \frac{\sigma^2(e_{unk})}{\sigma^2(e_{ref})} \tag{5}$$

where γ is the index calculated and $\sigma^2(.)$ is the variance operator. The hypothesis test investigates the probability distribution behavior of the damage index, described by the null H_0 and alternative H_1 hypotheses, according to the defined significance level α. It is assumed that the residuals in the reference condition have Gaussian distribution and, consequently, a ratio between two variables with Gaussian distribution is achieved, representing an $\mathcal{F}$-distribution. Meanwhile, the presence of structural damage is considered at the moment when the damage index no longer has an $\mathcal{F}$-distribution, as a consequence of the prediction errors does not exhibit Gaussian distribution in the unknown condition (KOPSAFTOPOULOS; FASSOIS, 2010). The hypothesis test can be described as:

$$\begin{cases} H_0 : \gamma \sim \mathcal{F}\text{-distribution (healthy) - Null hypothesis} \\ H_1 : \gamma \nsim \mathcal{F}\text{-distribution (damaged) - Alternative hypothesis} \end{cases} \tag{6}$$

For the classification of the structural condition, the damage index must be investigated to ensure if it belongs to the $\mathcal{F}$-distribution. A convenient alternative to confirm this possibility is through the determination of the p-value defined by the integration of the Probability Density Function (PDF) (BENDAT; PIERSOL, 2011). So, the p-value defined can be expressed as:

$$p(\gamma \mid \nu_1, \nu_2) = 1 - \int \frac{\gamma^{\nu_1/2-1}(\nu_1/\nu_2)^{\nu_1/2}\Gamma[(\nu_1+\nu_2)/2]}{\Gamma(\nu_1/2)\Gamma(\nu_2/2)[1+\gamma(\nu_1/\nu_2)]^{(\nu_1+\nu_2)/2}} d\gamma \tag{7}$$

where ν_1 and ν_2 are the degrees of freedom and $\Gamma(.)$ is the Gamma function. For the right-tailed hypothesis test applied in this dissertation, the p-value should be compared to the significance level α, to classify the structural condition through the null H_0 and alternative hypotheses H_1, in such way that:

$$\begin{cases} H_0 : p > \alpha \text{ (healthy) - Null hypothesis} \\ H_1 : p \leq \alpha \text{ (damaged) - Alternative hypothesis} \end{cases} \tag{8}$$

where the null hypothesis H_0 affirm that is more likely to have a damage index γ in the healthy condition, resulting in no structural condition modification. Otherwise, the alternative hypothesis H_1 is assumed as a consequence of the presence of structural damage.

2.3 Extrapolation of Coefficients

The polynomial interpolation represents an attractive numerical method, as a consequence of the simplicity of its manipulation and characteristics that facilitate the analysis. The interpolation is often applied for the approximation of complex functions into simpler ones, and for prediction through extrapolations as it can be seen in (MCCARTHY; O'HIGGINS; FRIZZELL, 2010; BI; GENG; ZHANG, 2013; ULRIKSEN *et al.*, 2016). However, this method presents some issues related to the instabilities resulting from single high-order polynomials fitting large data, and modification in the polynomials due to changes in the data (ASCHER; GREIF, 2011).

An alternative to overcome these difficulties is to segment the analyzed intervals into smaller fractions and determine their respective polynomials. These polynomials are called the piecewise polynomials and, in this dissertation, the cubic splines are investigated. The possibility of obtaining approximations through low-order polynomial functions, such as first, second, and third order polynomials, results in the reduction of computational cost and avoid function instabilities (KREYSZIG, 2007). The cubic splines must satisfy some conditions, assuming a continuous aspect of merging points and also ensuring the continuity of the first and second derivatives (WOLBERG; ALFY, 2002). In this work, the piecewise cubic splines are employed at each coefficient of the AR model ψ_j associated with their respective n damage severity condition x_j, where $j = 1, 2, \ldots, n$, and the polynomials are expanded to determine future damage severity states by the following equation (WANG, 2011):

$$\psi_j(x) = f_j(x) + \xi_j \tag{9}$$

where the cubic splines $f_j(x)$ are defined in the damage intervals investigated and present an independent random error ξ_j. The determination of the cubic splines can be performed by minimizing a penalized criterion, in such a way:

$$\frac{1}{n}\sum_{j=1}^{n}(\psi_j - f_j(x))^2 + \lambda \int (\frac{\partial^2 f}{\partial x^2})dx \tag{10}$$

where λ represents the smoothing parameter, which correlates the curve fitting and the polynomial function obtained directly from the data. The main novelty of this dissertation is based on the premise that the cubic splines estimated at initial damage conditions can be extrapolated to more severe scenarios. The new information of the structure improves the curve fitting of the defined function, mitigating the errors resulting from the extrapolation, and improving the prediction. It is relevant to mention that the extrapolation is feasible under the condition that the progression of the damage severity occurs smoothly in relation to the undamaged condition.

At first, all measured data sets of the experiment are used for the identification of AR models with the same order. According to the number of parameters estimated, each average AR coefficients are determined and the cubic splines are defined, considering each of the damage intervals investigated. Then, the given function is extrapolated to more severe predefined damage conditions and to predict the system behavior. In this dissertation, the abscissa axis defines the damage conditions while the average AR coefficients define their respective amplitudes. An AR model is determined with the average AR coefficients extrapolated by the cubic splines in the most severe condition, and the detection step is performed to validate the proposed methodology. The efficiency of the extrapolation is analyzed through the convergence criterion determined by the performance of linear, quadratic and cubic polynomials for extrapolation. The damage indices determined by the model estimated under the most severe condition, γ_{meas}, and by the extrapolation, γ_{ext}, are compared by equation 11, considering the relative percentage error of each index.

$$E_{[\%]} = \sum_{i=1}^{n} \frac{|\gamma_{\text{ext}}(i) - \gamma_{\text{meas}}(i)|}{\gamma_{\text{meas}}(i)} \times 100 \tag{11}$$

2.4 Conclusions

This chapter presented the proposed methodology in this work for the extrapolation of AR model coefficients. A flowchart highlighted the proposed methodology considering the detection and extrapolation steps applied for the structural condition analysis. A more formal description of the detection stage was shown, investigating the AR model characteristics, the defined damage index and the hypothesis test determined. Finally, the extrapolation methodology based on the piecewise cubic splines and the convergence criterion for the performance analysis were presented.

3 Application of Methodology

This chapter presents the application of the proposed methodology. The description of the three-story building structure investigated in section 3.1. The damage detection through AR models identified using vibration data of the structure is performed in section 3.2. Finally, section 3.3 describes the application of the cubic splines method for the extrapolation of the AR model coefficients.

3.1 Experimental Setup

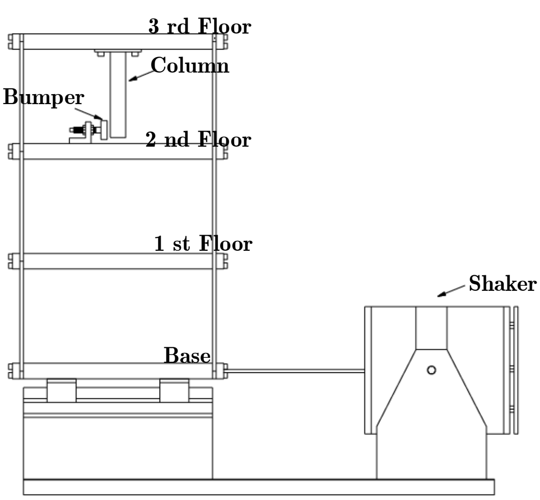
The three-story building structure investigated is shown in Figure 2. The behavior of this structure was extensively investigated in SHM applications by several researchers as seen in (BORNN; FARRAR; PARK, 2010; FIGUEIREDO *et al.*, 2011). The structure is composed of aluminum plates (305 x 305 x 25 mm^3) and columns (177 x 25 x 6 mm^3) assembled with bolted joints on each floor. The building is positioned on rails that limits its movement in x-direction only. The benchmark data and information can be accessed on the Los Alamos National Laboratory website[1].

An aluminum center column (150 x 25 x 25 mm^3) and a bumper placed on the top floor are used to simulate damage, by varying the gap distance between the components. First, the undamaged condition of the structure, denominated as reference condition, is analyzed. The damaged conditions, introduced by the gap distance, are progressively varied from 0.20 mm gap to more severe conditions with 0.15 mm, 0.13 mm and 0.10 mm gap. From this moment on the dissertation, the damage conditions will be denominated Damage I, II, III, and IV, respectively.

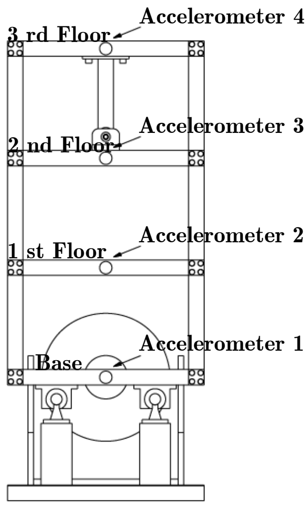
The data acquisition is performed through an electromagnetic shaker that excites the base of the structure using signals with random characteristics and band-limited frequency of 20-150 Hz. Four accelerometers are placed on the opposite side of the excitation on each floor to measure their responses, as shown in 3(b). In each condition, nine tests are performed considering a sampling frequency of 322.58 Hz and 8192 samples. The signals are segmented into eight sections with 1024 samples under the investigated conditions, due to the small amount of data. Thus, a total of 72 measurements are obtained.

[1] `http://www.lanl.gov/projects/national-security-education-center/engineering/ei-software-download/index.php`

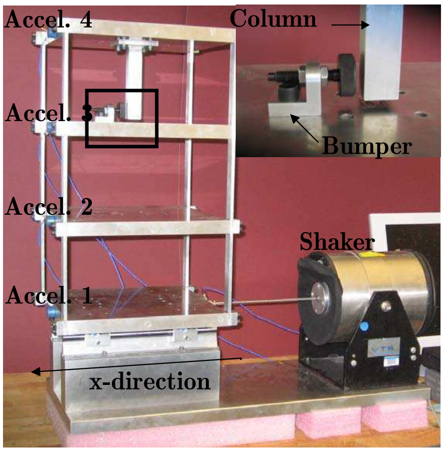
Figure 2 – Laboratory test structure.



(a) Modified schematic drawing of the structure - side view.



(b) Modified schematic drawing of the structure - front view.

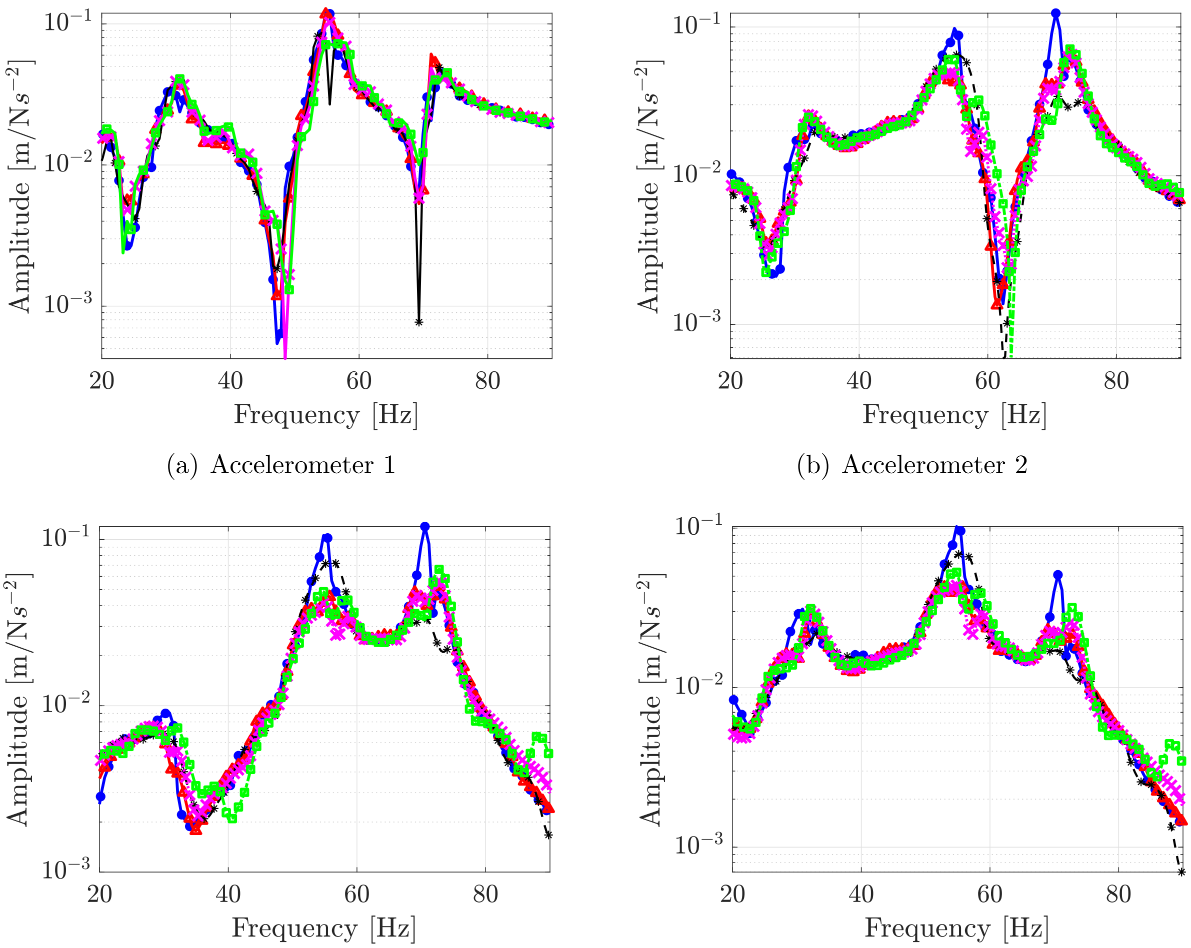


(c) Three-story building structure.

Source: Modified from Los Alamos National Laboratory website[1] (2007).

Figure 3 shows an example of FRFs analyzed each sensor over a frequency range of 20-90 Hz. It is evident the influence of damage introduced into the system due to the changes in the FRFs peaks close to 32 Hz, 55 Hz, and 71 Hz as it becomes more severe. The FRFs of accelerometers 3 and 4 reveal some differences related to the damage components localization and the sensor, resulting in a proper performance in the detection step. There is some evidence of the presence of another mode at frequencies lower than 20 Hz, which is neglected during the data acquisition procedure by the filter used, then, the existence of this mode cannot be defined.

Figure 3 – FRFs on each accelerometer. -•- represents the reference condition, -▲- 0.20 mm gap, -∗- 0.15 mm gap, -×- 0.13 mm gap, -■- 0.10 mm gap



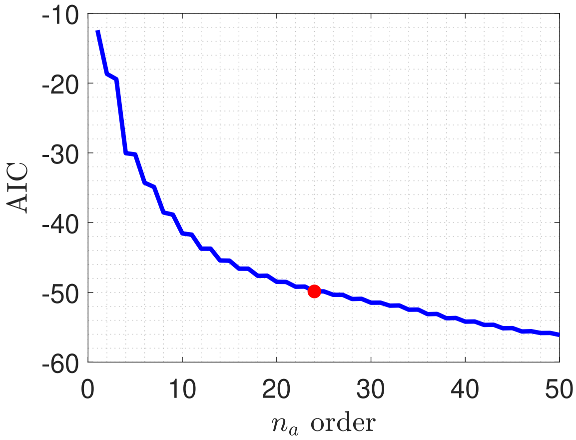
(a) Accelerometer 1 (b) Accelerometer 2 (c) Accelerometer 3 (d) Acelerometer 4

Source: Prepared by the author.

3.2 Damage Detection using AR model

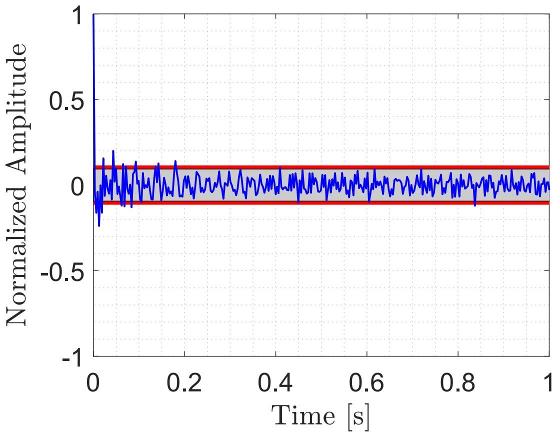
At first, the reference AR model is identified through signals from the reference data and, the determination of the polynomial order as 24 for n_a is done through AIC, as shown in Figure 4. The model validation is achieved through the autocorrelation functions of the residuals using another signal from the reference data, although different from the ones used during the identification procedure. Figure 5 illustrates the autocorrelation functions obtained from all accelerometers. The convergence of residuals within 99% of the confidence bounds results in the ability of the identified model to represent the features of the structural vibrational responses.

Figure 4 – Model order definition.

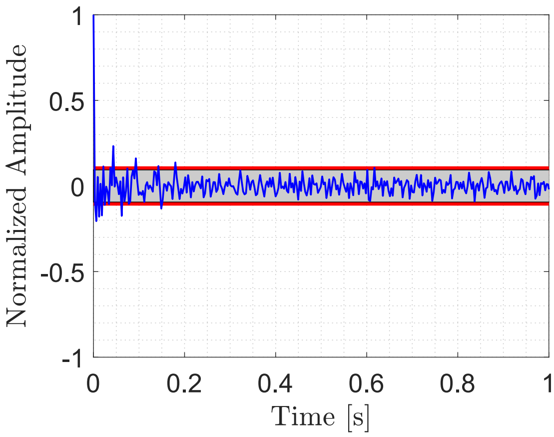


Source: Prepared by the author.

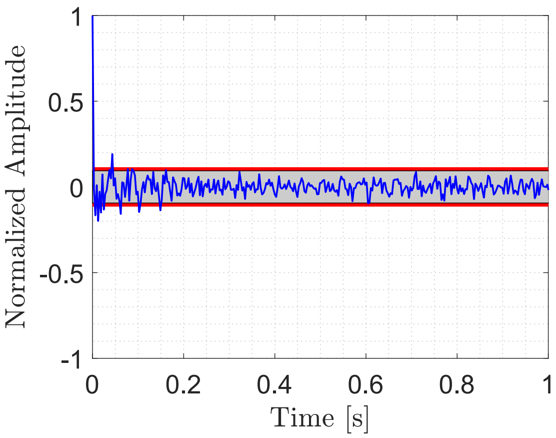
Figure 5 – Model validation. — represents the Autocorrelation function and — confidence interval bounds.



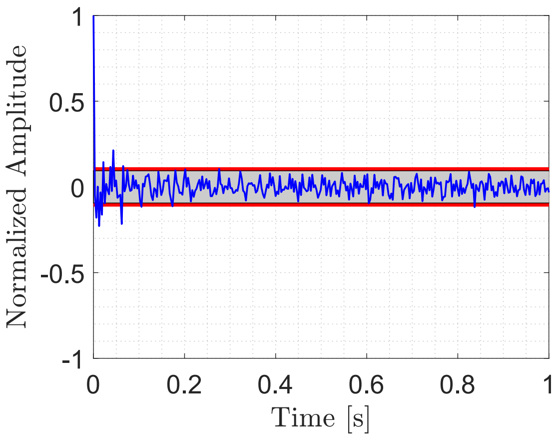
(a) Autocorrelation function of accelerometer 1



(b) Autocorrelation function of accelerometer 2



(c) Autocorrelation function of accelerometer 3

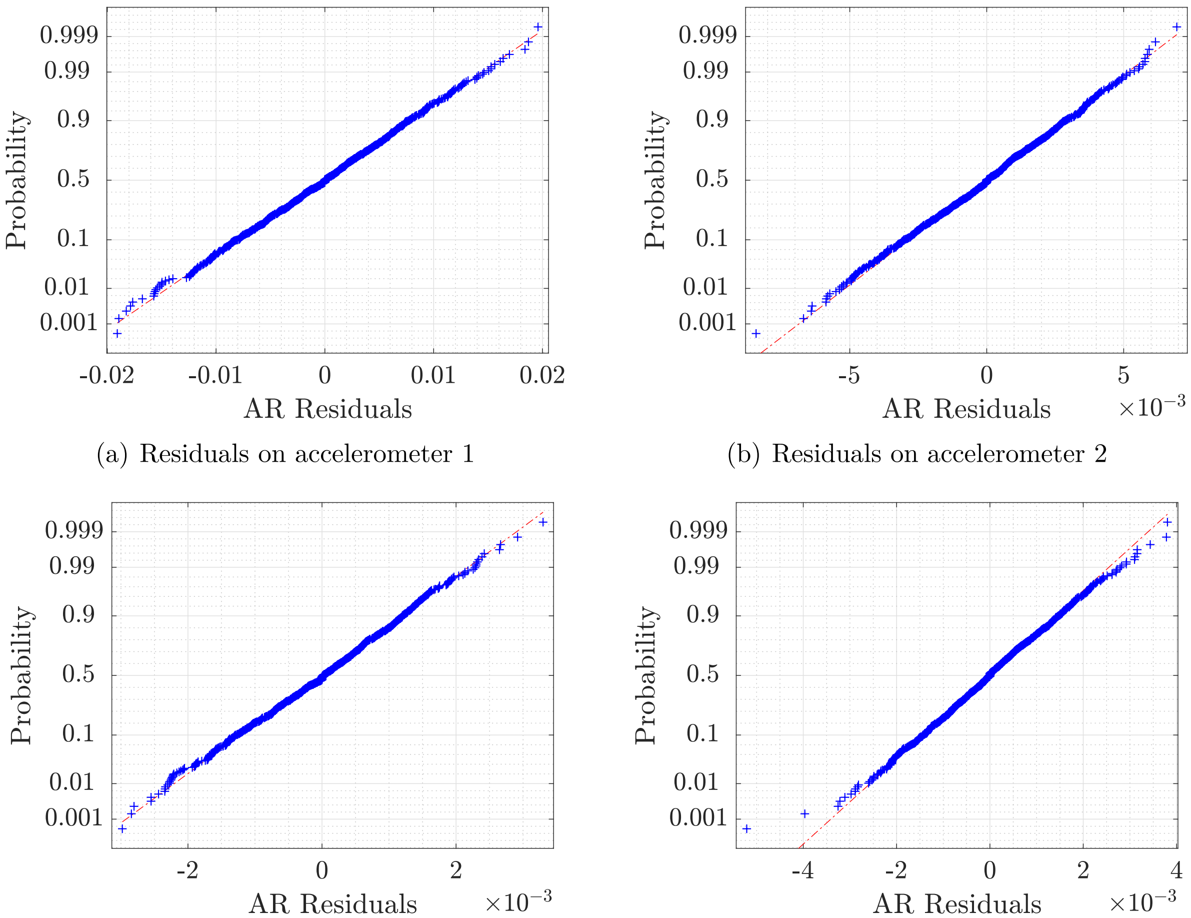


(d) Autocorrelation function of accelerometer 4

Source: Prepared by the author.

The probability distribution of the residuals on each accelerometer is shown in Figure 6. The residuals of the AR models have characteristics of Gaussian distributions in all accelerometers, which indicates that the defined AR model order is appropriate. Then, the proposed damage detection based on $\mathcal{F}$-distribution can be considered adequate for the analysis of the damage indices, γ, defined by equation 5. So, with the reference AR model identified, verified, and validated, and the metric for the damage detection step defined, it is possible to monitor the structural condition of the three-story building structure.

Figure 6 – Gaussian distribution of AR model residuals on each accelerometer in the healthy condition.



(a) Residuals on accelerometer 1

(b) Residuals on accelerometer 2

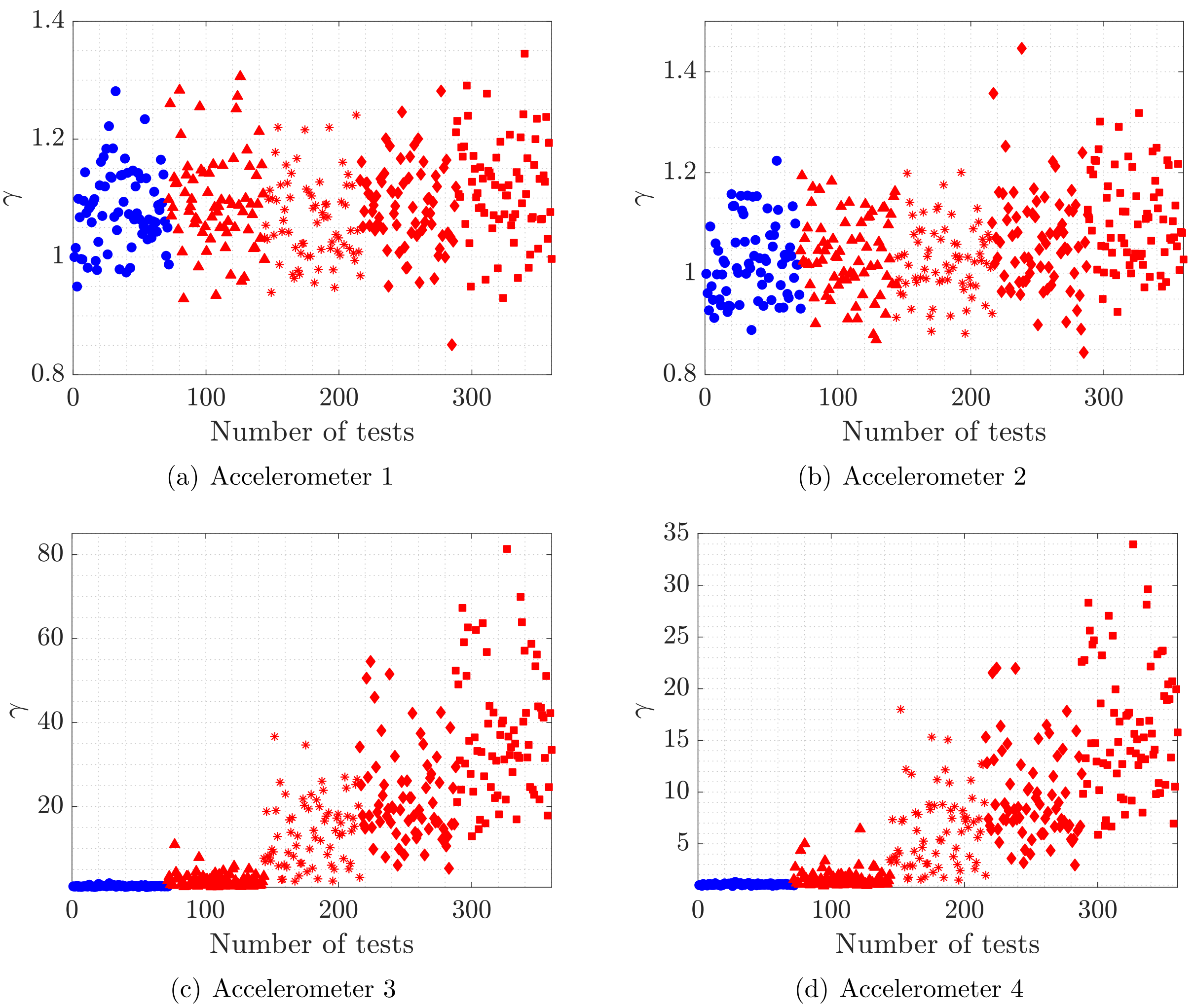
(c) Residuals on accelerometer 3

(d) Residuals on accelerometer 4

Source: Prepared by the author.

Figure 7 presents the damage-sensitive indices on each accelerometer, considering all investigated damage conditions. The prediction errors of the AR model tend to present higher amplitudes and higher dispersions as long as the model no longer represents the system properly. For this reason, the damage index presents higher values as the damage increases in severity. The defined damage index accentuates the model deficiency because of the variance of the prediction errors in the reference condition, which have minimum dispersion as a consequence of the adequate model fitting, is in its denominator.

Figure 7 – Damage-sensitive indices γ for all condition on each accelerometer. • represents the Reference condition, ▲ Damage I, ∗ Damage II, ♦ Damage III, ■ Damage IV.



Source: Prepared by the author.

Finally, the hypothesis test is applied considering a level of significance α of 1%, and the respective detection percentages obtained on each accelerometer is shown in Table 1. The accelerometers 3 and 4 show significant results in the damage detection step, presenting high percentages of true detection in the conditions in which the damage exist. In contrast, it cannot be stated in the same way for the accelerometers 1 and 2, positioned at the base and on the first floor of the structure, respectively. One possible explanation for this inability may be associated with the distance between the source of damage and the accelerometers. This fact is relevant due to the opportunity of performing damage localization in future analysis. It is important to mention the absence of false-alarm indicators, confirming the reliability of the estimated model and the proposed methodology.

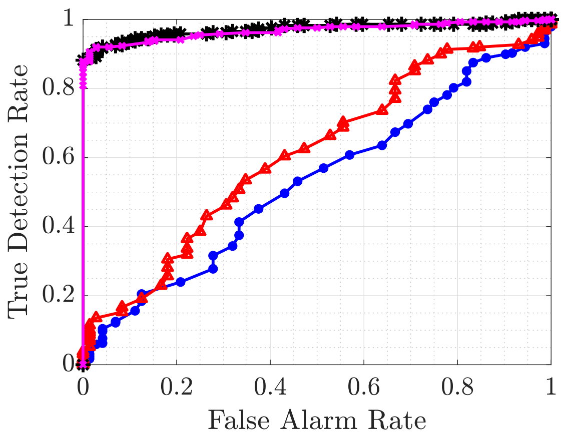
Table 1 – Percentage of detection of hypothesis testing for each accelerometer.

	False alarm [%]				True detection [%]			
Accelerometer	I	II	III	IV	I	II	III	IV
1	0	0	0	0	0	0	0	0
2	0	0	0	0	0	0	0	0
3	0	0	0	0	52.78	100	100	100
4	0	0	0	0	29.17	94.44	100	100

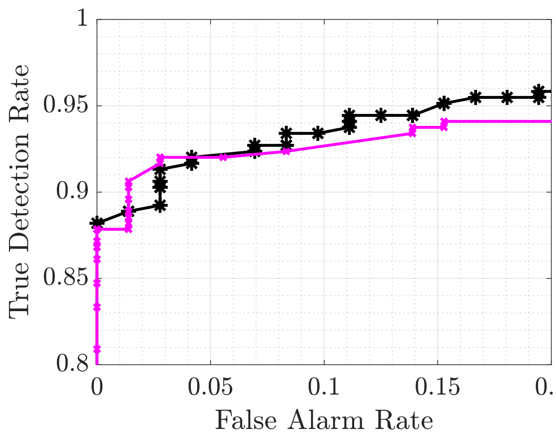
Source: Prepared by the author.

Figure 8 illustrates the receiver operating characteristic (ROC) curves to compare the efficiency in damage detection in a conventional approach. The ROC curves correlate the false-alarm and true detection rates, which is performed correctly by the identified model. Models with better performance in damage detection have ROC curves closer to the point (0.1), providing higher true detection and lower false-alarm rates. As expected, the accelerometers 1 and 2 present poor results due to the high false-positive rate. Meanwhile, the results of the accelerometers 3 and 4 through the conventional approach show ROC curves with significant true detection indices, confirming their ability to detect the existence of damage. Again, the requirement of identifying a new AR model in this new condition occurs from the moment the damage in the structure is detected.

Figure 8 – Receiver Operating Characteristic (ROC) curves on each accelerometer. -•- represents the accelerometer 1, -▲- accelerometer 2, -∗- accelerometer 3 and -×- accelerometer 4.



(a) ROC curves for all accelerometer



(b) ROC curves zoom

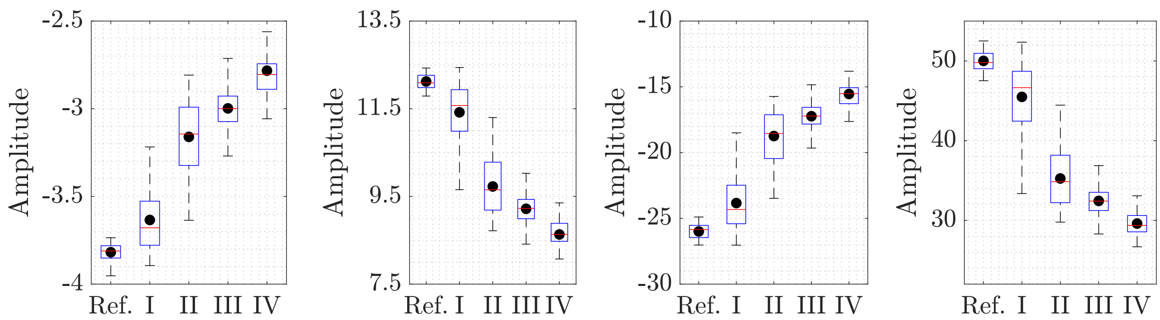
Source: Prepared by the author.

3.3 Extrapolation of AR model coefficients

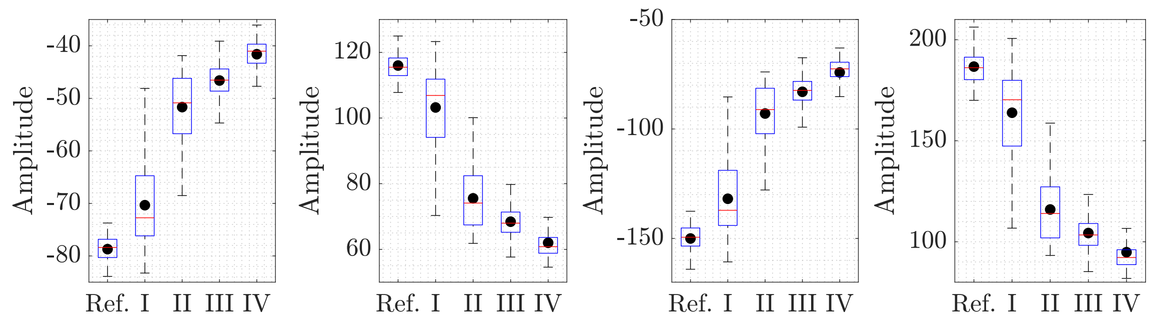
From this moment on the dissertation, the analysis will only be performed for accelerometers 3 and 4 because of their ability to detect damage. The Reference and Damage I states are extrapolated to the Damage II. The structural information obtained in the previous step is used alongside with further information, for the extrapolation until the Damage IV. Figure 9 shows the extrapolation of AR coefficients performed on the accelerometer 4, because of its distance to the damage mechanism. The boxplots represent the variation of the AR parameters estimated through the measured data of the structure, and to analyze the coefficients variation obtained via extrapolation.

The extrapolation method of the AR model coefficients shows satisfactory results, because the average coefficient extrapolated on each condition, present values within the expected interval of these parameters obtained traditionally. The results can be considered appropriate although some coefficients exceed the boxplots intervals, and a possible reason may be related to the small amount of information used during extrapolation. The influence of Damage I on extrapolation can be questioned, since the model was not fully capable of detecting damage and may have impaired the performance of extrapolation.

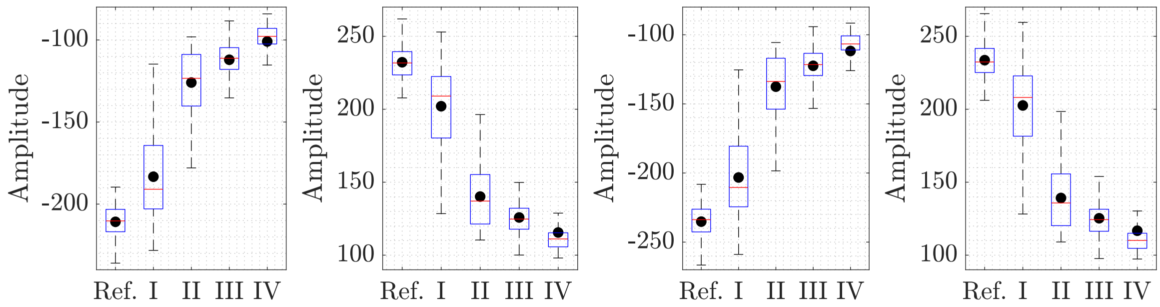
Figure 9 – AR model coefficients (•) extrapolation on accelerometer 4.



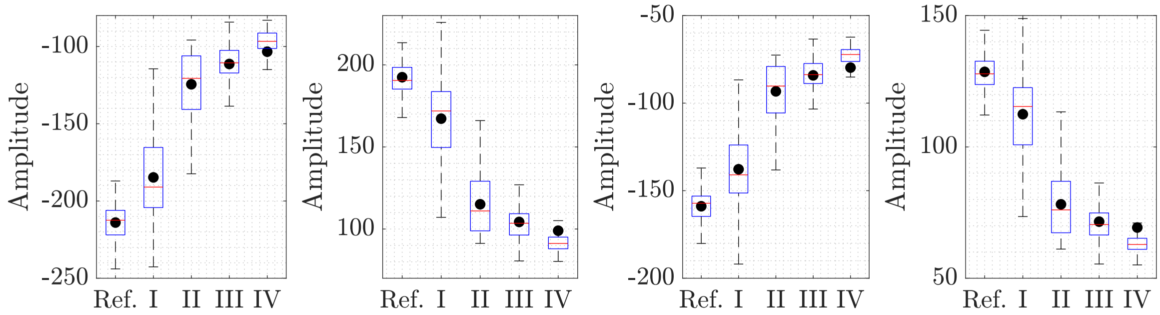
(a) Coefficients a_1-a_4



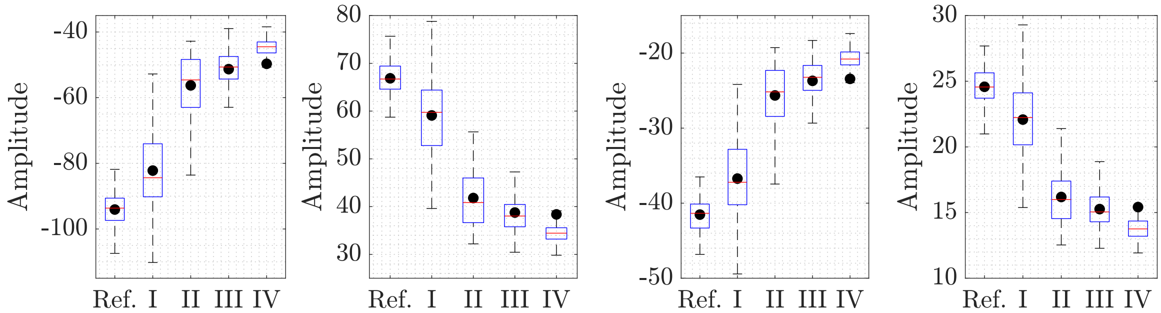
(b) Coefficients a_5-a_8



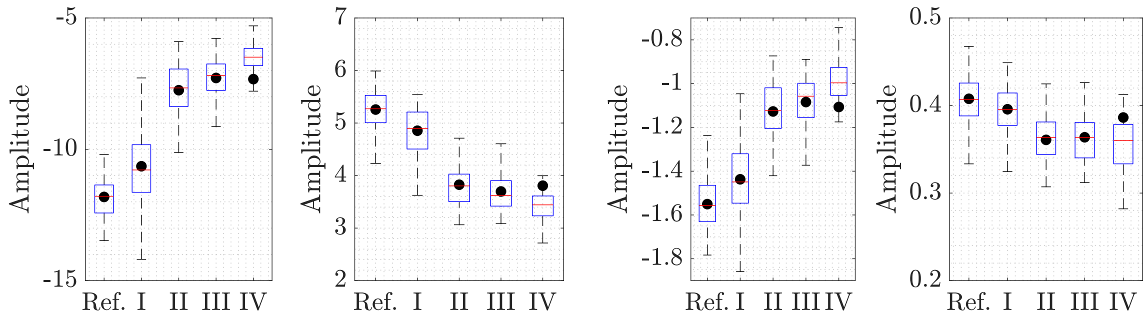
(c) Coefficients a_9-a_{12}



(d) Coefficients a_{13}-a_{16}



(e) Coefficients a_{17}-a_{20}

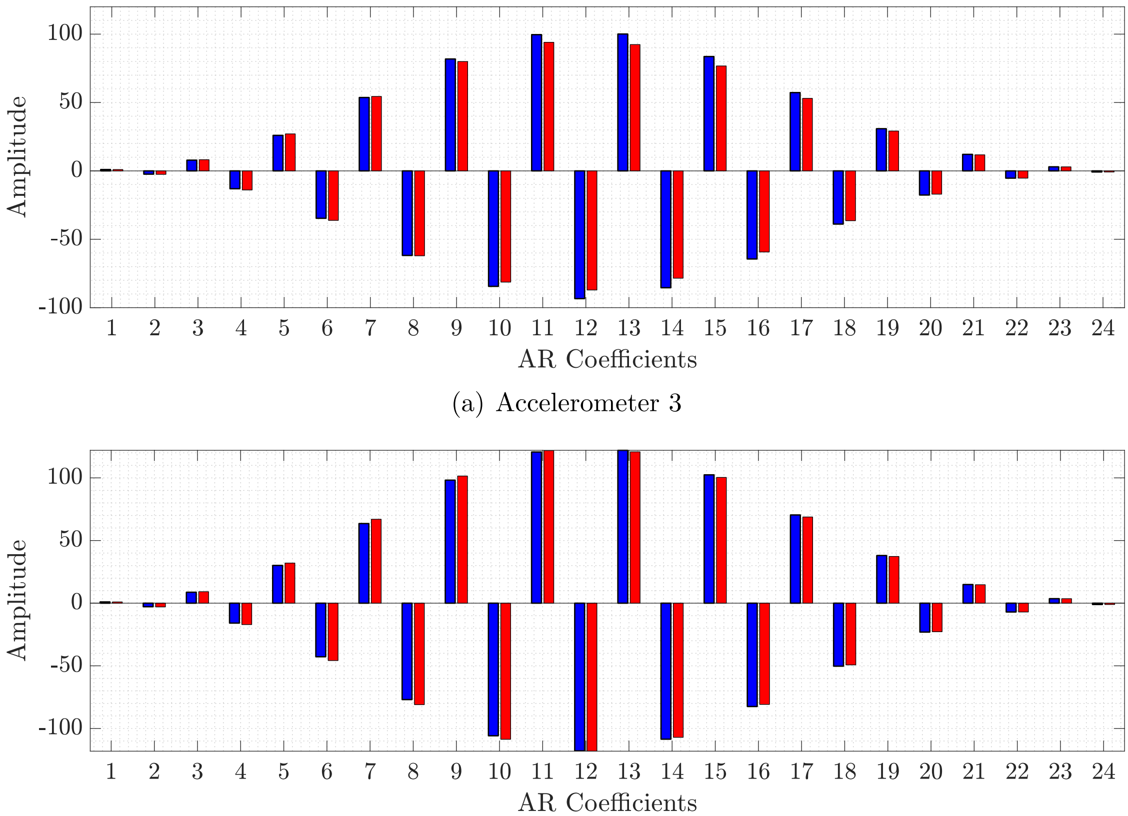


(f) Coefficients a_{21}-a_{24}

Source: Prepared by the author.

Figure 10 shows the pattern of the coefficients obtained in the Damage IV through the classical approach and via extrapolation methods. The coefficients present similar performances, confirming the efficiency of the method proposed. A new AR model is estimated with the extrapolated coefficients, and its responses are compared with the measured ones in the most severe state. The damage detection step is performed to validate the proposed methodology and, to analyze the damage severity in the structure.

Figure 10 – AR model coefficients variation on each accelerometer via extrapolation methods. ■ represents the classical approach and ■ extrapolation methodology.



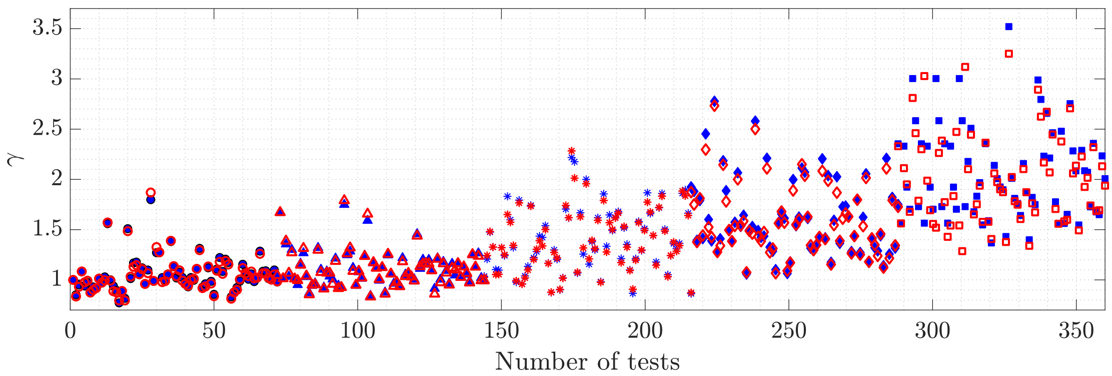
(a) Accelerometer 3

(b) Accelerometer 4

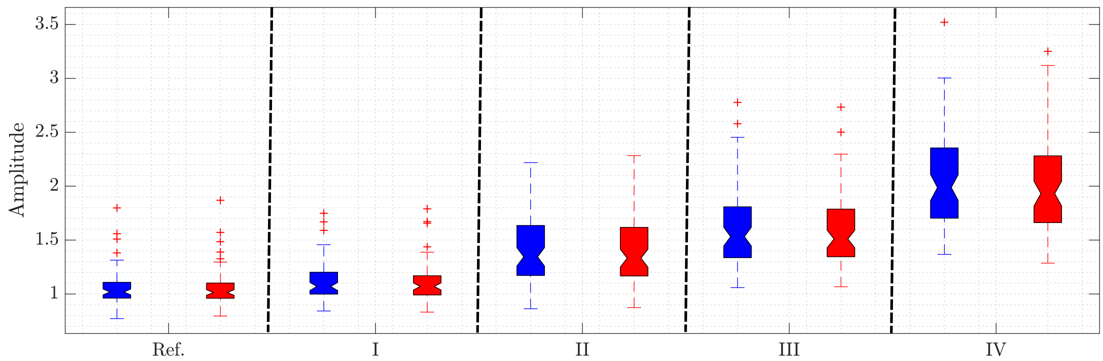
Source: Prepared by the author.

Figure 11 presents the damage indices obtained via extrapolation on each accelerometer. The accelerometers 3 and 4 presented a monotonic relationship between the damage indices and the progression of damage severity, proving the ability of the extrapolated model to classify the structural condition. The magnitude of the damage indices decreased compared to the results obtained previously despite maintaining its pattern. This fact occurs as a consequence of the AR model used, presenting lower dispersion of prediction errors in the condition in which it was identified. Also, the boxplots illustrate similar performances of damage indices variability, proving the efficiency of the proposed extrapolation method.

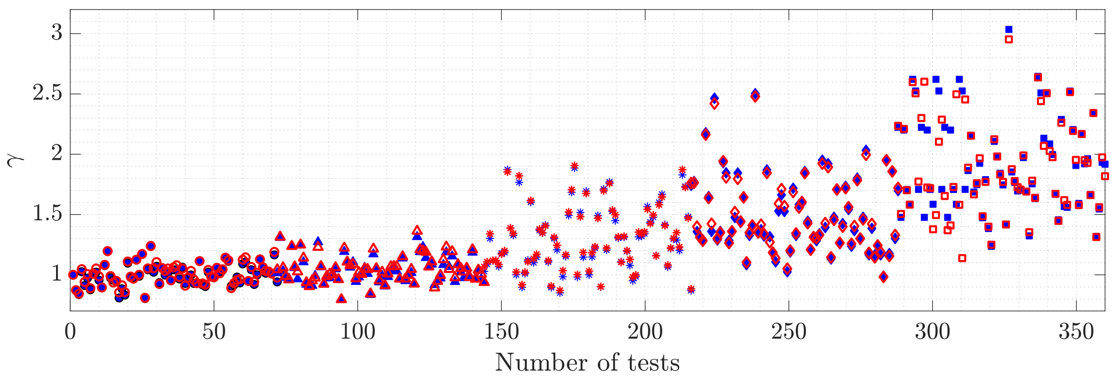
Figure 11 – Comparison of damage-sensitive indices γ obtained through classical (■) and extrapolation methods (■) on each accelerometer. • represents the Reference condition, △ Damage I, ∗ Damage II, ⋄ Damage III and □ Damage IV.



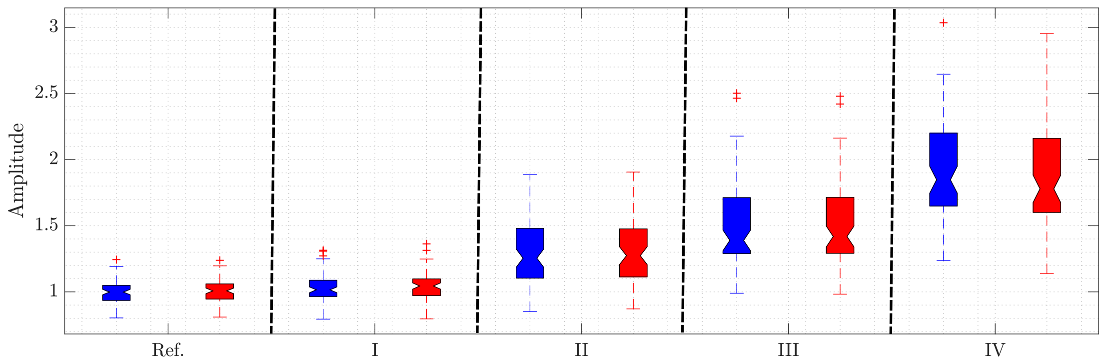
(a) Damage index of accelerometer 3



(b) Boxplots of damage index of accelerometer 3



(c) Damage index of accelerometer 4



(d) Boxplots of damage index of accelerometer 4

Source: Prepared by the author.

Table 2 shows a convergence criterion of the extrapolation methodology performed through linear, quadratic, and cubic polynomials based on the percentages of average relative errors as defined in equation 11. The extrapolation methodology presents an error of less than 5%, confirming that the methodology proposed in this work is satisfactory, although some differences in the indices of damages are observed. These differences may be related to the application of average coefficients on each condition for extrapolation and, perhaps, another approach may be more effective than the one applied in this dissertation. The application of the extrapolation methodology presents an inconvenience related to the applied polynomial. The third-order polynomials require, in theory, four states for their application. However, this is not always possible in real situations. An alternative to mitigate this issue is using linear and quadratic polynomials until more information about the structure is acquired, allowing the application of cubic splines.

Table 2 – Convergence criterion.

	Relative percentage error [%]		
Accelerometer	Linear	Quadratic	Cubic
1	2.50	1.39	1.59
2	3.02	1.99	2.05
3	96.98	7.80	3.95
4	83.27	6.61	3.02

Source: Prepared by the author.

3.4 Conclusions

This chapter presented the application of AR models for damage detection regarding damage indices defined by prediction errors, as well as the extrapolation of its coefficients. The proposed methodology was applied in a three-story building structure with a center column and a bumper placed on the top floor, to simulate the presence of damage. Moreover, the gap variation between both components modified its damage severity.

The results showed that the defined damage indices and severity have a monotonic relationship, showing appropriate results on the accelerometers near to the source of damage. Besides, the coefficient variation also confirmed adequate performances of these accelerometers. The hypothesis test applied showed relevant true detection rates, which were confirmed by the ROC curves. The application of the cubic splines method showed some coefficients that exceed the range of expected values. Nevertheless, the comparisons between the damage indices obtained through the extrapolated model showed minimal deviations, confirming the applicability of the methodology proposed in this work.

4 Final Remarks

This final chapter presents in the section 4.1 the main conclusions about the results obtained using the proposed approach in this dissertation. Furthermore, some suggestions for future works and the next steps are also depicted.

4.1 Conclusions

In this dissertation, damage detection and extrapolation of future state analysis through autoregressive models were presented. An innovative strategy was investigated based on the extrapolation of the coefficients of AR models, to predict the system behavior with damage progression. In this sense, it was investigated the behavior of a three-story building structure with a bumper and an aluminum center column, placed on its top floor, to simulate structural changes associated with damage. The severity was varied according to the gap between both components. The reference AR models were identified, verified and validated through the vibrational responses measured by four accelerometers placed on each floor of the structure. It is evident during the damage detection, that results achieved are relevant, allowing the evaluation of the damage progression and the existence of the damage in the structure according to the information from the ROC curves.

The extrapolation of AR coefficients performed by the cubic splines method showed significant results of the extrapolated model, although presenting minor differences in some coefficients. This issue can be mitigated by improving the fit of the trending curve of the AR coefficients through new measurements from the system. Also, the extrapolated model presents consistent results in reproducing the pattern of damage-sensitive indices as a result of the progression of the damage severity present in the structure.

4.2 Suggestions for future works

For the future work, the remaining useful life should be tackled in order to address the damage prognosis level of damage identification. The possibility of investigating simultaneous damage in two or more positions using machine learning algorithms will be discussed. In addition, the relationship between AR coefficients and modal parameters, or with finite element models, can be analyzed for a better understanding of the proposed extrapolation methodology. Finally, the practical application of extrapolation in real and specific cases would be interesting, to validate the methodology proposed in this work.

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