Reply to ''Comment on 'Validity of Feynman's prescription of disregarding the Pauli principle in intermediate states' ''

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In a recent paper, we raised a question on the validity of Feynman's prescription of disregarding the Pauli principle in intermediate states of perturbation theory. In the preceding Comment, Cavalcanti correctly pointed out that Feynman's prescription is consistent with the exact solution of the model that we used. This means that the Pauli principle does not necessarily apply to intermediate states. We discuss implications of this puzzling aspect.

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In a recent paper [1], we tested Feynman's prescription [2] of disregarding the Pauli principle (PP) in intermediate states of perturbation theory. The system of the model consists of a particle bound in a given potential together with its vacuum background. We used Dirac's hole theory (HT) in Ref. [1] and we continue to do so until we turn to quantum field theory (QFT) toward the end of this paper. We are interested in the second-order energy shift of the system caused by an external perturbation. We compared two methods of calculation, I and II. Method I takes account of PP in intermediate states whenever it is applicable. In method II, PP is completely ignored. We examined the one-dimensional bag model with a Hamiltonian of the form of $H=H_0+V(x)$, where H_0 is for a Dirac particle in the bag and $V(x)=\lambda x$ is an external perturbation. We found that the two methods lead to different results. We traced the source of the discrepancy to an alternating infinite series that is involved in method II; see Eq. (6) below.

In the preceding Comment [3], Cavalcanti showed that, if the mass m of the particle is zero, the Dirac equation $H\psi(x)=\epsilon\psi(x)$ of the model can be solved exactly and that the eigenvalue ϵ is independent of perturbation λx. This result itself, i.e., the energy shift is zero when $m=0$, is consistent with the result of our method II. As Cavalcanti correctly pointed out, method II in which Feynman's prescription is used is consistent with the exact HT solution of the model. Method I in which the PP is enforced in every step leads to an energy shift that differs from the exact one.

We can put Cavalcanti's comment in a more general perspective. Consider two systems that are, respectively, associated with the following two Dirac equations:

$$H_0\psi_{0,n}=\epsilon_{0,n}\psi_{0,n}, \tag{1}$$

$$H\psi_n=\epsilon_n\psi_n, \quad H=H_0+V. \tag{2}$$

Suffix $n(=\pm1,\pm2,\cdots)$ specifies eigenstates in the way as was done in Ref. [1]. The energy eigenvalues are labeled such that $0<\epsilon_1<\epsilon_2<\cdots$, and $0>\epsilon_{-1}>\epsilon_{-2}>\cdots$. We assume that the energy levels are discrete and nondegenerate. The systems do not have to be in one dimension and the mass does not have to be zero. We call the above two systems the unperturbed and perturbed systems. It is understood that there is one-to-one correspondence between perturbed and unperturbed solutions.

Consider the situation such that the lowest positive-energy state and all the negative-energy states are occupied. Then the energies of the two systems are, respectively, given by

$$E_0=\epsilon_{0,1}+\sum_j \epsilon_{0,-j}, \tag{3}$$

$$E=\epsilon_1+\sum_j \epsilon_{-j}. \tag{4}$$

The sum over the negative-energy states is the energy of the vacuum background. This sum diverges for both E and E_0. We are, however, only interested in the energy shift due to perturbation V, i.e., $E-E_0$. We define this difference as the sum of the differences of the corresponding terms, e.g., $\epsilon_{-j}-\epsilon_{0,-j}$. The energy shift so defined is finite for the model used in Ref. [1] but it may still diverge in other cases as we will point out later.

If Eqs. (1) and (2) are both solvable, the energies of the two systems are given by Eqs. (3) and (4). The E of Eq. (4) is the exact energy of the system in the presence of perturbation V. Now suppose (or pretend) that Eq. (1) is solvable, but not Eq. (2), and one attempts to solve Eq. (2) by perturbation theory. One starts with $\epsilon_{0,n}$ and attempts to reach ϵ_n. One follows the standard prescription of perturbation theory including *all* intermediate states. In going from $\epsilon_{0,n}$ to ϵ_n, the system is treated as a one-body system. One does not exclude the intermediate states that are already occupied by other particles. That is, one exactly follows method II of Ref. [1] and obtains

$$(\text{HT without PP})=(\text{HT with method II})=(\text{Exact HT}). \tag{5}$$

In Eq. (5) as such, it is understood that the perturbation calculation is done for all orders. In order to see the crux of the issue, however, we can focus on the second-order effects as

we did in Ref. [1]. Let us emphasize that the contributions from transitions between negative-energy states, from $-j$ to $-k$,

$$\sum_{k\neq j} |V_{-k,-j}|^2/(\epsilon_{0,-j}-\epsilon_{0,-k}) \tag{6}$$

appear in HT without PP.

Cavalcanti considered a special case of $m=0$ of the example of Ref. [1], but his observation that PP in intermediate states does not have to be considered is valid in general [beyond the specific case of $m=0$ and $V(x)=\lambda x$]. If one does perturbation calculation following method I, that is, by enforcing PP throughout, one may end up with a wrong result as we illustrated in Ref. [1],

$$(\text{HT with PP})=(\text{HT with method I})\neq(\text{Exact HT}). \tag{7}$$

In this case the series of Eq. (6) does not appear.

Apart from the problem concerning PP discussed above, the HT has another problem in the first-order effect. This has nothing to do with PP. The first-order energy shift is the expectation value of V in the unperturbed state. In the example of Ref. [1], we assumed $V(x)=\lambda x$ so that the first-order effect vanishes. For a $V(x)$ of a general form, the expectation value does not vanish. In fact it diverges because of the infinite number of particles in the vacuum. This is unphysical. This divergence cannot simply be subtracted away because it is a quantity that depends on the external perturbation.

Let us now turn to QFT. The HT and QFT are usually considered equivalent to each other. At the end of Sec. II of Ref. [1] we stated: ''In this way the whole language of HT can be transcribed into that of QFT.'' Cavalcanti's comment made us realize that HT and QFT are actually not equivalent for our example and probably also for some other examples. QFT has no difficulties that we found in HT. The first-order energy shift is zero. This is simply because the Hamiltonian of QFT is of the form of a normal product. No negative-energy particles appear in QFT. The QFT vacuum contains no particles or antiparticles. In the second order, unlike in HT with terms of Eq. (6), no transtitions between negative-energy states appear in QFT. Regarding PP, all PP-violating effects cancel. Hence one can disregard PP altogether. *If we have to choose between HT and QFT, we should take QFT*. Toward the end of Ref. [1] we said: '' . . . if we encounter ambiguity by disregarding PP, we should remain faithful to PP in every step of calculation.'' This statement, as it turned out, is consistent with choosing QFT over HT. If HT is in doubt, some of the calculations that were based on HT may have to be reexamined.

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[3] R. M. Cavalcanti, preceding paper, Phys. Rev. A **62**, 016101 (2000). When $m=0$, the Dirac equation can be solved exactly for an arbitrary $V(x)$ in a way similar to that of Cavalcanti. All eigenvalues are shifted by the same amount of $[1/(2a)]\int_{-\infty}^{\infty}V(x)dx$.