

An extension of the linear delta expansion to superspace

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We introduce and discuss the method of linear delta expansion for the calculation of effective potentials in superspace, by adopting the improved version of the super-Feynman rules. Calculations are carried out up to two loops and an expression for the optimized Kähler potential in the Wess-Zumino model is worked out.

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From among the most important tools of quantum field theory (QFT) one selects the study of the effective action and the effective potential. They have been used as a device to investigate important quantum properties of field theory, such as vacuum energy and symmetry breaking patterns.

There are well-known methods to compute effective actions and effective potentials to all finite orders in perturbation theory [1]. Especially, the issue of effective potential calculations has been discussed in a wide variety of models, due to the relevance of the problem in proposing unified models for the fundamental interactions [2–4]. Also, the important discussion of the gauge (in)dependence of the effective potential in the framework of Yang-Mills theories has received a great deal of attention in connection with the calculation of physical quantities in the electro-weak theory [5–7].

In the case of supersymmetric theories, a very powerful method to compute quantum corrections to the effective action is to use super-Feynman rules and then apply perturbation theory directly in superspace [8–11]. In general, the supereffective action is described by two functions of the chiral and the antichiral superfields; one is required to be a holomorphic function and the other one, called Kähler potential, less constrained, is required to be just a real function. The superspace techniques allow us to use various nonrenormalization theorems, which constrain the perturbative quantum corrections to the holomorphic part of the effective action and lead to all orders results [12,13]. On the other hand, the Kähler potential encodes the wave function renormalization for the chiral superfields and receives corrections to all orders in perturbation theory.

In general, the outstanding problems of QFT are typically nonperturbative. In this case it is necessary to develop a method to make possible a resummation of Feynman diagrams and derive a result which has all orders of the coupling constant. Those methods sum infinite Feynman

diagrams that belong to a specific set. The more traditional resummation result is the Coleman-Weinberg potential [14]. It is the sum of all one-loop diagrams. There are many ways to derive the Coleman-Weinberg potential, the two traditional ones are the diagrammatic calculation and the functional calculation [14,15]. If the fields are defined in an N -dimensional representation of any group, the Coleman-Weinberg potential can also be derived from a $1/N$ expansion in the large N limit. If we are interested in finite N results, which means to make a resummation that involves more than one-loop diagrams, both the diagrammatic and the functional methods are very difficult to use, mainly because it is necessary to work with infinite diagrams, which turns the renormalization procedure into a difficult task.

Over the past years, an alternative resummation method has been developed, namely, the linear delta expansion (LDE) [16]. This method can easily reproduce the Coleman-Weinberg potential, and the use of the LDE in various QFT models has proved to be a powerful tool to derive finite N results [17]. The main characteristic of the method is to use a traditional perturbative approach together with an optimization procedure. So, in order to derive a nonperturbative result, it is just necessary to work with a few diagrams and use perturbative renormalization techniques.

The main goal of this paper is to show that the LDE can be also a powerful method to derive nonperturbative results in supersymmetric theories. To this end, in Sec. I, we present the main steps of the method based on the LDE, in Sec. II, we further develop the method to be applied directly in superspace and, in Sec. III, we derive the Coleman-Weinberg potential plus two-loop corrections for the Wess-Zumino (WZ) model. Our concluding remarks are finally cast in Sec. IV.

I. THE LINEAR DELTA EXPANSION

In this section, we shall present a brief review of the LDE. Starting with a Lagrangian \mathcal{L} , let us define the following interpolated Lagrangian, \mathcal{L}^δ :

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$$\mathcal{L}^\delta = \delta \mathcal{L}(\mu) + (1 - \delta) \mathcal{L}_0(\mu), \quad (1)$$

where δ is an arbitrary parameter, $\mathcal{L}_0(\mu)$ is the free Lagrangian, and μ is a mass parameter. Notice that when $\delta = 1$, the original theory is retrieved, so δ is just a book-keeping parameter. The δ -parameter labels interactions and it is used as a perturbative coupling instead of the original coupling. The mass parameter appears in \mathcal{L}_0 and $\delta \mathcal{L}_0$. In fact, we are using the traditional trick consisting of adding and subtracting a mass term in the original Lagrangian. The μ dependence into \mathcal{L}_0 is absorbed into the propagators, whereas $\delta \mathcal{L}_0$ is regarded as a quadratic interaction vertex.

Let us now define the strategy of the method. We apply usual perturbation theory in δ and, at the very end, we set $\delta = 1$. Up to this stage, traditional perturbation theory was applied, working with finite Feynman diagrams, and the results are purely perturbative. However, quantities evaluated at finite order in δ explicitly depend on μ . So, it is necessary to fix the μ parameter. There are two ways of doing it. The first one is to use the principle of minimal sensitivity (PMS) [18]. Since μ does not belong to the original theory, we may require that a physical quantity, such as the effective potential $V^{(k)}(\mu)$, calculated perturbatively to order δ^k , must be evaluated at a point where it is less sensitive to the parameter μ . According to the PMS, $\mu = \mu_0$ is the solution of the equation

$$\left. \frac{\partial V^{(k)}(\mu)}{\partial \mu} \right|_{\mu=\mu_0, \delta=1} = 0. \quad (2)$$

After this procedure, the optimum value, μ_0 , will be a function of the original coupling and the fields. Then, we replace μ_0 into the effective potential $V^{(k)}$ and obtain a nonperturbative result, since the propagator depends on μ .

The second way to fix μ is known as the fastest apparent convergence (FAC) criterion [18]. It requires that, from any k -coefficient of the perturbative expansion

$$V^{(k)}(\mu) = \sum_{i=0}^k c_i(\mu) \delta^i, \quad (3)$$

the following relation be fulfilled:

$$[V^k(\mu) - V^{k-1}(\mu)]_{\delta=1} = 0. \quad (4)$$

Again, the μ_0 solution of the above equation will be a function of the original couplings and the fields and, when we replace $\mu = \mu_0$ into $V(\mu)$, we obtain a nonperturbative result. Equation (4) is equivalent to taking the k th coefficient of Eq. (3) equal to zero ($c_k = 0$). If we are interested in an order- δ^k result ($V^{(k)}(\mu)$) using the FAC criterion, it is just necessary to find the solution of the equation $c_{k+1}(\mu)|_{\mu=\mu_0} = 0$ and put it in $V^{(k)}(\mu)$. Reference [17] provides an extensive list of successful applications of the method.

To exemplify the above results, we calculate the effective potential for the Gross-Neveu model [19] at order δ . This is a model consisting of N fermions with quartic interaction, in $(1+1)$ dimension, and it is interesting for the study of some important issues in field theory, such as discrete chiral symmetry breaking, dimensional transmutation, and asymptotic freedom. It was first analyzed with LDE in [20], where the large N result was reproduced.¹ Important finite N results were derived in [22,23], where the emergence of a tricritical point was pointed out.

The original Lagrangian of the model is

$$\mathcal{L} = i\bar{\psi}^a \partial_m \gamma^m \psi^a + \frac{g}{2N} (\bar{\psi}^a \psi^a)^2, \quad (5)$$

with $a = 1, 2, \dots, N$. We now introduce the auxiliary field, σ , in the form

$$\mathcal{L} \rightarrow \mathcal{L} - \frac{N}{2g} \left(\sigma - \frac{g}{N} \bar{\psi}^a \psi^a \right)^2, \quad (6)$$

which allows us to make a $1/N$ expansion. Solving the Euler-Lagrange equations, we have

$$\sigma = \frac{g}{N} \bar{\psi}^a \psi^a, \quad (7)$$

which sets a constraint equation. Thus, the new Lagrangian is

$$\begin{aligned} \mathcal{L} &\rightarrow \mathcal{L} + \mu \bar{\psi}^a \psi^a - \mu \bar{\psi}^a \psi^a \\ &= i\bar{\psi}^a \partial_m \gamma^m \psi^a - \frac{N}{2g} \sigma^2 + \sigma \bar{\psi}^a \psi^a + \mu \bar{\psi}^a \psi^a - \mu \bar{\psi}^a \psi^a \\ &= \mathcal{L}_0(\mu) + \mathcal{L}_{\text{int}}(\mu), \end{aligned} \quad (8)$$

with

$$\begin{aligned} \mathcal{L}_0(\mu) &= i\bar{\psi}^a \partial_m \gamma^m \psi^a - \frac{N}{2g} \sigma^2 - \mu \bar{\psi}^a \psi^a, \\ \mathcal{L}_{\text{int}}(\mu) &= \sigma \bar{\psi}^a \psi^a + \mu \bar{\psi}^a \psi^a. \end{aligned} \quad (9)$$

The interpolated Lagrangian is

$$\begin{aligned} \mathcal{L}^\delta &= \delta \mathcal{L}(\mu) + (1 - \delta) \mathcal{L}_0(\mu) \\ &= i\bar{\psi}^a \partial_m \gamma^m \psi^a - \frac{N}{2g} \sigma^2 - \mu \bar{\psi}^a \psi^a + \delta \sigma \bar{\psi}^a \psi^a \\ &\quad + \delta \mu \bar{\psi}^a \psi^a. \end{aligned} \quad (10)$$

In this expression, the $-\mu \bar{\psi}^a \psi^a$ term is a mass term appearing in the propagator and the term $\delta \mu \bar{\psi}^a \psi^a$ represents an interaction with weight δ . Note that the

¹The $2+1$ Gross-Neveu model was also analyzed using LDE in [21].

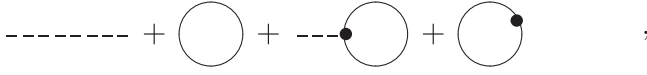


FIG. 1. Effective potential for the Gross-Neveu model at order δ .

σ -propagator carries the $1/N$ factor and each fermion loop contains an N factor.²

In general, when the effective potential is calculated in QFT, we do not worry about vacuum diagrams, since they do not depend on fields. However, the vacuum diagrams depend on μ and are important in the LDE, since the arbitrary mass parameter will depend on fields after the optimization procedure. So, in the LDE, it is necessary to calculate the vacuum diagrams order by order.

The diagrammatic representation of the effective potential at order δ is where the dashed line represents the σ -propagator. The second and the last diagrams are the vacuum contribution at order zero and at order δ . Using the corresponding Feynman rules, the diagrammatic sum yields

$$V_{\text{eff}}(\sigma_c) = \frac{N}{2g} \sigma_c^2 + iN \int \frac{d^2 p}{(2\pi)^2} \ln(p^2 - \mu^2) - 2iN\delta \int \frac{d^2 p}{(2\pi)^2} \frac{\mu \sigma_c}{(p^2 - \mu^2)} + 2iN\delta \int \frac{d^2 p}{(2\pi)^2} \frac{\mu^2}{(p^2 - \mu^2)}, \quad (11)$$

where σ_c is the classical field. The above expression corresponds to the classical potential ($N\sigma_c^2/2g$) plus the one-loop correction for the vacuum and one-point Green's function at order δ .

Now, we apply the PMS at order δ and take $\delta = 1$. We impose

$$\left. \frac{\partial V_{\text{eff}}}{\partial \mu} \right|_{\mu=\mu_0} = 0, \quad (12)$$

which implies

$$\mu_0 = \sigma_c. \quad (13)$$

Substituting this result in (11), the effective potential evaluated at $\mu = \mu_0$ is

$$\frac{V_{\text{eff}}(\sigma_c)}{N} = \frac{1}{2g} \sigma_c^2 + i \int \frac{d^2 p}{(2\pi)^2} \ln\left(1 - \frac{\sigma_c^2}{p^2 + i\varepsilon}\right), \quad (14)$$

which corresponds to the effective potential of the Gross-Neveu model in the $N \rightarrow \infty$ limit [24] and represents the

²We use here the traditional form of the interpolated Lagrangian because it seems more natural for our present application. However, in Ref. [23], the interpolation of the GN model is made in a different way, where the σ^2 term is multiplied by δ . We intend to analyze this possibility for future applications of the LDE in more general supersymmetric models.

sum of all one-loop diagrams for this theory. This result shows the simplicity of the method. We can reproduce an infinite sum of a class of diagrams, working with few diagrams, represented in Fig. 1.

II. LDE IN SUPERSPACE

Let us now further develop the LDE for superspace applications. We are going to use the WZ model. The superspace action is

$$\mathcal{S}[\Phi, \bar{\Phi}] = \int d^8 z \bar{\Phi} \Phi + \int d^6 z \left(\frac{m}{2} \Phi^2 + \frac{\lambda}{3!} \Phi^3 \right) + \int d^6 \bar{z} \left(\frac{m}{2} \bar{\Phi}^2 + \frac{\bar{\lambda}}{3!} \bar{\Phi}^3 \right), \quad (15)$$

where we have the notation $d^8 z = d^4 x d^2 \theta d^2 \bar{\theta}$, $d^6 z = d^4 x d^2 \theta$, $d^6 \bar{z} = d^4 x d^2 \bar{\theta}$. Using the nonrenormalization theorems, the more general effective action for the WZ model is written as

$$\Gamma[\Phi, \bar{\Phi}] = \int d^8 z [K(\bar{\Phi}, \Phi) + \dots], \quad (16)$$

where the first term is the Kähler potential and the other terms involve derivatives of the fields. The effective action can be calculated perturbatively directly in superspace, using superspace Feynman rules. Here we use the improved Feynman rules defined originally in [25].

Let us start to apply the LDE to the WZ model. The first modification is to implement two mass parameters, μ and $\bar{\mu}$, instead of just one. In order to fix these parameters, we use two optimization equations. In particular, we use the FAC criterion, so that we have one equation similar to (4) for μ and the other for $\bar{\mu}$. Using two mass parameters, the Lagrangian \mathcal{L}^δ can be written as

$$\mathcal{L}^\delta = \delta \mathcal{L} + (1 - \delta) \mathcal{L}_0 = \int d^4 \theta \bar{\Phi} \Phi + \int d^2 \theta \left(\frac{M}{2} \Phi^2 + \frac{\delta \lambda}{3!} \Phi^3 - \frac{\delta \mu}{2} \Phi^2 \right) + \int d^2 \bar{\theta} \left(\frac{\bar{M}}{2} \bar{\Phi}^2 + \frac{\delta \bar{\lambda}}{3!} \bar{\Phi}^3 - \frac{\delta \bar{\mu}}{2} \bar{\Phi}^2 \right), \quad (17)$$

where $M = m + \mu$ and $\bar{M} = m + \bar{\mu}$. The action \mathcal{S}^δ can be written using the matrix notation as [26]

$$\mathcal{S}^\delta[\Phi, \bar{\Phi}] = \frac{1}{2} \int dz dz' (\Phi(z) \bar{\Phi}(z')) H^{(M, \bar{M})} \begin{pmatrix} \Phi(z') \\ \bar{\Phi}(z') \end{pmatrix} + \int d^6 z \left(\frac{\delta \lambda}{3!} \Phi^3 - \frac{\delta \mu}{2} \Phi^2 \right) + \int d^6 \bar{z} \left(\frac{\delta \bar{\lambda}}{3!} \bar{\Phi}^3 - \frac{\delta \bar{\mu}}{2} \bar{\Phi}^2 \right), \quad (18)$$

where

$$H^{(M,\bar{M})} = \begin{pmatrix} M & -\frac{1}{4}\bar{D}^2 \\ -\frac{1}{4}D^2 & \bar{M} \end{pmatrix} \begin{pmatrix} \delta_+(z, z') & 0 \\ 0 & \delta_-(z, z') \end{pmatrix}. \quad (19)$$

Now, one has a new chiral and antichiral quadratic interaction proportional to $\delta\mu$ and $\delta\bar{\mu}$. Also, the superpropagator $G^{(M,\bar{M})}$, defined in terms of the chiral and antichiral delta functions ($\delta_+(z, z')$, $\delta_-(z, z')$) by the equation

$$G^{(M,\bar{M})}H^{(M,\bar{M})} = \begin{pmatrix} \delta_+(z, z') & 0 \\ 0 & \delta_-(z, z') \end{pmatrix}, \quad (20)$$

has shifted mass terms $M = m + \mu$ and $\bar{M} = m + \bar{\mu}$. The new Feynman rules can now be read off; they are cast below:

(1) Propagators:

$$\begin{aligned} \Phi\bar{\Phi}: & -\frac{i}{k^2 + M\bar{M}}\delta^4(\theta - \theta'), \\ \Phi\Phi: & \frac{iM}{k^2(k^2 + M\bar{M})}\frac{1}{4}D^2(k)\delta^4(\theta - \theta'), \\ \bar{\Phi}\bar{\Phi}: & \frac{i\bar{M}}{k^2(k^2 + M\bar{M})}\frac{1}{4}\bar{D}^2(k)\delta^4(\theta - \theta'); \end{aligned} \quad (21)$$

(2) Vertices:

$$\begin{aligned} \bullet &= i\delta\lambda \int d^8z, & \circ &= i\delta\bar{\lambda} \int d^8z, \\ \text{---}\times\text{---} &= -i\delta \int \mu d^8z, \\ \text{---}\otimes\text{---} &= -i\delta \int \bar{\mu} d^8z. \end{aligned} \quad (22)$$

The perturbative effective Kähler potential can be calculated in powers of δ using the one particle irreducible (OPI) functions. It is well known that, in superspace, vacuum superdiagrams are identically zero, by virtue of Berezin integrals. To avoid this, we have to consider, from the very beginning, the parameters μ , $\bar{\mu}$ as superfields and keep the vacuum supergraphs until the optimization procedure. From the generating superfunctional in the presence of the chiral (J) and antichiral (\bar{J}) sources:

$$\begin{aligned} \tilde{Z}[J, \bar{J}] &= \exp\left[iS_{\text{INT}}\left(\frac{1}{i}\frac{\delta}{\delta J}, \frac{1}{i}\frac{\delta}{\delta \bar{J}}\right)\right] \\ &\times \exp\left[\frac{i}{2}(J, \bar{J})G^{(M,\bar{M})}\begin{pmatrix} J \\ \bar{J} \end{pmatrix}\right], \end{aligned} \quad (23)$$

we can write the supereffective action:

$$\begin{aligned} \Gamma[\Phi, \bar{\Phi}] &= -\frac{i}{2} \ln[s \det(G^{(M,\bar{M})})] - i \ln \tilde{Z}[J, \bar{J}] \\ &- \int d^6z J(z)\Phi(z) - \int d^6\bar{z} \bar{J}(z)\bar{\Phi}(z), \end{aligned} \quad (24)$$

where $s \det(G^{(M,\bar{M})})$ is the superdeterminant of the matrix propagator, which, in general, is equal to one; but, here we keep it, because $G^{(M,\bar{M})}$ depends on μ and $\bar{\mu}$. Also, due to the μ and $\bar{\mu}$ dependence, the vacuum diagrams supergenerator $\tilde{Z}[0, 0]$ is not identically equal to one. We can define the normalized functional generator as $Z_N = \frac{\tilde{Z}[J, \bar{J}]}{\tilde{Z}[0, 0]}$ and write the effective action as

$$\Gamma[\Phi, \bar{\Phi}] = -\frac{i}{2} \ln[s \det(G)] - i \ln \tilde{Z}[J_0, \bar{J}_0] + \Gamma_N[\Phi, \bar{\Phi}], \quad (25)$$

where the sources J_0 and \bar{J}_0 are defined by the equations

$$\begin{aligned} \frac{\delta W[J, \bar{J}]}{\delta J(z)} \Big|_{J=J_0} &= \frac{\delta W[J, \bar{J}]}{\delta \bar{J}(z)} \Big|_{\bar{J}=\bar{J}_0} = \frac{\delta \tilde{Z}[J, \bar{J}]}{\delta J(z)} \Big|_{J=J_0} \\ &= \frac{\delta \tilde{Z}[J, \bar{J}]}{\delta \bar{J}(z)} \Big|_{\bar{J}=\bar{J}_0} = 0. \end{aligned} \quad (26)$$

In Eq. (25) the first two terms represent the vacuum diagrams (which are usually zero) and $\Gamma_N[\Phi, \bar{\Phi}]$ is the usual contribution to the effective action.

III. KÄHLER POTENTIAL IN THE LDE USING THE FAC CRITERION

Let us now calculate the Kähler potential by using the LDE up to the order δ^2 . We are going to show that this corresponds to the Coleman-Weinberg potential plus a sum of infinite two-loop supergraphs.

In Fig. 2 one can see the diagrammatic sum of the effective Kähler potential up to the order δ^2 ($\mathcal{V}_{\text{eff}}^{\delta^2}$). The tadpole diagrams are zero, as usual, since they have quadratic terms of Grassmann delta functions.

The diagrammatic sum of Fig. 2 corresponds to the terms:

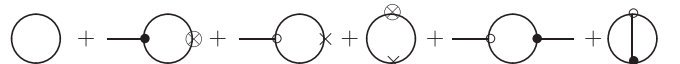


FIG. 2. One and two-loop diagrams up to the order δ^2 .

$$\begin{aligned} \mathcal{V}_{\text{eff}}^{\delta^2} = & -\frac{i}{2} s \text{Tr} \ln[H^{(M, \bar{M})}] - \frac{i}{2} \delta^2 \lambda \int \frac{d^4 k}{(2\pi)^4} \int d^4 \theta \frac{\bar{\mu} \Phi(0, \theta)}{(k^2 + M\bar{M})^2} + -\frac{i}{2} \delta^2 \bar{\lambda} \int \frac{d^4 k}{(2\pi)^4} \int d^4 \theta \frac{\mu \bar{\Phi}(0, \theta)}{(k^2 + M\bar{M})^2} \\ & + \frac{i}{2} \delta^2 \int \frac{d^4 k}{(2\pi)^4} \int d^4 \theta \frac{\mu \bar{\mu}}{(k^2 + M\bar{M})^2} + \frac{i}{2} \delta^2 \lambda \bar{\lambda} \int \frac{d^4 k}{(2\pi)^4} \int d^4 \theta \frac{\bar{\Phi}(0, \theta) \Phi(0, \theta)}{(k^2 + M\bar{M})^2} + \frac{1}{6} \delta^2 \lambda \bar{\lambda} \int \frac{d^4 k d^4 q}{(2\pi)^8} \int d^4 \theta \frac{1}{A}, \end{aligned} \quad (27)$$

where $A = (k^2 + M\bar{M})(q^2 + M\bar{M})((q - k)^2 + M\bar{M})$. We wrote the first term of Eq. (24) as a supertrace in the full superspace (see Ref. [26] for details). Now we have to fix the mass parameters and write the Kähler potential of Eq. (27) at $\delta = 1$ in terms of the optimized parameters μ_0 and $\bar{\mu}_0$. The FAC criterion is employed as an optimization procedure. In order to calculate the Kähler effective potential up to order 2 in δ ($\mathcal{V}_{\text{eff}}^{\delta^2}$) we have to solve, for μ_0 and $\bar{\mu}_0$, the equation

$$c^3(\mu, \bar{\mu}) = 0, \quad (28)$$

at $\delta = 1$, where $c^3(\mu, \bar{\mu})$ corresponds to the δ^3 coefficients in the perturbative expansion of the Kähler potential.

The order δ^3 ($\mathcal{V}_{\text{eff}}^{\delta^3}$) diagrams are drawn in Fig. 3 below:

Using the corresponding Feynman rules, they correspond to the following terms of the Kähler potential:

$$\begin{aligned} \frac{i}{2} \delta^3 \int \frac{d^4 k}{(2\pi)^4} \int d^4 \theta \frac{1}{(k^2 + M\bar{M})^3} [& -\lambda^2 \bar{\lambda} M \Phi^2 \bar{\Phi} + \lambda \bar{\lambda} M \mu \Phi \bar{\Phi} - \bar{\lambda} M \mu^2 \bar{\Phi} + M \mu^2 \bar{\mu} - \lambda M \mu \bar{\mu} \Phi + \lambda^2 M \bar{\mu} \Phi^2 \\ & - \lambda \bar{\lambda}^2 \bar{M} \Phi \bar{\Phi}^2 + \lambda \bar{\lambda} \bar{M} \bar{\mu} \Phi \bar{\Phi} - \lambda \bar{M} \bar{\mu}^2 \Phi + \bar{M} \mu \bar{\mu}^2 - \bar{\lambda} \bar{M} \mu \bar{\mu} \bar{\Phi} + \bar{\lambda}^2 \bar{M} \mu \bar{\Phi}^2] + \frac{1}{4} \delta^3 \lambda \bar{\lambda} \int \frac{d^4 k d^4 q}{(2\pi)^8} \\ & \times \int d^4 \theta \left[\frac{1}{B} (-\lambda M \Phi - \bar{\lambda} \bar{M} \bar{\Phi} + \mu M + \bar{\mu} \bar{M}) + \frac{1}{C} (\mu M + \bar{\mu} \bar{M} - \lambda M \Phi - \bar{\lambda} \bar{M} \bar{\Phi}) \right], \end{aligned} \quad (29)$$

where each term corresponds to its respective diagram in Fig. 3, in the same order. Rearranging the terms in the above equation we obtain that $\mathcal{V}_{\text{eff}}^{\delta^3}$ corresponds to

$$\begin{aligned} \frac{i}{2} \delta^3 \int \frac{d^4 k}{(2\pi)^4} \int d^4 \theta \frac{1}{(k^2 + M\bar{M})^3} [& \lambda \bar{\lambda} M \Phi \bar{\Phi} (-\lambda \Phi + \mu) + M \mu^2 (-\bar{\lambda} \bar{\Phi} + \bar{\mu}) + \lambda M \bar{\mu} \Phi (-\mu + \lambda \Phi) \\ & + \lambda \bar{\lambda} \bar{M} \Phi \bar{\Phi} (-\bar{\lambda} \bar{\Phi} + \bar{\mu}) + \bar{M} \bar{\mu}^2 (-\lambda \Phi + \mu) + \bar{\lambda} \bar{M} \mu \bar{\Phi} (-\bar{\mu} + \bar{\lambda} \bar{\Phi})] + \frac{1}{4} \delta^3 \lambda \bar{\lambda} \int \frac{d^4 k d^4 q}{(2\pi)^8} \\ & \times \int d^4 \theta \left[\frac{1}{B} (M(-\lambda \Phi + \mu) + \bar{M}(-\bar{\lambda} \bar{\Phi} + \bar{\mu})) + \frac{1}{C} (M(\mu - \lambda \Phi) + \bar{M}(\bar{\mu} - \bar{\lambda} \bar{\Phi})) \right], \end{aligned} \quad (30)$$

where $C = (k^2 + M\bar{M})(q^2 + M\bar{M})((q - k)^2 + M\bar{M})^2$ and $B = (k^2 + M\bar{M})^2(q^2 + M\bar{M})((q - k)^2 + M\bar{M})$. From the above equation, we can derive the following simple solution for Eq. (28), before calculating the integrals

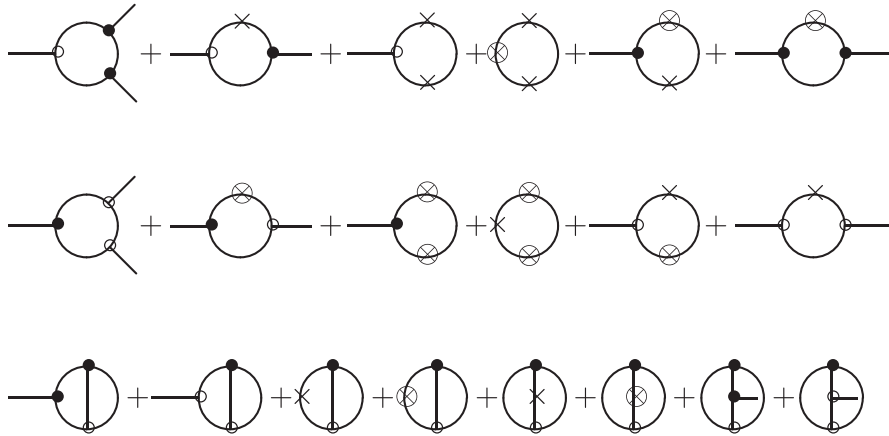


FIG. 3. One and two-loop diagrams up to the order δ^3 .

$$\mu_0 = \lambda \Phi, \quad \bar{\mu}_0 = \bar{\lambda} \bar{\Phi}. \quad (31)$$

Finally, substituting this solution into Eq. (27) at $\delta = 1$, we have only two terms for the optimized Kähler potential. They read as follows:

$$\mathcal{V}_{\text{eff}} = -\frac{i}{2} s \text{Tr} \ln[H^{(M_0, \bar{M}_0)}] + \frac{1}{6} \lambda \bar{\lambda} \int \frac{d^4 k d^4 q}{(2\pi)^8} \int d^4 \theta \frac{1}{(k^2 + M_0 \bar{M}_0)(q^2 + M_0 \bar{M}_0)((q - k)^2 + M_0 \bar{M}_0)}, \quad (32)$$

where $H^{(M_0, \bar{M}_0)}$ is the same as $H^{(M, \bar{M})}$, defined in (19), where we replace M by $M_0 = m + \lambda \Phi$ and \bar{M} by $\bar{M}_0 = m + \bar{\lambda} \bar{\Phi}$. The first piece corresponds to the Coleman-Weinberg potential and it represents the sum of all one-loop supergraphs. The second one represents a sum of two-loop supergraphs, and the same result was derived in [27], using functional methods in superspace.

We are then left with the usual procedure of regularizing the loop integrals and adopting renormalization approaches compatible with supersymmetry [28]. In particular we can use the same strategy of [27].

IV. CONCLUDING REMARKS

We have applied superspace and supergraph techniques to carry out the LDE for the WZ model. We have shown that the LDE can be a powerful method to compute quantum corrections in the framework of supersymmetric field theories, since it reproduces an infinite sum of supergraphs using a traditional perturbative approach.

Our main effort in this paper consisted in avoiding the lengthy component-field evaluations, by proposing to implement the powerful superspace techniques in the framework of the LDE. Supersymmetry perturbation theory greatly benefits from superfield manipulations and it would be a pity if these were not brought to match with the LDE; this is why we have been motivated to carry out the present work.

In the example we have used, it was possible to find an analytical solution for the optimization procedure in order to reproduce a nonperturbative result. Notice that it was not necessary to regularize the integrals before making the optimization. In this way, we easily reproduce well-known results. However, as shown in [22,29], in more involved situations, we can make the renormalization procedure before the optimization and find numerical solutions that reproduce new results. These solutions are hard to find by using traditional methods, mainly owing to difficulties to

take into account an infinite set of superdiagrams with more than two loops. In LDE, we do not have this kind of problem because we always work with a finite set of diagrams.

The next natural systems to probe the method are the O’Raifeartaigh [30] and Fayet-Iliopoulos [31] models, which realize the spontaneous breaking of supersymmetry. We have interesting nonrenormalization theorems [28] to be satisfied and these calculations may be a good test for the LDE in the situation of supersymmetry spontaneous breaking, mainly because we have techniques to keep on working in superspace even if supersymmetry is broken, and we know how to deal with explicit θ -dependent terms [32].

More relevant for phenomenology are the models in which the soft explicitly breaking terms are introduced. The latter have been carefully classified and studied by Girardello and Grisaru, in the work of Ref. [33]. With those (phenomenologically interesting) terms, supergraph techniques become less obvious, but they have anyhow been extended to include the effect of the supersymmetry breaking parameters to all orders [9]. We believe that the application of the LDE to such a class of models may be relevant for better control of the method and also for the sake of application in phenomenologically realistic models based on supersymmetry. We shall be reporting on these results in a forthcoming work [32].

Finally, we point out that it would be of interest to investigate the control of the convergence of the LDE in superspace. This question has not yet been addressed, but its development could bring a new insight and new elements in the extension of the LDE to superspace.

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