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Superspace approach to the renormalization of the O'Raifeartaigh model up to the second order in the linear delta expansion parameter

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We adopt a superspace/supergraph formalism to pursue the investigation of the structure of one- and two-loop divergences in the frame of the minimal O'Raifeartaigh model that realizes the F-term spontaneous supersymmetry breaking. The linear delta expansion (LDE) procedure is introduced and renormalization is carried out up to the second order in the LDE expansion parameter. In agreement with the nonrenormalization theorem for the (chiral/antichiral) matter potential of $\mathcal{N} = 1$, $D = 4$ supersymmetry, our explicit supergraph calculations confirm that only the Kähler potential is actually renormalized.

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I. INTRODUCTION

Supersymmetry (SUSY) and viable procedures to investigate its explicit, spontaneous and dynamical breaking mechanisms [1] are topics of constant and renewed interest in the literature, in view of the structural role SUSY plays in the construction of field-theoretic frameworks for fundamental interactions and realistic models for elementary particle physics. In the early days of the dawn of SUSY in high-energy particle theories, it was realized that spontaneous breakdown of this new fermion/boson symmetry was a topic of major relevance in order to make contact between SUSY and observations and communicate high-energy SUSY imprints with the low energy sector of the standard model. Ever since, in different scenarios like the minimal supersymmetric standard model [2], Kaluza-Klein supergravities [3], string inspired models for elementary particle interactions [4], Seiberg-Witten super-Yang-Mills dualities [5], and brane-world scenarios [6], the problem of SUSY breaking has been suitably reassessed. More recently, SUSY breaking in connection with brane physics in general and M2-brane modeling [7], more specifically, has triggered a great deal of attention to the understanding of a number of issues related to $(2 + 1)$-dimensional supersymmetric field theories [8], like the so-called Aharony-Bergman-Jafferis-Maldacena models [9].

If, on the one hand, SUSY breaking mechanisms and their consequences are relevant for connecting high-energy fundamental physics to the regime of accelerator energies and for establishing the consistency of more formal field-theoretic models, on the other hand, one must develop technical methods to suitably carry out the SUSY breaking program and to pursue its investigation perturbatively and, hopefully, by means of some nonperturbative scheme. In this case, if it is not possible to derive an exact result, one could somehow come over this problem by devising some sort of semiperturbative scheme based upon the resummation of a certain class of (perturbative) Feynman diagrams and attain, thereby, a result that incorporates all orders in some coupling parameter.

In the frame of perturbative and resummation methods, the calculation and study of the effective potential is a viable path to be followed. In this context, different methods for effective potential calculation have been proposed in the literature soon after the idea of spontaneous symmetry breaking and the Higgs mechanism were adopted to realize the breakdown of gauge symmetries and to introduce the hierarchy of energy scales in unified models for elementary particle interactions. Here, we shall be concentrating our efforts to apply the so-called linear delta expansion (LDE) [10], suitably extended [11,12] to incorporate superspace and superfield techniques, to compute corrections at the one- and two-loop orders to the effective potential of the minimal O'Raifeartaigh model [13] which spontaneously breaks $\mathcal{N} = 1$ SUSY in four space-time dimensions. We stress that, even though SUSY is broken, a superfield approach is still the most appropriate setup to describe the problem. In a previous contribution [14], it was actually shown, in a systematic way, how all order spurion corrected superpropagators and super-Feynman rules can be employed to compute explicit 1- and 2-loop corrections to effective potentials in a scenario with an F-term breaking of $\mathcal{N} = 1$, $D = 4$ supersymmetric matter self-coupled model. The main characteristic of the LDE is to use a traditional perturbative approach together with an optimization procedure. So, in order to derive a result in all orders of the coupling constant, it is just necessary to work with a few diagrams and use perturbative renormalization techniques.

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Having in mind the ever increasing importance of understanding and proposing new scenarios for SUSY breaking, our paper sets out to tackle a specific problem—the application of the LDE at the second order to the minimal O'Raifeartaigh model—mainly motivated by the reasons that follow below:

(i) to exploit superfield techniques and superspace methods in connection with the LDE procedure to compute higher-order corrections to the effective potential of a supersymmetric field model, even if SUSY is spontaneously broken and we are obliged to deal with terms that explicitly break SUSY in superspace. This may show us nontrivial technicalities and features whenever we insist in performing superfield calculations to describe SUSY breaking. Our viewpoint is that superspace, with its corresponding tensor calculus expressed in terms of superfields, is still the most suitable tool to deal with even if SUSY is lost;

(ii) to use the supergraph approach to carry out the superspace renormalization of supersymmetric models if one adopts the LDE procedure to compute the loop corrected effective potential. In the present paper, we shall be concerned with the structure of divergences and the renormalization of the model we pick out to work with. The optimization procedure and the attainment of the full (LDE) two-loop corrected effective potential shall be reported on in a forthcoming work [15];

(iii) once the whole treatment is understood for this more traditional case of F-type breaking, we shall be able to apply it to other interesting situations, such as SUSY breaking by a D-term in supersymmetric gauge theories, R symmetry spontaneous breakdown in connection with the existence of metastable SUSY breaking vacua [16] and the computation of loop quantum contributions to the effective Kähler and chiral potentials [17].

Keeping in mind this whole framework and the motivations mentioned above, we organize the present paper according to the following outline: in Sec. II, we recall the main features of the LDE in superspace, highlighting the superfield techniques in the presence of terms that explicitly break SUSY. Section III is devoted to report and discuss the superspace evaluation of the results attained at order one in the δ parameter; next, in Sec. IV, we go a step further and analyze the divergent structure of the order two contributions, which encompass one- and two-loop supergraphs, to show how to renormalize the model to the first and second order in the parameter of LDE. With our calculations, we shall see that only the Kähler potential is actually renormalized, as expected by the nonrenormalization of the chiral potential in $\mathcal{N}=1$, $D=4$ SUSY. Finally, our concluding remarks are cast in Sec. V. Two Appendices follow: in Appendix A, we work out the main results in connection with the integration over the Grassmannian sector of superspace; in Appendix B, we present the explicit answers to the momentum-space loop integrals that appear in the course of our calculations.

II. THE LINEAR DELTA EXPANSION IN SUPERSPACE

In this section, we are going to make a brief review of the application of the linear delta expansion to supersymmetric theories. We follow Refs. [11,12]. Starting with a Lagrangian $\mathcal{L}$, let us define the following interpolated Lagrangian $\mathcal{L}^{\delta}$:

$$\mathcal{L}^{\delta} = \delta\mathcal{L}(\mu) + (1-\delta)\mathcal{L}_0(\mu), \tag{1}$$

where δ is an arbitrary parameter, $\mathcal{L}_0(\mu)$ is the free Lagrangian, and μ is a mass parameter. Note that, when $\delta = 1$, the original theory is retrieved. The δ parameter labels interactions and is used as a perturbative coupling instead of the original one. The mass parameter appears in $\mathcal{L}_0$ and $\delta\mathcal{L}_0$. The μ dependence of $\mathcal{L}_0$ is absorbed into the propagators, whereas $\delta\mathcal{L}_0$ is regarded as a quadratic interaction.

Let us now define the strategy of the method. We apply a usual perturbative expansion in δ and, at the end of the calculation, we set $\delta = 1$. Up to this stage, traditional perturbation theory is applied, working with finite Feynman diagrams, and the results are purely perturbative. However, quantities evaluated at finite order in δ explicitly depend on μ. So it is necessary to fix the μ parameter. There are two ways to do that. The first one is to use the principle of minimal sensitivity (PMS) [18]. It requires that a physical quantity, such as the effective potential $V^{(k)}(\mu)$, calculated perturbatively to order δ^k, must be evaluated at a point where it is less sensitive to the parameter μ. According to the PMS, $\mu = \mu_0$ is the solution to the equation

$$\frac{\partial V^{(k)}(\mu)}{\partial\mu}\bigg|_{\mu=\mu_0,\delta=1} = 0. \tag{2}$$

After this procedure, the optimum value, μ_0, will be a function of the original coupling and fields. Then, we replace μ_0 into the effective potential $V^{(k)}$ and obtain a nonperturbative result, since the propagator depends on μ.

The second way to fix μ is known as the fastest apparent convergence (FAC) criterion [18]. It requires that, for any k coefficient of the perturbative expansion

$$V^{(k)}(\mu) = \sum_{i=0}^{k} c_i(\mu)\delta^i, \tag{3}$$

the following relation must be fulfilled:

$$[V^{(k)}(\mu) - V^{(k-1)}(\mu)]|_{\delta=1} = 0. \tag{4}$$

Again, the μ_0 solution of the above equation will be a function of the original couplings and fields, and whenever we replace $\mu = \mu_0$ into $V(\mu)$, we obtain a nonperturbative result. Equation (4) is equivalent to taking the kth coefficient of (3) equal to zero ($c_k = 0$). If we are interested in an order-δ^k result $[V^{(k)}(\mu)]$ using the FAC criterion, it is just necessary to find the solution to the equation $c_{k+1}(\mu)|_{\mu=\mu_0} = 0$ and plug it into $V^{(k)}(\mu)$. References [19,20] provide an extensive list of successful applications of the method.

Let us now further develop the LDE for superspace applications. Following Ref. [11], for general models with chiral and antichiral superfields, we need to implement two mass parameters, μ and $\bar{\mu}$, instead of just one. In order to fix these parameters, we employ two optimization equations. Also, we need to take care of the vacuum diagrams. In general, when the effective potential is calculated in quantum field theory, we do not worry about vacuum diagrams, since they do not depend on fields. However, the vacuum diagrams depend on μ and are important to the LDE, since the arbitrary mass parameter will depend on fields after the optimization procedure. So, in the LDE, it is necessary to calculate the vacuum diagrams order by order. On the other hand, it is well known that, in superspace, vacuum superdiagrams are identically zero, by virtue of Berezin integrals. To avoid this, we have to consider, from the very beginning, the parameters μ, $\bar{\mu}$ as superfields and keep the vacuum supergraphs until the optimization procedure is carried out. In order to make the procedure clear, let us write the interpolated Lagrangian, $\mathcal{L}^\delta$, for the Wess-Zumino model discussed in [11]:

$$\begin{aligned}\mathcal{L}^\delta &= \delta\mathcal{L}(\mu,\bar{\mu}) + (1-\delta)\mathcal{L}_0(\mu,\bar{\mu}) \\ &= \int d^4\theta\bar{\Phi}\Phi + \int d^2\theta\left(\frac{M}{2}\Phi^2 + \frac{\delta\lambda}{3!}\Phi^3 - \frac{\delta\mu}{2}\Phi^2\right) \\ &\quad + \int d^2\bar{\theta}\left(\frac{\bar{M}}{2}\bar{\Phi}^2 + \frac{\delta\bar{\lambda}}{3!}\bar{\Phi}^3 - \frac{\delta\bar{\mu}}{2}\bar{\Phi}^2\right), \end{aligned} \tag{5}$$

where m is the original mass, $M = m + \mu$ and $\bar{M} = m + \bar{\mu}$. Now, one has a new chiral and antichiral quadratic interaction proportional to $\delta\mu$ and $\delta\bar{\mu}$. Also the superpropagator will have a dependence on μ and $\bar{\mu}$. From the generating superfunctional in the presence of the chiral (J) and antichiral ($\bar{J}$) sources

$$\begin{aligned}\tilde{Z}[J,\bar{J}] &= \exp\left[iS_{\rm INT}\left(\frac{1}{i}\frac{\delta}{\delta J}, \frac{1}{i}\frac{\delta}{\delta\bar{J}}\right)\right] \\ &\quad\times \exp\left[\frac{i}{2}(J,\bar{J})G^{(M,\bar{M})}\begin{pmatrix}J\\ \bar{J}\end{pmatrix}\right], \end{aligned} \tag{6}$$

we can write the supereffective action:

$$\begin{aligned}\Gamma[\Phi,\bar{\Phi}] &= -\frac{i}{2}\ln[s\,{\rm Det}(G^{(M,\bar{M})})] - i\ln\tilde{Z}[J,\bar{J}] \\ &\quad - \int d^6z J(z)\Phi(z) - \int d^6\bar{z}\,\bar{J}(z)\bar{\Phi}(z), \end{aligned} \tag{7}$$

where $G^{(M,\bar{M})}$ is the matrix propagator and $s\,{\rm Det}(G^{(M,\bar{M})})$ is the superdeterminant of $G^{(M,\bar{M})}$, which, in general, is equal to one; but here we keep it, because $G^{(M,\bar{M})}$ depends on μ and $\bar{\mu}$. Also, due to the μ and $\bar{\mu}$ dependence, the supergenerator of the vacuum diagrams, $\tilde{Z}[0,0]$, is not identically equal to one. We can define the normalized functional generator as $Z_N = \frac{\tilde{Z}[J,\bar{J}]}{\tilde{Z}[0,0]}$, and write the effective action as

$$\Gamma[\Phi,\bar{\Phi}] = -\frac{i}{2}\ln[s\,{\rm Det}(G)] - i\ln\tilde{Z}[J_0,\bar{J}_0] + \Gamma_N[\Phi,\bar{\Phi}], \tag{8}$$

where the sources J_0 and $\bar{J}_0$ are defined by the equations

$$\begin{aligned}\frac{\delta W[J,\bar{J}]}{\delta J(z)}\bigg|_{J=J_0} &= \frac{\delta W[J,\bar{J}]}{\delta\bar{J}(z)}\bigg|_{\bar{J}=\bar{J}_0} = \frac{\delta\tilde{Z}[J,\bar{J}]}{\delta J(z)}\bigg|_{J=J_0} \\ &= \frac{\delta\tilde{Z}[J,\bar{J}]}{\delta\bar{J}(z)}\bigg|_{\bar{J}=\bar{J}_0} = 0. \end{aligned} \tag{9}$$

In (8), the first two terms represent the vacuum diagrams (which are usually zero) and $\Gamma_N[\Phi,\bar{\Phi}]$ is the usual contribution to the effective action.

Let us now derive the interpolated Lagrangian and the new Feynman rules for the O'Raifeartaigh model. The simplest O'Raifeartaigh model is described by the following Lagrangian:

$$\begin{aligned}\mathcal{L} &= \int d^4\theta\bar{\Phi}_i\Phi_i \\ &\quad - \left[\int d^2\theta(\xi\Phi_0 + m\Phi_1\Phi_2 + g\Phi_0\Phi_1^2) + {\rm H.c.}\right], \end{aligned} \tag{10}$$

where $i = 0, 1, 2$.

Following Ref. [12], in order to take into account the nonperturbative contributions of all fields of the model, we need to implement the LDE with the matrix mass parameters μ_{ij} and $\bar{\mu}_{ij}$. Adding and subtracting these mass terms in the Lagrangian of a general O'Raifeartaigh model we obtain

$$\mathcal{L}(\mu,\bar{\mu}) = \mathcal{L}_0(\mu,\bar{\mu}) + \mathcal{L}_{\rm int}(\mu,\bar{\mu}), \tag{11}$$

where

$$\begin{aligned}\mathcal{L}_0(\mu,\bar{\mu}) &= \int d^4\theta\bar{\Phi}_i\Phi_i \\ &\quad - \left[\int d^2\theta\left(\xi_i\Phi_i + \frac{1}{2}M_{ij}\Phi_i\Phi_j\right) + {\rm H.c.}\right], \end{aligned} \tag{12}$$

$$\begin{aligned}&\mathcal{L}_{\rm int}(\mu,\bar{\mu}) \\ &\quad = -\left[\int d^2\theta\left(\frac{1}{3!}g_{ijk}\Phi_i\Phi_j\Phi_k - \frac{1}{2}\mu_{ij}\Phi_i\Phi_j\right) + {\rm H.c.}\right], \end{aligned} \tag{13}$$

with $M_{ij} = m_{ij} + \mu_{ij}$ and $i, j, k = 0, 1, 2$ are symmetrical indices.

Let us expand the arbitrary mass parameters as chiral and antichiral superfields:

$$\mu_{ij} = \lambda_{ijk}\varphi_k = \lambda_{ijk}(\rho_k + \theta^2\chi_k) = \lambda_{ijk}\rho_k + \lambda_{ijk}\chi_k\theta^2 = \rho_{ij} + b_{ij}\theta^2, \tag{14}$$

so that

$$M_{ij} = m_{ij} + \mu_{ij} = (m_{ij} + \rho_{ij}) + b_{ij}\theta^2 = a_{ij} + b_{ij}\theta^2. \tag{15}$$

Now, the interpolated Lagrangian (1) becomes

$$\mathcal{L}^{\delta} = \mathcal{L}_0^{\delta} + \mathcal{L}_{\rm int}^{\delta}, \tag{16}$$

where the free Lagrangian, $\mathcal{L}_0^{\delta}$, is

$$\mathcal{L}_0^{\delta} = \int d^4\theta \bar{\Phi}_i\Phi_i - \bigg[\int d^2\theta\Big(\xi_i\Phi_i + \frac{1}{2}a_{ij}\Phi_i\Phi_j + \frac{1}{2}b_{ij}\theta^2\Phi_i\Phi_j\Big) + \text{H.c.}\bigg], \tag{17}$$

and the interaction Lagrangian reads as follows:

$$\mathcal{L}_{\rm int}^{\delta} = -\bigg[\int d^2\theta\Big(\frac{\delta}{3!}g_{ijk}\Phi_i\Phi_j\Phi_k - \frac{\delta}{2}\mu_{ij}\Phi_i\Phi_j\Big) + \text{H.c.}\bigg]. \tag{18}$$

Notice that the interaction Lagrangian has now soft breaking terms proportional to the μ components. We are going to treat these terms perturbatively in δ, like all interactions.

Now, in order to get the simplest O'Raifeartaigh model when $\delta = 1$ [10], we make the choices

$$\xi_0 = \xi; \quad M_{01} = a_{01} = \rho_{01} = a; \quad M_{11} = b_{11}\theta^2 = b\theta^2;$$
$$M_{12} = a_{12} = m_{12} + \rho_{12} = m + \rho = M; \quad g_{011} = g, \tag{19}$$

and all other ξ_i and M_{ij} set to zero. With that, we obtain

$$\mathcal{L}_0^{\delta} = \int d^4\theta\bar{\Phi}_i\Phi_i - \bigg[\int d^2\theta\Big(\xi\Phi_0 + M\Phi_1\Phi_2 + a\Phi_0\Phi_1 + \frac{1}{2}b\theta^2\Phi_1^2\Big) + \text{H.c.}\bigg],$$
$$\mathcal{L}_{\rm int}^{\delta} = -\bigg[\int d^2\theta\Big(\delta g\Phi_0\Phi_1^2 - \delta\rho\Phi_1\Phi_2 - \delta a\Phi_0\Phi_1 - \frac{\delta}{2}b\theta^2\Phi_1^2\Big) + \text{H.c.}\bigg]. \tag{20}$$

As is well known, this O'Raifeartaigh model has an R symmetry. The R charges of chiral superfields Φ_0, Φ_1, Φ_2 are respectively $R_0 = 2$, $R_1 = 0$, and $R_2 = 2$. In order to preserve the R symmetry in the interpolated Lagrangian, the R charges of the parameters a and b are $R_a = 0$ and $R_b = 0$, which must be preserved after the optimization procedure.

The new propagators can be derived from the free Lagrangian, which also has an explicit dependence on θ and $\bar{\theta}$ from the μ and $\bar{\mu}$ components. Using the techniques developed in [21], the propagators can be written as

$$\begin{aligned}
\langle\Phi_0\bar{\Phi}_0\rangle &= (k^2 + |M|^2)A(k)\delta_{12}^4 + |a|^2|b|^2B(k)\theta_1^2\bar{\theta}_1^2\delta_{12}^4;\\
\langle\Phi_0\bar{\Phi}_1\rangle &= \bar{a}bC(k)\frac{1}{16}D_1^2\bar{D}_1^2\theta_1^2\delta_{12}^4;\\
\langle\Phi_0\bar{\Phi}_2\rangle &= -M\bar{a}A(k)\delta_{12}^4 + M\bar{a}|b|^2B(k)\theta_1^2\bar{\theta}_1^2\delta_{12}^4;\\
\langle\Phi_1\bar{\Phi}_1\rangle &= E(k)\delta_{12}^4 + |b|^2B(k)\frac{1}{16}D_1^2\theta_1^2\bar{\theta}_1^2\bar{D}_1^2\delta_{12}^4;\\
\langle\Phi_1\bar{\Phi}_2\rangle &= -M\bar{b}F(k)\bar{\theta}_1^2\delta_{12}^4;\\
\langle\Phi_2\bar{\Phi}_2\rangle &= (k^2 + |a|^2)A(k)\delta_{12}^4 + |M|^2|b|^2B(k)\theta_1^2\bar{\theta}_1^2\delta_{12}^4;\\
\langle\Phi_0\Phi_0\rangle &= -|a|^2\bar{b}C(k)\frac{1}{4}D_1^2\theta_1^2\delta_{12}^4;\\
\langle\Phi_0\Phi_1\rangle &= \bar{a}A(k)\frac{1}{4}D_1^2\delta_{12}^4 - \bar{a}|b|^2B(k)\frac{1}{4}\theta_1^2\bar{\theta}_1^2D_1^2\delta_{12}^4;\\
\langle\Phi_0\Phi_2\rangle &= -\bar{M}\,\bar{a}\,\bar{b}\,C(k)\frac{1}{4}D_1^2\theta_1^2\delta_{12}^4;\\
\langle\Phi_1\Phi_1\rangle &= \bar{b}F(k)\frac{1}{4}\bar{\theta}_1^2D_1^2\delta_{12}^4;\\
\langle\Phi_1\Phi_2\rangle &= \bar{M}A(k)\frac{1}{4}D_1^2\delta_{12}^4 - \bar{M}|b|^2B(k)\frac{1}{4}D_1^2\theta_1^2\bar{\theta}_1^2\delta_{12}^4;\\
\langle\Phi_2\Phi_2\rangle &= -|M|^2\bar{b}C(k)\frac{1}{4}D_1^2\theta_1^2\delta_{12}^4,
\end{aligned} \tag{21}$$

with

$$\begin{aligned}
A(k) &= \frac{1}{k^2(k^2 + |M|^2 + |a|^2)},\\
B(k) &= \frac{1}{(k^2 + |M|^2 + |a|^2)[(k^2 + |M|^2 + |a|^2)^2 - |b|^2]},\\
C(k) &= \frac{1}{k^2[(k^2 + |M|^2 + |a|^2)^2 - |b|^2]},\\
E(k) &= \frac{1}{k^2 + |M|^2 + |a|^2},\\
F(k) &= \frac{1}{(k^2 + |M|^2 + |a|^2)^2 - |b|^2}.
\end{aligned}$$

In usual quantum field theories, the optimized parameters appear at the poles of the propagators, as mass terms. Here, it should be emphasized the nontrivial dependence on the parameters a, b, and ρ, which appear not only at the poles, but also in the numerators.

We can also write the new Feynman rules for the vertices:

$$\Phi_0\Phi_1^2 \text{ vertex}: 2\delta g\int d^4\theta; \quad \Phi_1\Phi_2 \text{ vertex}: -\delta\rho\int d^4\theta;$$
$$\Phi_0\Phi_1 \text{ vertex}: -\delta a\int d^4\theta; \Phi_1\Phi_1 \text{ vertex}: -\frac{\delta b}{2}\int d^4\theta\theta^2. \tag{22}$$

Now we have all the necessary ingredients to calculate the effective potential using perturbation theory in δ. In the next section we show the order one results.

III. ORDER ONE RESULTS

The perturbative effective potential can now be calculated in powers of δ using the one particle irreducible functions, defined in the expansion of the effective action, taking into account vacuum diagrams. In Ref. [12], it was shown that, after the optimization procedure at order one, the optimized effective potential provides the sum of all one-loop diagrams. In that case, analytical solutions were obtained for the optimization procedure before calculating the superspace and momentum integrals. However, in order to go beyond the one-loop approximation, it is necessary to go beyond the order δ^1. In this case, it is not possible to find solutions before evaluating the superspace and momentum integrals and the optimization procedure must be carried out after the renormalization of the theory. Owing to the nontrivial dependence of the propagators on the optimized parameters, it is not clear that the method does not alter the divergences structure of the model. We investigate this fact here.

In Fig. 1, one can see the diagrammatic sum of the effective potential up to the order δ^1 ($\mathcal{V}^{(1)}_{\rm eff}$).

Note that, by virtue of the θ-dependent propagators, the tadpole diagrams are not identically zero, as usual in superspace. The first diagram is of order δ^0 and corresponds to the first term of the effective action expansion, defined in (8)
.

Using the Feynman rules and the results of [12], the expressions for the diagrams of the figure above, in the order they appear are:

(i) Order δ^0 (vacuum diagram):

$$\mathcal{G}^{(1)\delta^0} = \frac{1}{2}\int d^4\theta_{12}\delta^4_{21}\,\mathrm{Tr}\ln[P^T K]\delta^4_{21}, \qquad (23)$$

where $d^4\theta_{12} = d^4\theta_1 d^4\theta_2$ and Tr is the trace over the chiral multiplets defined in the real basis by $(\Phi^T, \bar{\Phi})^T$. Details of this calculation can be seen in Refs. [12,22]. The matrix P is defined by the chiral projectors $P_+ = \frac{\bar{D}^2 D^2}{16\Box}$ and $P_- = \frac{D^2\bar{D}^2}{16\Box}$ as

$$P = \begin{pmatrix} 0 & P_- \\ P_+ & 0 \end{pmatrix}, \qquad (24)$$

and

$$K = \begin{pmatrix} \left(AP_- + B\frac{1}{\Box^{1/2}}\eta_-\right)\frac{D^2}{4\Box} & \mathbf{1}_3 \\ \mathbf{1}_3 & \left(\bar{A}P_+ + \bar{B}\frac{1}{\Box^{1/2}}\bar{\eta}_+\right)\frac{\bar{D}^2}{4\Box} \end{pmatrix}, \qquad (25)$$

with

$$A = \begin{pmatrix} 0 & a & 0 \\ a & 0 & M \\ 0 & M & 0 \end{pmatrix}, \qquad B = \begin{pmatrix} 0 & 0 & 0 \\ 0 & b & 0 \\ 0 & 0 & 0 \end{pmatrix},$$
$$\eta_- = \Box^{1/2}P_-\theta^2 P_-, \qquad \bar{\eta}_+ = \Box^{1/2}P_+\bar{\theta}^2 P_+, \qquad (26)$$

is the quadratic part of the free Lagrangian (there are also quadratic terms in the interaction Lagrangian, which depend on the optimization parameters). Using the results of [12], Eq. (23) can be written as

$$\mathcal{G}^{(1)\delta^0} = \frac{1}{2}\int \frac{d^4k}{(2\pi)^4}\ln\left[1 - \frac{|b|^2}{(k^2 + |M|^2 + |a|^2)^2}\right]. \qquad (27)$$

(ii) Order δ^1:

$$\mathcal{G}_1^{(1)\delta^1} = -2\delta g\bar{b}\int\frac{d^4k}{(2\pi)^4}F(k)\int d^4\theta\bar{\theta}^2\Phi_0 + \text{H.c.};$$
$$\mathcal{G}_2^{(1)\delta^1} = \frac{1}{2}\delta|b|^2\int\frac{d^4k}{(2\pi)^4}F(k)\int d^4\theta\theta^2\bar{\theta}^2 + \text{H.c.};$$
$$\mathcal{G}_3^{(1)\delta^1} = 4\delta g\bar{a}|b|^2\int\frac{d^4k}{(2\pi)^4}B(k)\int d^4\theta\theta^2\bar{\theta}^2\Phi_1 + \text{H.c.};$$
$$\mathcal{G}_4^{(1)\delta^1} = -\delta|a|^2|b|^2\int\frac{d^4k}{(2\pi)^4}B(k)\int d^4\theta\theta^2\bar{\theta}^2 + \text{H.c.};$$
$$\mathcal{G}_5^{(1)\delta^1} = -\delta\rho\bar{M}|b|^2\int\frac{d^4k}{(2\pi)^4}B(k)\int d^4\theta\theta^2\bar{\theta}^2 + \text{H.c.}. \qquad (28)$$

Note that all the expressions for the superdiagrams were written as an integral over $d^4\theta$, according to the nonrenormalization theorem. In Appendix A we give the final results for the superspace integrals. Using the results

$$\int d^4\theta\bar{\theta}^2\Phi_0 = \int d^2\theta\Phi_0,$$
$$\int d^4\theta\theta^2\bar{\theta}^2 = 1\,\mathrm{e}$$
$$\int d^4\theta\theta^2\bar{\theta}^2\Phi_1 = \int d^2\theta\theta^2\Phi_1, \qquad (29)$$

the effective potential up to the order δ^1 is given by

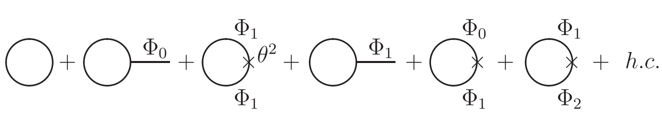

FIG. 1. Effective potential up to the order δ^1.

$$\mathcal{V}_{\rm eff}^{(1)} = \mathcal{G}^{(1)\delta^0} + \sum_{i=1}^{5} \mathcal{G}_i^{(1)\delta^1} = \frac{1}{2}\int d^4\theta_{12}\delta_{12}^4 \,\mathrm{Tr}\ln[P^T K]\delta_{12}^4$$
$$+ \delta\int \frac{d^4k}{(2\pi)^4} F(k)\Big\{-2g\bar{b}\int d^2\theta\Phi_0 + \frac{1}{2}|b|^2 + \mathrm{H.c.}\Big\}$$
$$+ \delta\int \frac{d^4k}{(2\pi)^4} B(k)\Big\{4g\bar{a}|b|^2\int d^2\theta\theta^2\Phi_1 - |a|^2|b|^2 - \rho\bar{M}|b|^2 + \mathrm{H.c.}\Big\}. \tag{30}$$

Now we are going to regularize the integrals that appear in the effective potential up to the order δ^1. The diagrams of order δ^1 are shown in Fig. 2.

We are going to use the notation defined in (B9). Using the results of Appendix B, these one-loop diagrams are written as

$$\mathcal{G}_1^{(1)\delta^1} = 2\delta g\int \frac{d^4kd^4\theta_{12}}{(2\pi)^4}\Phi_0(1)\delta_{12}^4 \times\Big[-\frac{1}{4}\bar{D}_1^2(k)\langle\Phi_1\Phi_1\rangle\Big] + \mathrm{H.c.}$$
$$= -2\delta g\bar{b}\int\frac{d^4k}{(2\pi)^4}F(k)\int d^4\theta\bar{\theta}^2\Phi_0 + \mathrm{H.c.}$$
$$= -\frac{4\delta g b\langle F_0\rangle}{\kappa}\frac{1}{\epsilon} + \frac{2\delta g\langle F_0\rangle}{\kappa} \times(\eta^+\overline{\ln}\eta^+ - \eta^-\overline{\ln}\eta^- - 2b); \tag{31}$$

$$\mathcal{G}_2^{(1)\delta^1} = -\frac{\delta b}{2}\int\frac{d^4kd^4\theta_{12}}{(2\pi)^4}\theta_1^2\delta_{12}^4\Big[-\frac{1}{4}\bar{D}_1^2(k)\langle\Phi_1\Phi_1\rangle\Big] + \mathrm{H.c.}$$
$$= \frac{\delta|b|^2}{2}\int\frac{d^4k}{(2\pi)^4}F(k)\int d^4\theta\theta^2\bar{\theta}^2 + \mathrm{H.c.}$$
$$= \frac{\delta b^2}{\kappa}\frac{1}{\epsilon} - \frac{\delta b}{2\kappa}(\eta^+\overline{\ln}\eta^+ - \eta^-\overline{\ln}\eta^- - 2b); \tag{32}$$

$$\mathcal{G}_3^{(1)\delta^1} = 4\delta g\int\frac{d^4kd^4\theta_{12}}{(2\pi)^4}\Phi_1(1)\delta_{12}^4 \times\Big[-\frac{1}{4}\bar{D}_1^2(k)\langle\Phi_0\Phi_1\rangle\Big] + \mathrm{H.c.}$$
$$= 4\delta g\bar{a}|b|^2\int\frac{d^4k}{(2\pi)^4}B(k)\int d^4\theta\theta^2\bar{\theta}^2\Phi_1 + \mathrm{H.c.}$$
$$= -\frac{4\delta g a\langle\varphi_1\rangle}{\kappa}(2\eta^2\overline{\ln}\eta^2 - \eta^+\overline{\ln}\eta^+ - \eta^-\overline{\ln}\eta^-); \tag{33}$$

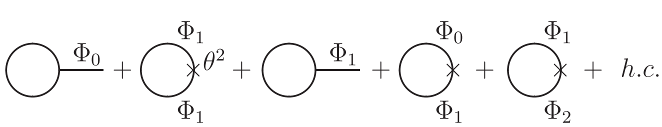

FIG. 2. One-loop diagrams of order δ^1.

$$\mathcal{G}_4^{(1)\delta^1} = -\delta a\int\frac{d^4kd^4\theta_{12}}{(2\pi)^4}\delta_{12}^4 \times\Big[-\frac{1}{4}\bar{D}_1^2(k)\langle\Phi_0\Phi_1\rangle\Big] + \mathrm{H.c.}$$
$$= -\delta|a|^2|b|^2\int\frac{d^4k}{(2\pi)^4}B(k)\int d^4\theta\theta^2\bar{\theta}^2 + \mathrm{H.c.}$$
$$= \frac{\delta a^2}{\kappa}(2\eta^2\overline{\ln}\eta^2 - \eta^+\overline{\ln}\eta^+ - \eta^-\overline{\ln}\eta^-); \tag{34}$$

$$\mathcal{G}_5^{(1)\delta^1} = -\delta\rho\int\frac{d^4kd^4\theta_{12}}{(2\pi)^4}\delta_{12}^4\Big[-\frac{1}{4}\bar{D}_1^2(k)\langle\Phi_1\Phi_2\rangle\Big] + \mathrm{H.c.}$$
$$= -\delta\rho\bar{M}|b|^2\int\frac{d^4k}{(2\pi)^4}B(k)\int d^4\theta\theta^2\bar{\theta}^2 + \mathrm{H.c.}$$
$$= \frac{\delta\rho M}{\kappa}(2\eta^2\overline{\ln}\eta^2 - \eta^+\overline{\ln}\eta^+ - \eta^-\overline{\ln}\eta^-), \tag{35}$$

with $\Phi_0(1) = \Phi_0(p = 0, \theta_1, \bar{\theta}_1)$.

Let us deal with the divergent ones, $\mathcal{G}_1^{(1)\delta^1}$ and $\mathcal{G}_2^{(1)\delta^1}$, and apply the renormalization procedure. We are going to use the $\overline{MS}$ scheme. The diagram $\mathcal{G}_2^{(1)\delta^1}$ is a vacuum diagram, so its renormalization is trivial. We just need to cancel the divergence with a constant counterterm, which implies a redefinition of the vacuum energy. Since the tadpole diagrams are not identically zero, the interpolated theory has a new divergence, which apparently is not canceled by a counterterm of the Kahleriań-type potential. To renormalize the divergence in $\mathcal{G}_1^{(1)\delta^1}$, we need the counterterm

$$\frac{2\delta g b}{\kappa\epsilon}\int d^4\theta\bar{\theta}^2\Phi_{0R} + \mathrm{H.c.}$$
$$= \frac{2\delta g b}{\kappa\epsilon}\int d^2\theta\Phi_{0R} + \mathrm{H.c.}, \tag{36}$$

where Φ_{0R} denotes the renormalized superfield. It seems that the chiral potential is renormalized, which is strange because the renormalization structure of the theory should not be modified by the soft terms introduced by the method. However, this counterterm depends on b, which must be a solution of the optimization procedure.

The renormalized effective potential up to the order δ^1 is

$$\mathcal{V}^{(1)}_{\rm eff}=\frac{1}{(4\pi)^2}\bigg\{\frac{1}{4}(M^2+a^2)^2\ln\bigg[1-\frac{b^2}{(M^2+a^2)^2}\bigg]+\frac{b}{2}(M^2+a^2)\ln\bigg[\frac{M^2+a^2+b}{M^2+a^2-b}\bigg]+\frac{b^2}{4}\ln\bigg[\frac{(M^2+a^2)^2-b^2}{\mu^4}\bigg]-\frac{3b^2}{4}\bigg\}$$
$$+\frac{\delta}{(4\pi)^2}\bigg\{b(b-4g\langle F_0\rangle)+2[a(a-4g\langle\varphi_1\rangle)+\rho M](M^2+a^2)\overline{\ln}[M^2+a^2]+\bigg[a(4g\langle\varphi_1\rangle-a)+\frac{1}{2}(4g\langle F_0\rangle-b)-\rho M\bigg]$$
$$\times(M^2+a^2+b)\overline{\ln}[M^2+a^2+b]+\bigg[a(4g\langle\varphi_1\rangle-a)-\frac{1}{2}(4g\langle F_0\rangle-b)-\rho M\bigg](M^2+a^2-b)\overline{\ln}[M^2+a^2-b]\bigg\}. \quad (37)$$

At this stage, we have a perturbative result for the effective potential. In order to get a nonperturbative result, we apply the optimization procedure. Since we split the parameters M_{ij} into a θ-independent (a_{ij}) and a θ-dependent (b_{ij}) part, and recalling (19), the optimized parameters will be $a_{01}=a$, $b_{11}=b$, and $\rho_{12}=\rho$. Using the PMS criterion to find the optimized parameters a, b, and ρ, we have to solve the three coupled equations

$$\frac{\partial\mathcal{V}^{\delta^2}_{\rm eff}}{\partial a}\bigg|_{a=a_0}=\frac{\partial\mathcal{V}^{\delta^2}_{\rm eff}}{\partial b}\bigg|_{b=b_0}=\frac{\partial\mathcal{V}^{\delta^2}_{\rm eff}}{\partial\rho}\bigg|_{\rho=\rho_0}=0, \quad (38)$$

at $\delta=1$, and plug the optimized values a_0, b_0, and ρ_0 into (37). We find the following analytical solutions:

$$a_0=4g\langle\varphi_1\rangle=\bar{a}_0$$
$$b_0=4g\langle F_0\rangle=\bar{b}_0, \quad (39)$$
$$\rho_0=0=\bar{\rho}_0.$$

This result shows that the optimized parameters are functions of the original coupling and fields, as we expected. It is easy to see that this is also a solution of the FAC criterion. However, as we have three optimized parameters, it is not clear how we may write three optimization equations using this criterion. Replacing these values in (30), all the δ^1 terms vanish and the optimized potential is written as

$$\mathcal{V}^{(1)}_{\rm eff}=\mathcal{G}^{(1)\delta^0}$$
$$=\frac{1}{(8\pi)^2}\bigg\{(m^2+16g^2\langle\varphi_1\rangle^2)^2\ln\bigg[1-\frac{16g^2\langle F_0\rangle^2}{(m^2+16g^2\langle\varphi_1\rangle^2)^2}\bigg]$$
$$+8g\langle F_0\rangle(m^2+16g^2\langle\varphi_1\rangle^2)\ln\bigg[\frac{m^2+16g^2\langle\varphi_1\rangle^2+4g\langle F_0\rangle}{m^2+16g^2\langle\varphi_1\rangle^2-4g\langle F_0\rangle}\bigg]$$
$$+16g^2\langle F_0\rangle^2\ln\bigg[\frac{(m^2+16g^2\langle\varphi_1\rangle^2)^2-16g^2\langle F_0\rangle^2}{\mu^4}\bigg]-48g^2\langle F_0\rangle^2\bigg\}. \quad (40)$$

This is the Coleman-Weinberg potential for the O'Raifeartaigh model [21] and represents the sum of all one-loop diagrams, i.e., a nonperturbative result, because it takes into account infinite orders of the original coupling constant.

Let us come back to the counterterm defined in (36). Since Φ_0 and Φ_1 are classical superfields, the optimized parameters can be written as

$$a_0=4g\int d^2\bar{\theta}\bar{\Phi}_1=4g\int d^2\theta\Phi_1=4g\langle\varphi_1\rangle,$$
$$b_0=4g\int d^2\bar{\theta}\bar{\Phi}_0=4g\int d^2\theta\Phi_0=4g\langle F_0\rangle. \quad (41)$$

This is in agreement with the R charges of the parameters a and b. Thus, putting the expression above for b_0 into (36) we see that the counterterm needed to renormalize the divergence in $\mathcal{G}^{(1)\delta^1}_1$ is of the form

$$\frac{8\delta g^2}{\kappa\epsilon}\int d^4\theta\bar{\Phi}_{0R}\Phi_{0R}+\text{H.c.}=\frac{16\delta g^2}{\kappa\epsilon}\int d^4\theta\bar{\Phi}_{0R}\Phi_{0R}, \quad (42)$$

and we note that in fact, after the optimization procedure, only the Kähler potential is renormalized, in agreement with the nonrenormalization theorem. In the next section we discuss the divergences at order δ^2.

IV. ORDER TWO RESULTS

At order two we have one- and two-loop diagrams. Let us start with the one-loop diagrams. They have three topologies: diagrams with two external legs and no insertions, diagrams with one external leg and one insertion, and vacuum diagrams with no external legs and two insertions. There are 42 such diagrams (plus the Hermitian conjugates). However, since our main concern in the present work is to analyze the divergent structure of the theory, we write below only the three divergent diagrams, shown in Fig. 3:

$$\mathcal{G}_1^{(1)\delta^2} = 2(2\delta g)(2\delta g)\int \frac{d^4kd^4\theta_{12}}{(2\pi)^4}\Phi_0(1)\bar{\Phi}_0(2)\Big[-\frac{1}{4}\bar{D}_1^2(k)\langle\Phi_1\bar{\Phi}_1\rangle\Big]\Big[-\frac{1}{4}D_2^2(-k)\langle\bar{\Phi}_1\Phi_1\rangle\Big]$$
$$= \frac{8\delta^2g^2}{16}\int\frac{d^4k}{(2\pi)^4}\Big\{E(k)E(k)\mathcal{J}_1(\theta,\bar\theta) + 2\frac{|b|^2}{16}E(k)B(k)\mathcal{J}_2(\theta,\bar\theta) + \frac{|b|^4}{(16)^2}B(k)B(k)\mathcal{J}_3(\theta,\bar\theta)\Big\}$$
$$= \frac{8\delta^2g^2\langle F_0\rangle^2}{\kappa}\Big(\frac{1}{\epsilon} - \overline{\ln}\eta^2\Big) + \frac{2\delta^2g^2\langle F_0\rangle^2}{\kappa b}[4b\overline{\ln}\eta^2 - (\eta^2+2b)\overline{\ln}\eta^+ + (\eta^2-2b)\overline{\ln}\eta^- + 2b]. \tag{43}$$

$$\mathcal{G}_2^{(1)\delta^2} = 2(2\delta g)\Big(-\frac{\delta\bar{b}}{2}\Big)\int\frac{d^4kd^4\theta_{12}}{(2\pi)^4}\Phi_0(1)\bar\theta_2^2\Big[-\frac{1}{4}\bar{D}_1^2(k)\langle\Phi_1\bar{\Phi}_1\rangle\Big]\Big[-\frac{1}{4}D_2^2(-k)\langle\bar{\Phi}_1\Phi_1\rangle\Big] + \text{H.c.}$$
$$= -\frac{2\delta^2g}{16}\bar{b}\int\frac{d^4k}{(2\pi)^4}\Big\{E(k)E(k)\mathcal{J}_4(\theta,\bar\theta) + 2\frac{|b|^2}{16}E(k)B(k)\mathcal{J}_5(\theta,\bar\theta) + \frac{|b|^4}{(16)^2}B(k)B(k)\mathcal{J}_6(\theta,\bar\theta)\Big\} + \text{H.c.}$$
$$= -\frac{4\delta^2gb\langle F_0\rangle}{\kappa}\Big(\frac{1}{\epsilon} - \overline{\ln}\eta^2\Big) - \frac{\delta^2gb\langle F_0\rangle}{\kappa b}[4b\overline{\ln}\eta^2 - (\eta^2+2b)\overline{\ln}\eta^+ + (\eta^2-2b)\overline{\ln}\eta^- + 2b]. \tag{44}$$

$$\mathcal{G}_3^{(1)\delta^2} = 2\Big(\frac{\delta b}{2}\Big)\Big(\frac{\delta\bar b}{2}\Big)\int\frac{d^4kd^4\theta_{12}}{(2\pi)^4}\theta_1^2\bar\theta_2^2\Big[-\frac{1}{4}\bar{D}_1^2(k)\langle\Phi_1\bar{\Phi}_1\rangle\Big]\Big[-\frac{1}{4}D_2^2(-k)\langle\bar{\Phi}_1\Phi_1\rangle\Big]$$
$$= \frac{\delta^2}{2(16)}|b|^2\int\frac{d^4k}{(2\pi)^4}\Big\{E(k)E(k)\mathcal{J}_7(\theta,\bar\theta) + 2\frac{|b|^2}{16}E(k)B(k)\mathcal{J}_8(\theta,\bar\theta) + \frac{|b|^4}{(16)^2}B(k)B(k)\mathcal{J}_9(\theta,\bar\theta)\Big\}$$
$$= \frac{\delta^2b^2}{2\kappa}\Big(\frac{1}{\epsilon} - \overline{\ln}\eta^2\Big) + \frac{\delta^2b^2}{8\kappa b}[4b\overline{\ln}\eta^2 - (\eta^2+2b)\overline{\ln}\eta^+ + (\eta^2-2b)\overline{\ln}\eta^- + 2b]. \tag{45}$$

There are four two-loop diagrams (plus the Hermitian conjugates). They are all vacuum diagrams and differ from each other by the propagators appearing in the loops. Out of these four diagrams, only one gives a divergent contribution, and we show it in Fig. 4.

As the one-loop diagram $\mathcal{G}_3^{(1)\delta^2}$, this is a vacuum diagram, and its renormalization is trivial. Again, we just need to cancel the divergence with a constant counterterm, which implies a redefinition of the vacuum energy.

To renormalize the divergent term in $\mathcal{G}_1^{(1)\delta^2}$ we introduce the counterterm

$$-\frac{8\delta^2g^2}{\kappa\epsilon}\int d^4\theta\Phi_{0R}\bar\Phi_{0R}, \tag{46}$$

and, for the divergent term in $\mathcal{G}_2^{(1)\delta^2}$, we introduce the counterterm

$$\frac{2\delta^2g}{\kappa\epsilon}\int d^4\theta\bar\theta^2b\Phi_{0R} + \text{H.c.} \tag{47}$$

As in the previous section, when we renormalized the effective potential up to the order δ^1, the counterterm introduced for the divergent term in $\mathcal{G}_2^{(1)\delta^2}$ is proportional to b, and, at one-loop, the optimized parameter b_0 is given by Eq. (41). It can be shown that this solution is valid for all orders in δ at one-loop level. Thus, the counterterm is proportional to

$$\delta^2\int d^4\theta\Phi_{0R}\bar\Phi_{0R}, \tag{48}$$

showing that in fact, at one-loop, only the wave function is renormalized. However, owing to the two-loop vacuum diagrams, for the effective potential up to the order δ^2, the optimized parameters defined in (39) are no longer solutions to the PMS equations. In fact, when diagrams like the one in Fig. 4 are taken into account, we can derive two-loop nonperturbative corrections to the Coleman-Weinberg potential. To this end it is necessary to evaluate all the order δ^2 diagrams and solve numerically a complicated set of equations. This is a work in progress [15]. Here, we are interested in the renormalization structure of the theory. Since the counterterms depend only on b, let us concentrate on this parameter. Although it is not possible to find an analytical solution for the b parameter at order δ^2, we can write the general form for it. Based on R charge

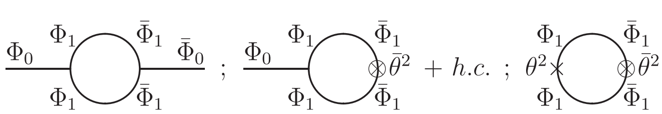

FIG. 3. Divergent diagrams $\mathcal{G}_1^{(1)\delta^2}$, $\mathcal{G}_2^{(1)\delta^2}$, and $\mathcal{G}_3^{(1)\delta^2}$.

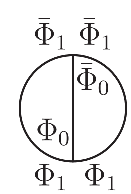

FIG. 4. Divergent diagrams $\mathcal{G}_1^{(1)\delta^2}$, $\mathcal{G}_2^{(1)\delta^2}$, and $\mathcal{G}_3^{(1)\delta^2}$.

considerations, on the fact that b_0 must be a function of the original component fields and on the Lagrangian dependence on b, we can argue that the optimized parameter up to the order δ^2 must be

$$b_0 = \int d^2\bar{\theta}^2[4g\bar{\Phi}_0 + \bar{A}_{2\text{-loop}}], \tag{49}$$

where $\bar{A}_{2\text{-loop}}$ is a two-loop correction with R charge equal to -2. Since all the counterterms are proportional to $\int d^4\theta\Phi_{0R}\bar{\Phi}_{0R}$ or $\int d^2\theta b\Phi_{0R}$, it can be seen that only the Kähler potential is renormalized.

V. CONCLUDING REMARKS

Our efforts in the present work have been focused on the application of superfield techniques and supergraph calculations to study the renormalization of the minimal O'Raifeartaigh model in the LDE scheme to the second order in the expansion parameter. Our calculations show that only the Kähler potential gets renormalized, according to what should be expected from the $\mathcal{N} = 1$, $D = 4$ SUSY nonrenormalization theorem for the chiral potential. We point out that, in this paper, we are actually interested in understanding and mastering the superfield approach if we adopt the LDE procedure to compute higher-order corrections to the effective potential in the case of spontaneously broken SUSY.

Here, we have not yet concentrated on the task of effective potential calculation. As already stated in the Introduction, this is the matter of a forthcoming work. Our main purpose in the present paper was to check the consistency and the efficacy of superfield and supergraph methods to deal with SUSY explicitly breaking terms in a higher-order loop computation in superspace. We have checked, with our explicit supergraph computations, that the extended super-Feynman rules are perfectly consistent even though these explicitly breaking terms show up. The structure of divergences has suitably been treated and the final result of the Kähler potential renormalization is a good check of our manipulations. Once the renormalization task is accomplished, it remains to be done—and we believe this requires a forthcoming work—the complete two-loop calculation to allow us to proceed to the next step, namely, the optimization in the LDE parameter, to finally write down the full two-loop corrected effective potential. This demands a nontrivial work in terms of Feynman supergraph computation and numerical computation and we shall be soon reporting on our results [15].

Once this whole program of higher-order corrected effective potentials has been accomplished for the F-term SUSY breaking, we believe it would be worthwhile to concentrate efforts on the LDE effective potential calculation in the case of D-term SUSY breakings in the gauge sector and to pay attention to the problem of metastable SUSY breaking vacua and its connection with R symmetry spontaneous breaking, which has direct consequences to the physics of the so-called lightest supersymmetric particle.

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APPENDIX A: SUPERSPACE INTEGRALS

This Appendix is devoted to the evaluation of some of the superspace one-loop integrals which arise in order δ^2. The integrals that appear in the expressions for the divergent one-loop diagrams of order δ^2 are

$$\mathcal{J}_1(\theta,\bar{\theta}) = \int d^4\theta_{12}\Phi_0(1)\bar{\Phi}_0(2)[\bar{D}_1^2(k)\delta_{12}^4] \times [D_2^2(-k)\delta_{12}^4] = 16\langle F_0\rangle^2; \tag{A1}$$

$$\begin{aligned}\mathcal{J}_2(\theta,\bar{\theta}) &= \int d^4\theta_{12}\Phi_0(1)\bar{\Phi}_0(2)[\bar{D}_1^2(k)\delta_{12}^4] \\ &\quad\times [D_2^2(-k)\bar{D}_2^2(-k)\bar{\theta}_2^2\theta_2^2D_2^2(-k)\delta_{12}^4] \\ &= (16)^2\langle F_0\rangle^2;\end{aligned} \tag{A2}$$

$$\begin{aligned}\mathcal{J}_3(\theta,\bar{\theta}) &= \int d^4\theta_{12}\Phi_0(1)\bar{\Phi}_0(2) \\ &\quad\times [\bar{D}_1^2(k)D_1^2(k)\theta_1^2\bar{\theta}_1^2\bar{D}_1^2(k)\delta_{12}^4] \\ &\quad\times [D_2^2(-k)\bar{D}_2^2(-k)\bar{\theta}_2^2\theta_2^2D_2^2(-k)\delta_{12}^4] \\ &= (16)^3\langle F_0\rangle^2;\end{aligned} \tag{A3}$$

$$\begin{aligned}\mathcal{J}_4(\theta,\bar{\theta}) &= \int d^4\theta_{12}\Phi_0(1)[\bar{D}_1^2(k)\delta_{12}^4][\bar{\theta}_2^2D_2^2(-k)\delta_{12}^4] \\ &= 16\langle F_0\rangle;\end{aligned} \tag{A4}$$

$$\begin{aligned}\mathcal{J}_5(\theta,\bar{\theta}) &= \int d^4\theta_{12}\Phi_0(1)[\bar{D}_1^2(k)\bar{\theta}_1^2D_1^2(k)\delta_{12}^4] \\ &\quad\times [\theta_2^2\bar{D}_2^2(-k)\bar{\theta}_2^2D_2^2(-k)\delta_{12}^4] \\ &= (16)^2\langle F_0\rangle;\end{aligned} \tag{A5}$$

$$\begin{aligned}\mathcal{J}_6(\theta,\bar{\theta}) &= \int d^4\theta_{12}\Phi_0(1)[\bar{D}_1^2(k)D_1^2(k)\theta_1^2\bar{\theta}_1^2\bar{D}_1^2(k)\delta_{12}^4] \\ &\quad\times [\bar{\theta}_2^2D_2^2(-k)\bar{D}_2^2(-k)\bar{\theta}_2^2\theta_2^2D_2^2(-k)\delta_{12}^4] \\ &= (16)^3\langle F_0\rangle;\end{aligned} \tag{A6}$$

$$\mathcal{J}_7(\theta,\bar{\theta}) = \int d^4\theta_{12}[\theta_1^2\bar{D}_1^2(k)\delta_{12}^4][\bar{\theta}_2^2D_2^2(-k)\delta_{12}^4] = 16; \tag{A7}$$

$$\mathcal{J}_8(\theta,\bar\theta) = \int d^4\theta_{12}[\theta_1^2\bar D_1^2(k)\bar\theta_1^2 D_1^2(k)\delta_{12}^4] \times[\theta_2^2\bar D_2^2(-k)\bar\theta_2^2 D_2^2(-k)\delta_{12}^4] = (16)^2; \tag{A8}$$

$$\mathcal{J}_9(\theta,\bar\theta) = \int d^4\theta_{12}[\theta_1^2\bar D_1^2(k)D_1^2(k)\theta_1^2\bar\theta_1^2\bar D_1^2(k)\delta_{12}^4] \times[\bar\theta_2^2 D_2^2(-k)\bar D_2^2(-k)\bar\theta_2^2\theta_2^2 D_2^2(-k)\delta_{12}^4] = (16)^3. \tag{A9}$$

APPENDIX B: MOMENTUM-SPACE INTEGRALS

In this Appendix we collect the expressions for the one-loop integrals, which are not so easy to obtain directly. We use dimensional regularization.

We begin with the simple integrals [22]:

$$\begin{aligned}
J(m^2) &= \frac{(\mu^2)^\epsilon}{(2\pi)^D}\int\frac{d^Dk}{k^2+m^2} = \frac{(\mu^2)^\epsilon}{(4\pi)^{2-\epsilon}}\Gamma(-1+\epsilon)(m^2)^{1-\epsilon}\\
J_n(m^2) &= \int\frac{d^Dk}{(2\pi)^D\mu^{D-4}}\frac{1}{k^2}\frac{1}{(k^2+m^2)}\\
&= \frac{(m^2)^{1-n}}{16\pi^2}\left(4\pi\frac{\mu^2}{m^2}\right)^{2-(D/2)}\frac{\pi}{\Gamma(D/2)\sin\pi(D/2-n)}\\
L_n(m^2) &= \int\frac{d^Dk}{(2\pi)^D\mu^{D-4}}\frac{1}{k^{2n}}\ln\left(1+\frac{m^2}{k^2}\right) = \frac{m^2}{D/2-n}J_n(m^2),
\end{aligned} \tag{B1}$$

where $D = 4-2\epsilon$ and μ is the renormalization scale. Expanding these expressions, we obtain the useful equalities:

$$J(m^2) = \frac{1}{16\pi^2}\left[-\frac{m^2}{\epsilon} + m^2\left(\ln\frac{m^2}{\mu^2}-1\right)\right], \tag{B2}$$

$$J_0(m^2) = -\frac{m^2}{16\pi^2}\left[\frac{1}{\epsilon}+1-\ln\frac{m^2}{\mu^2}\right], \tag{B3}$$

$$J_1(m^2) = \frac{1}{16\pi^2}\left[\frac{1}{\epsilon}+1-\ln\frac{m^2}{\mu^2}\right], \tag{B4}$$

$$L_0(m^2) = -\frac{1}{2}\frac{m^4}{16\pi^2}\left[\frac{1}{\epsilon}+\frac{3}{2}-\ln\frac{m^2}{\mu^2}\right], \tag{B5}$$

$$L_1(m^2) = \frac{m^2}{16\pi^2}\left[\frac{1}{\epsilon}+2-\ln\frac{m^2}{\mu^2}\right]. \tag{B6}$$

In the calculation of the order δ^0 diagram, we have to solve the integral

$$K(m^2,M^2) = \int\frac{d^Dk}{(2\pi)^D\mu^{D-4}}\,\mathrm{tr}\ln\left(1-\frac{M^2\bar M^2}{(k^2+m^2\bar m^2)^2}\right), \tag{B7}$$

which can be written as a sum of integrals of the type (B5)

$$K(m^2,M^2) = \mathrm{tr}[L_0(m^2+\tilde m^2)+L_0(m^2-\tilde m^2)-2L_0(m^2)], \tag{B8}$$

with $\tilde m^2 = (M^2\bar M^2)^{1/2}$.

In the following, we define

$$\eta^2 = M^2+a^2, \qquad \eta^\pm = M^2+a^2\pm b, \tag{B9}$$

and we adopt the same notation of [23–25].

Using the definition of the propagators we now show the results for the integrals appearing in the one-loop diagrams of orders δ^1 and δ^2. To solve them we use the method of partial fraction, splitting each integral as a sum of other integrals with just one propagator in the integrand. Doing this, we obtain

$$\begin{aligned}
I_1(k) &= \int\frac{d^4k}{(2\pi)^4}F(k) = \int\frac{d^4k}{(2\pi)^4}\frac{1}{(k^2+M^2+a^2)^2-b^2}\\
&= -\frac{1}{2b}J(\eta^+)+\frac{1}{2b}J(\eta^-)\\
&= \frac{1}{\kappa\epsilon}-\frac{1}{2\kappa b}(\eta^+\overline{\ln}\eta^+-\eta^-\overline{\ln}\eta^--2b),
\end{aligned} \tag{B10}$$

$$\begin{aligned}
I_2(k) &= \int\frac{d^4k}{(2\pi)^4}B(k)\\
&= \int\frac{d^4k}{(2\pi)^4}\frac{1}{(k^2+M^2+a^2)[(k^2+M^2+a^2)^2-b^2]}\\
&= -\frac{1}{b^2}J(\eta^2)+\frac{1}{2b^2}J(\eta^+)+\frac{1}{2b^2}J(\eta^-)\\
&= -\frac{1}{2\kappa b^2}(2\eta^2\overline{\ln}\eta^2-\eta^+\overline{\ln}\eta^+-\eta^-\overline{\ln}\eta^-),
\end{aligned} \tag{B11}$$

$$\begin{aligned}
I_3(k) &= \int\frac{d^4k}{(2\pi)^4}E(k)E(k)\\
&= \int\frac{d^4k}{(2\pi)^4}\frac{1}{(k^2+M^2+a^2)(k^2+M^2+a^2)} = \tilde J(\eta^2)\\
&= \frac{1}{\kappa\epsilon}-\frac{1}{\kappa}\overline{\ln}\eta^2,
\end{aligned} \tag{B12}$$

$$I_4(k) = \int \frac{d^4k}{(2\pi)^4} B(k)E(k) = \int \frac{d^4k}{(2\pi)^4} \frac{1}{(k^2+M^2+a^2)[(k^2+M^2+a^2)^2-b^2](k^2+M^2+a^2)}$$
$$= \frac{1}{b^4}\tilde{J}(\eta^2) + \frac{1}{4b^4}\tilde{J}(\eta^+) + \frac{1}{4b^4}\tilde{J}(\eta^-) + \frac{3}{4b^5}J(\eta^+) - \frac{3}{4b^5}J(\eta^-) = \frac{1}{2\kappa b^3}(2b\overline{\ln}\eta^2 - \eta^+\overline{\ln}\eta^+ + \eta^-\overline{\ln}\eta^- + 2b), \tag{B13}$$

$$I_5(k) = \int \frac{d^4k}{(2\pi)^4} B(k)B(k) = \int \frac{d^4k}{(2\pi)^4} \frac{1}{(k^2+M^2+a^2)[(k^2+M^2+a^2)^2-b^2](k^2+M^2+a^2)[(k^2+M^2+a^2)^2-b^2]}$$
$$= \frac{1}{b^4}\tilde{J}(\eta^2) + \frac{1}{4b^4}\tilde{J}(\eta^+) + \frac{1}{4b^4}\tilde{J}(\eta^-) + \frac{3}{4b^5}J(\eta^+) - \frac{3}{4b^5}J(\eta^-)$$
$$= -\frac{1}{4\kappa b^5}[4b\overline{\ln}\eta^2 - (3\eta^+ - b)\overline{\ln}\eta^+ + (3\eta^- + b)\overline{\ln}\eta^- + 6b]. \tag{B14}$$

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