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Decoherence of Quantum Superpositions In Relativistic Settings

Levy Bruno do Nascimento Batista

Orientador
Dr. George E. A. Matsas

Co-orientador
Dr. André G. S. Landulfo
Dr. Robert B. Mann

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Orientador: Gerge E. A. Matsas
Coorientador: André G. S. Landulfo
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DECOHERENCE OF QUANTUM SUPERPOSITIONS IN RELATIVISTIC SETTINGS

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Comissão Examinadora:

Prof. Dr. GEORGE EMANUEL AVRAAM MATSAS(Orientador)
Instituto de Física Teórica/UNESP

Prof. Dr. DANIEL AUGUSTO TUROLLA VANZELLA
Instituto de Física de São Carlos da Universidade de São Paulo-IFSC-USP

Prof. Dr. GABRIEL SANTOS MENEZES
Instituto de Física da Universidade de São Paulo-IFUSP

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"So if you say you take an ordinary person
who's willing to devote a great deal of time,
and study, and work, and thinking, and Mathematics,
then he's become a scientist!"

Richard P. Feynman, *Ways of Thinking*

Resumo

Recentemente, Danielson, Satishchandran e Wald propuseram uma nova forma de decoerência de cargas e massas em um estado de superposição espacial quando colocadas em um espaço-tempo com horizontes de Killing. Eles argumentaram que o efeito é causado principalmente pela emissão de partículas soft através desses horizontes, o que eventualmente ocorrerá mesmo se o processo for assumido como adiabático. Acontece que essa decoerência está intimamente relacionada à presença de quanta de baixa frequência na radiação térmica, conforme descrita por observadores estacionários nos espaços de Schwarzschild e de Rindler, que surge dependendo da escolha do estado de vácuo.

Investigamos essa afirmação em um cenário diferente, com um detector sem gap interagindo com um campo escalar massivo. Apresentamos como tal interação é descrita e como quantificar a decoerência resultante. Em seguida, analisamos nosso modelo proposto em dois casos seminais: quando os componentes da superposição espacial são inerciais e quando estão uniformemente acelerados no espaço de Minkowski. Destacamos as semelhanças e diferenças entre nossos resultados e os de estudos anteriores quando o sistema é ajustado em um regime análogo.

Por fim, discutimos o que buscamos aprimorar com nosso modelo e como ele pode esclarecer algumas questões deixadas em aberto no trabalho original.

Palavras Chaves: Decoerência; Detectores de Unruh-DeWitt; Efeito Unruh; Partículas Soft; Campo Escalar Massivo.

Áreas do conhecimento: Física; Física Teórica; Teoria Quântica de Campos; Relatividade Geral.

Abstract

Recently, Danielson, Satishchandran, and Wald have proposed a novel source of decoherence of charges and masses in a spatial superposition state when placed in a spacetime with Killing horizons. They argued that the effect is primarily caused by the emission of soft particles through these horizons, which will eventually occur even if the process is assumed to be adiabatic. It turns out this decoherence is intimately related to the presence of low-frequency quanta in the thermal radiation, as described by stationary observers in the Schwarzschild and Rindler spaces, which arises depending on the choice of vacuum state. We investigate this claim in a rather different setting, with a gapless detector interacting with a massive scalar field. We introduce how such interaction is described and how to quantify the resulting decoherence. Then, we investigate our proposed model in two seminal cases: We consider that the components of the spatial superposition are inertial and that they are uniformly accelerating in the Minkowski space. We highlight the similarities and differences between our results and those of previous studies when the system is tuned in an analogous regime. Finally, we discuss what we sought to improve with our model and how it might clarify some questions left open in the original work.

Keywords: Decoherence; Unruh-DeWitt Detectors; Unruh Effect; Soft Particles; Massive Scalar Field.

Knowledge areas: Physics; Theoretical Physics; Quantum Field Theory; General Relativity.

Contents

Chapter 1

Introduction

Quantum mechanics and special relativity are undoubtedly two of the most successful physical theories of the twentieth century. Both were crucial to provide a reasonable explanation for the biggest open problems by the beginning of the last century, e.g., the UV catastrophe of the black body radiation and the nature of the Ether, respectively. Quantum mechanics presents a powerful framework for addressing issues on the microscopic level, enabling a consistent description of many atomic phenomena. On the other hand, special relativity, when it postulates that the physical laws are the same in all inertial frames and that the speed of light is absolute, represents a huge change of paradigm from the long-standing Newtonian mechanics, in particular ruling out the notion of the universality of simultaneity.

The features introduced by relativity showed that the Newtonian theory of gravity was no longer consistent, mostly because it allowed a blatant violation of causality in its field equations. In that regard, Einstein proposed general relativity in 1915 as a new description of gravity, which had incorporated the fundamental aspects of special relativity. That replaced the idea of gravitational interactions as a force with the assumption that matter distributions curve the spacetime, and that is the resultant geometry that determines how free bodies move. The first accomplishment of general relativity was correctly predicting the precession of Mercury's perihelion, which matched the observational data. However, its most remarkable achievement in those early times was the prediction of the bending of light when it propagates next to a massive body, like a star, and this was confirmed by observations in 1917. All of that contributed to elevating general relativity to the position of one of the cornerstones of theoretical physics.

However, it was not the end of the story. Subsequent developments of these theories led to new phenomena that required increasingly complex models to investigate them. Physicists reached the unavoidable point where both quantum and relativistic features were expected to play an important role in describing these models. The attempt to insert causality and locality into quantum mechanics led to what one calls quantum field theory (QFT). In this new framework, the fundamental object is a field, and its interactions with itself and other fields enable a proper description of a broad range of phenomena involving elementary particles, such as scatterings and decays.

The influence of gravity in these cases is usually assumed to be negligible; thus, quantum field theory could be entirely formulated in Minkowski spacetime. The interest in such a theory in different spacetimes gained force as many models were proposed to investigate the role of fundamental particles in cosmological problems. In this context, Fulling [1] identified that the canonical quantization of a scalar field in flat space in terms of Rindler coordinates led to some ambiguities regarding the definition of particles, in the sense that the field quanta in that description were not the same as what one finds when the procedure is done using inertial coordinates. Ultimately, the Hilbert spaces for the field states defined in each case are different, and that raised some doubts concerning the physical reality of these Rindler modes.

Hawking's work [2] on field quantization in the vicinity of a spherically symmetric body undergoing gravitational collapse, resulting in a black hole, added a new point of view on this issue. He demonstrated that, despite what was known from the classical theory, black holes emit thermal radiation at a steady rate after a sufficiently long time as if they were ordinary bodies with temperature $\kappa/2\pi$ such that κ is the surface gravity. The prediction of Hawking radiation was one of the foundational arguments for the study of black hole thermodynamics, once it supported Bekenstein's proposition [3] that the area of a black hole's event horizon represents a measure of its entropy. Davies [4] interpreted these results in the Rindler space as if an observer undergoing uniform acceleration a could see a fixed wall irradiating at temperature $a/2\pi$. The presence of an inertial surface to explain the creation of particles in a flat spacetime seemed rather paradoxical, as no external agent provided the required energy for it.

Unruh addressed all these questions [5], and these observations led to the precise definition of what was later known as the Unruh effect. In fact, the field prescription according to inertial and uniformly accelerated observers is not equivalent, and as a consequence, the definition of particles can no longer be universal. Rather, it depends on the observer's worldline, which is directly related to the orbits of the timelike Killing vectors in a given region of spacetime. Remarkably, it was shown that the field state which a family of inertial observers in flat space would describe as a no-particle state, characterized by the Minkowski vacuum state, is seen as a thermal state by a uniformly accelerated observer; in other words, they feel as being immersed in a thermal bath at the same temperature predicted by Davies. Hence, the importance of a suitable particle detector becomes clear, as the interpretation of physical phenomena is somewhat more intuitive when one describes them in terms of particles.

In this context, Unruh and Wald [6], and DeWitt independently, proposed a simple yet powerful semiclassical model for a particle detector consisting of a 2-level point monopole that is linearly coupled to the field. There is an energy gap between these two internal states, and one defines the detection event as the detector being found in the excited state at a certain time after the interaction has occurred. The authors looked into the field response to a uniformly accelerated detector, showing that inertial and accelerated observers have different interpretations of a detection event. They also shed light on possible paradoxes that it might bring. It is important to emphasize that the detector used in our work is rather gapless; the reason and subtleties of its description will be provided in the next section.

Therefore, one might naively conclude that understanding key aspects of quantum field theory in curved spaces allowed theoretical physicists to construct a complete quantum theory of gravity. Unfortunately, that is not true, as in all works cited previously, one has the assumption that spacetime is essentially determined by a classical metric whose corresponding geometry determines the causal structure to be followed by the fields. Nevertheless, it is possible to highlight two great reasons why it is such an important framework. First, QFT in curved spaces has a wide range of applicability, as Hawking argued [2] that its breakdown might occur when the space's curvature is of the order of the Planck scale. This is expected to be observed only near a black hole singularity and in the early universe.

Also, it is expected to provide a low-energy limit for any correct quantum gravity proposal. This is important when it comes to experimentally probing quantum-mechanical effects in which gravity may play a decisive role. In particular, a promising route is probing the decoherence of quantum systems in different setups [7–9], where both massive particles and photons were considered, and an interesting discussion on whether gravity, as described by Newtonian theory, could lead to the same results is raised. Recently, a *gedankenexperiment* has been proposed [10–12], where a potential paradox involving violations of causality and complementarity is avoided if one assumes that the gravitational field is quantized and admits local vacuum fluctuations.

In light of this scenario, it was shown [13] that a charge in a spatial quantum superposition placed near a Killing horizon will invariably decohere due to the radiation of soft photons into the horizon. We investigate this claim in a rather different setting: we analyze the loss of information of a qubit in a spatial superposition in Rindler space interacting with a massive scalar field.

The present work is structured as follows: first, a brief review of the quantization of scalar fields in globally hyperbolic spacetimes, particularly in the Rindler space, is provided. Next, we discuss the model of a degenerate Unruh-DeWitt detector and how the time evolution operator of interest can be computed non-perturbatively. Then, we review Danielson, Satishchandran, and Wald's work [13–15] in more detail for the electromagnetic case, and briefly comment on the gravitational analog phenomenon. Finally, we investigate how the decoherence in our proposed model depends on the relevant parameters for the situations where one has an inertial superposition in flat space and a stationary one in Rindler space. We observe that our model yields some results that differ from those previously obtained; we then discuss why these differences are reasonable in our context. A situation outside this regime is also explored, and it is verified that the system exhibits an interesting behavior when the components are far apart.

Chapter 2

Quantum scalar fields in globally hyperbolic spacetimes

2.1 Overview

Let M be a 4-dimensional Lorentzian manifold equipped with a metric g_{ab}, whose signature is assumed to be $(-,+,+,+)$. (M, g_{ab}) is said to be globally hyperbolic if there exists a 3-surface Σ such that its Cauchy development corresponds to M, e.g., $D(\Sigma) = M$. Σ is called a Cauchy surface of M. Recall that, for an arbitrary subset $A \subset M$, the Cauchy development, also known as the domain of dependence, of A is defined as [16]

$$D(A) \equiv \{p \in M \mid \text{every inextensible causal curve} \\ \text{that passes through p intercepts A}\}.$$

An important feature satisfied by all globally hyperbolic spacetimes is that they are diffeomorphic to $(\mathbb{R} \times \Sigma, g_{ab})$, so there exists a differentiable function $t : M \to \mathbb{R}$ and a family of smooth Cauchy surfaces Σ_t such that $\Sigma_{t=const} = \{p \in M \mid t(p) = const\}$ [17]. In other words, M can be completely foliated by a family of Cauchy surfaces indexed by a real parameter t, and if t is associated with a given coordinate time, $\{\Sigma_t\}$ become spacelike 3-surfaces.

Globally hyperbolicity is an essential condition for turning the initial value problem for any dynamical equation into a well-posed problem. In particular, the quantization of scalar fields starts from finding the classical solutions to the KG equation

$$(\nabla_a \nabla^a - m^2)\phi = 0, \tag{2.1}$$

with ϕ being the scalar field, m its mass and ∇_a is covariant derivative operator compatible with the metric g_{ab}. The existence and uniqueness of such a solution are summarized by the following theorem [18].

Theorem: Let (M, g_{ab}) be a globally hyperbolic spacetime with a Cauchy surface Σ and $(\phi_0, \pi_0) \in C^\infty(\Sigma) \times C^\infty(\Sigma)$ be a pair of smooth real functions defined on Σ. There exists one, and only one, solution $\phi \in S \subset C^\infty(M)$ to Eq. (2.1) such that $\phi|_\Sigma = \phi_0$ and $n^a \nabla_a \phi|_\Sigma = \pi_0$. n^a is the future-directed normal vector to Σ.

Moreover, one defines the anti-symmetric bilinear form

$$\Omega(\phi_1, \phi_2) \equiv \int_{\Sigma_t} d\mathbf{x} \sqrt{h} (\phi_2 n^a \nabla_a \phi_1 - \phi_1 n^a \nabla_a \phi_2), \tag{2.2}$$

$h = |\det(h_{ab})|$, h_{ab} is the induced metric on Σ_t, formally defined as the pullback $h = i^* g$ of g_{ab} under the inclusion map $i : \Sigma \hookrightarrow M$, $\mathbf{x}$ are coordinates on Σ_t and ϕ_1, ϕ_2 are smooth solutions of Eq. (2.1) whose initial values have compact support on Σ_t; this last condition guarantees that Eq. (2.2) converges.

It's important to emphasize that Ω does not depend on the choice of Cauchy surface Σ_t, and extending it to the space of complex solutions of Klein-Gordon equation $S_\mu^{\mathbb{C}}$[1], one defines the Klein-Gordon inner product

$$(f_1, f_2)_{KG} \equiv -i\Omega(\overline{f}_1, f_2), \tag{2.3}$$

such that $\overline{f}_1$ denotes the complex conjugate of f_1. Note that $(.,.)_{KG}$ is not positive-definite, so $(S_\mu^{\mathbb{C}}, (.,.)_{KG})$ cannot be a Hilbert space. Then, the quantization scheme proceeds by finding a subspace $\mathcal{H} \subset S_\mu^{\mathbb{C}}$ such that

i) $S_\mu^{\mathbb{C}} = \mathcal{H} \oplus \overline{\mathcal{H}}$, where $\overline{\mathcal{H}}$ is the complex conjugate space of $\mathcal{H}$;

ii) $(.,.)_{KG}$ is positive-definite in $\mathcal{H}$;

iii) Let $f_1 \in \mathcal{H}$ and $f_2 \in \overline{\mathcal{H}}$ be a positive and negative frequency solution, respectively, then $(f_1, f_2)_{KG} = 0$.

A crucial remark at this point is that there are infinitely many choices of $\mathcal{H}$, and, a priori, there is no physical reason to privilege a given representation over the others. This holds even in Minkowski spacetime, however, Poincaré invariance provides an additional criterion to obtain a preferred representation [19]. In general, the existence of spacetime symmetries is useful to select a space $\mathcal{H}$ as it seems a natural choice for a given family of observers.

[1] In order to properly perform this Cauchy-completion procedure in an infinite-dimensional space, it is necessary first to introduce a real inner product μ and use it to enlarge the original space of solutions S to S_μ and then complexify it, obtaining $S_\mu^{\mathbb{C}}$ [19].

The next step is to construct the space of field states, which is given by the symmetric Fock space $\mathcal{F}_S(\mathcal{H})$ associated with $\mathcal{H}$

$$\mathcal{F}_S(\mathcal{H}) \equiv \bigoplus_{n=0}^{\infty} \left(\bigotimes_{k=0}^{n} {}_S\mathcal{H} \right) \tag{2.4}$$

In analogy with the standard canonical quantization procedure, one defines the creation $a^\dagger(\sigma) : \mathcal{F}_S(\mathcal{H}) \to \mathcal{F}_S(\mathcal{H})$ and annihilation $a(\overline{\tau}) : \mathcal{F}_S(\mathcal{H}) \to \mathcal{F}_S(\mathcal{H})$ operators such that their action upon $\Psi = (c, \psi_1, \psi_2, ...)$ is given by

$$a^\dagger(\sigma)\Psi \equiv (0, c\sigma, \sqrt{2}\sigma \otimes_S \psi_1, \sqrt{3}\sigma \otimes_S \psi_2, ...) \tag{2.5}$$

and

$$a(\overline{\tau})\Psi \equiv ((\tau, \psi_1)_{KG}, \sqrt{2}\tau \cdot \psi_2, \sqrt{3}\tau \cdot \psi_3, ...), \tag{2.6}$$

respectively. Note that $\sigma, \tau \in \mathcal{H}$, $\psi_1 \otimes_s \psi_2 \equiv \frac{1}{2}(\psi_1 \otimes \psi_2 + \psi_2 \otimes \psi_1)$ and, as $\psi_n \in \bigotimes_{k=0}^{n} {}_S\mathcal{H}$, one finds that $\sigma \cdot \psi_n \equiv (\sigma, \psi_n)_{KG} \in \bigotimes_{k=0}^{n-1} {}_S\mathcal{H}$, where the inner product is taken with respect to the first Hilbert space of the tensor product. Also, $a(\overline{\tau})$ is defined as a function of an element of $\overline{\mathcal{H}}$ rather than an element of $\mathcal{H}$, so it is linear on these vectors.

These operators are responsible for creating σ modes and annihilating τ modes, respectively. From their definition, Eqs. (2.5) and (2.6), we can obtain their commutator

$$[a(\overline{\tau}), a^\dagger(\sigma)] = (\tau, \sigma)_{KG}\mathbb{I}, \tag{2.7}$$

with $\mathbb{I}$ being the identity operator. Additionally, the vacuum state $|0\rangle \in \mathcal{F}_S(\mathcal{H})$ is defined such that $a(\overline{\tau})\,|0\rangle = 0$ for all $\tau \in \mathcal{H}$. Now, it is convenient to introduce an orthonormal basis on $\mathcal{H}$ $\{u_j \mid j \in \mathbb{N}\}$, with $(u_i, u_j)_{KG} = \delta_{ij}$, and consequently $\{\overline{u}_j \mid j \in \mathbb{N}\}$ is an orthonormal basis on $\overline{\mathcal{H}}$ equipped with the inner product $-(.,.)_{KG}$, e.g., $(\overline{u}_i, \overline{u}_j)_{KG} = -\delta_{ij}$, hence the field operator can be defined as

$$\phi \equiv \sum_j (u_j a(\overline{u}_j) + \overline{u}_j a^\dagger(u_j)). \tag{2.8}$$

The expression above has some issues, such as the infinite sum over the modes j does not converge, and to make it mathematically well defined, the field operator has to be defined as an operator-valued distribution [19]. Let $f \in C_0^\infty(M)$ be a test function with compact support on M, and let Af and Rf be advanced and retarded solutions, respectively, to the KG equation with source f

$$\begin{cases} (\nabla_a \nabla^a - m^2)(Af) = f, \\ (\nabla_a \nabla^a - m^2)(Rf) = f. \end{cases} \tag{2.9}$$

Thus, one defines the map $E : C_0^\infty(M) \to S_\mu^{\mathbb{C}}$ as

$$Ef \equiv Af - Rf = \int_M d^4x' \sqrt{-g(x')}(G^{adv}(x,x') - G^{ret}(x,x'))f(x'),$$

such that $G^{adv}(x,x')$ and $G^{ret}(x,x')$ are the corresponding advanced and retarded Green functions [20]. Moreover, let $K : S_\mu^{\mathbb{C}} \to \mathcal{H}$ be an operator that takes the part of ϕ with positive Klein-Gordon norm, e.g., $\phi = \phi^+ + \phi^- \Rightarrow K\phi = \phi^+$; analogously, $\overline{K} : S_\mu^{\mathbb{C}} \to \overline{\mathcal{H}}$, then $\overline{K}\phi = \phi^-$. In particular, if $\phi \in S \subset S_\mu^{\mathbb{C}}$, ϕ is real and, consequently, $\overline{K}\phi = \phi^- = \overline{\phi^+} = \overline{K\phi}$.

It is possible to rewrite Eq. (2.8) in terms of the KG inner product

$$\phi = \sum_j \left((u_j, \phi)_{KG} u_j - (\overline{u}_j, \phi)_{KG} \overline{u}_j\right), \tag{2.10}$$

so, the action of K and $\overline{K}$ upon ϕ results in $K\phi = \sum_j (u_j, \phi)_{KG} u_j$ and $\overline{K}\phi = -\sum_j (\overline{u}_j, \phi)_{KG} \overline{u}_j$, respectively. Now, we have all the tools needed to define the smeared quantum field operator using a test function f

$$\phi(f) \equiv \int_M d^4x \sqrt{-g}\phi f(x). \tag{2.11}$$

Using Eq. (2.8) and the linearity of the creation and annihilation operators in their argument, one ends up with

$$\begin{aligned} \phi(f) &= ia(-\sum_j (\overline{u}_j, Ef)_{KG} \overline{u}_j) - ia^\dagger (\sum_j (u_j, Ef)_{KG} u_j) \\ &= ia(\overline{KEf}) - ia^\dagger (KEf). \end{aligned} \tag{2.12}$$

It is worth noting that an important step to obtain Eq. (2.12) consists of using the following relations, which will be crucial throughout this work

$$\begin{cases} \int_M d^4x \sqrt{-g} u_j f(x) = -i(\overline{u}_j, Ef)_{KG}, \\ \int_M d^4x \sqrt{-g} \overline{u}_j f(x) = -i(u_j, Ef)_{KG}. \end{cases} \tag{2.13}$$

Additionally, for the sake of completeness, we also present a covariant version of the canonical commutation relations satisfied by the smeared quantum fields, as a consequence of Eqs. (2.7) and (2.13)

$$[\phi(f_1), \phi(f_2)] = -i\Delta(f_1, f_2)\mathbb{I}, \tag{2.14}$$

with $f_1, f_2 \in C_0^\infty(M)$ and $\Delta(f_1, f_2) = \int_M d^4x\sqrt{-g} f_1 E f_2$ [21].

2.2 Rindler space and the Unruh effect

Let us now derive one of the most remarkable results that the field quantization in globally hyperbolic spacetimes has unveiled: the Unruh effect. First, let us explicitly show the usefulness of spacetime symmetries for choosing a specific $\mathcal{H}$. Let (M, g_{ab}) be a static spacetime, e.g., there exists a coordinate system such that the line element can be written as

$$ds^2 = -f(\mathbf{x})dt^2 + h_{ij}(\mathbf{x})dx^i dx^j, \tag{2.15}$$

with $f(\mathbf{x}) > 0$ [16]. Naturally, $K^a \equiv \left(\frac{\partial}{\partial t}\right)^a$ is a timelike Killing field whose orbits represent possible worldlines of a family of observers. It is worth mentioning that interpreting a QFT in terms of particles is extremely difficult in nonstationary spacetimes[2] [22]. With this metric, Eq. (2.1) becomes[3]

$$\left(-\frac{\partial^2}{\partial t^2} - \mathcal{K}\right)\phi = 0, \tag{2.16}$$

with

$$\mathcal{K}\phi \equiv f\left(\frac{1}{\sqrt{-g}}\partial_i(\sqrt{-g}h^{ij}\partial_j\phi) + m^2\phi\right). \tag{2.17}$$

$\mathcal{K}$ is a Hermitian and positive operator, and since we are considering only Cauchy-complete spaces, $\mathcal{K}$ is self-adjoint [1]. Hence, let $\{\psi_j\}$, j is the set of quantum numbers that fully describe a given state, be eigenfunctions of $\mathcal{K}$ with the corresponding eigenvalues $\omega_j^2 \geq 0$ such that

$$\mathcal{K}\psi_j = \omega_j^2\psi_j. \tag{2.18}$$

[2]Static spacetimes are also stationary, the converse is not true though.

[3]Recall that, in term of components, the operator $\nabla_a\nabla^a$ is given by $\nabla_\mu\nabla^\mu = -\frac{1}{\sqrt{-g}}\partial_\mu(\sqrt{-g}g^{\mu\nu}\partial_\nu)$ in any coordinate system [18].

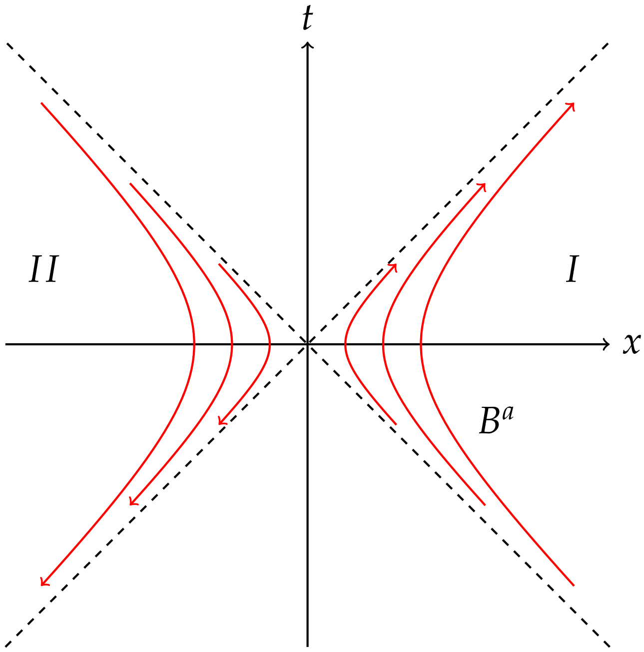

Figure 2.1: Minkowski spacetime diagram with regions I, the right Rindler wedge, and II, the left Rindler wedge. The orbits of the boost Killing field B^a, which is future-oriented in I and past-oriented in II, in these regions are depicted in red.

In Minkowski space with cartesian coordinates, $\mathcal{K}$ reduces to $\sum_k p_k^2 + m^2$, with $p_k \equiv -i\dfrac{\partial}{\partial x^k}$. Therefore, $j = \mathbf{k} \in \mathbb{R}^3$, $\omega_{\mathbf{k}} = \sqrt{\mathbf{k}^2 + m^2}$ and $\psi_j \equiv \dfrac{e^{i\mathbf{k}\cdot\mathbf{x}}}{(2\pi)^{3/2}}$, which lead us to the standard Hilbert space $\mathcal{H}_M$ for scalar fields in QFT

$$\mathcal{H}_M \equiv \left\{ \int \frac{d\mathbf{k}}{(2\omega_{\mathbf{k}})^{1/2}} \varphi(\mathbf{k}) e^{-i\omega_{\mathbf{k}}t} \frac{e^{i\mathbf{k}\cdot\mathbf{x}}}{(2\pi)^{3/2}} \mid \varphi \in L^2(\mathbb{R}^3, d\mathbf{k}) \right\}. \tag{2.19}$$

This is the natural choice of positive-frequency solutions for inertial observers, and the Minkowski vacuum state is denoted by $|0_M\rangle \in \mathcal{F}_s(\mathcal{H}_M)$. For convenience, let us define the Minkowski modes as

$$u_{\mathbf{k}} \equiv \frac{e^{-i(\omega_{\mathbf{k}}t - \mathbf{k}\cdot\mathbf{x})}}{\sqrt{16\pi^3\omega_{\mathbf{k}}}}. \tag{2.20}$$

It turns out that we can formulate the field theory in a rather different way. Assume that the coordinates (τ, ξ, y, z) cover the region I in Fig. 2.1, corresponding to $|t| < x$, such that

$$\begin{cases} t = a^{-1}e^{a\xi}\sinh(a\tau), \\ x = a^{-1}e^{a\xi}\cosh(a\tau). \end{cases} \tag{2.21}$$

Thus, the line element is now

$$ds^2 = e^{2a\xi}(-d\tau^2 + d\xi^2) + dy^2 + dz^2, \tag{2.22}$$

what shows that it describes a static space with a timelike Killing field $B^a = \left(\frac{\partial}{\partial \tau}\right)^a$, or, in terms of the Cartesian coordinates, $B^a = a\Big[x\left(\frac{\partial}{\partial t}\right)^a + t\left(\frac{\partial}{\partial t}\right)^a\Big]$. That corresponds to the boost Killing field whose orbits are hyperbolae in a Minkowski spacetime diagram. Each hyperbola is assigned with a $\xi \in \mathbb{R}$, and the proper acceleration of an observer whose worldline is given by one of these orbits is $ae^{-a\xi}$. Thus, we say that these orbits are associated with a family of uniformly accelerated observers in Minkowski space.

With these new coordinates, $\mathcal{K}$ becomes $\mathcal{K} = \Big(-\dfrac{\partial^2}{\partial \xi^2} + m^2 e^{2a\xi}\Big) + e^{2a\xi}(p_y^2 + p_z^2)$. Also, the states are now labeled by $j = (\omega, \mathbf{k}_\perp) \in (0, \infty) \times \mathbb{R}^2$, with $\mathbf{k}_\perp \equiv (k_y, k_z)$, and assuming that we can use separation of variables to solve Eq. (2.18) by writing the eigenfunctions as

$$\psi_{\omega \mathbf{k}_\perp} = g_{\omega \mathbf{k}_\perp}(\xi) \frac{e^{i\mathbf{k}_\perp \cdot \mathbf{x}_\perp}}{2\pi},$$

where $g_{\omega \mathbf{k}_\perp}(\xi)$ satisfies

$$\Big(-\frac{\partial^2}{\partial \xi^2} + e^{2a\xi}(m^2 + \mathbf{k}_\perp^2)\Big) g_{\omega \mathbf{k}_\perp}(\xi) = \omega^2 g_{\omega \mathbf{k}_\perp}(\xi). \tag{2.23}$$

The solutions of the equation above are of the form

$$g_{\omega \mathbf{k}_\perp}(\xi) = \Big[\frac{2\omega \sinh\left(\frac{\pi\omega}{a}\right)}{\pi^2 a}\Big]^{1/2} K_{i\omega/a}\left(\frac{\kappa}{a} e^{a\xi}\right),$$

with $\kappa \equiv \sqrt{\mathbf{k}_\perp^2 + m^2}$ and $K_\nu(x)$ being a modified Bessel function of second kind. We assume suitable boundary conditions [23]. Hence, the Hilbert space $\mathcal{H}_R^I$ associated to the region I is

$$\mathcal{H}_R^I \equiv \Big\{ \int d\omega d\mathbf{k}_\perp \varphi(\omega, \mathbf{k}_\perp) e^{-i\omega\tau + i\mathbf{k}_\perp \cdot \mathbf{x}} \Big[\frac{\sinh\left(\frac{\pi\omega}{a}\right)}{4\pi^4 a}\Big]^{1/2} K_{i\omega/a}\left(\frac{\kappa}{a} e^{a\xi}\right) \\ \mid \varphi \in L^2((0, \infty) \times \mathbb{R}^2, d\omega d\mathbf{k}_\perp) \Big\},$$

where we also define the right Rindler modes

$$v^R_{\omega \mathbf{k}_\perp} \equiv e^{-i\omega\tau + i\mathbf{k}_\perp \cdot \mathbf{x}} \Big[\frac{\sinh\left(\frac{\pi\omega}{a}\right)}{4\pi^4 a}\Big]^{1/2} K_{i\omega/a}\Big(\frac{\kappa}{a} e^{a\xi}\Big). \tag{2.24}$$

Observe that they vanish in the region II.

One can proceed in the exact same manner in order to obtain a Hilbert space $\mathcal{H}_R^{II}$ corresponding to the region II in Fig. 2.1, e.g., $|t| < -x$, as long as the new coordinates are $(\tilde{\tau}, \tilde{\xi}, y, z)$ such that

$$\begin{cases} t = a^{-1}e^{a\tilde{\xi}}\sinh(a\tilde{\tau}), \\ x = -a^{-1}e^{a\tilde{\xi}}\cosh(a\tilde{\tau}). \end{cases}$$

The left Rindler modes $v^L_{\omega\mathbf{k}_\perp}$ assume the same form as Eq. (2.24) in terms of these coordinates. In turn, they vanish in the region I. Thus, the Hilbert space $\mathcal{H}_R \equiv \mathcal{H}_R^I \oplus \mathcal{H}_R^{II}$ represents the natural construction of positive-frequency solutions for uniformly accelerated observers in Minkowski spacetime. A particle interpretation can be given to the theory with the Fock space $\mathcal{F}_s(\mathcal{H}_R) \simeq \mathcal{F}_s(\mathcal{H}_R^I) \otimes \mathcal{F}_s(\mathcal{H}_R^{II})$. We define the Rindler vacuum state as $|0_R\rangle \in \mathcal{F}_s(\mathcal{H}_R)$.

We have found two equally valid constructions for the quantum scalar field theory in Minkowski space. In particular, if both Minkowski and (right and left) Rindler modes form a complete set, e.g., we can expand the field, Eq. (2.8), in terms of either of them, we should be able to write one as a linear combination of the other. To illustrate that, let us define two arbitrary sets of positive-frequency modes, $\{f_i^{(1)}\}$ and $\{f_I^{(2)}\}$, that are orthonormal with respect to the KG inner product, hence

$$f_I^{(2)} = \sum_i \left(\alpha_{Ii} f_i^{(1)} + \beta_{Ii} \overline{f_i^{(1)}}\right), \tag{2.25}$$

where α_{Ii} and β_{Ii} are the so-called Bogoliubov coefficients [23]. They are fully determined by the inner product between elements of different sets, since $\alpha_{Ii} = (f_i^{(1)}, f_I^{(2)})_{KG}$ and $\beta_{Ii} = -(\overline{f_i^{(1)}}, f_I^{(2)})_{KG}$. We can expand the field in terms of $\{f_I^{(2)}\}$, plug Eq. (2.25), and then compare the resulting expression with the same expansion in terms of $\{f_i^{(1)}\}$. Hence, one obtains the creation and annihilation operators of i modes as a function of the corresponding operators for the I modes. These expressions can be inverted, resulting in

$$a_I^{(2)\dagger} = \sum_i \left(\alpha_{Ii} a_i^{(1)\dagger} - \beta_{Ii} a_i^{(1)}\right), \tag{2.26}$$

and its Hermitian conjugate gives an analogous expression for the annihilation operator.

Remarkably, one notices that the expectation value of the number operator

$N_I^{(2)} \equiv a_I^{(2)\dagger} a_I^{(2)}$ on the vacuum state of the other construction, e.g., $\left|0_{(1)}\right\rangle$ such that $a_i^{(1)} \left|0_{(1)}\right\rangle = 0$ for any mode i, does not vanish, rather

$$\left\langle 0_{(1)}\right| N_I^{(2)} \left|0_{(1)}\right\rangle = \sum_i |\beta_{Ii}|^2. \tag{2.27}$$

This is a key concept for deriving the Unruh effect. Recall that the right Rindler modes vanish in the region II, whereas the left Rindler modes vanish in the region I. In order to explicitly find the Bogoliubov coefficients, one needs to extend them to the whole Minkowski space; that can be done by writing all the modes in terms of the corresponding advanced,

$$\begin{cases} V = t + x, \\ v = \tau + \xi, \end{cases} \tag{2.28}$$

and retarded,

$$\begin{cases} U = t - x, \\ u = \tau - \xi, \end{cases} \tag{2.29}$$

null coordinates. Also, note that one can easily find U and V as a function of u and v as $U = -\dfrac{1}{a} e^{-au}$ and $V = \dfrac{1}{a} e^{av}$.

For free fields, with (2) corresponding to right Rindler modes and (1) to Minkowski modes, Eq. (2.25) becomes

$$v^R_{\omega \mathbf{k}_\perp} = \int_{-\infty}^{\infty} \frac{dk_z}{\sqrt{4\pi k_0}} \left[\alpha^R_{\omega k_x \mathbf{k}_\perp} e^{-ik_0 t + ik_x x} + \beta^R_{\omega k_x \mathbf{k}_\perp} e^{ik_0 t - ik_x x} \right] \frac{e^{i\mathbf{k}_\perp \cdot \mathbf{x}_\perp}}{2\pi}, \tag{2.30}$$

where we relabel $\omega_{\mathbf{k}} \equiv k_0$ to avoid ambiguities. From the similarity of the construction of the right and left Rindler modes, it follows the relations

$$\begin{cases} \alpha^R_{\omega k_x \mathbf{k}_\perp} = \alpha^L_{\omega - k_x \mathbf{k}_\perp}, \\ \beta^R_{\omega k_x \mathbf{k}_\perp} = \beta^L_{\omega - k_x \mathbf{k}_\perp}. \end{cases} \tag{2.31}$$

At $t = x = \frac{V}{2}$, $t > 0$, which corresponds to $\xi \to -\infty$, $v^R_{\omega \mathbf{k}_\perp}$ assumes the form [23]

$$v^R_{\omega \mathbf{k}_\perp} \approx \frac{i}{4\pi} \left(a \sinh\left(\frac{\pi\omega}{a}\right)\right)^{-1/2} e^{i\mathbf{k}_\perp \cdot \mathbf{x}_\perp} \left[\frac{\left(\frac{\kappa}{2a}\right)^{i\omega/a} e^{-i\omega u}}{\Gamma(1 + i\omega/a)} - \frac{\left(\frac{\kappa}{2a}\right)^{-i\omega/a} e^{-i\omega v}}{\Gamma(1 - i\omega/a)} \right].$$

Note that, when $U = 0$, hence $u \to \infty$, the first term in the expression above oscillates too rapidly and can be disregarded. Therefore, one can multiply both sides of Eq. (2.30) by $e^{i(k_0 - k_x)V/2}$ and integrate over V to obtain $\alpha^R_{\omega k_x \mathbf{k}_\perp}$; conversely, multiplying by $e^{-i(k_0-k_x)V/2}$ and integrating over V gives $\beta^R_{\omega k_x \mathbf{k}_\perp}$. With these coefficients and Eq. (2.31), one finds that $\beta^R_{\omega k_x \mathbf{k}_\perp} = -e^{-\pi\omega/a}\overline{\alpha^L_{\omega k_x \mathbf{k}_\perp}}$ and $\beta^L_{\omega k_x \mathbf{k}_\perp} = -e^{-\pi\omega/a}\overline{\alpha^R_{\omega k_x \mathbf{k}_\perp}}$. From them, we define the Unruh modes, which are purely positive-frequency with respect to inertial time, even though they are written in terms of Rindler modes [21]

$$\begin{cases} w^1_{\omega \mathbf{k}_\perp} \equiv \dfrac{v^R_{\omega \mathbf{k}_\perp} + e^{-\pi\omega/a}\overline{v^L_{\omega -\mathbf{k}_\perp}}}{\sqrt{1 - e^{-2\pi\omega/a}}}, \\ w^2_{\omega \mathbf{k}_\perp} \equiv \dfrac{v^L_{\omega \mathbf{k}_\perp} + e^{-\pi\omega/a}\overline{v^R_{\omega -\mathbf{k}_\perp}}}{\sqrt{1 - e^{-2\pi\omega/a}}}. \end{cases} \tag{2.32}$$

Representing the scalar field in terms of these modes results in two operators that must annihilate the Minkowski vacuum state. These new operators are linear combinations of ladder operators on $\mathcal{F}_s(\mathcal{H}_R)$.

Before writing them down, let us consider that the field has discrete energy levels, e.g., the free field is now put in a "box". Moreover, we reduce the dimensionality of the problem to $(1+1)d$, thus we can choose a new label $\omega \mathbf{k}_\perp \to \omega_i$. Under that, the operators that annihilate $|0_M\rangle$ become

$$\begin{cases} (a^R_{\omega_i} - e^{-\pi\omega/a} a^{L\dagger}_{\omega_i}) |0_M\rangle = 0, \\ (a^L_{\omega_i} - e^{-\pi\omega/a} a^{R\dagger}_{\omega_i}) |0_M\rangle = 0. \end{cases} \tag{2.33}$$

With the commutation relations $[a^R_{\omega_i}, a^{R\dagger}_{\omega_j}] = [a^L_{\omega_i}, a^{L\dagger}_{\omega_j}] = \delta_{ij}$, one finally finds that

$$\langle 0_M | a^{R\dagger}_{\omega_i} a^R_{\omega_i} |0_M\rangle = \langle 0_M | a^{L\dagger}_{\omega_i} a^L_{\omega_i} |0_M\rangle = (e^{2\pi\omega_i/a} - 1)^{-1}, \tag{2.34}$$

which means that the expected number of Rindler particles in the Minkowski vacuum is that of bosons in a thermal bath of temperature $T = a/2\pi$. To conclude the derivation, let us show that $|0_M\rangle$ restricted to one of the wedges corresponds to the grand canonical ensemble. From Eq. (2.34), one infers that the number of left and right Rindler particles is the same for each ω_i. Hence

$$|0_M\rangle = \prod_i \frac{1}{\sqrt{1 - e^{-2\pi\omega_i/a}}} \sum_{n_i=0}^{\infty} \frac{e^{-\pi n_i \omega_i/a}}{n_i!} (a^{R\dagger}_{\omega_i} a^{L\dagger}_{\omega_i})^{n_i} |0_R\rangle . \tag{2.35}$$

Defining the n_i Rindler-particle state as $|n_i, R\rangle \otimes |n_i, L\rangle \equiv \frac{(a^{R\dagger}_{\omega_i} a^{L\dagger}_{\omega_i})^{n_i}}{n_i!} |0_R\rangle$ and tracing out the left Rindler states, we end up with the reduced density matrix for the right wedge

$$\rho_R = \prod_i \frac{1}{1 - e^{-2\pi\omega_i/a}} \sum_{n_i=0}^{\infty} e^{-2\pi n_i \omega_i/a} |n_i, R\rangle \langle n_i, R| \,. \tag{2.36}$$

That precisely describes free bosons of temperature $T = a/2\pi$.

2.3 Particle detectors and decoherence

The model introduced by Unruh and Wald [6] consists of a detector whose internal state pertains to a Hilbert space $\mathcal{H}_D$ and has the proper Hamiltonian defined as $H_D = \Omega D^\dagger D$. Here, Ω is the detector's energy gap and $D|0\rangle = D^\dagger |1\rangle = 0$, $D|1\rangle = |0\rangle$ and $D^\dagger |0\rangle = |1\rangle$. Also, $|0\rangle, |1\rangle \in \mathcal{H}_D$ are the ground and excited energy eigenstates, respectively. The detector is set to interact with a real scalar field ϕ whose free propagation in a globally hyperbolic spacetime (M, g_{ab}) is generally described by the action

$$S = -\frac{1}{2} \int_M d^4x \sqrt{-g} (\nabla_a \phi \nabla^a \phi + m^2 \phi^2 + \xi R \phi^2), \tag{2.37}$$

with R being the scalar curvature, and $\xi \in \mathbb{R}$ is the coupling between the field and the curvature of the manifold. Note that the extremization of Eq. (2.37), assuming that $\xi = 0$, will lead to Eq. (2.1). In analogy to classical discrete systems, one writes down the free field Hamiltonian as

$$H_\phi(t) = \int_{\Sigma_t} d^3\mathbf{x} (\pi(t, \mathbf{x}) \dot{\phi}(t, \mathbf{x}) - \mathcal{L}[\phi, \nabla_a \phi]), \tag{2.38}$$

such that $\mathbf{x} \equiv (x^1, x^2, x^3)$ are the spatial coordinates on Σ_t, which are Cauchy surfaces that foliate the spacetime and are labeled by a real parameter t, $\dot{\phi} \equiv \partial_0 \phi$, $\pi \equiv \frac{\delta S}{\delta \dot{\phi}}$ is the conjugate momentum and $\mathcal{L}$ is the corresponding Lagrangian density [24].

Now, let us introduce the interaction Hamiltonian

$$H_{int}(t) = \epsilon(t) \int_{\Sigma_t} d^3\mathbf{x} \sqrt{-g} \phi(x) [\psi(\mathbf{x}) D + \overline{\psi}(x) D^\dagger], \tag{2.39}$$

where $\epsilon(t) \in C_0^\infty(\mathbb{R})$ is a smooth real-valued function with compact support, which models the fact that the detector is switched on for a finite proper time interval, and $\psi(t,\mathbf{x})|_{\Sigma_t} \in C_0^{\mathbb{C}\infty}(\Sigma_t)$ is a complex-valued function with compact support on Σ_t that introduces the range of interaction of the detector in the vicinity of its worldline, that is, in turn, described by a timelike isometry of the spacetime [25].

Thus, the total Hamiltonian of the system is $H_{D\phi} = H_D + H_\phi + H_{int}$, and we define $H_0 \equiv H_D + H_\phi$ as the detector-field free Hamiltonian. It is pertinent to investigate the system's evolution in the interaction picture, where the interaction Hamiltonian becomes $H^I_{int}(t) = U_0^\dagger(t)H_{int}(t)U_0(t)$, $U_0(t)$ being the time evolution operator associated with H_0. Thence, the total state $|\Psi_\infty\rangle$ at a long time after the interaction has ended is given by

$$|\Psi_\infty\rangle = T\exp\Big[-i\int_{-\infty}^{\infty} dt' H^I_{int}(t')\Big]\,|\Psi_{-\infty}\rangle\,. \tag{2.40}$$

Using Eq. (2.39), and also $D^I = e^{-i\Omega t}D$, one finds that

$$|\Psi_\infty\rangle = T\exp\Big[-i\int d^4x\sqrt{-g}\phi(x)(fD+\overline{f}D^\dagger)\Big]\,|\Psi_{-\infty}\rangle\,, \tag{2.41}$$

such that $f = f(x) \equiv e^{-i\Omega t}\epsilon(t)\psi(\mathbf{x})$. In first order, and recalling Eq. (2.11), the expression above reduces to $|\Psi_\infty\rangle \approx [\mathbb{I} - i(\phi(f)D + \phi(f)^\dagger D^\dagger)]\,|\Psi_{-\infty}\rangle$. Furthermore, assuming that $\epsilon(t)$ is a slowly-varying function that is non-vanishing only in a time interval T such that $T \gg \Omega^{-1}$, it can be shown [25] that f is roughly a positive-frequency solution, which implies that Eq. (2.12) becomes $\phi(f) \approx ia^\dagger(\lambda)$, with $\lambda \equiv -KEf$. Therefore, we can finally cast Eq. (2.41) as

$$|\Psi_\infty\rangle = (\mathbb{I} + a^\dagger(\lambda)D - a(\overline{\lambda})D^\dagger)\,|\Psi_{-\infty}\rangle\,. \tag{2.42}$$

This remarkable result shows that when the detector is excited, an observer co-moving with the detector sees the absorption of a quantum of the field. In contrast, they observe that the emission of a particle follows the de-excitation of the detector. We can make this statement more precise by computing a simple example. Suppose that, initially, the detector is set to be in its ground state and the field is prepared in the state with n particles in a mode γ, e.g., $|\Psi_{-\infty}\rangle = |n\rangle_\gamma \otimes |0\rangle$.

Thus

$$\begin{aligned} |\Psi_\infty\rangle &= |n\rangle_\gamma \otimes |0\rangle - (a(\overline{\lambda})\,|n\rangle_\gamma) \otimes |1\rangle \\ &= |n\rangle_\gamma \otimes |0\rangle - \sqrt{n}(\lambda,\gamma)_{KG}\,|n-1\rangle_\gamma \otimes |1\rangle\,, \end{aligned} \tag{2.43}$$

saying that the probability of finding the detector in the excited state at the end is $P_{exc} = n|(\lambda,\gamma)_{KG}|^2$. It is manifest that, even in first order, the detector will be excited if and only if a particle has been absorbed, e.g., the modes λ and γ overlap. One should expect that from a suitable particle detector model [6].

The major drawback of this approach is that we have to employ an approximation in order to obtain significant results regarding the system's evolution. As we are ultimately interested in investigating the decoherence of a 2-level quantum system due to its interaction with a real scalar field, we can slightly modify this model in a way that the two levels of the detector have the same energy. This simple yet powerful change will allow us to evaluate the final state and trace out the field degrees of freedom nonperturbatively. Now, the terminology "detector" might be a bit misleading, so we will also refer to it as a qubit henceforth.

The total Hamiltonian becomes $H = H_\phi + \tilde{H}_{int}$, such that the new interaction term, in the interaction picture, is given by

$$\tilde{H}^I_{int}(t) \equiv \epsilon(t)\int_{\Sigma_t} d^3\mathbf{x}\sqrt{-g}\,\psi(t,\mathbf{x})\phi(t,\mathbf{x}) \otimes \sigma^z, \tag{2.44}$$

where we redefine $\psi\,|_{\Sigma_t} \in C_0^\infty(\Sigma_t)$ to be a smooth real-valued function with compact support and σ^z is one of the Pauli matrices. We made the tensor product explicit here to avoid ambiguities, but just as in the last interaction Hamiltonian, we have an operator acting on field states and one acting on the qubit states. In the spirit of Eq. (2.40), we have to find the time evolution operator associated with $\tilde{H}^I_{int}(t)$. Now, it can be cast as $U_{\phi Q} = \exp(\Omega)$, where $\Omega = \sum_{n=1}^\infty \Omega_i$ is the so-called Magnus expansion. We refer to [26] for addressing important issues concerning the existence and convergence of such an expansion as a solution for $\dot{U}_{\phi Q}(t) = -i\tilde{H}^I_{int}(t)U_{\phi Q}(t)$, with $U_{\phi Q}(0) = \mathbb{I}$. Assuming that all conditions are met, the first terms of the expansion are sufficient for our problem, namely

$$\Omega_1 = -i\int_{-\infty}^{\infty} dt\tilde{H}^I_{int}(t) = -i\phi(f) \otimes \sigma^z, \tag{2.45}$$

$$\Omega_2 = -\frac{1}{2}\int_{-\infty}^{\infty} dt\int_{-\infty}^{t} dt'[\tilde{H}^I_{int}(t), \tilde{H}^I_{int}(t')] = i\Xi\mathbb{I}, \tag{2.46}$$

$$\Omega_3 = \frac{i}{6}\int_{-\infty}^{\infty} dt \int_{-\infty}^{t} dt' \int_{-\infty}^{t'} dt'' \Big([\tilde{H}^I_{int}(t), [\tilde{H}^I_{int}(t'), \tilde{H}^I_{int}(t'')] + [\tilde{H}^I_{int}(t''), [\tilde{H}^I_{int}(t'), \tilde{H}^I_{int}(t)] \Big) = 0. \tag{2.47}$$

A few remarks on these expressions: The recursive character of the Magnus expansion is manifest, which implies that $\Omega_k = 0$ for $k \geq 4$. The result for Ω_1 follows from Eq. (2.11), where $f \equiv \epsilon(t)\psi(t,\mathbf{x})$ carries the information about the qubit's worldline [24]. Moreover, to evaluate Ω_2, we explicitly apply the identity $[A \otimes C, B \otimes D] \equiv [A,B] \otimes CD + AB \otimes [C,D]$, such that A and B pertain to the same operator space, whereas C and D pertain to the same space but different from the former. The quantity Ξ is defined as

$$\Xi \equiv \frac{1}{2}\int_{-\infty}^{\infty} dt \epsilon(t) \int_{-\infty}^{t} dt' \epsilon(t') \Delta(t,t'),$$

and $\Delta(t,t')$, in turn, is given by

$$\Delta(t,t') \equiv \int_{\Sigma_t} d^3\mathbf{x}\sqrt{-g} \int_{\Sigma_{t'}} d^3\mathbf{x}' \sqrt{-g'} \psi(t,\mathbf{x}) \Delta(x,x') \psi(t',\mathbf{x}').$$

Note that $\Delta(x,x')$ is the position-space (or unsmeared) representation of what appears in the right-hand side of Eq. (2.14). This is directly related to the operator E defined in Section 2.1. Thus, one casts the time evolution operator $U_{\phi Q}$ as

$$U_{\phi Q} = \exp(i\Xi)\exp(-i\phi(f) \otimes \sigma^z). \tag{2.48}$$

Provided that, we introduce the situation of main interest in the present work: Suppose that the qubit also has a spatial degree of freedom, e.g., it is set to be in a spatial superposition state, such that each superposition is associated with a classical worldline. Our setting consists of preparing the qubit in a pure internal state, then we can create a spatial superposition using an external device. The qubit interacts with the field through Eq. (2.44) for a finite time, and the superpositions are recombined afterwards. Thus, one can probe the final internal state and verify whether it corresponds to a statistical ensemble. If this is the case, then part of the initial information now lies in the correlations with the field, and we say that the qubit has decohered.

For an example of how that could be implemented, consider that the qubit is a spin-1/2 particle initially prepared in the eigenstate of σ^x with positive eigenvalue. One makes this particle go through a Stern-Gerlach apparatus in the z direction, creating a spatial superposition where the position and spin states are entangled. After the interaction occurred and the particle had gone through a reverse Stern-Gerlach apparatus, one can measure its spin in the x direction. If the experiment is executed repeatedly and the outcome is always positive, it means the particle keeps its coherence; otherwise, the spin state is mixed [10]. We are interested in how such decoherence depends on the different parameters related to the problem. In particular, we investigate two special cases, consisting of the situations where the detector is inertial and is uniformly accelerating.

Therefore, our next step is to define an observable associated with decoherence. Let $\mathcal{H}_{path}$ be a 2-dimensional Hilbert space with an orthonormal basis $\{|\gamma_1\rangle, |\gamma_2\rangle\}$. These states describe the possible worldlines, or more precisely, they represent coherent states centered at γ_1 and γ_2, respectively. We do not want to impose any particular dynamics on this degree of freedom, hence we naturally extend Eq. (2.48) to this new scenario as

$$U = U^{(1)}_{\phi Q} \otimes |\gamma_1\rangle \langle\gamma_1| + U^{(2)}_{\phi Q} \otimes |\gamma_2\rangle \langle\gamma_2| . \tag{2.49}$$

The introduction of a superscript leads to redefine f as $f_j \equiv \epsilon(t)\psi_j(t, \mathbf{x})$, such that ψ_j has support around γ_j, $j = 1, 2$. Suppose we prepare the qubit in the state $\frac{1}{\sqrt{2}}(|0\rangle \otimes |\gamma_1\rangle + |1\rangle \otimes |\gamma_2\rangle)$ and set it up to interact with the field which is initially in the Minkowski vacuum $|0_M\rangle$. The total state is

$$|\Psi_{-\infty}\rangle = \frac{1}{\sqrt{2}}(|0_M\rangle \otimes |0\rangle \otimes |\gamma_1\rangle + |0_M\rangle \otimes |1\rangle \otimes |\gamma_2\rangle), \tag{2.50}$$

and evolving it through Eq. (2.49), one obtains

$$\begin{aligned} U|\Psi_{-\infty}\rangle &= \frac{1}{\sqrt{2}}\Big[(U^{(1)}_{\phi Q}|0_M\rangle \otimes |0\rangle) \otimes |\gamma_1\rangle + (U^{(2)}_{\phi Q}|0_M\rangle \otimes |1\rangle) \otimes |\gamma_2\rangle\Big] \\ &= \frac{1}{\sqrt{2}}\Big[(e^{i\Xi_1}e^{-i\phi(f_1)}|0_M\rangle) \otimes |0\rangle \otimes |\gamma_1\rangle + (e^{i\Xi_2}e^{i\phi(f_2)}|0_M\rangle) \otimes |1\rangle \otimes |\gamma_2\rangle\Big]. \end{aligned} \tag{2.51}$$

Where we define the corresponding fields' final states as $|\Phi_1\rangle \equiv e^{i\Xi_1}e^{-i\phi(f_1)}|0_M\rangle$ and $|\Phi_2\rangle \equiv e^{i\Xi_2}e^{i\phi(f_2)}|0_M\rangle$. Inspired by [13], we define a measure $\mathcal{D}$ of decoherence as

$$\mathcal{D} \equiv 1 - |\langle\Phi_1|\Phi_2\rangle|. \tag{2.52}$$

This expression has a clear interpretation in terms of particle emission as seen by an inertial observer: If this interaction occurs in such a way that, up to a phase, the field state remains unchanged, then there is no decoherence, e.g., $\mathcal{D} \approx 0$; in contrast, if there were emission of entangling particles [14], the field evolves to orthogonal states along each path and $\mathcal{D} \approx 1$.

Note that, for our case, Eq. (2.52) becomes

$$\mathcal{D} = 1 - | \langle 0_M | e^{i\phi(f_1)} e^{i\phi(f_2)} | 0_M \rangle |. \tag{2.53}$$

To evaluate this inner product, let us briefly introduce an alternative approach to QFT, known as the algebraic approach [19]. It has the advantage of formulating the theory without requiring the choice of a specific representation in curved spaces, as shown in the canonical formulation. In this scheme, the field quantization is represented by the set of $\phi(f)$, which forms the *-algebra $\mathcal{A}(M)$, e.g., the algebra of observables of the Klein-Gordon field. Exponentiating each element of $\mathcal{A}(M)$, we define the Weyl algebra $W(Ef) \equiv e^{i\phi(f)} \in \mathcal{W}(M)$ whose important properties are summarized as follows [24]

$$\overline{W(Ef)} = W(-Ef), \tag{2.54}$$

$$W(E[-\nabla^a \nabla_a + m^2 + \xi R]f) = \mathbb{I}, \tag{2.55}$$

$$W(Ef_1)W(Ef_2) = e^{\frac{i}{2}\Delta(f_1,f_2)} W(E(f_1 + f_2)). \tag{2.56}$$

In particular, the E in the elements of $\mathcal{W}(M)$ is important to make Eq. (2.55) consistent with the fact that the field is a solution of Eq. (2.1). Additionally, field states are now defined as positive $\mathbb{C}$-linear functionals on $\mathcal{W}(M)$ that satisfy a normalization condition. An important subclass of states consists of the quasifree, or Gaussian, states, which are defined by

$$\omega_\mu[W(Ef)] \equiv e^{-\mu(Ef,Ef)/2}, \tag{2.57}$$

where $\mu : S \times S \to \mathbb{R}$, in turn, is a real inner product satisfying

$$|\Omega(Ef_1, Ef_2)|^2 \leq 4\mu(Ef_1, Ef_1)\mu(Ef_2, Ef_2), \tag{2.58}$$

with Ω being the bilinear form given by Eq. (2.2). We can restrict ourselves to the quasifree states as they comprise the Minkowski vacuum state. It can be shown [27] that these states give a unique prescription to construct a one-particle Hilbert space $\mathcal{H}$. For the Minkowski vacuum, this connection ends up being $\mu(Ef, Ef) = (K_M Ef, K_M Ef)_{KG} = \|K_M Ef\|^2$, where K_M takes the positive-frequency part of the solutions to Eq. (2.1) with respect to inertial time.

The last piece required to link the algebraic formalism to Eq. (2.53) is an expression that relates expectation values in the Minkowski vacuum and its action as a quasifree state on an element of $\mathcal{W}(M)$, which is provided by

$$\omega_\mu[W(Ef)] = \langle 0_M | \, e^{i\phi(f)} \, |0_M\rangle \, . \tag{2.59}$$

Therefore, using Eq. (2.56), Eq. (2.53) becomes

$$\begin{aligned} \mathcal{D} &= 1 - | \, \langle 0_M | \, e^{i\phi(f_1+f_2)} \, |0_M\rangle \, | \\ &= 1 - e^{-\|K_M E(f_1+f_2)\|^2/2}. \end{aligned} \tag{2.60}$$

The algebraic QFT approach is interesting for more general cases (any arbitrary quasifree state); as ours is simple enough, we can also expand the smeared fields in terms of creation and annihilation operators and make use of Eq. (2.7). Thus, applying the BCH formula

$$e^{\mathcal{A}}e^{\mathcal{B}} = e^{\mathcal{A}+\mathcal{B}+\frac{1}{2}[\mathcal{A},\mathcal{B}]+\frac{1}{12}[\mathcal{A},[\mathcal{A},\mathcal{B}]]+\frac{1}{12}[\mathcal{B},[\mathcal{B},\mathcal{A}]]+\dots}, \tag{2.61}$$

such that $\mathcal{A}$ and $\mathcal{B}$ are any two bounded operators, to evaluate Eq. (2.53). We note that the linear terms vanish, as we are interested in the expected value in the Minkowski vacuum, hence

$$\begin{aligned} [i\phi(f_1), i\phi(f_2)] &= [a^\dagger(K_M E f_1) - a(\overline{K_M E f_1}), a^\dagger(K_M E f_2) - a(\overline{K_M E f_2})] \\ &= -[a(\overline{K_M E(f_1+f_2)}), a^\dagger(K_M E(f_1+f_2))] \\ &= -\|K_M E(f_1+f_2)\|^2. \end{aligned} \tag{2.62}$$

It is straightforward to visualize that the contributions from the next terms in the BCH formula vanish for similar reasons as the linear ones, so we end up with the same result, as one should expect.

Now, it is possible to investigate the implications of this formula in the context of a qubit in a spatial superposition state of two inertial worldlines in Minkowski space and two stationary ones in Rindler space. Still, first, we provide a detailed review of Danielson, Satishchandran, and Wald's work.

Chapter 3

Review on the decoherence in Minkowski vacuum

3.1 Killing horizon viewpoint

The configuration we are interested in can be described as a charged particle with charge q (let us focus on the electromagnetic case; then we briefly address the gravitational one afterwards) with some internal degree of freedom. For simplicity, this degree of freedom is associated with a quantum state which lies in a two-dimensional Hilbert space $\mathcal{H}$, such that, after putting the particle through a Stern-Gerlach apparatus over a time T_1, the resulting state is

$$\frac{1}{\sqrt{2}}(|\psi_1\rangle + |\psi_2\rangle), \tag{3.1}$$

where $|\psi_1\rangle, |\psi_2\rangle \in \mathcal{H}$ are normalized, spatially separated states. The experiment is realized in a stationary laboratory in a stationary spacetime, and Alice, the experimenter, lets this superposition state be stationary with respect to her lab over a proper time T. After that, she recombines it, passing the particle through a reverse Stern-Gerlach apparatus over a time T_2. Then, knowing the initial state, she can verify whether it has lost coherence by simply measuring the final state. Note that the particle is not supposed to be stationary during the creation and recombination times.

As usual, a charged particle will emit electromagnetic radiation throughout the experiment. The radiation state, however, will depend on which component emitted a given photon; hence, the total system is better described by the following joint state

$$\frac{1}{\sqrt{2}}(|\psi_1\rangle \otimes |\Psi_1\rangle + |\psi_2\rangle \otimes |\Psi_2\rangle), \tag{3.2}$$

such that $|\Psi_1\rangle$ and $|\Psi_2\rangle$ are coherent states describing the out radiation state associated with the particle state $|\psi_1\rangle$ and $|\psi_2\rangle$, respectively. Then, we quantify the particle's decoherence as

$$\mathcal{D} \equiv 1 - |\langle \Psi_1 | \Psi_2 \rangle|, \tag{3.3}$$

so we need to find the precise form of these coherent states in terms of the classical solutions to Maxwell's equations in order to compute Eq. (3.3).

We assume that the initial state of the electromagnetic field is a vacuum state $|\Omega\rangle$ invariant under time translation symmetries; ultimately, we are only interested in situations where this vacuum state is the Minkowski vacuum $|0_M\rangle$, but let us keep it general at this point. Moreover, we define the charge-current operator $\mathbf{j}^a$ in such a way that its fluctuations in the particle states are negligible when compared to its expectation value; thus, $j_1^a = \langle\psi_1| \mathbf{j}^a |\psi_1\rangle$ and $j_2^a = \langle\psi_2| \mathbf{j}^a |\psi_2\rangle$ can be treated as c-numbers and act as sources in Maxwell's equations. The states are assumed to be sufficiently far apart, e.g., $\langle\psi_1| \mathbf{j} |\psi_2\rangle = 0$. On the other hand, the electromagnetic field operator corresponding to the particle state $|\psi_n\rangle$, $n = 1, 2$, is

$$\mathbf{A}_{n,a} = \mathbf{A}_a^{\text{in}} + G_a^{\text{ret}}(j_n)\mathbb{I}, \tag{3.4}$$

where $\mathbf{A}_a^{\text{in}}$ is the unperturbed field operator (recall that before the superposition state is created, we can just assume that $j_1^a = j_2^a$), and $G_a^{\text{ret}}(j_n)$ is the retarded classical solution associated with j_n^a. The final radiative field is readily obtained by subtracting the Coulomb contribution C_a from the recombined particle

$$\mathbf{A}_{n,a}^{\text{out}} = \mathbf{A}_a^{\text{in}} + \mathcal{A}_{n,a}\mathbb{I}, \tag{3.5}$$

such that $\mathcal{A}_{n,a} \equiv G_a^{\text{ret}}(j_n) - C_a$. The idea is that, from Eq. (3.5), one notices that the correlation functions of the field operator at late times can be obtained from the vacuum ones by shifting the field by a multiple of the identity operator. Therefore, the out state of the radiation is given by a coherent state of the form

$$|\Psi_n\rangle = e^{-\|K\mathcal{A}_n\|^2/2} e^{a^\dagger(K\mathcal{A}_n)} |\Omega\rangle, \tag{3.6}$$

where K projects classical solutions into the one-particle Hilbert space $\mathcal{H}_{in}$. Naturally, K depends on the choice of $|\Omega\rangle$. Thus, using the BCH formula, Eq. (3.3) becomes

$$\mathcal{D} = 1 - e^{-\|K(\mathcal{A}_1 - \mathcal{A}_2)\|^2/2}. \tag{3.7}$$

Remarkably,

$$\langle N \rangle \equiv \|K(\mathcal{A}_1 - \mathcal{A}_2)\|^2 = \left\|KG^{\text{ret}}(j_1 - j_2)\right\|^2 \tag{3.8}$$

represents the expected number of photons in the coherent state whose classical solutions at late times are sourced by $j_1 - j_2$. Thence, we cast the expression above as

$$\mathcal{D} = 1 - e^{-\langle N \rangle /2}. \tag{3.9}$$

If the emission of entangling photons had occurred significantly, e.g., $\langle N \rangle \gg 1$, the decoherence of Alice's particle will be basically complete ($\mathcal{D} \approx 1$). The main difference between the present section and the next one lies on the way we compute the $\langle N \rangle$; here, we will focus on finding the classical solutions on the future Killing horizon and quantize them applying the usual prescription [19], whereas later we will show that similar results can be obtained by only looking into the two-point correlation functions of the electromagnetic field operator. This has the advantage of making clear that, what at first sight looks like depends on the global structures of spacetime, actually depends only on the interaction of the system with soft modes present in the thermally populated states, such as the one characterized by the Minkowski vacuum as seen by a congruence of uniformly accelerating observers restricted to move in the right Rindler wedge and the Hartle-Hawking vacuum as described by stationary observers near an eternal black hole. This second approach is more powerful, as it enables a more direct generalization for other spacetimes and vacuum states. Additionally, when we investigate our proposed model, we will focus on that interpretation.

Given that, we start exploring Eq. (3.9) with the simplest case: Alice's laboratory is inertial in Minkowski spacetime, and the initial state of the electromagnetic field is $|0_M\rangle$. It is clear that the particle can only emit entangling photons during the creation and recombination times, as a resulting acceleration to perform these processes is required. We can estimate $\langle N \rangle$ at the null infinity using the Larmor formula [10]

$$E \sim q^2 a^2 [\min(T_1, T_2)], \tag{3.10}$$

therefore

$$\langle N \rangle \sim q^2 a^2 [\min(T_1, T_2)]^2 = q^2 d^2 / [\min(T_1, T_2)]^2, \tag{3.11}$$

where a is the acceleration of the particle during the time intervals T_1 and T_2, and d is the distance between the superposition components. Note that the emitted

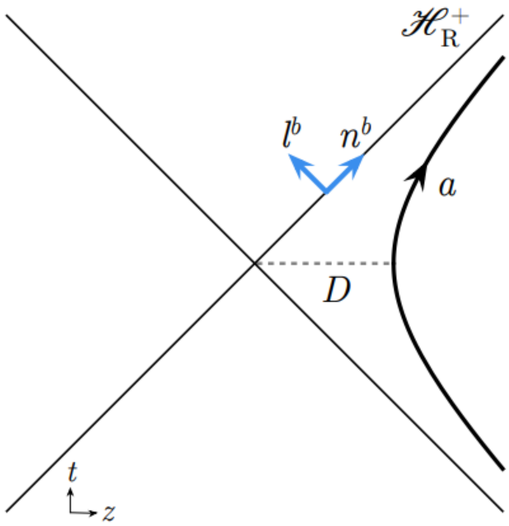

Figure 3.1: Schematic representation of Alice's lab moving along a stationary worldline in the right Rindler wedge. We assume that its distance from the horizon bifurcation is D, and remember that it is related to the lab's proper acceleration $a = 1/D$. Note that $\mathscr{H}_R^+$ is the future Killing horizon, with $l^b \equiv \left(\frac{\partial}{\partial U}\right)^b$ and $n^b \equiv \left(\frac{\partial}{\partial V}\right)^b$ being future-directed null vectors satisfying $l_a n^a = -1$. Figure taken from [13].

photons have a frequency on the order of $1/\min(T_1, T_2)$. Now, we will see how this picture changes when one considers that Alice's lab is uniformly accelerating in Minkowski space, e.g., its trajectory is stationary in Rindler space.

Recall that the planes $t = z$ and $t = -z$ are called Killing horizons because they are null surfaces that are normal to the boost Killing field [27]. We define null coordinates (U, V) and (u, v) in the same spirit as Eqs. (2.28) and (2.29), just switching from x to z. They are more suitable for analyzing the electromagnetic tensor on $\mathscr{H}_R^+$. Henceforth, we will also refer to V as "affine time" and v as "boost Killing time", as they provide a reasonable notion of time on the future Killing horizon. Remember that V and v are related by $V = \dfrac{1}{a}e^{av}$.

One can solve Maxwell's equations for a stationary charge in Rindler space, finding out that the only non-vanishing component in the region $V > 0$ of $\mathscr{H}_R^+$ [28]

$$E_U \equiv F_{ab}n^a l^b = \frac{2a^2 q}{\pi(1 + a^2\rho^2)^2}, \tag{3.12}$$

where $\rho^2 \equiv x^2 + y^2$. We borrowed the definition of the electric field as experienced by an observer with 4-velocity u^b, $E_a = F_{ab}u^b$, to compute the field on the horizon, even though n^b is a null vector in the present case. The radiation field E_A, where the index A refers to the spatial coordinates defined across the horizon, x and y, is given by the pullback of E_a to $\mathscr{H}_R^+$. According to Eq. (3.12), we observe that $E_A = 0$, thence the flux of "boost energy", $T_{ab}B^a$, vanishes at $\mathscr{H}_R^+$. Recall that T_{ab} is the energy-momentum tensor of the electromagnetic field. However, the situation changes if one displaces the particle in the z-direction such that its distance from the bifurcation of the Killing horizons goes from $D = 1/a$ to $D' = 1/a'$, e.g., it is moved to a different orbit of the same Killing field, such that $d \equiv D' - D \ll D$. On the horizon, Maxwell's equations enable us to write

$$\tilde{\nabla}^A E_A = \partial_V E_U, \tag{3.13}$$

where $\tilde{\nabla}^A$ is the covariant derivative operator on the cross sections of the horizon, and the corresponding indices can be raised and lowered using the standard metric δ_{AB} on $\mathbb{R}^2$.

We fix our gauge by assuming that $A_a n^a = 0$ on $\mathscr{H}_R^+$, which lets us write

$$E_A = -\partial_V A_A. \tag{3.14}$$

On the other hand, the transverse components of the Coulomb field vanish; thus, we can replace the vector potential in the expression above with its Coulomb-subtracted version $\mathcal{A}_A$. Therefore, integrating both sides of Eq. (3.13) by V results in

$$\tilde{\nabla}^A(\Delta\mathcal{A}_A) = -\Delta E_U. \tag{3.15}$$

One computes $\Delta E_U \equiv E'_U - E_U$ from Eq. (3.12), which leads to the following magnitude for $\Delta\mathcal{A}_A$

$$|\Delta\mathcal{A}_A| = \Delta\mathcal{A}_\rho \sim \frac{qda^3\rho}{(1+a^2\rho^2)^2}. \tag{3.16}$$

It is interesting to point out that, despite the radiative part of the electromagnetic field being zero at early and late times, the vector potential does not return to its initial value, regardless of how the particle is displaced to the new position. This can be readily understood as a memory effect, and we will see that it implies infrared divergencies analogous to those that occur at null infinity in QED [29]. Hence, quantizing these classical solutions following pretty much the same prescription as introduced in Section 2.1, and also recalling that the free data of the electromagnetic field on $\mathscr{H}_R^+$ in the chosen gauge is simply $\mathcal{A}_A$, we write down the inner product of their one-particle states as

$$\langle K\mathcal{A}_1|K\mathcal{A}_2\rangle_{\mathscr{H}_R^+} = 2\int_{\mathbb{R}^2} dxdy \int_0^\infty d\omega \frac{\omega}{2\pi}\delta^{AB}\overline{\hat{\mathcal{A}}_{1,A}}\hat{\mathcal{A}}_{2,B}. \tag{3.17}$$

Note that $\hat{\mathcal{A}}_A(\omega, x^B)$ is the Fourier transform of $\mathcal{A}_A(V, x^B)$. Naturally, the expected number of entangling photons in our case is $\langle N\rangle = \|K\Delta\mathcal{A}_\rho\|^2$. As $\Delta\mathcal{A}_\rho$ does not depend on V, its Fourier transform depends on ω as $1/\omega$, which will make $\langle N\rangle$ logarithmically diverge as $\omega \to 0$. It means that an infinite number of soft photons will be radiated across the horizon when the particle remains in its new position forever.

Now, suppose that, after a proper time T, Alice moves the particle back to its initial position. Observe that the assumption $d \ll D$ allows us to affirm that the proper time along the two worldlines is nearly the same. The vector potential returns to its initial value and, therefore, no memory effect is observed. As a consequence, the infrared divergence in $\langle N\rangle$ is fixed, thence the number of entangling photons propagating to $\mathscr{H}_R^+$ is expected to be finite. In the regime where T is very large, or more precisely, $T \gg T_1, T_2$, the asymptotic behavior of Eq. (3.17) is determined by the soft modes emitted when the particle lies at a distance D' from the bifurcation. Mathematically, this is implemented by imposing an infrared cut-off at frequency $\omega \sim 1/V$ such that $V = \frac{1}{a}e^{aT}$. In the same spirit, $1/\min(V_1, V_2)$ plays the role of an ultraviolet cut-off, such that V_1 and V_2 are the duration of the creation and recombination times as seen along the horizon. Note that we need to use the affine parameters as we are evaluating the norm on $\mathscr{H}_R^+$. It is shown that

$$\langle N\rangle = q^2d^2a^2\log\left(\frac{V}{\min(V_1, V_2)}\right) \sim q^2d^2a^3T. \tag{3.18}$$

Comparing the result above with the previous one for the case where the lab is inertial, we immediately conclude that now making the process adiabatic will not prevent the superposition from radiating soft photons across the horizon. Furthermore, we estimate the maximum time T_D that Alice can keep the coherence of her particle by observing that the decoherence rate is given by $\Gamma_{rad} = q^2d^2a^3$.

Thence, restoring the constants

$$\begin{aligned} T_D &= \frac{\epsilon_0 \hbar c^6}{a^3 q^2 d^2} \\ &\sim 10^{33} \text{ years} \left(\frac{g}{a}\right)^3 \cdot \left(\frac{e}{q}\right)^2 \cdot \left(\frac{1\,\text{m}}{d}\right)^2. \end{aligned} \tag{3.19}$$

In other words, if Alice's lab is accelerating with g, the gravitational acceleration on Earth's surface, in Minkowski space, and she creates a spatial superposition of an electron such that the components are one meter apart, then Alice cannot maintain the electron's coherence for more than 10^{33} years.

To conclude this section, we turn our attention to the gravitational case. The only thing we will change from the previous setting is that, instead of a charged particle, now we consider a massive body of mass m. Naturally, this massive body in a spatial superposition state sources a long-range gravitational field, so when the lab is inertial, the emission of entangling gravitons comes entirely from the creation and recombination times. We assume that the center of mass position is conserved, so there is no dipole radiation as in the electromagnetic case. The first nontrivial contribution comes from the quadrupole radiation, which results in the following expected number of gravitons radiated to null infinity

$$\langle N \rangle \sim m^2 d^4 / [\min(T_1, T_2)]^4. \tag{3.20}$$

If the lab is taken to be uniformly accelerating in flat space, we have to look into the "electric part" of the perturbed Weyl tensor $E_{ab} = C_{acbd} n^c n^d$, which plays the role of the electromagnetic field E_a. Recall that the Weyl tensor only contains the propagating degrees of freedom, in contrast to the Riemann tensor, which also carries matter and gauge parts [16, 18]. It represents the conformal curvature of the spacetime, e.g., the one that is not directly related to local matter. In $4D$, one writes it down in terms of components as

$$C_{abcd} = R_{abcd} + g_{b[c} R_{d]a} - g_{a[c} R_{d]b} + \frac{1}{3} R g_{a[c} g_{d]b}. \tag{3.21}$$

The indices between square brackets are anti-symmetrized. Observe that the Weyl tensor has the same symmetries with respect to its indices as the Riemann tensor. Moreover, C_{abcd} is exactly the traceless part of R_{abcd}. The classical solutions for a uniformly accelerating point mass imply that the only non-vanishing component of E_{ab} on the horizon is $E_{UU} = C_{acbd} l^a l^b n^c n^d$.

As before, the radiative part of the gravitational field is described by the pullback E_{AB} of E_{ab} to $\mathscr{H}_R^+$, and again, one observes that a static particle does not produce a flux of boost energy across the horizon. The situation changes if the body is moved by a distance $d \ll D$ away in the z-direction. Now, the Bianchi identity on $\mathscr{H}_R^+$ imply that

$$\tilde{\nabla}^A E_{AB} = \partial_V E_{UB}, \tag{3.22}$$

$$\tilde{\nabla}^A E_{UA} = \partial_V E_{UU}. \tag{3.23}$$

Therefore, we end up with

$$\tilde{\nabla}^A \tilde{\nabla}^B E_{AB} = \partial_V^2 E_{UU}. \tag{3.24}$$

At this point, we quantize the linearized gravitational field. Let h_{ab} be a metric perturbation in such a gauge that $h_{ab}n^a = 0$ and $\delta^{AB}h_{AB} = 0$. The free data on $\mathscr{H}_R^+$ is h_{AB}, which is related to E_{AB} as follows

$$E_{AB} = -\frac{1}{2}\partial_V^2 h_{AB}. \tag{3.25}$$

As in the electromagnetic case, we construct the Fock space associated with the Minkowski vacuum, and the inner product in the corresponding one-particle Hilbert space is given by

$$\langle K h_1 | K h_2 \rangle_{\mathscr{H}_R^+} = \frac{1}{8}\int_{\mathbb{R}^2} dxdy \int_0^\infty d\omega \frac{\omega}{2\pi} \delta^{AB}\delta^{CD}\overline{\hat{h}_{1,AC}}\hat{h}_{2,BD}, \tag{3.26}$$

where $\hat{h}_{AB}(\omega, x^C)$ is the Fourier transform of $h_{AB}(V, x^C)$. Eqs. (3.24) and (3.25) show that if the body remains in the new position forever, then $\Delta E_{UU} \neq 0 \to \Delta h_{AB} \neq 0$. Again, this memory effect leads to a logarithmic infrared divergence in the expected number of soft gravitons emitted to the future Killing horizon. However, we expect that the amount of radiated entangling gravitons becomes finite when one moves the body to its initial position after a long proper time T. A similar computation of $\langle N \rangle$ will result in the same linear dependence on T

$$\langle N \rangle = \|K\Delta h\|^2_{\mathscr{H}_R^+} \sim m^2 d^4 a^5 T. \tag{3.27}$$

Restoring the constants, we can estimate the corresponding decoherence time

$$\begin{aligned} T_D^{GR} &= \frac{\hbar c^{10}}{G m^2 d^4 a^5} \\ &\sim 2\text{ fs}\left(\frac{M_{\text{moon}}}{m}\right)^2 \cdot \left(\frac{R_{\text{moon}}}{d}\right)^4 \cdot \left(\frac{g}{a}\right)^5. \end{aligned} \tag{3.28}$$

So, consider a body whose mass is of the order of the Moon mass. If it is put in a superposition state in such a way that the components are one Moon radius apart and they are accelerating at one g, then Alice cannot keep its coherence for more than 2 femtoseconds.

The authors still argue that, although at first sight it seems that the acceleration of Alice's lab is causing the decoherence, some examples show that the principal responsible is, in fact, the presence of Killing horizons. For instance, they explicitly showed that, in de Sitter space, the cosmological horizons will eventually decohere any spatial superposition of charges and masses, even for non-accelerating components [13]. They later realized that the fundamental ingredient for such decoherence is the presence of low-frequency modes in the Unruh thermal bath that interact with the superposition [15] when the problem is analyzed in flat space. This becomes evident when we turn our attention to the two-point functions of the field operators introduced in this section.

3.2 Local viewpoint

The construction of a local approach in the electromagnetic case starts from generalizing Eq. (3.8) by noting that, at late times, one can replace G^{ret} by $-\Delta \equiv G^{\text{ret}} - G^{\text{adv}}$. This happens because, after recombining the superposition, we assume that $j_1^a = j_2^a$, Thence, we now have

$$\langle N \rangle = \|K\Delta(j_1 - j_2)\|^2, \tag{3.29}$$

and there is no need to evaluate the classical solutions only at late times as before. Again, we assume that the electromagnetic field is initially in some vacuum state $|\Omega\rangle$, so we can quantize the unperturbed field operators and express them in terms of creation and annihilation operators on $\mathcal{F}(\mathcal{H}_{in})$, the Fock space constructed from the corresponding one-particle Hilbert space $\mathcal{H}_{in}$ [13, 15]

$$\mathbf{A}_a^{\text{in}}(f^a) = ia(\overline{K\Delta(f)}) - ia^\dagger(K\Delta(f)), \tag{3.30}$$

where f^a is a divergence-free vector field. Naturally, the expression above leads to a direct connection between propagators and two-point functions of smeared fields

$$\langle\Omega| [\mathbf{A}_a^{\text{in}}(f^a)]^2 |\Omega\rangle = \|K\Delta(f)\|^2, \tag{3.31}$$

which, in turn, enables us to interpret $\langle N\rangle$ as vacuum fluctuations of the unperturbed electromagnetic field smeared by the difference of sources $j_1^a - j_2^a$

$$\langle N\rangle = \langle\Omega| [\mathbf{A}_a^{\text{in}}(j_1^a - j_2^a)]^2 |\Omega\rangle . \tag{3.32}$$

Observe that we have just obtained a description of the decoherence that does not refer to the existence of a horizon but rather depends only on the expected currents. Now, we compute these currents explicitly so that we can evaluate the corresponding two-point function. The 4-current of a point-like charged particle can be generically described as

$$j_1^a(t, x^i) = \frac{q}{\sqrt{-g}}\delta^{(3)}[x^i - X_1^i(t)]u_1^a\frac{d\tau_1}{dt}, \tag{3.33}$$

such that t is a Killing time coordinate, x^i are spatial coordinates on a Cauchy surface Σ_t which is perpendicular to the timelike Killing field t^a, $X_1^i(t)$ describes the trajectory taken by the particle, u_1^a is its 4-velocity and τ_1 is the proper time along its worldline. We can write an analogous expression for j_2^a. For a nonrelativistic motion, $\frac{d\tau_1}{dt} \approx \sqrt{-g_{tt}}$, hence

$$j_1^a(t, x^i) = \frac{q}{\sqrt{-g}}\delta^{(3)}[x^i - X_1^i(t)](t^a + v_1^a), \tag{3.34}$$

where v_1^a is the corresponding coordinate velocity, defined as $v_1^i \equiv \frac{dX_1^i}{dt}$ with $v_1^t \equiv 0$. The displacement between the components at a time t is given by the vector $S^a(t) \equiv d(t)s^a(t)$. $s^a(t)$ is a unit vector that is Lie transported along t^a and $d(t)$ represents the proper distance, so we define $d(t)$ as a smooth function such that

$$d(t) \equiv \begin{cases} d, -T/2 \leq t \leq T/2, \\ 0, t < -T/2 - T_1 \text{ and } t > T/2 + T_2. \end{cases} \tag{3.35}$$

Equipped with that, we can compute the difference $j_1^a - j_2^a$, which has a term that depends on the difference of charge densities and one that depends on the difference of spatial currents

$$\begin{aligned} j_1^a - j_2^a &\approx \frac{q}{\sqrt{-g}} t^a (\delta^{(3)}(x^i - X_1^i) - \delta^{(3)}(x^i - X_2^i)) - \frac{q}{\sqrt{-g}} \delta^{(3)}(x^i - X^i)(v_2^a - v_1^a) \\ &\approx \frac{qd(t)}{\sqrt{-g}} t^a s^b \nabla_b (\delta^{(3)}(x^i - X^i)) - \frac{q}{\sqrt{-g}} \delta^{(3)}(x^i - X^i) s^a t^b \nabla_b (d(t)), \end{aligned} \tag{3.36}$$

where X^i describes the path of Alice's lab. Therefore

$$j_1^a - j_2^a = \frac{2q}{\sqrt{-g}} t^{[a} s^{b]} \nabla_b [d(t) \delta^{(3)}(x^i - X^i)]. \tag{3.37}$$

Recall that the electric field on Σ_t is $E_a = F_{ab} t^b = (\nabla_a A_b - \nabla_b A_a) t^b$, so the smeared field operator given by

$$\mathbf{A}_a^{\text{in}} (j_1^a - j_2^a) = 2q \int d^4x t^{[a} s^{b]} \mathbf{A}_a^{\text{in}} \nabla_b [d(t) \delta^{(3)}(x^i - X^i)] \tag{3.38}$$

can be rewritten using integration by parts, and switching the indices in such a way that a F_{ab} arises in the integrand, so that we end up with

$$\mathbf{A}_a^{\text{in}} (j_1^a - j_2^a) = q \int dt d(t) s^a \mathbf{E}_a^{\text{in}} (t, X^i). \tag{3.39}$$

The expected number of entangling photons then depends on the two-point function of the component of the electric field in the direction of separation $s^a \mathbf{E}_a^{\text{in}}$ measured at Alice's lab

$$\langle N \rangle = q^2 \int dt dt' d(t) d(t') \langle s^a \mathbf{E}_a^{\text{in}} (t, X^i) s^{a'} \mathbf{E}_{a'}^{\text{in}} (t', X^i) \rangle_{\Omega}. \tag{3.40}$$

We will explicitly evaluate Eq. (3.40) in the Schwarzschild spacetime, assuming that the vacuum state is the Unruh vacuum. Close to the horizon, this result is analogous to what one would obtain if the same computations were done in Rindler space, considering the Minkowski vacuum. Note that stationary observers in the presence of an evaporating black hole describe the Unruh vacuum as a thermal flux of particles, the Hawking radiation, just as uniformly accelerating observers in flat space describe the Minkowski vacuum as a thermal bath. That accounts for many similarities concerning some two-point functions and detector responses in both cases, apart from redshift factors. For simplicity, we only consider a radial separation, so we can compute the expression in the integrand above using a mode expansion [30]

$$
\begin{aligned}
\langle \mathbf{E}_r(x)\mathbf{E}_r(x')\rangle_\Omega = \sum_{l=1}^{\infty} \frac{C_l P_l(\hat{r}\cdot\hat{r}')}{r^2 r'^2} \int_{-\infty}^{\infty} d\omega \frac{e^{-i\omega(t-t')}}{\omega} \\
\times \left[G^{\mathscr{H}^-}(\omega) R^{\mathscr{H}^-}_{\omega l}(r) \overline{R^{\mathscr{H}^-}_{\omega l}}(r') + G^{\mathscr{I}^-}(\omega) R^{\mathscr{I}^-}_{\omega l}(r) \overline{R^{\mathscr{I}^-}_{\omega l}}(r') \right],
\end{aligned}
\tag{3.41}
$$

where

$$
C_l \equiv \frac{1}{16\pi^2} l(l+1)(2l+1), \tag{3.42}
$$

P_l is the lth Legendre Polynomial, $R^{\mathscr{H}^-}_{\omega l}(r)$ and $R^{\mathscr{I}^-}_{\omega l}(r)$ are radial mode functions, with the former corresponding to incoming modes from the past Killing horizon and the latter corresponding to the incoming ones from the past null infinity, and $G^{\mathscr{H}^-}(\omega)$ and $G^{\mathscr{I}^-}(\omega)$ are coefficients that depend on the choice of vacuum state. We are interested in the case where $x^i = x'^i$, therefore, Eq. (3.40) becomes

$$
\begin{aligned}
\langle N \rangle =& q^2 \sum_{l=1}^{\infty} \frac{C_l (1-2M/r)}{r^4} \int_{-\infty}^{\infty} \frac{d\omega}{\omega} |\hat{d}(\omega)|^2 \\
& \times \left[G^{\mathscr{H}^-}(\omega) |R^{\mathscr{H}^-}_{\omega l}(r)|^2 + G^{\mathscr{I}^-}(\omega) |R^{\mathscr{I}^-}_{\omega l}(r)|^2 \right],
\end{aligned}
\tag{3.43}
$$

such that $\hat{d}(\omega)$ is the Fourier transform of $d(t)$. In the limit of $T \gg T_1, T_2$, we can approximate $|\hat{d}(\omega)|$ as

$$
|\hat{d}(\omega)| \sim \begin{cases} d/|\omega|, 1/T \leq |\omega| \leq 1/\min(T_1, T_2), \\ 0, |\omega| < 1/T \text{ and } |\omega| > 1/\min(T_1, T_2). \end{cases} \tag{3.44}
$$

For the Schwarzschild space, there is an extra factor of $(1-2M/r)$ in Eq. (3.43) arising when one goes from $s^a E_a$ to E_r [15]. For the Unruh vacuum, the coefficients $G^{\mathscr{H}^-}(\omega)$ and $G^{\mathscr{I}^-}(\omega)$ assume the form

$$
G^{\mathscr{H}^-}(\omega) = \frac{1}{1-e^{-2\pi\omega/\kappa}}, \tag{3.45}
$$

$$
G^{\mathscr{I}^-}(\omega) = \Theta(\omega), \tag{3.46}
$$

where $\kappa = 1/4M$ is the surface gravity of a static black hole of mass M and $\Theta(\omega)$ is the Heaviside function. This means that the Unruh vacuum is positive-frequency with respect to the Killing time at $\mathscr{I}^-$ and is positive-frequency with respect to the affine time at $\mathscr{H}^-$. Observe that, in the limit of T large, Eq. (3.43) is dominated by the low-frequency contribution, so we will end up using $G^{\mathscr{H}^-}(\omega) \approx \kappa/2\pi\omega$.

Now, we turn our attention to the radial mode functions. They satisfy the following differential equation

$$\frac{d^2 R_{\omega l}}{dr^{*2}} + [\omega^2 - V(r)]R_{\omega l} = 0, \tag{3.47}$$

with

$$V(r) \equiv (1 - 2M/r)\frac{l(l+1)}{r^2} \tag{3.48}$$

and

$$r^* = r + 2M \log\left(\frac{r}{2M} - 1\right) \tag{3.49}$$

being the so-called radial tortoise coordinate. Naturally, the incoming modes from the past horizon and from the past null infinity are subject to different asymptotic conditions, namely

$$R^{\mathscr{H}^-}_{\omega l}(r) = \begin{cases} e^{i\omega r^*} + A^{\mathscr{H}^-}_{\omega l} e^{-i\omega r^*}, \text{ as } r \to 2M, \\ B^{\mathscr{H}^-}_{\omega l} e^{i\omega r^*}, \text{ as } r \to \infty, \end{cases} \tag{3.50}$$

and

$$R^{\mathscr{I}^-}_{\omega l}(r) = \begin{cases} B^{\mathscr{I}^-}_{\omega l} e^{-i\omega r^*}, \text{ as } r \to 2M, \\ e^{-i\omega r^*} + A^{\mathscr{I}^-}_{\omega l} e^{i\omega r^*}, \text{ as } r \to \infty, \end{cases} \tag{3.51}$$

where $A_{\omega l}$ and $B_{\omega l}$ are initially unknown constants. Then, assume that Alice's lab is located somewhere in the region $M \ll r \ll 1/\omega$. The corresponding normalized solutions for $R^{\mathscr{I}^-}_{\omega l}(r)$ are

$$R^{\mathscr{I}^-}_{\omega l}(r) = -2i^{3l+1}\omega r^* j_l(\omega r^*), \tag{3.52}$$

such that j_l are the spherical Bessel functions. Ultimately, we are first interested in the case where Alice's lab is inertial in flat space, so we can ignore the contribution from the past horizon modes and take the $M \to 0$ limit in Eq. (3.52), e.g., $r^* \to r$. Furthermore, recalling that $\omega r \ll 1$, Eq. (3.52) becomes

$$R^{\mathscr{I}^-}_{\omega l}(r) \approx -\frac{i^{3l+1}2^{l+1}l!}{(2l+1)!}(\omega r)^{l+1}. \tag{3.53}$$

The dominant contribution to Eq. (3.43) comes from the $l = 1$ modes, thus, we finally arrive at

$$\begin{aligned}\langle N\rangle^{\text{EM}}_{\text{inertial}} &\sim \frac{q^2}{r^4}\int_{1/T}^{1/\min(T_1,T_2)}\frac{d\omega}{\omega}\frac{d^2}{\omega^2}(\omega r)^4\\ &\sim q^2d^2/[\min(T_1,T_2)]^2,\end{aligned} \tag{3.54}$$

where we have disregarded all numerical factors. We just recovered Eq. (3.11) from a completely local point of view.

Now, for the case where the lab is uniformly accelerating in flat space, we need to account for the past horizon modes contribution in Eq. (3.43). The suitable radial solutions in the region $M \ll r \ll 1/\omega$ for these modes are

$$R^{\mathscr{H}^-}_{\omega l}(r) \approx -\frac{i2^{l+2}(l-1)!(l+1)!}{(2l+1)(2l)!}\left(\frac{M}{r}\right)^l(M\omega). \tag{3.55}$$

Note that the low-frequency modes can reach Alice's lab before being reflected back to the black hole, so they will eventually decohere a quantum superposition constructed there. Again, the relevant contribution to Eq. (3.43) comes from the $l = 1$ modes, hence, assembling all pieces together

$$\begin{aligned}\langle N\rangle &\sim \frac{q^2M^2}{r^6}\int_{1/T}^{1/\min(T_1,T_2)}\frac{d\omega}{\omega}\frac{d^2}{\omega^2}(M\omega)^2\frac{\kappa}{\omega}\\ &\sim \frac{q^2d^2M^3}{r^6}T.\end{aligned} \tag{3.56}$$

To conclude, remind that a stationary observer at a distance r from a static black hole is uniformly accelerating at $a \sim M/r^2$. Therefore, we recover Eq. (3.18) for stationary observers in Rindler space

$$\langle N\rangle^{\text{EM}}_{\text{accel}} \sim q^2d^2a^3T. \tag{3.57}$$

It is important to emphasize that no explicit mention to a Killing horizon was made in this approach. Although they determine whether one has to consider past horizon modes or not, no classical solutions on them were evaluated in order to obtain Eq. (3.57).

Henceforth, we discuss a similar treatment for the gravitational case. First of all, we have seen in the previous section that, in the linearized gravity regime, the metric perturbation h_{ab} plays the role of the vector potential. Thus, one can naturally extend Eq. (3.32) to

$$\langle N \rangle = \langle \Omega | \, [\mathbf{h}^{\text{in}}_{ab}(T_1^{ab} - T_2^{ab})]^2 \, |\Omega\rangle \,, \tag{3.58}$$

where T_1^{ab} and T_2^{ab} are the energy-momentum tensor associated to each component of the superposition. Note that their difference is the test function which smears out the metric perturbation. Additionally, in analogy to Eq. (3.33), we construct the tensor corresponding to a point-like massive particle

$$T_1^{ab}(t, x^i) = \frac{m}{\sqrt{-g}} \delta^{(3)}(x^i - X_1^i(t)) u_1^a u_1^b \frac{d\tau_1}{dt}. \tag{3.59}$$

There are some subtleties regarding the backreaction in Alice's lab due to the external force acting on the components during creation and recombination times, and an extra contribution to keep the lab stationary is required. Nevertheless, in leading order, the difference $T_1^{ab} - T_2^{ab}$ can be presented in such a way that closely resembles Eq. (3.37)

$$T_1^{ab} - T_2^{ab} = \frac{2m}{\sqrt{-g}} \frac{dt}{d\tau} t^{[a} s^{c]} t^{[b} s^{d]} \nabla_c \nabla_d [d^2(t) \delta^{(3)}(x^i - X^i)]. \tag{3.60}$$

Eqs. (3.21) and (3.60) enable us to readily write down the smeared metric perturbation in terms of the electric part of the Weyl tensor field operator $\mathbf{E}^{\text{in}}_{ab}$ evaluated at Alice's lab

$$\mathbf{h}^{\text{in}}_{ab}(T_1^{ab} - T_2^{ab}) = m \int dt d^2(t) s^a s^b \mathbf{E}^{\text{in}}_{ab}(t, X^i). \tag{3.61}$$

Observe that now we define the classical field E_{ab} as $E_{ab} = C_{acbd} t^c t^d$. Thus, Eq. (3.40) becomes

$$\langle N \rangle = m^2 \int dt dt' d^2(t) d^2(t') \langle s^a s^b \mathbf{E}^{\text{in}}_{ab}(t, X^i) s^{a'} s^{b'} \mathbf{E}^{\text{in}}_{a'b'}(t, X^i) \rangle_\Omega \tag{3.62}$$

Again, to obtain the desired results of decoherence on the Minkowski vacuum in flat space, we will evaluate Eq. (3.62) in Schwarzschild spacetime, assuming a radial separation and that the gravitons are initially found at the Unruh vacuum. However, we do not need to perform all the corresponding computations once we realize that, apart from numerical factors, they are directly related to their analog in the electromagnetic case through the substitution $q \to m$ and $d \to d^2$. Furthermore, as gravitons are spin-2 particles, the sum over l has to start from

$l = 2$, hence the dominant contributions are

$$\begin{aligned}\langle N\rangle^{\text{GR}}_{\text{inertial}} &\sim \frac{m^2}{r^6}\int_{1/T}^{1/\min(T_1,T_2)}\frac{d\omega}{\omega}\frac{d^4}{\omega^2}(\omega r)^6 \\ &\sim m^2 d^4/[\min(T_1,T_2)]^4,\end{aligned} \tag{3.63}$$

and

$$\begin{aligned}\langle N\rangle &\sim \frac{m^2M^4}{r^{10}}\int_{1/T}^{1/\min(T_1,T_2)}\frac{d\omega}{\omega}\frac{d^4}{\omega^2}(M\omega)^2\frac{\kappa}{\omega} \\ &\sim \frac{m^2d^4M^5}{r^{10}}T.\end{aligned} \tag{3.64}$$

Using one more time that $a \sim M/r^2$, we also recover the result for the uniformly accelerating case

$$\langle N\rangle^{\text{GR}}_{\text{accel}} \sim m^2d^4a^5T. \tag{3.65}$$

The powerfulness of this approach allows one to analyze the experiment for different vacua, such as the Hartle-Hawking and Boulware vacua. Also, as a horizon is no longer required, one can think of what would happen if the experiment was realized near a static star or if there is a way to devise a body with internal degrees of freedom whose interaction with the system could mimic this decoherence due to soft emission. These questions were discussed in detail in [15]. Now, we focus on the model of a gapless finite-time detector interacting with a massive scalar field.

Chapter 4

Results

In the Section 2.3, we presented a form to quantify the decoherence of a qubit put in spatial superposition interacting with a scalar field. Now, we explore Eq. (2.60) for the cases where the qubit is inertial and uniformly accelerating in flat space. We also comment on the situations where one has a single spatial component, which provides important insights to interpret the physical significance of our results. In particular, we compare the decoherence of a single component in the accelerating case in the limit that the acceleration vanishes with the inertial result, showing that they are directly related. The choice of a qubit ensures that the emitted radiation consists exclusively of zero-energy particles. Furthermore, observe that $\|K_M E(f_1 + f_2)\|^2$ describes the expected number of scalar particles created throughout the experiment; hence, Eq. (2.60) immediately resembles the expression found by Danielson, Satishchandran, and Wald for their decoherence, e.g.,

$$\mathcal{D} = 1 - e^{-\langle N \rangle/2}. \tag{4.1}$$

The reason we opted for a massive field instead of a massless one will become clear in the next section. The field mass is a physical regulator that turns the number of created particles finite. Note that when we assume that the qubit components are inertial, the decoherence effect comes entirely from switching them on and off. On the other hand, even though the contribution from the switching remains when they are uniformly accelerating, we can choose the parameters in such a way that the soft emission dominates.

4.1 Inertial case

Initially, considering only a single worldline, we can assume that $f_2 \equiv 0$. On the other hand, $f_1 = \epsilon(t)\psi_1(t,\mathbf{x})$ is such that

$$\psi_1(t,\mathbf{x}) = \delta(x)\delta(y)\delta(z), \tag{4.2}$$

e.g., a stationary pointlike detector placed at the origin of the coordinate system, and $\epsilon(t) = \varepsilon C(t)$ with

$$C(t) = \begin{cases} e^{\alpha(t+T)}, t < -T, \\ 1, \; -T \leq t \leq T, \\ e^{-\alpha(t-T)}, t > T, \end{cases} \tag{4.3}$$

e.g., the detector is kept on from $t = -T$ to $t = T$ but it is continuously switched on and off. An instantaneous switching leads to UV divergences [31], hence α is the inverse of a timescale of how fast the detector is switched on, whereas ε indicates how strongly the detector is coupled to the field.

To evaluate $\|K_M E f_1\|^2$, recall that Ef_1 is a solution to the Klein-Gordon equation, which means that it can be expanded in terms of positive- and negative-frequency modes for any congruence of stationary observers who move along orbits of timelike Killing fields. In particular, for our case

$$Ef_1 = \int d^3\mathbf{k}[(u_\mathbf{k}, Ef_1)_{KG} u_\mathbf{k} - (\overline{u}_\mathbf{k}, Ef_1)_{KG} \overline{u}_\mathbf{k}]. \tag{4.4}$$

Note that the expression above is just the free field version of Eq. (2.10). Thus,

$$\|K_M E f_1\|^2 = \int d^3\mathbf{k} |(u_\mathbf{k}, Ef_1)_{KG}|^2, \tag{4.5}$$

and the integrated, computed using Eq. (2.13), is

$$|(u_\mathbf{k}, Ef_1)_{KG}|^2 = \frac{\varepsilon^2}{8\pi^2 \omega_\mathbf{k}} |\tilde{C}(\omega_\mathbf{k})|^2, \tag{4.6}$$

where $\tilde{C}(\omega) \equiv \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} dt e^{i\omega t} C(t)$ is the Fourier transform of $C(t)$. One finds that

$$|\tilde{C}(\omega_\mathbf{k})|^2 = \frac{2\alpha^2}{\pi \omega_\mathbf{k}^2 (\omega_\mathbf{k}^2 + \alpha^2)^2} \big(\omega_\mathbf{k} \cos(\omega_\mathbf{k} T) + \alpha \sin(\omega_\mathbf{k} T)\big)^2, \tag{4.7}$$

then, using the dispersion relation, Eq. (4.5) becomes

$$\begin{aligned} \|K_M E f_1\|^2 = {} & \frac{\varepsilon^2 \alpha^2}{\pi^2} \\ & \times \int_0^\infty dk \frac{k^2 \left(\sqrt{k^2+m^2} \cos\left(\sqrt{k^2+m^2} T\right) + \alpha \sin\left(\sqrt{k^2+m^2} T\right)\right)^2}{(k^2+m^2)^{3/2} (k^2+m^2+\alpha^2)^2}, \end{aligned} \tag{4.8}$$

such that $k = |\mathbf{k}|$.

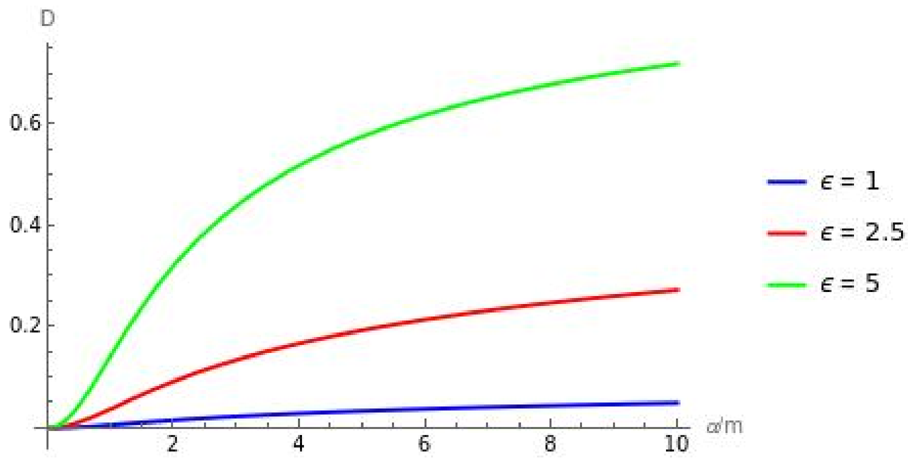

Figure 4.1: Decoherence as a function of the ratio α/m for different couplings.

An interesting regime to look into is when T is very large, e.g., $T \gg 1/m, 1/\alpha$, so we are going to take the average values of the trigonometric functions in Eq. (4.8) as the resulting integral converges. Thus

$$\begin{aligned} \|K_M E f_1\|^2 &= \frac{\varepsilon^2 \alpha^2}{2\pi^2} \int_0^\infty dk \frac{k^2}{(k^2+m^2)^{3/2}(k^2+m^2+\alpha^2)} \\ &= \frac{\varepsilon^2}{2\pi^2} \Big(\sqrt{1+(\frac{m}{\alpha})^2} \text{arcsinh} \frac{\alpha}{m} - 1 \Big). \end{aligned} \tag{4.9}$$

Note that, for a fixed α/m, the larger the coupling, the greater is the decoherence effect. Also, for a fixed coupling constant, switching the detector on/off faster increases the decoherence. In particular when $\alpha/m \gg 1$, one finds that Eq. (4.9) reduces to

$$\|K_M E f_1\|^2 \sim \frac{\varepsilon^2}{2\pi^2} \log \frac{\alpha}{m}.$$

A similar log-type divergence appears when one computes the transition probability for detectors with an energy gap [31]. Physically, this effect arises exclusively from the switching, as it requires some external work. It was shown [21] that, for an inertial qubit, the amount of work needed grows with the coupling strength ε and the inverse timescale α. Hence, increasing these parameters results in more particles being created, which intensifies the information loss.

Conversely, let us consider a detector in a spatial superposition state; hence, besides the ψ_1 defined in Eq. (4.2), we introduce ψ_2 such that $\psi_2(t, \mathbf{x}) = \delta(x)\delta(y)\delta(z - L)$, so it represents a stationary detector displaced by L in the z-direction from the origin. Recall that neither the coupling strength nor the switching function

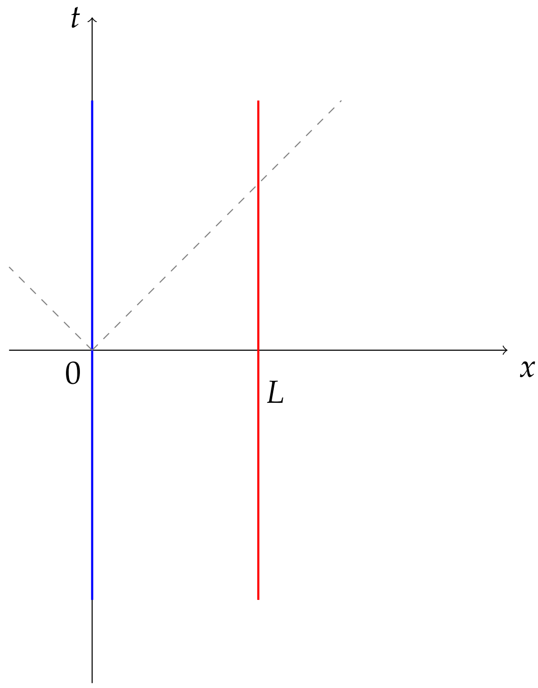

Figure 4.2: Diagram containing the stationary trajectories in superposition for the inertial case.

changes, so Eq. (4.6) is now cast as

$$|(u_{\mathbf{k}}, E(f_1 + f_2))_{KG}|^2 = \frac{\epsilon^2(1 + \cos(kL\cos\theta))}{4\pi^2\omega_{\mathbf{k}}}|\tilde{C}(\omega_{\mathbf{k}})|^2, \tag{4.10}$$

which results in, knowing that $\text{sinc}(x) \equiv \sin x/x$

$$\|K_M E(f_1 + f_2)\|^2 = \frac{\epsilon^2\alpha^2}{\pi^2}\int_0^\infty dk \frac{k^2\left[1 + \text{sinc}(\sqrt{k^2 + m^2}L)\right]}{(k^2 + m^2)^{3/2}(k^2 + m^2 + \alpha^2)}. \tag{4.11}$$

Again, we have considered the limit where T is very large. The first term in the expression above is simply what we computed before, so let us focus on the remaining one. An analytical approach with general assumptions is too difficult once the integrand slowly oscillates. One may be interested in the regime $T \gg 1/\alpha \gg 1/m \gg L$, e.g., light rays emitted from one worldline reach the other faster than switching the interaction. In this case, we can find a closed analytic form for Eq. (4.11), then we first rescale all quantities in Eq. (4.11) by m

$$\|K_M E(f_1 + f_2)\|^2 = \frac{\epsilon^2\alpha^2}{\pi^2 m^2}\int_0^\infty d\tilde{k} \frac{\tilde{k}^2\left[1 + \text{sinc}(\sqrt{\tilde{k}^2 + 1}mL)\right]}{(\tilde{k}^2 + 1)^{3/2}(\tilde{k}^2 + 1 + \alpha^2/m^2)}, \tag{4.12}$$

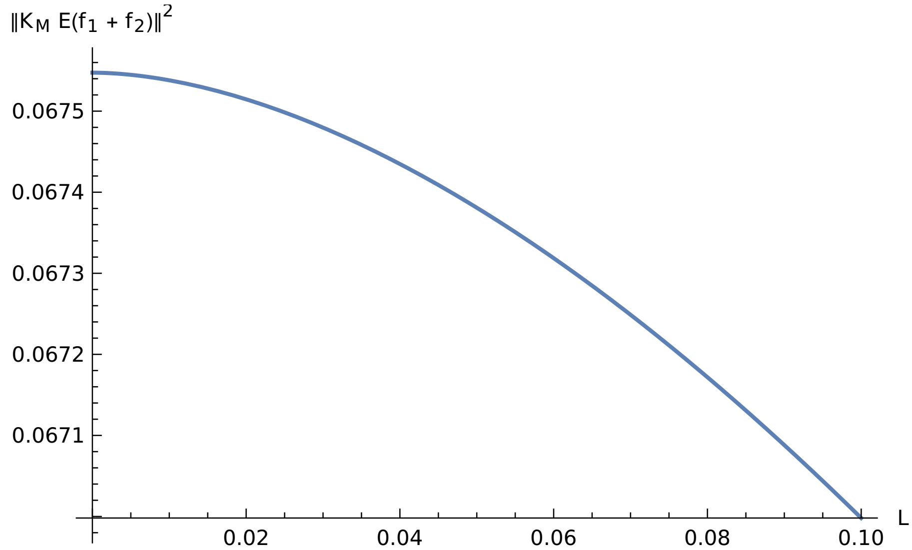

Figure 4.3: $\|K_M E(f_1 + f_2)\|^2$ as a function of L. We set $\varepsilon = 10$, $\alpha = 0.1$, and $m = 1$.

but for $\alpha/m \ll 1$, we approximate the expression above as

$$\|K_M E(f_1 + f_2)\|^2 \sim \frac{\varepsilon^2 \alpha^2}{\pi^2 m^2}\Big[\frac{1}{3} + \int_0^\infty d\tilde{k} \frac{\tilde{k}^2 \mathrm{sinc}(\sqrt{\tilde{k}^2+1}mL)}{(\tilde{k}^2+1)^{5/2}}\Big]. \tag{4.13}$$

From Figure 4.3, we infer that the decoherence is not very sensitive to variations in L, as $\|K_M E(f_1 + f_2)\|^2$ decreases for only $\sim 0.6\%$ when we go from $L = 0$ to $L = 0.1$ (in units of $1/m$). We are in a regime where the work required to turn the qubit on and off is small compared to the rest energy of the scalar particles. Therefore, varying L must not change the result significantly.

Note that making $L \to 0$ does not recover Eq. (4.9) in the corresponding regime, as $\mathrm{sinc}(x)$ goes to 1 when $x \to 0$. We know that the detector's internal state plays a role in the dynamics, which can be seen in Eq. (2.44). So, "collapsing" the worldlines but letting the internal state be different in each one must lead to additional decoherence. To further interpret this result physically, let us focus on the first non-trivial term of Eq. (4.13)

$$\|K_M E(f_1 + f_2)\|^2 \sim \frac{1}{3\pi^2}\frac{\varepsilon^2 \alpha^2}{m^2} + \mathcal{O}(L, \alpha^2). \tag{4.14}$$

Recall that, in [10,11,13,15], the authors introduce a similar setting where a charged particle is put in a spatial superposition state such that its spin is entangled with its position. The experiment can be divided into three eras, consisting of the moment when the superposition is created, followed by the time at which the charge is let in this state, and then it ends when the worldlines are recombined. In flat space, with the components of that superposition being inertial trajectories, the emission of entangling radiation might occur during the first and last eras, during which the system cannot be held stationary. The acceleration of the particle over these time intervals is said to be equal to a. This emission, widely discussed in the previous chapter, is governed by dipole radiation, and the expected number of photons emitted is, per Eq. (3.10)

$$\langle N \rangle \sim q^2 a^2 [\min(T_1, T_2)]^2. \tag{4.15}$$

Now, comparing this result to Eq. (4.14), we observe that ε plays the role of the charge q, informing in each case how strongly the particle is coupled to the corresponding field, α corresponds to the acceleration a, e.g., they represent the contribution of the external agent, which is responsible for the decoherence, and $1/m$ sets a characteristic timescale, just like $\min(T_1, T_2)$ does, for which the phenomenon becomes relevant. The smaller the mass m of the scalar field, the more information is lost due to the interaction, which can be attributed to the fact that lighter particles require less energy to be created. In the limit $m \to 0$, we end up with infinitely many massless particles that lead to complete decoherence.

Our result allows us to go further and obtain more corrections due to the interference between the components of the superposition. Thus, we conclude that, if the first and last eras are implemented adiabatically, e.g., sufficiently slow, the particle keeps its coherence and no entangling radiation is emitted (refer to Eq. (3.11)). We will see that this is not the case when one analyzes a scenario in which the detector experiences a thermal bath containing soft particles.

4.2 Uniformly accelerating case

Analogously to what was done in the inertial case, we start with a single worldline, assigning it the coordinate $\xi = 0$. Besides that, our switching function is the same as defined in Eq. (4.3), but now $\psi_1(t, \mathbf{x})$ becomes $\psi_1(t, \mathbf{x}) = \frac{1}{\sqrt{-g(x)}}\delta(\xi)\delta(y)\delta(z)$. The $\sqrt{-g(x)}$ factor is important to define the delta distribution covariantly. The expansion made in Eq. (4.4) is now taken in terms of the Unruh modes defined in Eq. (2.32), which are linear combinations of the Rindler modes that correspond to positive-frequency solutions with respect to inertial

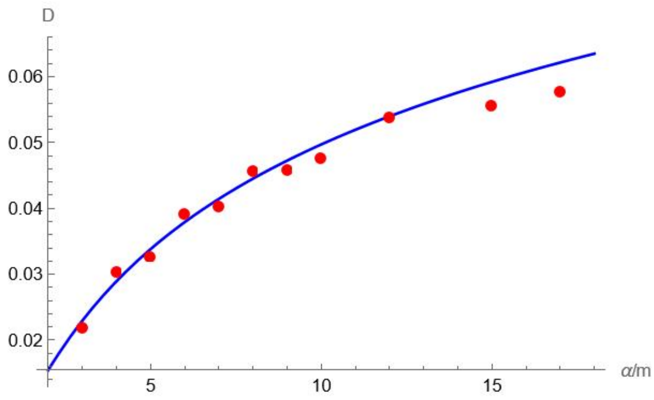

Figure 4.4: Decoherence as a function of the ratio α/m, considering the coupling $\varepsilon = 1$, for the inertial case (Eq. (4.9)) and uniformly accelerated case (Eq. (4.19)) when we set $a = 0.1$, $m = 1$ and $T = 100$.

time. Hence, Eq. (4.5) becomes

$$\|K_M E f_1\|^2 = \int_{\mathbb{R}} d\omega \int_{\mathbb{R}^2} d^2\mathbf{k}_\perp |(w^1_{\omega\mathbf{k}}, Ef_1)_{KG}|^2. \tag{4.16}$$

It is worth mentioning that $w^1_{-\omega\mathbf{k}_\perp} = w^2_{\omega\mathbf{k}_\perp}$ [21], and recalling Eq. (2.13), one writes down

$$(w^1_{\omega\mathbf{k}_\perp}, Ef_1)_{KG} = \frac{i\varepsilon}{\sqrt{1-e^{-2\pi\omega/a}}}\left(\frac{\sinh\left(\frac{\pi\omega}{a}\right)}{2\pi^3 a}\right)^{1/2} K_{-i\omega/a}\left(\sqrt{k_\perp^2+m^2}/a\right)\tilde{C}(\omega), \tag{4.17}$$

and

$$(w^2_{\omega\mathbf{k}_\perp}, Ef_1)_{KG} = \frac{i\varepsilon e^{-\pi\omega/a}}{\sqrt{1-e^{-2\pi\omega/a}}}\left(\frac{\sinh\left(\frac{\pi\omega}{a}\right)}{2\pi^3 a}\right)^{1/2} K_{i\omega/a}\left(\sqrt{k_\perp^2+m^2}/a\right)\tilde{C}(\omega), \tag{4.18}$$

therefore one finds that

$$\|K_M E f_1\|^2 = \frac{\varepsilon^2 a}{\pi^2}\int_0^\infty d\omega \cosh\left(\frac{\pi\omega}{a}\right)|\tilde{C}(\omega)|^2 \int_{m/a}^\infty dx x |K_{i\omega/a}(x)|^2, \tag{4.19}$$

where $|\tilde{C}(\omega)|^2$ is given by Eq. (4.7). We also used the fact that $\tilde{C}^*(\omega) = \tilde{C}(\omega)$.

An interesting consistency check one can make with the expression is verifying if the inertial result can be obtained in the limit $a \to 0$. Numerically, it can be implemented by making a be the smallest scale in the problem, while T is set to be the largest one, as the same hypothesis was considered when we were analyzing the inertial case. In other words, we are working in the regime $T \gg 1/a \gg$

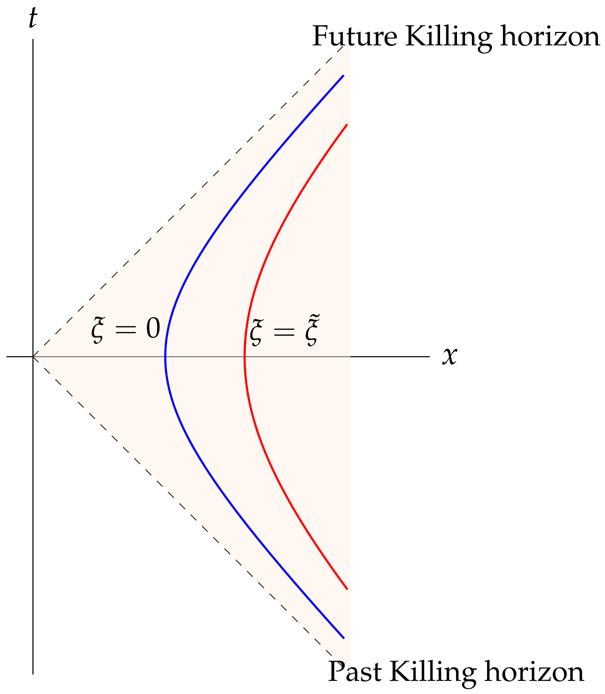

Figure 4.5: Diagram containing the stationary trajectories in superposition for the uniformly accelerating case. We will assume that $\tilde{\xi} \ll 1/a$.

$1/\alpha,\ 1/m$. These results are compared in Figure 4.4.

Now, we consider the situation with the spatial superposition in Rindler space, as depicted in Figure 4.5. We modify the switching function $C(t)$ for the component associated with the trajectory $\xi = \tilde{\xi}$ such that $T \to T' = Te^{-a\tilde{\xi}}$ and $\alpha \to \alpha' = \alpha e^{a\tilde{\xi}}$. Thus, co-moving observers with each component see the detector switched on during the same time interval, and the switching itself occurs at the same proper time along each worldline. We refer to this new function as $C_2(t)$, whereas the former becomes $C_1(t)$. Besides that, we also have that $\psi_2(t, \mathbf{x}) \equiv \frac{1}{\sqrt{-g(x)}}\delta(\xi - \tilde{\xi})\delta(y)\delta(z)$. Provided that, we now cast Eq. (4.17) as

$$
\begin{aligned}
(w^1_{\omega\mathbf{k}_\perp}, E(f_1 + f_2))_{KG} &= \frac{i\epsilon}{\sqrt{1 - e^{-2\pi\omega/a}}}\left(\frac{\sinh\left(\frac{\pi\omega}{a}\right)}{2\pi^3 a}\right)^{1/2} \\
&\times \left[K_{-i\omega/a}\left(\sqrt{k_\perp^2 + m^2}/a\right)\tilde{C}_1(\omega) + K_{-i\omega/a}\left(\sqrt{k_\perp^2 + m^2 e^{a\tilde{\xi}}}/a\right)\tilde{C}_2(\omega)\right],
\end{aligned}
\tag{4.20}
$$

and an analogous expression can be obtained from Eq. (4.18). Therefore, we can

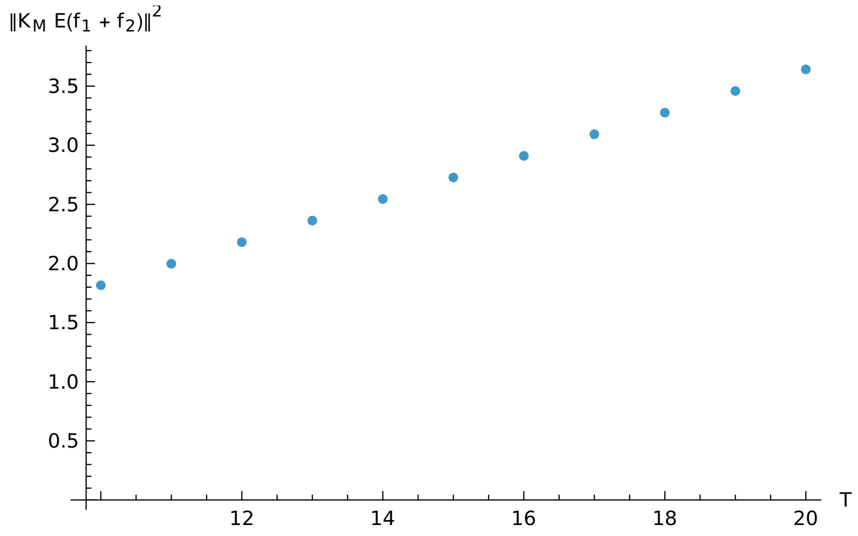

Figure 4.6: $\|K_M E(f_1 + f_2)\|^2$ as a function of time T. We set $\alpha = 0.001$, $m = 0.01$, $a = 1$ and $\tilde{\xi} = 0.1$.

split up the corresponding norm squared into three terms

$$\|K_M E(f_1 + f_2)\|^2 = \frac{\epsilon^2 a}{\pi^2}(I_{\xi=0} + I_{\xi=\tilde{\xi}} + I_{int}), \tag{4.21}$$

such that $I_{\xi=0}$ is given by the right-hand side of Eq. (4.19), whereas $I_{\xi=\tilde{\xi}}$ represents the contribution of the second component

$$I_{\xi=\tilde{\xi}} \equiv e^{-2a\tilde{\xi}} \int_0^\infty d\omega \cosh\left(\frac{\pi\omega}{a}\right) |\tilde{C}_2(\omega)|^2 \int_{me^{a\tilde{\xi}}/a}^\infty dx x |K_{i\omega/a}(x)|^2, \tag{4.22}$$

and I_{int} represents the contribution that arises from the interference between the trajectories

$$I_{int} \equiv 2 \int_0^\infty d\omega \cosh\left(\frac{\pi\omega}{a}\right) \tilde{C}_1(\omega)\tilde{C}_2(\omega) \int_{m/a}^\infty dx x \mathrm{Re}[K_{i\omega/a}(x) K_{-i\omega/a}(xe^{a\tilde{\xi}})]. \tag{4.23}$$

We will look into the regime where the distance between the worldlines is relatively small compared to their distance to the bifurcation of the Killing horizons, e.g., $1/a \gg \tilde{\xi}$; hence, the coordinate $\tilde{\xi}$ can be interpreted as the proper distance between them. We also want to minimize the effect of the switching on the decoherence, and from the inertial result, we conclude that it can be achieved by making α small.

In particular, we are interested in the regime where $1/\alpha \gg 1/m \gg T \gg 1/a \gg \tilde{\xi}$. Hence, we minimize the soft emission due to switching the detector

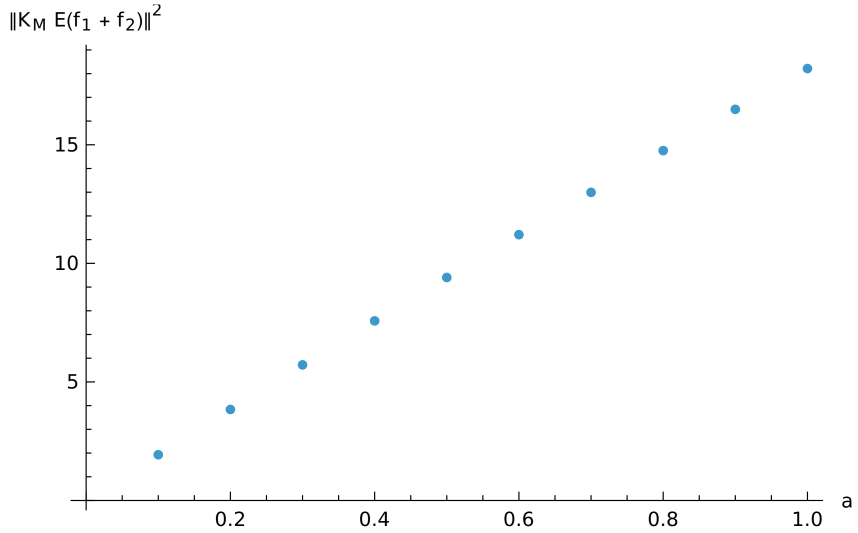

Figure 4.7: $\|K_M E(f_1 + f_2)\|^2$ as a function of the acceleration a. We set $\alpha = 0.0001$, $m = 0.001$, $T = 100$, $\tilde{\xi} = 0.1$.

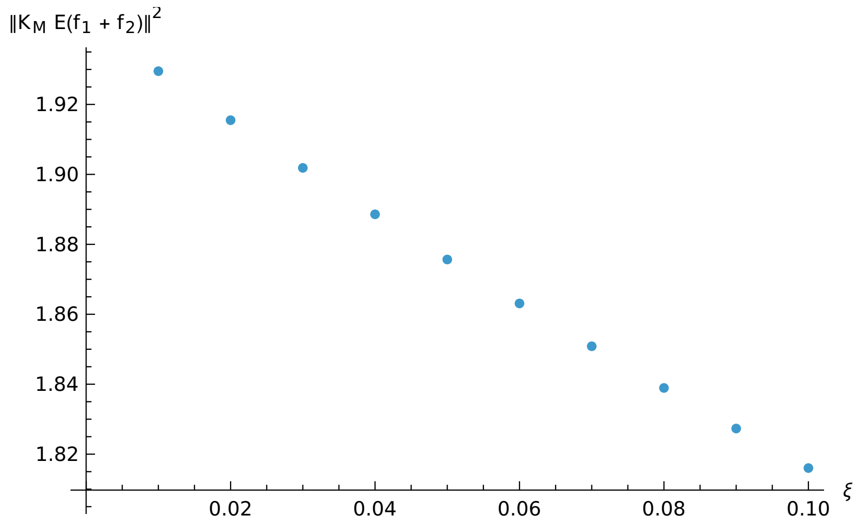

Figure 4.8: $\|K_M E(f_1 + f_2)\|^2$ as a function of the distance $\tilde{\xi}$. We set $\alpha = 0.001$, $m = 0.01$, $T = 10$, $a = 1$.

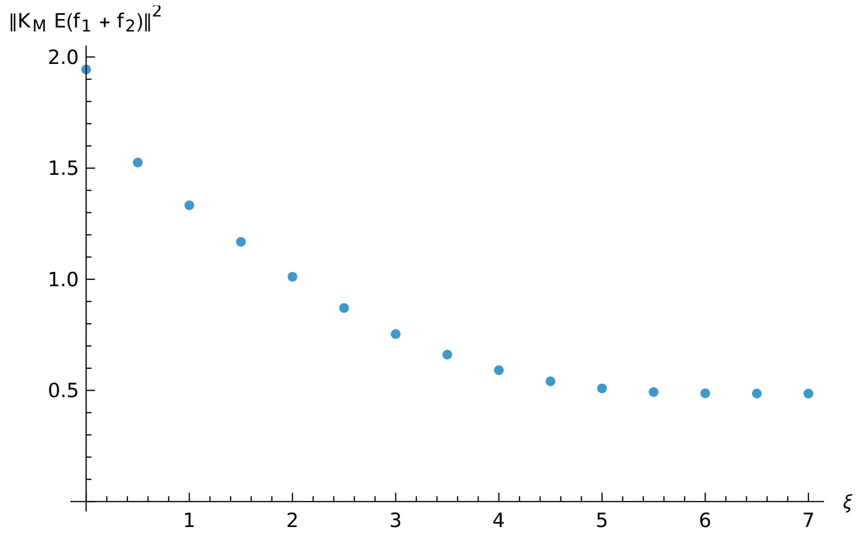

Figure 4.9: $\|K_M E(f_1 + f_2)\|^2$ as a function of the spatial coordinate $\tilde{\xi}$. We relaxed the condition $1/a \ll \tilde{\xi}$, so one can look into the regime of large distances. We set $\alpha = 0.001$, $m = 0.01$, $T = 10$, $a = 1$.

on and off, although $1/\alpha \gg T$ makes the interaction with the Unruh bath during the switching times relevant; nevertheless, we can disregard this contribution by subtracting the number of particles created at such times. Ultimately, we are looking into an analogous regime to [13], where they found that

$$\langle N \rangle \sim q^2 a^3 d^2 T. \tag{4.24}$$

The authors assumed that the proper time at which the components are stationary is very large (since their case is electromagnetic, there is no field mass, thus $T \gg 1/a, d$); additionally, the authors considered that the distance between the components is very small when compared to the distance from the laboratory to the horizon's bifurcation ($d \ll D = 1/a$).

In Figs. 4.6, 4.7, and 4.8, we show how the number of soft particles changes as we increase the time of interaction, the proper acceleration of the components, and the distance between them, respectively.

Regarding the dependence on T, we found that the effect grows linearly, in agreement with the work of Danielson, Satischandran, and Wald. That demonstrates that both phenomena are of the same nature, e.g., as we increase the time interval at which the detector is interacting with the field, more soft modes present in the Unruh bath are absorbed, stimulating the emission of more soft particles.

On the other hand, although our result also grows with a, it does not go as a^3, but rather increases linearly as well. Note that the temperature of the thermal bath for each component is $\Theta = a/2\pi$ and $\Theta = ae^{-a\tilde{\xi}}/2\pi \approx a/2\pi$, so increasing a, we turn the bath hotter, which results in a larger population of soft modes. Hence, one should expect that the emission of particles would be enhanced, increasing the decoherence.

Finally, we found that the detector's decoherence decreases as one moves the second component away from the first one, which is kept fixed, in contrast to the previous results. In our context, the displaced component experiences a colder bath, so it interacts with fewer soft particles compared to the component at $\xi = 0$. It might be the case that quantum interference effects compensate for this temperature decrease in [13], but it is clear that the same does not occur in the present work.

We can further study the phenomenon in the limit of large distances, where we cannot approximate the proper distance between the worldlines with the spatial coordinate $\tilde{\xi}$ anymore. It is interesting that when the thermal energy of the particles present in the bath experienced by the displaced component is of the order of its rest energy (e.g., $\tilde{\xi} = \log(a/2\pi m)/a$), the curve in Fig. 4.9 forms a plateau, and the converging value corresponds to the particle emission associated with the fixed component only. That means that, for large distances, the displaced component essentially sees no soft modes in the thermal bath.

Chapter 5

Closing remarks

The decoherence of quantum superposition in relativistic configurations is a relevant topic that has sparked recent interest, as it seems to provide a promising path towards a better comprehension of the quantum nature of gravity. Since experiments that can probe gravitons directly are not believed to be reached in the near future, and it is not clear that their detection is possible even in principle [32,33], the decoherence of spatial superpositions induced by gravity may be an easier alternative to observe such regime in a laboratory. This experimental approach, however, cannot happen without a complete understanding of this sort of phenomenon from a theoretical perspective.

In this sense, Wald and collaborators have proposed a conceptually simple but extremely enlightening setting discussed throughout the present work. They argued that the spatial superposition of sources of long-range fields, such as charges for electromagnetic fields and masses for gravitational fields, will eventually decohere when placed in a spacetime that contains Killing horizons. Such spacetimes comprise not only the Rindler space, but also the Schwarzschild and de Sitter spaces. A key aspect of their claim is that this effect arises primarily from the emission of soft particles through the Killing horizons, because even if the experiment is taken adiabatically, the expected number of particles on the horizons is not negligible. A more careful examination of this problem, on the other hand, revealed that it is caused by the presence of low-frequency Unruh and Hawking quanta that appear in Minkowski and Schwarzschild spacetime, respectively, depending on the choice of vacuum state. Particles of arbitrarily small energy are directly related to Weinberg's soft theorems [34] and hence are fundamental to understanding the infrared behavior of field theories. Despite that, the existence and physical significance of soft particles have been disputed, and the situation proposed by these authors might shed light on the issue, providing a scenario where they are absolutely necessary to explain the results obtained.

However, the cited work also brings new questions that can be better addressed. For instance, the influence of the external agent, required to construct and recombine the spatial components, besides keeping them stationary in Rindler space, is not completely under control, and it might give rise to extra contributions that have not been taken into consideration. Also, working with only leading terms makes the results reliable only in a certain regime. In this sense, we proposed a setting where the contribution of the external agent, e.g., the one responsible for switching the detector on and off, is precisely determined. Moreover, our model is not restricted to a specific regime, and it is supposed to work out for any choice of parameters.

We have shown that, in the inertial case, our model produces results that can be interpreted in line with the ones obtained by [13]. In this scenario, the switching of the detector provides enough energy to excite the massive scalar field and hence create particles, whereas in the uniformly accelerating case, turning the process adiabatic, e.g., by making $\alpha \to 0$, does not prevent the system from losing its coherence. Tuning our system in an analogous regime to that of the referred authors, we observed that the soft modes present in the Unruh thermal bath induced the same behavior regarding the dependence on the time of interaction, whereas the dependence on the proper acceleration can be related only qualitatively, in the sense that the decoherence in both scenarios grows with a. The main difference lies on the results concerning the dependence on the distance between the components, where we verified that it monotonically decreases in that regime. Moreover, for large distances, the contribution from the displaced component is suppressed, once the thermal bath experienced by it does not contain enough soft modes. Ultimately, we are investigating a scalar theory, in contrast to the original works that are intimately related to the nuances of electromagnetic and gravitational theories.

Furthermore, we demonstrated that our approach allows us to recover the inertial result in the limit that $a \to 0$, which is not manifest in the previous setting. We might explore new regimes that can test the validity of our model and bring new insights into the problem. Another possibility is changing the initial field state to other vacuum states in different spacetimes and then verifying whether the effect remains [15]. We hope that our discussion helps to improve the comprehension of such a rich topic that involves concepts of quantum information, quantum field theory, and gravity.

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