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Left–right asymmetry and minimal coupling

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In this paper we deal with an alternative approach to the description of massless particles of arbitrary spin. Within this scheme chiral components of a spinor field are regarded as fundamental quantities and treated as independent field variables. The free field Lagrangian is built up from the requirement of chiral invariance. This formulation is parallel to the neutrino theory and allows for a formulation that generalizes, to particles of arbitrary spin, the two-component neutrino theory. We achieve a spinor formulation of electrodynamics. In the case of the photon, the nonzero helicity components satisfy Weyl's equations and are associated to observables (electromagnetic fields) whereas the zero helicity components are related to nonobservables (electromagnetic potentials). Within the spinor formulation of electrodynamics the minimal coupling substitution follows as a consequence of the linearity of the interaction and the preference of nature for chiral components, that is, of the left–right asymmetry of nature. © *1996 American Institute of Physics.* [S0022-2488(96)01310-4]

I. INTRODUCTION

Nowadays there is plenty of evidence that nature is asymmetric with respect to chirality (left and right handedness).[1] Within the realm of the fundamental interactions this asymmetry has been realized since the V-A theory[2] of weak interactions and has been verified empirically in 1957 in a parity violating process occurring as a result of weak processes.[3] In this paper we will show that, within an alternative scheme for treating massless particles, electrodynamics is also a left–right asymmetric theory. In this way we extend even further the notion that fundamental interactions violate left–right symmetry.

Within the usual formulation of electrodynamics (the tensorial method) the seemingly asymmetrical magnetism[1] cannot be entirely understood in view of the fact that the electromagnetic fields are fundamentally symmetrical (the Lagrangians, in terms of these fields, do not exhibit any left–right asymmetry). Within the spinor method, the one that will be employed in this paper, one has an equivalent formulation of electrodynamics allowing us to understand that magnetism and electrodynamics is, in fact, a basic departure from the left–right asymmetry of nature.

The spinorial formulation proposed here allows us to formulate theories involving massless spin 1 particles in close analogy with massless spin $\frac{1}{2}$ particles. The requirement of chiral invariance, at the free field level, leads to Maxwell's equation in vacuum. The requirement of chiral asymmetry for the linear interaction of the spinor field with matter leads to Maxwell's equation in the presence of matter. Previous spinorial formulations of electrodynamics can be found in Refs. 4 and 5.

In order to generalize the left–right asymmetry to other interactions, as will be done in this paper, let us recall that for massless spin $\frac{1}{2}$ particles one can define chiral components ψ_L and ψ_R of a basic field ψ as follows:

$$\psi_R = \tfrac{1}{2}(I+\gamma^5)\psi, \quad \psi_L = \tfrac{1}{2}(I-\gamma^5)\psi.$$

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Furthermore, under space reflection (P) these components transform as

$$\psi_R \xrightarrow{P} \psi'_R(x') = \xi\gamma_0\psi_L(-\mathbf{x},t), \quad \psi_L \xrightarrow{P} \psi'_L(x') = \xi\gamma_0\psi_L(-\mathbf{x},t).$$

The intriguing aspect of the weak interactions is that, at low energies, the right and left components interact in a different way with ordinary matter. As a matter of fact, there is no evidence at all that the right-hand component interacts with ordinary matter. Weak interactions are definitely asymmetric with regard to left and right. This follows from the fact that right and left components interact with different strength to matter.

At the Lagrangian level this preference of nature for chiral components can be formulated in a simple way by stating that the Lagrangian is not invariant under the transformation

$$\psi_R \to \psi_L .$$

Due to the transformation properties of $\psi_R(\psi_L)$ under space reflections, the noninvariance of the Lagrangian under the left–right transformation implies breakdown of space reflection.

The first question that we deal with in this paper is the possibility of extending, to massless particles of arbitrary spin, the usual neutrino theory. We shall see that the spinor method provides an approach that permits us such a generalization. Within the spinor method it is possible to generalize the notion of chiral invariance for particles of arbitrary spin as well as to define chiral components analogous to the ones associated to massless particles of spin $\frac{1}{2}$. Within the spinor method one assigns to a particle of spin s a symmetric spinor ψ of rank $2s$. The definition of chiral components, for a particle of arbitrary spin, is the tensor product of the usual ones.

We propose that these chiral components should be treated as independent variables and that they satisfy extended Dirac equations analogous to the ones satisfied by the left and right neutrino field components. These equations, as will be shown later, follow from the requirement that the free field Lagrangian be chiral invariant.

The generalized chiral components provide a very simple criterion on whether a theory involving masslessness is left–right symmetric or not. The theory is left–right symmetric if the Lagrangian is invariant under the subscript interchange $L \to R$ (or $R \to L$). Otherwise the theory is $L-R$ asymmetric. The theory is left–right asymmetric if the coupling of the chiral components to the matter fields occurs with different strengths.

As a byproduct of the formulation of zero-mass particles in terms of chiral components, we will show that it is possible to generalize the two-component neutrino theory to particles of arbitrary spin. The generalized two-component fields satisfy generalized Weyl's equations.

It seems to be worthwhile to analyze whether, besides the neutrinos, there is other evidence in nature of an asymmetric coupling of chiral components of zero-mass particles to ordinary matter. The next, nontrivial, zero-mass particles that couple to ordinary particles are the photons. We find that the coupling of photons reflects some kind of asymmetry between the coupling of chiral components to ordinary matter. We will show that, as in other theories, in electrodynamics chiral asymmetry and parity nonconservation are intimately connected. We shall see that, in electrodynamics, the only consequence of violation of these symmetries is a dynamical one, namely, the minimal substitution way of coupling the electromagnetic fields to matter.

In Sec. II we present a novel approach to the study of massless particles of arbitrary spin. In close analogy with the spin $\frac{1}{2}$ particles we generalize, to arbitrary spin s particles, the usual spin $\frac{1}{2}$ chiral components. One can also generalize the two-component neutrino theory to massless particles of arbitrary spin. This extension is possible in the context of the Bargmann–Wigner (BW) theory. The totally right (left) components have generalized helicity components s $(-s)$ and obey the generalized Weyl's equation.

In order to illustrate how the spinor method works we present in Sec. III the BW theory for spin 1 massive particles. The interesting point here is that clearly BW theory leads to a complete description of massive particles by associating to these particles a symmetric rank 2 tensor field instead of associating particles to a rank 1 tensor (the usual procedure). The subsidiary condition $\partial^\mu B_\mu = 0$, for instance, follows naturally from the decomposition of the basic BW field into the spinor space and the BW equation.

In Sec. IV we present an alternative approach to spin 1 massless particles. We show how Maxwell's equation in vacuum emerges from the requirement of chiral invariance and the treatment of the chiral components as independent variables.

In Sec. V we formulate electrodynamics in terms of the chiral components ψ_{RR}, ψ_{RL}, ψ_{LR}, and ψ_{LL}. Here we show explicitly that one can formulate QED as long as matter couples only to some components of the chiral fields. That is, QED is manifestly left–right asymmetric.

In Sec. VI we touch on the question of the quantization of the BW fields. This is achieved by imposing an appropriate commutation relation among the BW components.

We end this paper with a section dedicated to conclusions.

II. ALTERNATIVE METHOD FOR THE DESCRIPTION OF MASSLESS PARTICLES

A. Bargmann–Wigner method—Massive particles

It has long been recognized that spinor quantities can be regarded as fundamental in any particle description within the field theoretical approach. In this chapter we will describe the spinor method, proposed by Bargmann and Wigner,[6] for studying massive particles and an alternative method, based on treating the chiral components of a spinor field as independent variables, for the description of massless particles.

Before setting the framework it is important to recall that the description of processes involving particles, within the field theoretic approach, requires the assignment to every particle a set of basic fields. In order to have Lorentz covariance explicitly the fields should have a well defined Lorentz transformation property. It just happens that, as in the case of spin 1 particles, the fields contain more degrees of freedom than the particles they are describing. These extra degrees of freedom of the field are eliminated by imposing complementary conditions on the fields. The case of massive (mass m) spin 1 particles is a very good example of this situation. In this case one associates to these particles (three polarization states) a four-component vector field B_μ. In order to eliminate the extra degree of freedom one imposes the covariant restriction

$$\partial_\mu B^\mu = 0. \tag{2.1}$$

The free field equation is

$$\partial_\mu G^{\mu\nu} = 0, \tag{2.2}$$

where

$$G_{\mu\nu} = \partial_\nu B_\mu - \partial_\mu B_\nu . \tag{2.3}$$

It was pointed out by Bargmann and Wigner[6] that the assignment particle→field is not unique. In fact, within the Bargmann–Wigner method one assigns to a massive particle of mass m and spin s a spinor field of rank $2s$:

$$\psi_{a_1 a_2 \cdots a_{2s}}(x), \quad a_k = 1,2,3,4. \tag{2.4}$$

Since the rank $2s$ spinor field contains, for particles of spin larger than $\frac{1}{2}$, much more degrees of freedom than the particle description requires, one has to impose further restrictions on the field ψ. One requires, as proposed by Bargmann–Wigner, that ψ be symmetric in its spin indices and imposes further that ψ satisfies a set of $2s$ Dirac-like equations; that is,

$$i\partial\!\!\!/_{a_k a_k'}\psi_{a_1\cdots a_k'\cdots a_{2s}}(x)=m\psi_{a_1\cdots a_k\cdots a_{2s}}(x),\qquad k=1...2s. \tag{2.5}$$

The set of equations (2.5) can be written under the equivalent form, or permutation of it,

$$i\partial\!\!\!/\otimes I\otimes\cdots\otimes I\psi=m\psi. \tag{2.6}$$

It is then evident, from (2.6), that the Bargmann–Wigner approach is just an extension of Dirac's theory to particles of arbitrary spin.

As a final remark on the whole framework, not only valid for the Bargmann–Wigner method but also for the generalized four-component and two-component theory that will be presented next, we would like to emphasize the need for an explicit representation for $2s$ rank spinors. The method that will be used is to express the $2s$ rank spinor fields as linear combinations of symmetric 4×4 matrices (within the four-spinor framework), or symmetric 2×2 matrices (for the two-spinor framework). The coefficients in these expansions are new fields. The properties of the new fields allow us to establish the connection of our approach to the usual electrodynamics.

In the four-spinor case the symmetric matrices are, by using the notation of Bjorken and Drell,[7]

$$(\sigma_{\mu\nu}C)\quad\text{and }(\gamma_\mu C), \tag{2.7}$$

whereas in the two-spinor case the symmetric matrices are

$$(\sigma^k C_1),\quad k=1,2,3, \tag{2.8}$$

where C is charge conjugation matrix and C_1 is the 2×2 charge conjugation matrix (see Appendix B).

As an example, let us write the decomposition of the rank 2 spinor field $\psi_{a_1a_2}$ associated to a mass m and spin 1 particle in terms of the symmetric matrices $(\gamma^\mu C)$ and $(\sigma^{\mu\nu}C)$. $\psi_{a_1a_2}$ admits the following decomposition:

$$\psi_{a_1a_2}(x)=\sqrt{m}\left\{B_\mu(x)(\gamma^\mu C)_{a_1a_2}-\frac{1}{2m}G_{\mu\nu}(x)(\sigma^{\mu\nu}C)_{a_1a_2}\right\}, \tag{2.9}$$

where the field $B_\mu(x)$ is a vector field (which, as will be shown later, is the usual spin 1 field), and $G_{\mu\nu}(x)$ is a field not yet determined and that, in principle, should involve derivatives of the $B_\mu(x)$ field. This is actually what happens. As will be shown in the next section, from (2.6) and (2.9) it follows that $G_{\mu\nu}$ in (2.9) can be written as

$$G_{\mu\nu}(x)=\partial_\nu B_\mu(x)-\partial_\mu B_\nu(x). \tag{2.10}$$

B. Massless particles—Chiral components

For the description of spin $\frac{1}{2}$ massless particles (neutrinos) one defines, as usual, the right and left components of a four-component spinor $\tilde{\psi}$ as

$$\tilde{\psi}_R=\tfrac{1}{2}(I+\gamma^5)\tilde{\psi}, \tag{2.11a}$$

$$\tilde{\psi}_L=\tfrac{1}{2}(I-\gamma^5)\tilde{\psi}. \tag{2.11b}$$

The free field Lagrangian, in terms of the right and left components, is

$$L=\bar{\bar{\psi}}_R i\ \partial\!\!\!/\psi_L+\bar{\bar{\psi}}_L i\ \partial\!\!\!/\psi_R\equiv\bar{\psi} i\ \partial\!\!\!/\psi. \tag{2.12}$$

By treating the left and right components independently one gets as equations of motion the following equations:

$$i\partial\!\!\!/\psi_R=0, \tag{2.13a}$$

$$i\partial\!\!\!/\psi_L=0. \tag{2.13b}$$

For the description of massless particles of arbitrary spin we assume, in analogy with the BW theory for massive particles, that a particle of spin s is associated to a symmetric spinor field of rank $2s$:

$$\tilde{\psi}_{a_1\cdots a_k\cdots a_{2s}}(x),\quad a_k=1,2,3,4. \tag{2.14}$$

For the rank $2s$ spinor field (2.14) one can define a set of chiral components as follows:

$$\tilde{\psi}_{R\cdots R}(x)\equiv\tilde{\psi}_{R_{a_1}\cdots R_{a_{2s}}}(x)=\tfrac{1}{2}(I+\gamma^5)_{a_1a_1'}\tfrac{1}{2}(I+\gamma^5)_{a_2a_2'}\cdots\tfrac{1}{2}(I+\gamma^5)_{a_{2s}a_{2s}'}\tilde{\psi}_{a_1'\cdots a_{2s}'}(x),$$

$$\tilde{\psi}_{R\cdots RL}(x)\equiv\tilde{\psi}_{R_{a_1}\cdots R_{a_{2s-1}}L_{a_{2s}}}(x)=\tfrac{1}{2}(I+\gamma^5)_{a_1a_1'}\cdots\tfrac{1}{2}(I+\gamma^5)_{a_{2s-1}a_{2s-1}'}\cdot\tfrac{1}{2}(I-\gamma^5)_{a_{2s}a_{2s}'}\tilde{\psi}_{a_1'\cdots a_{2s}'}(x),$$

$$\vdots$$

$$\tilde{\psi}_{L\cdots L}(x)\equiv\tilde{\psi}_{L_{a_1}\cdots L_{a_{2s}}}(x)=\tfrac{1}{2}(I-\gamma^5)_{a_1a_1'}\cdots\tfrac{1}{2}(I-\gamma^5)_{a_{2s}a_{2s}'}\tilde{\psi}_{a_1'\cdots a_{2s}'}(x). \tag{2.15}$$

We propose that these components satisfy an equation analogous to (2.13); that is,

$$i\partial\!\!\!/_{a_1a_1'}\tilde{\psi}_{R_{a_1'}R_{a_2}\cdots R_{a_{2s}}}(x)=0,$$

$$i\partial\!\!\!/_{a_1a_1'}\tilde{\psi}_{R_{a_1'}R_{a_2}\cdots L_{a_{2s}}}(x)=0,$$

$$\vdots$$

$$i\partial\!\!\!/_{a_1a_1'}\tilde{\psi}_{L_{a_1'}L_{a_2}\cdots L_{a_{2s}}}(x)=0. \tag{2.16}$$

We shall see in Sec. IV A that these equations follows from chiral invariance of the free Lagrangian.

C. Massless particles—Generalized two-component theory[8]

It is well known that massless particles of spin $\frac{1}{2}$ can be described by a two-component theory. We will show that this is also true for massless particles of arbitrary spin. That is, we will show that one can describe massless particles of arbitrary spin by means of a generalized two-component theory. Furthermore, we will show that these components satisfy equations analogous to Weyl's equations for particles of spin $\frac{1}{2}$.

In order to extend the two-component neutrino theory we just recall that one can write a four-component spinor as

$$\tilde{\psi}(x)=\begin{bmatrix}\xi(x)\\ \dot{\chi}(x)\end{bmatrix}=\psi_R+\psi_L, \tag{2.17}$$

where $\xi(x)$ and $\dot{\eta}(x)$ are two-component spinors, and ψ_R, ψ_L are defined in (2.11).

The set of equations (2.13) for ψ_R and ψ_L is equivalent to the following Weyl's equations for the two-component spinors ξ and $\dot{\eta}$:

$$(-i\sigma_0\partial_0 - i\boldsymbol{\sigma}\cdot\boldsymbol{\nabla})\xi(x)=0, \quad (-i\sigma_0\partial_0 + i\boldsymbol{\sigma}\cdot\boldsymbol{\nabla})\dot{\eta}(x)=0. \tag{2.18}$$

The two-component spinors ξ and $\dot{\chi}$ are eigenstates of the helicity operator $\frac{1}{2}\boldsymbol{\sigma}\cdot\mathbf{n}(\mathbf{n}=\mathbf{p}/|\mathbf{p}|)$ with eigenvalues $\pm\frac{1}{2}$.

The formulation of massless particles, in terms of generalized two-component spinors, can now be implemented by recalling that if $\tilde{\psi}$ transforms, under a Poincaré transformation, as a tensor product of $2s$ bispinors,

$$\tilde{\psi} \sim \begin{bmatrix} \xi_I \\ \dot{\chi}_I \end{bmatrix} \otimes \cdots \otimes \begin{bmatrix} \xi_{IIS} \\ \dot{\chi}_{IIS} \end{bmatrix}, \tag{2.19}$$

then, in chiral representation, the one that we will use from now on in the massless case, $\tilde{\psi}_{R\cdots R}, \tilde{\psi}_{R\cdots RL}, \tilde{\psi}_{L\cdots L}$, will transform like

$$\tilde{\psi}_{R\cdots R} \sim \begin{bmatrix} \xi_I \\ 0 \end{bmatrix} \otimes \cdots \otimes \begin{bmatrix} \xi_{IIS} \\ 0 \end{bmatrix} = \begin{bmatrix} \xi_I \cdots \xi_{IIS} \\ 0 \\ \vdots \\ 0 \end{bmatrix},$$

$$\tilde{\psi}_{R\cdots RL} \sim \begin{bmatrix} \xi_I \\ 0 \end{bmatrix} \otimes \cdots \otimes \begin{bmatrix} \xi_{IIS_{-1}} \\ 0 \end{bmatrix} \otimes \begin{bmatrix} 0 \\ \dot{\chi}_{IIS} \end{bmatrix} = \begin{bmatrix} 0 \\ \xi_I \cdots \xi_{IIS_{-1}} \dot{\chi}_{IIS} \\ 0 \\ \vdots \\ 0 \end{bmatrix}, \tag{2.20}$$

$$\vdots$$

$$\tilde{\psi}_{L\cdots L} \sim \begin{bmatrix} 0 \\ \dot{\chi}_I \end{bmatrix} \otimes \cdots \otimes \begin{bmatrix} 0 \\ \dot{\chi}_{IIS} \end{bmatrix} = \begin{bmatrix} 0 \\ \vdots \\ 0 \\ \dot{\chi}_I \cdots \dot{\chi}_{IIS} \end{bmatrix}.$$

From (2.20), one can write, by using (2.15),

$$\tilde{\psi}_{R\cdots R} \sim \begin{bmatrix} \varphi^{(1)} \\ 0 \\ \vdots \\ 0 \end{bmatrix},$$

$$\tilde{\psi}_{R\cdots RL} \sim \begin{bmatrix} 0 \\ \varphi^{(2)} \\ 0 \\ \vdots \\ 0 \end{bmatrix}, \tag{2.21}$$

$$\vdots$$

$$\tilde{\psi}_{L\cdots L} \sim \begin{bmatrix} 0 \\ \vdots \\ 0 \\ \varphi^{(2s)} \end{bmatrix},$$

where $\varphi^{(1)},\varphi^{(2)},...,\varphi^{(2s)}$, are, respectively, the only nonzero components of $\tilde{\psi}_{R\cdots R},\tilde{\psi}_{R\cdots RL},\ldots,\tilde{\psi}_{L\cdots L}$ and are two-component spinors of rank $2s$:

$$\begin{aligned} \varphi^{(1)} &= \varphi_{b_1 b_2 \cdots b_{2s}}, \\ \varphi^{(2)} &= \varphi'_{b_1 b_2 \cdots b_{2s}}, \\ &\vdots \\ \varphi^{(2s)} &= \varphi^{(2s-1)}_{b_1 b_2 \cdots b_{2s}}, \\ b_1, b_2, &\ldots, b_{2s} = 1,2. \end{aligned} \tag{2.22}$$

Using Eqs. (2.16) we can see that $\varphi^{(1)}$ and $\varphi^{(2s)}$ satisfy generalized Weyl's equations:

$$(-i\sigma^0\partial_0 - i\boldsymbol{\sigma}\cdot\boldsymbol{\nabla})\otimes\mathbf{1}\otimes\cdots\otimes\mathbf{1}\varphi^{(1)}(x)=0, \tag{2.23a}$$

$$(-i\sigma^0\partial_0 + i\boldsymbol{\sigma}\cdot\boldsymbol{\nabla})\otimes\mathbf{1}\otimes\cdots\otimes\mathbf{1}\varphi^{(2s)}(x)=0, \tag{2.23b}$$

where $\varphi^{(1)}$ and $\varphi^{(2s)}$ are symmetric two-component spinors of rank $2s$. Equation (2.23b) can be derived from (2.23a) by space reflection, so one can consider just one of the equations (2.23).

The other components $\varphi^{(2)},...,\varphi(2s-1)$ satisfy equations analogous to Weyl's.

Since the spin operator, in chiral representation, is given by

$$\boldsymbol{\Sigma} = \tfrac{1}{2}(\boldsymbol{\Sigma}\otimes\mathbf{1}\otimes\mathbf{1}\cdots\otimes\mathbf{1}+\cdots+\mathbf{1}\otimes\mathbf{1}\otimes\mathbf{1}\cdots\otimes\boldsymbol{\Sigma}), \tag{2.24}$$

then the generalized helicity operator W is

$$W = \mathbf{n}\cdot\boldsymbol{\Sigma}. \tag{2.25}$$

The eigenvalues of $W(\omega)$ lie in the range $-s \leqslant \omega \leqslant s$. In particular, we have

$$W\psi_{R\cdots R} = s\psi_{R\cdots R}, \quad W\psi_{L\cdots L} = -s\psi_{L\cdots L}. \tag{2.26}$$

Our proposal for treating massless particles is then parallel to the neutrino theory. For the free field theory one requires chiral invariance. For massless particles it is sensible to define chiral components and treat them as independent variables. For free fields, by using four-spinors, these equations are (2.16), whereas, by using two-spinors, the basic equations are (2.23). We shall see that this leads to Maxwell's equation in vacuum in the case of spin 1 particles.

III. BW THEORY FOR MASSIVE SPIN 1 PARTICLES

A. Free fields

This chapter shows how one describes massive spin 1 particles with the spinor method proposed by Bargmann and Wigner.

Within the BW method a spin 1 massive particle of mass m is described, in the noninteracting case, by a rank 2 symmetric spinor $\psi_{a_1 a_2}(x)$ obeying a system of two Dirac-type equations:

$$(i\partial\!\!\!/\otimes I)\psi = m\psi, \quad (I\otimes i\partial\!\!\!/)\psi = m\psi. \tag{3.1}$$

Equations (3.1) may be derived from the following Lagrangian:[6]

$$\mathscr{L}_0 = \bar{\psi}\left\{ \frac{i}{2}\left[\gamma^\mu \otimes I + I\otimes \gamma^\mu \right]\partial_\mu - mI\otimes I \right\}\psi. \tag{3.2}$$

If we treat the field $\psi_{a_1 a_2}$ as the independent variable, one gets, in particular,

$$\{ i\partial\!\!\!/ - m\}_{a_1 a_s'} \psi_{a_1' a_2}(x) = 0. \tag{3.3}$$

We replace $\psi_{a_1' a_2}$ in (3.3) by its decomposition (2.9) and obtain

$$i\gamma^\alpha_{a_1 a_1'}\partial_\alpha \left\{ B_\mu(x)(\gamma^\mu C)_{a_1' a_2} - \frac{1}{2m} G_{\mu\nu}(x)(\sigma^{\mu\nu}C)_{a_1' a_2} \right\}$$
$$= m\left\{ B_\mu(x)(\gamma^\mu C)_{a_1 a_2} - \frac{1}{2m} G_{\mu\nu}(x)(\sigma^{\mu\nu}C)_{a_1 a_2} \right\}. \tag{3.4}$$

In order to see that (3.4) leads to the usual equations one has to make some simple operations involving γ matrices. For instance, if one multiplies (3.4) by $(C^{-1})_{a_2 a_1}$ and sums over $a_1 a_2$, one gets

$$i\partial_\alpha B_\mu \ \mathrm{Tr}(\gamma^\alpha \gamma^\mu) = 0,$$

from which it follows that

$$\partial_\mu B^\mu = 0. \tag{3.5}$$

It is interesting to see that the subsidiary condition $\partial_\mu B^\mu = 0$ follows directly from the BW equation. If, on the other hand, one multiplies (3.4) by $(C^{-1}\gamma^\beta)_{a_2 a_1}$, one gets

$$-\frac{1}{2m}\partial_\alpha G_{\mu\nu} \ \mathrm{Tr}(\gamma^\alpha \sigma^{\mu\nu}\gamma^\beta) = mB_\mu \ \mathrm{Tr}(\gamma^\mu\gamma^\beta),$$

from which one gets

$$-\partial^\mu G_{\mu\beta} + m^2 B_\beta = 0. \tag{3.6}$$

Finally, if we multiply (3.4) by $(C^{-1}\gamma^\beta\gamma^\lambda)_{a_2 a_1}$, one gets

$$i\partial_\alpha B_\mu \ \mathrm{Tr}(\gamma^\alpha\gamma^\mu\gamma^\beta\gamma^\lambda) = -\frac{1}{2m} G_{\mu\nu} \ \mathrm{Tr}(\sigma^{\mu\nu}\gamma^\beta\gamma^\lambda),$$

from which it follows that

$$\partial_\mu B^\mu g^{\beta\lambda} + \partial^\lambda B^\beta - \partial^\beta B^\lambda = G^{\beta\lambda}. \tag{3.7}$$

By using (3.5) in (3.7) one gets

$$G^{\beta\lambda} = \partial^\lambda B^\beta - \partial^\beta B^\lambda; \tag{3.8}$$

that is, in the decomposition (2.9) of the field ψ, the only acceptable tensor is $\partial^\nu B^\mu - \partial^\mu B^\nu$.

We have then seen that BW equations lead to the following restrictions upon the fields B^μ and $G^{\mu\nu}$:

$$\partial^\mu B_\mu = 0, \quad G_{\mu\nu} = \partial_\nu B_\mu - \partial_\mu B_\nu, \quad -\partial^\mu G_{\mu\nu} + m^2 B_\nu = 0. \tag{3.9}$$

In order to see the equivalence between the BW method and the usual approach, in which we assign to a vector field B_μ a spin 1 particle, let us now write $\mathscr{L}_0$ in terms of B_μ.

By using decomposition (2.9) one gets

$$\mathscr{L}_0 = 4m^2 B^\mu B_\mu - 2G^{\mu\nu} G_{\mu\nu} \tag{3.10}$$

or, using (3.8),

$$\mathscr{L}_0 = 4m^2 B^\mu B_\mu + 4\partial^\mu B^\nu (\partial_\nu B_\mu - \partial_\mu B_\nu). \tag{3.11}$$

If we consider B_μ as the independent field, then one gets from (3.11) the usual Euler–Lagrange for the B_μ field; that is,

$$m^2 B_\nu - \partial^\mu G_{\mu\nu} = 0, \tag{3.12}$$

where

$$G_{\mu\nu} = \partial_\nu B_\mu - \partial_\mu B_\nu .$$

Equation (3.12) is the same as Eq. (3.6).

B. Interacting fields

Let us consider the interaction of massive spin 1 particles with massive spin $\frac{1}{2}$ particles, described, as usual, by a rank 1 spinor field η.

If we restrict ourselves to Lagrangians that are linear in the ψ fields (which leads, ultimately, to renormalizable models), then the forms that are compatible with Lorentz invariance are

$$\mathscr{L}_{\text{int}} = g_1 \bar{\psi}_{a_1 a_2} \eta^c_{a_1} \eta^c_{a_2} + g_2 \bar{\psi}_{a_1 a_2} \eta^c_{a_1} \eta^c_{a_2} + \text{h.c.} = g_1 \eta^c \bar{\psi} \eta + g_2 \eta \bar{\psi} \eta + \text{h.c.}, \tag{3.13}$$

where g_1 and g_2 are constants with dimension $[L]^{1/2} = 1/[M]^{1/2}$.

In the following we shall take $g_2 = 0$. As we shall see later in the zero-mass limit nature seems to prefer this type of coupling.

We shall study the following total Lagrangian for a spin 1 massive field interacting with a spin $\frac{1}{2}$ massive field:

$$\mathscr{L} = \bar{\psi} \{ \tfrac{1}{2} (\gamma^\mu \otimes I + I \otimes \gamma^\mu) \partial_\mu - mI \otimes I \} \psi + g_1 \eta^c \bar{\psi} \eta + \bar{\eta} (i \partial\!\!\!/ - m_f) \eta. \tag{3.14}$$

Treating now $\bar{\psi}_{a_1 a_2}$, $\psi_{a_1 a_2}$, η as independent fields, we obtain using (3.9) and Lagrange equations

$$\partial_\mu G^{\rho\mu} = -m^2 B^\rho + g_1 \frac{\sqrt{m}}{4} \bar{\eta} \gamma^\rho \eta. \tag{3.15}$$

Model (3.15) leads to equations analogous to Maxwell's. In fact, in analogy with the massless case, we name

$$G^{0k} = E^k, \quad G^{jk} = \epsilon^{ljk} H^l. \tag{3.16}$$

We obtain from (3.15) the following set of equations:

$$\nabla\cdot\mathbf{E}+m^2B^0=g_1\frac{\sqrt{m}}{4}\,\bar{\eta}\gamma^0\eta,$$

$$\partial^0\mathbf{E}-\nabla\wedge\mathbf{H}-m^2\mathbf{B}=-g_1\frac{\sqrt{m}}{4}\,\bar{\eta}\gamma^0\eta,$$

$$\nabla\cdot\mathbf{H}=0,\quad \partial^0\mathbf{H}+\nabla\wedge\mathbf{E}=0. \tag{3.17}$$

Equations (3.17) are analogous to Maxwell's. That is, we get a set of coupled equations of first order in terms of the observables **E** and **H**.

C. Hamiltonian

We take the form (3.14) of $\mathscr{L}$ as a starting point. The two fields η and ψ are independent. We construct conjugate momenta from $\mathscr{L}$ by the standard prescription, so we obtain

$$\pi_{a_1a_2}=\frac{\partial\mathscr{L}}{\partial(\partial^0\psi)_{a_1a_2}}=\bar{\psi}_{a_1'a_2'}\frac{i}{2}\{\gamma^0_{a_1'a_1}\delta_{a_2'a_2}+\delta_{a_1'a_1}\gamma^0_{a_2'a_2}\},$$

$$\pi_{a_1}=\frac{\partial\mathscr{L}}{\partial(\partial^0\eta)_{a_1}}=\bar{\eta}_{a_2}i\gamma^0_{a_2a_1}. \tag{3.18}$$

The Hamiltonian is defined by

$$\mathbf{H}=\pi_{a_1a_2}\partial^0\psi_{a_1a_2}+\pi_{a_1}\partial^0\eta_{a_1}-\mathscr{L}. \tag{3.19}$$

Replacing $\psi_{a_1a_2}(\bar{\psi}_{a_1a_2})$ by its decomposition (2.9), one gets

$$\mathbf{H}=4\{G^{0\mu}\partial_0B_\mu+\partial_0B_\mu G^{0\mu}\}-4\{m^2B^\mu B_\mu-\tfrac{1}{2}G^{\mu\nu}G_{\mu\nu}\}$$
$$+g_1\sqrt{m}\left\{B_\mu\bar{\eta}\gamma^\mu\eta-\frac{1}{2m}G_{\mu\nu}\bar{\eta}\sigma^{\mu\nu}\eta\right\}-\bar{\eta}(i\gamma^k\partial_k-m_f)\eta, \tag{3.20}$$

where

$$k,j=1,2,3,$$

$$\mu,\nu=0,1,2,3.$$

Now, following Bjorken–Drell[7] we adopt the notation

$$E^j=E^j_l+E^j_t=-\partial_jB^0-\partial_0B^j$$

and we assume that the fields **E** and **H** are real, so we obtain

$$\mathbf{H}=4(\mathbf{H}^2+\mathbf{E}_t^2)-4\mathbf{E}_l^2-4m^2B^\mu B_\mu+g_1\sqrt{m}\left\{B^0\bar{\eta}\gamma^0\eta+B_j\bar{\eta}\gamma^j\eta-\frac{1}{2m}G_{\mu\nu}\bar{\eta}\sigma^{\mu\nu}\eta\right\}$$
$$-\bar{\eta}(i\gamma^k\partial_k-m_f)\eta. \tag{3.21}$$

IV. ALTERNATIVE APPROACH TO SPIN 1 MASSLESS PARTICLES

A. Spin 1 massless particles

In this section we will see how one can apply the two-component and the four-component spinorial formalism in the description of spin 1 massless particles. In this case one works with a rank 2 spinor field ψ_{ab}. Our starting point could be the zero-mass limit of (3.2). The Lagrangian that one gets in this limit is

$$\mathscr{L}_0=\bar{\psi}\left\{\frac{i\partial\!\!\!/}{2}\otimes\mathbf{1}+\mathbf{1}\otimes\frac{i\partial\!\!\!/}{2}\right\}\psi. \tag{4.1}$$

Lagrangian (4.1) is not, however, the appropriate Lagrangian for spin 1 massless particles. The reason why the extension of the Bargmann–Wigner theory, in this case, is not straightforward is chiral symmetry. $\mathscr{L}_0$ defined in (4.1) is not invariant under the generalized chiral transformation

$$\psi\rightarrow\psi'=e^{i\alpha\gamma_5}\otimes e^{i\alpha\gamma_5}\psi. \tag{4.2}$$

Only the equivalent Lagrangians

$$\mathscr{L}=\bar{\psi}(i\partial\!\!\!/\otimes 1)\psi \quad [\mathscr{L}'=\bar{\psi}(1\otimes i\partial\!\!\!/)\psi] \tag{4.3}$$

are invariant under the chiral transformations

$$\psi\rightarrow\psi'=(e^{i\alpha\gamma_5}\otimes 1)\psi \quad [\psi'=(1\otimes e^{i\alpha\gamma_5})\psi]. \tag{4.4}$$

Expressions (4.3) suggest that the Lagrangian for massless spin 1 particles is not uniquely defined. This would be the case if the field ψ is asymmetric. However, for a symmetric ψ field both expressions in (4.3) are equivalent. The problem is that we are not able to impose, at the Lagrangian level, the symmetry properties of the ψ field.

The need to work with asymmetric ψ fields lead us eventually to difficulties in dealing with discrete symmetries. This is due to the fact that depending on the Lagrangian we take we might get a different transformation law for the fields ψ. These transformation laws are equivalent only in the case of symmetric ψ fields. This is just to call attention to the fact that some care is needed when dealing with discrete symmetries.

For the Lagrangian $\mathscr{L}$ ($\mathscr{L}'$) one can introduce, in close analogy with the spin $\frac{1}{2}$ case, the chiral components

$$\begin{aligned}\psi_{RR}&=\frac{(1+\gamma_5)}{2}\otimes\frac{(1+\gamma_5)}{2}\psi, \quad \psi_{RL}=\frac{(1+\gamma_5)}{2}\otimes\frac{(1-\gamma_5)}{2}\psi,\\ \psi_{LR}&=\frac{(1-\gamma_5)}{2}\otimes\frac{(1+\gamma_5)}{2}\psi, \quad \psi_{LL}=\frac{(1-\gamma_5)}{2}\otimes\frac{(1-\gamma_5)}{2}\psi,\end{aligned} \tag{4.5}$$

where, by definition,

$$\psi=\psi_{RL}+\psi_{RR}+\psi_{LR}+\psi_{LL}. \tag{4.6}$$

These chiral components are eigenstates of the chirality operator $\gamma_5\otimes 1(1\otimes\gamma_5)$ with eigenvalues $+1$ or (-1). That is, they are polarized either to the right or to the left.

For massless particles the most general decomposition of the ψ field is now

$$\psi_{a_1a_2}=C_1A_\mu(\gamma^\mu C)_{a_1a_2}-C_2F_{\mu\nu}(\sigma^{\mu\nu}C)_{a_1a_2}, \tag{4.7}$$

where C_1 and C_2 in (4.7) are arbitrary constants that, in the massless case, cannot be related to each other as in the massive case [expression (2.9)], since C_1 and C_2 are dimensional constants having different dimensions.

The chiral components defined in (4.5) assume, after inserting (4.7) in (4.5), the following form:

$$\begin{aligned}(\psi_{RR}(x))_{a_1a_2}&=-C_2[\tfrac{1}{2}(1+\gamma_5)\sigma^{\mu\nu}C]_{a_1a_2}F_{\mu\nu},\\(\psi_{RL}(x))_{a_1a_2}&=C_1[\tfrac{1}{2}(1+\gamma_5)\gamma^{\mu}C]_{a_1a_2}A_{\mu},\\(\psi_{LR}(x))_{a_1a_2}&=C_1[\tfrac{1}{2}(1-\gamma_5)\gamma^{\mu}C]_{a_1a_2}A_{\mu},\\(\psi_{LL}(x))_{a_1a_2}&=-C_2[\tfrac{1}{2}(1-\gamma_5)\sigma^{\mu\nu}C]_{a_1a_2}F_{\mu\nu}.\end{aligned}\tag{4.8}$$

That is, ψ_{RL},ψ_{RR} are related to potentials whereas ψ_{RR},ψ_{LL} are related to observables (electromagnetic fields).

Our proposal for treating massless spin 1 particles is, in close analogy with spin $\frac{1}{2}$ particles, to treat all chiral components $(\psi_{RL},\psi_{LR},\psi_{LL},\psi_{RR})$ as independent field variables. These chiral components describe, in principle, different species of spin 1 massless particles. In this way we consider all chiral components as dynamical variables. This approach leads to a formulation of QED in which potentials and observable fields are treated on equal footing. It is simple to check that the substitution of (4.6) into (4.3) leads to the following Lagrangian density:

$$\tilde{\mathscr{L}}_0=\bar{\tilde{\psi}}_{RL}(i\partial\!\!\!/\otimes I)\tilde{\psi}_{RR}+\bar{\tilde{\psi}}_{LR}(i\partial\!\!\!/\otimes I)\tilde{\psi}_{LL}+\bar{\tilde{\psi}}_{RR}(i\partial\!\!\!/\otimes I)\tilde{\psi}_{RL}+\bar{\tilde{\psi}}_{LL}(i\partial\!\!\!/\otimes I)\tilde{\psi}_{LR}.\tag{4.9}$$

By treating all chiral components as independent field variables one gets the following equations of motion:

$$\begin{aligned}(i\partial\!\!\!/\otimes I)\tilde{\psi}_{RR}=0,\quad (i\partial\!\!\!/\otimes I)\tilde{\psi}_{RL}=0,\\(i\partial\!\!\!/\otimes I)\tilde{\psi}_{LR}=0,\quad (i\partial\!\!\!/\otimes I)\tilde{\psi}_{LL}=0.\end{aligned}\tag{4.10}$$

We will be interested in analyzing whether the interaction of photons with matter exhibits any preference of nature with regard to chiral components and, if this occurs, if there is violation of parity. The free field Lagrangian is invariant under space reflection ($\mathbf{x}\rightarrow-\mathbf{x}$) if the chiral fields transform as

$$\begin{aligned}\psi_{LP}&\overset{P}{\rightarrow}\psi'_{LR}(x')=\xi(\gamma_0\otimes 1)\psi_{RR}(+\mathbf{x},t),\\\psi_{LL}&\overset{P}{\rightarrow}\psi'_{LL}(x')=\xi(\gamma_0\otimes 1)\psi_{RL}(+\mathbf{x},t),\\\psi_{RR}&\overset{P}{\rightarrow}\psi'_{RR}(x')=\xi(\gamma_0\otimes 1)\psi_{LR}(+\mathbf{x},t),\\\psi_{RL}&\overset{P}{\rightarrow}\psi'_{RL}(x')=\xi(\gamma_0\otimes 1)\psi_{LL}(+\mathbf{x},t).\end{aligned}\tag{4.11}$$

From (4.11) and (4.8) one would conclude that the mirror fields of the potentials are the observable fields (**E** and **H**) and vice-versa. This example illustrates, as pointed out before, the difficulty which we might run into when dealing with discrete symmetries within the spinors method.

There are two alternatives in dealing with discrete symmetries that can easily be checked taking the explicit example of space reflection. In the first case one can use the explicitly symmetric representation (4.7) in any of the alternative forms in (4.3) and analyze the symmetry properties of the Lagrangian in terms of the fields A_μ and $F_{\mu\nu}$. The Lagrangian is the standard one of electromagnetism as we shall see later.

The other alternative is to impose, from the very beginning, that ψ is symmetric. Under these circumstances Lagrangians (4.3) and (4.1) are equivalent. These equivalent Lagrangians would now be invariant under space reflections if the chiral fields transform as

$$\psi'_{RR}(x')=\xi(\gamma^0\otimes\gamma^0)\psi_{LL}(x),\quad \psi'_{RL}(x')=\xi(\gamma^0\otimes\gamma^0)\psi_{LR}(x),$$

$$\psi'_{LR}(x')=\xi(\gamma^0\otimes\gamma^0)\psi_{RL}(x),\quad \psi'_{LL}(x')=\xi(\gamma^0\otimes\gamma^0)\psi_{RR}(x).$$

It can be checked explicitly that these transformations lead to the proper transformation properties of the A_μ and $F_{\mu\nu}$ fields.

The generalized helicity operator W is now

$$\tfrac{1}{2}(\mathbf{\Sigma}\cdot\mathbf{n}\otimes I+I\otimes\mathbf{\Sigma}\cdot\mathbf{n})\equiv W,\quad \mathbf{n}=\frac{\mathbf{p}}{|\mathbf{p}|}. \tag{4.12}$$

It is straightforward to show that $\psi_{RR},\psi_{RL},\psi_{LR},\psi_{LL}$ are eigenstates of W with eigenvalues $+1,0,-1$; that is,

$$\begin{aligned}\tfrac{1}{2}(\mathbf{\Sigma}\cdot\mathbf{n}\otimes I+I\otimes\mathbf{\Sigma}\cdot\mathbf{n})\tilde{\psi}_{RR}=\psi_{RR},\quad \tfrac{1}{2}(\mathbf{\Sigma}\cdot\mathbf{n}\otimes I+I\otimes\mathbf{\Sigma}\cdot\mathbf{n})\tilde{\psi}_{RL}=0,\\ \tfrac{1}{2}(\mathbf{\Sigma}\cdot\mathbf{n}\otimes I+I\otimes\mathbf{\Sigma}\cdot\mathbf{n})\tilde{\psi}_{LR}=0,\quad \tfrac{1}{2}(\mathbf{\Sigma}\cdot\mathbf{n}\otimes I+I\otimes\mathbf{\Sigma}\cdot\mathbf{n})\tilde{\psi}_{LL}=-\psi_{LL}.\end{aligned} \tag{4.13}$$

B. Free electrodynamics

Let us check, finally, that the Lagrangian density (4.9) provides an alternative formulation of free electrodynamics. By substituting (4.8) into Eqs. (4.10) and furthermore multiplying (4.10) by (C) and taking the trace, we get

$$\partial^\mu A_\mu=0; \tag{4.14}$$

that is, we get a Lorentz condition. Furthermore, multiplying (4.10) by $(C^{-1}\gamma^\beta)$ we get the equation of motion for the $F^{\mu\nu}$ field,

$$\partial^\mu F_{\mu\nu}=0. \tag{4.15}$$

We shall see, by employing the two-spinor approach, that $F_{\mu\nu}$ can be written, in terms of an antisymmetric field tension $\mathscr{F}_{\mu\nu}$, as

$$F_{\mu\nu}=\mathscr{F}_{\mu\nu}+i\tilde{\mathscr{F}}_{\mu\nu}, \tag{4.16}$$

where

$$\tilde{\mathscr{F}}^{\mu\nu}=\tfrac{1}{2}\epsilon^{\mu\nu\alpha\beta}\mathscr{F}_{\alpha\beta}. \tag{4.17}$$

The equation of motion for $\mathscr{F}_{\mu\nu}$ and $\tilde{\mathscr{F}}_{\mu\nu}$ is, from (4.15),

$$\partial_\mu \mathscr{F}^{\mu\nu}=0, \quad \partial_\mu \tilde{\mathscr{F}}^{\mu\nu}=0. \tag{4.18}$$

It should be pointed out at this point that since in the massless case ψ_{RL} and ψ_{RR} components are treated as independent variables there is no connection between $\mathscr{F}^{\mu\nu}$ and the derivatives of the field A_μ. That is, in this case A_μ and $F_{\mu\nu}$ are independent field variables. We shall see, however, that within the generalized two-component theory of massless spin 1 particles this connection emerges from Maxwell's equations for the observable fields. We shall see that, also in this case, one can write

$$\mathscr{F}^{\mu\nu}=\partial^\mu A^\nu-\partial^\nu A^\mu \tag{4.19}$$

and, as a result, from (4.19) and (4.14), it follows that the A^μ field satisfies, by using (4.18),

$$\Box A^\mu=0. \tag{4.20}$$

C. Two-component formulation of free electrodynamics

We shall now consider the formulation of free electrodynamics within the generalized two-component theory presented in Sec. I. In the case of spin 1 particles the field ψ transforms as

$$\psi\sim\begin{bmatrix}\xi^I\\ \dot{\chi}^I\end{bmatrix}\otimes\begin{bmatrix}\xi^{II}\\ \dot{\chi}^{II}\end{bmatrix} \tag{4.21}$$

so that the fields $\psi_{RL},\psi_{LR},\psi_{RR},\psi_{LL}$ defined in (4.5) transform like

$$\begin{aligned}\tilde{\psi}_{RR}&\sim\begin{bmatrix}\xi^I\\ 0\end{bmatrix}\otimes\begin{bmatrix}\xi^{II}\\ 0\end{bmatrix}, \quad \tilde{\psi}_{RL}\sim\begin{bmatrix}\xi^I\\ 0\end{bmatrix}\otimes\begin{bmatrix}0\\ \dot{\chi}^{II}\end{bmatrix},\\ \tilde{\psi}_{LR}&\sim\begin{bmatrix}0\\ \dot{\chi}^I\end{bmatrix}\otimes\begin{bmatrix}\xi^{II}\\ 0\end{bmatrix}, \quad \tilde{\psi}_{LL}\sim\begin{bmatrix}0\\ \dot{\chi}^I\end{bmatrix}\otimes\begin{bmatrix}0\\ \dot{\chi}^{II}\end{bmatrix}.\end{aligned} \tag{4.22}$$

The generalized Weyl's equations satisfied by the nonzero components of ψ_{RR} and ψ_{LL} are

$$\begin{aligned}(-i\sigma^0\partial_0-i\boldsymbol{\sigma}\cdot\boldsymbol{\nabla})_{aa'}\xi^I_{a'}\xi^{II}_b&=0,\\ (-i\sigma^0\partial_0+i\boldsymbol{\sigma}\cdot\boldsymbol{\nabla})_{aa'}\dot{\chi}^I_{a'}\dot{\chi}^{II}_b&=0,\end{aligned} \tag{4.23}$$

where symmetrization in the spin indices is assumed in Eq. (4.23).

Equations (4.23) can be written, in terms of $\varphi^{(1)}$ and $\varphi^{(4)}$ defined in (2.21), as

$$(-i\sigma^0\partial_0-i\boldsymbol{\sigma}\cdot\boldsymbol{\nabla})\varphi^{(1)}(\mathbf{x},t)=0, \tag{4.24a}$$

$$(-i\sigma_0\partial_0+i\boldsymbol{\sigma}\cdot\boldsymbol{\nabla})\varphi^{(4)}(\mathbf{x},t)=0. \tag{4.24b}$$

We are now ready to get the field equations of electrodynamics. As in the generalized four-component approach, one tries to write $\varphi^{(1)}$ and $\varphi^{(4)}$ as a linear combination of 2×2 symmetric matrices. In this case, the candidate matrices are the matrices (2.8), $\boldsymbol{\sigma}C_1$, where C_1 is the 2×2 charge conjugation matrix. One can then write

$$\varphi^{(1)}(x)=\mathbf{f}(x)(\boldsymbol{\sigma}\cdot C_1), \tag{4.25}$$

where $\mathbf{f}$ is a vector that satisfies, after substituting (4.25) into (4.24), the following equations:

$$\nabla \cdot \mathbf{f} = 0, \tag{4.26}$$

$$i\partial^0 \mathbf{f} = \nabla \times \mathbf{f}. \tag{4.27}$$

We have shown in Sec. IV A that $\varphi^{(1)}$ and $\varphi^{(4)}$ (or, equivalently, ψ_{RR} and ψ_{LL}) should be built from an antisymmetric tensor $F_{\mu\nu}$. More specifically, it follows from (4.8) and (4.25) that the ith component f_i can be written as

$$f_i = \tfrac{1}{2}\, \epsilon_{0i\alpha\beta} F^{\alpha\beta} = \tfrac{1}{2}\, \epsilon_{i\alpha\beta} F^{\alpha\beta}, \tag{4.28}$$

with $i,\alpha,\beta = 1,2,3$.

As pointed out in Ref. 5 $F_{\mu\nu}$ in (4.28) should be self-dual. In this way if one assumes that $\mathcal{F}_{\mu\nu}$ is an antisymmetrical tensor, then one can construct from this tensor a self-dual one by writing

$$F^{\mu\nu} = \mathcal{F}^{\mu\nu} + i\tilde{\mathcal{F}}^{\mu\nu}, \tag{4.29}$$

where

$$\tilde{\mathcal{F}}^{\mu\nu} = \tfrac{1}{2}\epsilon^{\mu\nu\alpha\beta}\mathcal{F}_{\alpha\beta}. \tag{4.30}$$

If one defines further,

$$E^i \equiv \mathcal{F}_0 i, \quad H^i = \tilde{\mathcal{F}}_0 i. \tag{4.31}$$

It follows that Eqs. (4.26) and (4.27) for **f** imply Maxwell's equations for the **E** and **H** fields defined in (4.31). That is,

$$\begin{aligned} \nabla \cdot \mathbf{E} = 0, &\quad \nabla \cdot \mathbf{H} = 0, \\ \nabla \times \mathbf{E} - \partial_0 \mathbf{E} = 0, &\quad \nabla \times \mathbf{H} - \partial_0 \mathbf{H} = 0. \end{aligned} \tag{4.32}$$

Finally, it follows from Maxwell's equation (4.32) and from (4.31) that $\mathcal{F}_{\mu\nu}$ can be written as

$$\mathcal{F}_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu. \tag{4.33}$$

It is worth commenting, at this point, on the connection between $\mathcal{F}_{\mu\nu}$ and the derivatives of the field A_μ in the case of massless particles. In the massive case the connection analogous to (4.33) follows from the restriction imposed by BW equations. In the massless case the possibility of writing $\mathcal{F}_{\mu\nu}$ under the form (4.33) follows from Maxwell's equations that allows us to write the observable fields in terms of derivatives of the four-potentials. We will show, in the interaction case, how expression (4.33) can be derived by adding chiral violating terms to the Lagrangian.

The conclusion is that, within the spinor method proposed here, Maxwell's equation in vacuum emerges from the requirement of chiral invariance of the free field Lagrangian. Maxwell's equations for the fields **E** and **H** emerge from the equations analogous to Weyl's equation in the two-component neutrino theory.

V. INTERACTING FIELDS—QED

A. Linear interactions

Let us now consider the interaction of massless spin 1 fields. We will mainly be concerned with the interaction of these particles with ordinary matter. That is, we will be concerned with the most general interaction Lagrangian describing massless spin 1 particles interacting with spin $\frac{1}{2}$ massive (m_f) particles.

The most general Lagrangian involving the interaction of the chiral fields (4.5) with ordinary matter (here represented by the Fermi on field η) is

$$\mathscr{L}=\bar{\psi}_{RL}(i\partial\!\!\!/\otimes 1)\psi_{RR}+\bar{\psi}_{LR}(i\partial\!\!\!/\otimes 1)\psi_{LL}+\bar{\psi}_{RR}(i\partial\!\!\!/\otimes 1)\psi_{RL}+\bar{\psi}_{LL}(i\partial\!\!\!/\otimes 1)\times\psi_{LR}+\bar{\eta}(i\partial\!\!\!/-m)\eta+\mathscr{L}_{\text{int}}(\psi_{RR},\ \psi_{RL},\ \psi_{LR},\ \psi_{LL},\ \eta). \tag{5.1}$$

By imposing that the Lagrangian is linear in the ψ fields, then the most general form that is bilinear in the matter field η that we can construct with the four independent fields ψ_{RR}, ψ_{RL}, ψ_{LR}, and ψ_{LL} is

$$\mathscr{L}_{\text{int}}=A\bar{\bar{\psi}}_{R_{a_1}R_{a_2}}\eta_{a_1}\eta_{a_2}+B\bar{\bar{\psi}}_{L_{a_1}L_{a_2}}\eta_{a_1}\eta_{a_2}+D\bar{\bar{\psi}}_{R_{a_1}L_{a_2}}\eta_{a_1}\eta_{a_2}+E\bar{\bar{\psi}}_{L_{a_1}R_{a_2}}\eta_{a_1}\eta_{a_2}+F\bar{\bar{\psi}}_{R_{a_1}R_{a_2}}\eta^c_{a_1}\eta_{a_2}+J\bar{\bar{\psi}}_{L_{a_1}L_{a_2}}\eta^c_{a_1}\eta_{a_2}+K\bar{\bar{\psi}}_{R_{a_1}L_{a_2}}\eta^c_{a_1}\eta_{a_2}+L\bar{\bar{\psi}}_{L_{a_1}R_{a_2}}\eta^c_{a_1}\eta_{a_2}+\text{h.c.}, \tag{5.2}$$

where A, B, D, E, F, J, K, and L are arbitrary constants.

The reason for so many terms is that we shall assume, to start with, that ordinary matter couples with different strengths to the chiral components of the field ψ. As a matter of fact, we shall see that, in this case, only the zero helicity components couple to ordinary matter.

It is trivial to check that if nature is asymmetric with regard to left and right (that is, if it prefers chiral components), then the theory violates parity [in the sense that the Lagrangian is not invariant under transformations (4.11)]. The physical consequences, however, depend on how this symmetry is broken. We will show that within the field theoretical context the breakdown of parity, and consequently of left–right symmetry, does not lead always to processes occurring with different probabilities in the actual system or in its mirror image. That is, for theories in which not all the fields are observables, the parity symmetry breakdown might not be accompanied by observable effects. Electrodynamics follow into this category of field theories. In order to illustrate this relevant aspect let us consider two parity nonconserving Lagrangians:

$$\mathscr{L}_1^{\text{int}}=D(\psi_{LR})\eta_c\eta, \tag{5.3}$$

$$\mathscr{L}_2^{\text{int}}=K(\psi_{RL}+\psi_{LR})\eta_c\eta, \tag{5.4}$$

where K and D are coupling constants.

The first Lagrangian leads to left–right asymmetry leading to observable effects as far as parity breaking is concerned. In fact, Lagrangian (5.3) describes essentially the interaction of a vector field A_μ coupled to a vector and axial vector current $[\bar{\eta}(1-\gamma_5)\gamma_\mu\eta]$. As will be shown in the next section, the Lagrangian (5.4) is an equivalent formulation of QED.

If one wants to ensure that there are no observable effects in the breakdown of parity of Lagrangian (5.2), one has to require that

$$A=B,\quad D=E,\qquad F=J,\quad K=L. \tag{5.5}$$

Under restrictions (5.5), the most general interaction Lagrangian is then

$$\mathscr{L}_{\text{int}}=A(\bar{\bar{\psi}}_{RR}\eta\eta+\bar{\bar{\psi}}_{LL}\eta\eta)+D(\bar{\bar{\psi}}_{RL}\eta\eta+\bar{\bar{\psi}}_{LR}\eta\eta)+F(\bar{\bar{\psi}}_{RR}\eta^c\eta+\bar{\bar{\psi}}_{LL}\eta^c\eta)+K(\bar{\bar{\psi}}_{RL}\eta^c\eta+\bar{\bar{\psi}}_{LR}\eta^c\eta). \tag{5.6}$$

We now consider the particular interaction Lagrangian obtained by taking $A=D=K=0$:

$$\mathcal{L}_{\text{int}}=F(\bar{\tilde{\psi}}_{RR}\eta^c\eta+\bar{\tilde{\psi}}_{LL}\eta^c\eta). \tag{5.7}$$

The complete Lagrangian is then

$$\mathcal{L}=\bar{\tilde{\psi}}_{RL}(i\partial\!\!\!/\otimes I)\bar{\tilde{\psi}}_{RR}+\bar{\tilde{\psi}}_{LR}(i\partial\!\!\!/\otimes I)\bar{\tilde{\psi}}_{LL}+\bar{\tilde{\psi}}_{RR}(i\partial\!\!\!/\otimes I)\bar{\tilde{\psi}}_{RL}$$

$$+\bar{\tilde{\psi}}_{LL}(i\partial\!\!\!/\otimes I)\bar{\tilde{\psi}}_{LR}+F(\bar{\tilde{\psi}}_{RR}\eta^c\eta+\bar{\tilde{\psi}}_{LL}\eta^c\eta)+\bar{\eta}(i\partial\!\!\!/-m_f)\eta. \tag{5.8}$$

Writing Lagrange equations explicitly, one has

$$i\partial\!\!\!/\otimes I\bar{\tilde{\psi}}_{LR}+F\eta^c\eta=0, \tag{5.9a}$$

$$i\partial\!\!\!/\otimes I\bar{\tilde{\psi}}_{RL}+F\eta^c\eta=0, \tag{5.9b}$$

$$i\partial\!\!\!/\otimes I\bar{\tilde{\psi}}_{RR}=0, \tag{5.9c}$$

$$i\partial\!\!\!/\otimes I\bar{\tilde{\psi}}_{LL}=0. \tag{5.9d}$$

Equations (5.9) and (4.8) imply, in particular, the following relations:

$$2iC_1\partial_\mu A^\mu=F\bar{\eta}\eta, \tag{5.10}$$

$$\partial^\mu F_{\mu\nu}=0. \tag{5.11}$$

The conclusion is that Lagrangian (5.8) describes the interaction of massless spin 1 particles with matter but this theory is not electrodynamics.

B. Electrodynamics and chiral asymmetry

Let us consider now the interaction of massless particles with ordinary matter described by the interaction Lagrangian:

$$\mathcal{L}_{\text{int}}=K\{\bar{\tilde{\psi}}_{RL}\eta^c\eta+\bar{\tilde{\psi}}_{LR}\eta^c\eta\}. \tag{5.12}$$

By adding the free field Lagrangians we end up with the following total Lagrangian:

$$\mathcal{L}=\bar{\tilde{\psi}}_{RL}(i\partial\!\!\!/\otimes I)\tilde{\psi}_{RR}+\bar{\tilde{\psi}}_{LR}(i\partial\!\!\!/\otimes I)\tilde{\psi}_{LL}+\bar{\tilde{\psi}}_{RR}(i\partial\!\!\!/\otimes I)\tilde{\psi}_{RL}+\bar{\tilde{\psi}}_{LL}(i\partial\!\!\!/\otimes I)\tilde{\psi}_{LR}$$

$$+K\{\bar{\tilde{\psi}}_{RL}\eta^c\eta+\bar{\tilde{\psi}}_{LR}\eta^c\eta\}+\bar{\eta}(i\partial\!\!\!/-m_f)\eta+\alpha\bar{\tilde{\psi}}_{RR}\psi_{LL}+\alpha\bar{\tilde{\psi}}_{LL}\psi_{RR}. \tag{5.13}$$

One of the basic features of this approach is that although only some chiral components couple with the usual matter, all chiral components should be considered as dynamical variables. That is, one should write five Lagrange equations, one for each of the independent fields (ψ_{RR}, ψ_{RL}, ψ_{LR}, ψ_{LL}, and η).

With $\mathcal{L}$ given by (5.13), they lead to

$$i\partial\!\!\!/\otimes I\tilde{\psi}_{RL}+\alpha\tilde{\psi}_{LL}=0, \tag{5.14a}$$

$$i\partial\!\!\!/\otimes I\tilde{\psi}_{RR}+K\eta^c\eta=0, \tag{5.14b}$$

$$i\partial\!\!\!/\otimes I\tilde{\psi}_{LL}+K\eta^c\eta=0, \tag{5.14c}$$

$$i\partial\!\!\!/\otimes I\tilde{\psi}_{LR}+\alpha\tilde{\psi}_{RR}=0, \tag{5.14d}$$

$$K\eta^c\{\bar{\tilde{\psi}}_{RL}+\bar{\tilde{\psi}}_{LR}\}-m_f\bar{\eta}-i\partial_\mu(\bar{\eta})\gamma^\mu=0. \tag{5.14e}$$

From equation (5.14e) it follows that

$$K\eta^c_{a_1}\{\bar{\tilde{\psi}}_{RL}+\bar{\tilde{\psi}}_{LR}\}_{a_1a_2}-m_f\bar{\eta}_{a_2}-i\partial_\mu\bar{\eta}_{a_1}\gamma^\mu_{a_1a_2}=0. \tag{5.15}$$

Or, replacing $\bar{\tilde{\psi}}_{RL}$ and $\bar{\tilde{\psi}}_{LR}$ by expression (4.8),

$$-KC_1\bar{\eta}_{a_1}A_\mu(\gamma^\mu)_{a_1a_2}-m_f\bar{\eta}_{a_2}-i\partial_\mu\bar{\eta}_{a_1}\gamma^\mu_{a_1a_2}=0. \tag{5.16}$$

Taking the Hermitian conjugate of (5.16) one gets

$$(i\partial\!\!\!/-m_f)_{a_1a_2}\eta_{a_2}=KC_1(\gamma^\mu)_{a_1a_2}A_\mu\eta_{a_2}. \tag{5.17}$$

Now adding (5.14b) and (5.14c) and replacing $\tilde{\psi}_{RR}$ and $\tilde{\psi}_{LL}$ by expressions (4.8) we get

$$\partial_\mu F^{\beta\mu}=-\frac{K}{4C_2}\bar{\eta}\gamma^\beta\eta. \tag{5.18}$$

Similarly, adding (5.14a) and (5.14d) and replacing $\tilde{\psi}_{LR}$ and $\tilde{\psi}_{RL}$ by expressions (4.8) we get the following constraint:

$$\partial^\mu A_\mu=0; \tag{5.19}$$

that is, one gets the Lorentz condition and

$$\mathscr{F}^{sr}=\frac{C_1}{2\alpha C_2}(\partial^s A^r-\partial^r A^s). \tag{5.20}$$

This implies the usual relationship if one takes

$$\frac{C_1}{2\alpha C_2}=1. \tag{5.21}$$

The conclusion is that, by adding chiral asymmetric terms in the Lagrangian, one can get the usual relation (4.33).

We have seen in Sec. IV that the most general form for $F_{\mu\nu}$ is

$$F_{\mu\nu}=\mathscr{F}_{\mu\nu}+i\tilde{\mathscr{F}}_{\mu\nu}. \tag{5.22}$$

Replacing $F_{\mu\nu}$ given by (5.22) in Eq. (5.18) we get an alternative form for Eq. (5.18). This alternative form is

$$\begin{cases}\partial_\mu\mathscr{F}^{\beta\mu}=-\dfrac{K}{4C_2}\bar{\eta}\gamma^\beta\eta,\\ \partial_\mu\tilde{\mathscr{F}}^{\beta\mu}=0.\end{cases} \tag{5.23}$$

These are Maxwell's equations in Lorentz gauge if one writes $\mathscr{F}_{\mu\nu}$ under the form (4.33) and imposes the restriction

$$-\frac{K}{4C_2}=e. \tag{5.24}$$

In order to see this equivalence in terms of the observables **E** and **H** one writes

$$f^{ok}=E^k,$$
$$f^{jk}=e^{ljk}H^l, \quad k,j,l=1,2,3. \tag{5.25}$$

Equations (5.23) give rise to the following equations;

$$\nabla\cdot\mathbf{E}=e\,\bar{\eta}\gamma^0\eta, \quad \partial^0\mathbf{E}-\nabla\wedge\mathbf{H}=-e\,\bar{\eta}\boldsymbol{\gamma}\eta,$$
$$\nabla\cdot\mathbf{H}=0, \quad \partial^0\mathbf{H}+\nabla\wedge\mathbf{E}=0. \tag{5.26}$$

Equations (5.26) are Maxwell's equation in the presence of matter.

As a final remark we would like to show the equivalence between the spinor method and the usual tensor method. In order to show this all one has to do is to substitute in Lagrangian (5.13) the expression for the chiral components (4.8). In terms of the fields $F_{\mu\nu}$ and A_μ the Lagrangian density can be written as

$$\mathscr{L}[F^{\mu\nu},\ A^\mu]=-8\alpha C_2^2\{F^{*\mu\nu}(\partial_\nu A_\mu-\partial_\mu A_\nu)-A_\mu^*(\partial_\nu F^{\mu\nu}-\partial_\nu F^{\mu\nu})\}+2K\alpha C_2\,\bar{\eta}\gamma^\mu\eta A_\mu^*$$
$$+\bar{\eta}(i\partial\!\!\!/-m_f)\,\eta-8\alpha C_2^2F_{\mu\nu}^*F^{\mu\nu}, \tag{5.27}$$

where we have used condition (5.21).

By treating $F_{\mu\nu}^*$, A_μ^*, and η as independent field variables one gets the following equations:

$$\partial_\nu F^{\mu\nu}=-\frac{K}{4C_2}\,\bar{\eta}\gamma^\mu\eta, \tag{5.28}$$

$$(i\partial\!\!\!/-m_f)\,\eta=-2K\alpha C_2\gamma^\mu\eta A_\mu, \tag{5.29}$$

$$\mathscr{F}_{\mu\nu}=\partial^\mu A_\nu-\partial^\nu A^\mu. \tag{5.30}$$

Finally, one gets the usual Lagrangian of electrodynamics in terms of A_μ if one uses (5.30) and makes a proper choice of the constant α.

VI. QUANTIZATION

In this section we will consider the quantization of massive fields within the BW theory. We also propose an extension to the zero-mass case. We show that there is no basic distinction between this method and the usual approach. The relevant point is that it is possible to quantize the spinor fields by imposing appropriate commutation relations for these fields.

We quantize the BW fields by imposing the following commutation relations for BW's massive fields:

$$[\psi_{a_1a_2\cdots a_{2s}}(x),\ \overset{+}{\psi}_{a_1'a_2'\cdots a_{2s}'}(y)]_s=(-i)^{2s-1}\kappa\sum_{\mathscr{P}}\ i(i\partial\!\!\!/_x+m)_{a_1a_1'}\cdots i(i\partial\!\!\!/_x+m)_{a_{2s}a_{2s}'}\Delta(x-y),$$
$$a_1,\ldots,a_1',\ldots=1,2,3,4. \tag{6.1}$$

$\Delta(x-y)$ is the Jordan Pauli function, κ is a constant to be determined, $\mathscr{P}$ denotes all possible permutations among the spinor indices, and we use, in (6.1), the following notation:

$$[\phi,\psi]_s=\phi\psi+(-1)^{2s-1}\phi\psi, \tag{6.2}$$

where s is the spin of the fields in (6.2).

In the case of massless particles, and as pointed out in Sec. II, the fundamental objects are two-component spinors of rank $2s$ $\varphi^{(1)}\cdots\varphi^{(2s)}$. For the massless case the quantization is carried out by imposing the following commutation relations for the two-component Weyl spinors φ:

$$[\varphi_{b_1\cdots b_{2s}}(x),\ \varphi^{+}_{b_1'\cdots b_{2s}'}(y)]_x=(-i)^{2s-1}\kappa\sum_{\mathscr{P}}\ i(i\sigma^0\partial_0-i\boldsymbol{\sigma}\cdot\boldsymbol{\nabla})_{b_1b_1'}\cdots i(i\sigma^0\partial_0-i\boldsymbol{\sigma}\cdot\boldsymbol{\nabla})_{b_{2s}b_{2s}'}\times D(x-y), \tag{6.3}$$

where $D(x-y)$ is the Jordan Pauli function for massless particles, κ is a constant to be determined, $\mathscr{P}$ denotes all possible permutations among the spinor indices, and where we use convention (6.2).

One can now use, for spin 1 massive particles, the decomposition (2.9) and write the usual plane wave expansion for the field B_μ. In the case of the photon this decomposition for the A_μ field is

$$A^\mu(x)=\frac{1}{(2\pi)^3}\int\ \frac{d^3\mathbf{p}}{\sqrt{2p^0}}\sum_{\lambda=1}^{2}\ \epsilon^\mu(p,\ \lambda)[A(p,\ \lambda)e^{-ipx}+A^+(p,\ \lambda)e^{ipx}], \tag{6.4}$$

where $p=(p^0,\,\mathbf{p})$.

If one uses the decomposition (4.25) with $\mathbf{f}$ given by (4.28), then the commutation relations (6.3) imply the following commutation relations for the creation (A^+) and annihilation (A) operators:

$$[A(p,\ \lambda),A^+(p',\ \lambda')]=\delta^3(p-\mathbf{p}')\,\delta_{\lambda\lambda'}\,. \tag{6.5}$$

By analyzing the behavior of observables such as the energy, as we did in Sec. III C, and momentum we would realize that these fields describe particles.

We shall not further analyze the question of the quantization of the spinor fields. We just wanted to point out that one can provide a quantization method for spinor fields as well Feynman rules for the interaction of these fields. Most of the questions on computing cross sections for specific processes and the analysis of renormalization can be carried out in a simpler manner within the usual approach (tensor method).

VII. CONCLUSIONS

In this paper we have presented the description of electrodynamics in terms of spinors. The spinor method provides a description of massless particles in terms of chiral components when they are treated as independent field variables.

We have shown that, within the spinorial approach proposed here, photons can be described in close analogy with neutrinos. The requirement of chiral invariance, at the free field level, leads to Maxwell's equation in vacuum. The requirement of chiral asymmetry for the linear interaction of the spinor field with matter fields leads, for a proper choice of chiral components, to Maxwell's equations in the presence of matter.

One of the advantages of this approach is that it allows one to formulate electrodynamics in terms of potentials or, by using two-component spinors, in terms of the observable fields (Maxwell equation) E and H.

The question is whether electrodynamics is a left–right asymmetric theory. If matter does not distinguish between chiral components, then it should couple with the field ψ. That is not, however, the case. On the other hand, if ordinary matter couples only with $\psi_{RR}\psi_{LL}$ we would have electrodynamics formulated entirely in terms of observables. That is not the alternative that nature chooses either.

The usual QED is compatible with a theory in which only $\psi_{LR}+\psi_{RL}$ couples with ordinary matter. From this point of view QED is another example of an asymmetric interaction between chiral components. The conclusion is that the Lagrangian describing electrodynamics, in this four-component (chiral components) framework, is

$$\mathscr{L}_{\text{QED}}=\bar{\psi}_{RL}(i\partial\!\!\!/\otimes 1)\psi_{RR}+\bar{\psi}_{LR}(i\partial\!\!\!/\otimes 1)\psi_{LL}+\bar{\psi}_{RR}(i\partial\!\!\!/\otimes 1)\psi_{RL}$$
$$+\bar{\psi}_{LL}(i\partial\!\!\!/\otimes I)\psi_{LR}+\bar{\eta}(i\partial\!\!\!/-m)\eta+F(\bar{\psi}_{RL}+\bar{\psi}_{LR})\eta_c\eta.$$

Within this spinor framework it is easy to see that electrodynamics is a parity nonconserving theory. Although electrodynamics is asymmetric with regard to left and right and usually chiral asymmetry is connected to parity nonconservation, it is obvious that in this case the consequence of symmetry does not lead to processes occurring differently in the actual systems or in its mirror image. There are no observable consequences for the breakdown of parity.

There is a dynamical consequence for the particular way in which nature chooses to break chiral symmetry. Violation of chiral symmetry in this case reflects the preference of matter in interacting through the electromagnetic potentials rather than interacting through the electromagnetic fields. The linear nature of the interaction and the breakdown of the symmetry implies the minimal coupling substitution.

Our conclusion is that the linearity of the theory and left–right asymmetry implies the minimal substitution way of coupling the electromagnetic fields to matter.

The extension of this method to spin 2 field is under preparation.[9,10]

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APPENDIX A: RANK 2*s* FOUR SPINORS

Remembering that a rank one spinor transforms, under Poincaré transformation, as

$$\eta(x)\rightarrow\eta'(x')=D(l)\,\eta(x),$$

a $2s$ rank spinor $\psi(x)$ transforms as

$$\psi(x)\rightarrow\psi'(x')=\underbrace{D(l)\otimes\cdots\otimes D(l)}_{2s\ \text{factors}}\psi(x).$$

Analogously, the $2s$ rank spinor $\bar{\psi}(x)$ defined by

$$\bar{\psi}(x)=\psi^{+}(x)\underbrace{\gamma^0\otimes\cdots\otimes\gamma^0}_{2s\ \text{factors}}$$

transforms like

$$\bar{\psi}(x)\rightarrow\bar{\psi}'(x')=\bar{\psi}(x)\underbrace{D^{-1}(l)\otimes\cdots\otimes D^{-1}(l)}_{2s\ \text{factors}},$$

where we have used the property that

$$\gamma^0 D^+(l)\gamma^0 = D^{-1}(l).$$

APPENDIX B: NOTATION AND γ MATRICES

(1) The metric used is $g_{\mu\nu}=(1,-1,-1,-1)$.
(2) Dirac's matrices commutation rules are those of Bjorken and Drell.[7]
(3) In chiral representation

$$\gamma^5=\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix},\quad \boldsymbol{\Sigma}=\begin{bmatrix}\boldsymbol{\sigma} & 0\\ 0 & \boldsymbol{\sigma}\end{bmatrix},\quad \gamma^{\mu}_{\alpha\dot{\beta}}=\begin{bmatrix}0 & -(\sigma_\mu)_{\alpha\dot{\beta}}\\ -(\sigma^\mu)^{\alpha\dot{\beta}} & 0\end{bmatrix}.$$

(4) Charge conjugation matrix

$$(C)_{\alpha\dot{\beta}}=\begin{bmatrix}-i\sigma^2_{\alpha\dot{\beta}}=C_1 & 0\\ 0 & i\sigma^2_{\alpha\dot{\beta}}=C_1\end{bmatrix}.$$

(5) Pauli matrices are defined by

$$\sigma^0=\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix},\quad \sigma^1=\begin{bmatrix}0 & 1\\ 1 & 0\end{bmatrix},\quad \sigma^2=\begin{bmatrix}0 & -i\\ i & 0\end{bmatrix},\quad \sigma^0=\begin{bmatrix}1 & 0\\ 0 & -1\end{bmatrix}.$$

By convention $\sigma^\mu=(\sigma^0=1,\ \boldsymbol{\sigma})$ designates $(\sigma^\mu)_{\alpha\dot{\beta}}$.

We have used also the property that

$$(\sigma^0)_{\alpha\dot{\beta}}=(\sigma_0)_{\alpha\dot{\beta}},\quad (\sigma^k)_{\alpha\dot{\beta}}=-(\sigma_k)_{\alpha\dot{\beta}}.$$

C_1 denotes the charge conjugation matrix in (2,2) space and obeys ${}^tC_1=-C_1$, $C_1^+=C_1^{-1}$.

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