Physica A 390 (2011) 2562–2570

Contents lists available at ScienceDirect

Physica A

journal homepage: www.elsevier.com/locate/physa

A Markovian model market—Akerlof's lemons and the asymmetry of information

Paulo F.C. Tilles [a], Fernando F. Ferreira [b,*], Gerson Francisco [a], Carlos de B. Pereira [b], Flavia M. Sarti [b]

[a] Instituto de Física Teórica UNESP-Universidade Estadual Paulista, Caixa Postal 70532-2, 01156-970 São Paulo, SP, Brazil
[b] EACH-Universidade de São Paulo, Av. Arlindo Bettio 1000, 03828-000 São Paulo, Brazil

ARTICLE INFO

Article history:
Received 18 September 2010
Received in revised form 21 February 2011
Available online 16 March 2011

Keywords:
Markovian market model
Asymmetric information
Technological evolution

ABSTRACT

In this work we study an agent based model to investigate the role of asymmetric information degrees for market evolution. This model is quite simple and may be treated analytically since the consumers evaluate the quality of a certain good taking into account only the quality of the last good purchased plus her perceptive capacity β. As a consequence, the system evolves according to a stationary Markov chain. The value of a good offered by the firms increases along with quality according to an exponent α, which is a measure of the technology. It incorporates all the technological capacity of the production systems such as education, scientific development and techniques that change the productivity rates. The technological level plays an important role to explain how the asymmetry of information may affect the market evolution in this model. We observe that, for high technological levels, the market can detect adverse selection. The model allows us to compute the maximum asymmetric information degree before the market collapses. Below this critical point the market evolves during a limited period of time and then dies out completely. When β is closer to 1 (symmetric information), the market becomes more profitable for high quality goods, although high and low quality markets coexist. The maximum asymmetric information level is a consequence of an ergodicity breakdown in the process of quality evaluation.

1. Introduction

The asymmetry of information is a very important concept in the study of marketplace exchanges with a remarkable consequence in finance, accounting, organizational behavior, economics and marketing. In 1970 Akerlof introduced and analyzed the impact of asymmetric information in the market failure problem [1]. The basic idea is that agents trade in the market because they share mutual benefits. However when the information asymmetry increases up to a certain level, the high quality goods drop out from the market and only low quality goods can be traded. This situation is called *adverse selection* [2,3]. Later Spence and Stiglitz addressed the role of asymmetry of information in the context of job market [4,5] and insurance [6] showing that agents can send signals to mitigate the effect of the informational asymmetry. Despite it has been introduced three decades ago, the complexity of both the market and human behavior make it very difficult to build mathematical models able to be applied in real world situations. The challenge is to adequately incorporate the asymmetric information into mainstream of the standard economic theory. To attack this problem one should find the correct way to address the modeling issue.

* Corresponding author.
E-mail address: fernff@gmail.com (F.F. Ferreira).

0378-4371/$ – see front matter © 2011 Elsevier B.V. All rights reserved.
doi:10.1016/j.physa.2011.03.007

In attempt to capture the market mechanism many researchers have been working with probabilistic models [7–9] or agent based models [10–15]. In special, some of those models discuss the role and the economical implications of the asymmetric information. For instance, the supply and demand law has been revisited by Zhang [16] by introducing the product's quality and the imperfect information as the key ingredients. Although he obtained many classical results known from Neoclassical theory others results are in discordance and they have deep implications in economic theory as the impossibility for the economy to reach maximal welfare. Also, the asymmetric information appears in Ref. [17] to explain the emergence of product differentiation. When heterogeneous buyers collect and process information, somehow vendors are forced to diversify their products [18]. More recently Frieden et al. [19,20] have been shown a framework for the micro-foundation of the economy by trying to link the micro-dynamics and the macro-observables using the Fisher information. Asymmetric information is defined as the difference between intrinsic Fisher information and the information that is in the data. By positing this measure for computing the information, they have shown how to derive macroeconomic dynamical law from the asymmetry of information.

On the other front, Wang et al. [11] have introduced an agent based model to investigate the impact of asymmetric information on the market's evolution. They have shown the emergence of adverse selection from the simulation of a very simple Markovian model. To improve the understanding of the market functioning, they introduced another variable called *valuation ratio*, defined as the ratio between consumers' valuation (consumers' willingness to pay) and the firms' valuation (cost) about the same good under the situation of complete information. As a result they observe that the coexistence of high and low quality is a consequence of a trade-off between consumers' perception of product quality and large valuation ratios.

In the present work, we modify the Wang model by proposing a nonlinear production cost function. Now, to compute the minimum selling price, firms take into account the technology used on the manufacture of goods to reduce the production cost. Besides that, we realize that this model could be analyzed by a Markov chain approach. As a result it was found that the model exhibits stationary solutions and presents adverse selection occurring in different degrees of informational asymmetry and technological levels. The profit of firms increases in the region of low quality products, and some asymmetric information level will imply higher profits in the production of the low quality goods. However, firms should avoid a very high asymmetric information level in order to maximize their profits. The effect of the nonlinearity introduced throughout the technology parameter is revealed by the strong feedback mechanism observed in the average number of transaction and in the relative profit obtained by firms. Lastly, the model has predicted a market collapse caused by opportunistic agents.

This study is organized as follows: in Section 2 we describe the model and set the condition of market existence; in Section 3 we use the Markov chain approach to obtain the probability distribution function and statistics describing the transaction process; in Section 4 we explain the market's behavior predicted by the model; the presence of agent that act opportunistically causes the market breakdown is shown in Section 5; we close in Section 6 with the conclusions.

2. Model

Based on the model proposed by Wang et al. [11] we consider a system composed of N_s sellers that offer goods with different quality labeled by integers in the range $[\kappa_{\min}, \ldots, \kappa_{\max}]$ and N_b buyers which have access to all goods in the market. Following the scheme designed by Laureti and Zhang [8], the trading process is modeled as a random matching between buyers and sellers whose outcome is determined by the individual evaluation of each part of the process: the sellers evaluate their products based on their quality while the buyers use the product information given by the seller associated with their own expectations formulated before the transaction takes place (it is based on their own history of transactions). At each step the consumer i chooses one firm at random and evaluates the quality q_i of the product she is buying according to the expression

$$q_i(k; k_a^{(i)}) = \beta k + (1 - \beta)k_a^{(i)}, \tag{1}$$

where $\beta \in [0, 1]$ is the degree of asymmetric information, k is an integer that corresponds to the quality of the good assigned by the firm and $k_a^{(i)}$ represents the buyer's expectation on the product, which in the model is assumed to be the quality of the last item purchased by the agent i. The consumer recognizes the real quality of the item only after purchasing it, and this quality is used as the next $k_a^{(i)}$. For $\beta = 1$ the information is both perfect and symmetric: the two groups share the same quality perception. As β decreases the consumers are no longer able to identify the products' quality correctly. The asymmetry of information emerges from the different perceptions between firms and consumers about the same product. The consumers' inability to assign a product its true quality leads the system to an imperfect state of information (although it is still perfect on the seller's side). The asymmetry reaches its maximum when β equals zero, making it impossible for the consumers to have any idea about the quality of the item they are purchasing.

Consumers and firms may disagree on how much the good is worth, so it is not expected that they would agree on a value assigned to it or even on how much a difference in the quality of a product should cost. In this sense, we propose two distinct ways for consumers and firms to assign a value to the product, depending on its quality. The consumers have a valuation function

$$V_i^b(k; k_a^{(i)}) = a_1 q_i(k; k_a^{(i)}) + a_2, \tag{2}$$

where $a_1 > 0$ is the rate of the buyer's willingness to pay and $a_2 \geq 0$ gives the valuation scale. The firms compute the minimum value of their goods by taking into account the actual manufacturing technological state of the economy $\alpha > 0$

$$V^s(k) = b_1 k^\alpha + b_2, \tag{3}$$

and the monetary scale of sales via $b_1 > 0$ and $b_2 \geq 0$. This function was proposed to establish a nonlinear relation between product quality and production cost. The aim is to capture the changes observed in the production function of the most commonly observed markets, according to the technology evolution, and also to keep the original formulation and richness as a limit of this generalization ($\alpha = 1$ and $a_1 = a_2 = 0$) as proposed by Wang et al. [11]. Empirically, it is observed that there are higher relative prices associated to high quality goods produced in certain period.

As both parts of the negotiation have their own valuations with opposite limiting boundaries about the product in hand - a seller will not sell its product for less than its costs while a consumer will not pay more than she thinks it is worth — there must exist a closed set of transaction values for the two parts to find an agreement. The necessary condition for the existence of this closed set is

$$V^b(k; k_a^{(i)}) \geq V^s(k). \tag{4}$$

In the model whenever this condition in not satisfied no transaction takes place and the consumer chooses another firm randomly in the next step.

To understand the implications of the nonlinear generalization on the sellers behavior, we find it useful to analyze the marginal valuation $\partial_k V^s(k)$. It is possible to see that three different regimes appear as the value of the parameter α is changed. For $\alpha < 1$ the technological state of the economy is marginally more efficient, i.e., the production cost of an additional unity is given by a decreasing function in contrast with $\alpha > 1$ where the derivative increases for increasing quality (the marginal value is constant for $\alpha = 1$). Due to this feature it is possible to associate α with the state of technology in the economy because a firm may produce the same product with different value profile according to the value assumed by α.

Since we have defined how the individual agents interact in the economy, we now focus our attention on the analysis of the sufficient conditions for the market to exist and on how the parameters influence the market picture. For this we consider the cases with $\beta = 1$ and $V^b < V^s$ (no transactions allowed). By inserting Eqs. (2) and (3) in $V^b < V^s$ one may define the parameters $\chi = (a_2 - b_2)/a_1$ and $\lambda = b_1/a_1$ and end up with the inequality $k + \chi < \lambda k^\alpha$. A proper choice of the scale difference χ guarantees that this inequality is never satisfied and any nonlinear effect on the market existence conditions is ruled out. On the other hand, it is always possible to define χ in a way that forbidden quality regions appear in the system, depending on the proper choice of the technological state α. These regions are defined by the firms whose products may not be sold due to the consumers' undervaluation. We will discuss these situations in more detail ahead when we deal with the presence of opportunism in the market in Section 5.

Under the assumption that any scale difference is absent in the market ($\chi = 0$), the nonlinear parameter α defines what are the market existence conditions. As the model presents a trivial dynamics for the case of a monopolist market (it is independent of β), we will consider only the cases where at least two vendors are able to sell their products. This market condition is achieved whenever (4) is satisfied for at least two values in the $\beta = 1$ case. A simple calculation will show that for $\alpha \geq 1$ the sufficient condition is $\lambda < 1$ and for the $\alpha < 1$ case the condition $\lambda < \kappa_{\max}^{1-\alpha}$ must hold. Given that λ satisfies either of these conditions, three different market pictures arise, depending on the technological state: $\alpha > 1$ gives rise to a forbidden high quality region above $\hat{k} = \lambda^{\frac{1}{1-\alpha}}$, independently of β; for $\alpha < 1$ a forbidden low quality region below $\hat{k}$ emerges when $\beta = 1$, but it becomes accessible for lower values of β; lastly, if $\alpha = 1$ no prohibitive region exists.

Before we enter the analysis of the model dynamics we think it is important to discuss a few points concerning the price. In a bargaining trading process the price will be somewhere between the consumers willingness to pay $V^b(k; k_a)$ and the firm's costs $V^s(k)$. The discussion on the differences between price and value involves lots of controversy, including the differentiation between value in use and value in exchange, according to Adam Smith [21], "no trade occurs if the value in use of a good is lower than its value in exchange". In other words, whenever a trade occurs, the value in use (utility of the good for the consumer) is usually significantly higher than value in exchange (market price), and the latter should be at least equal to or higher than the production cost. Based on this observation we define the relative profit as $\gamma = \frac{V^b}{V^s}$ just to avoid introducing a sophisticated price dynamics model.

Our focus is on the impact of asymmetric information under a fixed technological level in the evolution of the market. The buyer's quality evaluation mechanism follows a Markovian process since only the more recent state (the last purchased product) is relevant for computing the next quality. Due to the feature that the buyers do not interact with each other, this agent based model may be treated as if there was just one representative agent whose behavior is supposed to be determined by the dynamic interaction with the sellers. The group behavior of a collection of such agents should only affect the statistical properties of the system, since each agent should randomly choose one of the sellers to make the deal. In this way, a Markovian chain approach emerges as the best candidate to explain the behavior of the market as it reaches the stationary state. So, in the next section we consider the statistical properties of the system and set up the stochastic process to understand the economic properties of the market.

3. Stationary probability distribution and statistics

In each round, a consumer chooses one firm at random, who will offer her a product of quality k. The probability $p_v(k, k_a)$ for the transaction to occur is given by

$$p_v(k, k_a) = \Pr[V^b(k, k_a) \geq V^s(k)] = \Pr[\beta k + (1-\beta)k_a + \chi \geq \lambda k^\alpha]. \tag{5}$$

For this buyer's next round, her k_a will either be the same or change to k, depending on the result of this transaction. In this way, one can describe the consumer's behavior as a stochastic process, where her state n is defined as the quality of her last purchased good, which may remain the same if condition (4) is not fulfilled (or if she is trying to purchase the same quality) or undergo a transaction $n \to m$ whenever she tries to acquire a new good with quality m. Since the products are ordered on a discrete set of qualities the process is described by a Markovian chain, where the quality is mapped to the state label n via

$$k(n) = \left(\frac{\kappa_{\max} - \kappa_{\min}}{N_s - 1}\right)(n-1) + \kappa_{\min}, \tag{6}$$

with states $n = 1, 2, \ldots, N_s$. To guarantee that only accessible states are reached we use Heaviside's step function $\Theta(x)$ and write the transition probabilities as

$$\Pr(n \to m) = \frac{1}{N_s}\Theta[\beta k(m) + (1-\beta)k(n) - \lambda k(m)^\alpha + \chi], \tag{7}$$

$$\Pr(n \to n) = \frac{\Theta(k(n) - \lambda k(n)^\alpha + \chi) + \sum_{w=1}^{N_s} \Theta[\lambda k(w)^\alpha - \beta k(w) - (1-\beta)k(n) - \chi]}{N_s}. \tag{8}$$

With Eqs. (7) and (8) one may define a transition matrix T with elements $T_{nm} = \Pr(m \to n)$, and set a vector $\mathbf{P}_t$ describing the probability of the buyer to be found in each state of the system. The expression for the stochastic process given by $\mathbf{P}_{t+1} = T\mathbf{P}_t$, where the stationary solution $\mathbf{\Pi} = T\mathbf{\Pi}$ may be obtained by either diagonalizing the T matrix (in order to find the eigenvector with eigenvalue equal to unity) or numerically by iterating the equation until the convergence is reached.

In the stationary state, the probability $\Pi(k)$ for a consumer to be on the kth state may be associated with the probability of her last purchased item to be k, since $\mathbf{P}_{t+1} = \mathbf{P}_t = \mathbf{\Pi}$. This allows one to calculate the firm's probability of selling a product to one unspecified consumer using (5),

$$p_k = \sum_{k'=1}^{\kappa} p_v(k, k') = \sum_{k'=1}^{\kappa} \Theta[\beta k + (1-\beta)k' - \lambda k^\alpha + \chi]\Pi(k'), \tag{9}$$

that is, the probability of the firm k to perform a transaction given that it was chosen by one buyer.

To obtain all statistical quantities of interest we need to take into account the probability of a firm to be chosen by n consumers in one single run. Let X_k be a random variable that represents the number of consumers that choose the seller k in some round. As each consumer chooses only one seller at each step, the probability of a seller to be chosen by exactly n buyers is described by a binomial distribution

$$P(X_k = n) = \Pr(n) = \frac{N_b!}{n!(N_b - n)!}\frac{(N_s - 1)^{N_b - n}}{N_s^{N_b}}. \tag{10}$$

Since the probability of a seller k to actually sell to $Y_k = j$ buyers follows a binomial process where the Bernoulli trial has the weights p_k (9), the probability of some seller k performing exactly j transactions, given that she was chosen by n consumers ($n \geq j$), is

$$P(Y_k = j | X_k = n) = \frac{n!}{j!(n-j)!}p_k^j(1-p_k)^{n-j}. \tag{11}$$

Note that, when the system reaches the stationary state, we can use the ergodic property to calculate the temporal average number of transaction per run due to the seller k as a sum over all states of the system:

$$\begin{aligned}\langle \mathrm{trans}(k)\rangle &= \sum_{n=0}^{N_b} nP(Y_k = n) = \sum_{n=0}^{N_b} n \sum_{i=n}^{N_b} \Pr(i)P(Y_k = j | X_k = n) \\ &= \frac{N_b!}{N_s^{N_b}} \sum_{n=0}^{N_b}\sum_{i=n}^{N_b} \frac{(N_s - 1)^{N_b - i} p_k^n (1-p_k)^{i-n}}{(N_b - i)!(i-n)!(n-1)!}\end{aligned} \tag{12}$$

where $P(Y_k = n)$ is the probability of a seller to sell exactly n units of goods.

Let $P(k|k')$ be the probability that, given some consumer state k', some seller k will do business. We write this as

$$P(k|k') = \frac{\Theta[\beta k + (1-\beta)k' - \lambda k^{\alpha}]\Pi(k')}{\sum_{k'=1}^{N_b} \Theta[\beta k + (1-\beta)k' - \lambda k^{\alpha}]\Pi(k')}. \quad (13)$$

So, the average of any quantity written as a function of k and k', denoted by $\hat{O}(k; k')$, may be obtained via

$$\langle O(k)\rangle = \sum_{k'=1}^{N_b} \hat{O}(k; k')P(k|k'). \quad (14)$$

This expression may be used to calculate any economic observable one could be interested in. For example, the valuation ratio assumes the form

$$\langle \gamma(k;\beta)\rangle = \sum_{k'=1}^{N_b} \frac{a_1[\beta k + (1-\beta)k'] + a_2}{b_1 k^{\alpha} + b_2} P(k|k'). \quad (15)$$

4. Economic behavior and market failures

In economic theory the concept of market failure refers to the inefficiency in the allocation of goods and services by a free market. It means that there is still a certain amount of potential transactions that are not performed by the market. In order to have a good picture of how this inefficiency is generated in the market we set up the system in the symmetric information state ($\beta = 1$) under complete market conditions: all products are accessible to the consumers.

Variations on β will surely affect the market's behavior, but we would also like to understand what are the effects of the nonlinearity introduced in the dynamics. For this we set the parameters as follows: the system presents the same number of buyers and sellers $N_s = N_b = 1000$; the buyer's valuation function has minimum and maximum values $V^b_{\min} = V^b(\kappa_{\min})$ and $V^b_{\max} = V^b(\kappa_{\max})$ when $\beta = 1$, so $a_1 = \frac{V^b_{\max} - V^b_{\min}}{\kappa_{\max} - \kappa_{\min}}$ and $a_2 = V^b_{\max} - a_1\kappa_{\max}$; the seller's evaluation function is defined analogous to the consumer, $V^s_{\min} = V^s(\kappa_{\min})$ and $V^s_{\max} = V^s(\kappa_{\max})$, with $b_1 = \frac{V^s_{\max} - V^s_{\min}}{\kappa^{\alpha}_{\max} - \kappa_{\min}}$ and $b_2 = V^s_{\min} - b_1$. We will take $\kappa_{\min} = 1$ and $\kappa_{\max} = 1000$ and assume that the valuation ratio $\gamma(k)$ is equal in both ends, $\gamma(\kappa_{\min}) = \gamma(\kappa_{\max}) = 2$, with $V^b_{\max} = 10V^b_{\min} = 20$ and $V^s_{\max} = 10V^s_{\min} = 10$. With this set up the parameters of the model are $a_1 = 0.018$, $a_2 = 2.0$ (b_1 and b_2 depend on the parameter α).

Under these assumptions we have a complete market condition for $\alpha \geq 1/2$, and the fixed ends condition allows us to study the effect of a change in the technological state on the economic observables.

4.1. General aspects

As stated before, our main interests are the averages in the number of transactions and the valuation ratio for each firm. The average number of transactions (per round) gives a measure of the firms efficiency to sell their products, but it must be analyzed together with the valuation rate $\gamma(k) = V^b(k)/V^s(k)$, because companies, in general, tend to maximize their expected profit according to a strategy that may last longer than a single run, i.e., a firm may choose either to make fewer transactions on a time step with a higher profit or to sell its product to as many consumers as possible with a lower margin. As far as the buyers perception plays an important role in the model, each firm's observed average quality tells us how the buyers evaluate the quality of its product. If the consumer overestimates the quality of the product, trade will occur only if the willingness to pay is high enough to afford it, otherwise we expect the buyer to give up the purchase. Quality underestimation may also occur, but it leads to a lower profit margin. So asymmetric information is not always advantageous to firms, since the average number of transactions and (or) the relative gain may decrease. A similar conclusion was found in Ref. [7].

The Fig. 1(a) and (b) show the average number of transactions $\langle \text{trans}(k)\rangle$ and relative profit $\langle\gamma(k)\rangle$ as a function of β for several product qualities. In both figures we see a poor performance of firms in the highly asymmetric region ($\beta < 0.4$). Although there is some advantage for low quality products, it is not the best for anyone. On the other hand, the low asymmetric information level (high β values) presents the best scenario for high quality goods in terms of $\langle \text{trans}(k)\rangle$ and $\langle\gamma(k)\rangle$. A rich picture emerges in the intermediate region of β. High quality products ($k > 500$) have a good performance in terms of sales but they present a low relative profit. Firms that produce very low quality goods enjoy the best benefits. Lastly, firms with products of an intermediate level have the worst performance in terms of sales but they have a higher relative profit compared to high quality goods.

To understand the dynamical mechanism that drives the systems to the stationary behavior we need to look first at the buyer's expected valuation function $\langle V^b(k)\rangle = a_1[\beta k + (1-\beta)\langle k_a\rangle_k] + a_2$ for a given quality k. The expected value of the last purchased item $\langle k_a\rangle_k$ determines the profit structure of the system: if a product k' is less than $\langle k_a\rangle_k$ then the consumer

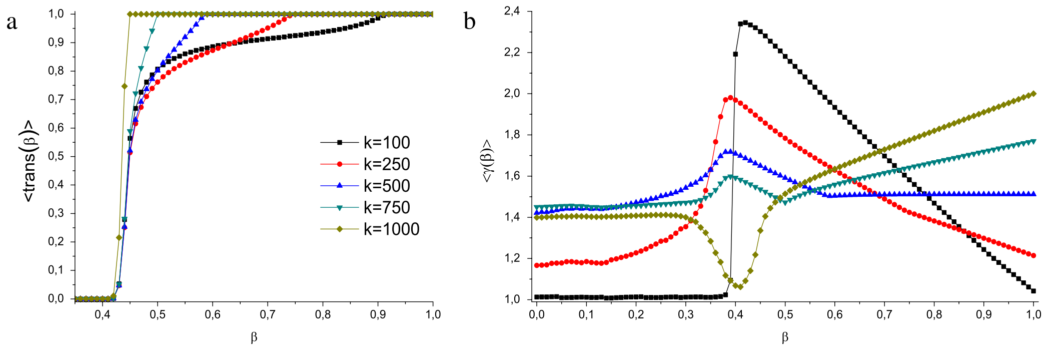

Fig. 1. Statistical description of economic observables for some products in the medium and high quality region as a function of the asymmetry parameter β for $\alpha = 1/2$: (a) average number of transactions; (b) average value ratio. The discrepancy between the analytical and simulation approaches to the average number of transactions is less than 10^{-3} for 100.000 interactions, becoming smaller as the number of interactions is increased.

will overpay this item, while for every $k' > \langle k_a \rangle_k$ the product will be under-appreciated by the buyer. This is true for every $\beta < 1$. If one starts to decrease β from a value close to 1, all states are still accessible and equally probable, so we have $\langle k_a \rangle_k = 500$. The effect of decreasing β affects $\langle V^b(k) \rangle$ in two ways:

(i) Transferring value effect: low quality products ($k < 500$) get an increase in the buyers expected valuation function $\langle V^b(k) \rangle$ due to the majority portion of products with a quality higher than k, while high quality items ($k > 500$) suffer the inverse effect.
(ii) Closing channels: some transition rates $\Pr(k' \to k)$ go to zero as the buyer cannot buy k after purchasing k', so the size of the set $\{k_a\}_k$ of available states for some given k is reduced. As it happens for lower qualities k', these states no longer contribute to $\langle k_a \rangle_k$, increasing the consumer's expectation.

The combination of these two processes (for β close to one) may be visualized in Fig. 1: the valuation ratio $\langle \gamma(k) \rangle$ increases for low quality products and decreases for $k > 500$; some products present a reduction of $\langle \text{trans}(k) \rangle$ due to the loss of some transition channels[1]. The value of β where the reduction on the average number of transactions is observed (denoted as β_r) may be calculated to give[2]

$$\beta_r(k, \alpha) = \frac{V^s(k, \alpha) - a_2 - a_1 \kappa_{\min}}{a_1(k - \kappa_{\min})}, \tag{16}$$

where behavior of this function is shown in Fig. 2 (left).

The closing channels affect the average number of transactions in different ways, depending on the actual value of α. For $\alpha < 1$ the first product that presents a reduction on $\langle \text{trans}(k) \rangle$ is always on the low quality region, defined by $k^* = \left(\frac{a_1}{\alpha b_1}\right)^{\frac{1}{\alpha - 1}}$. As β goes lower than $\beta_r(k^*)$ there is a redistribution on the probability values from $\Pi(k^*)$ to $\Pi(k')$ for all $k' < k^*$ that satisfy $\Pr(k' \to k^*) = 0$. Products with quality nearby k^* will experience the same process (left of Fig. 2), also increasing the stationary probability of the items on the extreme low quality region. The reduction of $\Pi(k)$ has not only an effect on $\langle \text{trans}(k) \rangle$, but it also diminishes $\langle k_a \rangle_{k'}$ for all products $k' > k$ such that $\Pr(k \to k') > 0$. As long as k' have all transition channels open there will be no effect on its average number of transaction, since the redistribution does not change the sum of all probabilities, but as soon as they lose some channels the reduction on $\langle \text{trans}(k') \rangle$ has a positive feedback from all k that have already undergone this reduction. This collective phenomenon may be seen on Fig. 1a: the first product shows a decreasing average number of transactions and has a small β contribution (decreasing is slow), but all other products with more quality experience a greater impact from β (due to the feedback), presenting a faster decrease. The chain reaction keeps getting fueled as β decreases, until the last item $\kappa_{\max}$ is affected with the greater impact from the asymmetry parameter. The final picture is an extreme market failure due to a strong feedback mechanism: all products with $k \geq k^*$ present $\langle \text{trans}(k) \rangle \to 0$ as the asymmetry enters the region $\beta \lesssim \beta_r(\kappa_{\max})$ and *only* low quality products are commercialized.

Now let us analyze how the strong feedback mechanism affects the valuation ratio behavior, as shown in Fig. 1b. Products with $k < 500$ present an increase in $\langle \gamma(k) \rangle$ for high values of β, but as the high quality states become less probable only low quality states are accessible, so they all present a maximum valuation ratio for some middle range asymmetry level. As $\beta \to 0$ products with $k \leq k^*$ present $\langle \gamma(k) \rangle \sim 1$ because the consumers get trapped in the low quality region, accounting

1 As the number of sellers and buyers are the same, the average number of times a seller is chosen by any consumer (in each round) is 1. Since the consumer may not select another seller on the same round when she does not make a deal it is not possible for any firm to sell more products in each round. These two features account for the upper bound in the average number of transactions shown in Fig. 1.

2 To obtain this expression one needs to consider the situation where the product k may no longer be bought by $\kappa_{\min}$.

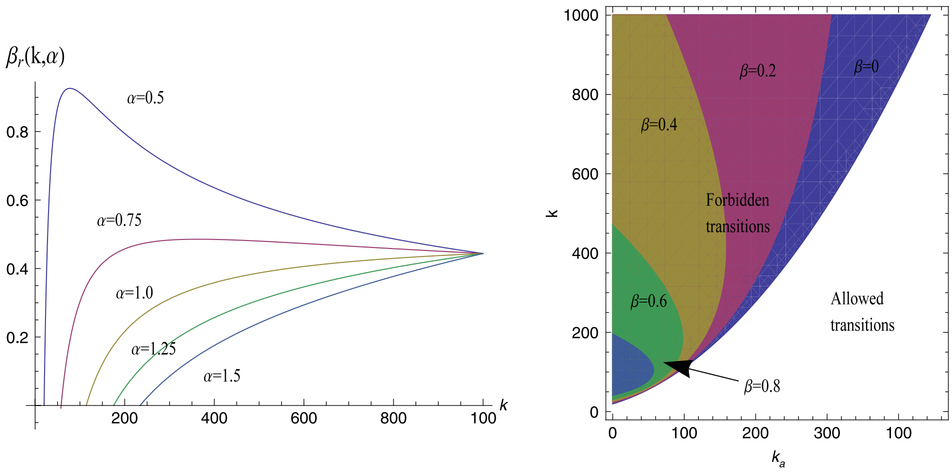

Fig. 2. Left: asymmetry level reduction β_r for all products as a function of α. Right: available transition channels $k_a \to k$ for some asymmetry parameter with $\alpha = 1/2$.

for $\langle k_a \rangle_k \lesssim k$. For the ones with $k^* < k \leq 500$, their probability of selling its products is almost null, as all the possible transactions come from states between middle to high level qualities (proportional to the probability of these states). They have $\langle k_a \rangle_k \sim k$ and their expected profit is $\langle \gamma(k) \rangle \sim \langle \gamma(k; \beta = 1) \rangle$. Qualities above $k = 500$ obey the same rule, with an attenuation due to natural undervaluation of high quality goods, so they have $\langle \gamma(k) \rangle \lesssim \langle \gamma(k; \beta = 1) \rangle$.

According to our simulations other technological scenarios with $\alpha \geq 1$ also present market failures, but much less intense than the previous ones. As k^* is always equal to $\kappa_{\max}$ and the rest of the products lose their transition channels in an ordered way proportional to β (Fig. 2 left), there is no strong feedback mechanism: the average number of transactions for high qualities present a decay, but the rate $\partial_\beta \langle \text{trans}(k; \beta) \rangle$ is much smaller than the ones with $\alpha < 1$. A lower technological state (higher α) implies a larger gap between seller and buyer valuation functions for almost all products (except for $\kappa_{\min}$ and $\kappa_{\max}$) and a weak feedback mechanism account for standard results: high quality products are always under-appreciated under asymmetry of information with average number of transactions significantly different from zero, while every good with $k < 500$ has a maximum valuation at some nonzero β' inversely proportional to α.

4.2. Optimum asymmetry scenario

As the dynamical mechanism that drives the market has shown possible payoff situations for the sellers, depending on the asymmetry parameter β, we focus our attention on the optimum asymmetry scenarios for each technological state.

Each seller has an expected valuation ratio and a average number of transactions well-defined for each value of asymmetry, but the changes in the behavior induced by variations of β do not affect these quantities on the same way, i.e., it was observed that for some quality values a decrease in β causes an increase in the relative profit but also a decrease in the average number of transactions. The best scenario for a firm is the one that maximizes its expected profit over time (on a time average), balancing the two effects. As the time average valuation ratio is obtained by a direct multiplication of the average number of transactions and the valuation ratio, we extracted the β_{opt} as the one that maximizes this quantity and calculated the ratio between the optimum valuation and the valuation for the $\beta = 1$ case to obtain a scale free observable, as shown in Fig. 3.

As expected, qualities above the median present $\beta_{opt} = 1$ while firms with quality under the median take advantage of the asymmetric information to obtain its highest profits with $\beta_{opt} < 1$. The unexpected feature is the effect of the technological state on the low quality products. On one hand one may observe that higher values of α decrease the value of β_{opt}, making it more difficult to achieve such scenario, since the maintenance of a high asymmetric state should be very costly in real economies. It means that even if an economy reaches a satisfactory technological level that allows the consumers to pursue good quality products, if they do not have sufficient information about the products they are purchasing it would open a niche for low quality products to take advantage of. On the other hand, the maximum amount of profit a firm could obtain at an optimum scenario only decreases with higher technological states. The interpretation of the two effects combined is that the economy tends to balance the effect of the asymmetry, generating a payoff between different technological levels for low quality products: higher (lower) technological states allow lower (higher) optimum profit margin with lower (higher) information asymmetry.

Another feature worth mentioning is the absence of a $\beta_{opt} = 0$ for any quality on all technology states. It means that even if a seller may take profit from the consumer's ignorance, too much ignorance will not benefit the firm because the buyer will not be able to purchase high quality products to overestimate the low quality ones. So a more balanced asymmetry of information is not only better for the low quality products, but it is also a better situation for the high quality goods too (in the sense that it is better than the $\beta = 0$ case).

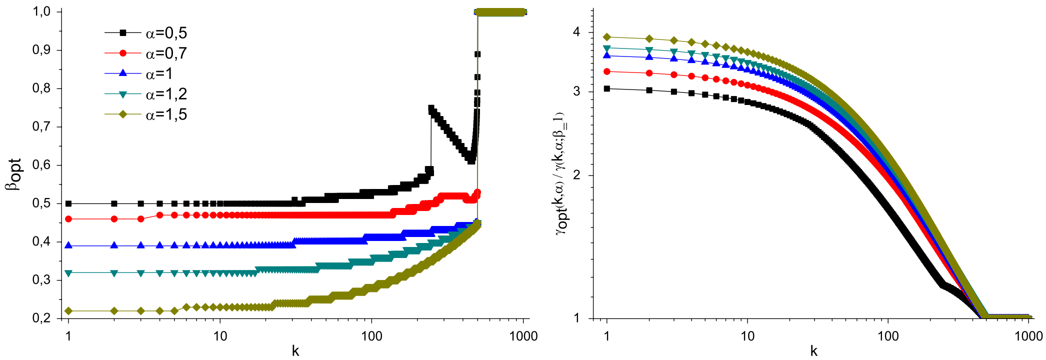

Fig. 3. Left: optimum asymmetry parameter β_{opt} for each firm. Right: relative profit between optimum value ratio (for β_{opt}) and the value ratio under perfect information scenario $\beta = 1$.

5. Opportunism and market breakdowns

We define opportunism as a group of sellers who have $V^s(k) > V^b(k; \beta = 1)$ and try to sell their products by taking advantage of the incomplete information passed to buyers as $\beta < 1$. Due to the assumption that both valuation functions are restricted to obey $\partial_k V^{s,b}(k) > 0$ for all k (a higher quality must always be better evaluated than a lower one), there are only three ways that these opportunists may be inserted into the market: on the high quality end with $k \in [k_1, \kappa_{max}]$ for some $k_1 > \kappa_{min}$; on the low quality end with $k \in [\kappa_{min}, k_2]$ for some $k_2 < \kappa_{max}$; or on some interval $\left[k', k''\right]$ with $k' > \kappa_{min}$ and $k'' < \kappa_{max}$. The first case is trivial since there is no value of β that allows consumers to buy this products, and the only effect on the market is the reduction $\kappa_{max} \to \kappa'_{max} = k_1 - 1$. The third case presents some interest, but as most of its behavior is qualitatively similar to the $\alpha < 1$ case shown in the previous section (except for β equal or close to 1), we will focus on the second type which presents a completely different phenomenon.

Let us assume first that $\alpha \leq 1$. Every product below a quality threshold θ defined by $V^s(k = \theta) = V^b(k = \theta; \beta = 1)$ may only be sold for $\beta < 1$. If one considers the case where $k_a = \kappa_{min}$, then for some value of β the only possible transitions $\kappa_{min} \to k$ have $k \in [k^*(\beta), \kappa_{max}]$ with $\partial_\beta k^*(\beta) < 0$. As one starts decreasing β there is a special value β_c that satisfies $k^*(\beta_c) = \kappa_{max}$, so any reduction for $\beta < \beta_c$ kills this last transition channel and turns κ_{min} into an absorbing state. This means that if a consumer buys the product κ_{min}, she may no longer buy any product in the market. As eventually every consumer will buy this product, the system presents a maximum degree of information asymmetry. For every asymmetry configuration below the critical level β_c the market will completely collapse (market breakdown). This critical parameter may be calculated to give

$$\beta_c = \frac{\lambda \kappa_{max}^{\alpha} + \chi - \kappa_{min}}{\kappa_{max} - \kappa_{min}}, \quad \alpha \leq 1. \tag{17}$$

To show that the absorbing state must appear at κ_{min}, let us consider two states $k \in [\kappa_{min}, \kappa_{max}]$ and $k' \in (\kappa_{min}, \theta)$. The condition $V^b(k; k_a = k') > V^b(k; k_a = \kappa_{min})$ is always satisfied, so if there is a transition $\kappa_{min} \to k$ for some k then there must exist at least one transition $k' \to k$. By considering the case where $V^b(\kappa_{max}; k_a = \kappa_{min}) = V^s(\kappa_{max})$, then for any state $k > \kappa_{min}$ we have $V^b(\kappa_{max}; k_a = k) > V^s(\kappa_{max})$, which shows that if $k_a = \kappa_{min}$ is not an absorbing state then there are no absorbing states in the system.

For the $\alpha > 1$ cases the market breakdown phenomenon is also present but it is independent of κ_{max}, as the last possible transition channel $\kappa_{min} \to k^*$ is defined by $V^b(k^*; k_a = \kappa_{min}, \beta_c) = V^s(k^*)$ and $\partial_k V^b(k = k^*; k_a = \kappa_{min}, \beta_c) = \partial_k V^s(k = k^*)$. The critical asymmetry parameter β_c is defined by the transcendental equation

$$\beta_c \left[\left(\frac{\beta_c}{\lambda \alpha} \right)^{\frac{1}{\alpha - 1}} - \kappa_{min} \right] = \lambda \left(\frac{\beta_c}{\lambda \alpha} \right)^{\frac{1}{\alpha - 1}} + \chi - \kappa_{min}, \quad \alpha > 1. \tag{18}$$

The presence of opportunists on the low quality region creates a maximum asymmetry level β_c under which the market may exist. This critical level is the point of a phase transition that destroys the market dynamics, as it was proposed by Akerlof.

6. Conclusions

This model addresses the interaction between firms through the choice of consumers whose behavior is described by a Markov chain. The simplicity of the model allowed us to obtain a semi-analytical expression for the transition probability matrix using the Markovian chain approach. Given the parameters α, N_s, κ_{min} and κ_{max} it was possible to obtain the probability of a consumer to be in the state k denoted by $\Pi(k)$. With this probability distribution we were able to explain the adverse selection phenomenon (low quality products have higher probability to perform transactions) via two different feedback mechanisms depending on the technological state α: a strong feedback for $\alpha < 1$ that leads to a extreme market failure where *only* low quality products are traded and a weak feedback for $\alpha \geq 1$ that enables high quality goods to be traded under much smaller inefficiency conditions. Also on some scenarios it was shown the optimal asymmetry levels for the firms to perform their transactions under maximum expected profit conditions (higher expected valuation ratio), which on real markets should be responsible for the competition among firms to control the consumers knowledge about their products.

As it was observed that under complete market conditions every asymmetry level would allow at least a portion of the market to perform transactions and finally we analyzed the case where opportunists are present in the system. It was shown that their presence is responsible for a phase transition that cause the market to collapse (market breakdown) on a critical asymmetry level β_c analytically calculated, and this is what really represents the effect of asymmetric information on the market collapse as proposed by Akerlof.

Acknowledgements

P.F.C.T. acknowledges fellowship by CAPES (Brazil). FFF thanks the Conselho Nacional de Pesquisa (CNPq) for financial support and Instituto de Física Teórica for hospitality.

References

[1] G.A. Akerlof, The market for 'lemons': quality uncertainty and the market mechanism, Quarterly Journal of Economics 84 (3) (1970) 488.
[2] J.P. Bigelow, Efficiency and adverse selection, Journal of Economic Theory 52 (2) (1990) 380.
[3] G. Biglaiser, J.W. Friedman, Adverse selection with competitive inspection, Journal of Economics and Management Strategy 8 (1) (1999) 1.
[4] M. Spence, Job market signaling, Quarterly Journal of Economics 87 (3) (1973) 355.
[5] J.E. Stiglitz, The theory of 'screening', education and the distribution of income, American Economic Review 65 (3) (1975) 283.
[6] M. Rothschild, J. Stiglitz, Equilibrium in competitive insurance markets: an essay on the economics of imperfect information, Quarterly Journal of Economics 90 (4) (1976) 629.
[7] Y.-C. Zhang, Happier world with more information, Physica A 299 (1–2) (2001) 104.
[8] P. Laureti, Y.-C. Zhang, Matching games with partial information, Physica A 324 (1–2) (2003) 49.
[9] Y. Wang, H.E. Stanley, Statistical approach to partial equilibrium analysis, Physica A 388 (2009) 1173.
[10] D. Challet, M. Marsili, Y.-C. Zhang, Modeling market mechanism with minority game, Physica A 276 (2000) 284.
[11] Y. Wang, Y. Li, M. Liu, Impact of asymmetric information on market evolution, Physica A 373 (2007) 665.
[12] M. Medo, Y.-C. Zhang, Market model with heterogeneous buyers, Physica A 387 (2008) 2889.
[13] L. Lu, M. Medo, Y.-C. Zhang, The role of a matchmaker in buyer-vendor interactions, The European Physical Journal B 71 (2009) 565.
[14] F.F. Ferreira, V.M. de Oliveira, A.F. Crepaldi, P.R.A. Campo, Agent-based model with heterogeneous fundamental prices, Physica A 357 (3–4) (2005) 534.
[15] A. De Martino, M. Marsili, Statistical mechanics of socio-economic systems with heterogeneous agents, Journal of Applied Physics A 39 (43) (2006) 465.
[16] Y.-C. Zhang, Supply and demand law under limited information, Physica A 350 (2005) 500.
[17] L. Lu, M. Medo, Y.-C. Zhang, D. Challet, Emergence of product differentiation from consumer heterogeneity and asymmetric information, The European Physical Journal B 64 (2) (2008) 293.
[18] H.-Y. Tsao, L.F. Pittb, P. Berthonc, An experimental study of brand sifnal quality of products in an asymmetric information environment, Omega 34 (4) (2006) 397.
[19] B.R. Frieden, R.J. Hawkins, Asymmetric information and economics, Physica A 389 (2010) 287.
[20] R.J. Hawkins, M. Aoki, B.R. Frieden, Asymmetric information and macroeconomic dynamics, Physica A 389 (2010) 3565.
[21] A. Smith, An Inquiry into the Nature and Causes of the Wealth of Nations, Clardenon Press, Oxford, 1979,1776.

Further reading

[1] D. Soberman, H. Gatignon, Research issues at the boundary of competitive dynamics and market evolution, Marketing Science 24 (1) (2005) 165.