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Gerson de Oliveira Barbosa

On The Stability Limits For Moons Orbiting Planets

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Gerson de Oliveira Barbosa

On The Stability Limits For Moons Orbiting Planets

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I dedicate this thesis to my entire family.

CURRICULAR DATA

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"O sonho é que leva a gente para a frente. Se a gente for seguir a razão, fica aquietado, acomodado."

Ariano Suassuna

Abstract

This thesis investigates the dynamical stability limits of exomoons in the planar circular restricted three-body problem, with emphasis on the outer boundary of circumplanetary confinement and on the phase-space structures that enable long-term survival. The study is based on high-resolution numerical simulations in which exomoons are modeled as massless test particles orbiting a planet and perturbed by a star. The planet–star orbit is kept circular throughout; the eccentricity varied in the simulations is the initial planetocentric eccentricity of the satellite, denoted by $e_{\rm sat}$. The integrations were carried out with the `MERCURY6` package using the Bulirsch–Stoer integrator in pure mode, and all results were compared through a uniform normalization based on the exact Hill radius, defined as the planet–L_1 distance. In the first stage, the critical semimajor axis for prograde satellites was determined empirically for five planet-to-star mass ratios, $\mu = 10^{-5}$ to 10^{-1}. The results show that the outer stability boundary decreases approximately linearly with the initial satellite eccentricity, $e_{\rm sat}$, recovering the classical trend reported in the literature while extending its validity over a much broader mass-ratio interval. Once expressed in units of the adopted Hill radius, the dependence on μ becomes weak, although still measurable. The analysis also demonstrates that stability is sensitive to the initial orbital phase, defined here by the satellite mean anomaly $M_{\rm sat}$: configurations started near $M_{\rm sat} = 180°$ remain stable at larger distances than those initialized at $M_{\rm sat} = 0°$. Thus, within the explored set of initial geometries, the $M_{\rm sat} = 180°$ case provides an upper-envelope estimate of the outermost stable semimajor axis, here denoted by a_E, whose dependence on $e_{\rm sat}$ is systematically mapped. In the second stage, extended stability maps were constructed for both prograde and retrograde orbits, considering different initial geometries and initial satellite eccentricities, $e_{\rm sat}$, reaching formally unbound values. These maps revealed that prograde stability cannot be described solely by an outer radial cutoff, but also by an upper satellite eccentricity limit for long-term survival. At high mass ratios, detached long-lived islands emerge and are shown to be associated with the evection resonance, as confirmed by resonant-angle libration and bounded oscillations of the orbital elements. In the retrograde case, the stable domain extends to substantially larger planetocentric distances, in some cases beyond one Hill radius, although it also depends on the initial orbital phase and is naturally connected to families of periodic and quasi-periodic orbits. The results show that exomoon stability cannot be reduced to a single empirical boundary. Rather, it is a structured phase-space problem governed by the combined action of semimajor axis, eccentricity, mass ratio, initial orbital phase, orbital sense, and resonant dynamics. This perspective provides a more complete dynamical framework for assessing exomoon survivability and for interpreting observational candidates in a wide variety of exoplanetary systems.

Keywords: exomoons; orbital stability; restricted three-body problem; evection resonance; prograde and retrograde orbits; Hill radius.

Resumo

Esta tese investiga os limites de estabilidade dinâmica de exoluas no problema circular restrito de três corpos planar, com ênfase na fronteira externa de confinamento circumplanetário e nas estruturas do espaço de fases que permitem a sobrevivência em longo prazo. O estudo é baseado em simulações numéricas de alta resolução, nas quais as exoluas são modeladas como partículas-teste sem massa orbitando um planeta e perturbadas por uma estrela. A órbita planeta–estrela é mantida circular ao longo de todo o trabalho; a excentricidade variada nas simulações é a excentricidade planetocêntrica inicial do satélite, denotada por $e_{\rm sat}$. As integrações foram realizadas com o pacote `MERCURY6`, utilizando o integrador Bulirsch–Stoer em modo puro, e todos os resultados foram comparados por meio de uma normalização uniforme baseada no raio de Hill exato, definido como a distância entre o planeta e o ponto L_1.

Na primeira etapa, o semieixo maior crítico para satélites prógrados foi determinado empiricamente para cinco razões de massa planeta–estrela, $\mu = 10^{-5}$ a 10^{-1}. Os resultados mostram que a fronteira externa de estabilidade decresce aproximadamente de forma linear com a excentricidade inicial do satélite, $e_{\rm sat}$, recuperando a tendência clássica reportada na literatura, ao mesmo tempo em que estende sua validade para um intervalo muito mais amplo de razões de massa. Quando expressa em unidades do raio de Hill adotado, a dependência com μ torna-se fraca, embora ainda mensurável. A análise também demonstra que a estabilidade é sensível à fase orbital inicial, definida aqui pela anomalia média do satélite $M_{\rm sat}$: configurações iniciadas próximas de $M_{\rm sat} = 180°$ permanecem estáveis a distâncias maiores do que aquelas inicializadas em $M_{\rm sat} = 0°$. Assim, dentro do conjunto de geometrias iniciais exploradas, o caso $M_{\rm sat} = 180°$ fornece uma estimativa do envelope superior do semieixo maior estável mais externo, aqui denotado por a_E, cuja dependência com $e_{\rm sat}$ é mapeada sistematicamente.

Na segunda etapa, mapas estendidos de estabilidade foram construídos para órbitas prógradas e retrógradas, considerando diferentes geometrias iniciais e excentricidades iniciais do satélite, $e_{\rm sat}$, alcançando valores formalmente não ligados. Esses mapas revelaram que a estabilidade prógrada não pode ser descrita apenas por um corte radial externo, mas também por um limite superior de excentricidade do satélite para a sobrevivência em longo prazo. Em altas razões de massa, ilhas destacadas de longa sobrevivência emergem e mostram-se associadas à ressonância de evection, conforme confirmado pela libração do ângulo ressonante e por oscilações limitadas dos elementos orbitais. No caso retrógrado, o domínio estável se estende a distâncias planetocêntricas substancialmente maiores, em alguns casos além de um raio de Hill, embora também dependa da fase orbital inicial e esteja naturalmente conectado a famílias de órbitas periódicas e quase-periódicas.

Os resultados mostram que a estabilidade de exoluas não pode ser reduzida a uma única fronteira empírica. Em vez disso, trata-se de um problema estruturado no espaço de fases, governado pela ação combinada do semieixo maior e excentricidade do satélite, razão de massa entre o planeta e a estrela, fase orbital inicial do satélite, sentido orbital e dinâmica ressonante. Essa perspectiva fornece um arcabouço dinâmico mais completo para avaliar a sobrevivência de exoluas e para interpretar candidatos observacionais em uma ampla variedade de sistemas exoplanetários.

Palavras-chave: exoluas; estabilidade orbital; problema restrito de três corpos; ressonância de evection; órbitas prógradas e retrógradas; raio de Hill.

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List of abbreviations and acronyms

$M_\star$	Stellar mass
M_p	Planet mass
μ	Planet-to-star mass ratio, $\mu \equiv M_p/M_\star$
a_p	Planet semimajor axis around the star
$a_{\rm sat}$	Satellite semimajor axis around the planet
a_E	Critical outer semimajor axis for long-term stability
e_p	Eccentricity of the planet's orbit around the star
$e_{\rm sat}$	Eccentricity of the satellite's orbit around the planet
$R_{H,p}$	Planetary Hill radius adopted in this work, defined as the planet–L_1 distance (i.e., $R_{H,p} \equiv r_{L1}$ from the CR3BP quintic equilibrium equation)
$R_{H,p}^{\rm simp}(\mu)$	Classical Hill-radius approximation, $R_{H,p}^{\rm simp} = a_p\,(\mu/3)^{1/3}$
$T_{\rm sat}$	Orbital period of the satellite
$T_{{\rm sat},1H}$	Orbital period of a satellite located at one Hill radius from the planet
$\Omega_{\rm sat}$	Longitude of the ascending node of the satellite orbit
$\omega_{\rm sat}$	Argument of pericentre of the satellite orbit
$\varpi_{\rm sat}$	Longitude of pericentre of the satellite orbit, $\varpi_{\rm sat} = \Omega_{\rm sat} + \omega_{\rm sat}$
$\lambda_\star$	Mean longitude of the star in the planet-centred frame, $\lambda_\star = \Omega_\star + \omega_\star + M_\star$
$\phi_{\rm sat}$	Resonant angle used to diagnose evection, defined as $\phi_{\rm sat} = \varpi_{\rm sat} - \lambda_\star$
$\alpha(\mu)$, $\beta(\mu)$	Fit coefficients in $a_E/R_{H,p} = \alpha(\mu) + \beta(\mu)\,e_{\rm sat}$
$\bar{\alpha}$, $\bar{\beta}$	Mean fit coefficients used to define the representative relation in Table 2
Δa_E	Deviation in the critical semimajor axis relative to our fit (in Hill units), $\Delta a_E \equiv a_{E,\rm model} - a_{E,\text{this work}}$

Contents

1 Introduction

1.1 The Exoplanet Era and the Exomoon Frontier

The past three decades have witnessed a revolution in planetary astrophysics, transitioning from the theoretical postulation of extrasolar planets to their routine demographic characterization. As documented by the NASA Exoplanet Archive, the number of confirmed exoplanets has surpassed 5,500, revealing a striking diversity of planetary architectures ranging from hot Jupiters to compact multi-planetary systems (NASA Exoplanet Archive, 2026). This long-term growth in the confirmed exoplanet census, together with the relative contribution of the main detection techniques, is illustrated in Figure 1. This vast demographic catalog is primarily the legacy of dedicated space-based transit surveys, most notably the *Kepler* space telescope and its extended *K2* mission (Borucki et al., 2010; Howell et al., 2014), which revolutionized our understanding of planetary occurrence rates. Furthermore, the Transiting Exoplanet Survey Satellite (*TESS*) (Ricker et al., 2015) continues to expand this population, identifying optimal targets around bright, nearby stars that are particularly amenable to the high-precision photometric follow-up required for exomoon searches. However, despite this wealth of planetary discoveries, the detection of natural satellites orbiting these bodies, exomoons, remains one of the most elusive frontiers in modern astronomy. In our own Solar System, moons vastly outnumber planets and play crucial roles in the dynamical and geological evolution of their host planets. Statistically and formation-wise, it is a natural corollary that exomoons should be ubiquitous across the galaxy, potentially hosting the necessary conditions for habitability in systems where the host planet resides in the habitable zone but is a gas giant (Heller and Williams, 2014).

The primary observational pathway to identifying exomoons relies on high-precision transit photometry. While a direct transit of a moon might produce a subtle dip in the stellar light curve (Sartoretti and Schneider, 1999; Szabó et al., 2006), the most robust detection framework combines Transit Timing Variations (TTV) and Transit Duration Variations (TDV) (Simon et al., 2007; Kipping, 2009). An exomoon perturbs its host planet, causing the planet to wobble around their common barycenter. This barycentric motion induces periodic deviations in the exact time the planet crosses the stellar disk (TTV) and alters the apparent velocity of the planet during the transit, thereby changing the transit duration (TDV). The unambiguous detection of an exomoon typically requires identifying both TTV and TDV signals with a 90° phase shift, which demands complex, computationally expensive photodynamical modeling of the entire system's light curve (Kipping et al., 2012).

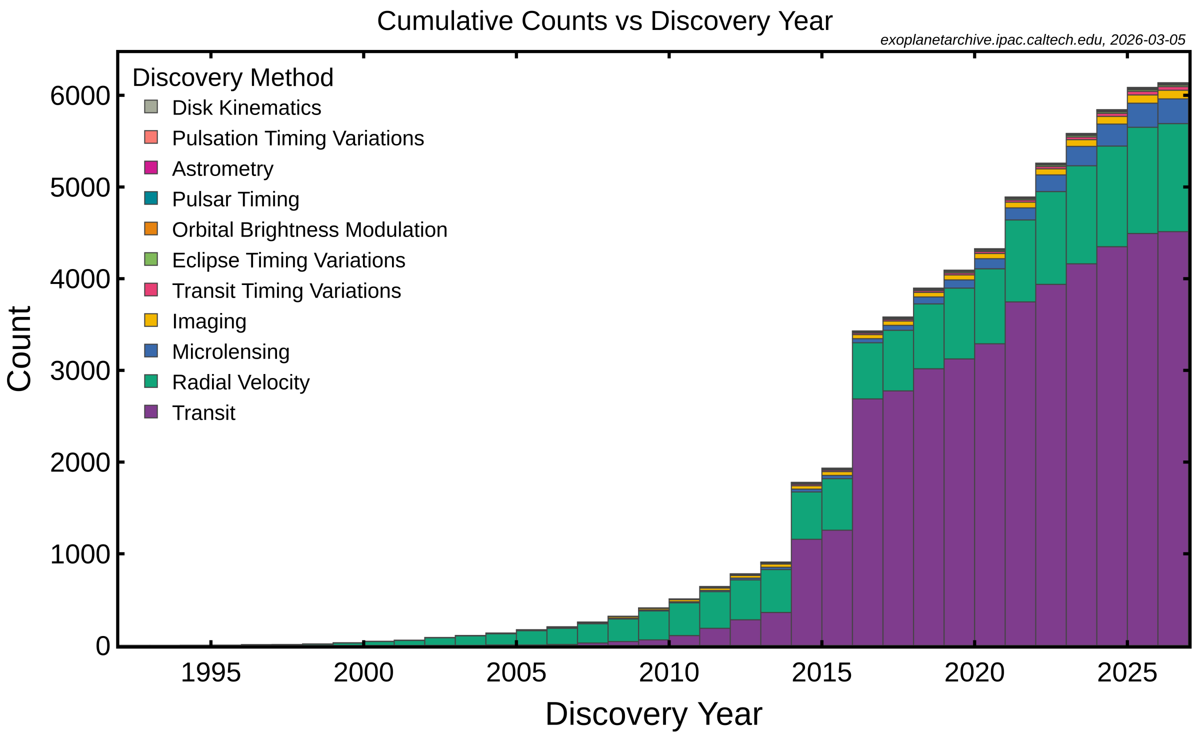

Figure 1 – Cumulative distribution of confirmed exoplanets by discovery year from the NASA Exoplanet Archive. The stacked histogram is color-coded by detection method. The figure illustrates the rapid increase in the number of confirmed exoplanets since the mid-1990s, particularly driven by transit and radial-velocity discoveries (NASA Exoplanet Archive, 2026).

This fragility in the observational data has led to intense scientific debate regarding the current exomoon candidates. To date, the most prominent candidates discussed in the literature are Kepler-1625b-i (Teachey and Kipping, 2018) and Kepler-1708b-i (Kipping et al., 2022), both postulated as unusually massive, Neptune-sized moons orbiting Jupiter-like planets. However, the confirmation of these candidates remains highly controversial. Independent re-analyses of the same Kepler and Hubble Space Telescope datasets by different research groups have frequently resulted in conflicting conclusions. Some authors argue that the observed light curve anomalies can be equally well explained by data detrending artifacts or unocculted starspots (Kreidberg et al., 2019; Kipping et al., 2024). Alternative physical explanations have also been proposed, such as extended circumplanetary ring systems or low-density planetary companions (Heller, 2018). Recently, the reality of both signals was strongly challenged by Heller and Hippke (2024), who argued that the detections are likely spurious, which immediately prompted a detailed rebuttal from Kipping et al. (2025) showing that the original evidence can still be recovered.

Beyond the Kepler candidates, the ambiguity of exomoon signals is further illustrated by planets like HIP-41378 f, a temperate Jovian planet with an unusually low apparent density (Vanderburg et al., 2016; Berardo et al., 2019). Harada et al. (2023) examined the detectability and stability of hypothetical exomoons around this planet, showing that large moons could remain dynamically stable and might affect transmission spectra, thereby complicating the interpretation of planetary atmospheric properties. As these observational ambiguities compound, it becomes clear that relying solely on photometric data is insufficient to validate a candidate. This highlights the absolute necessity of establishing robust dynamical stability boundaries that go beyond simple, single-averaged estimates. To confidently evaluate candidate systems, theoretical stability models must rigorously incorporate the complex dependencies on orbital eccentricity, extreme mass ratios, orbital phase (mean anomaly), and resonant effects.

1.2 The Zero-Order Problem of Dynamical Stability

To ascertain whether an exomoon candidate is physically viable, one must first address the zero-order problem of dynamical stability: determining the maximum orbital distance at which a satellite can remain gravitationally bound to its host planet before stellar perturbations strip it away. While the Hill radius (R_H) defines the absolute theoretical sphere of planetary gravitational dominance, the actual region of long-term orbital survival is substantially smaller. The foundation for quantifying this boundary was laid by Holman and Wiegert (1999), who investigated the stability of test particles in S-type orbits within binary star systems. Although their work focused on circumstellar planets in binaries, the hierarchical nature of the restricted three-body problem allows their empirical limit, expressed as fractions of the binary separation and mass ratio, to be broadly analogous to the star-planet-moon architecture.

Building directly upon the planetary satellite context, Domingos et al. (2006) conducted extensive numerical integrations to define the critical semimajor axis (a_E) for both prograde and retrograde exomoons. They showed that stability is not solely a function of the Hill sphere, but depends steeply on the eccentricities of both the planet (e_p) and the satellite (e_{sat}). For circular, prograde orbits, Domingos et al. found that the stability region extends to approximately $0.49R_H$, but this boundary contracts significantly as orbital eccentricities increase. Retrograde orbits were found to be stable at considerably greater distances (up to $\sim 0.93R_H$ for circular orbits), largely due to the stabilizing influence of quasi-periodic orbital families. These empirical fits provided the standard theoretical filter for early exomoon formation and survival models.

However, subsequent high-resolution numerical studies revealed that the stability boundary is highly sensitive to the initial orbital phase, a factor often averaged out in earlier approximations. Rosario-Franco et al. (2020) revisited the limits proposed by Domingos et al. by varying the initial mean anomaly of the satellites and extending the integration timescales. They showed that when stability is approached probabilistically across different orbital phases, the critical semimajor axis for a circular prograde exomoon shrinks to approximately $0.40R_H$, roughly 20% lower than previously estimated. This tightening of the stability envelope implies that many massive satellites once deemed stable in older models might actually be vulnerable to ejection or collision on secular timescales.

1.3 The Evection Resonance and Stable Orbital Families

Beyond the classical empirical boundaries, the dynamical architecture of circumplanetary space is intricately shaped by resonant phenomena, most notably the evection resonance. Originally identified in the context of the early lunar orbit, the evection resonance occurs when the secular precession rate of the satellite's pericenter ($\dot{\varpi}_{sat}$) becomes commensurate with the mean motion of the external perturber, the host star (n_*) (Touma and Wisdom, 1998; Ćuk and Burns, 2004). The dynamical consequence of this commensurability is the libration of the resonant angle $\phi = \varpi_{sat} - \lambda_*$ (where λ_* is the star's mean longitude). As showed by the seminal works of Yokoyama et al. (2008) and Frouard et al. (2010a), the evection resonance plays a dual and highly complex role in satellite dynamics: while it can drive secular eccentricity excitation leading to collisions or ejections, it can also act as a powerful protective mechanism. Under specific initial phase configurations, resonant trapping shields outer satellites from chaotic diffusion, preventing rapid eccentricity growth and allowing them to survive well beyond the conventional continuous stability limits.

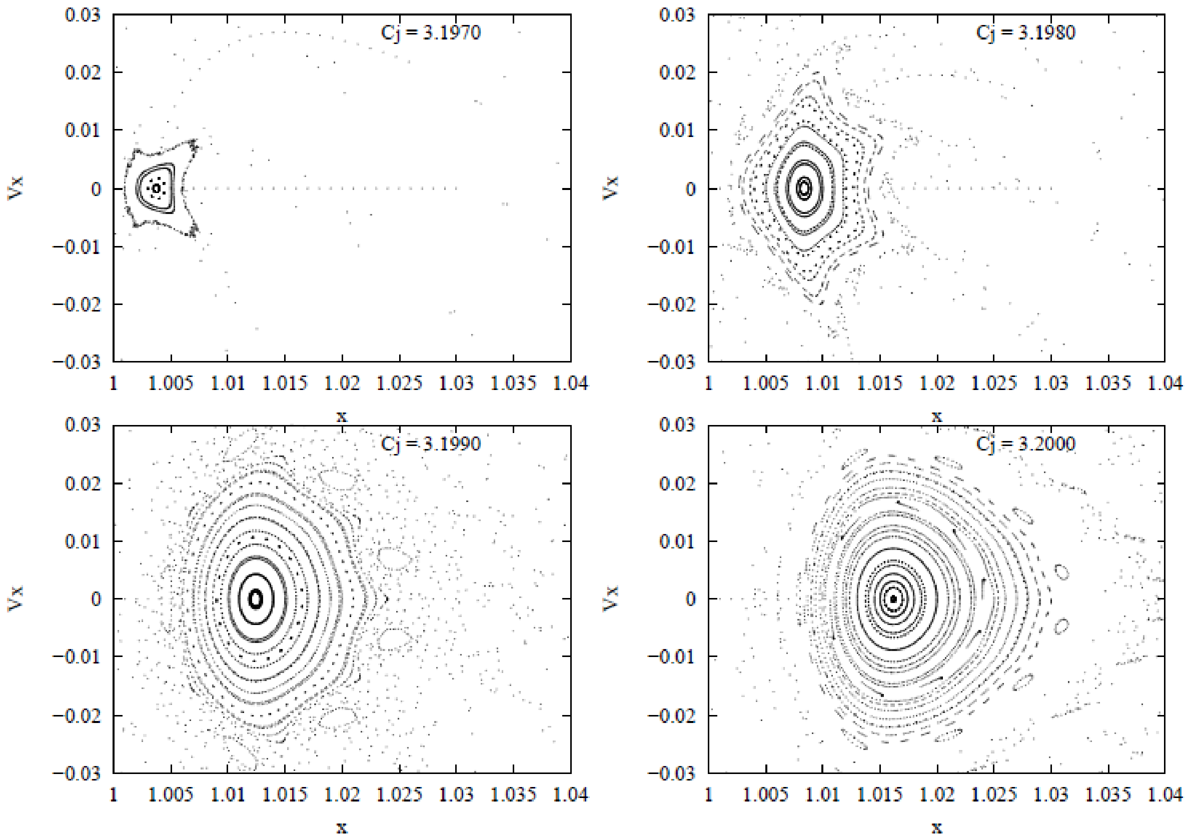

Figure 2 – Poincaré surface of section mapping the phase space topology associated with the h_1 family of direct periodic orbits. The exact periodic orbit is located at the center of the stable island, surrounded by invariant closed curves that represent quasi-periodic orbits. These structures effectively bound the eccentricity and semimajor axis variations, shielding the satellite from chaotic diffusion and subsequent escape. Figure from Winter and Vieira Neto (2002).

The underlying topological structure that enables such extended survival is best understood through the mapping of the phase space using Poincaré surfaces of section. Foundational investigations by Winter and Vieira Neto (2002) into the stability of distant direct orbits revealed that stability at these extreme boundaries is anchored by specific families of simple periodic orbits. Specifically, they identified that the outermost stable regions are governed by the h_1 and h_2 families of direct periodic orbits (see Figures 2 and 3), originally classified by Broucke (1962). Satellites that reside in quasi-periodic orbits, those oscillating closely around these exact periodic trajectories, are geometrically protected from the large-scale chaotic variations induced by the third body.

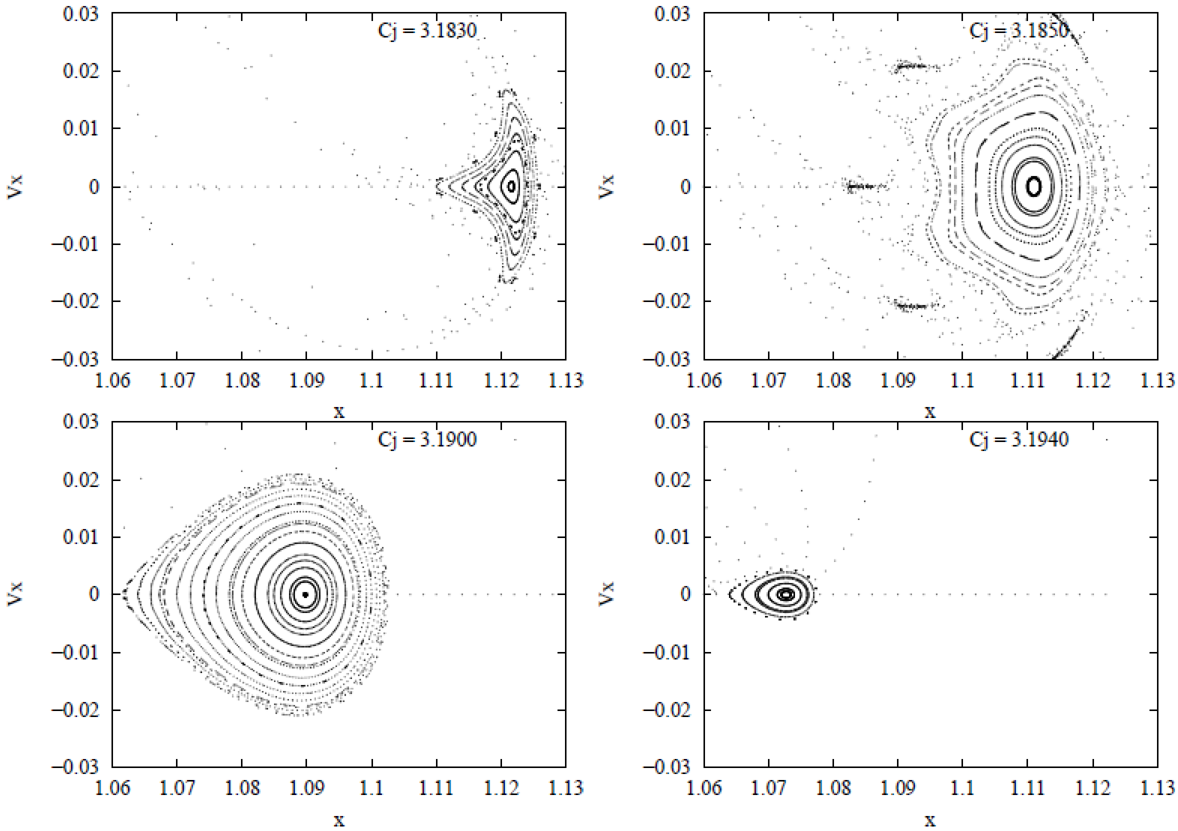

Figure 3 – Phase space structure illustrating the stability region governed by the h_2 periodic orbit family. The presence of these stable islands at large planetocentric distances shows that long-term survival is possible well beyond the traditional continuous stability boundary, provided the satellite is captured within these resonant quasi-periodic trajectories. Figure from Winter and Vieira Neto (2002).

The geometric configuration of these periodic orbits dictates the precise location and width of the stable islands, showing that stability is not merely a strictly monotonic function of radial distance, but is deeply rooted in the phase space topology of the restricted three-body problem. In the exoplanetary context, the protective nature of these resonant islands has profound implications for the detectability and validation of massive exomoons. For systems with large planet-to-star mass ratios, the continuous stability boundary truncates closer to the planet, but the evection resonance generates distinct, detached islands of stability corresponding to these h-family orbits. Exomoons captured or formed within these resonant islands exhibit bounded, quasi-periodic oscillations in both semimajor axis and eccentricity, ensuring long-term survivability. Consequently, when evaluating the viability of distant exomoon candidates, theoretical models must account for these resonant safe harbors, which provide a dynamical pathway for the existence of wide-orbit satellites that would otherwise be prematurely dismissed as unstable.

1.4 Tidal Effects and the Inner Stability Limit

While the zero-order dynamical boundaries and resonant phenomena define the outermost radial limits for exomoon survival, the long-term existence of a satellite is equally dictated by secular tidal interactions. Gravitational tides define the inner stability limit and impose a strict temporal lifespan on the satellite. In a star-planet-moon system, the orbital evolution is driven by a complex interplay of two primary tidal forces: the tides raised by the star on the planet, and the tides raised by the moon on the planet (Barnes and O'Brien, 2002). For close-in exoplanets, stellar tides rapidly dissipate the planet's rotational energy, slowing its spin and pushing the planetary corotation (synchronous) radius outward. If an exomoon orbits interior to this expanding synchronous radius, the tidal bulge it raises on the planet lags behind the moon. This configuration transfers angular momentum from the moon's orbit to the planet's rotation, forcing the satellite to undergo an inevitable inward orbital decay (Alvarado-Montes et al., 2017; Piro, 2018).

The ultimate fate of such inward-migrating exomoons is catastrophic. As the semimajor axis shrinks, the satellite eventually crosses the planetary Roche limit, where tidal shear forces exceed its self-gravity, leading to tidal disruption. This process is theorized to leave behind dense circumplanetary ring systems, offering a potential physical explanation for anomalous transit signatures observed in some candidates (Alvarado-Montes, 2022). In their foundational work, Barnes and O'Brien (2002) showed that for many close-in giant planets, this tidal lifespan is significantly shorter than the main-sequence lifetime of the host star. Consequently, even if a moon forms or is captured within a dynamically stable exterior orbit, tidal dissipation acts as a strict temporal filter. Therefore, when evaluating the viability of exomoon candidates, it is imperative to consider not only their current orbital architecture but also the age of the system, as tides efficiently clear the inner circumplanetary space over secular timescales.

1.5 Research Motivation

The motivation for this research is connected to the observational and theoretical challenges involved in the search for exomoons. As illustrated by the discussions surrounding candidates such as Kepler-1625b-i and Kepler-1708b-i, current photometric detections are often based on signals close to the limits of observational precision. In this context, features such as shallow transit-like dips or timing variations may admit more than one interpretation and therefore require additional constraints. Dynamical stability provides one such constraint, allowing proposed satellite configurations to be tested against the requirement that they remain bound to their host planet over long timescales. Thus, before the physical properties or possible habitability of an exomoon candidate are further assessed, it is necessary to examine whether its proposed orbit is dynamically viable.

This thesis is based on two original research studies conducted during the course of the doctoral work, which together constitute its central scientific foundation. The first (Barbosa et al., 2026), examines the dependence of the critical outer stability limit of exomoons on orbital eccentricity and on the planet-to-star mass ratio through a broad numerical investigation. The second (Barbosa and Winter, 2026), expands this framework by incorporating the role of the initial mean anomaly and by comparing prograde and retrograde orbital configurations. Together, these studies organize the main results presented in this thesis and support its broader analysis of exomoon survivability near the external boundary of circumplanetary stability.

1.6 Thesis Objectives

The general objective of this thesis will be to comprehensively investigate the dynamical stability, phase-space topology, and orbital evolution of exomoons orbiting extrasolar planets, thereby providing a robust theoretical framework for evaluating the survivability of current and future observational candidates.

To accomplish this overarching goal, the thesis will pursue the following specific objectives:

- **Map the Continuous Stability Boundaries:** It will determine the critical semimajor axis for exomoon survival across a broad and largely unexplored range of planet-to-star mass ratios (from $\mu = 10^{-5}$ up to extreme cases of $\mu = 10^{-1}$), thereby refining classical empirical stability limits.
- **Characterize Resonant Islands of Stability:** It will analyze the phase-space topology associated with the evection resonance, with particular emphasis on mapping detached islands of stability related to the h_1 and h_2 families of periodic orbits at high eccentricities and large mass ratios.
- **Evaluate Orbital Phase Sensitivity:** It will quantify how the stability limits are modulated by the satellite's initial mean anomaly, thereby clarifying the probabilistic nature of long-term survival near the chaotic boundary.
- **Investigate the Possibility of Exomoon Capture:** It will examine whether bodies initialized on formally unbound trajectories, with eccentricities greater than or equal to 1, may evolve into long-lived bound configurations within the system.

1.7 Outline of the Thesis

To systematically address these objectives, this document is structured as follows:

Chapter 2 provides the theoretical foundation for the research, reviewing the fundamentals of orbital dynamics, including the restricted three-body problem, the Hill and Roche limits, and the mechanisms of mean-motion and secular resonances that dictate satellite dynamics.

Chapter 3 details the methodology and numerical setup, describing the N-body symplectic integrators, the integration timescales, and the extensive parameter space explored in our numerical simulations.

Chapter 4 presents the first core results regarding the Islands of Stability and Resonances, mapping the phase space to reveal how the evection resonance creates isolated protective zones for wide-orbit satellites.

Chapter 5 extends the study of the critical stability limit by presenting a phase-resolved numerical survey of exomoon survival in the planar circular restricted problem. It details the high-resolution mapping of continuous and detached stability regions, examines the effects of the initial mean anomaly and orbital direction through prograde and retrograde configurations, and discusses the absence of long-lived capture from formally unbound initial conditions.

Finally, **Chapter 6** summarizes the general conclusions of this work, highlighting the main contributions to the field of exoplanetology and outlining future perspectives for the intersection between dynamical theory and observational searches.

During the preparation of this work, the author used *ChatGPT* to improve readability. After using this tool, the author reviewed and edited the content as needed and takes full responsibility for the content of the publication.

6 Conclusions

This thesis investigated the long-term stability of exomoons in the planar circular restricted three-body problem, with emphasis on the outer circumplanetary stability boundary and on the phase-space structures that support survival near that limit. The analysis was developed in two stages. First, a baseline numerical survey was used to determine the empirical outer stability boundary for prograde satellites over the interval $\mu = 10^{-5}$ to 10^{-1}. Second, the analysis was extended to examine the full morphology of the stability maps, including the effects of initial mean anomaly, the contrast between prograde and retrograde motion, and the role of resonant and quasi-periodic structures.

A central methodological aspect of this work is the use of a uniform Hill-scale normalization based on the exact planet–L_1 distance in the circular restricted three-body problem. This choice avoids the loss of accuracy introduced by the usual asymptotic Hill-radius approximation at high mass ratio and allows direct comparison of stability limits across a broad interval in μ. All integrations were carried out over 10^4 orbital periods of a reference satellite at one Hill radius, providing a homogeneous basis for the comparison of long-term outcomes.

For prograde motion, the main result of the baseline survey is that the outer stability boundary decreases approximately linearly with increasing satellite eccentricity. This behavior is recovered for all five mass ratios explored. When expressed in units of the adopted Hill radius, the dependence on μ is weak but measurable, allowing both individual fits for each mass ratio and a compact representative relation for the prograde outer boundary. Comparison with previous prescriptions shows that the relation of Domingos et al. (2006) remains accurate over much of the interval examined here once the same Hill normalization is adopted, although it becomes mildly optimistic at high μ. The prescription of Rosario-Franco et al. (2020), in contrast, is systematically more conservative and underestimates the extent of the prograde stable region when extrapolated beyond the regime for which it was calibrated.

The extended maps show that this empirical boundary is not determined by semimajor axis and eccentricity alone. In the prograde problem, the initial orbital phase relative to the star modifies both the location and the morphology of the stable domain. In particular, the $M_{\mathrm{sat}} = 180°$ configuration systematically allows survival at larger semimajor axes than the $M_{\mathrm{sat}} = 0°$ configuration. The prograde domain is therefore better described by a phase-dependent stability envelope than by a single universal cutoff. The enlarged eccentricity interval explored in the extended survey also shows that no long-lived population of planet-centered survivors emerges from the initially unbound regime. In this sense, the present calculations do not support persistent capture as a relevant source of long-term exomoon survival in the planar circular problem.

A further result of the extended prograde analysis is that the stable domain is not fully characterized by a single radial boundary. In addition to the outer semimajor-axis limit, the maps reveal an upper eccentricity boundary for long-term survival. Together, these two limits provide a more complete empirical description of the prograde stable region than the critical semimajor axis alone. They also show that the appropriate empirical fit depends on the adopted initial phase configuration.

The extended maps also reveal detached islands of long-lived prograde survival, especially at high mass ratio. Targeted integrations of representative particles show bounded oscillations of semimajor axis and eccentricity together with libration of the evection angle, indicating that these structures are dynamically associated with evection resonance. These islands are therefore not numerical artifacts of the grid, but resonantly organized regions of phase space. In the present problem, evection is not only a source of secular excitation; under appropriate conditions, it also provides phase protection and allows survival beyond the main continuous prograde domain.

For retrograde motion, the stability region is much more extended than in the prograde case and, in some configurations, reaches beyond one Hill radius. The retrograde maps also show a richer morphology, including distant survivor branches that are absent or much less developed in the prograde problem. At the same time, the retrograde domain is not phase invariant: the $M_{\mathrm{sat}} = 180°$ configuration is systematically more restrictive than the corresponding $M_{\mathrm{sat}} = 0°$ case. The retrograde results therefore show that orbital sense, initial phase, and mass ratio all contribute to the structure of the long-lived circumplanetary domain.

Comparison with the framework proposed by Winter and Vieira Neto (2001) suggests that the distant retrograde survivor branches identified here are qualitatively consistent with organization by retrograde periodic-orbit families and the quasi-periodic trajectories around them. This interpretation is necessarily qualitative, since the present problem spans a different mass-ratio interval and a different physical context, but it provides a plausible dynamical explanation for the geometry and persistence of the outer retrograde structures. More generally, the results obtained in this thesis show that long-term circumplanetary survival is controlled not only by empirical radial limits, but also by the underlying phase-space architecture of the restricted three-body problem.

Taken together, the results show that the outer circumplanetary stability limit cannot be reduced to a single universal semimajor axis. Its location and morphology depend on eccentricity, mass ratio, orbital sense, initial phase, and resonant dynamics. For exomoon studies, this implies that simple Hill-based criteria should be used with caution: they capture the leading scale of the problem, but they do not describe the full structure of the long-term stable domain.

The present conclusions are restricted to the planar circular restricted three-body problem with massless satellites and without tides. This framework is appropriate for isolating the fundamental dynamical structure of the problem, but real systems may differ substantially once planetary eccentricity, mutual inclination, finite satellite mass, and tidal evolution are included. Extending the present analysis to the elliptic and spatial restricted problems, and coupling it with tidal evolution, are natural next steps. These extensions are necessary to determine how much of the phase-space structure identified here survives in more realistic exomoon configurations.

In summary, this thesis refines the empirical determination of the prograde outer stability boundary, shows that the boundary is phase dependent, identifies detached prograde islands associated with evection resonance, and demonstrates that retrograde stability is both more extended and more structurally complex than its prograde counterpart. The main conclusion is that exomoon stability is a structured phase-space problem. Any realistic assessment of long-term survival must therefore account not only for radial distance from the planet, but also for the dynamical mechanisms that organize motion near the circumplanetary stability limit.

References

J. A. Alvarado-Montes. Tidally induced migration of tess gas giants orbiting m dwarfs. *Monthly Notices of the Royal Astronomical Society*, 517(2):2831–2841, 2022. doi: 10.1093/mnras/stac2741.

J. A. Alvarado-Montes, J. I. Zuluaga, and M. Sucerquia. The effect of close-in giant planets' evolution on tidal-induced migration of exomoons. *Monthly Notices of the Royal Astronomical Society*, 471(3):3019–3027, 2017. doi: 10.1093/mnras/stx1745.

R. Araujo, O. Winter, A. Prado, and R. Vieira Martins. Sphere of influence and gravitational capture radius: a dynamical approach. *Monthly Notices of the Royal Astronomical Society*, 391(2):675–684, 2008.

G. O. Barbosa and O. C. Winter. On the stability limits for moons orbiting planets: A numerical investigation ii. Unpublished manuscript, 2026.

G. O. Barbosa, T. Santana, and O. C. Winter. On the stability limits for moons orbiting planets: A numerical investigation. *Icarus*, 448:116924, 2026. doi: 10.1016/j.icarus.2025.116924.

J. W. Barnes and D. P. O'Brien. Stability of satellites around close-in extrasolar giant planets. *The Astrophysical Journal*, 575(2):1087–1093, 2002. doi: 10.1086/341477.

D. Berardo, I. J. M. Crossfield, M. Werner, E. Petigura, J. L. Christiansen, J. E. Schlieder, D. W. Latham, E. M. Bryant, J. C. Becker, H. A. Knutson, A. W. Mayo, C. D. Dressing, J. H. Livingston, and A. Vanderburg. Revisiting the hip 41378 system with k2 and spitzer. *The Astronomical Journal*, 157(5):185, 2019. doi: 10.3847/1538-3881/ab100c.

W. J. Borucki, D. Koch, G. Basri, N. Batalha, T. M. Brown, D. Caldwell, J. Christensen-Dalsgaard, W. D. Cochran, E. DeVore, E. W. Dunham, T. N. Gautier, J. C. Geary, R. Gilliland, A. Gould, S. B. Howell, J. M. Jenkins, D. W. Latham, J. J. Lissauer, G. W. Marcy, D. Monet, D. Sasselov, A. Boss, D. Charbonneau, L. Doyle, A. K. Dupree, E. B. Ford, J. Fortney, M. J. Holman, S. Seager, J. H. Steffen, J. Tarter, W. F. Welsh, J. Rowe, H. Anderson, L. Buchhave, D. Ciardi, L. Walkowicz, W. Sherry, E. Horch, H. Isaacson, M. E. Everett, D. Fischer, G. Torres, J. A. Johnson, M. Endl, P. MacQueen, S. T. Bryson, J. Dotson, M. Haas, J. Kolodziejczak, and J. Van Cleve. Kepler planet-detection mission: Introduction and first results. *Science*, 327(5968):977–980, 2010. doi: 10.1126/science.1185402.

R. Broucke. *Recherches d'orbites périodiques dans le problème restreint plan (Système Terre-Lune)*. PhD thesis, Université de Louvain, Louvain, Belgium, 1962.

R. Broucke. Periodic orbits in the restricted three-body problem with earth-moon masses. Technical Report 32-1168, Jet Propulsion Laboratory, Pasadena, CA, 1968.

J. E. Chambers. A hybrid symplectic integrator that permits close encounters between massive bodies. *Monthly Notices of the Royal Astronomical Society*, 304(4):793–799, 1999.

J. E. Chambers. *MERCURY: A Software Package for Orbital Dynamics*, 2001. User manual, last modified 1 March 2001.

M. Ćuk and J. A. Burns. On the secular behavior of irregular satellites. *The Astronomical Journal*, 128(5):2518–2541, 2004. doi: 10.1086/424937.

R. C. Domingos, O. C. Winter, and T. Yokoyama. Stable satellites around extrasolar giant planets. *Monthly Notices of the Royal Astronomical Society*, 373(3):1227–1234, 2006. doi: 10.1111/j.1365-2966.2006.11104.x.

J. Eberle, M. Cuntz, and Z. Musielak. The instability transition for the restricted 3-body problem-i. theoretical approach. *Astronomy & Astrophysics*, 489(3):1329–1335, 2008.

E. Everhart. An efficient integrator that uses gauss-radau spacings. In *Dynamics of Comets: Their Origin and Evolution*, volume 115 of *Astrophysics and Space Science Library*, pages 185–202. D. Reidel Publishing Company, 1985. doi: 10.1007/978-94-009-5400-7_17.

J. Frouard, M. Fouchard, and A. Vienne. About the dynamics of the evection resonance. *Astronomy & Astrophysics*, 515:A54, 2010a.

J. Frouard, M. Fouchard, A. Vienne, and N. Petrova. Influence of the evection resonance on the dynamics of martian satellites. *Astronomy & Astrophysics*, 515:A54, 2010b. doi: 10.1051/0004-6361/200913048.

R. Gomes, H. F. Levison, K. Tsiganis, and A. Morbidelli. Origin of the cataclysmic late heavy bombardment period of the terrestrial planets. *Nature*, 435(7041):466–469, 2005.

N. N. Gorkavyi. A unified model for the formation of the satellite systems of jupiter, saturn, and uranus. *Astronomy Letters*, 19(6):448, 1993.

D. P. Hamilton and J. A. Burns. Orbital stability zones about asteroids. *Icarus*, 92(1): 118–131, 1991. doi: 10.1016/0019-1035(91)90039-V.

D. P. Hamilton and A. V. Krivov. Dynamics of distant moons of asteroids. *Icarus*, 128(1): 241–249, 1997. doi: 10.1006/icar.1997.5738.

C. K. Harada et al. Stability and detectability of exomoons orbiting hip 41378 f, a temperate jovian planet with an anomalously low apparent density. *The Astrophysical Journal*, 949(1):22, 2023. doi: 10.3847/1538-4357/ad011c.

R. Heller. The nature of the giant exomoon candidate kepler-1625 b-i. *Astronomy & Astrophysics*, 610:A39, 2018. doi: 10.1051/0004-6361/201731760.

R. Heller and M. Hippke. Large exomoons unlikely around kepler-1625 b and kepler-1708 b. *Nature Astronomy*, 8:193–206, 2024. doi: 10.1038/s41550-023-02148-w.

R. Heller and D. Williams. Formation environments, lithospheric evolution, and habitabilities of kepler-1625b-like exomoons. *Astrobiology*, 14(9):798–835, 2014. doi: 10.1089/ast.2014.1147.

M. Hénon. Numerical exploration of the restricted problem, v. *Astronomy and Astrophysics, vol. 1, p. 223-238 (1969).*, 1:223–238, 1969.

M. Hénon. Numerical exploration of the restricted problem. vi. hill's case: Non-periodic orbits. *Astronomy & Astrophysics*, 9:24–36, 1970.

M. J. Holman and P. A. Wiegert. Long-term stability of planets in binary systems. *The Astronomical Journal*, 117(1):621–628, 1999. doi: 10.1086/300695.

S. B. Howell, C. Sobeck, M. Haas, M. Still, T. Barclay, F. Mullally, J. Troeltzsch, S. Aigrain, S. T. Bryson, D. Caldwell, W. J. Chaplin, W. D. Cochran, D. Huber, G. W. Marcy, A. Miglio, J. R. Najita, M. Smith, J. D. Twicken, and J. J. Fortney. The k2 mission: Characterization and early results. *Publications of the Astronomical Society of the Pacific*, 126(938):398–408, 2014. doi: 10.1086/676406.

D. Kipping, S. Bryson, C. Burke, J. Christiansen, K. Hardegree-Ullman, B. Quarles, B. Hansen, J. Szulágyi, and A. Teachey. An exomoon survey of 70 cool giant exoplanets and the new candidate kepler-1708 b-i. *Nature Astronomy*, 6(3):367–380, 2022. doi: 10.1038/s41550-021-01539-1.

D. Kipping, A. Teachey, D. A. Yahalomi, B. Cassese, B. Quarles, S. Bryson, B. Hansen, J. Szulágyi, C. Burke, and K. Hardegree-Ullman. A reply to: Large exomoons unlikely around kepler-1625 b and kepler-1708 b, 2024.

D. Kipping, A. Teachey, D. A. Yahalomi, B. Cassese, B. Quarles, S. Bryson, B. Hansen, J. Szulagyi, C. Burke, and K. Hardegree-Ullman. Concerning the possible exomoons around kepler-1625 b and kepler-1708 b. *Nature Astronomy*, 9(6):795–798, 2025. doi: 10.1038/s41550-025-02547-1.

D. M. Kipping. Transit timing effects due to an exomoon. *Monthly Notices of the Royal Astronomical Society*, 392(1):181–189, 2009. doi: 10.1111/j.1365-2966.2008.13999.x.

D. M. Kipping, G. Á. Bakos, L. Buchhave, D. Nesvorný, and A. Schmitt. The hunt for exomoons with kepler (hek). i. description of a new observational project. *The Astrophysical Journal*, 750(2):115, 2012. doi: 10.1088/0004-637X/750/2/115.

W. S. Koon, M. W. Lo, J. E. Marsden, and S. D. Ross. Dynamical systems, the three-body problem and space mission design. In *Equadiff 99: (In 2 Volumes)*, pages 1167–1181. World Scientific, 2000.

L. Kreidberg, R. Luger, and M. Bedell. No evidence for lunar transit in new analysis of hubble space telescope observations of the kepler-1625 system. *The Astrophysical Journal Letters*, 877(2):L15, 2019. doi: 10.3847/2041-8213/ab20c8.

A. Morbidelli. *Modern Celestial Mechanics: Aspects of Solar System Dynamics*. Taylor & Francis, London, 2002.

A. Morbidelli, H. F. Levison, K. Tsiganis, and R. Gomes. Chaotic capture of jupiter's trojan asteroids in the early solar system. *Nature*, 435(7041):462–465, 2005.

F. R. Moulton. On the stability of direct and retrograde satellite orbits. *Monthly Notices of the Royal Astronomical Society*, 75(2):40–57, 1914.

C. D. Murray and S. F. Dermott. *Solar system dynamics*. Cambridge university press, 1999.

NASA Exoplanet Archive. Nasa exoplanet archive. https://exoplanetarchive.ipac.caltech.edu/, 2026. Accessed on 2026-03-05.

P. Pires and O. C. Winter. Location and stability of distant retrograde orbits around the moon. *Monthly Notices of the Royal Astronomical Society*, 494(2):2727–2735, 2020. doi: 10.1093/mnras/staa887.

A. L. Piro. Exoplanets torqued by the combined tides of a moon and parent star. *The Astronomical Journal*, 156(2):54, 2018.

H. Poincaré. *Les méthodes nouvelles de la mécanique céleste*, volume 1. Gauthier-Villars, Paris, 1892.

H. Poincaré. *Les méthodes nouvelles de la mécanique céleste*, volume 2. Gauthier-Villars, Paris, 1893.

H. Poincaré. *Les méthodes nouvelles de la mécanique céleste*, volume 3. Gauthier-Villars, Paris, 1899.

B. Quarles, J. Eberle, Z. Musielak, and M. Cuntz. The instability transition for the restricted 3-body problem-iii. the lyapunov exponent criterion. *Astronomy & Astrophysics*, 533:A2, 2011.

B. Quarles, G. Li, V. Kostov, and N. Haghighipour. Orbital stability of circumstellar planets in binary systems. *The Astronomical Journal*, 159(3):80, 2020a.

B. Quarles, G. Li, and M. Rosario-Franco. Application of orbital stability and tidal migration constraints for exomoon candidates. *The Astrophysical Journal Letters*, 902 (1):L20, 2020b.

G. Rabl and R. Dvorak. Satellite-type planetary orbits in double stars-a numerical approach. *Astronomy and Astrophysics (ISSN 0004-6361), vol. 191, no. 2, Feb. 1988, p. 385-391.*, 191:385–391, 1988.

G. R. Ricker, J. N. Winn, R. Vanderspek, D. W. Latham, G. Á. Bakos, J. L. Bean, Z. K. Berta-Thompson, T. M. Brown, L. Buchhave, N. R. Butler, R. P. Butler, W. J. Chaplin, D. Charbonneau, J. Christensen-Dalsgaard, M. Clampin, D. Deming, J. Doty, N. De Lee, C. Dressing, E. W. Dunham, M. Endl, F. Fressin, J. Ge, T. Henning, M. J. Holman, A. W. Howard, S. Ida, J. M. Jenkins, G. Jernigan, J. A. Johnson, L. Kaltenegger, N. Kawai, S. Kawaler, H. Kjeldsen, G. Laughlin, A. Levine, D. Lin, J. J. Lissauer, P. MacQueen, G. Marcy, P. R. McCullough, T. D. Morton, N. Narita, M. Paegert, E. Palle, F. Pepe, J. Pepper, A. Quirrenbach, S. A. Rinehart, D. Sasselov, B. Sato, S. Seager, A. Sozzetti, K. G. Stassun, P. Sullivan, A. Szentgyorgyi, G. Torres, S. Udry, and J. Villasenor. Transiting exoplanet survey satellite (tess). *Journal of Astronomical Telescopes, Instruments, and Systems*, 1(1):014003, 2015. doi: 10.1117/1.JATIS.1.1.014003.

M. Rosario-Franco, B. Quarles, Z. E. Musielak, and M. Cuntz. Orbital stability of exomoons and submoons with applications to kepler 1625b-i. *The Astronomical Journal*, 159(6): 260, 2020. doi: 10.3847/1538-3881/ab89a7.

R. Rufu and R. M. Canup. Tidal evolution of the evection resonance/quasi-resonance and the angular momentum of the earth-moon system. *Journal of Geophysical Research: Planets*, 125(8):e2019JE006312, 2020.

P. Sartoretti and J. Schneider. On the detection of satellites of extrasolar planets with the method of transits. *Astronomy and Astrophysics Supplement Series*, 134(3):553–560, 1999. doi: 10.1051/aas:1999148.

D. J. Scheeres. *Orbital motion in strongly perturbed environments: applications to asteroid, comet and planetary satellite orbiters*. Springer, 2016.

A. Simon, K. Szatmáry, and G. M. Szabó. Determination of the size, mass, and density of "exomoons" from photometric transit timing variations. *Astronomy & Astrophysics*, 470 (2):727–731, 2007. doi: 10.1051/0004-6361:20066560.

G. M. Szabó, K. Szatmáry, Z. Divéki, and A. Simon. Possibility of a photometric detection of "exomoons". *Astronomy & Astrophysics*, 450(1):395–398, 2006. doi: 10.1051/0004-6361:20054555.

V. Szebehely. *Theory of orbit: The restricted problem of three Bodies*. Elsevier, 2012.

A. Teachey and D. M. Kipping. Evidence for a large exomoon orbiting kepler-1625b. *Science Advances*, 4(10):eaav1784, 2018. doi: 10.1126/sciadv.aav1784.

J. Touma and J. Wisdom. Resonances in the early evolution of the earth-moon system. *The Astronomical Journal*, 115(4):1653–1663, 1998. doi: 10.1086/300312.

T. Vaillant and A. C. Correia. Eviction-like resonances for satellite orbits-application to phobos, the main satellite of mars. *Astronomy & Astrophysics*, 657:A103, 2022.

M. J. Valtonen and H. Karttunen. *The three-body problem*. Cambridge University Press, 2006.

A. Vanderburg, J. C. Becker, M. H. Kristiansen, S. A. Rappaport, H. M. Schwengeler, A. W. Mayo, M. de Val-Borro, P. Boyd, J. M. Brewer, D. R. Ciardi, L. T. Elkins-Tanton, M. E. Everett, S. B. Howell, H. Isaacson, S. R. Kane, J. H. Livingston, G. W. Marcy, J. M. Schmitt, E. Sinukoff, G. Vanderas, and L. M. Weiss. Five planets transiting a ninth magnitude star. *The Astrophysical Journal Letters*, 827(1):L10, 2016. doi: 10.3847/2041-8205/827/1/L10.

E. Vieira Neto, O. Winter, and T. Yokoyama. The effect of jupiter's mass growth on satellite capture-retrograde case. *Astronomy & Astrophysics*, 414(2):727–734, 2004.

E. Vieira Neto, O. Winter, and T. Yokoyama. Effect of jupiter's mass growth on satellite capture-the prograde case. *Astronomy & Astrophysics*, 452(3):1091–1097, 2006.

O. C. Winter and E. Vieira Neto. Time analysis for temporary gravitational capture: Stable orbits. *Astronomy & Astrophysics*, 377:1119–1127, 2001. doi: 10.1051/0004-6361:20011165.

O. C. Winter and E. Vieira Neto. Distant stable direct orbits around the moon. *Astronomy & Astrophysics*, 393(2):661–671, 2002. doi: 10.1051/0004-6361:20021080.

J. Wisdom and M. Holman. Symplectic maps for the N-body problem. *The Astronomical Journal*, 102(4):1528–1538, 1991. doi: 10.1086/115978.

J. Wisdom, M. Holman, and J. Touma. Symplectic correctors. In J. E. Marsden, G. W. Patrick, and W. F. Shadwick, editors, *Integration Algorithms and Classical Mechanics*, volume 10 of *Fields Institute Communications*, pages 217–244. American Mathematical Society, 1996.

T. Yokoyama, E. Vieira Neto, O. C. Winter, D. M. Sanchez, and P. I. O. Brasil. On the evection resonance and its connection to the stability of outer satellites. *Mathematical Problems in Engineering*, 2008:251978, 2008. doi: 10.1155/2008/251978.