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NEURAL PARAMETER CALIBRATION FOR RELIABLE HYSTERESIS PREDICTION IN BOLTED JOINT ASSEMBLIES

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Abstract. Bolted joints are a common way for connecting multiple structures, and ensuring their safe operation is crucial. Changes in operational conditions, such as variations in tightening torque, can introduce hysteresis mechanisms and complex nonlinearities, making it challenging to analyze and diagnose any issues. Vibration measurements can be used to calibrate a reduced-order model that captures the effects of energy dissipation in bolted joints, such as a Bouc-Wen oscillator. However, the nonlinearities inherent in these systems make calibration problematic, typically requiring ad-hoc knowledge and considerations. In this work, we propose to use a new neural parameter calibration paradigm for this computational model, utilizing a physics-informed neural network to estimate the Bouc-Wen model parameters from time series data. The approach involves using a neural differential equation to represent the hysteresis effect and extracting coefficients from the vibration time series to inform the model. The method generates accurate predictions for the hysteresis loop in a matter of minutes, demonstrating the potential of this approach for real-time monitoring and diagnosis of bolted joint assemblies.

Keywords: bolted joints, hysteresis mechanisms, model calibration, neural differential equation

1. INTRODUCTION

The accurate calibration of dynamic models is fundamental to predicting the behavior of mechanical systems, especially those that exhibit non-linear and hysteretic behavior, such as bolted joints. Among the various models available for capturing these dynamics, the Bouc-Wen model has proven highly effective for describing hysteresis in structural systems (Ismail *et al.*, 2009; Miguel *et al.*, 2020; Delgado-Trujillo *et al.*, 2023). However, precise parameter estimation in these models, particularly in complex applications like bolted joints, remains a challenge due to their non-linearities (Tomlinson, 2000) and frictional effects (Bograd *et al.*, 2011).

Traditional approaches to parameter estimation, such as Bayesian inference and least-squares methods, have proven highly effective, particularly in capturing complex dynamics in mechanical systems. These techniques, as shown in the works of Teloli *et al.* (2021) and Miguel *et al.* (2022), have been invaluable for identifying parameters in higher-order, non-linear systems, such as the Bouc-Wen model applied to bolted joints. However, despite their robustness, these methods can be computationally demanding, mainly when applied to real-time applications or large-scale systems like bolted joints. As mechanical systems grow in complexity, there is a strong motivation to explore more efficient and scalable methods to reduce computational costs while maintaining accuracy in parameter calibration.

In the last years, machine learning, especially neural networks, became a compelling alternative for everything related to the calibration of parameters (Ferreira *et al.*, 2024). Neural networks are flexible and scalable; hence, they can efficiently handle big datasets and complex models. For example, Gaskin *et al.* (2023b) proposed neural parameter calibration for large-scale multi-agent systems, exploiting neural differential equations to approximate parameter probability densities. This approach improves computational efficiency and maintains or enhances the accuracy of traditional calibration methods.

Moreover, integrating neural networks with physics-based models, such as the Bouc-Wen model, completely opens new perspectives in structural dynamics. Physics-informed neural networks (PINNs), which incorporate physical laws directly into the learning process, have shown promise in ensuring that the neural network's predictions remain consistent with established mechanical principles (Cuomo *et al.*, 2022; Gaskin *et al.*, 2023b; Jagtap and Karniadakis, 2023; Liu and Meidani, 2023). This framework is particularly beneficial in systems exhibiting hysteresis, such as bolted joints, where factors like friction, preload, and material properties introduce additional complexity (Song *et al.*, 2004; Olejnik and Ayankoso, 2023).

The Bouc-Wen model's flexibility in capturing non-linear hysteretic behavior makes it well-suited for such applications, and by employing machine learning techniques, we can further improve the efficiency and accuracy of parameter estimation. Neural networks have demonstrated their ability to handle the dynamic interaction between components in

bolted joints, offering a scalable solution to the parameter calibration problem. In particular, neural differential equation frameworks provide a powerful tool for calibrating these complex systems while reducing the computational burden associated with traditional methods. This is further supported by the successful applications that techniques for neural parameter calibration find in other domains. For example, epidemic forecasting (Gaskin *et al.*, 2023b,a) has shown the versatility of those techniques in handling a wide range of complex and/or nonlinear systems.

In this work, based on the recent development in neural parameter calibration, we apply the techniques to the Bouc-Wen model and estimate its parameters with high accuracy. We aim to extend this methodology to a real bolted joint structure, where frictional and hysteretic effects influence the dynamic behavior. By leveraging neural networks within a differential equation framework, we aim to offer a computationally efficient and scalable solution for parameter estimation in non-linear mechanical systems, paving the way for further research into machine learning-driven calibration in structural dynamics.

2. FRAMEWORK FOR NEURAL PARAMETER CALIBRATION

Figure 1 shows a graphical abstract of the proposed framework for the neural parameter calibration of a Bouc-Wen model, in which a neural differential equation is used to estimate the model parameters from synthetic noisy time series data.

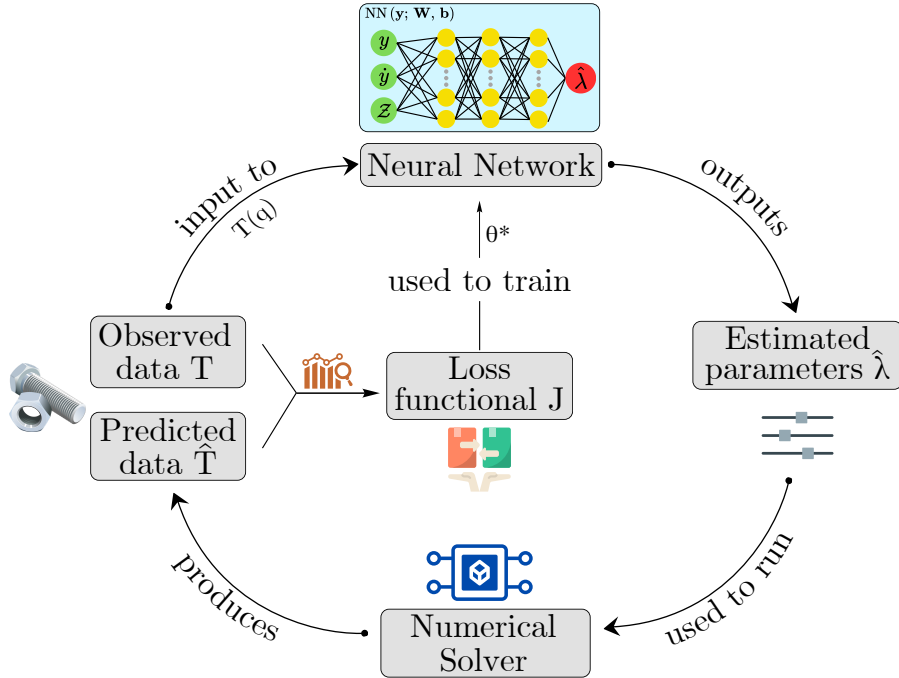


Figure 1. Graphical abstract of the proposed framework for neural parameter calibration of a Bouc-Wen model.

We start with a time series data that serves as the neural network input. The network predicts the Bouc-Wen model parameters, which are then used to run a numerical simulation of the model. The simulated data is compared with the original time series data, and the loss is backpropagated to update the network weights. This process is repeated for multiple epochs until the network converges to accurate parameter estimates.

2.1 Bouc-Wen model

The Bouc-Wen model is a well-established system of equations that describes the hysteretic behavior of structural systems, particularly in bolted joints. The model parameters are crucial for accurately capturing the system's dynamics, but their estimation can be challenging due to the model's non-linearities. The following coupled nonlinear differential equations define the governing equations of the Bouc-Wen oscillator:

$$m\ddot{y}(t) + c\dot{y}(t) + ky(t) + \mathcal{Z}(y, \dot{y}) = u(t), \quad (1)$$

$$\dot{\mathcal{Z}}(y, \dot{y}) = \alpha\dot{y}(t) - \beta(\gamma|\dot{y}(t)||\mathcal{Z}(y, \dot{y})|^{\nu-1}\mathcal{Z}(y, \dot{y}) + \delta\dot{y}(t)|\mathcal{Z}(y, \dot{y})|^\nu), \quad (2)$$

where m is the mass, c is the damping coefficient, k is the stiffness, $u(t)$ is the excitation force, $y(t)$ is the displacement, $\dot{y}(t)$ is the velocity and $\mathcal{Z}(t)$ is the hysteretic restoring force. In addition, α , β , γ , δ , and ν are called Bouc-Wen parameters and, in brief, they are responsible for inducing and controlling the memory effects and elastoplastic behavior of the model (Ismail *et al.*, 2009).

2.2 Observed data – generation and processing

As part of our training data generation, we consider an excitation scenario where the system is subjected to a harmonic sinusoidal force, given by:

$$u(t) = A \sin(2\pi t), \quad (3)$$

where A is the amplitude of the force and t is the time.

The synthetic data is generated by solving the Bouc-Wen set of equations using a tensor approach via `torchdiffeq` package from Python using a Runge-Kutta of fourth order (a necessary approach in order to work with the physics-informed neural network) with initial conditions $y(0) = 0$, $\dot{y}(0) = 0$ and $\mathcal{Z}(0) = 0$. Additionally, to simulate real-world conditions, Gaussian noise was added to the generated data, with a signal-to-noise ratio (SNR) of 40 dB. Table 1 shows the simulation parameters used for data generation, and Fig. 2 shows the synthetic time series data. It is important to note that we analyze only the last 2 seconds of the time series data for training the neural network as the system reaches a steady-state response.

Table 1. Simulation parameters used for data generation.

m [kg]	c [Ns/m]	k [N/m]	α [N/m]	β [N/m]	γ [m ⁻¹]	δ [m ⁻¹]	ν [-]	A [N]	t_{sim} [s]	Δt [s]
2.0	8.0	2000.0	1500.0	100.0	0.8	-1.1	1.0	60.0	10.0	0.01

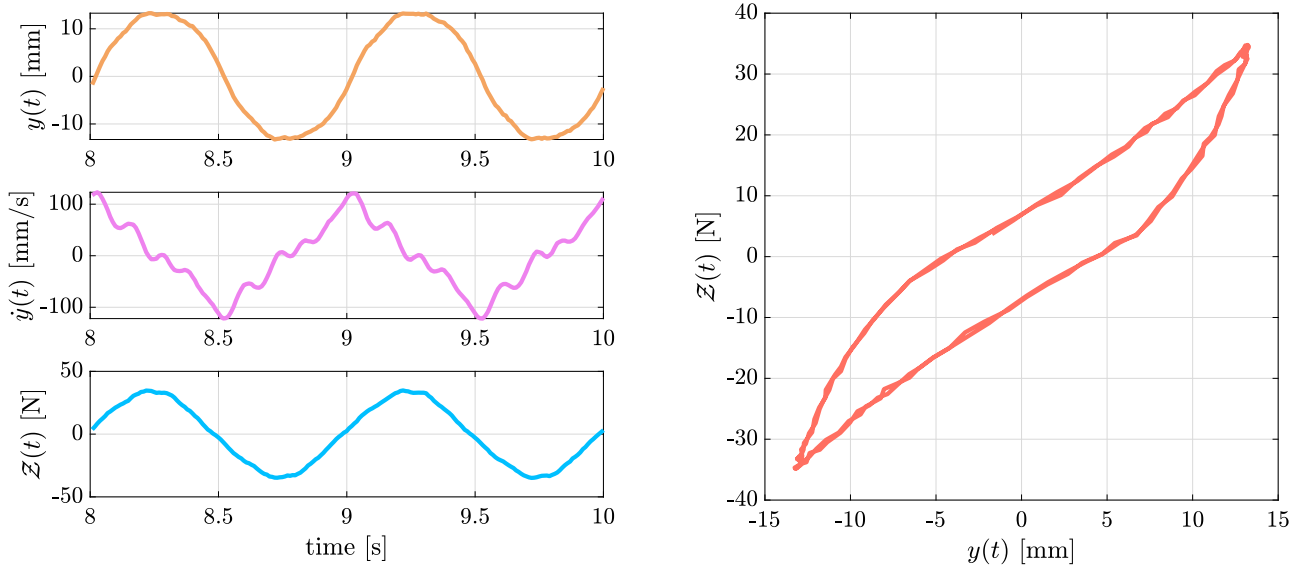


Figure 2. Synthetic time series data generated by the Bouc-Wen model with added Gaussian noise.

2.3 Training the neural network

A neural network was designed to predict six key parameters of the Bouc-Wen model: c , k , α , β , γ , and δ . As known values, we keep m , ν , and A fixed during training. The network architecture consists of an input layer for the initial state (displacement, velocity, and force), followed by three hidden layers with 20 neurons, each using the tanh activation function, and the final layer outputs the six estimated parameters. The network was trained for 800 epochs using the Adam optimizer with a learning rate scheduler that reduces the learning rate when the loss plateaus. The loss function minimized during training was the mean squared error (MSE) between the estimated and actual system response.

As the network is trained to produce normalized values between -1 and 1, due to the tanh activation function, the output values are scaled in a uniform set of values for each parameter. We use a sigmoid function to scale the output to the desired range if the range is positive. If the range is negative, we use a hyperbolic tangent function to scale the output to the desired range. In this way we ensure that the network predicts physically meaningful values. The bounds for each parameter are presented in Eqs. (4a) – (4b), and finally Fig. 3 shows a graphical abstract of how is the training process of the neural network.

$$c \in [5.0, 12.0], \quad k \in [1000.0, 3000.0], \quad \alpha \in [1000.0, 3000.0] \quad (4a)$$

$$\beta \in [50.0, 150.0], \quad \gamma \in [0.0, 1.0], \quad \delta \in [-2.0, 2.0] \quad (4b)$$

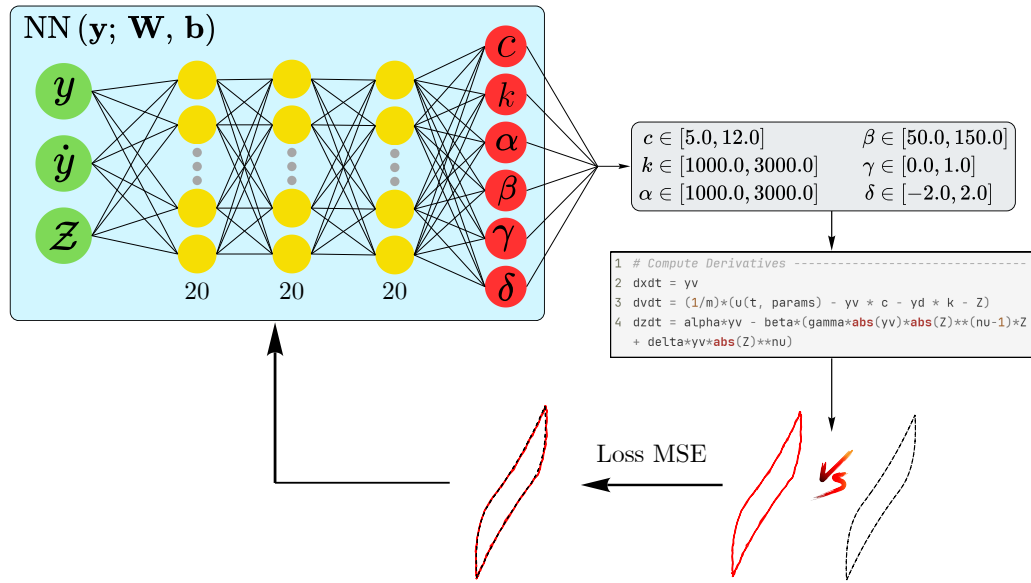


Figure 3. Graphical abstract of the training process of the neural network.

3. RESULTS AND DISCUSSION

Figure 4 shows the training loss curve for the neural network. The loss decreases rapidly in the first 200 epochs and then stabilizes, indicating that the network has converged to a solution. The total training time was approximately 16 minutes on a DELL G15¹. The network was able to predict the Bouc-Wen model parameters that best fit the observed data, demonstrating the effectiveness of the neural parameter calibration approach.

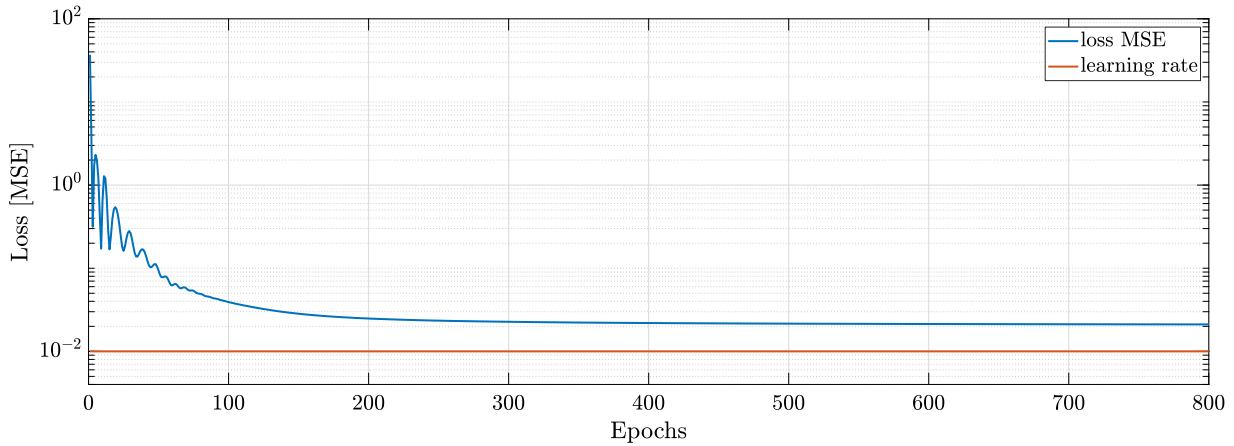


Figure 4. Training loss and learning rate over epochs for the neural network.

Next we present the evolution of the estimated parameters during training. Figure 5 shows the estimated values for each parameter, which converge to the final values after 800 epochs, and Tab. 2 shows the comparison between the true and estimated parameters. Finally, Fig. 6 shows the responses estimated by the calibrated Bouc-Wen model, which closely matches the observed data.

Table 2. Comparison between the true and estimated parameters.

	c [Ns/m]	k [N/m]	α [N/m]	β [N/m]	γ [m^{-1}]	δ [m^{-1}]
Real	8.00	2000.00	1500.00	100.00	0.80	-1.10
Estimated	6.90	2022.51	1554.12	111.65	0.71	-0.95

As can be seen in Fig. 6, the model is almost similar to the observed data, being able to predict the hysteresis loop with high accuracy. However, the estimated parameters are not the same as the real ones, as Tab. 2 states.

¹i5-12500H, 16GB RAM DDR5, RTX 3050 4GB DDR6

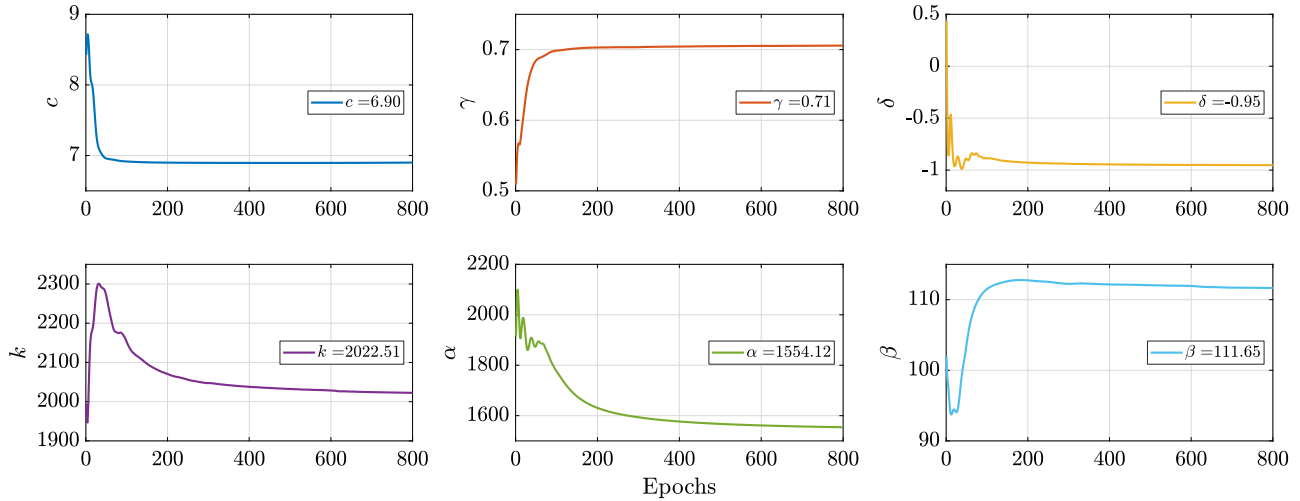


Figure 5. Evolution of the estimated parameters during training.

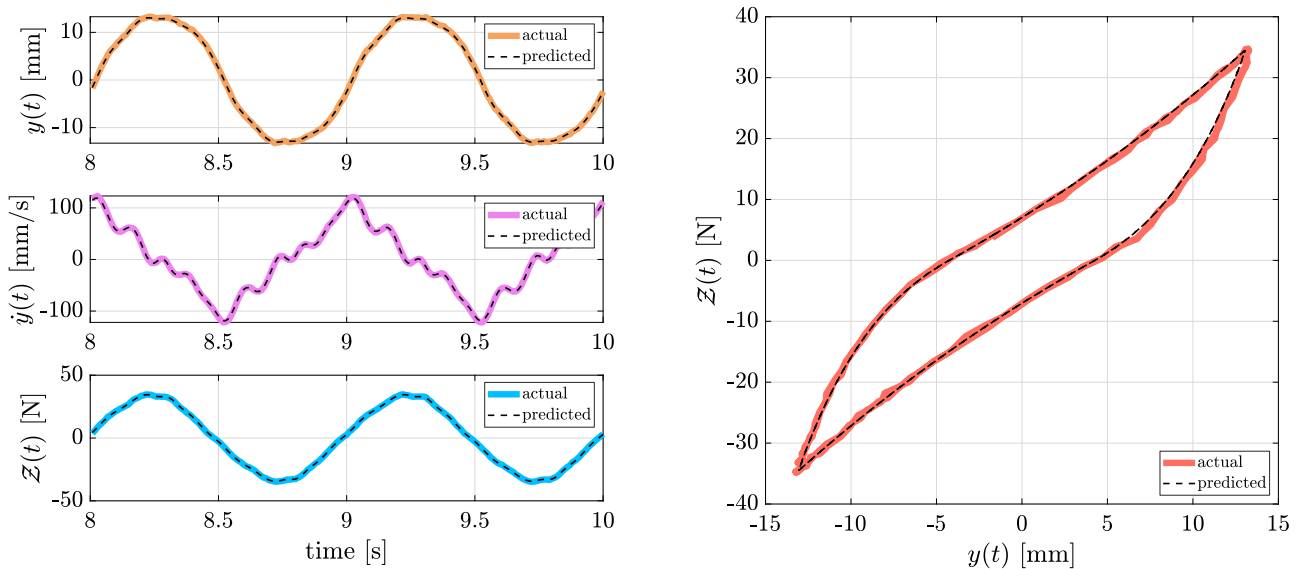


Figure 6. Comparison between the observed data and the estimated by the calibrated Bouc-Wen model.

This is because the Bouc-Wen parameters are not unique, and different combinations of parameters can produce similar responses. This is a common issue in parameter estimation, and it is important to consider the model's sensitivity to parameter changes when interpreting the results.

Despite the minor discrepancies in the estimated parameters, the neural network could accurately predict the system's response, demonstrating the effectiveness of the neural parameter calibration approach. The network's ability to capture the system's dynamics and predict the hysteresis loop with high accuracy is a promising result, showing the potential of this method for parameter estimation in non-linear mechanical systems, mainly considering the computational efficiency in scenarios where traditional methods are impractical.

4. FINAL REMARKS

This work presented a new approach for calibrating the Bouc-Wen model's neural parameters. It used a physics-informed neural network to estimate the model parameters from synthetic noisy time series data. The network was trained to predict the six key parameters of the Bouc-Wen model, which were then used to simulate the system's response. The results showed that the network could accurately predict the hysteresis loop, demonstrating the effectiveness of the neural parameter calibration approach.

In future work, we plan to extend this methodology to real bolted joint structures, where frictional and hysteretic effects influence the dynamic behavior. By leveraging neural networks within a differential equation framework, we aim to offer a computationally efficient and scalable solution for parameter estimation in non-linear mechanical systems, paving the way for further research into machine learning-driven calibration in structural dynamics.

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