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Scattering process in the Scalar Duffin-Kemmer-Petiau gauge theory

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Abstract. In this work we calculate the cross section of the scattering process of the Duffin-Kemmer-Petiau theory coupling with the Maxwell's electromagnetic field. Specifically, we find the propagator of the free theory, the scattering amplitudes and cross sections at Born level for the Moeller and Compton scattering process of this model. For this purpose we use the analytic representation for free propagators and take account the framework of the Causal Perturbation Theory of Epstein and Glaser.

1. Introduction

One of the main restriction to construct field equations is the Lorentz covariance and the causality principle. Thus, elementary particles are described by Lorentz-covariant field equations and they do not interact at a distance. Furthermore, in order to analyse the relativistic effects on the physical systems, relativistic wave equations, such the Dirac, Klein-Gordon-Fock (KGF) and Duffin-Kemmer-Petiau (DKP) equations, are constructed. The solutions of the relativistic wave equations with arbitrary spin have a long history. Besides the KGF and Dirac equations, the Dirac-like first order Lorentz invariant Kemmer equation supplemented with the β-matrices algebra, for both spin-0 and spin-1 particles was first derived by Kemmer in 1939 [1] and they are called the DKP[1] equations. Exact solutions of the DKP equation in the presence of an external field have been investigated by some authors, such as the quantum oscillator in the framework of the DKP theory [2, 3] during the last few decades.

It is known that in the free field case the DKP and KGF theories are equivalent, both in classical and quantum pictures. For instance, it was shown that both theories are equivalent in the classical level for the cases of minimal interaction with electromagnetic [4] and gravitational fields [5]. Strict proofs of equivalence between both theories were also given for cases of interaction of quantized scalar field with classical and quantized electromagnetic, Yang-Mills and external gravitational fields [6]. However, there are still no general equivalent proofs of equivalence between these theories when interactions and decays of unstable particles are taken into account.

Perhaps one of the most evident advantages in working with this theory is the fact that derivative couplings do not appear between DKP and the gauge field. This property has been

[1] There were two independent previous works given by Duffin and Petiau.

used by Gribov, who employed the spin-1 DKP theory to study the quark confinement problem [7]. Such property will result in manifestly covariant expressions for the interaction Hamiltonian and the vacuum expectation values of time ordered products of fields. Other advantages are the formal similarity with SQED and QED, what allows an unified treatment of the scalar and vector fields.

For spin zero particles it is known as the Duffin-Kemmer-Petiau theory for scalar fields (SDKP). One of the difficulties in working with SQED is the presence of a term of second order in the coupling constant in the interaction Hamiltonian, which causes trouble in proving gauge invariance. In SDKP theory it was achieved, by an effective approach, that this second order term does not contribute to the S-matrix, and thus it can be neglected when we construct the Feynman rules for the theory.

The Epstein-Glaser-Scharf causal perturbative method [8, 9] applied to SQED [10, 11] is a finite ultraviolet (UV) theory which differs from the conventional formalism because the perturbative construction starts from the first order interaction, and the quadratic term appears in the process of distribution splitting as a consequence of gauge invariance. Following the causal approach, J.T. Lunardi, B.M. Pimentel and J.S. Valverde [12] studied the SDKP coupled with the electromagnetic field and found the UV free radiative corrections.

Moreover, when the causal approach is supplemented with the analytical representation, this framework gives us a powerful method to approach formally Light-front Dynamics. In particular, it has been obtained a Light-Front Quantum Electrodynamics without prescriptions [13]. The same idea was applied for Generalized Quantum Electrodynamics [14], in the usual dynamics, to calculate the Podolsky's corrections to the Bhabha scattering at the Born level.

In this work we will approach the SDKP coupled with the electromagnetic field in the framework of the causal perturbative method supplemented with the analytical representation. Our goals are to calculate the free electromagnetic and scalar propagators and, to calculate the amplitude transitions and the cross section, at Born level, for the Moeller and Compton scattering.

2. The S-Matrix inductive Causal Program

The axiomatic construction of perturbative Quantum Field Theory is reviewed from a causal point of view introduced by Epstein and Glaser in 1973 [8]. In this axiomatic approach the scattering operator S can be written in the following formal perturbative series:

$$S\left[g\right]=1+\sum_{n=1}^{\infty}\frac{1}{n!}\int dx_1dx_2...dx_nT_n\left(x_1,x_2,...,x_n\right)g\left(x_1\right)g\left(x_2\right)...g\left(x_n\right), \tag{1}$$

where we can identify the quantity T_n as an operator-valued distribution and $g^{\otimes n}$ its test function[16]. The operator S can be constructed using *Causality* and *Poincaré invariance* as the principal physical properties. From *Causality* we can prove the following property

$$T_n\left(x_1,...,x_m,x_{m+1},...,x_n\right)=T_m\left(x_1,...,x_m\right)T_{n-m}\left(x_{m+1},...,x_n\right), \tag{2}$$

if $\left\{x_1^0,...,x_m^0\right\}>\left\{x_{m+1}^0,...,x_n^0\right\}$.

The first step in the construction of S is to define $T_1(x)$. In the KGF Formalism we define $T_1^{KG}(x)\equiv i:\mathcal{L}_{\text{int}}^{1^o}:$, where $\mathcal{L}_{\text{int}}^{1^o}$ is the interacting lagrangian in first order on the coupling constant avoiding the e^2 term obtained by minimal coupling prescription for getting gauge invariance[11]. For DKP formalism the e^2 term does not exist, and we can follow the definition $T_1(x)\equiv i:\mathcal{L}_{\text{int}}:$ as for Quantum Electrodynamics (QED)[9].

The second step is to construct the *intermediary distributions* $A_2'(x_1,x_2)\equiv -T_1(x_1)T_1(x_2)$ and $R_2'(x_1,x_2)\equiv -T_1(x_2)T_1(x_1)$ with the goal to obtain advanced A_2 and retarded R_2

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distributions as

$$A_2(x_1, x_2) \equiv -T_1(x_1)T_1(x_2) + T_2(x_1, x_2) = A'_2(x_1, x_2) + T_2(x_1, x_2), \tag{3}$$

$$R_2(x_1, x_2) \equiv -T_1(x_2)T_1(x_1) + T_2(x_1, x_2) = R'_2(x_1, x_2) + T_2(x_1, x_2). \tag{4}$$

We can see from (2) that the support of A_2 is the past of light cone centered in x_2, and for R_2 the support is the future part. Unlike the usual way to get the advanced and retarded part of propagators using the step function, the Epstein-Glasser's method does not multiply distributions in the same variable because such multiplication is not always well defined.

Now, from (3) and (4) we can compute explicitly the *causal distribution* $D_2(x_1, x_2) \equiv R'_2(x_1, x_2) - A'_2(x_1, x_2) = R_2(x_1, x_2) - A_2(x_1, x_2)$. In the computation of D_2 we can use Wick's theorem to obtain the following form for the causal distribution

$$D_2(x_1, x_2) = \sum_k : \left[\prod_j \phi^\dagger(x_j)\right] d_2^k(x_1, x_2) \left[\prod_l \phi^\dagger(x_l)\right] :: \left[\prod_m A(x_m)\right] :, \tag{5}$$

where $d_2^k(x_1, x_2)$ are numerical distributions with the light cone centered in x_2 as support. Using the distribution theory, we can compute R_2 as a retarded part of D_2. This process is called *causal splitting* and it is not a trivially process. In *causal splitting* we need to find the *order of singularity* ω of $d_2^k(x_1, x_2)$. In function of ω, we have two solutions for the retarded part $\hat{r}_2(p)$ of $d_2^k(x_1, x_2)$ in momentum space[17]

$$\hat{r}(p) = \frac{i}{2\pi} sgn(p^0) \int dt \frac{\hat{d}(tp)}{1 - t + sgn(p^0)i0^+}, \quad \text{when } \omega < 0 \tag{6}$$

$$\hat{r}(p) = \left[\frac{i}{2\pi} sgn(p^0) \int dt \frac{\hat{d}(tp)}{t^{\omega+1}(1 - t + sgn(p^0)i0^+)}\right] + \sum_{a=0}^{\omega} C_a p^a, \quad \text{when } \omega \geqslant 0 \tag{7}$$

The solution (6) is the trivial splitting using a step function $\Theta(x^0)$. The second solution (7) is carefully computed with the distribution theory and in that computation all steps are well defined. In the second solution, the constant C_a appears because of the uniqueness of the solution. Those constants will be computed using physical properties as gauge invariance, charge invariance, etc. Finally, we compute the second term $T_2(x_1, x_2)$ making the necessary substitutions. With $T_2(x_1, x_2)$ we compute all we need from the scattering operator until second order.

3. Free DKP causal propagators

The Epstein-Glasser causal approach follows the Heisenberg program[15]. For that reason we work with the causal propagators obtained from the free field theory. In this work we are going to use Wightman's formalism to compute those propagators[14][17]. Wightman's formalism tell us that if we know the green function $\hat{G}(k)$ associated to some free field equation, then the analytical representation of the positive and negative propagators $\hat{D}^{(\pm)}$ are

$$\left\langle \hat{D}^{(\pm)}, \varphi \right\rangle = (2\pi)^{-2} \oint_{c_\pm} \hat{G}(k)\varphi(k_0)dk_0. \tag{8}$$

where $c_{+(-)}$ is a counterclockwise closed path which contains only the positive (negative) poles of the Green function $\hat{G}(k)$, and $\varphi(k_0)$ are test functions of $\hat{D}^{(\pm)}$. Using (8), we may define the *causal* propagator:

$$\hat{D}(k) = \hat{D}^{(+)}(k) + \hat{D}^{(-)}(k), \tag{9}$$

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furthermore, for local field theories, this propagator must have causal support in the configuration space

$$suppD\left(x_1,x_2\right)\subseteq\Gamma_2^+\left(x_2\right)\cup\Gamma_2^-\left(x_2\right). \tag{10}$$

This property must be proven explicitly for each case.

Now we are going to apply Wightman's formalism for DKP's fields. The free DKP theory is considered as a Lagrangian [4]

$$\mathcal{L}_{DKP}=\frac{i}{2}\bar{\psi}\beta^{\mu}\overleftrightarrow{\partial}_{\mu}\psi-m\bar{\psi}\psi, \tag{11}$$

where ψ is a multi-component wave function, $\bar{\psi}=\psi^{\dagger}\eta^0$, and $\eta^0=2\left(\beta^0\right)^2-1$. We have that $\{\beta^{\mu}\}$ are a set of matrices ($\mu=0,1,2,3$) satisfying the algebraic relations

$$\beta^{\mu}\beta^{\nu}\beta^{\rho}+\beta^{\rho}\beta^{\nu}\beta^{\mu}=\beta^{\mu}g^{\nu\rho}+\beta^{\rho}g^{\mu\nu}. \tag{12}$$

The equations of motion are then

$$\mathcal{D}\left(\partial\right)\psi=\left(i\beta.\partial-m\right)\psi=0\ ,\ \bar{\psi}\overleftarrow{\mathcal{D}}\left(\partial\right)=\bar{\psi}\left(i\beta.\overleftarrow{\partial}+m\right)=0. \tag{13}$$

It is known that the algebra of the β matrices has only three irreducible representations, whose degrees are 1, 5 and 10 [1]. The first one is trivial, having no physical content. The second and third ones correspond, respectively, to the scalar and vectorial representations. In this work we shall restrict us to the scalar case.

To obtain the causal propagator, we can find that the Green function of the DKP field equation (13) is given by:

$$\hat{G}\left(k\right)=\frac{1}{m}\frac{\beta.p\left(\beta.p+m\right)}{p^2-m^2}-\frac{I}{m}, \tag{14}$$

then, using the analytical representation (8), the positive (PF) and negative (PF) frequency propagators are

$$\hat{S}^{(\pm)}\left(p\right)=\frac{1}{m}\beta.p\left(\beta.p+m\right)\hat{D}_m^{(\pm)}\left(p\right). \tag{15}$$

where $\hat{D}_m^{(\pm)}$ are the PF and NF scalar propagators[17]. These PF and NF propagators are related to the contractions of DKP fields as follows

$$\overbrace{\psi_a\left(x\right)\bar{\psi}_b\left(y\right)}\equiv\left[\psi_a^{(-)}\left(x\right),\bar{\psi}_b^{(+)}\left(y\right)\right]=\frac{1}{i}S_{ab}^{(+)}\left(x-y\right), \tag{16}$$

$$\overbrace{\bar{\psi}_a\left(x\right)\psi_b\left(y\right)}\equiv\left[\bar{\psi}_a^{(-)}\left(x\right),\psi_b^{(+)}\left(y\right)\right]=\frac{1}{i}S_{ba}^{(-)}\left(x-y\right), \tag{17}$$

Moreover, the *causal DKP propagator* is obtained as $\hat{S}=\hat{S}^{(+)}+\hat{S}^{(-)}$, with the following result

$$\hat{S}\left(p\right)=\frac{1}{m}\beta.p\left(\beta.p+m\right)\hat{D}_m\left(p\right), \tag{18}$$

where $\hat{D}_m$ is the known scalar massive Pauli-Jordan propagator given by

$$\hat{D}_m\left(p\right)=\hat{D}_m^{(+)}+\hat{D}_m^{(-)}=\frac{i}{2\pi}sgn\left(p_0\right)\delta\left(p^2-m^2\right). \tag{19}$$

It is straightforward confirm that $\hat{S}(p)$ has a causal support as required.

For electromagnetic field quantization by Wightman's formalism we recommend the reference [17]. The most important result for our calculation is the contraction between two electromagnetic fields

$$\overbrace{A_\mu(x)\,A_\nu(y)} \equiv \left[A_\mu^{(-)}(x)\,,A_\nu^{(+)}(y)\right] = iD_{\mu\nu}^{(+)}(x-y)\,, \tag{20}$$

where $D_{\mu\nu}^{(+)}(x_1-x_2)$ is given by the following expression

$$D_{\mu\nu}^{(+)}(x) = g_{\mu\nu}D_0^{(+)}(x)\,, \tag{21}$$

and D_0 is given by (19) with $m=0$.

4. Moeller and Compton scattering

As we said in section 2. for DKP gauge theory we define $T_1(x) \equiv ie : \bar{\psi}(x)\beta^\mu\psi(x) : A_\mu(x)$. Then, in the Epstein-Glasser approach the causal distribution D_2 has the following structure

$$D_2(x,y) = D_2^{(1)} + D_2^{(2)} + D_2^{(3)} + D_2^{(4)} + D_2^{(5)} \tag{22}$$

where each term in (22) could be associated with different processes. In this work we are interested in Moeller and Compton scattering $D^{(1)}$ and $D^{(2)}$ respectively. Those terms explicitly are

$$D_2^{(1)} = -ie^2 : \overline{\psi}(x_2)\beta^\mu\psi(x_2)\overline{\psi}(x_1)\beta^\nu\psi(x_1) : D_{\mu\nu}(x_1-x_2) \tag{23}$$

$$\begin{aligned} D_2^{(2)} =& ie^2 : \overline{\psi}(x_1)\beta^\nu S(x_1-x_2)\beta^\mu\psi(x_2) :: A_\mu(x_2)A_\nu(x_1) : \\ &- ie^2 : \overline{\psi}(x_2)\beta^\mu S(x_2-x_1)\beta^\nu\psi(x_1) :: A_\mu(x_2)A_\nu(x_1) : \end{aligned} \tag{24}$$

4.1. Moeller Scattering

The differential cross section for Moeller scattering is obtained from (23). For *splitting* (23) the computation of the order of singularity gives us $\omega = -2$, what means that the retarded part is given by the solution (6). Making the necessary substitutions we get for $T_2^{(1)}$

$$T_2^{(1)}(x_1,x_2) = -e^2 i g_{\mu\nu}\beta_{ab}^\mu\beta_{cd}^\nu : \overline{\psi}_a(x_2)\psi_b(x_2)\overline{\psi}_c(x_1)\psi_d(x) : D_0^F(x_1-x_2), \tag{25}$$

where $D_0^F(x_1-x_2)$ is the massless Feynman propagator which in the momentum space is

$$D_0^F(k) = -(2\pi)^{-2}\frac{1}{k^2+i0^+} \tag{26}$$

Because the distributional nature of S-matrix, we make our computation of cross section smearing S-matrix with the wave packets

$$|\psi_i\rangle = \int d^3p_1 d^3q_1 \psi_i(\mathbf{p}_1,\mathbf{q}_1)\,a^\dagger(\mathbf{p}_1)\,a^\dagger(\mathbf{q}_1)\,|\Omega\rangle\,, \tag{27}$$

$$|\psi_f\rangle = \int d^3p_2 d^3q_2 \psi_f(\mathbf{p}_2,\mathbf{q}_2)\,a^\dagger(\mathbf{p}_2)\,a^\dagger(\mathbf{q}_2)\,|\Omega\rangle\,, \tag{28}$$

where $a^\dagger(\mathbf{p}_l)$ is a creation operator of a DKP particle with 3-momentum $\mathbf{p}_l$ and if $\mathbf{p}_i$ and $\mathbf{p}_f$ are the ingoing and outgoing 3-momentum of the DKP's particles, then $|\psi_i\rangle$ and $|\psi_f\rangle$ are the wave-packet functions sharply peaked in $\mathbf{p}_i$ and $\mathbf{p}_f$ respectively [9]. Then, by definition we have for the transition amplitude $\mathcal{P}_{fi}$

$$\mathcal{P}_{fi} \equiv \left(\left|\psi_f\right\rangle, \left|\psi_i\right\rangle\right)^2 = \left|S_{fi}\right|^2 = \left|\left\langle \psi_f \left| S_2^{(1)} + \dots \right| \psi_i \right\rangle\right|^2. \tag{29}$$

In the laboratory frame, the incoming particles are a beam of particles with a cylinder form of radius R and velocity $\mathbf{v}$, then summing over all final states we obtain [9]

$$\sum_f \mathcal{P}_{fi}(R) = \frac{1}{\pi R^2}\left[(2\pi)^2 \frac{1}{|\mathbf{v}|}\int d^3p_2 d^3q_2 \left|\mathcal{M}(p_i, q_i, p_2, q_2)\right|^2 \delta(p_2 + q_2 - p_i - q_i)\right], \tag{30}$$

where $\mathcal{M}$ is a distributional quantity related to the *S-Matrix* as it follows

$$S_{fi} = ie^2\delta(p_f + q_f - p_i - q_i)\mathcal{M}. \tag{31}$$

The scattering cross section in the laboratory frame is then given by

$$\sigma_{\text{lab}} \equiv \lim_{R\to\infty} \pi R^2 \overline{\sum_f P_{if}(R)}. \tag{32}$$

where $\overline{\sum_f P_{if}(R)}$ is the average of $\sum_f P_{if}(R)$ over the cylinder of radius R defined by the incoming particles.

Finally, after some no trivial computation, we end with the following differential cross section

$$\frac{d\sigma_{c.m}}{d\Omega} = \frac{\alpha^2}{4s}\left|\frac{s-t}{u} + \frac{s-u}{t}\right|^2, \tag{33}$$

where α is the fine-structure constant and $\{s, t, u\}$ the Mandelstam's variables. The solution (33) is in accordance with the one obtained using the KGF equation by the *causal approach* [11] and the one obtained using the usual approach[19][18].

4.2. Compton Scattering

For computing the differential cross section for Compton scattering we use (24). The order of singularity of (24) is $\omega = 0$. This value tell us that to obtain the retarded part in the causal splitting process we must use the solution (7) where a non fixed constants will appear. Then, making the necessary substitutions we get the 2-points distribution $T_2^{(2)}$

$$\begin{aligned} T_2^{(2)} &= e^2 i : \overline{\psi}_c(x)\beta^\nu_{cd}(-S^F_{da}(x-y) + C_{da}\delta(x-y))\beta^\mu_{ab}\psi_b(y) :: A_\mu(y)A_\nu(x) : \\ &+ e^2 i : \overline{\psi}_a(y)\beta^\mu_{ab}(-S^F_{bc}(y-x) - C'_{bc}\delta(x-y)\beta^\nu_{cd}\psi_d(x) :: A_\mu(y)A_\nu(x) : \end{aligned} \tag{34}$$

where C_{da} and C'_{bc} can not be fixed by causality or causal splitting process. To obtain them we need to use other physical properties or symmetries of S-Matrix. Using ***charge invariance*** we get $C = C'$ and using ***Gauge invariance*** we get $C = \frac{\mathbb{I}}{m}$. We want to emphasize here that if we make the integration in Delta Dirac distribution in the S-matrix we are going to obtain the multiplication of two DKP fields and two electromagnetic fields in the same variable. This result is consistent with the term e^2 placed in the usual approach to get gauge invariance by minimal coupling prescription.

In order to compute the differential cross section we could follow the same steps as for the Moeller scattering, but here we need to smear the S-Matrix with the following wave packets

$$\left|\psi_i\right\rangle = \int d^3p_1 d^3k_1 \psi_i(\mathbf{p}_1, \mathbf{q}_1)\, a^\dagger(\mathbf{p}_1)\, \varepsilon_{i\nu} \mathrm{b}^\dagger_\nu(\mathbf{k}_1)\left|\Omega\right\rangle, \tag{35}$$

$$\left|\psi_f\right\rangle = \int d^3p_2 d^3k_2 \psi_f(\mathbf{p}_2, \mathbf{q}_2)\, a^\dagger(\mathbf{p}_2)\, \varepsilon_{f\nu} \mathrm{b}^\dagger_\nu(\mathbf{k}_2)\left|\Omega\right\rangle, \tag{36}$$

where $\varepsilon_{i\nu}$ and $\varepsilon_{f\nu}$ are the 4-polarization vectors for the incoming and outgoing photons, and the $\mathrm{b}_\nu^\dagger(\mathbf{k}_l)$ are the creation operator valued distributions for photons. The other steps in the computation are the same which were used for Moeler scattering.

With the help of the polarization conditions

$$\varepsilon_i k_i = 0, \quad \varepsilon_f k_f = 0 \tag{37}$$

and using the reference system where

$$p_i \varepsilon_i = 0, \quad p_i \varepsilon_f = 0 \tag{38}$$

we get

$$\begin{aligned} \frac{d\sigma_{\text{lab}}}{d\Omega} &= \frac{e^4}{16\pi^2}\frac{\omega_f^2}{m^2\omega_i^2}\left[\varepsilon_f^\mu(\mathbf{k}_f)\varepsilon_{i\mu}(\mathbf{k}_i)\right]^2 \\ &= \frac{\alpha^2\omega_f^2}{m^2\omega_i^2}\left[\varepsilon_f^\mu(\mathbf{k}_f)\varepsilon_{i\mu}(\mathbf{k}_i)\right]^2. \end{aligned} \tag{39}$$

where α is the fine-structure. The solution (39) is in accordance with the one obtained using the KGF equation by the Causal Approach [11] and the one obtained using the usual approach[19].

5. Conclusions

We could note that the combination of the analytical representation technique with the Epstein-Glasser causal approach gives us a well define theory for the study of the SDKP.

We have calculated the DKP free propagator using analytical representation, and the cross section for the Moeller and Compton scattering process for the SDKP.

For the Moeller and Compton scattering process we find, at Born level, whose the results of the differential cross sections are identical to that obtained in the usual treatment of Scalar Quantum Electrodynamics. These results provide us an evidence that the SDKP and the KGF theory are equivalent, at Born level, even when they are coupled with the electromagnetic field for these processes.

For future works, we will calculate the radiative corrections and determine some important general results such as the renormalizability of the theory.

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