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A state-space description for perfect-reconstruction wavelet FIR filter banks with special orthonormal basis functions

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ABSTRACT

This paper presents a state space description for wavelet FIR filter banks with perfect reconstruction using special orthonormal basis functions. The FIR structure guarantees the BIBO stability, robustness and improves the filter divergence problem while orthonormal basis functions have characteristics that make them attractive in the modeling of dynamic systems. The state space description presented in this paper has all of those advantages and is minimal.

1. Introduction

Dynamic system models are important to study problems that arise in closed-loop systems, systems in which conditions are converted into information that can be observed and controlled [1]. The modeling of real systems is of great importance to engineers, since models are usually needed for the design of new processes and the analysis of existing ones. Generally, advanced techniques of controller design, optimization and supervision are based on process models, and the quality of the model directly influences the quality of the final solution to the problem [2–4].

Several techniques are proposed for modeling dynamic systems. There are models obtained by using orthogonal functions that form a complete basis for the Lebesgue $L_2[0, \infty)$ space and orthonormal basis functions [2,3]. Such models have some characteristics that make them attractive for dynamic systems modeling: absence of output recursion, not requiring prior knowledge of the exact structure of the vector of regression; possibility to increase the capacity of representation of the model by increasing the number of orthonormal functions employed; natural uncoupling of the outputs in multivariable models; tolerance to unmodeled dynamics, and others [3].

On the other hand, the FIR (Finite Impulse Response) structure does not only guarantee both BIBO (Bounded Input/Bounded Output) stability and robustness to some parameter changes, but also improves the filter divergence problem [5–7].

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This paper presents a state space description for an orthonormal basis of functions developed in [8], which are used as wavelet FIR filter banks with perfect reconstruction. This description holds the advantages presented by orthonormal basis functions and the FIR structure, culminating in a minimal realization.

The contribution of this paper consists in developing a realization in state space for a wavelet filter bank, with the explicit presence of parameters that can be freely adjusted keeping the guarantees of perfect reconstruction and orthogonality. For this purpose, the parameterization described in [8] is adopted as a starting point. The novelty of the present work with respect to [8] is the proposal of a realization of the resulting filter in the state space.

Other studies about state space using FIR filters present some properties in common in relation to the proposed parameterization, and other aspects that can be checked in [9–13,5–7]. Unlike these studies, this work employs orthonormal wavelet filter banks, rather than FIR filters in a general way. An advantage of the wavelet parameterization in the state space is the presence of a certain number of free parameters that can be adjusted by the designer.

Within the scope of possible applications and extensions of the present paper, it is worth mentioning some representative contributions about innovative filtering and auxiliary model based estimation methods [14], as well as identification of dual-rate systems [15,16] using the hierarchical identification principle. Identification methods for a Hammerstein system with its linear dynamic subsystem being an observer canonical state space model were exploited in [17], with an approach that can effectively solve the identification problem of a class of bilinear identification systems. A discussion about parameter estimation algorithm to establish the mathematical models for dynamic systems and an estimated state based on recursive least squares algorithm can be found in [18]. A study considering modeling and identification problems for linear systems based on canonical state space models with d-step state-delay is given in [19]. A study about state filtering and parameter estimation problems for state space systems with scarce output availability using least squares based algorithm and an observer based parameter estimation algorithm is developed by [20]. About reconstruction, some related issues of non-uniformly sampled systems, including model derivation, controllability and observability, computation of single-rate models with different sampling periods, reconstruction of continuous-time systems, and parameter identification of non-uniformly sampled discrete-time systems are discussed in [21]. These works can link the results of this paper to other possible extensions.

2. Background

Sherlock and Monro [8] developed a formulation to parameterize the space of orthonormal wavelets by using a set of angular parameters, adapting the work about factorization of paraunitary matrices of [4]. This formulation is presented below.

Consider the low-pass filter analysis in a two-channel orthonormal filter bank with $2N$ coefficients $\{h_i\}$ and its z-transform

$$H(z) = \sum_{i=0}^{2N-1} h_i z^{-i} = H_0(z^2) + z^{-1} H_1(z^2),$$

where H_0 and H_1 are the polyphase components of $H(z)$, namely,

$$H_0(z) = \sum_{i=0}^{N-1} h_{2i} z^{-1} \quad \text{and} \quad H_1(z) = \sum_{i=0}^{N-1} h_{2i+1} z^{-1}. \tag{1}$$

In [4], Vaidyanathan proposed the factorization of paraunitary matrices $H_p(z)$ in the following manner

$$\begin{aligned} H_p(z) &= \begin{pmatrix} H_0(z) & H_1(z) \\ G_0(z) & G_1(z) \end{pmatrix} \\ &= \begin{pmatrix} C_0 & S_0 \\ -S_0 & C_0 \end{pmatrix} \prod_{i=1}^{N-1} \begin{pmatrix} 1 & 0 \\ 0 & z^{-1} \end{pmatrix} \begin{pmatrix} C_i & S_i \\ -S_i & C_i \end{pmatrix}, \end{aligned} \tag{2}$$

where $C_i = \cos(\alpha_i)$, $S_i = \sin(\alpha_i)$ and $G_0(z)$ and $G_1(z)$ are the polyphase components of the high-pass filter analysis $G(z)$. This factorization generates all perfect reconstruction two-channel orthonormal filter banks with $2N$ coefficients, i.e., any filter can be written in terms of N parameters $\alpha_i \in [0, 2\pi)$. However, in order to the filter bank be built in terms of these parameters correspond to a basis of orthonormal wavelets, it is necessary that

$$\sum_{i=0}^{2N-1} h_i = \sqrt{2}. \tag{3}$$

The formulation of Sherlock and Monro [8] is developed from (2). In fact this formulation consists in rewriting (2) in a recursive form expressing the polyphase matrices corresponding to filters with length $2(N+1)$ in terms of the polyphase matrices corresponding to the filters of length $2N$

$$H_p^{(N+1)}(z) = H_p^{(N)}(z) \begin{pmatrix} 1 & 0 \\ 0 & z^{-1} \end{pmatrix} \begin{pmatrix} C_N & S_N \\ -S_N & C_N \end{pmatrix} \tag{4}$$

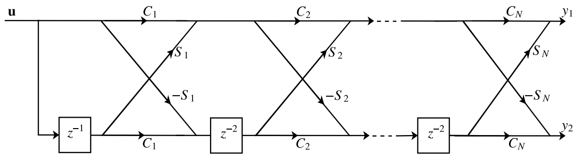

Fig. 1. Procedure for parameterizing the filter bank. Adapted from [26].

where $N = 1, 2, \ldots$ and $H_p^{(1)} = \begin{pmatrix} C_0 & S_0 \\ -S_0 & C_0 \end{pmatrix}$. Therefore, a recursive formula can be obtained by expressing the coefficients $\{h_i^{N+1}\}$ of the filters of length $2(N+1)$ in terms of the coefficients $\{h_i^{(N)}\}$ for the filters of length $2N$:

$$\begin{aligned}
h_0^{(1)} &= \cos(\alpha_1) \\
h_1^{(1)} &= \sin(\alpha_1) \\
h_0^{(N+1)} &= \cos(\alpha_{N+1}) h_0^{(N)} \\
h_{2i}^{(N+1)} &= \cos(\alpha_{N+1}) h_{2i}^{(N)} - \sin(\alpha_{N+1}) h_{2i-1}^{(N)} \qquad i = 0, 1, \ldots, N-1 \\
h_{2N}^{(N+1)} &= -\sin(\alpha_{N+1}) h_{2N-1}^{(N)} \\
h_1^{(N+1)} &= \sin(\alpha_{N+1}) h_0^{(N)} \\
h_{2i+1}^{(N+1)} &= \sin(\alpha_{N+1}) h_{2i}^{(N)} + \cos(\alpha_{N+1}) h_{2i-1}^{(N)} \\
h_{2N+1}^{(N+1)} &= \cos(\alpha_{N+1}) h_{2N-1}^{(N)}.
\end{aligned} \tag{5}$$

The parameterization based on (5) satisfies the orthonormality conditions for filter banks presented in [22],

$$\sum_{i=0}^{2N-1} \left[h_i^{(N)}\right]^2 = 1, \quad N \geq 1, \tag{6}$$

$$\sum_{i=0}^{2N-1-2m} h_i^{(N)} h_{i+2m}^{(N)} = 0, \quad m = 1, 2, \ldots, N-1,\ N \geq 2, \tag{7}$$

the proof of these facts can be checked in [23].

There are extensions to the formulation of (5) to ensure greater regularity, in this sense it can be mentioned the works that present these extensions: additional restrictions to ensure at least two vanishing moments are proposed in [24] and additional restrictions to ensure at least three vanishing moments are proposed in [23,25].

3. Proposed state space description

For the transfer function characterized by a FIR filter used in this paper we have

$$H(z) = \sum_{i=0}^{2N-1} h_i z^{-i} \tag{8}$$

and

$$G(z) = \sum_{i=0}^{2N-1} g_i z^{-i} \tag{9}$$

with

$$g_i = (-1)^{i+1} h_{2N-1-i} \tag{10}$$

where the coefficients $\{h_i\}$ are given by (5). Eqs. (8) and (9) are the transfer functions of the low-pass and high-pass filters, respectively [8].

Equations (5), (8) and (9) follow the procedure described in Fig. 1, where the first delay operator is z^{-1} and the remainders are z^{-2}. C_i and S_i denote $\sin(\theta_i)$ and $\cos(\theta_i)$, respectively, where $\theta = \begin{bmatrix} \theta_1 & \theta_2 & \cdots & \theta_N \end{bmatrix}$ and $\theta_1 = \alpha_N, \theta_2 = \alpha_{N-1}, \ldots, \theta_N = \alpha_1$. In Fig. 1, y_1 is the output signal corresponding to the input signal $\mathbf{u}$ filtered by the low-pass filter and y_2 corresponds to the input signal $\mathbf{u}$ filtered by the high-pass filter.

The model in the state space is obtained as

$$\mathbf{x}[k+1] = \mathbf{A}\mathbf{x}[k] + \mathbf{B}\mathbf{u}[k], \tag{11}$$

$$\mathbf{y}[k] = \mathbf{C}\mathbf{x}[k] + \mathbf{D}\mathbf{u}[k], \tag{12}$$

where $\mathbf{x} = [x_1 \quad x_2 \quad \cdots \quad x_{2N-1}]^T$ and k denotes the kth sampling instant.

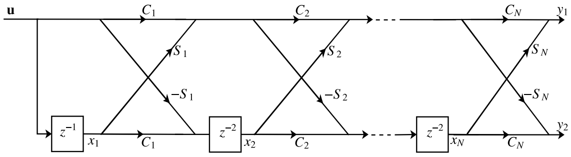

Fig. 2. Parameterization of the filter bank in the state space.

The definition of x_i presented in the previous paragraph is shown in Fig. 2.
From Fig. 2, it follows:

$$\begin{cases} x_1[k+1] = u[k], \\ x_2[k+2] = C_1 x_1[k] - S_1 u[k], \\ x_3[k+2] = C_2 x_2[k] - S_2 S_1 x_1[k] - S_2 C_1 u[k], \\ x_4[k+2] = C_3 x_3[k] - S_3 S_2 x_2[k] - S_3 C_2 S_1 x_1[k] - S_3 C_2 C_1 u[k] \\ \vdots \\ x_{N-1}[k+2] = C_{N-2} x_{N-2}[k] - S_{N-2} S_{N-3} x_{N-3}[k] - \sum_{j=2}^{N-3} \left[S_{N-2} S_{j-1} x_{j-1}[k] \prod_{i=j}^{N-3} C_i \right] - S_{N-2} \prod_{i=1}^{N-3} C_i u[k], \\ x_N[k+2] = C_{N-1} x_{N-1}[k] - S_{N-1} S_{N-2} x_{N-2}[k] - \sum_{j=2}^{N-2} \left[S_{N-1} S_{j-1} x_{j-1}[k] \prod_{i=j}^{N-2} C_i \right] - S_{N-1} \prod_{i=1}^{N-2} C_i u[k]. \end{cases} \tag{13}$$

Defining

$$\begin{cases} x_{N+1}[k] = x_2[k+1], \\ x_{N+2}[k] = x_3[k+1], \\ x_{N+3}[k] = x_4[k+1], \\ \vdots \\ x_{2N-1}[k] = x_N[k+1], \end{cases} \tag{14}$$

implying

$$\begin{cases} x_{N+1}[k+1] = x_2[k+2], \\ x_{N+2}[k+1] = x_3[k+2], \\ x_{N+3}[k+1] = x_4[k+2], \\ \vdots \\ x_{2N-1}[k+1] = x_N[k+2], \end{cases} \tag{15}$$

it follows from (13)–(15) that

$$\begin{cases} x_1[k+1] = u[k], \\ x_2[k+1] = x_{N+1}[k], \\ x_3[k+1] = x_{N+2}[k], \\ \vdots \\ x_N[k+1] = x_{2N-1}[k], \\ x_{N+1}[k+1] = C_1 x_1[k] - S_1 u[k], \\ x_{N+2}[k+1] = C_2 x_2[k] - S_2 S_1 x_1[k] - S_2 C_1 u[k], \\ x_{N+3}[k+1] = C_3 x_3[k] - S_3 S_2 x_2[k] - S_3 C_2 S_1 x_1[k] - S_3 C_2 C_1 u[k], \\ \vdots \\ x_{2N-2}[k+1] = C_{N-2} x_{N-2}[k] - S_{N-2} S_{N-3} x_{N-3}[k] - \sum_{j=2}^{N-3} \left[S_{N-2} S_{j-1} x_{j-1}[k] \prod_{i=j}^{N-3} C_i \right] - S_{N-2} \prod_{i=1}^{N-3} C_i u[k], \\ x_{2N-1}[k+1] = C_{N-1} x_{N-1}[k] - S_{N-1} S_{N-2} x_{N-2}[k] - \sum_{j=2}^{N-2} \left[S_{N-1} S_{j-1} x_{j-1}[k] \prod_{i=j}^{N-2} C_i \right] - S_{N-1} \prod_{i=1}^{N-2} C_i u[k]. \end{cases} \tag{16}$$

The state matrix (or system matrix) **A** presented in (11) is given by

$$\mathbf{A} = \begin{bmatrix} 0 & 0 & 0 & \cdots & 0 & 0 & 0 & 0 & 0 & \cdots & 0 \\ 0 & 0 & 0 & \cdots & 0 & 0 & 1 & 0 & 0 & \cdots & 0 \\ 0 & 0 & 0 & \cdots & 0 & 0 & 0 & 1 & 0 & \cdots & 0 \\ 0 & 0 & 0 & \cdots & 0 & 0 & 0 & 0 & 1 & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \cdots & 0 & 0 & 0 & 0 & 0 & \cdots & 1 \\ C_1 & 0 & 0 & \cdots & 0 & 0 & 0 & 0 & 0 & \cdots & 0 \\ -S_2S_1 & C_2 & 0 & \cdots & 0 & 0 & 0 & 0 & 0 & \cdots & 0 \\ -S_3C_2S_1 & -S_3S_2 & C_3 & \cdots & 0 & 0 & 0 & 0 & 0 & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \cdots & \vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\ S_{N-1}S_1\prod_{i=2}^{N-2}C_i & S_{N-1}S_2\prod_{i=3}^{N-2}C_i & S_{N-1}S_3\prod_{i=4}^{N-2}C_i & \cdots & C_{N-1} & 0 & 0 & 0 & 0 & \cdots & 0 \end{bmatrix}, \tag{17}$$

and the input matrix **B** is given by

$$\mathbf{B} = \begin{bmatrix} 1 \\ 0 \\ \vdots \\ 0 \\ -S_1 \\ -S_2C_1 \\ \vdots \\ -S_{N-1}\prod_{i=1}^{N-2}C_i \end{bmatrix}. \tag{18}$$

The output matrix **C** and the feedthrough (or feedforward) matrix **D** presented in (12) are given by

$$\mathbf{C} = \begin{bmatrix} S_1\prod_{i=2}^{N}C_i & S_2\prod_{i=3}^{N}C_i & \cdots & S_{N-2}\prod_{i=N-1}^{N}C_i & S_{N-1}C_N & S_N & 0 & \cdots & 0 \\ -S_NS_1\prod_{i=2}^{N-1}C_i & -S_NS_2\prod_{i=3}^{N-1}C_i & \cdots & -S_NS_{N-2}\prod_{i=N-1}^{N-1}C_i & -S_NS_{N-1} & C_N & 0 & \cdots & 0 \end{bmatrix} \tag{19}$$

and

$$\mathbf{D} = \begin{bmatrix} \prod_{i=1}^{N}C_i \\ -S_N\prod_{i=1}^{N-1}C_i \end{bmatrix}, \tag{20}$$

respectively.

Let ($\mathbf{A}$, $\mathbf{B}$, $\mathbf{C}$, $\mathbf{D}$) be the minimal realization for $H(z)$, then it can be verified that λ is a pole of the $H(z)$ if, and only if, it is an eigenvalue for **A**. Then the stability condition is equivalent to the condition that $|\lambda_i| < 1$ [4]. When $H(z)$ is a FIR filter, all eigenvalues of **A** are zero (i.e., all these eigenvalues are inside the unit circle). The definition of minimal realization is presented from the following lemmas, with the purpose of verifying if this realization is minimal. Proofs and further discussion of these lemmas can be found in [4].

Lemma 1 ([4]). *A structure with n delays (i.e., n state variables) is reachable if, and only if, matrices* **A** *and* **B** *are such that matrix*

$$\mathcal{R}_{\mathbf{A},\mathbf{B}} = \begin{bmatrix} \mathbf{B} & \mathbf{AB} & \mathbf{A}^2\mathbf{B} & \cdots & \mathbf{A}^{n-1}\mathbf{B} \end{bmatrix} \tag{21}$$

has full rank n.

Lemma 2 ([4]). *A structure with n delays (i.e., n state variables) is observable if, and only if, matrices* $\mathbf{C}$ *and* $\mathbf{A}$ *are such that matrix*

$$\mathcal{S}_{\mathbf{C},\mathbf{A}} = \begin{bmatrix} \mathbf{C} \\ \mathbf{CA} \\ \mathbf{CA}^2 \\ \vdots \\ \mathbf{CA}^{n-1} \end{bmatrix} \tag{22}$$

has full rank n.

Lemma 3 ([4]). *Suppose a realization* ($\mathbf{A}, \mathbf{B}, \mathbf{C}, \mathbf{D}$) *is reachable and observable. Then it is minimal, i.e., there does not exist an equivalent structure for the same transfer function with fewer delays.*

The next step is to verify if the state space realization proposed in this paper using (17)–(20) is minimal, according to the previous lemmas.

The state space realization proposed is really minimal, according to the following theorem statement:

Theorem 4. *A realization* ($\mathbf{A}, \mathbf{B}, \mathbf{C}, \mathbf{D}$) *given by* (17)–(20), *respectively, with all angles different of* $0, \pi/2, \pi$ *and* $3\pi/2$, *is minimal because it is reachable and observable.*

Proof. From (21), (17) and (18) a $(2N-1) \times (2N-1)$ matrix is obtained, where, for simplicity of notation, φ represents nonzero elements:

$$\mathcal{R}_{\mathbf{A},\mathbf{B}} = \begin{bmatrix}
\varphi & 0 & 0 & 0 & 0 & 0 & 0 & \cdots & 0 & 0 \\
0 & \varphi & \varphi & 0 & 0 & 0 & 0 & \cdots & 0 & 0 \\
0 & \varphi & \varphi & \varphi & \varphi & 0 & 0 & \cdots & 0 & 0 \\
0 & \varphi & \varphi & \varphi & \varphi & \varphi & \varphi & \cdots & 0 & 0 \\
\vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\
0 & \varphi & \varphi & \varphi & \varphi & \varphi & \varphi & \cdots & 0 & 0 \\
0 & \varphi & \varphi & \varphi & \varphi & \varphi & \varphi & \cdots & \varphi & \varphi \\
\varphi & \varphi & 0 & 0 & 0 & 0 & 0 & \cdots & 0 & 0 \\
\varphi & \varphi & \varphi & \varphi & 0 & 0 & 0 & \cdots & 0 & 0 \\
\varphi & \varphi & \varphi & \varphi & \varphi & \varphi & 0 & \cdots & 0 & 0 \\
\varphi & \varphi & \varphi & \varphi & \varphi & \varphi & \varphi & \cdots & 0 & 0 \\
\vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\
\varphi & \varphi & \varphi & \varphi & \varphi & \varphi & \varphi & \cdots & 0 & 0 \\
\varphi & \varphi & \varphi & \varphi & \varphi & \varphi & \varphi & \cdots & \varphi & 0
\end{bmatrix}.$$

All rows and columns of $\mathcal{R}_{\mathbf{A},\mathbf{B}}$ are linearly independent. Then the determinant is nonzero (i.e., $\mathcal{R}_{\mathbf{A},\mathbf{B}}$ is nonsingular) implying that this matrix has full rank. Therefore Lemma 1 is checked (the realization is reachable).

From (22), (17) and (18) a $(4N-2) \times (2N-1)$ matrix is obtained, again φ representing nonzero elements:

$$\mathcal{S}_{\mathbf{C},\mathbf{A}} = \begin{bmatrix}
\varphi & \varphi & \varphi & \cdots & \varphi & 0 & 0 & 0 & \cdots & 0 \\
\varphi & \varphi & \varphi & \cdots & \varphi & 0 & 0 & 0 & \cdots & 0 \\
0 & 0 & 0 & \cdots & 0 & \varphi & \varphi & \varphi & \cdots & \varphi \\
\vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\
\varphi & \varphi & \varphi & \cdots & 0 & 0 & 0 & 0 & \cdots & 0 \\
\varphi & \varphi & \varphi & \cdots & 0 & 0 & 0 & 0 & \cdots & 0 \\
0 & 0 & 0 & \cdots & 0 & \varphi & \varphi & \varphi & \cdots & 0 \\
0 & 0 & 0 & \cdots & 0 & \varphi & \varphi & \varphi & \cdots & 0 \\
\varphi & \varphi & \varphi & \cdots & 0 & 0 & 0 & 0 & \cdots & 0 \\
\varphi & \varphi & \varphi & \cdots & 0 & 0 & 0 & 0 & \cdots & 0 \\
0 & 0 & 0 & \cdots & 0 & \varphi & \varphi & 0 & \cdots & 0 \\
0 & 0 & 0 & \cdots & 0 & \varphi & \varphi & 0 & \cdots & 0 \\
\varphi & \varphi & 0 & \cdots & 0 & 0 & 0 & 0 & \cdots & 0 \\
\varphi & \varphi & 0 & \cdots & 0 & 0 & 0 & 0 & \cdots & 0 \\
0 & 0 & 0 & \cdots & 0 & \varphi & 0 & 0 & \cdots & 0 \\
0 & 0 & 0 & \cdots & 0 & \varphi & 0 & 0 & \cdots & 0 \\
\varphi & 0 & 0 & \cdots & 0 & 0 & 0 & 0 & \cdots & 0 \\
\varphi & 0 & 0 & \cdots & 0 & 0 & 0 & 0 & \cdots & 0
\end{bmatrix}.$$

All rows and columns of $\mathcal{S}_{\mathbf{C},\mathbf{A}}$ are linearly independent. Then this matrix has full rank. Therefore Lemma 2 is checked (the realization is observable). According to Lemma 3 and the previous paragraphs, the realization is minimal. □

As set out by Theorem 4, the proposed realization $(\mathbf{A}, \mathbf{B}, \mathbf{C}, \mathbf{D})$ is minimal (reachable and observable) and, as already mentioned, the characteristic polynomial of $\mathbf{A}$ with size $N \times N$ is S^N. This fact implies that, when $H(z)$ is a FIR filter, all eigenvalues of $\mathbf{A}$ are zero. In other words, all these eigenvalues are inside the unit circle, fulfilling the stability condition.

4. Illustrative examples

This section presents three examples of wavelets for the proposed parameterization, namely the Daubechies wavelets `db2`, `db3` and `db4`. The procedure proposed in [27] was employed to define the sets of angular parameters representing the filter coefficients.

- `db2`: The set of angular parameters $\alpha = \{-1.3090,\ 2.0944\}$ is obtained from the coefficients of the `db2` wavelet lowpass filter. Therefore, from (17)–(20), it follows that matrices A, B, C and D are

$$\mathbf{A} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ -0.5 & 0 & 0 \end{bmatrix}, \qquad \mathbf{B} = \begin{bmatrix} 1 \\ 0 \\ -0.866 \end{bmatrix},$$

$$\mathbf{C} = \begin{bmatrix} 0.2241 & -0.9659 & 0 \\ 0.8365 & 0.2588 & 0 \end{bmatrix} \quad \text{and} \quad \mathbf{D} = \begin{bmatrix} -0.1294 \\ -0.4830 \end{bmatrix}.$$

- `db3`: In an analogous manner to the previous case, with the set of angular parameters $\alpha = \{1.4653, 0.4998, -1.1797\}$, it is obtained

$$\mathbf{A} = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 0.3812 & 0 & 0 & 0 & 0 \\ 0.4431 & 0.8777 & 0 & 0 & 0 \end{bmatrix}, \qquad \mathbf{B} = \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0.9245 \\ -0.1827 \end{bmatrix},$$

$$\mathbf{C} = \begin{bmatrix} -0.0854 & 0.0505 & 0.9944 & 0 & 0 \\ 0.8069 & -0.4766 & 0.1053 & 0 & 0 \end{bmatrix} \quad \text{and} \quad \mathbf{D} = \begin{bmatrix} 0.0352 \\ -0.3327 \end{bmatrix}.$$

- `db4`: For `db4`, with the set of angular parameters $\alpha = \{-1.5248, -0.2537, 0.6814, 1.8826\}$:

$$\mathbf{A} = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ -0.3067 & 0 & 0 & 0 & 0 & 0 & 0 \\ -0.5995 & 0.7767 & 0 & 0 & 0 & 0 & 0 \\ 0.1856 & 0.1581 & 0.9680 & 0 & 0 & 0 & 0 \end{bmatrix}, \qquad \mathbf{B} = \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \\ -0.9518 \\ 0.1932 \\ -0.0598 \end{bmatrix},$$

$$\mathbf{C} = \begin{bmatrix} 0.0329 & 0.0280 & -0.0115 & -0.9989 & 0 & 0 & 0 \\ 0.7148 & 0.6091 & -0.2507 & 0.0460 & 0 & 0 & 0 \end{bmatrix} \quad \text{and} \quad \mathbf{D} = \begin{bmatrix} -0.0106 \\ -0.2304 \end{bmatrix}.$$

It is worth noting that the size of the matrices varies according to the number of angular parameters α.

5. Conclusions

In this paper a new state space description based on the formulation of Sherlock and Monro [8] was proposed, where the representation characterizes a FIR filter with coefficients given by orthonormal basis functions.

It was considered a perfect reconstruction two-channel orthonormal wavelet FIR filter bank, where the orthonormality condition is a property of the parameterization of Sherlock and Monro, and in the procedure for parameterizing the filter bank it was considered the lattice structure, such that the state space description is minimal.

Besides the minimality condition, reachability and observability conditions were achieved for the proposed state space description, allowing the design of controllers and observers, in addition to other important outcomes ensured by those properties. It is also important to mention that all eigenvalues of $\mathbf{A}$ are zero, i.e., all these eigenvalues are inside the unit circle, fulfilling the stability condition.

The proposed approach can also be valuable for use with nonlinear models. Exploring Wiener models [28–31] or Hammerstein [32,28] is a possibility, involving static nonlinearities in the model output (Wiener) or input (Hammerstein). In such cases, the proposed state-space formulation of the wavelet filter bank could be directly coupled to the linear side of the model under consideration. In the context of time-varying models, it is possible to explore the proposal in conjunction with system identification techniques that already use wavelet approaches [33,34].

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