UNIVERSIDADE ESTADUAL PAULISTA "JÚLIO DE MESQUITA FILHO"
FACULDADE DE ENGENHARIA
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DEVELOPMENT OF A ROBUST OPTIMIZATION METHOD FOR OPTIMAL OPERATION OF MICROGRIDS WITH ELECTRIC VEHICLES AND RENEWABLE ENERGY RESOURCES

Ilha Solteira
2023

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Tese apresentada à Faculdade de Engenharia do Câmpus de Ilha Solteira – UNESP como parte dos requisitos para a obtenção do título de Doutor em Engenharia Elétrica. Área de Conhecimento: Automação.

Prof. Dr. John Fredy Franco Baquero
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Ilha Solteira
2023

FICHA CATALOGRÁFICA
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Z27d

Zandrazavi, Seyed Farhad.
Development of a robust optimization method for optimal operation of microgrids with electric vehicles and renewable energy resources / Seyed Farhad Zandrazavi. -- Ilha Solteira: [s.n.], 2023
71 f. : il.

Tese (doutorado) - Universidade Estadual Paulista. Faculdade de Engenharia de Ilha Solteira. Área de conhecimento: Automação, 2023

Orientador: John Fredy Franco

Inclui bibliografia

1. Otimização convexa. 2. Veículos elétricos. 3. Gestão de energia. 4. Microrredes. 5. Geração renovável. 6. Modelagem de incerteza.

Amanda Sertori dos Santos
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Câmpus de Ilha Solteira

CERTIFICADO DE APROVAÇÃO

TÍTULO DA TESE: DEVELOPMENT OF A ROBUST OPTIMIZATION METHOD FOR OPTIMAL OPERATION OF MICROGRIDS WITH ELECTRIC VEHICLES AND RENEWABLE ENERGY RESOURCES

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‘Dedicated to my mom, dad, and sister – your endless love and unwavering support inspire every word of this thesis’

ACKNOWLEDGMENTS

I would like to express my heartfelt gratitude to my dearest family, my mom (Mahin), dad (Siavosh), and sister (Mitra), for their unwavering love, support, and encouragement throughout this journey. Your belief in me has been my greatest motivation, and I dedicate this achievement to you.

I extend my sincere thanks to my adorable supervisor, Prof. John Fredy Franco, for his invaluable guidance and support during these four years of research. Your friendship, mentorship, and leadership has been instrumental in shaping this thesis.

I am also deeply appreciative of the hospitality and assistance provided by Prof. João Soares and Prof. Zita Vale from GECAD during my internship at ISEP in Portugal. Their insights and suggestions were invaluable to my work.

I am profoundly grateful to Prof. Miadreza Shafiekhah for not only his help but also for treating me like a friend during my one-year visiting period at the University of Vaasa in Finland. His trust in me and the warm hospitality made my time there both productive and enjoyable.

Furthermore, I must express my deep appreciation to Prof. Alejandra Tabares. Despite her time constraints and the long distance that separated us, she graciously collaborated with me on numerous publications, and her guidance greatly enriched my research journey.

And finally, with thankful heart to all my colleagues and friends in the department of Electrical Engineering, and the LaPSEE laboratory. This research was supported by the Brazilian institution of Coordination for the Improvement of Higher Education Personnel (CAPES) – Finance Code 001.

This thesis would not have been possible without the support and encouragement of all these wonderful individuals, and for that, I am truly grateful.

RESUMO

As microrredes podem abrir caminho à descarbonização dos sistemas energéticos, fornecendo excelentes infraestruturas para a proliferação de recursos energéticos distribuídos e de veículos elétricos. No entanto, na presença da geração de energia renovável e de veículos elétricos, a gestão energética das microrredes parece inseparável das incertezas. Por um lado, as previsões imperfeitas da geração intermitente de energia renovável, da demanda da carga elétrica e do preço da eletricidade impõem um elevado nível de incertezas no funcionamento diário ótimo das microrredes. Por outro lado, negligenciar as incertezas por parte dos operadores de microrredes pode levar a soluções não ótimas ou mesmo a pontos operacionais inviáveis. A fim de modelar matematicamente o gerenciamento ótimo de energia de microrredes, inicialmente alguns conceitos fundamentais ligados à otimização matemática são brevemente introduzidos, incluindo otimalidade local, viabilidade, convexidade, programação linear, programação inteira, programação linear inteira mista, programação não linear não convexa e programação não linear convexa. Além disso, a formulação do fluxo de potência em microrredes é extraída passo a passo e então usada para modelar o gerenciamento determinístico de energia como um problema de programação não linear não convexo. Dessa forma, utiliza-se a relaxação e a linearização para transformar o referido modelo em um modelo convexo de forma a garantir a otimalidade global das soluções. Em seguida, para abranger a incerteza, são introduzidos os métodos mais conhecidos e amplamente utilizados na literatura para modelar a incerteza na operação de microrredes e as suas características são explicadas e comparadas. Para modelar a incerteza na prática, é apresentado um modelo de programação cônica estocástica inteira mista de dois estágios. As incertezas ligadas à geração fotovoltaica, geração de energia eólica, demanda elétrica e preços da eletricidade estão incluídas no modelo por meio de cenários. Vale ressaltar que os modelos de otimização são desenvolvidos em AMPL e resolvidos via solucionador CPLEX e testes são realizados em sistemas de teste IEEE de 33 barramentos e 69 barramentos, para avaliar a eficácia dos modelos propostos. Os resultados mostram como considerar a reconfiguração das microrredes pode afetar positivamente a operação, contribuindo para a redução de custos, desvios de tensão e perdas de energia. Além disso, os resultados mostram que a consideração de cenários pode contribuir para a modelagem da incerteza, mas afetará a solução ótima em comparação com métodos determinísticos, uma vez que as restrições devem ser satisfeitas para todos os cenários, simultaneamente.

Palavras-chave: otimização convexa; veículos elétricos; gestão de energia; microrredes; geração renovável; modelagem de incerteza.

ABSTRACT

Microgrids can pave the way for the decarbonization of power systems by providing excellent infrastructure for the proliferation of distributed energy resources and electric vehicles. Nevertheless, in the presence of renewable energy generation and electric vehicles, the energy management of microgrids seems inseparable from uncertainties. On one hand, imperfect forecasts of intermittent renewable energy generation, electric load demand, and electricity price impose a high level of uncertainties in the daily optimal operation of microgrids. On the other hand, neglecting uncertainties by microgrids' operators may lead to non-optimal solutions or even infeasible operational points. In order to mathematically model the optimal energy management of microgrids, firstly some fundamental concepts linked to mathematical optimization are briefly introduced, including local optimality, feasibility, convexity, linear programming, integer programming, mixed-integer linear programming, nonlinear nonconvex programming, and nonlinear convex programming. In addition, the power flow formulation in microgrids is extracted step by step, and then it is used to model deterministic energy management of microgrids as a nonlinear nonconvex programming problem. Then, relaxation and linearization are used to transform the aforementioned model into a convex model so as to guarantee the global optimality of the solutions. Secondly, to embrace uncertainty, the most well-known methods deployed widely in the literature for modeling uncertainty in the operation of microgrids are introduced and their characteristics are explained and compared. In order to model uncertainty in practice, a two-stage stochastic mixed-integer conic programming model is presented. Uncertainties linked to photovoltaic generation, wind power generation, electric demand, and electricity prices are included in the model via scenarios. It is noteworthy that the optimization models are developed in AMPL and solved via the CPLEX solver and tests are carried out on IEEE 33-bus and 69-bus test systems, to evaluate the effectiveness of the proposed models. The results show how considering the reconfigurability of microgrids can positively affect the operation by contributing to cost, voltage deviation and power loss reduction. Moreover, the results show considering scenarios can contribute to uncertainty modeling, yet it will affect the optimal solution compared to deterministic methods, as the constraints must be satisfied for every scenario, simultaneously.

Keywords: convex optimization; electric vehicles; energy management; microgrids; renewable generation; uncertainty modeling.

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NOMENCLATURE

Indices

ω	Index of block in piecewise linearization
bs	Index of battery
dg	Index of distributed generation
k, m, n	Indices of bus
km, mn	Indices of line
pv	Index of photovoltaic
s	Index of scenarios
t	Index of time interval
wt	Index of wind turbine

Parameters

π_s	Probability of scenario
β^{BS}	Self-discharge rate
Δt_t	Duration of time interval
η_{bs}^{BS}	Charging efficiency
η_{bs}^{BS}	Discharging efficiency
$a_{dg}^{DG}, b_{dg}^{DG}, c_{dg}^{DG}$	Cost coefficient of distributed generation
$C_{t,s}^{GR}$	Electricity price at the grid
$\overline{E}^{BS}$	Maximum stored energy of battery
$\underline{E}^{BS}$	Minimum stored energy of battery
$\overline{I}_{mn}$	Maximum current at line mn
$\overline{P}^{BS}$	Maximum charging/discharging of battery energy
$P_{m,t,s}^{D}$	Active power demand
$\overline{P}_{dg}^{DG}$	Maximum capacity of distributed generation
$\underline{P}_{dg}^{DG}$	Minimum capacity of distributed generation
$\tilde{P}_{pv,t,s}^{PV}$	Available generation of photovoltaic
$\tilde{P}_{wt,t,s}^{WT}$	Available generation of wind turbine
$Q_{m,t,s}^{D}$	Reactive power demand
φ_{dg}^{DG}	Power factor of distributed generation
R_{mn}	Resistance of line mn
S^{Tr}	Capacity of transformer
$\overline{V}$	Upper bound of voltage
$\underline{V}$	Lower bound of voltage
V^{ref}	Reference voltage
X_{mn}	Reactance of line mn
Z_{mn}	Impedance of line mn

Variables

$E_{bs,t,s}^{BS}$	Energy stored in battery
$I_{mn,t,s}^{sqr}$	Square of current in line mn
$P_{mn,t,s}$	Power at line mn
$P_{mn,t,s}^{+}$	Power at line mn in forward direction
$P_{mn,t,s}^{-}$	Power at line mn in backward direction
$P_{bs,t,s}^{BS}$	Power of battery
$P_{bs,t,s}^{BS-}$	Power injected by battery
$P_{bs,t,s}^{BS+}$	Power absorbed by battery
$P_{dg,t,s}^{DG}$	Power of distributed generation

$P_{t,s}^{GR}$	Active power exchanged between microgrid and power system
$P_{pv,t,s}^{CPV}$	Power of photovoltaic
$\Theta_{pv,t,s}^{PV}$	Power curtailment of photovoltaic
$P_{wt,t,s}^{CWT}$	Power of wind turbine
$\Theta_{wt,t,s}^{WT}$	Power curtailment of wind turbine
$Q_{dg,t,s}^{DG}$	Reactive power of distributed generation
$Q_{t,s}^{GR}$	Reactive power exchanged between microgrid and power system
$u_{dg,t}^{DG}$	Commitment of distributed generation
$V_{n,t}$	Voltage magnitude

CONTENTS

1 INTRODUCTION

1.1 MOTIVATION

Microgrids are defined as compact power systems comprising electric loads, control systems, and distributed energy resources. Typically, these microgrids are designed to operate in a grid-connected mode, allowing them to interact with the main power system, thereby enhancing overall reliability and resilience. Their controllability also makes them ideal for integrating renewable energy-based distributed generations and electric vehicles, facilitating the transition from fuel-based power systems to clean-energy-based alternatives. Nevertheless, it is crucial for microgrid operators to exercise caution and prudence when managing energy within these systems. Irresponsible energy management can lead to increased energy costs and, in the worst-case scenarios, may result in malfunctions and damage to microgrid components, including electric loads and distributed energy resources. Therefore, careful attention to technical limitations is essential to ensure the smooth and safe operation of microgrids.

The literature contains a plethora of high-quality papers exploring microgrid characteristics and energy management. Parhizi et al. (2015) offer a comprehensive review of microgrids, covering various aspects such as control, energy management, planning, demand side management, and protection. The paper also highlights the benefits microgrids provide to the main power grid, including improved reliability, resiliency, and power quality. Alsaidan et al. (2017) delve into distributed energy storage sizing and placement in microgrids, emphasizing the positive impact on optimal energy management and control, leading to operational cost reduction. Another examination of microgrid energy management is provided by Jirdehi et al. (2020). This review encompasses different objective functions typically considered in microgrid energy management, encompassing cost, pollution, power losses, and reliability indices. Additionally, various electric loads and technologies related to energy resources and energy storage systems are explained and categorized. Zia et al. (2018) contribute to the study of microgrid energy management, focusing on a range of metaheuristic optimization models, including differential evolution, ant colony optimization, gravitational search algorithm, imperialist competition algorithm, and tabu search algorithm, among others.

Considering the aforementioned papers, there are valuable review articles in the literature about the energy management of microgrids. Hence, by taking advantage of such references it is possible to distinguish and comprehend the different models for energy management of microgrids, such as mixed-integer nonlinear nonconvex, mixed-integer nonlinear convex, and mixed-integer linear programming. However, a profound understanding of these models depends on previous knowledge of mathematical optimization.

1.2 BASIC CONCEPTS IN OPTIMIZATION

The energy management of microgrids can be conceptualized as an optimization challenge. Hence, possessing a solid grasp of fundamental concepts in mathematical optimization, including feasibility, local optimality, global optimality, and convexity, can significantly enhance one's comprehension of the various models showcased in this thesis and other related references.

1.2.1 FEASIBILITY AND OPTIMALITY

A common form of a constrained optimization problem (minimization in this case) is presented in (1), wherein x denotes the decision variables. $f(x)$ is called the objective function, which is expected to be minimized, and $g(x)$ and $h(x)$ correspond to the inequality and equality constraints, respectively.

$$\begin{aligned} & \min f(x) \\ & \textit{s.to:} \\ & \begin{cases} g(x) \leq 0 \\ h(x) = 0 \end{cases} \end{aligned} \tag{1}$$

For the sake of simplicity and coherence, without loss of the generality, minimization is only considered through this study since by negating the objective function, a minimization problem can be transformed easily into an equivalent maximization problem as presented in (2), and vice versa.

$$\begin{aligned} & -\max(-f(x)) \\ & \textit{s.to:} \\ & \begin{cases} g(x) \leq 0 \\ h(x) = 0 \end{cases} \end{aligned} \tag{2}$$

A solution $\tilde{x}$ is a feasible solution if it does not violate any equality and inequality constraints. In other words, if all the feasible solutions are included, then the feasibility region Ω_x means that for any solution where $x \in \Omega_x$, x is an acceptable solution. However, in mathematical optimization, finding an acceptable solution is usually not sufficient; on the contrary, the object of solving an optimization problem generally is to find the optimal solution x^*, meaning that there is no other solution that leads to a smaller value of the objective function. In other words, if x^* is the best solution, then the inequality in (3) is valid.

$$f(x) \geq f(x^*) \quad \forall x \epsilon \Omega_x \tag{3}$$

It is noteworthy, that an optimization problem might have more than one optimal solution (i.e., x^*), yet the optimal value (i.e., $f(x^*)$) is the same for them all. On the other hand, it is possible that an optimization problem does not have any feasible solution, in this case, the problem is called infeasible. It is noteworthy that further information on mathematical optimization can be found in Sioshansi and Conejo, (2017) and Luenberger and Ye, (1984).

1.2.2 LINEAR PROGRAMMING

If all the aforementioned functions in (1) are linear functions and the decision variables (x) are all continuous, the constrained optimization problem is called linear optimization and can be written as follows:

$$\min f(x) = \sum_i C_i x_i$$

s.to:

$$\begin{cases} \sum_i A_{j,i}^g x_i - b_j^g \leq 0 \\ \sum_i A_{j,i}^h x_i - b_j^h = 0 \end{cases} \tag{4}$$

in which C_i, $A_{j,i}^g$, $A_{j,i}^h$, b_j^g, and b_j^h are all known constants (i.e. parameters). In linear programming, a feasible region, if exists, is always polyhedral and the optimal solution (or solutions) lay in the vertices (also known as extreme points). Linear programming problems are highly important in mathematical optimization and engineering, as there are so efficient methods available for solving them such as the simplex method and the interior points. Linear programming has the capability to be used for solving substantially large-scale problems, by hundreds of thousands of constraints and decision variables in a rational time taking advantage of commercial software such as CPLEX (ILOG, 2008) or Gurobi (BIXBY, 2007). The further information on linear programming can be found in Robert, (2021) or Bazaraa et al., (2008).

1.2.3 MIXED-INTEGER LINEAR PROGRAMMING

If some of the variables in (4) are integer variables the optimization problem is called mixed-integer linear programming and can be written as (5).

$$\min f(x) = \left(\sum_i C_i x_i + \sum_k C_k y_k \right)$$

s.to:

$$\begin{cases} \sum_i A^g_{j,i} x_i + \sum_k A^g_{j,k} y_k - b^g_j \leq 0 \\ \sum_i A^h_{j,i} x_i + \sum_k A^h_{j,k} y_k - b^h_j = 0 \\ \qquad\qquad y_k \in \{Z\} \end{cases} \tag{5}$$

Despite the existence of methods like branch-and-bound, cutting plane, branch-and-cut, and branch-and-price, specifically designed to address this type of optimization problem, the computation time and tractability of the problem suffer when the number of binary variables increases. As a result, if the number of binary variables becomes extremely large (e.g., 10,000), solving the model using current commercial solvers and standard computers within a reasonable timeframe may become unfeasible.

1.2.4 LOCAL OPTIMALITY, CONVEXITY, AND NONLINEAR PROGRAMMING

In (3), the definition of optimality is presented, which is known as global optimality as well. On the other hand, the aforementioned definition for the optimality can be extended to define another type of optimality, which is called the local optimality. In contrast to the global optimality, a local optimal solution x_l^* does not need to be the optimal solution (the minimum in this case) for all the feasible regions (i.e. $\forall x \in \Omega_x$), rather it is enough to be the best solution in a neighborhood (δ) of the feasible set. Therefore, the formulation presented in (3), can be rewritten to define a local optimal solution x_l^* as indicated in (6), in which the subscript l is added to emphasize on the local optimality of the solution.

$$f(x) \geq f(x_l^*) \quad \forall x \in \Omega_x | \{x - x_l^*\} \leq \delta \tag{6}$$

In a convex optimization problem, all local optimal solutions are also global optimal solutions. In (2), a general compact formula for a constrained optimization problem is presented. This optimization problem is called a convex optimization problem, if the objective function $f(x)$ is a convex function and the respective feasibility set is a convex set Boyd et al., (2004). These conditions are all satisfied for linear programming; thus any local solution in linear programming problems is a global solution as well. Nonetheless, the majority of nonlinear programming problems are not convex, so a local optimal solution is not necessarily the global optimal one.

Nonlinear programming (NLP) is simply an optimization problem that does not fall under the category of linear programming. If the objective function or any constraints involve non-linear functions, then the problem qualifies as nonlinear. To tackle such optimization challenges, several algorithms exist, though their performance heavily relies on the initial points and may lead to locally optimal solutions. Those interested in delving deeper into nonlinear programming can find valuable insights in Bazaraa et al. (2013).

Certain classes of nonlinear optimization problems, such as quadratic programming, second-order cone programming, and semidefinite programming, fall under the category of convex models. These specific types of nonlinear convex optimization models can be efficiently solved using interior point methods, as demonstrated by Nesterov and Nemirovskii (1994), guaranteeing the optimality of the solutions (global optimality). This makes them particularly suitable for mathematically modeling energy management in microgrids, as commercial solvers like CPLEX (ILOG, 2008) can efficiently solve them while ensuring optimal solutions.

1.2.5 MIXED-INTEGER NONLINEAR PROGRAMMING

Similar to mixed-integer linear programming, if at least one of the decision variables in NLP problem is an integer variable, the respective nonlinear problem is called mixed-integer nonlinear programming (MINLP) problem. The above discussions on local optimality, global optimality, and convexity are valid for MINLP as well; for example, mixed-integer second-order cone programming problems can still be solved efficiently by commercial solvers.

1.3 BASICS OF ENERGY MANAGEMENT IN MICROGRIDS

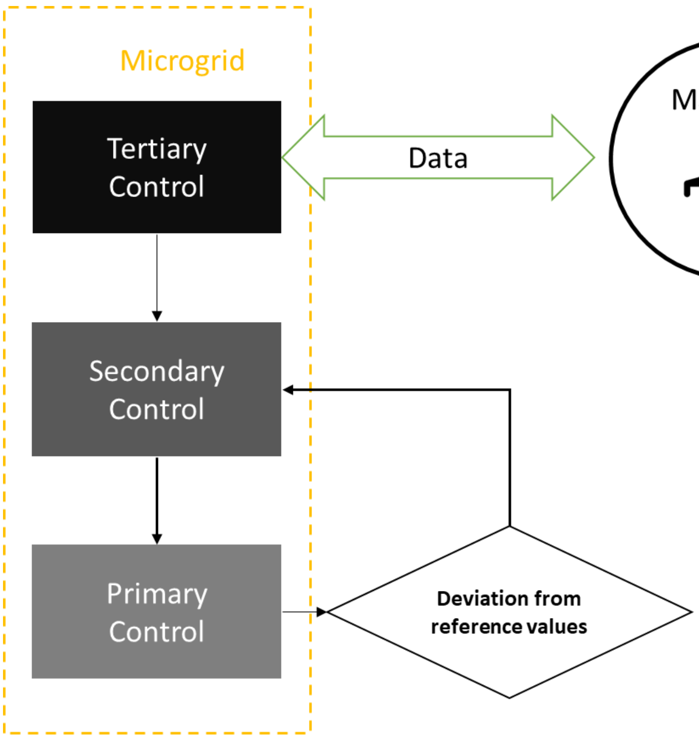
Microgrids, as small-scale distribution-level power systems, offer the advantage of operating independently and have played a pivotal role in facilitating the seamless integration of renewable generation and plug-in electric vehicles. By having their own control and management systems, microgrids effectively address the challenges posed by the high-level integration of intermittent renewable sources like photovoltaic and wind turbine units, as well as the variability of electric loads such as electric vehicles. One of the significant contributions of microgrids lies in their ability to provide ancillary services to the main power grid. They enhance the resiliency and reliability of the entire power system by supplementing extra power generation during specific time periods. This high capability and controllability make microgrids valuable partners in the decarbonization of power systems, offering essential infrastructure for the extended integration of renewable generation and electric vehicles.

Furthermore, microgrids play a vital role in the decentralization of power systems. With their own energy management systems in place, they can effectively coordinate and collaborate with the main power grid, ensuring a more balanced and robust energy distribution. Moreover, microgrids contribute to the democratization of power systems. They actively participate in electricity markets either directly or through aggregators, as discussed in Parhizi et al. (2015). This involvement allows for a more inclusive and diversified approach to power generation and distribution, benefiting both consumers and the overall energy landscape.

1.3.1 HIERARCHICAL STRUCTURE OF ENERGY MANAGEMENT OF MICROGRIDS

Microgrids offer numerous economic and societal advantages, but to fully capitalize on their benefits for consumers and power systems, microgrid operators and planners must make informed decisions. Consequently, achieving optimal energy management and planning is crucial for the successful implementation of microgrids. The hierarchical structure of microgrid energy management, outlined in Figure 1, consists of three distinct levels known as primary, secondary, and tertiary control levels, as described in the works of Guerrero et al. (2011) and Vasquez et al. (2010).

Figure 1 – Hierarchical structure of energy management in microgrids



Source: Created by the author

Primary Control Level: The primary control level plays a pivotal role in maintaining the stability and reliability of microgrids. By closely monitoring voltage, current injection, and frequency references, this level ensures that these essential parameters stay well within acceptable operational ranges. With its real-time operations, the primary control level acts as the first line of defense, promptly responding to any fluctuations or disturbances that may arise within the microgrid. One of the key techniques employed at this level is the droop control technique, which is renowned for its effectiveness in achieving stable and balanced operations. The droop control technique utilizes small variations in the output voltage and frequency to regulate the power flow and maintain the system's equilibrium. By employing this method, the Primary Control Level can quickly adapt to changing conditions and seamlessly address any mismatches between power generation and demand.

Secondary Control Level: The Secondary Control Level, with its focus on reducing deviations from reference values, plays a critical role in fine-tuning the performance of microgrids. Although it operates with a slower response time than the Primary Level, this deliberate pace allows it to carefully assess the system's condition and apply appropriate adjustments to restore voltages and frequencies to their desired levels. As a compensating process, the secondary control acts as a stabilizing force within the microgrid, ensuring that any fluctuations or disturbances are swiftly rectified. By continuously monitoring and comparing actual values with the predefined reference points, the secondary control can detect and address any steady-state errors that may arise.

Tertiary Control Level: The Tertiary Control Level represents the highest tier within the hierarchical energy management structure of microgrids, tasked with optimizing their energy usage. This control level establishes communication with the main power grid to govern and regulate the exchange of active and reactive power between the microgrid and the main grid. Within this level, various crucial decisions are made, including determining the generation of active and reactive power from dispatchable distributed generation units, managing the charging and discharging of energy storage systems, implementing demand response programs, overseeing electric vehicle charging, configuring the microgrid's setup, and controlling the curtailment of intermittent renewable generation. The effectiveness of the tertiary control level significantly impacts the reliability, resiliency, and efficiency of both the microgrid and the larger power systems it interacts with. To ensure seamless energy management, this level operates on an hourly basis for a 24-hour time horizon, generally executing its tasks one day in advance of the energy delivery day. By meticulously planning and optimizing energy usage, the tertiary control level plays a crucial role in the successful operation of microgrids and their integration into the broader energy infrastructure.

As it will be explained in-depth in the following chapters. Various models are presented in the literature for the energy management of microgrids considering different uncertainty modeling techniques. Notwithstanding, there is still a real need for developing robust methods for energy management of unbalanced, balanced, and multi-carrier microgrids for several reasons. Firstly, the majority of the previous works neglecting some important technical issue such as power flow equations or they only focus on balanced microgrids Zandrazavi et al., (2022). In addition, presenting robust models for energy management of microgrids is still challenging due to presence of intermittent renewable energy generation, sporadic loads such as plug-in electric vehicles Zandrazavi et al., (2022), since the models should be exact enough to guarantee the validity of the solution, while the model should be kept tractable.

The focus of this research is on this level of energy management of microgrids and different mathematical models, including mixed-integer linear programming, mixed-integer nonlinear nonconvex programming, and mixed-integer second-order cone programming, are considered. Technical aspects linked to energy management, such as objective function, system constraints, load shedding, and renewable energy curtailment are described and respective formulas are presented in the following chapters.

1.4 OBJETIVES AND CONTRIBUTIONS

- Developing a nonconvex mathematical model for the optimal energy management of balanced and unbalanced microgrids considering renewable and non-renewable distributed generation, batteries, and electric vehicles while including technical constraints and limits;
- Convexifying the aforementioned models so as to guarantee the global optimality of the obtained solutions via commercial solvers such as CPLEX or Gurobi;
- Extending the model to accommodate the uncertainties regarding electric loads, renewable energy generation, and electric vehicles.

1.5 ORGANIZATION OF CHAPTERS

This thesis is organized as below:

Chapter 2: Uncertainty modeling methods such as Monte Carlo simulation, robust optimization, and stochastic programming and their application in the optimal operation of microgrids are reviewed in this chapter.

Chapter 3: Power flow is the backbone of energy management of microgrids as a result in this chapter, power flow modeling for balanced microgrids is presented in detail.

Chapter 4: A deterministic model for optimal energy management of balanced microgrids including the reconfigurable ones is considered and results are depicted and discussed.

Chapter 5: Uncertainty modeling methods for the operation of microgrids are reviewed in this chapter and respective formulations for stochastic optimization of microgrids are presented.

Chapter 6: The conclusions linked to the results obtained in Chapter 4 and Chapter 5 and the perspectives for future work are presented in this chapter.

2 Review on uncertainty modeling in microgrids

Fuzzy method, information gap decision theory, adaptive robust optimization, chance-constrained optimization, and stochastic programming can be mentioned as the most well-known methods deployed widely in the literature for the optimal operation of microgrids. In this chapter, the aforementioned methods for uncertainty modeling are introduced focusing on the operation of microgrids, while their characteristics are explained.

2.1 BACKGROUND

Microgrids can pave the way for the decarbonization of power systems by providing an excellent infrastructure for the proliferation of distributed energy resources and electric vehicles. Nevertheless, considering renewable energy generation and electric vehicles, the operation of microgrids seems inseparable from uncertainties. On one hand, imperfect forecasts of intermittent renewable energy generation, electric load demand, electricity price, and availability of the main grid impose a high level of uncertainties in the daily optimal operation of microgrids. On the other hand, neglecting uncertainties by microgrids' operators may lead to non-optimal solutions or even infeasible operational points. As a result, many researchers have been trying to properly model and cope with uncertainty in the optimal operation of microgrids. Nevertheless, it is a complicated task, because the optimization model must be accurate enough to minimize the negative effects of the uncertainties in the real-world operation of the system, whereas the model must be tractable and time-efficient to be useful in practice. Therefore, due to the importance of uncertainty, it is not surprising that various methods are suggested in the literature for modeling uncertainties in microgrids.

Alqurashi et al. (2016) conducted a thorough review that delves into the application of stochastic programming in various classical power system problems, such as economic dispatch, optimal power flow, unit commitment, and market clearing. The authors also compared deterministic and stochastic methods, shedding light on the advantages and disadvantages of each approach. In another study by Soroudi and Amraee (2013), different methods for uncertainty modeling, such as information gap decision theory, possibilistic, hybrid, and robust optimization, were presented. They discussed the respective merits and drawbacks of these methods while reviewing their application in diverse power system research domains, including state estimation, electricity markets, transmission/generation planning, and operation. Likewise, Aien et al. (2016) contributed to the literature with a comprehensive review of uncertainty modeling methods, providing a classification to aid decision-makers in selecting the most suitable approach based on the specific characteristics of their problems. The authors also detailed the latest modifications and adjustments made to these methods, incorporating recent improvements. Overall, these review articles hold great value in the domain of power systems as they explore uncertainty modeling and its practical applications in-depth, offering valuable insights and guidance for researchers and decision-makers.

2.2 STOCHASTIC PROGRAMMING

Stochastic programming stands as the prevailing method in the literature for addressing uncertainty in the optimal operation of microgrids. Zandrazavi et al. (2022) introduce a stochastic model designed for multi-objective optimal energy management in unbalanced grid-connected microgrids, incorporating electric vehicles, photovoltaic generation, and wind power generation. The scenario generation employs a roulette wheel mechanism, and backward scenario reduction is utilized to streamline the number of scenarios. Rezaei et al. (2020) present a two-stage stochastic mixed integer linear programming model for optimizing energy management in islanded microgrids. Scenarios for load demand, wind turbine, and photovoltaic generation are generated and combined using a scenario tree. Baziar and Kavousi-Fard (2013) adopt the 2m point estimate method to model uncertainties in energy management for microgrids, considering various distributed generation units and batteries. In the works of Karimi et al. (2021) and Karimi and Jadid (2020), a seven-level scenario generation approach is employed to model uncertainties associated with wind speed, solar irradiance, power demand, and electricity prices. Ebadat-Parast et al., (2022) present a resilience-oriented multi-objective two-stage stochastic optimization model based on a mixed-integer linear programming framework for multi-microgrid systems. This model accounts for electric vehicles and emergency operations. These research papers offer diverse methods for incorporating uncertainties in microgrid energy management, showcasing the broad range of approaches utilized to tackle this critical aspect in microgrid optimization.

In reference to (7), a comprehensive form of a two-stage stochastic programming model is presented, featuring first-stage decision variables denoted as x and second-stage decision variables as y_s. The subscript s in y_s indicates that the second-stage variables depend on the actual scenarios that unfold, while the first-stage variables must be determined prior to the realization of uncertainties. The objective function, $C(x)$ contains the cost associated with the first-stage decision variables, while $D(y_s)$ represents the cost function related to the second-stage decisions. Notably, the presence of constraints that encompass both first-stage and second-stage variables establishes a reliance between these variables. In other words, once the first-stage decision variables are set, they impact the feasible set of choices for the second-stage decision variables. To illustrate this dependency, consider the example of dispatchable units' commitment in microgrid scheduling. The commitments of distributed generation units represent first-stage decision variables, while the output power amount is considered a second-stage decision variable. If a unit is not committed in the first stage to generate power, its generation in the second stage must be zero, irrespective of the uncertainties that unfold. This two-stage stochastic programming model exemplifies the interconnected nature of first-stage and second-stage decisions, where the choices made in the initial stage influence the possibilities available in the subsequent stage, playing a crucial role in the optimization of microgrid operations.

$$min\left(C(x) + \pi_s \sum_{s} D(y_s)\right) \tag{7}$$

s.to:

$$g(x) \leq b$$

$$h(x, y_s) \leq l_s$$

The significance of finding a balance between model accuracy and complexity should not be underestimated, especially when dealing with stochastic optimization models that can become unwieldy with a large number of scenarios. Opting for methods like the $2m$ point estimate or 7-level scenario generation offers the advantage of limiting the number of scenarios, thereby easing the computational burden. In cases where other techniques, such as Monte Carlo simulation, are employed for scenario generation, it becomes necessary to reduce the scenario count using tools like clustering or other scenario reduction methods. This step is crucial to maintain model manageability while still capturing the essential aspects of uncertainty for effective decision-making. By employing these strategies, researchers and practitioners can strike a practical balance between precision and simplicity in stochastic optimization models.

2.3 FUZZY PROGRAMMING METHOD

Humans have the ability to describe intricate systems using simple and imprecise terms like slow, fast, low, and high, reflecting the fuzzy and uncertain nature of information. Zadeh (1965) introduced fuzzy set theory and fuzzy logic as a means to model complex systems when dealing with imprecise and vague data. Fuzzy models go beyond acknowledging that available information is approximate and uncertain; they also allow scientists to incorporate human knowledge through linguistic terms, while still leveraging traditional mathematical techniques. Essentially, fuzzy models provide a way to handle uncertainties similar to how humans do in their daily lives. The applications of fuzzy sets and systems extend widely across engineering, offering benefits beyond just optimization. However, this chapter primarily focuses on the fuzzy approach's application in the optimal energy management of microgrids. By utilizing fuzzy methodologies, researchers can navigate the complexities of microgrid energy management, where imprecise information and human expertise play critical roles in making informed decisions.

Nasri et al. (2021) employed a hybrid fuzzy-stochastic approach for day-ahead scheduling of microgrids, encompassing both normal and emergency operations. The model takes into account uncertainties associated with photovoltaic and wind turbine unit generation, market price, and power demand. Banaei and Rezaee (2018) utilized fuzzy programming to address uncertainties related to photovoltaic and wind power generation, electric loads, and distribution line capacity. Notably, the study revealed a positive correlation between the level of fuzziness in uncertain factors and the total cost of microgrid operations. In the work by Ji et al. (2015), a hybrid fuzzy-stochastic method is proposed for optimal energy management in a regional microgrid. The model effectively manages the trade-off between the risk arising from uncertain factors and the overall cost of microgrid operation, demonstrating its efficiency in handling complex scenarios with uncertainties.

Fuzzy programming can be represented in both linear and non-linear programming forms. However, for ease of explanation, the concept is often better understood through a linear model. In (8)–(10), a concise representation of the linear programming problem is provided, known as the matrix representation of linear programming. In this representation, X denotes the vector of decision variables, and C represents the vector of cost coefficients, thereby forming a linear objective function. The linear inequality constraints are then represented by the matrix A and the vector B. This compact form simplifies the understanding and analysis of fuzzy programming within a linear context.

$$min C^T X \tag{8}$$

s.to:

$$AX \leq B \tag{9}$$

$$X \geq 0 \tag{10}$$

The optimization model discussed earlier assumes that all arrays in matrix A and vector B consist of crisp numbers without any uncertainty regarding their values. However, when uncertainty is present in these parameters, the constraint can be reformulated as follows by replacing crisp parameters with their corresponding fuzzy counterparts:

$$\tilde{A}X \leq \tilde{B} \tag{11}$$

To better comprehend the difference between (10) and (11), consider two simple constraints (12) and (13) depicted in Figure 2.

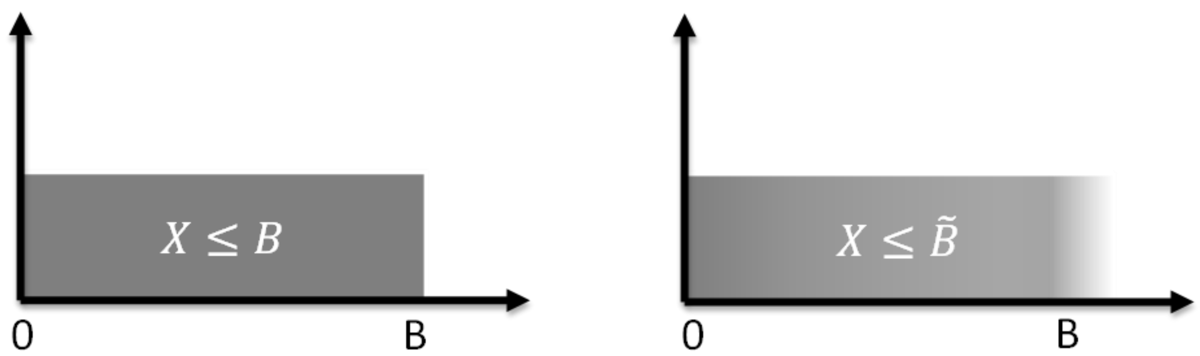
$$X \leq B \tag{12}$$

$$X \leq \tilde{B} \tag{13}$$

As it can be seen, fuzzy numbers differ from crisp parameters as their boundaries are not entirely distinct, necessitating the use of a fuzzy membership function to represent this ambiguity. Nevertheless, operators must determine the specific fuzzy membership function and degree of fuzziness, and these decisions heavily rely on their expertise and understanding of the uncertain parameters.

The literature offers a variety of methods to address fuzzy programming problems, as noted by Lodwick and Kacprzyk, (2010). One such approach, presented by Lai and Hwang (1992), efficiently tackles fuzzy linear programming models. In their reformulation, each fuzzy constraint is approximated using three equivalent normal constraints, enabling straightforward solutions through commercial software.

Figure 2 – Fuzzy constraint versus normal constraint



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2.4 INFORMATION GAP DECISION THEORY

Ben-Haim (2006) introduced Information Gap Decision Theory (IGDT), which sets itself apart from other uncertainty handling methods by not requiring a substantial amount of data associated with uncertain parameters. Despite having limited available data, IGDT empowers decision-makers to gain valuable insights into potential positive and negative effects stemming from uncertain parameters. As a result, decision-makers can more effectively evaluate optimization problems and make improved choices, whether adopting a risk-taking or risk-averse stance.

Sriyakul and Jermsittiparsert (2020) introduce a novel hybrid two-stage stochastic/IGDT method to effectively address uncertainty in optimizing microgrid operations, leveraging contract-based plug-in electric vehicle deployment. Mehdizadeh et al. (2018) apply IGDT to manage uncertainties associated with electricity prices in a renewable-based grid-connected microgrid, resulting in reduced power purchases from the main grid, thereby enhancing the microgrid's self-sufficiency. Aboli et al. (2019) propose a hybrid robust/IGDT model for optimizing energy management in multi-microgrid distribution networks. Risk-averse IGDT handles uncertainties concerning renewable energy generation and electric demand, while worst-case scenarios account for electricity price uncertainty. Zandrazavi et al. (2021) suggest a hybrid IGDT/stochastic model for optimal energy management of microgrids. They utilize scenario generation to model uncertainty related to photovoltaic and wind turbine power generation, while employing IGDT to address load demand uncertainty. The study demonstrates that integrating IGDT enhances the microgrid's resilience against load variations.

The literature introduces several information-gap models, including the Energy-bound model, envelope-bound models, Fourier-bound models, and Minkowski-norm models (Majidi et al. (2019)). Among these models, the Envelope-bound model is more commonly employed in articles addressing optimal energy management of microgrids using IGDT, primarily due to its simplicity (Majidi et al. (2019)). In (14), the scalar function for the Envelope-bound model is presented, involving uncertain parameters $x(t)$, nominal values $\tilde{x}(t)$, envelope shape $\varphi(t)$, and robustness factor α.

$$X(\alpha, \tilde{x}) = \left\{ x(t) \colon \left| \frac{x(t) - \tilde{x}(t)}{\varphi(t)} \right| \leq \alpha \right\}, \alpha \geq 0 \tag{14}$$

Despite being a valuable method for modeling and managing uncertainty in optimizing microgrid operations, IGDT is primarily intended for situations with limited information or data. Therefore, if there is an abundance of historical data on renewable energy generation or electric vehicle demand, it becomes challenging to justify the use of this method.

2.5 ADAPTIVE ROBUST OPTIMIZATION

The adaptive robust method is extensively utilized for uncertainty modeling in power system problems (Attarha et al. (2018); Choi et al. (2018); Ebrahimi and Amjady (2019); Zugno and Conejo (2015)) due to its strong mathematical foundation. Zugno and Conejo (2015) propose an adaptive robust optimization model to handle uncertainty in a day-ahead energy and reserve dispatch problem within an electricity market, considering wind power generation as an uncertain parameter. Attarha et al. (2018) suggest an adaptive robust optimization model for optimal power flow, incorporating uncertainties related to wind power generation and power demand. Ebrahimi and Amjady, (2019) present a framework based on adaptive robust optimization for day-ahead optimal energy management of balanced microgrids, encompassing uncertainties concerning solar power generation, wind power generation, electric demand, and electricity price. Choi et al., (2018) utilize adaptive robust optimization to model optimal energy management of microgrids, incorporating uncertainties related not only to renewable generation, power demand, and electricity price but also to electric vehicles.

In (15), a comprehensive tri-level formulation for the adaptive robust optimization problem is introduced. The first level involves decision variables (i.e., x) that represent here-and-now decisions, necessitating choices to be made before the actualization of uncertain parameters. Moving to the second level, the worst-case realization of uncertain parameters (i.e., y) is determined based on the objective function. Finally, in the third level, wait-and-see decisions (z) are made to counteract the adverse impact of the worst-case scenario on the optimal solution.

$$\min_{x\in X}\max_{y\in Y}\min_{z\in Z} f(x,y,z) \tag{15}$$

Different methods are suggested for solving adaptive robust optimization Sun and Conejo, (2021). The most widely recognized solution techniques include the Benders-dual cutting plane (Bertsimas et al. (2012)) and column-and-constraint generation (Bertsimas et al. (2012)). In contrast to other methods used to model uncertainties, readers may perceive robust optimization as more challenging to implement and more intricate to solve.

2.6 CHANCE-CONSTRAINED PROGRAMMING

The chance-constrained method is another significant approach for uncertainty modeling. Unlike the deterministic method, this technique involves setting some or all constraints as probabilistic constraints, meaning that these constraints must hold at a specified level of certainty. It is important to highlight that these constraints are considered probabilistic because they directly or indirectly depend on uncertain parameters. In other words, in certain instances, the occurrence of an uncertain parameter may result in a violation of the probabilistic constraints.

A chance-constrained model is employed by Aghdam et al. (2020b) for the hierarchical optimal energy management of a multi-microgrid network, enabling direct interaction between the microgrid operators and the distribution network operator. Uncertain parameters encompass solar and wind power generation, as well as electric demands. A day-ahead scheduling of microgrids is addressed in Aghdam et al. (2020a), where uncertainties associated with renewable generation and loads are managed through probabilistic constraints. In Hemmati et al., (2020), chance-constrained programming is utilized for optimal energy management of reconfigurable microgrids, accounting for the islanding capability constraint. The total cost is considered as the objective function, involving switching cost, reliability cost, and fuel cost.

To transform a deterministic optimization problem as presented in (16) into a chance-constrained programming model, one can rewrite it as (17). In this new formulation, the inequality constraint $h(x)$ becomes a probabilistic constraint with an occurrence probability greater than ϑ (known as the confidence level and often chosen between 0.90 and 0.99). For example, when ϑ is 0.95, it indicates that the probability of satisfying the constraint $h(x) \leq C$ is at least 95% of the time.

$$\begin{gathered} \min f(x) \\ s.to: \\ \begin{cases} g(x) = B \\ h(x) \leq C \end{cases} \end{gathered} \tag{16}$$

$$\begin{gathered} \min f(x) \\ s.to: \\ g(x) = B \\ \Pr\,(h(x) \leq C) \geq \vartheta \end{gathered} \tag{17}$$

Two distinct approaches are available for solving chance-constrained optimization problems. The first approach involves reformulating the problem as a deterministic one, known as the analytical approach. The second approach relies on approximation through a significant number of samples, where the constraint may be violated for a limited number of samples based on the chosen confidence level (Margellos et al. (2014)). In this latter case, the chance-constrained method exhibits considerable similarity with stochastic programming. However, there is ongoing discussion regarding the minimum number of scenarios required to accurately approximate the probabilistic constraint (Margellos et al. (2014)).

2.7 UNCERTAINTY MODELING VIA MONTE CARLO SIMULATION

The Monte Carlo simulation (MCS) finds extensive applications in science beyond just optimization or uncertainty modeling. Notably, it plays a significant role in modeling uncertainty in power system optimization. The uncertainties associated with renewable generation and power demand can be effectively modeled using probability density functions (PDFs) for uncertain parameters. A PDF represents the probability of a continuous random variable, enabling the generation of scenarios, such as wind power generation during a specific hour. Various methods exist to approximate PDFs, like the $2m$ and $2m + 1$ point estimate. However, when evaluating the accuracy of results in stochastic programming, MCS, as highlighted by Vergara et al. (2020), is the preferred approach. MCS uses a large number of samples to closely approximate the PDF, although in theory, an infinite number of samples would be required for an exact representation of a continuous function. Nevertheless, practical results indicate that even with a limited number of samples (e.g., 1,000 samples), MCS remains valid, with errors in results typically being negligible (Sadeghian et al. (2020)). The accuracy of results can be improved by increasing the number of samples, but a trade-off is necessary between accuracy and the computational effort involved in MCS (Sadeghian et al. (2020)). In the literature, various PDFs are used to model uncertainty related to solar irradiance, wind speed, and electric demand uncertainties, as demonstrated in the work of Ramadhani et al. (2020). Among these, Weibull, Beta, and Gaussian PDFs are widely accepted options for modeling wind speed, solar irradiance, and power demand, respectively, based on the research of Nikpour et al. (2021).

3 POWER FLOW IN BALANCED MICROGRIDS

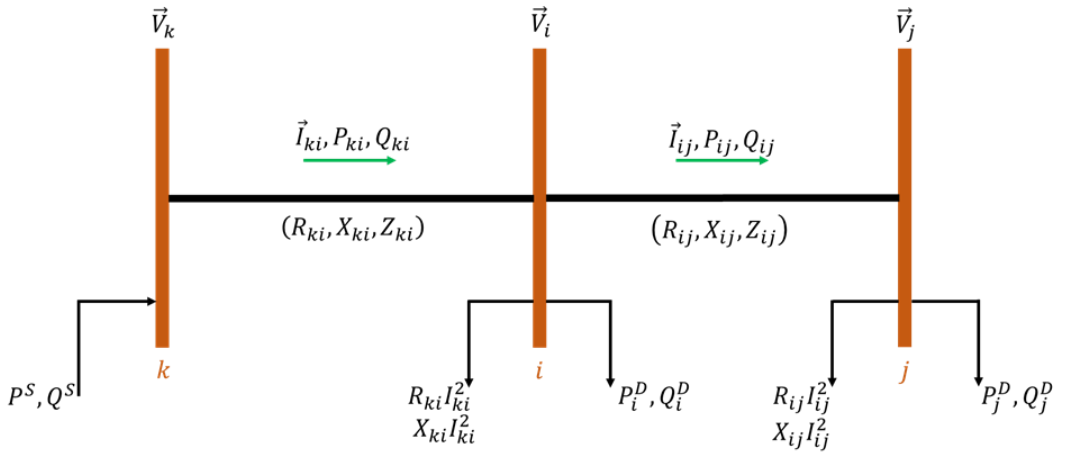
Power flow plays a crucial role in effectively managing microgrids as it determines their steady-state operation by calculating voltage values at buses, current flow through branches, and power losses in the system. To achieve this, we extract a nonlinear nonconvex model for power flow in a manner similar to the approach presented in Franco et al., (2012), as detailed in this chapter. To illustrate the mathematical modeling procedure, a simple passive three-bus distribution system is presented in Figure 3. At each bus i and branch ij, phasors $\vec{V}_i$ and $\vec{I}_{ij}$ (from i to j) representing voltages and currents, respectively. Similarly, at buses j and k, we have phasors of voltages, $\vec{V}_j$ and $\vec{V}_k$, along with vector $\vec{I}_{ki}$ for current through branch ki. The active and reactive powers provided by the substation are denoted as P_s and Q_s, while P_{Di}, Q_{Di}, P_{Dj}, and Q_{Dj} represent the active and reactive power demands at buses i and j, respectively. Impedance, reactance, and resistance of branch ij are denoted by Z_{ij}, X_{ij}, and R_{ij}, respectively. Moreover, we can calculate the active and reactive power losses in branch ki and ij using expressions $R_{ki}I_{ki}^2$, $X_{ki}I_{ki}^2$, $R_{ij}I_{ij}^2$, and $X_{ij}I_{ij}^2$, respectively.

According to Kirchhoff's voltage law, we can express the voltage drop in branch ij as given by equation (18):

$$\vec{V}_i - \vec{V}_j = \vec{I}_{ij}\left(R_{ij} + jX_{ij}\right) \tag{18}$$

Furthermore, $\vec{I}_{ij}$ can be expressed in terms of the voltage of bus i and active and reactive power flows through the branch ij as follows:

Figure 3 – Three-bus passive distribution network



Source: Created by the author

$$\vec{I}_{ij} = \left(\frac{P_{ij} + jQ_{ij}}{\vec{V}_i}\right)^* \tag{19}$$

Consequently, by substituting $\vec{I}_{ij}$ into equation (18), it can be reformulated as:

$$\vec{V}_i - \vec{V}_j = \left(\frac{P_{ij} + jQ_{ij}}{\vec{V}_i}\right)^* \left(R_{ij} + jX_{ij}\right) \tag{20}$$

After simplifications, (20) can be expressed as:

$$\left(\vec{V}_i - \vec{V}_j\right)\vec{V}_i^* = \left(P_{ij} - jQ_{ij}\right)\left(R_{ij} + jX_{ij}\right) \tag{21}$$

As $\vec{V}_i = V_i\angle\delta_i$, $\vec{V}_j = V_j\angle\delta_j$, and $\delta_{ij} = \delta_i - \delta_j$ where δ_i is the voltage phase angle at bus i, (22) is derived:

$$V_i^2 - V_iV_j\left[\cos\delta_{ij} - j\sin\delta_{ij}\right] = \left(P_{ij} - jQ_{ij}\right)\left(R_{ij} + jX_{ij}\right) \tag{22}$$

By decomposing the equation (22) into imaginary and real parts, expressions (23) and (24) can be obtained:

$$V_iV_j\cos\delta_{ij} = V_i^2 - \left(R_{ij}P_{ij} + X_{ij}Q_{ij}\right) \tag{23}$$

$$V_iV_j\sin\delta_{ij} = \left(X_{ij}P_{ij} - R_{ij}Q_{ij}\right) \tag{24}$$

By adding the square of (23) and (24), it can be written:

$$V_i^2 - V_j^2 = 2\left(P_{ij}R_{ij} + Q_{ij}X_{ij}\right) + \left(R_{ij}^2 + X_{ij}^2\right)I_{ij}^2 \tag{25}$$

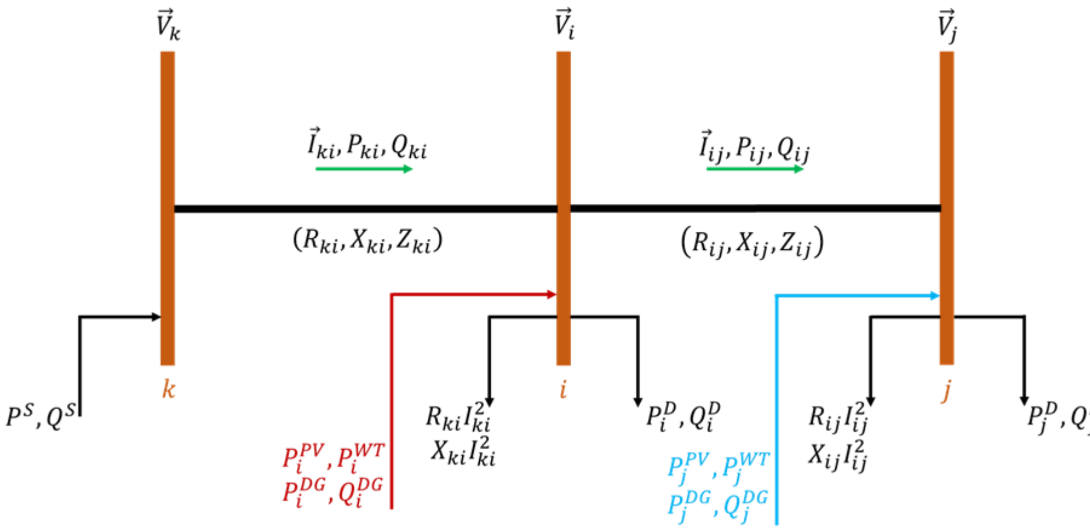
where the square of the current flowing at branch ij can be obtained as follows:

$$I_{ij}^2 = \left(\frac{P_{ij}^2 + Q_{ij}^2}{V_i^2}\right) \tag{26}$$

As per equations (25) and (26), it is feasible to determine the voltage at the terminal bus (V_j) based on the voltage at the initial bus (V_i). Although these equations are derived for a three-bus system, their applicability extends to the entire microgrid by taking into account all branches (represented by $\forall ij \in \Lambda$, where Λ is the set of all branches in the microgrid network).

Furthermore, apart from equations (25) and (26), it is essential to account for the active and reactive power balance for each bus in the system. For instance, referring to Figure 3, the active power balance for bus i can be expressed as follows:

Figure 4 – Three-bus balanced microgrid



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$$P_{ki} = P_i^D + P_{ij} + R_{ki}I_{ki}^2 \tag{27}$$

Similarly, the reactive power balance at bus i can be formulated as follows:

$$Q_{ki} = Q_i^D + Q_{ij} + X_{ki}I_{ki}^2 \tag{28}$$

The power balances mentioned earlier, denoted by equations (27) and (28), can be generalized to encompass all the buses in the system, resulting in the following formulations:

$$\sum_{ki \in \Lambda} (P_{ki} - R_{ki}I_{ki}^2) - \sum_{ij \in \Lambda} P_{ij} + P^S = P_i^D \quad \forall i \in \psi \tag{29}$$

$$\sum_{ki \in \Lambda} (Q_{ki} - X_{ki}I_{ki}^2) - \sum_{ij \in \Lambda} Q_{ij} + Q^S = Q_i^D \quad \forall i \in \psi \tag{30}$$

in which ψ is the set of all the buses in the system.

Expressions (25)–(26) and (29)–(30) are sufficient for representing the power flow in a radial passive distribution network without any generation, as demonstrated by Franco et al. (2012). Nonetheless, this model can be easily adapted to accommodate the unique characteristics of microgrids, which may involve dispatchable and renewable power generation. Thus, the equations (29)–(30) can be appropriately adjusted to be suitable for microgrids:

$$\sum_{ki \in \Lambda} (P_{ki} - R_{ki}I_{ki}^2) - \sum_{ij \in \Lambda} P_{ij} + P^S + P^{DG} + P^{PV} + P^{WT} = P_i^D \quad \forall i \in \psi \tag{31}$$

$$\sum_{ki\in\Lambda}(Q_{ki}-X_{ki}I_{ki}^{2})-\sum_{ij\in\Lambda}Q_{ij}+Q^{S}+Q^{DG}=Q_{i}^{D}\quad\forall i\in\ \psi \tag{32}$$

where, P^{DG} and Q^{DG} represent the active and reactive power injected by distributed generation units, while P^{PV} and P^{WT} indicate the active power generated by photovoltaic and wind turbine units, respectively. Figure 4 illustrates the modified three-bus system and unlike the passive distribution system model depicted in Figure 3, this model incorporates distributed generation, wind turbine, and photovoltaic units, making it a small-scale microgrid comprised of three buses.

4 DETERMINISTIC MODEL FOR OPTIMAL ENERGY MANAGEMENT OF MICROGRIDS

In this section, firstly, an optimal energy management model for grid-connected balanced microgrids is presented in the form of a mixed-integer nonlinear nonconvex optimization model. To this end, various objective functions such as operation cost, power losses, and voltage deviation index are introduced, and the respective formulations are presented. Moreover, a detailed explanation is given regarding the constraints related to the microgrid system, dispatchable distributed generation units, photovoltaic units, and wind turbine units. Secondly, an explanation is provided on how the presented model can be transformed into mixed-integer second-order cone programming and mixed-integer linear programming to ensure the global optimality of the solutions. Finally, instructions and explanations are included to assist readers in extending the proposed optimization model for the optimal energy management of reconfigurable microgrids.

4.1 OBJECTIVE FUNCTIONS

In the literature, researchers have proposed various objective functions for optimizing the energy management of microgrids. This subsection provides explanations and presents the formulas for some of the most common objective functions.

4.1.1 TOTAL COST OF MICROGRID

The most widely considered objective function in the literature for microgrid optimization is the total cost of the microgrid. In this chapter, the total cost function is represented by equation (33). The total cost is composed of two terms: the cost associated with the power generation of dispatchable distributed generation units and the cost linked to the power exchange between the microgrid and the main grid.

$$min(F_1) = \sum_{t}\sum_{dg} u_{dg,t}^{DG} a_{dg}^{DG} + \sum_{t} \Delta_t \left(C_t^{GR} P_t^{GR} + \sum_{dg} b_{dg}^{DG}\, P_{dg,t}^{DG} + \sum_{m} C_m^{LS} P_{m,t}^{D} \zeta_{m,t}^{LS} \right) \tag{33}$$

In (33), the first term captures the cost associated with the commitment of dispatchable units. It involves binary variable $u_{t,dg}^{DG}$, which indicates the commitment status of dispatchable units, and a_{dg}^{DG}, which represents the corresponding commitment cost. The second term models the cost related to the power purchased from the main grid. It incorporates the electricity price C_t^{GR} and the power exchanged between the microgrid and the main grid during time interval t, denoted by P_t^{GR}. The third term represents the generation cost of dispatchable units and is formulated as a linear function. It comprises the generation cost coefficients b_{dg}^{DG} and the output power of dispatchable units at time interval t, represented by $P_{dg,t}^{DG}$. The last term accounts for the load shedding cost. It involves C_m^{LS}, which represents the cost of load shedding, $\zeta_{m,t}^{LS}$, the load shedding factor, and $P_{m,t}^{D}$, the electric load that is subject to shedding.

Including load shedding in the model is crucial due to the potential occurrence of unintentional islanding in microgrids. In such situations, microgrids must independently maintain frequency and power balance without external assistance from the main system. Consequently, the microgrid should possess the capability to disconnect non-essential loads to prevent frequency instability in the system.

4.1.2 TOTAL POWER LOSS

As stated by Homayoun et al., (2021), electrical energy loss occurs in microgrids due to the resistance of the lines. Active power loss is an inherent aspect of microgrid energy management, but it can be mitigated through strategic scheduling of dispatchable units, batteries, and appropriate system reconfiguration. The formulation for active power loss in the microgrid is provided in equation (34).

$$min(F_2) = \sum_{t} \sum_{mn} R_{mn} I_{mn,t}^{sqr} \tag{34}$$

4.1.3 VOLTAGE DEVIATION INDEX

In assessing the performance of microgrids concerning energy provision, power quality plays a critical role. To ensure a high standard of power quality, one of the indices utilized is the voltage deviation index. Integrating the voltage deviation index as a secondary objective function allows microgrid operators to improve the voltage profile and prevent undesired deviations in voltage magnitude within the system as highlighted in Zandrazavi et al. (2022). This index can be expressed as follows:

$$min(F_3) = \sum_{t} \sum_{m} \left(V_{m,t} - V_m^{ref}\right)^2 \tag{35}$$

where $V_{m,t}$ is the voltage of bus m at time interval t and V_m^{ref} denotes the corresponding reference voltage for bus m.

4.2 POWER FLOW MODEL AND SYSTEM CONSTRAINTS

The nonlinear nonconvex power flow formulation for the microgrid is given by equations (36)–(39). As previously mentioned, ensuring a convex optimization model requires a convex feasibility set. To satisfy this condition, any existing equality constraints in the model must be linear. Unfortunately, the presence of the non-linear equality constraint (39) in the formulation causes nonconvexity in this optimization model.

$$\sum_{km} P_{km,t} - \sum_{mn}\left(P_{mn,t} + R_{mn} I_{mn,t}^{sqr}\right) + P_t^{GR} + \sum_{wt|wt=m} P_{wt,t}^{WT} + \sum_{pv|pv=m} P_{pv,t}^{PV} \quad (36)$$

$$+ \sum_{dg|dg=m} P_{dg,t}^{DG} = P_{m,t}^{D}\left(1 - \zeta_{m,t}^{LS}\right) + \sum_{bs|bs=m} P_{bs,t}^{BS}\ ;\ \forall m,t$$

$$\sum_{km} Q_{km,t} - \sum_{mn}\left(Q_{mn,t} + X_{mn} I_{mn,t}^{sqr}\right) + Q_t^{GR} + \sum_{dg|dg=m} Q_{dg,t}^{DG} = Q_m^{D};\ \ \forall m,t \quad (37)$$

$$\left(V_{m,t}^{sqr} - V_{n,t}^{sqr}\right) = 2\left(R_{mn} P_{mn,t} + X_{mn} Q_{mn,t}\right) + Z_{mn}^2 I_{mn,t}^{sqr};\ \ \forall mn,t \quad (38)$$

$$\left(V_{n,t}^{sqr} I_{mn,t}^{sqr}\right) = \left(P_{mn,t}^2 + Q_{mn,t}^2\right);\ \forall mn,t \quad (39)$$

$$\underline{V}^2 \le V_{m,t}^{sqr} \le \overline{V}^2;\ \ \forall m,t \quad (40)$$

$$0 \le I_{mn,t}^{sqr} \le \left(\overline{I}_{mn}^2\right);\ \ \forall mn,t \quad (41)$$

Expressions (36) and (37) ensure the active and reactive power balances in the system. Here, $P_{mn,t,s}$ and $Q_{mn,t,s}$ represent the active and reactive power flowing in circuit mn at time period t. The parameters R_{mn} and X_{mn} correspond to the resistance and reactance of circuit mn, respectively. Additionally, P_t^{GR} and Q_t^{GR} denote the active and reactive power transferred between the microgrid and the main grid at time period t, respectively. $P_{wt,t}^{WT}$ and $P_{pv,t}^{PV}$ represent the active output power of the wind turbine and photovoltaic units at time period t, respectively. Furthermore, $P_{m,t}^{D}$ and $Q_{m,t}^{D}$ are the active and reactive loads at bus m during time interval t, respectively.

Equations (38) and (39) establish the relationships between currents and voltages, as well as active power and reactive power flows in the microgrids. Here, Z_{mn} represents the magnitude of the impedance of circuit mn, and $V_{n,t}^{sqr}$ denotes the square of the voltage magnitude at bus n during time period t. Constraints (40) and (41) specify the limits associated with voltage and current magnitudes at bus m and circuit mn, respectively, during time period

t. $\overline{V}$ and $\overline{I}_{mn}$ represent the upper bounds for bus voltages and circuit currents, respectively, while $\underline{V}$ denotes the lower bound for voltage magnitude.

As previously discussed, constraint (39) is responsible for the nonconvex nature of the optimization model for three-phase power flow. However, there is a method to achieve convexity, as demonstrated by Farivar and Low (2013) by converting the equality constraint (39) into an equivalent inequality constraint (42). This transformed constraint is convex and referred to as a rotated cone, allowing the replacement of the nonlinear nonconvex model with a nonlinear convex second-order cone programming model.

$$V_{n,t}^{sqr} I_{mn,t}^{sqr} \geq P_{mn,t}^{2} + Q_{mn,t}^{2};\ \ \forall mn, t \tag{42}$$

In dealing with the nonconvexity of constraint (39), there is an alternative method apart from second-order cone programming models, which are efficiently solvable by commercial solvers. As previously mentioned, the presence of linear equality constraints is essential for convex optimization. Therefore, it is possible to retain the equality constraint while obtaining an equivalent linear model. The linearization process for this constraint is explained in Silveira et al. (2021). In this approach, instead of using the exact value of $V_{n,t}^{sqr}$, an approximated equivalent parameter denoted as $\tilde{V}_{n,t}^{sqr}$ is employed. This parameter, $\tilde{V}_{n,t}^{sqr}$, can be calculated by determining the midpoint between the upper and lower bounds of the voltage range, as shown in (43).

$$\tilde{V}_{n,t}^{sqr} = \frac{\overline{V}^{2} + \underline{V}^{2}}{2};\ \ \forall n, t \tag{43}$$

For the right-hand side of constraint (39), the quadratic terms involving the active and reactive powers flowing in microgrids (i.e., $P_{mn,t}^{2}$ and $Q_{mn,t}^{2}$) can be linearized using a piecewise linearization method. Consequently, equation (39) can be reformulated as follows (Silveira et al. (2021)):

$$\tilde{V}_{n,t}^{sqr} I_{mn,t}^{sqr} = \sum_{\omega} \Gamma_{mn,\omega} \left(\Delta_{mn,\omega,t}^{P} + \Delta_{mn,\omega,t}^{Q}\right);\ \ \forall mn, t \tag{44}$$

in this reformulated equation, $\Delta_{mn,\omega,t}^{P}$ and $\Delta_{mn,\omega,t}^{Q}$ represent auxiliary variables used to model the discretization blocks for the respective active and reactive powers. The parameter $\Gamma_{mn,\omega}$ denotes the slope of the lines employed in the piecewise linearization approach, while ω corresponds to the number of blocks considered in the discretization.

4.3 TECHNICAL CONSTRAINT RELATED TO THE TRANSFORMER

The microgrid is assumed to be connected to the main system via a transformer at the point of common coupling. As a result, there are limitations concerning the amount of power that can be exchanged between the microgrid and the main system. This technical constraint is modeled in equation (45), where $\overline{S}^{TR}$ represents the maximum apparent power that can flow through the transformer.

$$(P_t^{GR})^2 + (Q_t^{GR})^2 \leq \left(\overline{S}^{TR}\right)^2 ; \ \forall t \tag{45}$$

4.4 CONSTRAINTS RELATED TO DISPATCHABLE DISTRIBUTED GENERATION UNITS

Equations (46) and (47) represent the constraints related to the operation of dispatchable distributed generation (DG) units. In these equations, $\underline{P}_{dg}^{DG}$ and $\overline{P}_{dg}^{DG}$ represent the lower and upper bounds for active power generation, respectively. Additionally, φ_{dg}^{DG} and $Q_{dg,t}^{DG}$ refer to the power factor and reactive power generation of the DG units, respectively. To model the DG commitments, a binary variable $u_{dg,t}^{DG}$ is included. When this binary variable takes the value of 0, the output power of the DG unit becomes zero. Conversely, when the binary variable takes the value of 1, the output power of the DG unit is nonzero, but it is limited to the range presented in equation (46). The reactive power that can be generated by DG units is also limited as presented in (47) and it depends on $P_{dg,t}^{DG}$.

$$u_{dg,t}^{DG}\underline{P}_{dg}^{DG} \leq P_{dg,t}^{DG} \leq u_{dg,t}^{DG}\overline{P}_{dg}^{DG}; \ \forall dg, t \tag{46}$$

$$-\tan\left(\cos^{-1}\left(\varphi_{dg}^{DG}\right)\right) P_{dg,t}^{DG} \leq Q_{dg,t}^{DG} \leq \tan\left(\cos^{-1}\left(\varphi_{dg}^{DG}\right)\right) P_{dg,t}^{DG}; \ \forall dg, t \tag{47}$$

4.5 CONSTRAINTS RELATED TO WIND POWER UNITS

The growth of renewable generation is highly advantageous as it drives the transition toward greener energy-based power systems. However, certain situations may arise where the power generated by renewable energy-based units exceeds the demand. Consequently, this surplus power could lead to overvoltage or overcurrent in the system. To prevent such issues in microgrids, renewable energy curtailment can be employed. This means that the output power of renewable energy resources can be reduced when necessary to avoid violating technical constraints. Therefore, in equation (48), the wind power curtailment capability is formulated.

$\tilde{P}^{WT}_{wt,t}$ represents the available power generation based on the expected wind speed, and $\Psi^{WT}_{wt,t}$ denotes the amount of power generation that needs to be curtailed. To ensure that the maximum power curtailment does not exceed the available power, $\Psi^{WT}_{wt,t}$ is subject to constraint (49).

$$P^{WT}_{wt,t} = \tilde{P}^{WT}_{wt,t} - \Psi^{WT}_{wt,t};\ \ \forall wt, t \tag{48}$$

$$0 \leq \Psi^{WT}_{wt,t} \leq \tilde{P}^{WT}_{wt,t};\ \ \forall wt, t \tag{49}$$

4.6 CONSTRAINTS RELATED TO PHOTOVOLTAIC UNITS

Like wind power generation, the active power generated by photovoltaic units may also require reduction under certain circumstances. A corresponding formulation for power curtailment is given in equation (50), where $\tilde{P}^{PV}_{pv,t}$ represents the available solar power generation, and $\Psi^{PV}_{pv,t}$ indicates the amount of power generation that needs to be curtailed. To ensure that the power curtailment does not exceed the available solar power, $\Psi^{PV}_{pv,t}$ must be constrained by equation (51).

$$P^{PV}_{pv,t} = \tilde{P}^{PV}_{pv,t} - \Psi^{PV}_{pv,t};\ \ \forall pv, t \tag{50}$$

$$0 \leq \Psi^{PV}_{pv,t} \leq \tilde{P}^{PV}_{pv,t};\ \ \forall pv, t \tag{51}$$

4.7 CONSTRAINTS RELATED TO BATTERIES

Energy storage systems can enhance the efficiency, resilience, and reliability of microgrids by efficiently managing electrical energy. These systems charge the batteries when there is an excess of electrical energy available and discharge the batteries when electrical energy is needed. However, charging and discharging batteries result in power losses. To address this, two nonnegative continuous variables, $P^{BS+}_{bs,t}$ and $P^{BS-}_{bs,t}$, are introduced in equations (52) and (53) to separately model the charging and discharging powers of the batteries at time period t. The total power of the batteries, combining both charging and discharging, is represented as $P^{BS}_{bs,t}$.

$$0 \leq P^{BS+}_{bs,t} \leq \overline{P}^{BS};\ \ \forall bs, t \tag{52}$$

$$0 \leq P^{BS-}_{bs,t} \leq \overline{P}^{BS};\ \ \forall bs, t \tag{53}$$

$$P^{BS}_{bs,t} = P^{BS+}_{bs,t} - P^{BS-}_{bs,t};\ \ \forall bs, t \tag{54}$$

Equations (55) and (56), establish the connections between the charging and discharging powers and the energy stored in the batteries. Here, E_{ini}^{BS} represents the initial charge, and $E_{bs,t}^{BS}$ denotes the amount of energy stored in the batteries at time period t. Additionally, η_{ch}^{BS} and η_{dis}^{BS} refer to the charging and discharging efficiencies of the batteries, respectively, while β^{BS} represents the corresponding self-discharge rate.

$$E_{bs,t}^{BS} = E_{ini}^{BS} + \Delta_t \left(P_{bs,t}^{BS+} \eta_{ch}^{BS} - \left({P_{bs,t,s}^{BS-}}/{\eta_{dis}^{BS}} \right) - E_{bs,t}^{BS} \beta^{BS} \right); \ \forall bs, t | t = 1 \tag{55}$$

$$E_{bs,t}^{BS} = E_{bs,t-1}^{BS} + \Delta_t \left(P_{bs,t}^{BS+} \eta_{ch}^{BS} - \left({P_{bs,t}^{BS-}}/{\eta_{dis}^{BS}} \right) - E_{bs,t}^{BS} \beta^{BS} \right); \ \forall bs, t | t > 1 \tag{56}$$

In order to prolong the life span of batteries, it is crucial to prevent deep discharging or overcharging. To achieve this, constraints on the amount of energy stored in the batteries are introduced, as depicted by constraint (57). Here, $\underline{E_{bs}}^{BS}$ and $\overline{E}_{bs}{}^{BS}$ represent the lower and upper bounds of the battery capacity, respectively.

$$\underline{E_{bs}}^{BS} \leq E_{bs,t}^{BS} \leq \overline{E}_{bs}{}^{BS}; \ \forall bs, t \tag{57}$$

4.8 RECONFIGURABLE MICROGRIDS

Reconfigurability refers to the capability of altering the configuration of power systems through a switching process. In distribution network studies, reconfiguration is extensively employed to minimize power losses and enhance the voltage profile and reliability of distribution networks. To extend the proposed model for the optimal energy management of microgrids and incorporate reconfigurable microgrids, certain modifications and explanations are provided in this subsection. These adaptations allow for the consideration of changes in the network configuration.

To incorporate reconfigurability in the proposed model for optimal energy management of microgrids, constraints (38) and (41) need to be replaced by (58) and (59), respectively. The binary variables $y_{mn,t}^{+}$ and $y_{mn,t}^{-}$ are introduced to represent the status of the interconnection switches in the microgrid, where 'open' is denoted by 0 and 'closed' by 1. Additionally, the auxiliary variable $w_{mn,t}$ is utilized, which depends on the status of the switches (BORGES; FRANCO; RIDER, 2014).

$$\left(V_{m,t}^{sqr} - V_{n,t}^{sqr} \right) = 2\left(R_{mn} P_{mn,t} + X_{mn} Q_{mn,t} \right) + Z_{mn}^2 I_{mn,t}^{sqr} - w_{mn,t}; \ \forall mn, t \tag{58}$$

$$0 \leq I_{mn,t}^{sqr} \leq \overline{I}_{mn}^{2}\left(y_{mn,t}^{+} + y_{mn,t}^{-}\right); \quad \forall mn, t \tag{59}$$

Furthermore, the proposed model should be extended to incorporate new constraints (60)–(66). These constraints introduce nonnegative variables, $P_{mn,t}^{+}$ and $P_{mn,t}^{-}$, which represent the flow of active power in the forward direction (from bus m to bus n) and in the backward direction (from bus n to bus m), respectively, for circuit mn. Additionally, Λ_n denotes the number of buses in the microgrid.

$$\left|w_{mn,t}\right| \leq \left(\overline{V}^{2} - \underline{V}^{2}\right)\left(1 - \left(y_{mn,t}^{+} + y_{mn,t}^{-}\right)\right); \ \forall mn, t \tag{60}$$

$$P_{mn,t}^{+} - P_{mn,t}^{-} = P_{mn,t}; \quad \forall mn, t \tag{61}$$

$$0 \leq P_{mn,t}^{+} \leq \overline{V}\,\overline{I_{mn}} y_{mn,t}^{+}; \ \forall mn, t \tag{62}$$

$$0 \leq P_{mn,t}^{-} \leq \overline{V}\,\overline{I_{mn}} y_{mn,t}^{-}; \ \forall mn, t \tag{63}$$

$$-\overline{V}\,\overline{I_{mn}}\left(y_{mn,t}^{+} + y_{mn,t}^{-}\right) \leq Q_{mn,t} \leq \overline{V}\,\overline{I_{mn}}\left(y_{mn,t}^{+} + y_{mn,t}^{-}\right); \ \forall mn, t \tag{64}$$

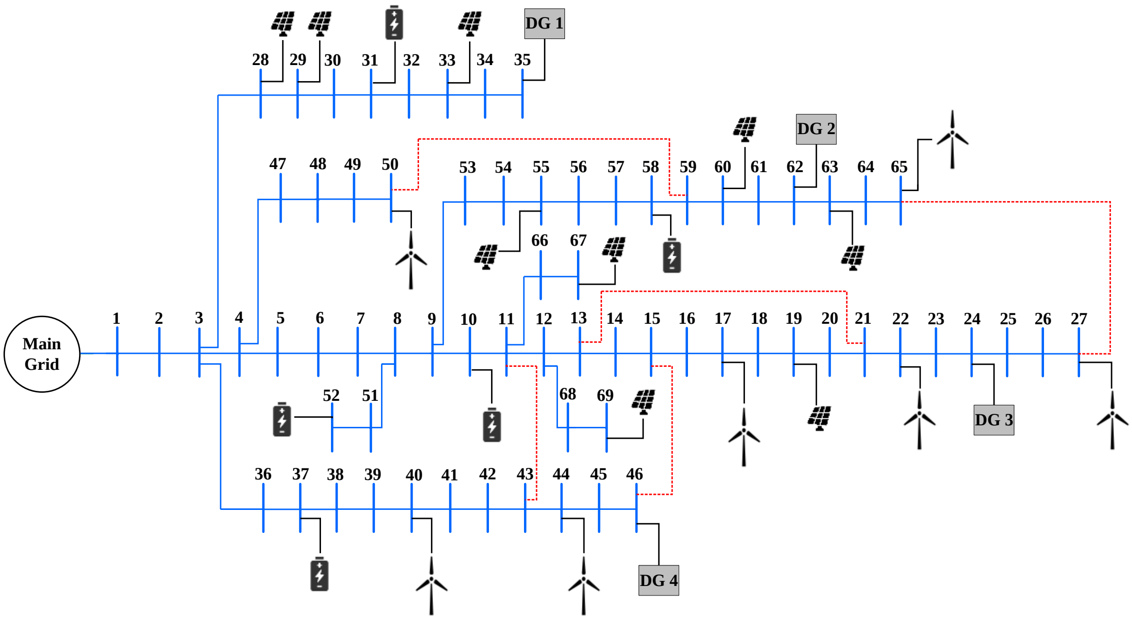
$$\sum_{mn}\left(y_{mn,t}^{+} + y_{mn,t}^{-}\right) = \Lambda_n - 1; \quad \forall t \tag{65}$$

$$y_{mn,t}^{+} + y_{mn,t}^{-} \leq 1; \ \forall mn, t \tag{66}$$

4.9 CASE STUDIES AND RESULTS

The grid-connected balanced microgrid is modeled using a modified IEEE 69-bus test system, which is illustrated in Figure 5. This microgrid comprises nine photovoltaic units with a capacity of 100 kW each, five batteries, four dispatchable distributed generation units, and seven wind turbine units with a capacity of 600 kW each. Further details regarding these distributed energy resources (DERs) can be found in Table 1 and Table 2. The study's time horizon is limited to 24 hours, focusing on day-ahead scheduling. To implement the proposed deterministic mixed-integer linear programming model, the AMPL environment (Fourer et al. (2003)) is utilized, and the commercial solver CPLEX (ILOG, 2008) is employed to solve the model.

Figure 5 – Microgrid based on the modified IEEE 69-bus test system



Source: Created by the author

Table 1 – Data on characteristics of dispatchable distributed generation units.

Dispatchable DG Units	**Bus**	$\underline{P}_{dg}^{DG}$ **(kW)**	$\overline{P}_{dg}^{DG}$ **(kW)**	a^{DG} **($)**	b^{DG} **($/MW)**
DG 1	35	100	350	27	87
DG 2	62	75	300	25	87
DG 3	24	75	300	28	92
DG 4	46	100	410	26	81

Source: Created by the author

Table 2 – Data on characteristics of batteries.

Battery Systems	**Bus**	$\overline{P}^{BS}$ **(kW)**	$\underline{E}^{BS}$ **(kWh)**	$\overline{E}^{BS}$ **(kWh)**	η_{ch}^{BS} & η_{dis}^{BS}	β^{BS}
BS 1	10	200	80	720	0.95	0.002
BS 2	31	200	80	720	0.94	0.001
BS 3	37	200	80	720	0.97	0.002
BS 4	52	200	80	720	0.95	0.001
BS 5	58	200	80	720	0.96	0.002

Source: Created by the author

In this thesis, it is assumed that the necessary data and forecasts required for day-ahead energy management of microgrids are available. Figure 6 illustrates the forecasted power generation from photovoltaic (PV) units and wind turbines (WT). In addition, electric demand and electricity price forecasts are depicted in Figure 7.

Figure 6 – Renewable energy generation forecast

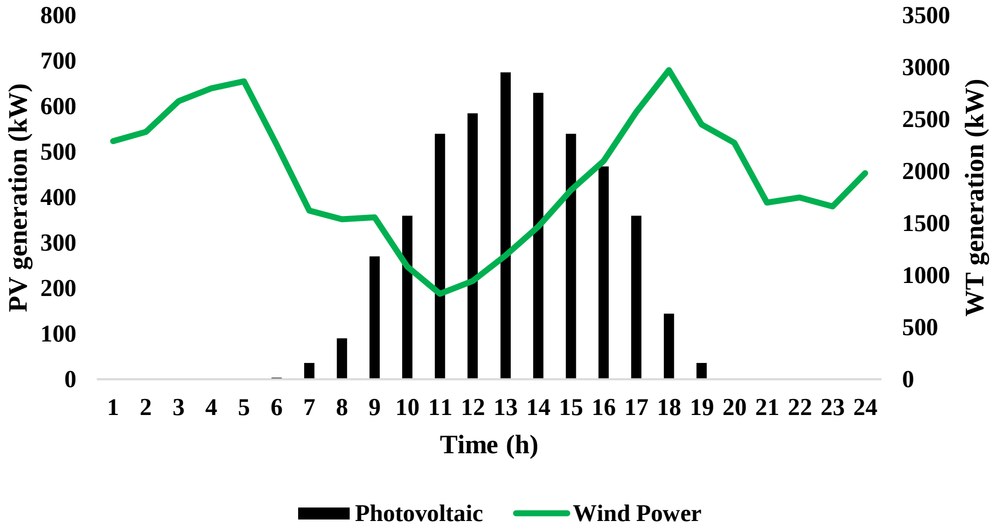
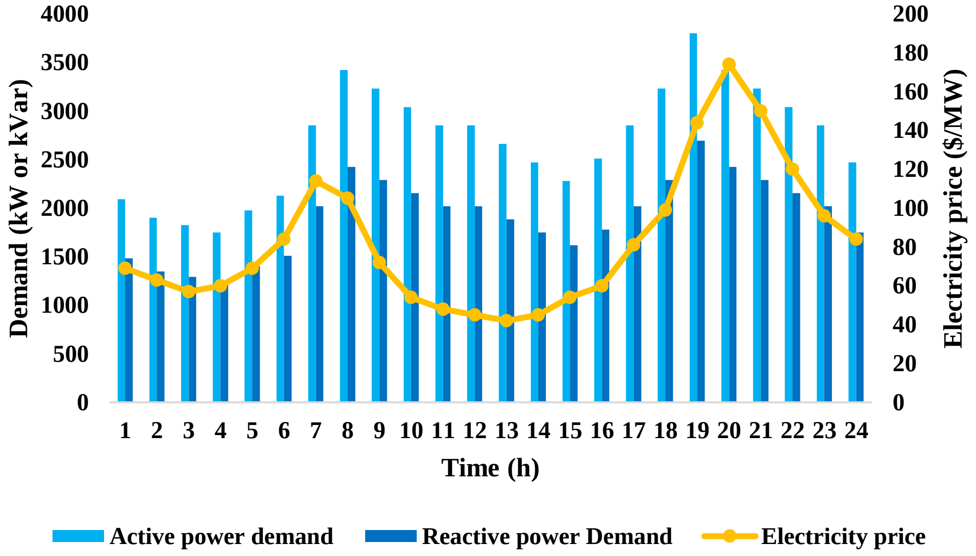


Figure 7 – Power demand and electricity price forecasts



4.9.1 CASE STUDY I

In this case study, the optimization model for the single objective optimal energy management of microgrids is solved without taking into account the reconfigurability of the microgrid. The objective function considered is the cost minimization presented in (33).

However, to assess the performance of the microgrid, the power loss and voltage deviation index presented in (34) and (35) are also calculated based on the optimal solution.

The decision variables in this study include the scheduling of dispatchable distributed generation units, the charging/discharging of batteries, the power exchanged between the microgrid and the main power system, load shedding, and renewable energy curtailment. The resulting values for the total cost, power losses, and voltage deviation index are $5797.91, 6538.14 kWh, and 1.11, respectively. In this case study, load shedding and renewable energy curtailments are both determined to be 0, indicating that the renewable generation does not exceed any technical limits, such as voltage or current constraints. However, as previously mentioned, it is advisable to include load shedding and renewable energy curtailments in the model to pre-emptively avoid potential issues related to the technical constraints of microgrids.

Figure 8 displays the generation of DG units. We can observe that the commitment and actual generation of the units differ, which is understandable since these generation units have distinct characteristics, such as different cost coefficients, and are connected to different buses. For instance, DG 3 does not generate any power, while DG 4 is committed to generating power at 15 hours. Furthermore, there are no committed units from $t = 1$ to $t = 6$, indicating the impact of lower electricity prices and reduced power demand on the generation units' commitment. In other words, during periods of lower power demand at night or when the electricity price from the main grid is lower, it is more cost-effective for microgrids to purchase energy from the main grid rather than utilizing their own generation units.

Figure 8 – Scheduling of dispatchable distributed generation units in Case Study I

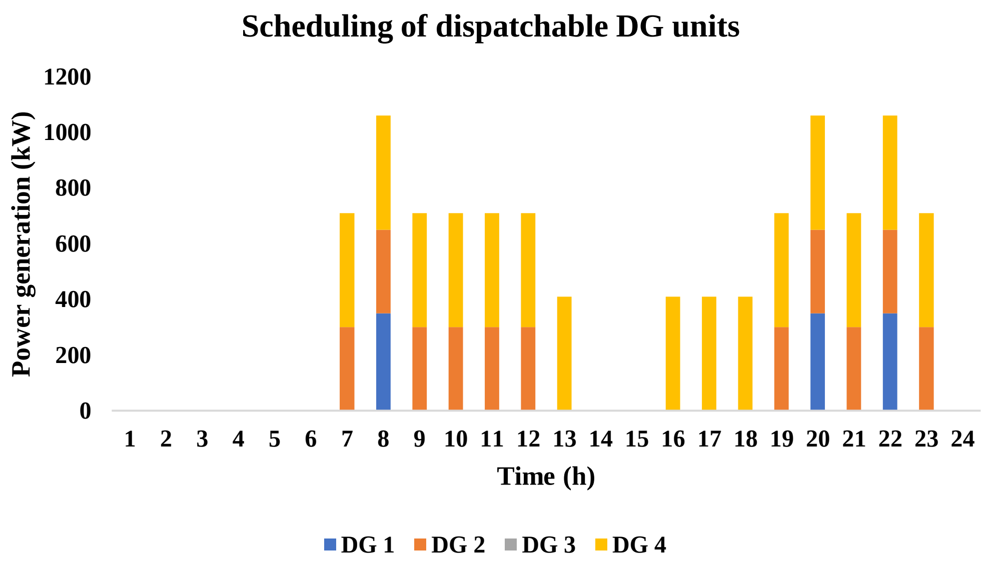


Figure 9 illustrates the graph depicting the charging and discharging patterns of batteries. As previously mentioned, batteries play a crucial role in the optimal energy management of microgrids. For instance, between $t = 2$ and $t = 4$, all the batteries are charging, while in the afternoon, from $t = 19$ to $t = 21$, they discharge to support the microgrid in meeting the power demand. These charging and discharging patterns align with expectations, as it is logical for batteries to charge during periods of lower electricity prices and to discharge when the energy prices are higher. This approach allows for cost-saving benefits associated with distributed generation, as batteries can economize on fuel costs by utilizing stored energy during times of higher electricity prices.

Displaying voltage profiles for all 69 buses of the microgrid over a 24-hour time horizon is not a suitable approach. Instead, in Figure 10, we visualize the maximum and minimum voltage magnitudes among all the microgrid buses. It is important to note that the acceptable range for voltage deviation is considered to be 10% (i.e. $\underline{V} = 0.9\ p.u.$ and $\overline{V} = 1.1\ p.u$).

Figure 9 – Charging and discharging of batteries in Case Study I

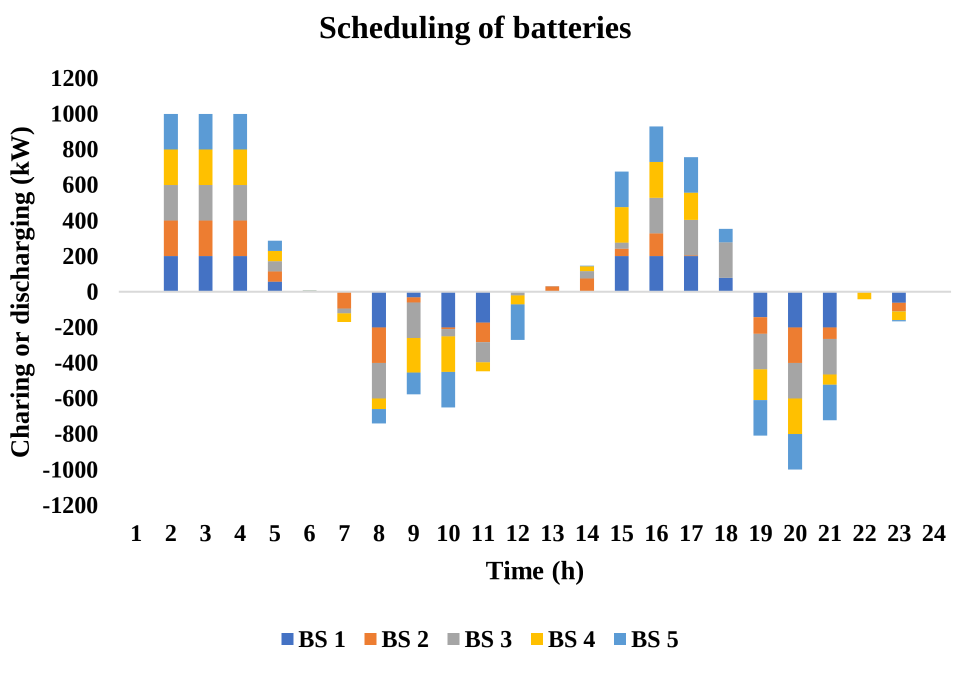
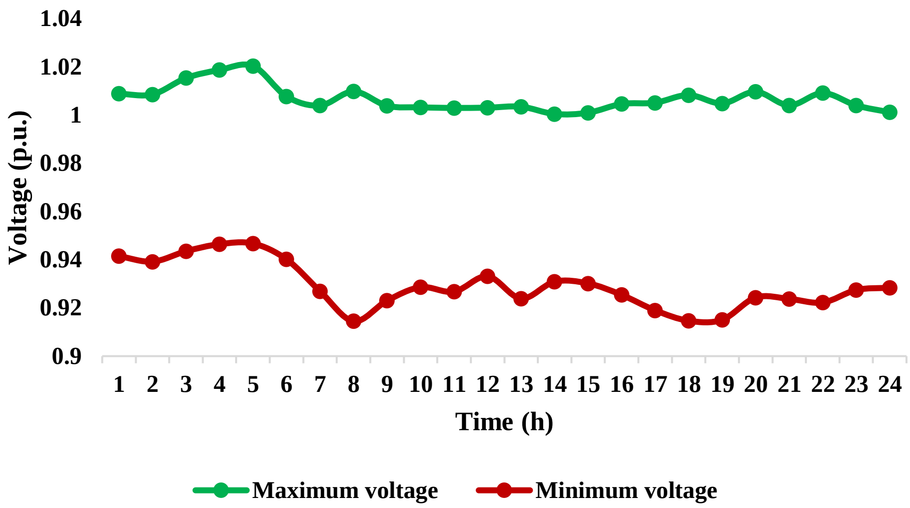


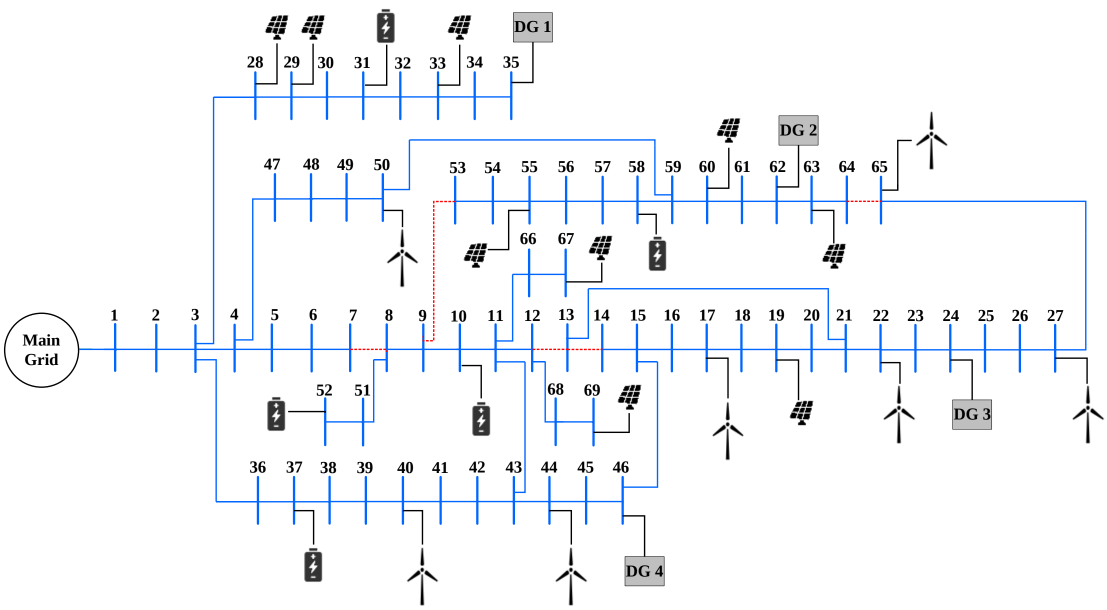
Figure 10 – Maximum and minimum voltage profiles in Case Study I



4.9.2 CASE STUDY II

In this case study, the optimal energy management of microgrids is solved, and simultaneously, the optimal configuration for the microgrid is determined. This means that in addition to DG and the charging and discharging of batteries, the state of the switches in the microgrid are also considered as decision variables. The optimal configuration is determined for the 24-hour time horizon (from 1 to 24), as presented in Figure 11. The resulting values for the total cost, power losses, and voltage deviation index are $5458.58, 5440.47 kWh, and 0.92, respectively. This indicates that not only the cost, but also other performance indices, have been improved due to reconfiguration in this case study.

Figure 11 – Optimal configuration of the microgrid in Case Study II for 24h



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Based on Figure 11, the optimal configuration of the microgrid shows that all the normally open switches are now closed, and switches associated with five lines, as indicated by red dashes, are opened following the reconfiguration. Similarly, in Case Study I, the power generation of DG units and the charging and discharging of batteries are illustrated in Figure 12 and Figure 13, respectively. Comparing the results between these two case studies reveals that considering the reconfigurability of the microgrid leads to changes in the optimal solution regarding other decision variables. This is reasonable since incorporating reconfiguration in the model can alter the feasibility sets and consequently impact the optimal solutions.

Figure 12 – Scheduling of dispatchable distributed generation units in Case Study II

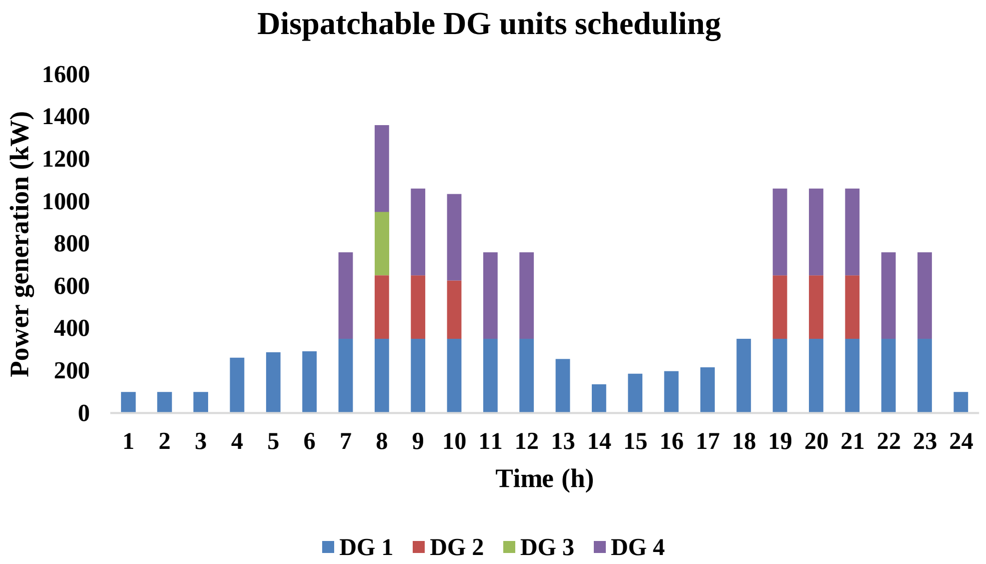


Figure 13 – Charging and discharging of batteries in Case Study II

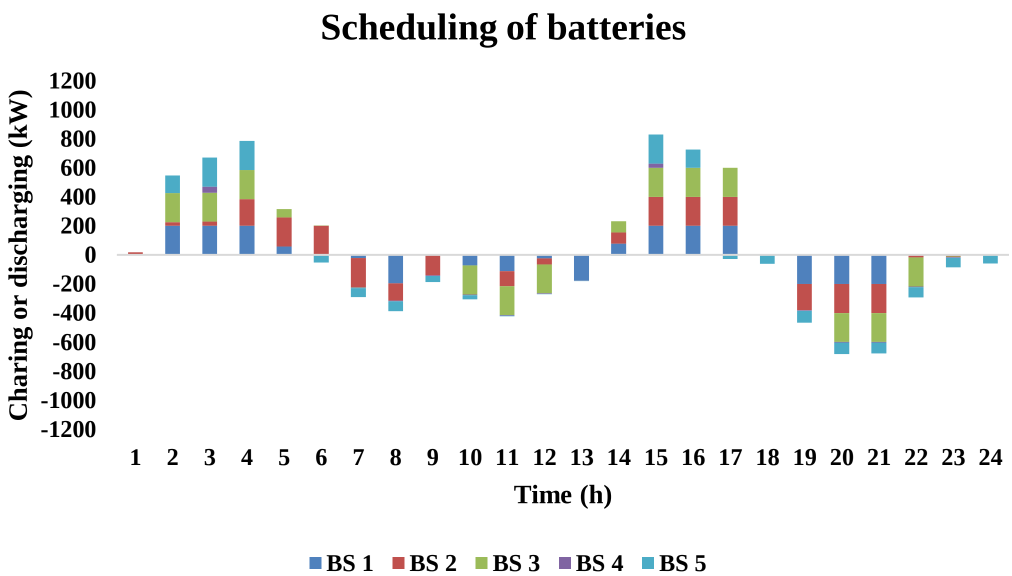
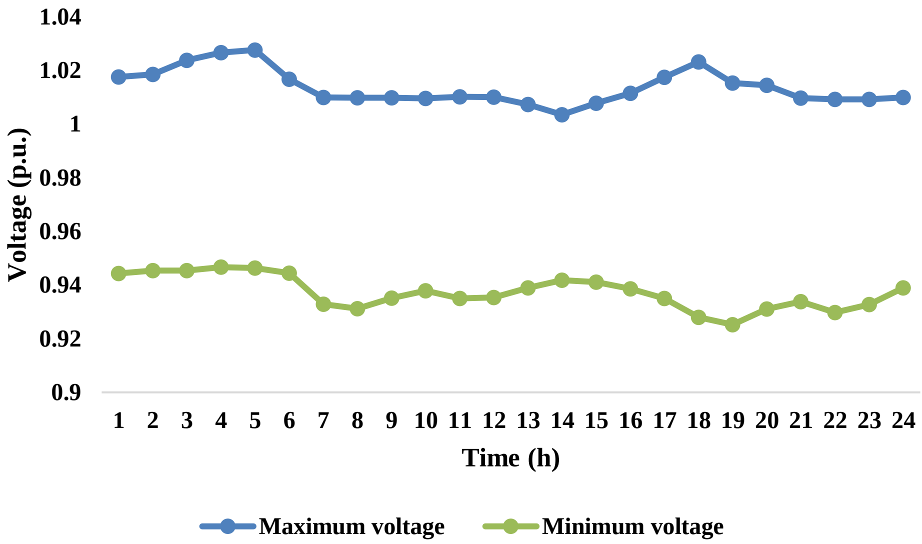


Figure 14 illustrates the maximum and minimum voltage profiles for Case Study II. A comparison between Figure 10 and Figure 14 reveals that the minimum voltage profile has increased. This improvement in voltage profile, where the voltage gets closer to the reference voltage, also results in a reduction of power losses from 6538.14 kWh in Case Study I to 5440.47 kWh in Case Study II. Notably, the reconfiguration, by altering the microgrid's structure, significantly influences the power flow, leading to a reduction in power losses and an improvement in the voltage profile.

Figure 14 – Maximum and minimum voltage profiles in Case study II



5 TWO-STAGE STOCHASTIC MODEL FOR OPTIMAL ENERGY MANAGEMENT OF MICROGRIDS

This section introduces the comprehensive mathematical model for a two-stage stochastic optimal operation of a grid-connected microgrid. The commitment of dispatchable DG units is regarded as the first-stage decision variable, while the second-stage decisions consist of the amount of power generated by dispatchable DGs, the charging/discharging of battery systems, and the power that is purchased from (or sold to) the main grid.

5.1 OBJECTIVE FUNCTION

Various objective functions can be taken into account for the optimal energy management of microgrids. However, to avoid complicating the model with a multi-objective problem, we consider the total cost of the microgrid as the single objective function, as presented in (67). The total cost comprises two terms: the cost associated with dispatchable DG generation and the cost related to the power exchanged between the microgrid and the main power system.

$$
\begin{aligned}
min(F) = &\sum_{t} \Delta_t u_t^{DG} a^{DG} \\
&+ \sum_{s}\sum_{t} \Delta_t \pi_s \left(C_{t,s}^{GR} P_{t,s}^{GR} + b^{DG} P_{dg,t,s}^{DG} + c^{DG} \left(P_{dg,t,s}^{DG}\right)^2 \right)
\end{aligned}
\tag{67}
$$

In equation (67), u_t^{DG} is a binary variable that indicates the commitment status of dispatchable DG units at each time interval t, while $P_{dg,t,s}^{DG}$ represents the power generated by DG units in each time interval t and scenario s. The cost function for DG units is incorporated using a quadratic function, with a^{DG}, b^{DG}, and c^{DG} denoting the cost coefficients. Additionally, $C_{t,s}^{GR}$ and $P_{t,s}^{GR}$ correspond to the electricity price and the power that is either purchased from or sold to the main grid each time interval t and scenario s.

5.2 CONSTRAINTS

In this section, the limits related to the stochastic optimal operation of microgrids are presented, encompassing the power flow of balanced microgrids and other operational restrictions related to the system components (e.g., DG units and batteries). It is worth noting that the presented power flow is valid for single-phase or balanced microgrids. Therefore, for three-phase unbalanced microgrids, different power flow models should be used. Further information on unbalanced power flow can be found in Zandrazavi et al. (2022).

5.2.1 NONLINEAR STOCHASTIC POWER FLOW MODEL

The microgrid's nonconvex stochastic power flow formulation is introduced in equations (68)–(73). The optimization model's nonconvexity is primarily enforced by constraint (71).

$$\sum_{km} P_{km,t,s} - \sum_{mn}\left(P_{mn,t,s} + R_{mn} I^{sqr}_{mn,t,s}\right) + P^{GR}_{t,s} + \sum_{wt|wt=m} P^{CWT}_{wt,t} + \sum_{pv|pv=m} P^{CPV}_{pv,t} \tag{68}$$

$$+ \sum_{dg|dg=m} P^{DG}_{dg,t} = P^{D}_{m,t,s} + \sum_{bs|bs=m} P^{BS}_{bs,t,s}\ ;\ \forall m,t,s$$

$$\sum_{km} Q_{km,t,s} - \sum_{mn}\left(Q_{mn,t,s} + X_{mn} I^{sqr}_{mn,t,s}\right) + Q^{GR}_{t,s} + \sum_{dg|dg=m} Q^{DG}_{dg,t,s} = Q^{D}_{m};\ \ \forall m,t,s \tag{69}$$

$$V^{sqr}_{m,t,s} - V^{sqr}_{n,t,s} = 2\left(R_{mn}P_{mn,t,s} + X_{mn}Q_{mn,t,s}\right) + Z^{2}_{mn} I^{sqr}_{mn,t,s};\ \ \forall mn,t,s \tag{70}$$

$$V^{sqr}_{n,t,s} I^{sqr}_{mn,t,s} = P^{2}_{mn,t,s} + Q^{2}_{mn,t,s};\ \forall mn,t,s \tag{71}$$

$$\underline{V}^{2} \le V^{sqr}_{m,t,s} \le \overline{V}^{2};\ \ \forall m,t,s \tag{72}$$

$$0 \le I^{sqr}_{mn,t,s} \le \overline{I}^{2}_{mn};\ \ \forall mn,t,s \tag{73}$$

Equations (68) and (69) pertain to the real and reactive power balances, where $P_{mn,t,s}$ and $Q_{mn,t,s}$ represent the real and reactive power flowing in line mn at time interval t and scenario s. The parameters R_{mn} and X_{mn} correspond to the resistance and reactance of line mn. Additionally, $P^{CWT}_{wt,t,s}$ and $P^{CPV}_{pv,t,s}$ denote the real power generated by WT and PV units at time interval t and scenario s, respectively. Moreover, $P^{GR}_{t,s}$ and $Q^{GR}_{t,s}$ represent the real and reactive power exchanged between the microgrid and the main network. Furthermore, $P^{D}_{m,t,s}$ and $Q^{D}_{m,t,s}$ represent the real and reactive power required by electric loads during time interval t and scenario s, respectively.

Equations (70) and (71) illustrate the relationship between currents, voltages, real power, and reactive power flowing through line mn. Here, Z_{mn} represents the impedance of line mn, and $V^{sqr}_{n,t,s}$ denotes the square of the voltage magnitude at bus n during time interval t and scenario s. Additionally, equations (72) and (73) impose restrictions on acceptable voltage and current ranges at each bus m and each line mn during time interval t and scenario s, respectively. The upper bounds for voltage and current magnitudes are denoted by $\overline{V}$ and $\overline{I}_{mn}$, while the lower bound for voltage magnitude is represented by $\underline{V}$. As mentioned previously, constraint (71) is responsible for the nonconvex nature of the optimization model for the three-phase power flow. Nevertheless, as demonstrated by Farivar and Low (2013), by replacing (71) with (74), the formulation can be transformed into a convex problem.

$$V_{n,t,s}^{sqr} I_{mn,t,s}^{sqr} \geq P_{mn,t,s}^{2} + Q_{mn,t,s}^{2}; \;\; \forall mn,t,s \tag{74}$$

5.2.2 CONSTRAINTS RELATED TO THE POINT OF THE COMMON COUPLING TRANSFORMER

The microgrid's connection to the main power system involves a transformer, leading to limitations on the apparent power exchange between the two systems. This technical constraint is represented by (75), with $\overline{S}^{TR}$ denoting the capacity of the transformer.

$$\left(P_{t,s}^{GR}\right)^2 + \left(Q_{t,s}^{GR}\right)^2 \leq \overline{S}^{TR}; \;\; \forall t,s \tag{75}$$

5.2.3 CONSTRAINTS RELATED TO DISPATCHABLE DG UNITS

As previously mentioned, the first-stage decision involves the commitment of DG units. Additionally, each DG unit has specific limitations on its real and reactive power generation capabilities. These operational constraints are presented in (76), where $\underline{P}_{dg}^{DG}$ and $\overline{P}_{dg}^{DG}$ represent the lower and upper bounds for the generation capacity of each DG unit, respectively. The power factor of DGs is denoted by θ^{DG}. It is evident that when the binary variable u_t^{DG}, representing the DG commitment, is set to (0), the DG unit does not generate any power. Conversely, when the binary variable is (1), the power generated by the DG unit is confined within the specified range. The reactive power that can be generated by DG units is also limited as presented in (77) and it depends on $P_{dg,t,s}^{DG}$.

$$u_t^{DG} \underline{P}_{dg}^{DG} \leq P_{dg,t,s}^{DG} \leq u_t^{DG} \overline{P}_{dg}^{DG}; \;\; \forall dg,t,s \tag{76}$$

$$-\tan(\cos^{-1}(\theta^{DG}))\, P_{dg,t,s}^{DG} \leq Q_{dg,t,s}^{DG} \leq \tan(\cos^{-1}(\theta^{DG}))\, P_{dg,t,s}^{DG}; \;\; \forall dg,t,s \tag{77}$$

5.2.4 CONSTRAINTS RELATED TO WIND POWER UNITS

Maximizing the utilization of renewable energy resources is a crucial goal. However, considering technical limitations, it is prudent to account for curtailment in renewable energy-based units, such as wind power generators, to prevent potential issues like overvoltage or overcurrent in the microgrid (Sabillon et al. (2018)). In other words, reducing renewable power generation can be a preventive measure to avoid contingencies in the system. Therefore, the model in equation (78) incorporates wind power curtailment capability, where $\tilde{P}_{wt,t,s}^{WT}$ denotes the available wind power generation, and $\Theta_{wt,t,s}^{WT}$ represents the amount of power generation that is curtailed. It is essential to ensure that the curtailed power does not exceed the available power, meaning wind power units cannot consume active power. Hence, constraint (79) limits $\Theta_{wt,t,s}^{WT}$ accordingly.

$$P_{wt,t,s}^{CWT} = \tilde{P}_{wt,t,s}^{WT} - \Theta_{wt,t,s}^{WT};\ \ \forall wt, t, s \tag{78}$$

$$0 \leq \Theta_{wt,t,s}^{WT} \leq \tilde{P}_{wt,t,s}^{WT};\ \ \forall wt, t, s \tag{79}$$

5.2.5 CONSTRAINTS RELATED TO PHOTOVOLTAIC UNITS

In a similar manner to wind power generation units, photovoltaic generation can also have its output power reduced when required through generation curtailment. The model is outlined in equation (80), where $\tilde{P}_{pv,t,s}^{PV}$ represents the available solar power generation, and $\Theta_{pv,t,s}^{PV}$denotes the amount of power generation to be curtailed. Just like with wind power, it is essential to ensure that the curtailed power does not exceed the available solar power, so constraint (81) limits $\Theta_{pv,t,s}^{PV}$ accordingly.

$$P_{pv,t,s}^{CPV} = \tilde{P}_{pv,t,s}^{PV} - \Theta_{pv,t,s}^{PV};\ \ \forall pv, t, s \tag{80}$$

$$0 \leq \Theta_{pv,t,s}^{PV} \leq \tilde{P}_{pv,t,s}^{PV};\ \ \forall pv, t, s \tag{81}$$

5.2.6 CONSTRAINTS RELATED TO BATTERIES

Batteries play a crucial role in optimizing microgrid operations, as they enable energy storage during periods of low demand or when electricity prices are lower, and they can release stored power into the microgrid during periods of high demand or when electricity prices are higher. This study takes into account the efficiency of batteries, meaning that a small portion of energy is lost during charging or discharging cycles. Therefore, two distinct continuous variables, defined in equations (82) and (83), are used to represent the charging and discharging power of the batteries. Specifically, $P_{bs,t,s}^{BS+}$ corresponds to the charging power, and $P_{bs,t,s}^{BS-}$ corresponds to the discharging power. Importantly, considering battery efficiency in the formulation helps prevent simultaneous charging and discharging, ensuring that $P_{bs,t,s}^{BS+}$ and $P_{bs,t,s}^{BS-}$ cannot be non-zero at the same time. The total power of the batteries, encompassing both charging and discharging, is represented by $P_{bs,t,s}^{BS}$ during time interval t and scenario s.

$$0 \leq P_{bs,t,s}^{BS+} \leq \overline{P}^{BS};\ \ \forall bs, t, s \tag{82}$$

$$0 \leq P_{bs,t,s}^{BS-} \leq \overline{P}^{BS}; \ \forall bs,t,s \tag{83}$$

$$P_{bs,t,s}^{BS} = P_{bs,t,s}^{BS+} - P_{bs,t,s}^{BS-}; \ \forall bs,t,s \tag{84}$$

Equations (85) and (86) establish the connection between the charging and discharging powers and the energy stored in the batteries. Here, E_{ini}^{BS} represents the initial charge of the batteries, and $E_{bs,t,s}^{BS}$ denotes the amount of energy stored in the batteries during time interval t and scenario s. The parameters η_{ch}^{BS}, η_{dis}^{BS}, and β^{BS} correspond to the charging efficiency, discharging efficiency, and self-discharge rate, respectively.

$$E_{bs,t,s}^{BS} = E_{ini}^{BS} + \Delta_t \left(P_{bs,t,s}^{BS+} \eta_{ch}^{BS} - {P_{bs,t,s}^{BS-}}\big/{\eta_{dis}^{BS}} - E_{bs,t,s}^{BS} \beta^{BS} \right); \ \forall bs,t,s | t = 1 \tag{85}$$

$$E_{bs,t,s}^{BS} = E_{bs,t-1,s}^{BS} + \Delta_t \left(P_{bs,t,s}^{BS+} \eta_{ch}^{BS} - {P_{bs,t,s}^{BS-}}\big/{\eta_{dis}^{BS}} - E_{bs,t,s}^{BS} \beta^{BS} \right); \ \forall bs,t,s | t > 1 \tag{86}$$

To prevent excessive discharging and overcharging of batteries, and also to account for their limited capacity, a range for battery energy is incorporated through constraint (87). In this constraint, $\underline{E}^{BS}$ represents the lower bound, and $\overline{E}^{BS}$ represents the upper bound of the battery energy. This ensures that the battery energy remains within a safe and acceptable range.

$$\underline{E}^{BS} \leq E_{bs,t,s}^{BS} \leq \overline{E}^{BS}; \ \forall bs,t,s \tag{87}$$

5.3 VALUE OF THE STOCHASTIC SOLUTIONS

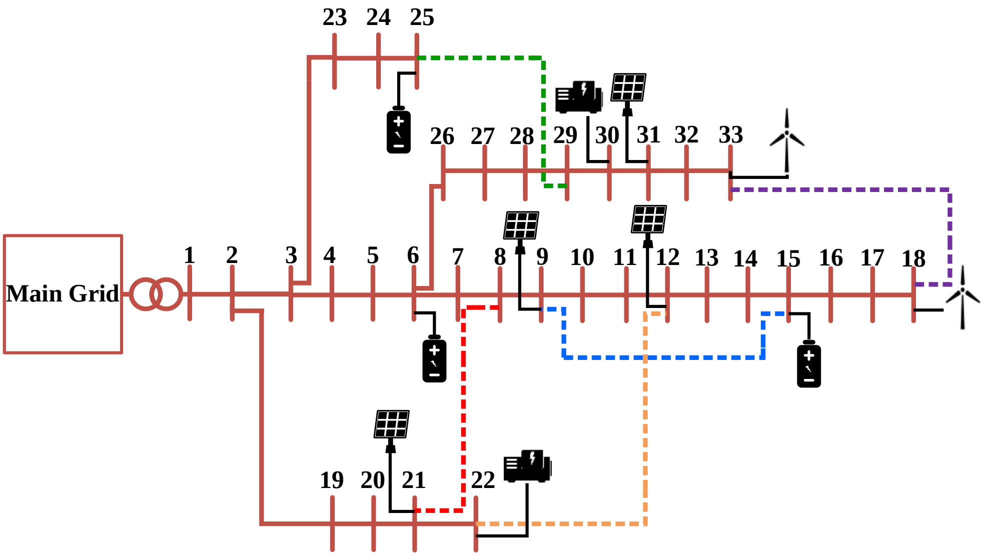
The value of the stochastic solutions (VSS) is an index to economically analyze the performance of stochastic optimization by analogy with the deterministic programming Conejo et al. (2010). The value of VSS can be obtained based on (88) Conejo et al. (2010), where obj^{S*} is the optimal value of the objective function in the stochastic optimization problem. To achieve the value of obj^{D*}, in the first step an equivalent deterministic model based on the average value of scenarios for all uncertain parameters must be solved. In the next step, the stochastic problem is solved for each scenario separately, yet all the first-stage decisions must be fixed to the solutions of the deterministic model (i.e. the one based on the average value).

$$VSS = obj^{D*} - obj^{S*} \tag{88}$$

5.4 CASE STUDY AND RESULTS

The grid-connected balanced microgrid, representing a modified IEEE 33-bus test system, serves as the test system for this study (Figure 15). This microgrid comprises 4 Photovoltaic systems (600 kW), 3 batteries, 2 dispatchable DG units, and 2 wind turbine units (1,500 kW). Additional details about these distributed energy resources can be found in Table 3 and Table 4. The study's time horizon is limited to 24 hours, focusing on day-ahead scheduling. To model the microgrid and its optimization, a two-stage stochastic mixed-integer second-order cone programming model is employed. The implementation of this model is done in the AMPL environment (Fourer et al., (2003)), and the commercial solver CPLEX (ILOG, 2008) is used to solve the optimization problem.

Figure 15 – Modified IEEE 33-bus as a grid-connected microgrid



Source: Created by the author

Table 3 – Data on the dispatchable DG units

Dispatchable DG units	Bus	$\underline{P}_{dg}^{DG}$(kW)	$\overline{P}_{dg}^{DG}$ (kW)	a^{DG}($)	b^{DG}($/MW)	c^{DG}($/MW^2)
DG 1	22	50	500	30	87	0.0025
DG 2	30	50	400	35	81	0.184

Source: Created by the author

Table 4 – Data on the battery systems

Battery Systems	Bus	$\overline{P}^{BS}$ (kW)	$\underline{E}^{BS}$(kWh)	$\overline{E}^{BS}$ (kWh)	η_{ch}^{BS} & η_{dis}^{BS}	β^{BS}
BS 1	6	100	60	360	0.95	0.002
BS 2	15	150	60	460	0.94	0.002
BS 3	25	120	60	430	0.96	0.002

Source: Created by the author

To account for uncertainties in wind power generation, photovoltaic generation, electric prices, and electric demand, a 7-level scenario generation method is adopted as presented by Karimi et al., (2021); Karimi and Jadid, (2020). The scenarios for photovoltaic generation and wind turbine power generation are shown in Figure 16 and Figure 17, respectively, with the black curve (Scenario 4) representing the expected values. Notably, these figures illustrate a negative correlation between wind power and solar power generation. Specifically, significant photovoltaic generation is expected during the noon, while wind power generation is predicted to be more substantial in the early morning and afternoon.

Figure 16 – Scenarios for photovoltaic generation

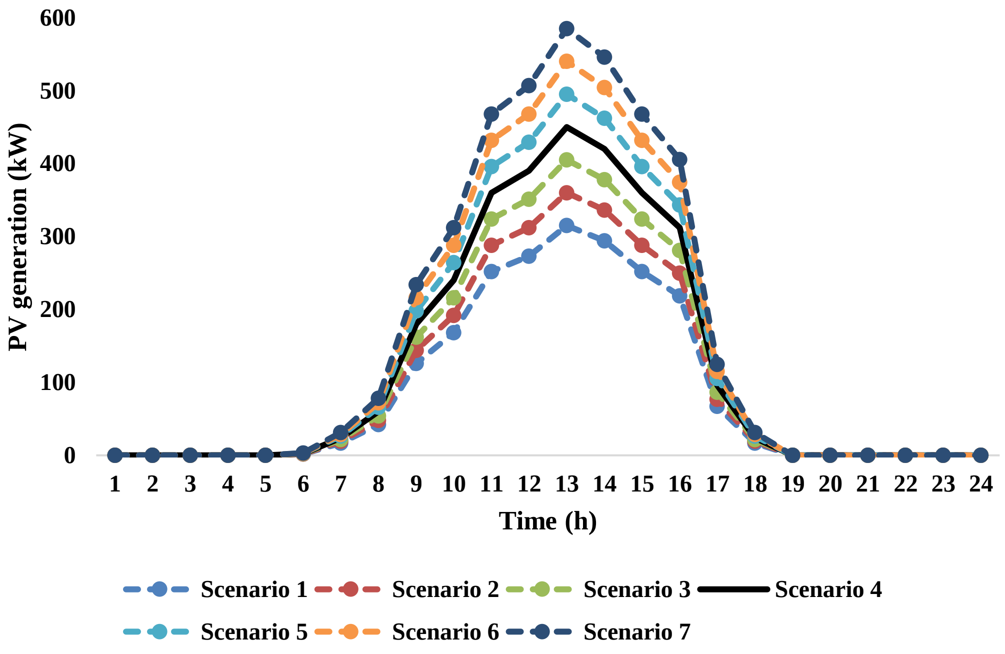


Figure 17 – Scenarios for wind turbine generation

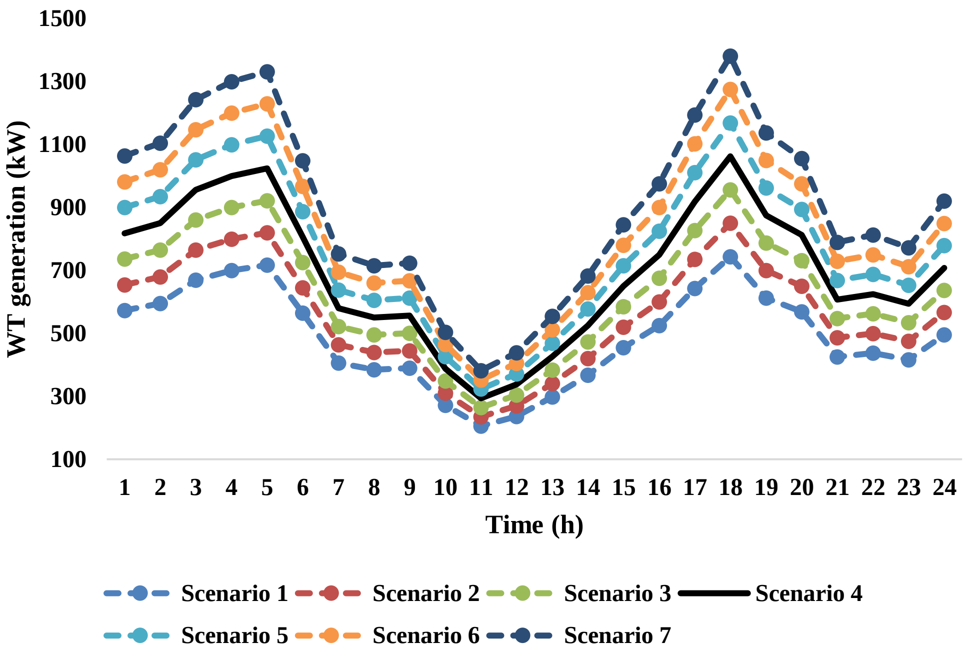
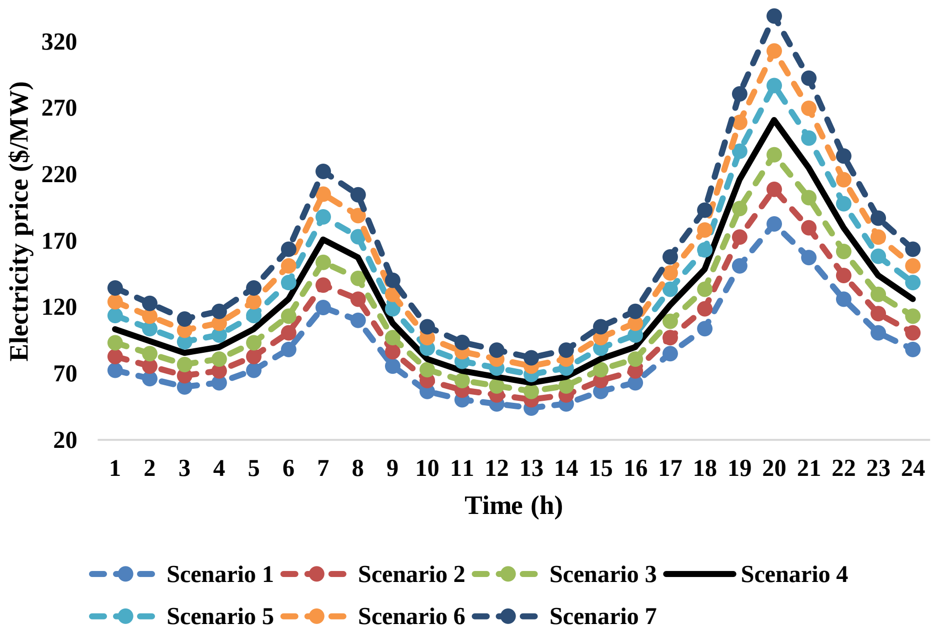
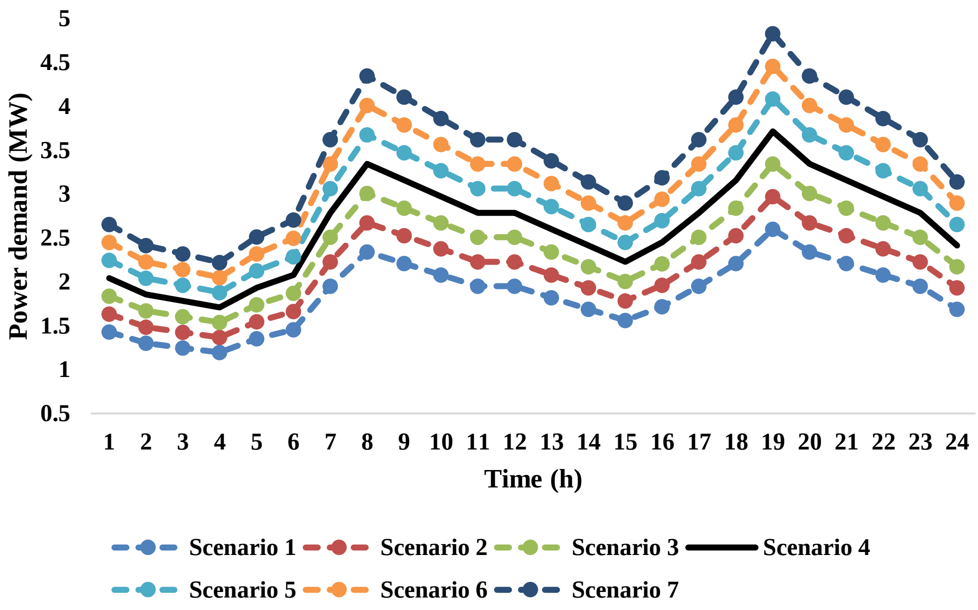


Figure 18 – Scenarios for electricity price



Likewise, Figure 18 and Figure 19 showcase the generated market price and electric demand scenarios, respectively. The anticipated values for the forecasts are represented by the black lines.

Figure 18 – Scenarios for electric power demand



It is essential to underscore that in this particular case study, the optimization model is addressed without accounting for the reconfigurability of the microgrid. This decision is motivated by the potential complexity introduced by a substantial number of binary variables, which could render the problem computationally intractable. Additionally, the problem is divided into two stages: the commitment of dispatchable distributed generation units is considered as the initial stage decision, while the charging/discharging of batteries, power generation by dispatchable distributed generation units, power exchange between the microgrid and the main grid, and renewable energy curtailment are treated as the second stage decision variables. Consequently, the solutions to these variables are contingent upon the realization of the various scenarios. The optimal day-ahead scheduling of the microgrid results in a total cost of $2,833. Furthermore, as previously mentioned, the value of the stochastic solution (VSS) index provides a means to assess the performance of stochastic programming in comparison to deterministic optimization. In this case study, the VSS is calculated at $456, indicating an approximate 16% reduction in the total cost (i.e., obj^{D*} = $3289). Such findings underscore the potential benefits of employing stochastic programming techniques in managing microgrid operations under uncertain conditions.

Figure 20 illustrates the commitment of DGs, where a value of one represents committed units, and zero denotes non-committed units. Based on the figure, DG 1 is committed to generating power for 7 hours during the day, while DG 2 is committed for 5 hours. The distinction in their commitments can be attributed to variations in their characteristics, such as chaoticities and generation costs. Additionally, their locations and operational constraints, such as voltage and current limits within the microgrid, can also influence their commitments and generation. Moreover, there exists a positive correlation between DG commitment and the higher price of electricity. In other words, it is economically sensible to maintain committed DG units, despite the commitment costs, to avoid purchasing electricity from the main power grid during periods of elevated market prices. This approach helps mitigate expenses and leverages the distinct capabilities of the committed DG units.

As previously stated, the charging and discharging of batteries are treated as real-time decision variables in this analysis. However, it is also feasible to represent their average values. This is demonstrated in Figure 21, which illustrates the behavior of the batteries. According to the figure, the batteries charge when the forecasted electricity price is lower and discharge power into the microgrid when the predicted price is higher. In essence, the batteries play a vital role in reducing the operational costs of the microgrid. They achieve this by storing energy during periods of lower prices and supplying it to the microgrid when the energy demand arises, thereby optimizing the overall energy usage and cost efficiency.

Figure 22 presents the variations in the maximum and minimum voltages within the microgrid, encompassing the voltage levels of all buses for different scenarios. Observably, an increase in wind power generation is accompanied by an augmentation in both the maximum and minimum voltage magnitudes. This observation can be attributed to the substantial contribution of wind power generation in comparison to photovoltaic generation. Furthermore, it is noted that when generation occurs towards the end of feeders, a higher voltage rise is encountered. This is due to the reverse power flow passing through a greater impedance compared to generation units located at the initial points of the feeders. Consequently, this phenomenon of voltage rise imposes limitations on the hosting capacity of renewable energy in distribution networks and microgrids. Nevertheless, by incorporating equation (72) into the model, the optimal solution ensures adherence to voltage limitations. This mathematical inclusion guarantees that voltage constraints are not violated during the optimization process, thereby maintaining the secure and stable operation of the microgrid.

Figure 19 – Commitment of dispatchable DGs compared to price scenarios

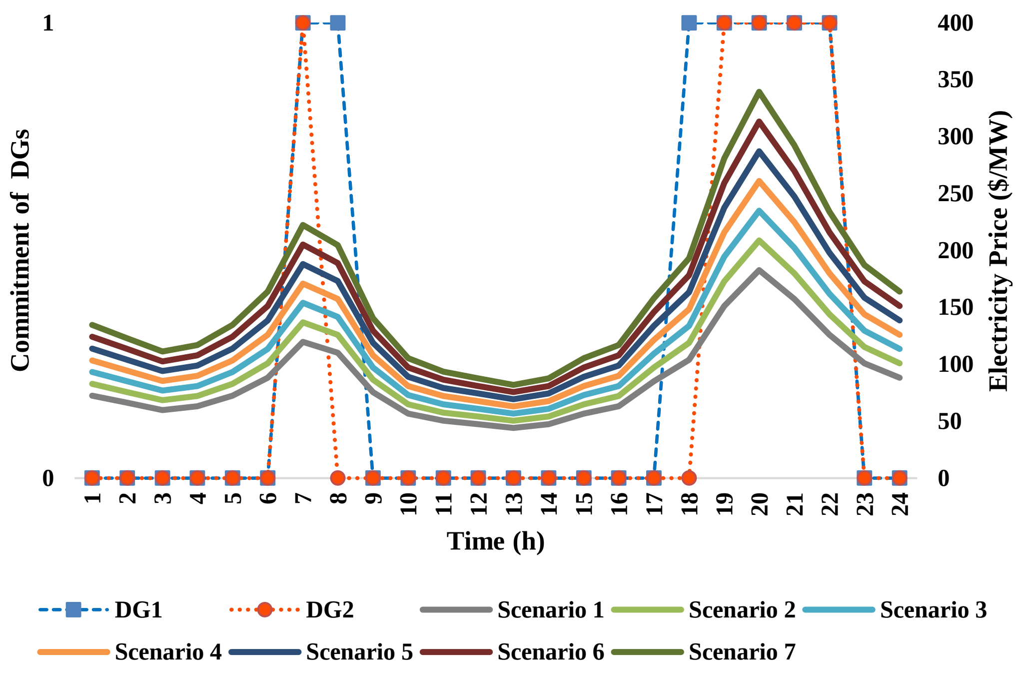


Figure 20 – Average charging and discharging of batteries compared to price scenarios

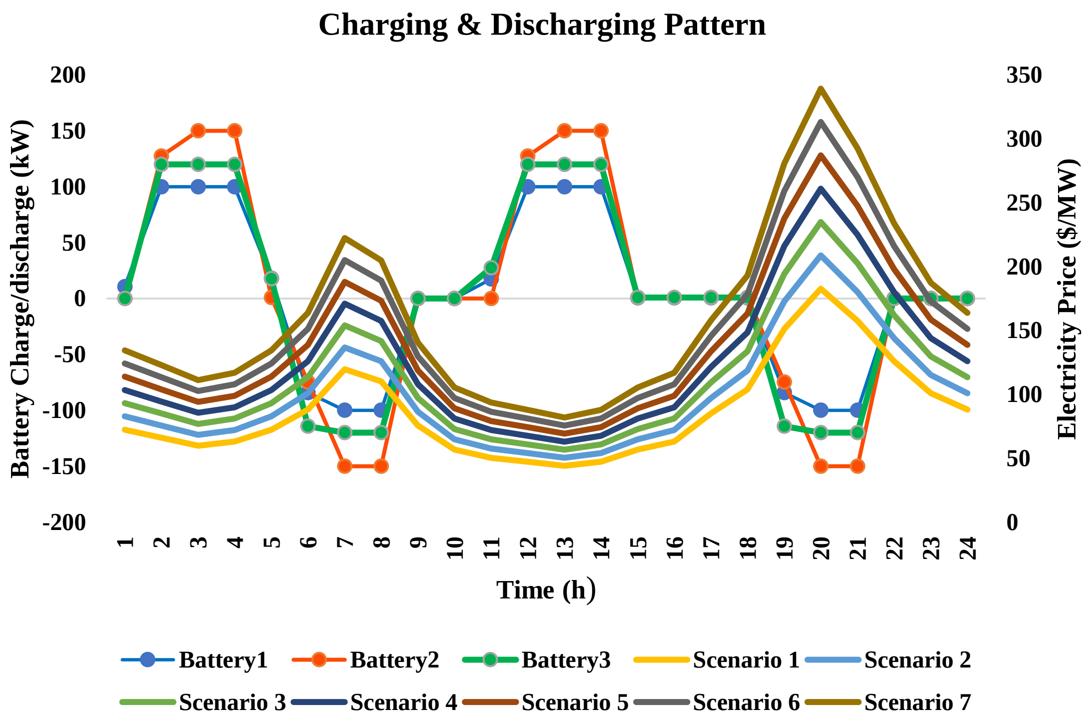
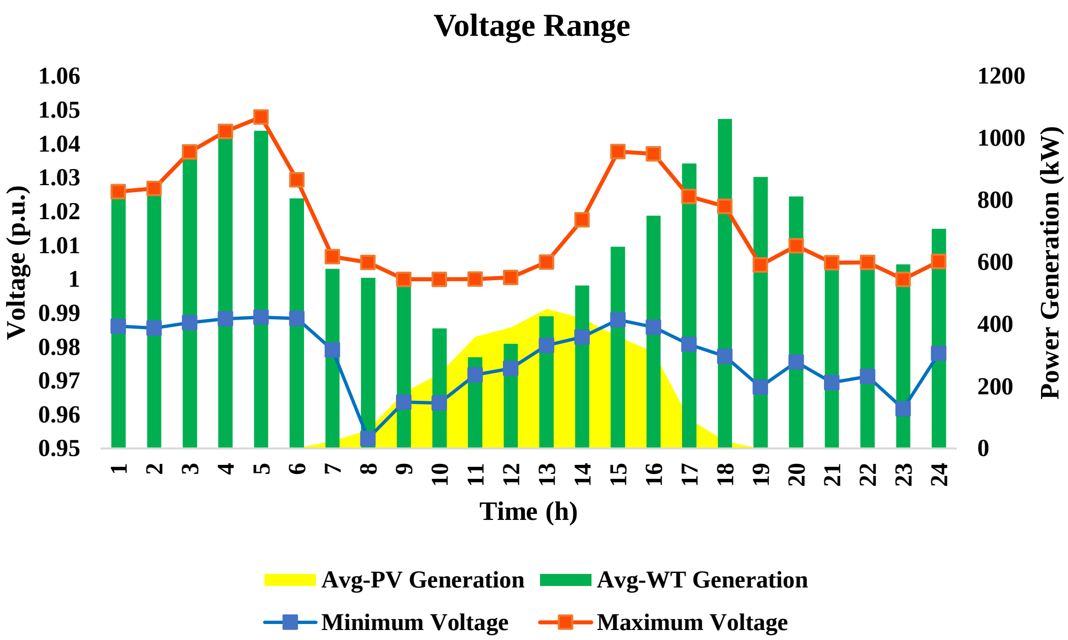


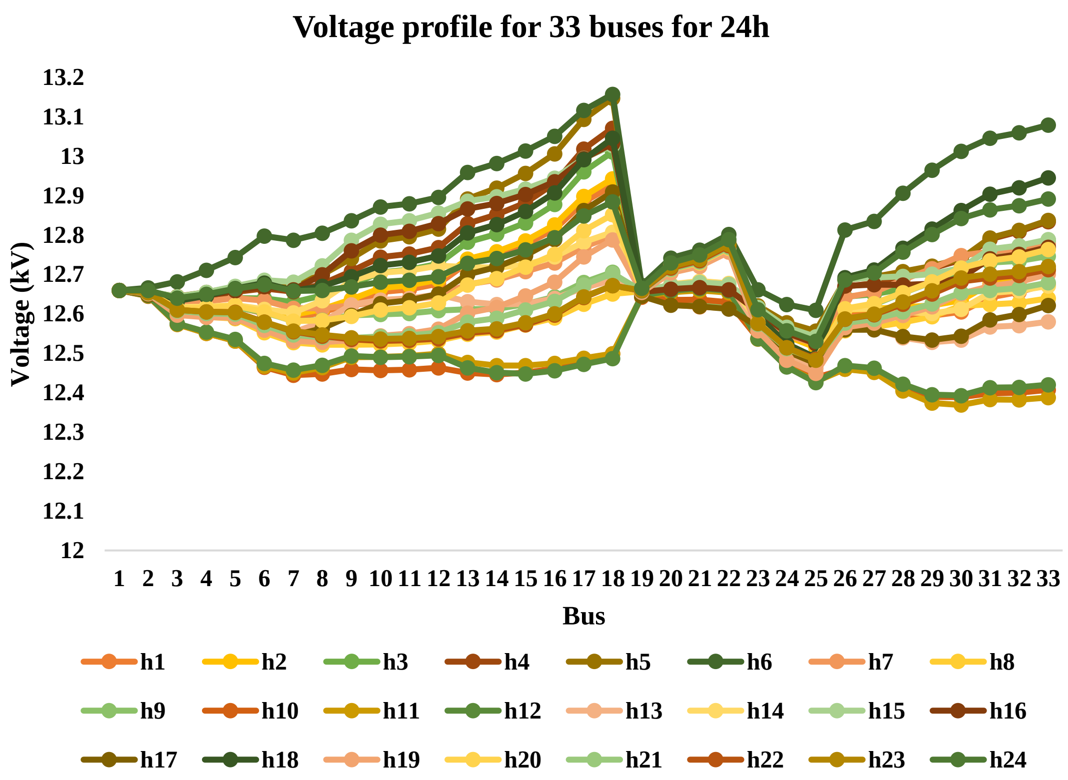
Figure 21 – Maximum and minimum voltage compared to renewable generation



Conversely, the impact of PV generation appears to have a more significant influence on the minimum voltage levels within the microgrid, while its effect on maximum voltage levels remains relatively marginal. This observation can be attributed to the relatively lower capacity of photovoltaic units compared to wind turbines, with PV units having a capacity of 600 kW as opposed to the 1500 kW capacity of wind turbine units. Furthermore, the positioning of these renewable energy sources differs within the microgrid. Unlike wind turbines, photovoltaic units are not located at the end of the microgrid feeders. Consequently, the reverse power flow from photovoltaics has a less pronounced effect on raising the voltage levels at the associated buses compared to the buses where wind turbines are connected.

To acquire deeper understanding, Figure 23 displays the voltage profile of the complete microgrid, comprising 33 buses, over different time intervals (hours) with consideration of the forecasted values in Scenario 4. The visualization showcases significant variations in bus voltages throughout the day, underscoring the dynamic nature of the microgrid's voltage behavior.

Figure 23 – Voltage profile of microgrid during the day for Scenario 4



It is important to note that stochastic programming considers all scenarios, leading to an overall optimal cost and solution, even though they may not be optimal for each individual scenario (from a deterministic perspective). The solutions obtained are feasible for the considered scenarios. However, it is crucial to acknowledge that scenario-based stochastic programming's computational time increases exponentially with the number of scenarios. If the model needs to be feasible for worst-case scenarios as well, an alternative approach such as adaptive robust optimization, as previously explained in Chapter 4, may be more advantageous for uncertainty modeling for three key reasons. Firstly, it only considers the worst-case scenario, making it more time efficient. Secondly, it does not require exact probability density information for uncertain parameters; knowing the interval is sufficient. Lastly, the results can be more reliable, as the solutions are guaranteed to be feasible for all scenarios falling within the specified range.

6 CONCLUSIONS AND FUTURE WORK

The optimal energy management of microgrids is modeled as a mathematical optimization problem to find the solutions guaranteeing the minimum total cost. Therefore, having good knowledge about fundamental concepts, such as convexity, feasibility, and linear and nonlinear programming is necessary for this purpose. To this end, some basic concepts in optimization are briefly introduced, involving local optimality, global optimality, feasibility, convexity, linear programming, integer programming, mixed-integer linear programming, nonlinear nonconvex programming, and nonlinear convex programming. Moreover, power flow which plays a key role in the steady-state study of microgrids is modeled step by step. Then, the power flow model is utilized to model the optimal energy management of microgrids as a nonlinear nonconvex programming problem. Some well-known objective functions such as operational cost, power losses, and voltage deviation index are introduced and many technical constraints are defined, and respective formulas are presented. The proposed model is convexified through two approaches, the relaxation of the nonconvex constraint and the linearization method. Finally, the convex model is modified to include reconfigurable microgrids as well. The optimal energy management of microgrid is solved by including and excluding the reconfigurability of the systems. The results show that reconfiguration not only can reduce the operational cost but also can improve other indices such as power losses and voltage deviation index.

The optimal operation of microgrids can be modeled as a deterministic optimization problem, in which all the parameters are considered certain and well-known. Nonetheless, due to the presence of parameters that are inherently uncertain, such as electric demand, plug-in electric vehicles, renewable energy generation, and the electricity market, there are always gaps between the real values of the parameters and the forecasted on. These respective errors can adversely affect the validity and accuracy of the optimal solutions and related decisions. Consequently, it is more practical and less risky to include uncertainty in the mathematical model so as to avoid any unfavorable consequences.

Various methods have been developed by intelligent researchers in the literature to cope with uncertainties and mitigate their negative effects on the conciseness of decisions. Hence, different methods, embracing stochastic programming, fuzzy programming, information gap decision theory, adaptive robust optimization, chance-constrained programming, and Monte Carlo simulation are explained. Each method has its own pros and cons, and probably there is nothing such a perfect method. Notwithstanding, a method should be chosen wisely based on the structure of the study problem and the availability of data regarding the uncertain parameters. For example, when there is substantial information available about uncertain parameters, selecting a method such as information gap decision theory may not be a rational decision. For another example, in fuzzy programming models, the fuzziness and the membership function regarding the constraints or objective functions should be determined in models, so if microgrid operators do not have enough knowledge linked to uncertain parameters, probably will be difficult to model uncertainty precisely.

In order to gain real experience in uncertainty modeling in the optimal operation of microgrids, a two-stage stochastic programming model including uncertainties related to wind power generation, photovoltaic generation, electric demand, and electricity price, is formulated in detail. In addition, the results are visualized and presented so as to not only stochastic programming but also the optimal operation of microgrids can be understood.

In this research, the primary focus has been placed on balanced microgrids within electric microgrid systems. It should be noted that the scope of this study has been restricted solely to the technical aspects of these microgrid configurations, with the exclusion of market-based energy management strategies. For future works, the investigation of unbalanced microgrids, particularly in the presence of electric vehicles, and the integration of multi-energy microgrids, including hydrogen-based systems, can be considered. Moreover, market-based energy management in the context of microgrids would be pursued. Valuable insights into addressing real-world grid conditions and the advancement of resilient and scalable energy systems can be provided by these avenues of future research, contributing to the implementation of sustainable microgrid technologies in the energy landscape.

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