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Analytical Prediction of the Spin Stabilized Satellite's Attitude Using The Solar Radiation Torque

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Abstract. The aim of this paper is to present an analytical solution for the spin motion equations of spin-stabilized satellite considering only the influence of *solar radiation torque*. The theory uses a cylindrical satellite on a circular orbit and considers that the satellite is always illuminated. The average components of this torque were determined over an orbital period. These components are substituted in the spin motion equations in order to get an analytical solution for the right ascension and declination of the satellite spin axis. The time evolution for the pointing deviation of the spin axis was also analyzed. These solutions were numerically implemented and compared with real data of the Brazilian Satellite of Data Collection – SCD1 and SCD2. The results show that the theory has consistency and can be applied to predict the spin motion of spin-stabilized artificial satellites.

1. Introduction

The mission accuracy of artificial satellite depends on the position and orientation in a coordinate system fixed in Earth. However the environmental forces and torques acting on the satellite affect the attitude and the orbit of the satellite. The rotational motion is described by the Euler's dynamic equations, which depend on the external torques. Then it is essential to study the influence of the torques in the satellite mission.

The goal of this paper is to analyze the rotational motion dynamics of spin stabilized Earth artificial satellites, through an analytical attitude prediction. Emphasis is given to modeling the solar radiation torque (SRT) and its influence on the spin axis of the spin-stabilized satellite. The direction of the spin axis in relation to the inertial system is specified by the right ascension (α) and the declination (δ).

The Solar Radiation Pressure is created by the continuous photons collisions with the satellite surface, which can be able to absorb or reflect on this flow. The total change of the momentum of all the incident photons on the satellite surface originates from the Solar Radiation Pressure Force, which can cause considerable disturbances in spacecraft orbits when the satellite has a great ratio area/mass. This force can produce a torque.

The most important radiation sources in space are the Sun and Earth. The provided radiation from the Earth has two significant portions: the solar radiation reflected diffusely on the Earth surface (*terrestrial albedo*), and the *terrestrial radiation*, that is the spontaneous emission in the infra-red



range, proportionally to the fourth power of the absolute terrestrial surface temperature. The albedo and the terrestrial radiation are respectively 90 and 93% smaller than the direct solar radiation at 700 km of altitude. Both decrease with the increase of the altitude. So, due to their magnitudes, they are worthless.

In this paper the model of SRT is given by [1] and their average components for an orbit period are included in the motion equations, therefore it's possible to obtain the analytical solution of these equations.

Two approaches were chosen and used to examine the influence of SRT acting during the evolution of rotational motion of the satellite. In the first approach, the attitude and orbit data were updated every 24 hours. In the second approach, the computed attitude and orbit data weren't updated in order to determine the analytical solution period of validity. In all numerical simulations the orbital elements were updated taking into account the main influence of the Earth oblateness.

Numerical simulations performed with data of Brazilian Satellites SCD1 and SCD2 show the period in which the analytical solution could be used to the attitude propagation, within the dispersion range of the attitude determination system performance of Brazilian Institute for Space Research - INPE.

2. Solar Radiation Torque

A Direct Solar Radiation Torque model was developed in Zanardi and Vilhena de Moares [1] for the case which the illuminated surfaces of the satellite are a circular flat surface, and a portion of the cylindrical surface. It is given by:

$$\vec{N} = N_x \hat{i} + N_y \hat{j} + N_z \hat{k} \quad (1)$$

$$N_x = -\frac{k}{R^4}(\beta_1\gamma_1 - \beta_2\gamma_2)\frac{h}{2}\pi\sigma^2 u_z^*(u_y^* \cos(\theta) + u_x^* \sin(\theta)) \quad (2)$$

$$N_y = -\frac{k}{R^4}(\beta_1\gamma_1 - \beta_2\gamma_2)\frac{h}{2}\pi\sigma^2 u_z^*(u_y^* \sin(\theta) - u_x^* \cos(\theta)) \quad (3)$$

$$N_z = 0 \quad (4)$$

$$u_x^* = (a_s R_x + r' a_1) \quad (5)$$

$$u_y^* = (a_s R_y + r' a_2) \quad (6)$$

$$u_z^* = (a_s R_z + r' a_3) \quad (7)$$

where k is a solar parameter with assumed value given by $1,01 \times 10^{17} \text{ kgm/s}$, a_s is Sun-Earth distance and here assume the value $1,49597870 \times 10^{11} \text{ m}$, r' is the satellite geocentric distance, R is the Sun-satellite distance, $\beta_i, \gamma_i, i=1,2$, are specular and has total reflection coefficients, respectively, for each satellite surface (which assume constant values), a_1, a_2 e a_3 are direction cosines which relate the Orbital System and the Satellite System (given in terms of orbital elements, right ascension and declination of the spin axis [2]), R_x, R_y e R_z give the Sun direction in the Satellite System and are obtained by:

$$R_x = b_1 \cos(\delta_s) \cos(\alpha_s) + b_2 \cos(\delta_s) \sin(\alpha_s) + b_3 \sin(\delta_s) \quad (8)$$

$$R_y = b_4 \cos(\delta_s) \cos(\alpha_s) + b_5 \cos(\delta_s) \sin(\alpha_s) + b_6 \sin(\delta_s) \quad (9)$$

$$R_z = b_7 \cos(\delta_s) \cos(\alpha_s) + b_8 \cos(\delta_s) \sin(\alpha_s) + b_9 \sin(\delta_s) \quad (10)$$

being b_1, b_2, \dots, b_9 the direction cosines which relate the Equatorial System and the Satellite System (given in terms of satellite's rotation angle, right ascension and declination of the spin axis [2]) and α_s, δ_s the Sun's right ascension and declination, respectively.

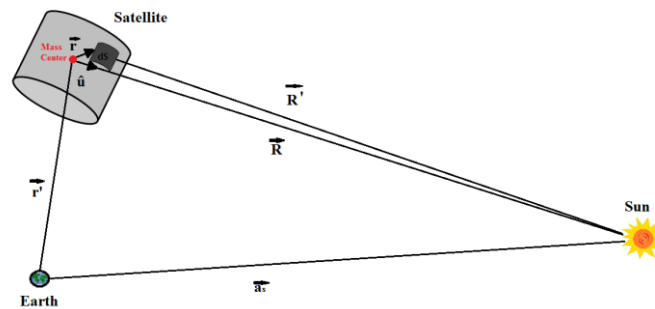


Figure 1. Satellite and Area Element Position

The Sun-satellite distance is represented in Fig. 1 and can be obtained by:

$$R^2 = a_s^2 + r'^2 + 2r'a_s(a_1R_x + a_2R_y + a_3R_z) \quad (11)$$

By the equation (4) it is possible to observe that the component of the satellite axis "Oz" is zero, due to the satellite geometric symmetry (cylindrical shape [1]). Then, the satellite oscillates around this axis.

3. Analytical Solution for the Equations of Rotational Motion

The variations of the spin velocity, the declination and the right ascension of the spin axis for spin stabilized artificial satellites are given by the Euler equations in spherical coordinates [3,4]:

$$\frac{d\alpha}{dt} = \frac{N_x}{I_z W \cos(\delta)} \quad (12)$$

$$\frac{d\delta}{dt} = \frac{N_y}{I_z W} \quad (13)$$

$$\frac{dW}{dt} = \frac{N_z}{I_z} \quad (14)$$

where I_z is the mean moment of inertia along the spin axis, N_x , N_y , N_z are the components of solar radiation torque.

At first moment it is hard to get the analytical solution for the equations (12) – (14), because the torque components depend on the time. Then in order to solve these equations, in this paper the SRT mean are the components which were used, and obtained by the integration of the instantaneous torque over one orbit period. Now the equations (12) – (14) can be integrated assuming that the orbital elements (I , Ω e ω) were held constant over one orbital period, with the SRT mean components being constant for one orbit. The spin velocity is also constant because the z-torque components are zero:

$$W = W_0 \quad (15)$$

where W_0 is the initial value for spin velocity.

Then the analytical solution for the declination of the spin axis is given by the integration of equation (13) [2]: (16)

$$\delta = \frac{N_{ym}}{I_z W_0} t + \delta_0$$

where δ_0 is the initial value for the declination of spin axis.

After that, by substituting the solution of declination δ in the equation (12), the solution of the right ascension is solved by Motta [2] and given by:

$$\alpha = \frac{N_{xm}}{N_{ym}} \ln \left[\frac{\sec\left(\frac{N_{ym}}{I_z W_0} t + \delta_0\right) + \tan\left(\frac{N_{ym}}{I_z W_0} t + \delta_0\right)}{\sec(\delta_0) + \tan(\delta_0)} \right] + \alpha_0 \quad (17)$$

where α_0 is the initial value for right ascension of spin axis.

The solutions presented in the equations (15), (16) and (17), for the spin velocity magnitude, declination and right ascension of the spin axis respectively, are valid for one orbital period. Thus, for every orbital period, the orbital data must be updated, taking into account at least the main influences of the Earth oblateness. With this approach, the analytical theory will be close to the real attitude behaviour of the satellite.

4. Applications

The theory developed has been applied to the spin stabilized Brazilian Satellites SCD1 and SCD2 for verification and comparison of the theory against data generated by the Satellite Control Center (SCC) of INPE. Two approaches were presented [3,5]. In the first one the propagated attitude is daily updated with the help of real satellite data, supplied by INPE. In the second approach daily updates of the attitude data were not performed in the propagation process.

The figures (1) – (4) show the results for the first approach during 40 days. The pointing deviation η is the angle between computed and provided spin axis, and its evolution time is presented in the figures (5) and (6). The results show that the region where the analytical solution is closer to the real data corresponds to the smallest decay of the spin velocity [2]. Over the test period the difference between theory and real data has a mean error deviation in right ascension and in declination of $-1,0812^\circ$ and $-0,0092^\circ$ respectively for the SCD1, and the SCD2 presents a mean error deviation of $0,1666^\circ$ in right ascension and $0,0584^\circ$ of mean error deviation in declination. The mean pointing deviation for the period test was $0,3704^\circ$ for SCD1 and $0,145033^\circ$ for SCD2 which is within the dispersion range of the attitude determination system performance by SCC (0.5°).

Table 1 presented the results for the second approach. The simulations were interrupted in the 5th day for SCD1 and SCD2 because the mean deviations errors between the computed values and real values for all variables were bigger than INPE's required precision for both satellites.

Table 1. Brazil Satellite's Data.

Day	SCD1's Data			Day	SCD2's Data		
	$\Delta\alpha$ (°)	$\Delta\delta$ (°)	η (°)		$\Delta\alpha$ (°)	$\Delta\delta$ (°)	η (°)
21/08/1993	0	0	0	24/02/2002	0	0	0
22/08/1993	-0,1292	0,2889	0,2898	25/02/2002	0,1744	0,1904	0,2082
23/08/1993	-0,1021	0,5783	0,5786	26/02/2002	0,3937	0,3932	0,4371
24/08/1993	-0,932	0,7084	0,7282	27/02/2002	0,6579	0,6049	0,6842
25/08/1993	-0,4393	0,9781	0,9814	28/02/2002	0,9637	0,8051	0,9321

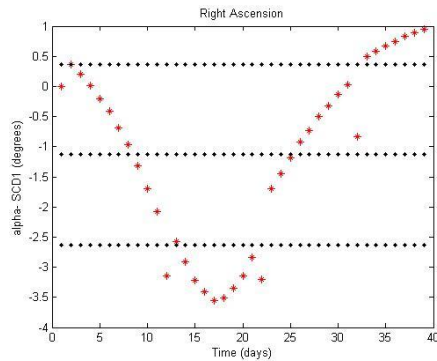


Figure 1. Time evolution for the difference between computed and provided right ascension to SCD1.

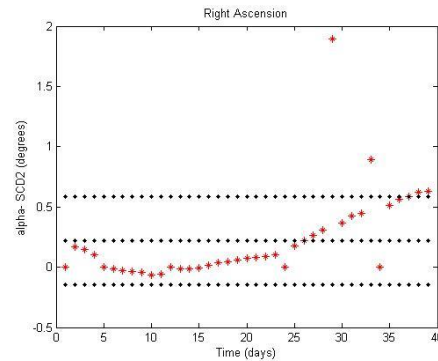


Figure 2. Time evolution for the difference between computed and provided right ascension to SCD2.

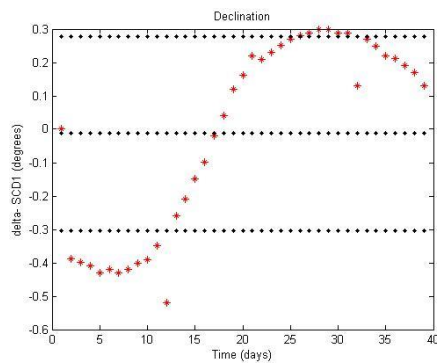


Figure 3. Time evolution for the difference between computed and provided declination to SCD1.

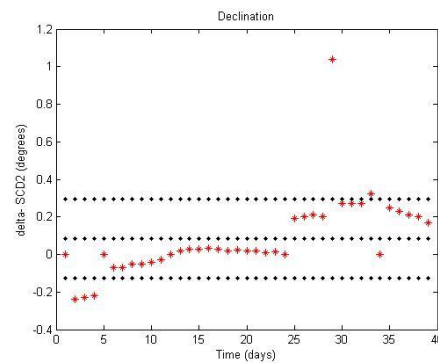


Figure 4. Time evolution for the difference between computed and provided declination to SCD2.

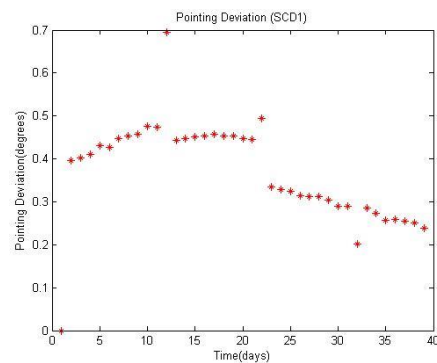


Figure 5. Time evolution for pointing deviation to SCD1.

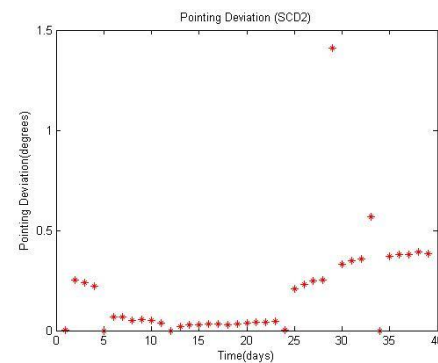


Figure 6. Time evolution pointing deviation to SCD2.

The mean pointing deviation was $0,5156^\circ$ for SCD1 and $0,4523^\circ$ for SCD2. The mean declination was $0,5107^\circ$ for the SCD1 and $0,3987^\circ$ for the SCD2, which are close to the dispersion range of the attitude determination system performance of SCC, even though the SCD1 surpassed this. The mean right ascension ($-0,3205^\circ$ for SCD1 and $0,4379^\circ$ for SCD2) is within the required precision by INPE, then it is possible to point out that the declination of the spin axis has great influence in the pointing deviation. The same observation can be applied for the first approach in 40 days for SCD1 simulation

where mean pointing deviation and mean declination were within the dispersion range required by INPE while the mean of right ascension did not satisfy the required precision by INPE.

5. Conclusion

In this paper an analytical approach for the spin-stabilized satellite rotational motion was presented taking into account the influence of the solar radiation torque.

The analytical solution shows that the effect of the radiation torque does not cause variation in the spin velocity magnitude because of the satellite's symmetry.

The theory was applied to the spin stabilized Brazilian's satellites SCD1 and SCD2. Results have shown the agreement between the analytical solution and the real satellite behaviour for specific time simulation and two approaches were presented.

In the first one the attitude and orbital data were daily updated with real attitude data supplied by INPE. The results showed a good agreement between the computed and real data during all simulation. The mean pointing deviation was of $0,3704^\circ$ for the SCD1 and $0,1450^\circ$ for the SCD2, which are within the dispersion range of the attitude determination system used for this satellite.

In the second approach the attitude and orbital data are not daily updated. The results presented a good agreement between the analytical solution and the actual satellite behaviour only for a five days simulation for SCD1 and SCD2. For more intervals the mean deviation of the right ascension, declination and pointing deviation were higher than the precision required by the SCC/INPE (0.5°).

For both approaches it is possible to note the influence of the declination of the spin axis in the calculation of the pointing deviation. In order to improve the results it is important to include the other external torques. However the procedures are useful for modelling the dynamics of spin stabilized satellite attitude disturbed by solar radiation torque.

6. Acknowledgments

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7. References

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