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To cite this article: T C F Carvalho *et al* 2017 *J. Phys.: Conf. Ser.* **911** 012025

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# Rendezvous and Quasi-Rendezvous Maneuvers With Space Debris

T C F Carvalho<sup>1</sup>, A D C Jesus<sup>1</sup>, L S Ferreira<sup>1</sup>, R R Sousa<sup>2</sup>

<sup>1</sup> UEFS, Universidade Estadual de Feira de Santana, Feira de Santana - BA, Brasil.

<sup>2</sup> UNESP, Universidade Estadual Paulista Júlio Mesquita Filho. Guaratinguetá, SP. Brasil.

E-mail: Tham\_mia@hotmail.com

**Abstract.** Rendezvous maneuvers are very useful in space missions operations. The encounters between spacecraft, between a spacecraft and a space debris, between a probe and a celestial body, between a spacecraft and a nearby object (NEO) as a mitigation measure to protect the Earth against collision and other applications, make use of these maneuvers. We define Quasi-Rendezvous maneuvers as those performed at low velocity, such that damage to the spacecraft is negligible. In this work, we study the distribution of these maneuvers as a function of the final relative velocities between a spacecraft and a spatial debris. The results were divided into two types: the first, for the dynamics subject only to the terrestrial gravitational force and the second, including the propulsion force acting on the vehicle. They show that the maneuvers propelled can approach the objects, overcoming the effect of the gravity of the earth for very small speeds.

## 1. Introduction

The Rendezvous between the two space objects must occur without collisions between them, that is, the relative position and velocity must be null in same time. This maneuver is one completely constrained, with several applications, mainly for the space station and space debris. Currently this maneuver has been designed for the strategies of mitigation of debris and the risk of collision of bodies near the Earth in 1960 [1]. After him, many authors studied this maneuver under many conditions and constraints. Stern [2] in 1984 approximated the Clohessy-Wiltshire equations to the small time transfer and obtained non-jointed rectilinear trajectories. The generalization to planar, minimum consumption Rendezvous case, in the general central force field was done by Humi [3] in 1993. In this year Abramovitz and Grunwald [4] developed an iterative graphical method to the optimal and planar Rendezvous inside many spacecraft of one space station environment, under several operational constraints saving more than 30 per cent fuel. In 1995, Yu [5] showed that an stable equilibrium state can occur in the relative motion between two close spacecrafts to Rendezvous inside a local coordinate system. Prado [6] also in 1995, derived an algorithm to solve optimal Rendezvous maneuvers with two impulses for a mono-revolution transfer or a multi-revolutions transfer, coplanar or non-coplanar. He found fits of the fuel consumption as function of transfer time. In 2001 Prado and Felipe [7] used impulsive control to study the Rendezvous maneuvers. A chaser-target rendezvous problem was solved by Carpenter [8] using a genetic algorithms. They used the Clohessy-Wiltshire equations as a linear approximation for preliminary mission planning. Another chaser-target type problem is studied by Kim and Spencer [9] with minimum fuel consumption as the objective function. Olsen and Fowler [10] also adopted the genetic algorithms to generate a near optimal solution to a rendezvous problem using elliptic orbits. Crispin [11],[12] obtained solutions to rendezvous problems as nonlinear discrete or continuous time optimal control problems with terminal



constraints. We studied Rendezvous maneuvers non-ideal, considering propulsion thrust deviations and mass variation [13]. We find the relative final position deviations w.r.t. the direction angles deviations, penalized by functions derived from the mass variation.

In 2015, we use the Rendezvous strategy for collision study of a space vehicle with a cloud of space debris [14]. In 2014 studies on the cloud of debris resonant with the earth's rotation angular velocity [15] and the modeling of the cloud of debris as fluid in an equation of continuity [16] were performed. The results obtained showed resonant movements for long periods, others irregular and an analytical expression for the evolution in the time of the density of the cloud. A cloud of debris spread by the interaction with the gravitational field and the obtaining of new trajectories of each debris belonging to the cloud by "Patched-Conics" can be found in [17]. This present study includes the possibility of Quasi-Rendezvous and our propelled maneuvers are continuous. Our approach is not optimal, but we use the propulsion system to control the terminal velocity of nearby objects in a Rendezvous or Quasi-Rendezvous maneuver around Earth.

## 2. Mathematical model

The dynamics between two space objects in the earth's gravitational field can be studied, using the relative motion equations found by Clohessy-Witshire [1]. These authors established the initial conditions in relative position and velocity favorable to a Rendezvous between two spacecraft. Jesus et al [18] included the propulsion force in the relative dynamics between two space objects and found a solution that can be used to perform an evasive maneuver against a collision with space debris. In this work, we will use the model of Jesus et al [18] to study the maneuvers of Rendezvous and Quasi-Rendezvous between a spacecraft and space debris. The coordinates of the relative velocity between these objects, subject to the earth's gravitational field, are:

$$\dot{x}_{(t)} = \dot{x}_o \cos(\omega t) + (2\dot{y}_o + 3\omega x_o) \sin(\omega t) \quad (1)$$

$$\dot{y}_{(t)} = -2\dot{x}_o \sin(\omega t) - (4\dot{y}_o + 6\omega x_o) \cos(\omega t) - (3y_o + 6\omega x_o) \quad (2)$$

$$\dot{z}_{(t)} = \dot{z}_o \cos(\omega t) - z_o \omega \sin(\omega t) \quad (3)$$

This solution was obtained for the situation of objects near each other with respect to the distance to the center of the Earth. The reference system is centered on the space vehicle that has circular orbit around the planet with angular velocity,  $\vec{\omega} = \omega \hat{k}$ , and the y-axis is in the direction radius vector  $r$  that links the objects. With these equations we can study the Rendezvous condition ( $r = 0$  and  $v = 0$ ) and the quasi-Rendezvous condition ( $r = 0$ ,  $v \sim 0$ ). These conditions can be obtained naturally in the gravitational field of the Earth. Nevertheless, a propelled maneuver can increase and control the possibilities of occurrence of these trajectories. This control will allow more safety and precision in the approximation of space objects. The solution of the relative dynamic equations with propulsion force was obtained by Jesus et al [18], whose Cartesian coordinates for the final relative velocity are:

$$\dot{x}(t) = 2Aw \cos(\omega t) + 2Bw \sin(\omega t) - \sum_{n=1}^{\infty} n\gamma F_n e^{-n\gamma t} + E \quad (4)$$

$$\dot{y}(t) = Bw \cos(\omega t) - Aw \sin(\omega t) - \sum_{n=1}^{\infty} n\gamma C_n e^{-n\gamma t} \quad (5)$$

$$\dot{z}(t) = Iw \cos(\omega t) - Hw \sin(\omega t) + \sum_{n=1}^{\infty} n\gamma J_n e^{-n\gamma t} \quad (6)$$

This dynamics obtained for a propulsion system, assuming exponential mass variation of the propellant. All coefficients depend on the relative initial conditions,  $(\vec{r}_o, \dot{\vec{r}}_o)$  and technological parameters characterize the propulsion system. These parameters are:  $\vec{v}_e$  - gas exhaust velocity;  $\chi > 1$  - mass factor, ie, the ratio of spacecraft mass ( $M_o$ ) to initial propellant mass ( $m_o$ ) and;  $\gamma > 0$  - motor power factor. With this model it is possible to control the approximation of the objects (the final values of  $x(t)$ ,  $y(t)$  and  $z(t)$ ) through the performance of the vehicle's propulsion system, establishing new Rendezvous and/or Quasi-Rendezvous conditions with propelled maneuvers. The coefficients in the Equations (4) – (6), in general, we can write them as  $L = (\vec{r}_o, \dot{\vec{r}}_o, \vec{v}_e, \gamma, \chi)$ . They are in the Appendix.

### 3. Numerical Simulation

In this section, we show the results of the simulations of the Rendezvous and Quasi-Rendezvous maneuvers, initially, under the exclusive action of the Earth's gravitational force and then for the maneuvers under the action of the propulsion force of the space vehicle. The Rendezvous condition is difficult to establish from the technical point of view if we consider infinite precision for the final relative position and velocity between the objects. The Quasi-Rendezvous condition is more likely, since depending on the range of small velocity we desire, the propulsion system can adjust the dynamics of the vehicle to obtain it. However, such propulsion systems are also limited and fail to implement any small speed ranges.

#### 3.1. Natural Rendezvous/Quasi-Rendezvous

In the simulations we used time Rendezvous (or collision time) equal to 3,000s. This time is sufficient for the processing operations of the vehicle's internal computer to calculate and implement the maneuver, escaping from the debris or coupling to it. The distributions of the Quasi-Rendezvous possibilities are shown in Figures 1(a), (b) and 2, below.

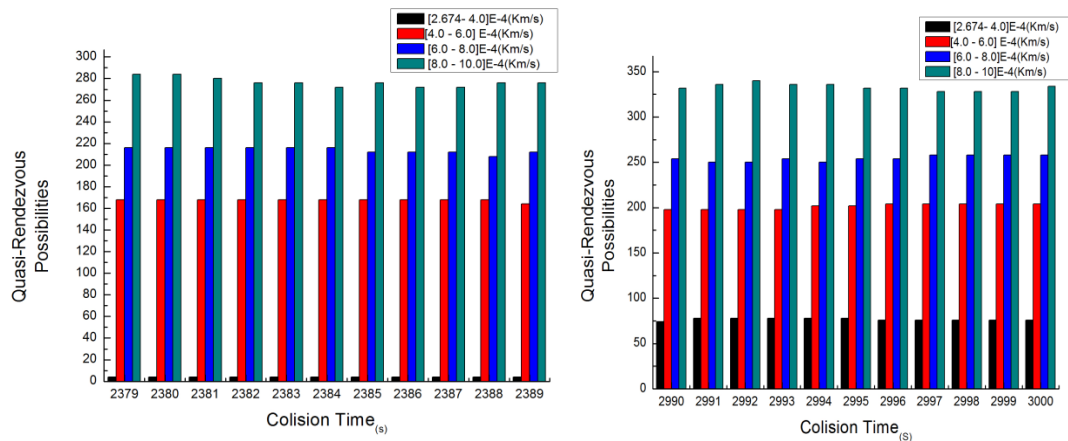


Figure 1- Quasi-Rendezvous Possibilities vs. Time Collision,  $v \approx 10^{-4}$  km/s: (a) [2,379 to 2,389]s, (b) [2,990 to 3,000]s

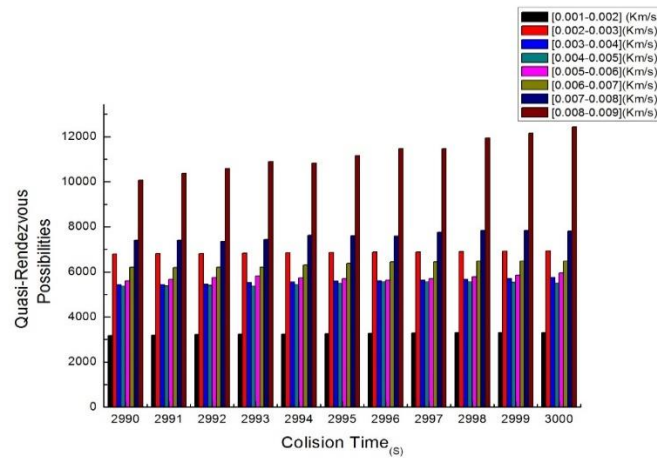


Figure 2- Quasi-Rendezvous Possibilities vs. Time Collision,  $v \approx 10^{-3}$  km/s: [2,990 to 3,000]s

These results show that the Quasi-Rendezvous (or Rendezvous approximate) possibilities increases with time maneuver and for velocities also increasing. That is, when objects are subject to the gravitational field, they tend to meet naturally, if their final relative velocities are increasing. Thus, the Rendezvous possibilities decrease significantly with increasing time. Obviously, bodies accelerated by the gravitational field over time of exposure to it. The results also show that depending on the accuracy required, a Quasi-Rendezvous be accepted as a Rendezvous between objects. Figure 3, below, shows the distribution of possibilities as a function of the final relative velocity between them.

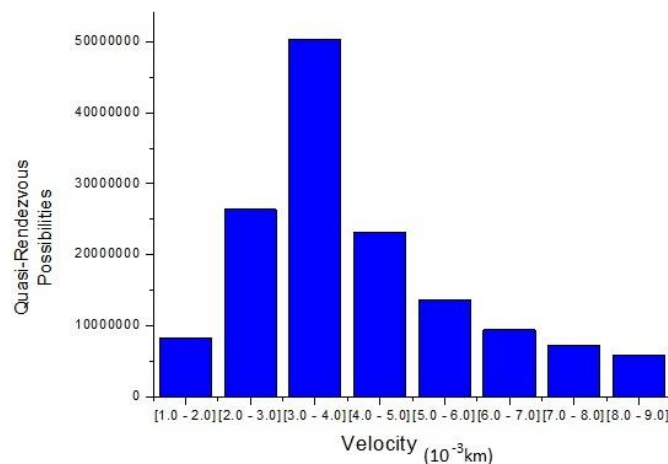


Figure 3- Quasi-Rendezvous Possibilities vs. Final Relative Velocity

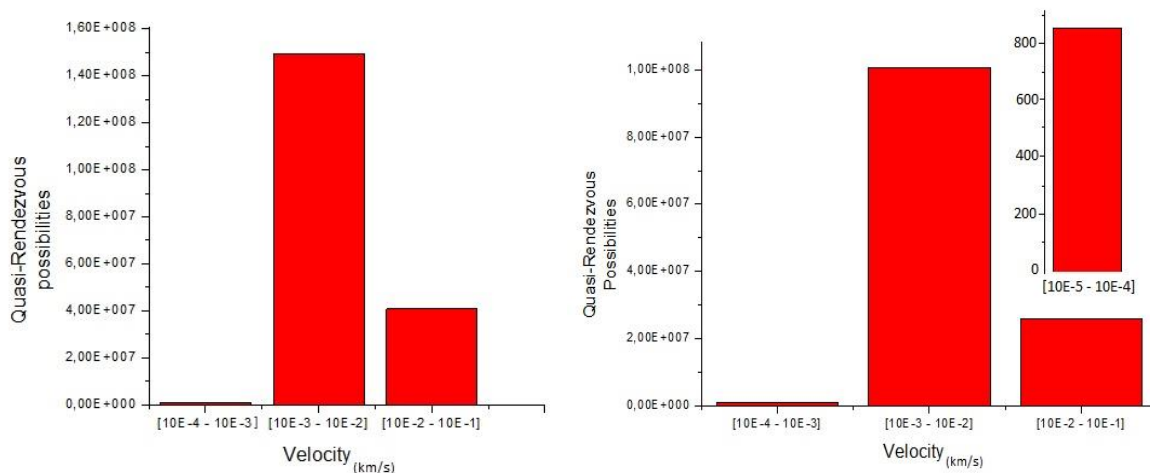
We observe a very large number of approximations between objects with small but not null velocities. There is a peak of possibilities in the range of  $[3,0 - 4,0] \times 10^{-3} \text{ km/s}$  and the distribution seems to obey an almost Gaussian law. Thus, with increasing velocity, the bodies have enough energy to escape from the Rendezvous condition, and therefore from the Quasi-Rendezvous.

### 3.2. Propelled Rendezvous/Quasi-Rendezvous

In this section we show the results for the case of Rendezvous and/or Quasi-Rendezvous maneuvers using the propulsion system. We used technological parameters by Jesus et al [18], ie,  $\chi = 10$ ,  $\gamma = 10^{-6} \text{ 1/s}$ ,  $v_e = 2.5 \text{ Km/s}$ . Table 1 below compares results of natural and propelled maneuvers. We observed that, in general, the natural maneuvers present more possibilities of Quasi-Rendezvous in relation to those propelled. As the velocity increases this trend is characterized. However, the propelled maneuvers allow Quasi-Rendezvous to velocities in  $[10^{-5} - 10^{-4}] \text{ km/s}$ , not occurring the natural ones. We understand that the propulsion controls the maneuvers, such that favor the Rendezvous (precision:  $10^{-5}$ ) that could not occur naturally under the terrestrial gravity. For higher velocities, propulsion reduces the possibilities of Rendezvous and Quasi-Rendezvous. Figures 1(a), (b), below show these results.

**Table 1 – Possibilities Natural and Propelled Quasi-Rendezvous**

Maneuver	[10e-6–10e-5] (km/s)	[10e-5 – 10e-4] (km/s)	[10e-4 – 10e-3] (km/s)	[10e-3 – 10e-2] (km/s)	[10e-2 – 10e-1] (km/s)
<b>Natural</b>	0.000	0.000	1046870.000	149326716.000	40674360.000
<b>Propelled</b>	0.000	851.000	895675.000	100653569.000	25835985.000



**Figure 4 - Quasi-Rendezvous Possibilities vs. Final Relative Velocity: (a) Natural;(b) Propelled Maneuvers**

#### 4. Conclusions

We studied Rendezvous and Quasi-Rendezvous natural maneuvers (subject only to the gravitational field) and propelled between a spacecraft and a space debris. Rendezvous maneuvers are rare, but are made possible by propulsion if we consider zero equal to  $10^{-5}$ . This accuracy can not be achieved naturally. In addition, propulsion reduces the possibility of Rendezvous for high velocities. Quasi-Rendezvous is more frequent at higher velocities if bodies are exposed to the gravitational field for a long time. This result also shows that in the space environment debris occurs with very small relative velocities different from those expected in LEO (7.76 km/s). Of course, this apparent discrepancy is explained by the fact that our model does not include other nongravitational forces

acting. In this environment, which would interfere in the distribution of the possibilities of encounters between objects.

## 5. Appendix A – Equations Coefficients

$$A = \left\{ \frac{2\dot{x}_0}{w} - 3y_0 + \frac{2v_{ex}}{w} \ln \left( \frac{\chi+1}{\chi} \right) - \sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n\chi^n} \left( \frac{2v_{ex}}{w} + \frac{n\gamma v_{ey}}{w^2} \right) \frac{1}{1 + \left( \frac{n\gamma}{w} \right)^2} \right\} \quad (A.1)$$

$$B = \left\{ \frac{\dot{y}_0}{w} + \frac{v_{ey}}{w} \ln \left( \frac{\chi+1}{\chi} \right) + \sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n\chi^n} \left( \frac{2n\gamma v_{ex}}{w^2} + \frac{v_{ey}}{w} \right) \frac{1}{1 + \left( \frac{n\gamma}{w} \right)^2} \right\} \quad (A.2)$$

$$E = \left\{ 6wy_0 - 3\dot{x}_0 - 3v_{ex} \ln \left( \frac{\chi+1}{\chi} \right) \right\} \quad (A.3)$$

$$F_n = \frac{(-1)^{n+1}}{n\chi^n} \left( \frac{4v_{ex}}{n\gamma} + \frac{2v_{ey}}{w} \right) \frac{1}{1 + \left( \frac{n\gamma}{w} \right)^2} - \frac{v_{ex}}{n\gamma} \quad (A.4)$$

$$G = x_0 + \frac{2\dot{y}_0}{w} + \frac{2v_{ey}}{w} \ln \left( \frac{\chi+1}{\chi} \right) + \sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n^2\chi^n} \cdot \frac{3v_{ex}}{w} \quad (A.5)$$

$$C_n = \frac{(-1)^{n+1}}{n\chi^n} \left( \frac{2v_{ex}}{2} + \frac{n\gamma v_{ey}}{w^2} \right) \frac{1}{1 + \left( \frac{n\gamma}{w} \right)^2} \quad (A.6)$$

$$D = 4y_0 - \frac{2\dot{x}_0}{w} - \frac{2v_{ex}}{w} \ln \left( \frac{\chi+1}{\chi} \right) \quad (A.7)$$

$$H = z_0 + \sum_{n=1}^{\infty} \frac{(-1)^{n+1} v_{ez} \gamma}{\chi^n w^2} \frac{1}{1 + \left( \frac{n\gamma}{w} \right)^2} \quad (A.8)$$

$$I = \frac{\dot{z}_0}{w} - \frac{v_{ez}}{w} \ln \left( \frac{\chi+1}{\chi} \right) + \sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n\chi^n w} v_{ez} \frac{1}{1 + \left( \frac{n\gamma}{w} \right)^2} \quad (A.9)$$

$$J_n = \sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n\chi^n w} v_{ez} \frac{1}{1 + \left( \frac{n\gamma}{w} \right)^2} \quad (A.10)$$

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