

Unitarity of spin-2 theories with linearized Weyl symmetry in $D = 2 + 1$ dimensions

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Here we prove unitarity of the recently found fourth-order (in derivatives) self-dual model of spin-2 by investigating the analytic structure of its propagator. The model describes massive particles of helicity $+2$ (or -2) in $D = 2 + 1$ dimensions and corresponds to the quadratic truncation of a higher derivative topologically massive gravity about a flat background. It is an intriguing example of a theory where a term in the propagator of the form $1/[\square^2(\square - m^2)]$ does not lead to ghosts. The crucial role of the linearized Weyl symmetry in getting rid of the ghosts is pointed out. We use a peculiar pair of gauge conditions which fix the linearized reparametrizations and linearized Weyl symmetries separately.

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I. INTRODUCTION

It is commonly believed that higher derivative theories, though they improve the ultraviolet behavior of field theories, lead to violations of unitarity. In the case of spin-2 particles in $D = 2 + 1$ an interesting exception is the third-order topologically massive gravity (TMG) of [1] which contains a second-order Einstein-Hilbert action with “wrong” sign and describes a massive particle of helicity $+2$ (or -2). A quantum and covariant way of understanding the absence of instabilities, about a flat background, in the TMG theory is to show that its quadratic truncation in the metric fluctuations $h_{\mu\nu}$, where $g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}$, is equivalent via master action [2] to some other (ghost-free) theory. In our notation the linearized version (quadratic truncation) of the TMG theory is called¹ $S_{\text{SD}}^{(3)}$ and the equivalent ghost-free model is the first-order self-dual model $S_{\text{SD}}^{(1)}$ of [3]. The corresponding master action relating $S_{\text{SD}}^{(1)}$ and $S_{\text{SD}}^{(3)}$ is constructed by the addition of trivial (no particle content) mixing terms to $S_{\text{SD}}^{(1)}$, thus implying that the original physical content of $S_{\text{SD}}^{(1)}$ is preserved in the dual theory $S_{\text{SD}}^{(3)}$ and that TMG is consequently ghost-free. The complete master action is given in [4].

Another interesting exception is the new massive gravity theory of [5] (the Bergshoeff-Hohm-Townsend or BHT theory) which also includes an Einstein-Hilbert action with “wrong” sign and a fourth-order term with curvature squares with fine-tuned coefficients (K term). This theory describes a parity doublet of massive particles of helicities $+2$ and -2 in $D = 2 + 1$. Also in this case, there is a master action [5] relating this model to the ghost-free

Fierz-Pauli theory by adding trivial mixing terms (linearized Einstein-Hilbert action) such that no ghosts are expected. Indeed, the unitarity of the BHT theory has been shown in [6] by an explicit analysis of the analytic structure of the propagator. In this work we are concerned with a new fourth-order self-dual model ($S_{\text{SD}}^{(4)}$) of spin-2 deduced in [7] by a Noether gauge embedment procedure and suggested also in [8]. Similarly to $S_{\text{SD}}^{(3)}$, the new model can be interpreted as the linearized version of a gravitational theory about a flat background. Such theory corresponds to a kind of higher derivative TMG without an Einstein-Hilbert term. In [7], a master action where the quadratic model $S_{\text{SD}}^{(4)}$ is obtained from $S_{\text{SD}}^{(3)}$ by the addition of a trivial (no particle content) mixing term is presented. Based on this explicitly covariant relationship we expect only one massive particle in the physical spectrum of $S_{\text{SD}}^{(4)}$ and no ghosts. The aim of this work is to present a detailed calculation of the residues about the poles of the propagator of the $S_{\text{SD}}^{(4)}$ theory, thus confirming our expectations. The crucial role of the linearized Weyl symmetry is made clear. In the next section we start with a bit more general Lagrangian and study also the subcases corresponding to the pure K term, addressed in [9] from another point of view, and the pure gravitational Chern-Simons term of [1], both at linearized level about a flat background.

II. COVARIANT GAUGE FIXING

We start with the action of the self-dual model of [7,8] which corresponds to the linearized version of a higher derivative topologically massive gravity:

$$S(a, b) = \int d^3x \left[b \sqrt{-g} \left(R_{\mu\nu} R^{\nu\mu} - \frac{3}{8} R^2 \right) - \frac{a}{2} \epsilon^{\mu\nu\rho} \Gamma_{\mu\gamma}^{\epsilon} \left(\partial_{\nu} \Gamma_{\epsilon\rho}^{\gamma} + \frac{2}{3} \Gamma_{\nu\delta}^{\gamma} \Gamma_{\rho\epsilon}^{\delta} \right) \right]_{hh}, \quad (1)$$

where the lower index hh represents the quadratic terms in

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¹In this work $S_{\text{SD}}^{(n)}$, with $n = 1, 2, 3, 4$, will stand for a spin-2 self-dual model containing, at most, n space-time derivatives of the fields. In all $S_{\text{SD}}^{(n)}$ models the equations of motion can be written as a self-duality condition $f_{\mu\nu} = f_{\mu\nu}^*$ where $f_{\mu\nu}^* = \epsilon_{\nu}^{\alpha\beta} \partial_{\alpha} f_{\beta\mu} / m$ and $f_{\mu\nu}$ is an appropriate second-rank tensor.

the fluctuations $h_{\mu\nu}$ about a flat background, i.e., $g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}$, where $\eta_{\mu\nu} = (-, +, +)$. The real constants (a, b) are arbitrary and have mass dimension -1 and -2 respectively. More explicitly, we can write the corresponding Lagrangian density in (1) as

$$\begin{aligned} \mathcal{L}(a, b) = & \frac{b}{4} h_{\lambda\mu} \square^2 \left(\theta^\lambda{}_\alpha \theta^\mu{}_\beta - \frac{\theta^{\lambda\mu} \theta_{\alpha\beta}}{2} \right) h^{\alpha\beta} \\ & + \frac{a}{2} h_{\lambda\mu} E^\lambda{}_\alpha \square \theta^\mu{}_\beta h^{\alpha\beta}. \end{aligned} \quad (2)$$

We use the following definitions throughout this work:

$$\square \theta_{\alpha\beta} = \square (\eta_{\alpha\beta} - \omega_{\alpha\beta}), \quad \omega_{\alpha\beta} = \frac{\partial_\alpha \partial_\beta}{\square}, \quad (3)$$

$$E_{\alpha\beta} = \epsilon_{\alpha\beta\gamma} \partial^\gamma, \quad \hat{E}_{\alpha\beta} = \frac{E_{\alpha\beta}}{\sqrt{\square}}, \quad (4)$$

with $\epsilon_{012} = 1$. All second-rank tensors are symmetric, e.g., $h_{\alpha\beta} = h_{\beta\alpha}$ except $E_{\alpha\beta} = -E_{\beta\alpha}$ (and $\hat{E}_{\alpha\beta}$). The action $S(a, b)$ is invariant under linearized reparametrizations and linearized Weyl symmetries respectively:

$$\delta_\Lambda h_{\mu\nu} = \partial_\mu \Lambda_\nu + \partial_\nu \Lambda_\mu, \quad (5)$$

$$\delta_\phi h_{\mu\nu} = \phi \eta_{\mu\nu}. \quad (6)$$

In order to obtain the propagator from (2) one first tries to add a gauge fixing term of the de Donder-like form $\mathcal{L}_{\text{GF1}} = f^\mu f_\mu / 2\lambda_1$ associated with the gauge condition:

$$f_\mu = r \partial^\nu h_{\mu\nu} + s \partial_\mu h = 0. \quad (7)$$

With (r, s) real constants and $h = h_\mu{}^\mu$. However, if we choose $\Lambda_\mu = A \partial_\mu \Omega$ and $\phi = B \square \Omega$ in (5) and (6) we find the gauge transformation

$$\begin{aligned} \delta_G f_\mu & \equiv (\delta_\phi + \delta_\Lambda) f_\mu \\ & = [2A(r+s) + B(r+3s)] \square \partial_\mu \Omega. \end{aligned} \quad (8)$$

Since Ω is an arbitrary function, it is clear that for any choice of r and s we can always find a real pair (A, B) such that $2A(r+s) + B(r+3s) = 0$. Thus, the gauge condition (7) leaves a residual symmetry. Consequently, another (scalar) gauge fixing condition $f = 0$ and a further gauge fixing term $\mathcal{L}_{\text{GF2}} = f^2 / 2\lambda_2$ are absolutely necessary. Since $\mathcal{L}(a, b)$ is a pure higher derivative theory we have found natural to use the higher-order gauge condition:

$$f = \tilde{r} \partial^\mu \partial^\nu h_{\mu\nu} + \tilde{s} \square h = 0. \quad (9)$$

The real constants (\tilde{r}, \tilde{s}) are arbitrary except for the forbidden choice $(\tilde{r}, \tilde{s}) = (r, s)$ which makes the second gauge condition (9) not independent of the first one (7). Although for the linearized theory considered here the ghosts decouple from the physical field $h_{\mu\nu}$, it may be useful in a more general situation to consider the following argument based on the Faddeev-Popov Lagrangian (\mathcal{L}_{FP}) in order to find a convenient choice for the couple (\tilde{r}, \tilde{s}) . Namely, from

the gauge transformations we have

$$\mathcal{L}_{\text{FP}} = \bar{c}^\mu (\delta_{\phi=c} + \delta_{\Lambda_\alpha=c_\alpha}) f_\mu + \bar{c} (\delta_{\phi=c} + \delta_{\Lambda_\alpha=c_\alpha}) f. \quad (10)$$

From (7)–(9) it is clear that only for $s = -r/3$ and $\tilde{s} = -\tilde{r}$ the Weyl ghosts and antighosts decouple from the reparametrization ghosts and antighosts. This means that the gauge conditions (7) and (9) fix separately the linearized reparametrization and linearized Weyl symmetries without any mixing. Moreover, the Faddeev-Popov Lagrangian becomes purely second-order: $\mathcal{L}_{\text{FP}} = \bar{c}^\mu (\square \eta_{\mu\nu} + \partial_\mu \partial_\nu / 3) c^\nu + 2\bar{c} \square c$ with the global symmetry $\delta \bar{c}^\nu = l \partial^\nu \bar{c}$ $\delta c = 2l \partial^\nu c_\nu / 3$ where l is a constant. The above gauge choices lead to the following gauge fixing term which will be used henceforth:

$$\begin{aligned} \mathcal{L}_{\text{GF}} & = \mathcal{L}_{\text{GF1}} + \mathcal{L}_{\text{GF2}} \\ & = \frac{1}{2\lambda_1} \left(\partial^\mu h_{\mu\nu} - \frac{1}{3} \partial_\nu h \right)^2 + \frac{1}{2\lambda_2} (\square h - \partial^\mu \partial^\nu h_{\mu\nu})^2. \end{aligned} \quad (11)$$

We can rewrite $\mathcal{L}(a, b) + \mathcal{L}_{\text{GF}}$ in terms of spin projection operators

$$\mathcal{L}(a, b) + \mathcal{L}_{\text{GF}} = h_{\lambda\mu} \mathcal{O}^{\lambda\mu}{}_{\alpha\beta} h^{\alpha\beta}, \quad (12)$$

where

$$\begin{aligned} \mathcal{O}^{\lambda\mu}{}_{\alpha\beta} & = \square \left[\left(\frac{b}{4} \square + \frac{a}{2} \sqrt{\square} \right) P_{+SS}^{(2)} + \left(\frac{b}{4} \square - \frac{a}{2} \sqrt{\square} \right) P_{-SS}^{(2)} \right. \\ & \quad - \frac{1}{4\lambda_1} P_{SS}^{(1)} - \frac{2}{9\lambda_1} P_{WW}^{(0)} + \frac{\sqrt{2}}{9\lambda_1} (P_{SW}^{(0)} + P_{WS}^{(0)}) \\ & \quad \left. + \left(\frac{\square}{\lambda_2} - \frac{1}{9\lambda_1} \right) P_{SS}^{(0)} \right]_{\alpha\beta}^{\lambda\mu}, \end{aligned} \quad (13)$$

where, following the notation of appendix B of [10], slightly modified, the projection operators of spin-2, -1, and -0 introduced above are given, respectively, by

$$\begin{aligned} (P_{\pm SS}^{(2)})^{\lambda\mu}{}_{\alpha\beta} & = \frac{1}{4} (\theta_\pm^\lambda{}_\alpha \theta^\mu{}_\beta + \theta_\pm^\mu{}_\alpha \theta^\lambda{}_\beta + \theta_\pm^\lambda{}_\beta \theta^\mu{}_\alpha \\ & \quad + \theta_\pm^\mu{}_\beta \theta^\lambda{}_\alpha - \theta^{\lambda\mu} \theta_{\alpha\beta}), \end{aligned} \quad (14)$$

$$\begin{aligned} (P_{SS}^{(1)})^{\lambda\mu}{}_{\alpha\beta} & = \frac{1}{2} (\theta^\lambda{}_\alpha \omega^\mu{}_\beta + \theta^\mu{}_\alpha \omega^\lambda{}_\beta + \theta^\lambda{}_\beta \omega^\mu{}_\alpha \\ & \quad + \theta^\mu{}_\beta \omega^\lambda{}_\alpha), \end{aligned} \quad (15)$$

$$(P_{SS}^{(0)})^{\lambda\mu}{}_{\alpha\beta} = \frac{1}{2} \theta^{\lambda\mu} \theta_{\alpha\beta}, \quad (P_{WW}^{(0)})^{\lambda\mu}{}_{\alpha\beta} = \omega^{\lambda\mu} \omega_{\alpha\beta}, \quad (16)$$

$$\begin{aligned} (P_{SW}^{(0)})^{\lambda\mu}{}_{\alpha\beta} &= \frac{1}{\sqrt{2}} \theta^{\lambda\mu} \omega_{\alpha\beta}, \\ (P_{WS}^{(0)})^{\lambda\mu}{}_{\alpha\beta} &= \frac{1}{\sqrt{2}} \omega^{\lambda\mu} \theta_{\alpha\beta}, \end{aligned} \quad (17)$$

where

$$\theta_{\pm}{}^{\alpha}{}_{\beta} = \frac{1}{2} (\theta^{\alpha}{}_{\beta} \pm \hat{E}^{\alpha}{}_{\beta}). \quad (18)$$

Note that the factors $\sqrt{\square}$ cancel out in (13). Taking² into account $P_{+SS}^{(2)} \cdot P_{-SS}^{(2)} = 0$, $P_{\pm SS}^{(2)} \cdot P_{SS}^{(1)} = 0$, $P_{\pm SS}^{(2)} \cdot P_{SS}^{(0)} = 0$, $P_{SS}^{(1)} \cdot P_{SS}^{(0)} = 0$, the spin-0 algebra $P_{IJ}^{(0)} \cdot P_{KL}^{(0)} = \delta_{JK} P_{IL}^{(0)}$ and the representation of the symmetric identity

$$\begin{aligned} (1_S)^{\lambda\mu}{}_{\alpha\beta} &= \frac{1}{2} (\delta^{\lambda}{}_{\alpha} \delta^{\mu}{}_{\beta} + \delta^{\mu}{}_{\alpha} \delta^{\lambda}{}_{\beta}) \\ &= [P_{+SS}^{(2)} + P_{-SS}^{(2)} + P_{SS}^{(1)} + P_{SS}^{(0)} + P_{WW}^{(0)}]^{\lambda\mu}{}_{\alpha\beta}, \end{aligned} \quad (19)$$

one can solve the equation $\mathcal{O} \cdot \mathcal{O}^{-1} = 1_S$ and find

$$\begin{aligned} (\mathcal{O}^{-1})^{\lambda\mu}{}_{\alpha\beta} &= \left\{ \frac{4}{b} \left[\frac{P_{+SS}^{(2)}}{\square(+m\sqrt{\square})} + \frac{P_{-SS}^{(2)}}{\square(-m\sqrt{\square})} \right] - \frac{4\lambda_1}{\square} P_{SS}^{(1)} \right. \\ &\quad + \frac{\lambda_2}{\square^2} P_{SS}^{(0)} + \left. \left(\frac{\lambda_2}{\square} - 9\lambda_1 \right) \frac{P_{WW}^{(0)}}{2\square} \right. \\ &\quad \left. + \frac{\lambda_2}{\sqrt{2}\square^2} (P_{SW}^{(0)} + P_{WS}^{(0)}) \right\}^{\lambda\mu}{}_{\alpha\beta}, \end{aligned} \quad (20)$$

where $m = 2a/b$. Note that \mathcal{O}^{-1} diverges as $\lambda_2 \rightarrow \infty$, confirming the need of the second (scalar) gauge condition.

III. THE PARTICLE CONTENT OF $S(a, b)$

Now we are ready to obtain the propagator in momentum space saturated with transverse, symmetric, and traceless (Weyl symmetry) sources along the lines of [11]. Using the reality condition of the sources in coordinate space, the desired result in the momentum space, called henceforth $A(k)$, can be written as

$$A(k) = T_{\alpha\beta}^*(k) \langle \tilde{h}^{\alpha\beta}(-k) \tilde{h}_{\lambda\mu}(k) \rangle T^{\lambda\mu}(k), \quad (21)$$

where $\tilde{h}_{\lambda\mu}(k)$ stand for the Fourier transform of $h_{\lambda\mu}(x)$ and the sources must satisfy

$$k_{\mu} T^{\mu\nu} = 0 = T^{\mu\nu} k_{\nu} \quad (22)$$

$$T^{\mu\nu} = T^{\nu\mu} \quad (23)$$

$$T^{\mu}{}_{\mu} = -T^{00} + T^{11} + T^{22} = 0. \quad (24)$$

When we sandwich the operator $(\mathcal{O}^{-1})^{\lambda\mu}{}_{\alpha\beta}$ with sources satisfying (22)–(24), only the contributions of the spin-2

operators survive, which is of course expected since the lower spin terms are gauge dependent and should not interfere in the analysis of the particle content of the model. Of course, we need to be careful in the neighborhood of the poles as we will stress later. More explicitly, if we replace \mathcal{L}_{GF2} by $\tilde{\mathcal{L}}_{\text{GF2}} = h^2/2\lambda_2$, which amounts to fix the scalar gauge condition $f = h = h_{\mu\nu} \eta^{\mu\nu} = 0$, we have

$$\tilde{\mathcal{L}}_{\text{GF2}} = \frac{1}{2\lambda_2} h_{\lambda\mu} \eta^{\lambda\mu} \eta_{\alpha\beta} h^{\alpha\beta}, \quad (25)$$

$$= \frac{1}{2\lambda_2} h_{\lambda\mu} [P_{WW}^{(0)} + \sqrt{2}(P_{WS}^{(0)} + P_{SW}^{(0)}) + 2P_{SS}^{(0)}]^{\lambda\mu}{}_{\alpha\beta} h^{\alpha\beta}. \quad (26)$$

Therefore, it is clear that there are changes only in the spin-0 sector of $\mathcal{O}^{\lambda\mu}{}_{\alpha\beta}$ and $(\mathcal{O}^{-1})^{\lambda\mu}{}_{\alpha\beta}$ (due to the algebra of the spin projection operators). Consequently, there are no effects on $A(k)$ and the particle content of the theory is preserved. More generally, we can say that, since each spin sector is inverted separately and the gauge conditions are expected to contribute only to the spin-1 and spin-0 sectors of $\mathcal{O}^{\lambda\mu}{}_{\alpha\beta}$, it follows that they cannot affect the particle content of the theory which lies entirely in the spin-2 branch due to the transverse (reparametrization symmetry) and traceless (Weyl symmetry) conditions on the sources.

Now, back to the calculation of the residues of $A(k)$, it is instructive to split the spin-2 operators in even and odd parity sectors and write

$$\begin{aligned} [(\mathcal{O}^{-1})^{\lambda\mu}{}_{\alpha\beta}]_{s=2} &= \frac{4}{b} \left[\frac{P_{SS}^{(2)}}{\square(\square - m^2)} - \frac{m}{\square^{3/2}(\square - m^2)} \right. \\ &\quad \left. \times (P_{+SS}^{(2)} - P_{-SS}^{(2)}) \right]^{\lambda\mu}{}_{\alpha\beta}. \end{aligned} \quad (27)$$

Where the parity even spin-2 operator is defined as $P_{SS}^{(2)} = P_{+SS}^{(2)} + P_{-SS}^{(2)}$. Using (22) we derive the following identities (in momentum space $\hat{E}_{\alpha\beta} = i\epsilon_{\alpha\beta\gamma} k^{\gamma} / \sqrt{-k^2}$):

$$T_{\alpha\beta}^*(P_{SS}^{(2)})^{\alpha\beta}{}_{\lambda\mu} T^{\lambda\mu} = T_{\alpha\beta}^* T^{\alpha\beta}, \quad (28)$$

$$\begin{aligned} T_{\alpha\beta}^*(P_{+SS}^{(2)} - P_{-SS}^{(2)})^{\alpha\beta}{}_{\lambda\mu} T^{\lambda\mu} &= T_{\alpha\beta}^* \hat{E}^{\alpha}{}_{\lambda} \theta^{\beta}{}_{\mu} T^{\lambda\mu} \\ &= T_{\alpha\beta}^* \hat{E}^{\alpha}{}_{\lambda} T^{\lambda\beta}. \end{aligned} \quad (29)$$

Then, after trivial rearrangements we can write

$$\begin{aligned} A(k) &= \frac{2i}{bm^2} \left[\left(T_{\alpha\beta}^* T^{\alpha\beta} - \frac{T_{\alpha\beta}^* E^{\alpha}{}_{\lambda} T^{\lambda\beta}}{m} \right) \left(\frac{1}{k^2 + m^2} - \frac{1}{k^2} \right) \right. \\ &\quad \left. - \frac{m}{(k^2)^2} (T_{\alpha\beta}^* E^{\alpha}{}_{\lambda} T^{\lambda\beta}) \right]. \end{aligned} \quad (30)$$

Next, we need to calculate the imaginary part of the residue about the poles of $A(k)$. We first analyze the massive pole. We choose the convenient frame $k_{\mu} = (m, 0, 0)$, so from the transverse condition (22) we have $T^{0\mu} = 0 = T^{\mu 0}$,

²We use the notation $(P \cdot Q)^{\lambda\mu}{}_{\alpha\beta} = P^{\lambda\mu}{}_{\gamma\delta} Q^{\gamma\delta}{}_{\alpha\beta}$.

$\mu = 0, 1, 2$. Therefore,

$$T_{\alpha\beta}^* T^{\alpha\beta} = |T_{11}|^2 + |T_{22}|^2 + 2|T_{12}|^2, \quad (31)$$

$$T_{\alpha\beta}^* E^\alpha{}_\lambda T^{\lambda\beta} = mi[T_{12}^*(T_{11} - T_{22}) - (T_{11}^* - T_{22}^*)T_{12}]. \quad (32)$$

From (30)–(32) we have

$$\begin{aligned} \text{Im Residue}[A(k)]_{k^2=-m^2} &= \lim_{k^2 \rightarrow -m^2} (k^2 + m^2)A(k) \\ &= \frac{2}{bm^2} (|T_{11} + iT_{12}|^2 + |T_{22} - iT_{12}|^2)_{k^2=-m^2}. \end{aligned} \quad (33)$$

Therefore we conclude that $\text{Im Residue}[A(k)]_{k^2=-m^2} > 0$ whenever $b > 0$ which proves, in agreement with the classical canonical analysis of [8], that we have, assuming of course $a \neq 0$, one physical massive particle in the spectrum of $\mathcal{L}(a, b)$. We remark that it is not necessary to use the traceless condition on the sources to derive (33).

Next we turn to the more subtle case of the massless poles. We use the frame $k_\mu = (-k_0, \epsilon, -k_0)$, which implies $k^2 = \epsilon^2$, and take afterward the limit $\epsilon \rightarrow 0$ assuming $\epsilon/k_0 < 1$. In fact, we need to calculate $\text{Im Residue}[A(k)]_{k^2=0} = \lim_{\epsilon \rightarrow 0} \epsilon^2 A(k)$ in the above frame where the transverse condition can be written as the following 3 equations:

$$k_0(T^{0\mu} + T^{2\mu}) = \epsilon T^{1\mu}, \quad \mu = 0, 1, 2. \quad (34)$$

It is clear from (30) that we need to evaluate the two quantities: $V(k) \equiv T_{\alpha\beta}^* E^\alpha{}_\lambda T^{\lambda\beta}$ and $U(k) \equiv T_{\alpha\beta}^* T^{\alpha\beta}$ in the limit $\epsilon \rightarrow 0$. We first look at $V(k)$. If we use the symmetric condition (23), the transverse condition and its complex conjugated we can write

$$\begin{aligned} V(k) &= i[k_0 T_\mu^{*1}(T^{2\mu} + T^{0\mu}) - k_0(T_\mu^{*2} + T_\mu^{*0})T^{1\mu} \\ &\quad + \epsilon T_\mu^{*2}T^{0\mu} - \epsilon T_\mu^{*0}T^{2\mu}], \end{aligned} \quad (35)$$

$$= i\epsilon(T_\mu^{*2}T^{0\mu} - T_\mu^{*0}T^{2\mu}). \quad (36)$$

So it is already clear, without the traceless condition (24), that the term $V(k)$ will not contribute to any residue at the simple massless pole in (30) but (36) is not enough to get rid of the massless double pole. Using (34) and the traceless condition (24) we can eliminate 4 out of 6 components of the symmetric tensor $T_{\mu\nu}$. It is convenient to choose T^{02} and T^{12} as the independent variables.³ Explicitly, without any approximation we have

³Other choices may require specific properties of some of the components of $T^{\mu\nu}$ at $\epsilon \rightarrow 0$ in order to guarantee that all $T^{\mu\nu}$ behave smoothly at $\epsilon \rightarrow 0$. However, by using such properties the leading behavior in (41) will not change.

$$T^{00} = -\frac{(\epsilon/k_0)(1 + \epsilon^2/k_0^2)}{(1 - \epsilon^2/k_0^2)} T^{12} - T^{02}, \quad (37)$$

$$T^{01} = -\frac{(1 + \epsilon^2/k_0^2)}{(1 - \epsilon^2/k_0^2)} T^{12}, \quad (38)$$

$$T^{11} = -\frac{(2\epsilon/k_0)}{(1 - \epsilon^2/k_0^2)} T^{12}, \quad (39)$$

$$T^{22} = \frac{\epsilon}{k_0} T^{12} - T^{02}. \quad (40)$$

It turns out that plugging all the above formulas in (36) we end up with the leading behavior

$$\begin{aligned} V(k) &= T_{\alpha\beta}^* E^\alpha{}_\lambda T^{\lambda\beta} = \frac{i2\epsilon(\epsilon^3/k_0^3)}{(1 - \epsilon^2/k_0^2)} (T_{02} T_{12}^* - T_{02}^* T_{12}) \\ &\approx C(k_0)\epsilon^4 + \dots, \end{aligned} \quad (41)$$

where $C(k_0)$ is real but has *a priori* no definite sign. Anyway, it is now clear that the massless double pole of (30) does not contribute to $\lim_{\epsilon \rightarrow 0} \epsilon^2 A(k)$ and drops out of the saturated propagator. The linearized Weyl symmetry has played a crucial role.

As a final step we have to evaluate the quantity $U(k) = T_{\alpha\beta}^* T^{\alpha\beta}$ at $\epsilon \rightarrow 0$. By using (37)–(40) we obtain

$$\begin{aligned} T_{\alpha\beta}^* T^{\alpha\beta} &= -\frac{2(\epsilon/k_0)^2(1 + \epsilon^2/k_0^2)}{(1 - \epsilon^2/k_0^2)} |T_{12}|^2 \\ &\quad - \frac{2(\epsilon/k_0)^3}{(1 - \epsilon^2/k_0^2)} (T_{02} T_{12}^* + T_{02}^* T_{12}) \\ &\approx -2\left(\frac{\epsilon}{k_0}\right)^2 |T_{12}|^2. \end{aligned} \quad (42)$$

So finally we completely get rid of the massless simple pole too. We conclude that the $S_{\text{SD}}^{(4)}$ model is free of ghosts, for $b > 0$, and only contains one (physical) massive particle, of helicity $+2$ (or -2), depending upon the sign of the constant a which is not fixed by unitarity.

Now we comment on two interesting subcases corresponding to $a = 0$ (linearized pure K term) and $b = 0$ (linearized pure gravitational Chern-Simons term). If we take $a \rightarrow 0$ ($m = 2a/b \rightarrow 0$) in (30) we obtain a massless double pole: $A(k) = -(2i/b)T_{\alpha\beta}^* T^{\alpha\beta}/(k^2)^2$. However, due to (42) we have a finite residue

$$\begin{aligned} \text{Im Residue}[A(k)]_{k^2=0} &= \lim_{\epsilon \rightarrow 0} \epsilon^2 A(k) = \epsilon^2 \left(\frac{2}{b}\right) \frac{2\epsilon^2}{k_0^2} \frac{|T_{12}|^2}{\epsilon^4} \\ &= \frac{4}{bk_0^2} |T_{12}|^2. \end{aligned} \quad (43)$$

Therefore for a positive coefficient ($b > 0$) the pure K term is ghost-free and contains only one massless physical particle in the spectrum, in agreement with the classical

canonical analysis of [9,12]. Regarding the second special case of the pure CS_3 theory, if we take $b \rightarrow 0$ ($m \rightarrow \infty$) only the last term of (30) survives: $A(k) = (i/a)V(k)/(k^2)^2$. Because of (41) the residue vanishes:

$$\lim_{\epsilon \rightarrow 0} \epsilon^2 A(k) = \epsilon^2 \left(\frac{i}{b} \right) \frac{C(k_0) \epsilon^4}{\epsilon^4} = 0, \quad (44)$$

which confirms the trivial (nonpropagating) nature of the pure gravitational Chern-Simons term (CS_3); see canonical analysis in [13].

Finally, a remark is in order regarding the covariance of our calculations. Since we have used specific reference frames in the analysis of both massive and the massless poles we have lost explicit covariance. However, based on similar calculations in the spin-1 Maxwell-Chern-Simons (MCS) theory of [1] we believe that explicit covariance can be recovered in principle. In the MCS theory we saturate the propagator with conserved currents $k_\mu J^\mu = 0$ such that the amplitude contains a parity-odd term with a massless pole: $A(k) = (i/m)J_\mu^* E_\nu^\mu J^\nu / k^2 + \dots$, where the dots stand for analytic terms at $k^2 = 0$. Although the simplest way to prove that the residue at $k^2 = 0$ vanishes is to choose a convenient frame, as we have done here, one can alternatively use the covariant identity $E^\mu{}_\nu k_\beta = E^\mu{}_\beta k_\nu - E_{\nu\beta} k^\mu + k^2 \epsilon^\mu{}_{\nu\beta}$ from which we can easily prove, using current conservation, that $(J_\mu^* E_\nu^\mu J^\nu) k_\beta / k^2 = \epsilon^\mu{}_{\nu\beta} J_\mu^* J^\nu$. Since, at least, one component of k_β must be nonvanishing, the residue of $A(k)$ at $k^2 = 0$ is shown to vanish in a explicit covariant way. We believe that similar identities for rank-two tensors can be used in order to make our calculations explicitly covariant without ever using polarization vectors as in [11].

IV. CONCLUSION

We have demonstrated here how the local linearized Weyl symmetry can help us in getting rid of massless ghosts and double poles in a purely higher derivative theory. The model in question corresponds to the newly found [7,8] self-dual model which describes massive particles of helicity $+2$ (or -2) in $D = 2 + 1$. Our results agree with the classical canonical analysis of [8]. This case should be contrasted with the BHT theory [5] which is also of fourth-order but includes the second-order Einstein-Hilbert action with a “wrong” sign and describes a mas-

sive parity-doublet of helicities $+2$ and -2 . In that case there are no double poles and the residue at the ghostlike massless pole vanishes in a quite different way [6]. In both cases it must be mentioned that unitarity was already expected from the master action point of view. The key ingredient is that both Lagrangians contain a trivial term with no particle content which allows a spectrum preserving relationship with a lower-order model as shown in [5,7] respectively.

Here we have also shown that the case of the linearized pure K term contains a massless particle with nonvanishing residue in the spectrum in agreement with the classical canonical analysis of [9]; see also [12], despite the double pole in the propagator.

Finally, we remark that a couple of gauge conditions (7) and (9) was necessary in order to obtain the propagator. It is not clear for us how both gauge conditions could be interpreted as linearizations of gauge conditions valid for the full higher derivative topologically massive (HDTMG) theory. To the best we know, the only local symmetry of HDTMG is general coordinate invariance for which one vector gauge condition should be expected. The same problem occurs in the pure K theory, $a = 0$ in (1), since the Weyl symmetry only appears after linearization. In fact, in that case the important role of the linearized Weyl symmetry in deriving the correct particle spectrum has been pointed out already in [9]. Only in the trivial case of the pure gravitational Chern-Simons term, $b = 0$ in (1), the Weyl symmetry is present in the full nonlinear theory, so there is no interpretation problem for our couple of gauge conditions. Clearly, this point demands a deeper investigation of the full (nonlinear) theory, specially its constraints structures which we are currently analyzing. Regarding the particle content of the theory, as argued in the last section, the linearized reparametrization and Weyl symmetries imply that only the spin-2 sector of the propagator is relevant for the spectrum analysis. Thus, the gauge conditions, which are expected to appear only in the spin-1 and spin-0 sectors, play no role in such analysis whatever is their origin.

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