Shear and rotation in Chaplygin cosmology

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We study the effect of shear and rotation on results previously obtained dealing with the application of the spherical collapse model (SCM) to generalized Chaplygin gas (gCg)-dominated universes. The system is composed of baryons and gCg and the collapse is studied for different values of the parameter α of the gCg. We show that the joint effect of shear and rotation is that of slowing down the collapse with respect to the simple SCM. This result is of utmost importance for the so-called unified dark matter models, since the described slowdown in the growth of density perturbations can solve one of the main problems of the quoted models, namely the instability described in previous papers [e.g., H. B. Sandvik *et al.*, Phys. Rev. D **69**, 123524 (2004)] at the linear perturbation level.

I. INTRODUCTION

During the 1990s, numerous results showed that the cold dark matter (CDM) model approach is not sufficient to describe the observed universe. Nowadays, the scenario that best describes our Universe is a flat cosmology with dark matter (DM) and an exotic component with a negative pressure, usually named dark energy (DE). This last component is, in the new picture, responsible for the accelerated rate of expansion of the Universe. This last conclusion—coming from the observations of high-redshift supernovae, which are dimmer than expected [1]—was also confirmed by several other independent observations (e.g., the baryon acoustic oscillations [2], the angular spectrum of the cosmic microwave background (CMB) radiation temperature fluctuations [3], and the integrated Sachs-Wolfe effect [4]). Nevertheless, after a decade of studies, the nature of the DE continues to remain a mystery, and as a consequence of this "ignorance" a large number of models have been proposed. The simplest

is to identify the DE with the cosmological constant Λ and the energy of the vacuum, so as to obtain the ΛCDM model, in which the equation of state (EoS) of the DE is simply given by $w = p/\rho = -1$. In order to alleviate one of the problems of the ΛCDM model, namely the cosmological constant concordance problem, several other alternative DE models have been proposed. Extensions of this model are based on a scalar field weakly interacting with matter (quintessence models) [5], K-essence, phantom models, or unified dark matter models (UDM) (see, e.g., Ref. [6]). In UDMs, DM and DE are described by the same physical entity. One peculiar case is the so-called *generalized Chaplygin gas* (gCg), introduced by Kamenshchik [7] and then developed in studies by Ref. [8].

The EoS describing the gCg is

$$p = -\frac{C}{\rho^{\alpha}}, \tag{1}$$

where C and α are positive constants, ρ is the density, and p is the pressure. When $\alpha = 1$, the gCg corresponds to the standard Chaplygin gas (sCg).[1]

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[1]The sCg is named after Sergey A. Chaplygin, the Russian physicist who studied it in a hydrodynamical context [9].

Avelino [10] showed that the gCg background density evolution is

$$\rho = \rho_0[\bar{C} + (1 - \bar{C})a^{-3(\alpha+1)}]^{\frac{1}{1+\alpha}}, \quad (2)$$

where a is the cosmic scale factor, related to the cosmological redshift by $1 + z = a_0/a$, $\bar{C} = C/\rho_0^{1+\alpha}$, and ρ_0 is the density at the present epoch.

The EoS parameter, w, is given by

$$w = -\bar{C}[\bar{C} + (1 - \bar{C})a^{-3(\alpha+1)}]^{-1}. \quad (3)$$

It is important to stress that Eq. (3) shows that the gCg behaves as DM at early times $(a \rightarrow 0)$, and at later times $w \rightarrow -1$, approaching a DE behavior.

Several theoretical [11] and observational consequences of the Chaplygin gas have been studied. Cosmological tests using CMB measurements [12], measurements of the X-ray luminosity of galaxy clusters [13], supernova Ia data [14], and lensing statistics [15] have been performed.

In the context of UDMs with $\alpha \neq 0$, observations of large-scale structure [16,17] and the comparison of the linear theory with observations have put in evidence some problems of the gCg UDM. Avelino [17] studied the onset of the nonlinear regime in gCg UDMs, showing that the transition from the DM behavior to the DE one is not smooth, and that in gCg UDM nonlinear effects generate a nontrivial backreaction in the background dynamics. This implies a breakdown of the linear theory at late times (even on large scales), for all $\alpha \neq 0$ models. They also pointed out the need to take into account nonlinear effects when comparing with cosmological observations.

However—notwithstanding that the linear perturbation theory has shown that not all α favor structure formation—there is a marginal degree of agreement between gCg UDM and large-scale structure observations [18].

In order to have a clearer idea of the importance of the gCg as an alternative to the ΛCDM, it is necessary to study the nonlinear evolution of DM and DE in the Chaplygin gas cosmology. This was partly performed by Ref. [19]. Moreover, a fully nonlinear analysis would require smoothed-particle hydrodynamics simulations (see, e.g., Refs. [20–23]). An alternative analytical approach to performing the quoted nonlinear analysis and studying the nonlinear evolution of perturbations of DM and DE is the popular spherical collapse model (SCM), introduced in the seminal paper of Ref. [24], and extended and improved upon in following papers [25–32].

The SCM proposed by Ref. [24] does not contain nonradial motions and angular momentum. The way to introduce angular momentum in the SCM, and its consequences, were studied in several papers [28,29,31–41].

Fernandes [42] used the SCM to perform the quoted nonlinear analysis. Their Friedmann-Lemaître-Robertson-Walker (FLRW) universe was endowed with two components:

gCg and baryons.[2] An interesting feature of Fernandes' treatment [42] is the fact that—differently from other works (e.g., Refs. [19,43,44])—they considered the collapse of both gCg and baryons. Moreover, they assumed, for the background and the collapsing region, a time-dependent equation-of-state parameter w, and derived a more accurate expression for the *effective sound speed*, c_{eff}^2, with respect to previous studies (e.g., Ref. [45]). However, their study did not consider two important factors, namely rotation (vorticity), ω, and shear, σ. Nevertheless, in any proper extension of the SCM the contraction effect produced by shear and the expansion effect produced by vorticity should be considered, as done by Ref. [46]. The previous authors studied the effect of shear and vorticity only in a DM-dominated universe, and only in Ref. [47] were shear and vorticity effects considered in the case of DM- and DE-dominated universes.

In the present paper, we study how the shear, σ, and the vorticity, ω, change the results obtained by Ref. [42].

The paper is organized as follows. Section II summarizes the model used. It reviews the derivation of the equation of the SCM in the presence of shear and vorticity, the effective sound speed used, and the way the equations are integrated. Section III deals with results and Sec. IV with conclusions.

II. MODEL

As we already reported, the SCM is a surrogate to N-body simulations to study the evolution of a density perturbation in the nonlinear phase. Because of the Birkhoff theorem, a slightly overdense sphere, embedded in the Universe, behaves exactly as a closed subuniverse. In the model the overdensity is divided into mass shells, each one expanding with the Hubble flow from an initial comoving radius x_i to a maximum one x_m (usually named the turn-around radius, x_{ta}), and eventually collapsing to a singularity (see Refs. [46,48] to see how the singularity can be eliminated). In practice, this collapse will never occur since dissipative physics and the process of violent relaxation will convert the kinetic energy of collapse into random motions, giving rise to a "virialized" structure (virialization occurs at $t \approx 2t_{\text{max}}$). Once a nonlinear object has formed, it will continue to attract matter in its neighborhood and its mass will grow by accretion of new material in the process of "secondary infall."

In the seminal paper of Ref. [24], the authors were interested in the formulation of a theory of infall of matter into clusters of galaxies. The equations of dynamics of the structure written by them are relativistic [Eq. (7) of Ref. [24]], but they continued the treatment in terms of Newtonian mechanics. Their treatment supposed that the structure collapsed radially and that nonradial motions were not present. Several following papers showed how

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[2]Radiation was neglected since the study considered only the post-recombination epoch.

to introduce nonradial motions and angular momentum, L, [28,29,31–40] preserving spherical symmetry.[3] The equations of the SCM with angular momentum can be written as (e.g., Refs. [37,40,49])

$$\frac{d^2R}{dt^2} = -\frac{GM}{R^2} + \frac{L^2}{M^2R^3}. \tag{4}$$

SCM equations can be written in terms of the overdensity δ, using general relativity [50] or in the pseudo-Newtonian (PN) approach to cosmology [51]. In the PN approach, the evolution equations of δ in the nonlinear regime have been obtained and used in the framework of the spherical and ellipsoidal collapse, and in structure formation by Refs. [52–56].

In order to obtain the quoted equations, we assume that the fluid satisfies the equation of state $P = w\rho$ (we assume that the velocity of light $c = 1$), and we use in the calculation the generalizations of the continuity equation, of Euler's equation (both valid for each fluid species j), and of Poisson's equation (which is valid for the sum of all fluids), given by [51,56]

$$\frac{\partial \rho_j}{\partial t} + \vec{\nabla}_{\vec{r}} \cdot (\vec{u}_j \rho_j) + p_j \vec{\nabla}_{\vec{r}} \cdot \vec{u}_j = 0, \tag{5}$$

$$\frac{\partial \vec{u}_j}{\partial t} + (\vec{u}_j \cdot \vec{\nabla}_{\vec{r}})\vec{u}_j = -\vec{\nabla}_{\vec{r}}\Phi - \frac{\vec{\nabla}_{\vec{r}} p_j}{\rho_j + p_j}, \tag{6}$$

$$\nabla_{\vec{r}}^2 \Phi = 4\pi G \sum_k (\rho_k + 3p_k), \tag{7}$$

where ρ_j, p_j, $\vec{u}_j$, and Φ denote, respectively, the density, pressure, velocity, and the Newtonian gravitational potential of the cosmic fluid.

It is important at this stage to recall that in order to derive our equations describing the evolution of perturbations, we assume the validity of the pseudo-Newtonian approach. This approach tries to include relativistic effects (like the inclusion of pressure) by adding additional terms to the usual hydrodynamical equations. This is particularly evident in Eqs. (5)–(7). Relativistic contributions play a role in all of the equations. Due to the equivalence principle, the pressure now acts as a source of the gravitational potential in Poisson's equation [Eq. (7)] and modifies the denominator of the last term on the rhs of Eq. (6) (Euler equation). This set of equations is valid only for subhorizon scales and they do not take into account possible effects at scales larger than the horizon. Despite their limitations, they have proven to be very useful for describing structure formation in quintessence models, and their predictions were in agreement with results of N-body simulations (see for example the Appendix in Ref. [44] and references

therein). The equations, as presented here, are not simply a generalization of Newtonian hydrodynamical equations, but they have a well-defined theoretical justification. As detailed in Ref. [44], to which we refer for more details, the pseudo-Newtonian equations can be derived directly from general relativity, assuming the stress-energy tensor of a perfect fluid characterized by density and pressure. Given the stress-energy tensor $T^{\mu\nu}$, one computes its four-divergence ($\nabla_\mu T^{\mu\nu} = 0$) and its contraction with the projection tensor $h_{\alpha\mu} = g_{\alpha\mu} + u_\alpha u_\mu$. The first, contracted with the four-velocity, gives the general relativistic continuity equation, and the second gives the Euler equation. Specifying the usual Newtonian metric and under the assumption of weak field and small velocities ($v \ll c$), we obtain the pseudo-Newtonian equations. Another confirmation of the validity of our approach is that our perturbed equations (11) and (12) coincide with the set of Eq. (30) in Ref. [57], assuming further that time derivatives of the gravitational potential are negligible with respect to spatial derivatives and that perturbations in the pressure term are not adiabatic. It might appear that there are differences between the two sets of equations, but this is due to the fact that we work in the configuration space while Ref. [57] worked in the Fourier space. The reasoning behind the derivation of Poisson's equation is slightly different. To derive it, we can proceed in two different ways: one can either combine Eqs. (23a) and (23d) in Ref. [57], or work it out directly with the assumed metric.

We would like to recall that in this work we want to generalize the work of Fernandes [42], taking into account the contribution of the shear and rotation terms. We therefore closely follow the derivation of their equations.

Introducing cosmological perturbations in the previous equations, using comoving coordinates, $\vec{x} = \vec{r}/a$, using $\delta_j = \delta\rho_j/\rho_j$, and assuming that w_j and $c_{\text{eff},j}^2$ are functions of time only, the equations for the perturbed quantities are

$$\dot{\delta}_j + 3H(c_{\text{eff},j}^2 - w_j)\delta_j = -[1 + w_j + (1 + c_{\text{eff},j}^2)\delta_j]$$
$$\times \frac{\vec{\nabla} \cdot \vec{v}_j}{a} - \frac{\vec{v}_j \cdot \vec{\nabla}\delta_j}{a}, \tag{8}$$

$$\dot{\vec{v}}_j + H\vec{v}_j + \frac{\vec{v}_j \cdot \vec{\nabla}}{a}\vec{v}_j = -\frac{\vec{\nabla}\phi}{a} - \frac{c_{\text{eff},j}^2 \vec{\nabla}\delta}{a[1 + w_j + (1 + c_{\text{eff},j}^2)\delta_j]}, \tag{9}$$

$$\frac{\nabla^2\phi}{a^2} = 4\pi G \sum_k \rho_{0_k} \delta_k (1 + 3c_{\text{eff},k}^2), \tag{10}$$

where $c_{\text{eff},j}^2 \equiv \delta p_j/\delta\rho_j$ is the effective sound speed of each fluid.[4]

The previous equations can be simplified, as in Ref. [56]:

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[3]Spherical symmetry is preserved if one assumes that the distribution of angular momenta of particles is random, implying a net null mean angular momentum [35].

[4]Note that in the previous equations, $\vec{\nabla}$ refers to the gradient with respect to the comoving coordinates $\vec{x}$.

$$\dot{\delta}_{\rm j} = -3H(c_{\rm eff,j}^2 - w_{\rm j})\delta_{\rm j} - [1 + w_{\rm j} + (1 + c_{\rm eff,j}^2)\delta_{\rm j}]\frac{\theta_{\rm j}}{a},$$

(11)

$$\dot{\theta}_{\rm j} = -H\theta_{\rm j} - \frac{\theta_{\rm j}^2}{3a} - 4\pi Ga\sum_{\rm k}\rho_{\rm 0k}\delta_{\rm k}(1 + 3c_{\rm eff,k}^2),$$

(12)

where $\theta_j \equiv \nabla \cdot \vec{v}_j$ and $\vec{v}_j$ is the peculiar velocity field.

If we do not discard the shear and vorticity, the equations read

$$\dot{\delta}_{\rm j} = -3H(c_{\rm eff,j}^2 - w_{\rm j})\delta_{\rm j} - [1 + w_{\rm j} + (1 + c_{\rm eff,j}^2)\delta_{\rm j}]\frac{\theta_{\rm j}}{a},$$

(13)

$$\dot{\theta}_{\rm j} = -H\theta_{\rm j} - \frac{\theta_{\rm j}^2}{3a} - 4\pi Ga\sum_{\rm k}\rho_{\rm 0k}\delta_{\rm k}(1 + 3c_{\rm eff,k}^2)$$
$$- \frac{\sigma_{\rm j}^2 - \omega_{\rm j}^2}{a}.$$

(14)

In Eq. (11) the number of equations is equal to the number of cosmological fluid components in the system. For a "top-hat" profile, resulting in $\vec{\nabla}\delta_j = 0$, the peculiar velocity is the same for all fluids ($\theta_j \equiv \theta$, $\sigma_j \equiv \sigma$, $\omega_j \equiv \omega$), resulting in only one Eq. (12) [or Eq. (14)]. The reason for this is that to preserve the top-hat profile, all fluids flow in the same way [58]. We remind the reader that shear and vorticity are already present in Eq. (9), via the term $(\vec{v} \cdot \vec{\nabla})\vec{v}$. To obtain σ and ω, one simply needs to take the divergence of Eq. (9).

In terms of the scale factor a, and recalling that $\Omega_j = \frac{8\pi G}{3H^2}\rho_j$, the previous equations can be expressed in the form

$$\delta_{\rm j}' = -\frac{3}{a}(c_{\rm eff,j}^2 - w_{\rm j})\delta_{\rm j} - [1 + w_{\rm j} + (1 + c_{\rm eff,j}^2)\delta_{\rm j}]\frac{\theta}{a^2 H},$$

(15)

$$\theta' = -\frac{\theta}{a} - \frac{\theta^2}{3a^2 H} - \frac{3H}{2}\sum_{\rm j}\Omega_{\rm j}\delta_{\rm j}(1 + 3c_{\rm eff,j}^2) - \frac{\sigma^2 - \omega^2}{a^2 H},$$

(16)

where the prime denotes the derivative with respect to a.

The evaluation of the term $\sigma^2 - \omega^2$ was discussed in Ref. [47] by defining the ratio α between the rotational and gravitational term in Eq. (4):

$$\beta = \frac{L^2}{M^3 RG}.$$

(17)

In the case of spiral galaxies like the Milky Way, $\beta \simeq 0.4$. Its value is larger for smaller-size perturbations (dwarf galaxy-size perturbations) and smaller for larger-size perturbations (for galaxy clusters the ratio is of the order of 10^{-6}).

In order to obtain a value for δ_c similar to the one obtained by Ref. [59], we set $\beta = 0.04$ for galactic masses (see also Ref. [60]).

In order to integrate Eq. (16) we need to make explicit the $\sigma^2 - \omega^2$ term. This was done in Ref. [47].

Based on the above outlined argument for rotation, one may calculate the same ratio between the gravitational and the extra term appearing in Eq. (16), thereby obtaining

$$\frac{\sigma^2 - \omega^2}{a^2 H^2} = -\frac{3}{2}\beta\sum_{\rm j}\Omega_{\rm j}\delta_{\rm j}(1 + 3c_{\rm eff,j}^2).$$

(18)

As previously told, our Eq. (18) is based on the assumption that the ratio of the acceleration due to the shear/rotation term to that of the gravitational field is constant during the collapse [Eq. (17)]. An objection to this argument could be that angular momentum, L, generated by tidal torques could reduce in the collapsing phase, producing a reduction of the value of β and consequently undermining the result of the calculation. This objection can be disproved as follows. As previously reported, according to the tidal torque theory, the large-scale structure exerts a torque on the forming structure, with the result of imparting angular momentum on the protohalo [61–70]. After the protostructure decouples from the Hubble flow, it turns around and starts to collapse, and tidal torquing is made almost inefficient because the length of the lever arms are reduced (see Fig. 7 in Ref. [67]) [69–71]. Consequently, angular momentum acquisition is maximum at turn-around, and later it remains constant since it is not lost in the collapse phase—as discussed by all of the previously cited papers, and as known from the comparison of the galaxies' rotation with the tidal torque theory (e.g., Refs. [68–70]). The term β is the ratio of the angular momentum acquired through tidal torques (which, as mentioned, remains constant after turn-around), the mass M, and radius R of the protostructure. While M remains constant, R is decreasing in the collapse, and in the case of a collapsing sphere its value at virialization is approximately $R_{\rm final} \simeq 1/2R_{\rm initial}$. This produces an increase in the term β. For precision's sake, we should add that in the protostructure formation two sources of angular momentum are present: (i) the angular momentum originated by tidal torques (as previously mentioned) connected to bulk streaming motions, and (ii) the angular momentum originated by random tangential motions, often refereed to as "random angular momentum" [28,29,31,32,35,37,67,72,73]. Random angular momentum increases the total value of the angular momentum of the protostructure.

In the following, we will consider $\beta = 0.04$—corresponding to spiral galaxies similar to the Milky Way—and $\beta = 0.02$ and $\beta = 0.01$, corresponding to systems in which rotation is less important.

At this point we want to stress out that combining Eqs. (15) and (16) will lead to a quite complicated

second-order ordinary differential equation that generalizes the usual evolution of the perturbations. To recover it, one has simply to identify c_{eff} with the unperturbed equation of state w. It is also important to notice that our derivation is very general and it should not be considered as an expansion in the overdensity parameter. The equations obtained have very broad validity, and they also hold in the case that $\delta \gg 1$.

The term c_{eff}^2 used is the same as that proposed by Ref. [42], namely

$$c_{\text{eff}}^2 = -\frac{C}{\rho^{1+\alpha}} \frac{(1+\delta)^{-\alpha} - 1}{\delta} = w \frac{(1+\delta)^{-\alpha} - 1}{\delta}. \quad (19)$$

It was obtained by the quoted authors by writing c_{eff}^2 using the EoS of the gCg, Eq. (1), and the relation between the densities in the background and in the collapsed region as follows:

$$c_{\text{eff}}^2 = \frac{\delta p}{\delta \rho} = \frac{p_{\text{c}} - p}{\rho_{\text{c}} - \rho}, \quad (20)$$

recalling that the perturbed quantities ρ_{c} and p_{c} are related to their background counterparts by

$$\rho_{\text{c}} = \rho + \delta\rho, \quad (21)$$

$$p_{\text{c}} = p + \delta p, \quad (22)$$

and using $\rho_{\text{c}} = \rho(1 + \delta)$ and Eq. (1).

Equation (19) shows that the effective sound speed depends on the collapsed region (through δ) and the background (through w). The w relative to the collapsed region, namely w_c, is given by Eq. (20) of Ref. [42]:

$$w_c = -\frac{C}{(\rho(1+\delta))^{1+\alpha}} = \frac{w}{(1+\delta)^{1+\alpha}}. \quad (23)$$

In order to study the effect of α on the growth of perturbations, we solved a system of two versions of Eqs. (15)
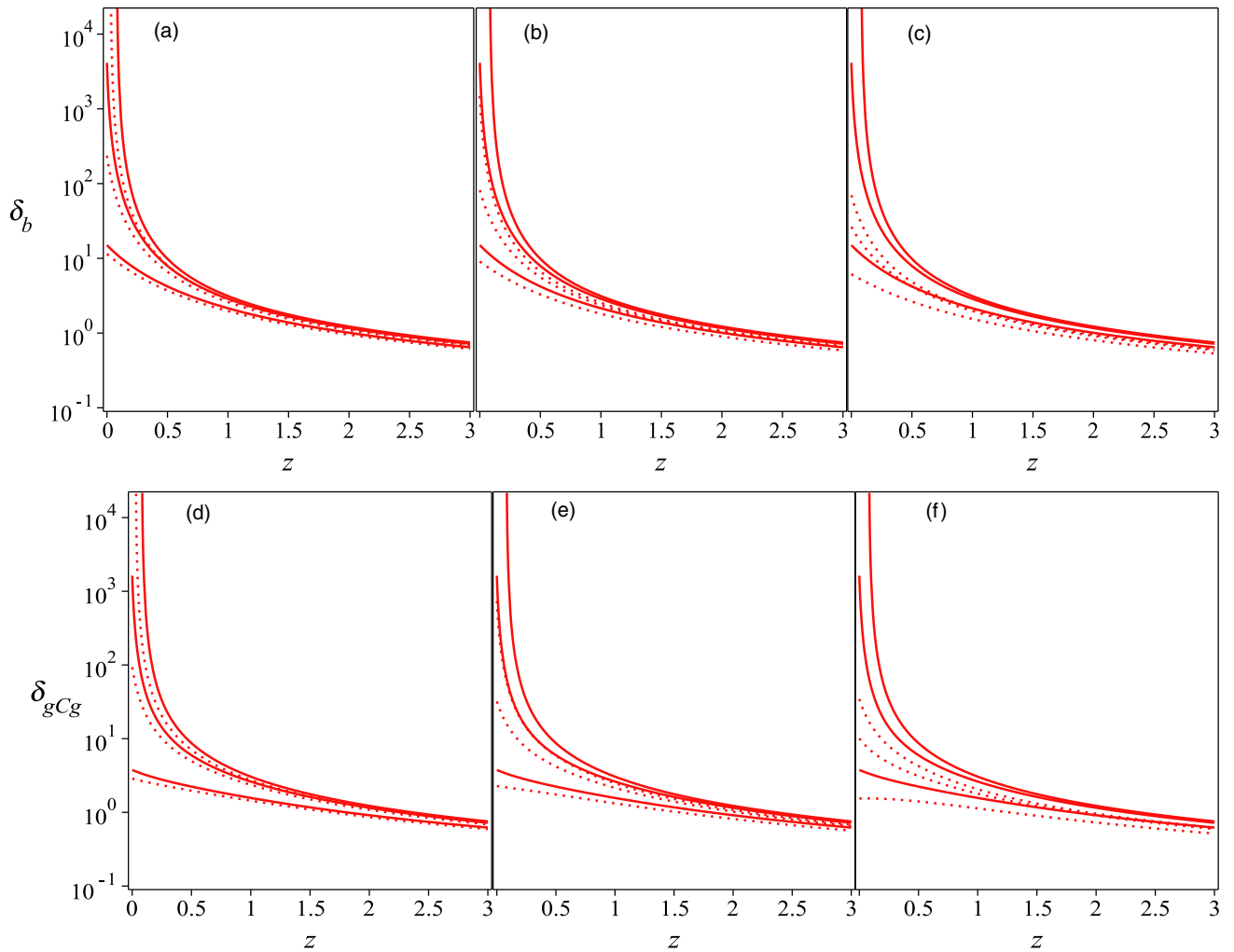

FIG. 1 (color online). Growth of perturbations for the SCM in gCg-dominated universes. *Top*: δ_b versus z [panels (a)–(c)]. *Bottom*: δ_{gCg} versus z [panels (d)–(f)]. Panels (a)–(c): from left to right α has the values 1, 0.5, 0. The solid line (dotted line) represents δ_b without (with) the effect of the additive term. The value of β is 0.01 in panel (a), 0.02 in panel (b), and 0.04 in panel (c). Panels (d)–(f): the same as panels (a)–(c) but for δ_{gCg}.
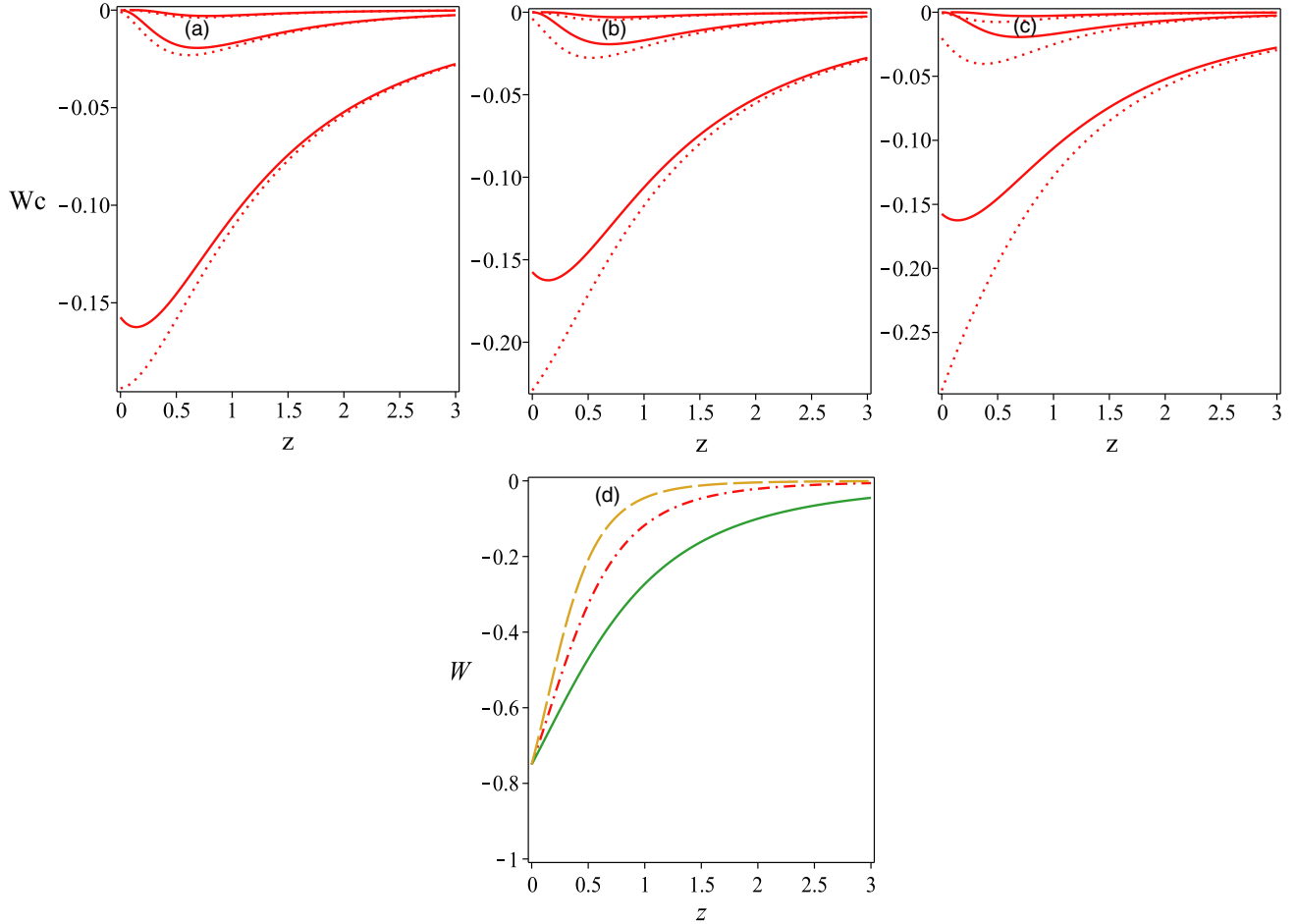
FIG. 2 (color online).   Evolution of w_c and w with z for gCg universes. Panels (a)–(c): from left to right α goes from 1, 0.5, 0. The solid line (dotted line) represents δ_b without (with) the effect of the additive term. The value of β is 0.01 in panel (a), 0.02 in panel (b), and 0.04 in panel (c). Panel (d): w versus z for $\alpha = 1$ (dashed line), 0.5 (dot-dashed line), and 0 (solid line).

(one for gCg and one for baryons) and one of Eq. (16). We used three values of α, namely $\alpha = 0$ (model equivalent to the ΛCDM), and $\alpha = 0.5, 1$, with $\bar{C} = 0.75$. As previously reported, we considered three values for β, namely $\beta = 0.01, 0.02$, and 0.04. The initial conditions (ICs) for the system, the values of the density parameters, and the Hubble constant are the same as in Ref. [42], and are in agreement with recent values for the ΛCDM [3]. As in Ref. [42], $p_b = w_b = c_{s,b}^2 = c_{eff,b}^2 = 0$.

III. RESULTS

In our calculations, we used the same ICs for all of the models. In Figs. 1(a)–1(c) [1(d)–1(f)], the solid lines represent the evolution of δ_b (δ_{gCg}) in the case that the term $\sigma^2 - \omega^2$ is not present (similarly to Ref. [42]), and from top to bottom in each plot the value of α changes from $\alpha = 1, 0.5, 0$.

As noticed by Ref. [42], larger values of α produce a faster collapse via larger values of the effective sound speed at lower z. The different behaviors of δ_b (δ_{gCg}) for different α reflect the different evolution of c_{eff}^2 and w on

the equations of evolution of δ. Moreover, at smaller z—when DE dominates—larger values of α produce a later transition from DM- to DE-dominated stages of the gCg universes.

The dotted line represents δ_b (δ_{gCg}) for the same values of α but when $\sigma^2 - \omega^2$ is different from zero. In Fig. 1(a) the value of β is 0.01, while in Figs. 1(b) and 1(c) it is 0.02 and 0.04, respectively, and similarly for Figs. 1(d)–1(f). The effect of $\sigma^2 - \omega^2$ is that of slowing down the collapse [74], so that the collapse acceleration produced by larger values of α is mitigated by the additive term. Somehow, the presence of the additive term can be mimicked by a reduction of α.

Before going on, we want to stress that it is not trivial to compare our calculations with other studies, e.g., Ref. [75], in order to understand if the instability shown in our Fig. 1 (when shear and vorticity are not taken into account) or their Fig. 1 is due to a term like $(kc_s/aH)^2\delta_k$ in their Eq. (7). A direct comparison of our calculations with theirs [e.g., Eq. (7)] shows that already at the linear level, we obtain different equations. This is due, as said, to the different approach followed (we followed Fernandes' approach).
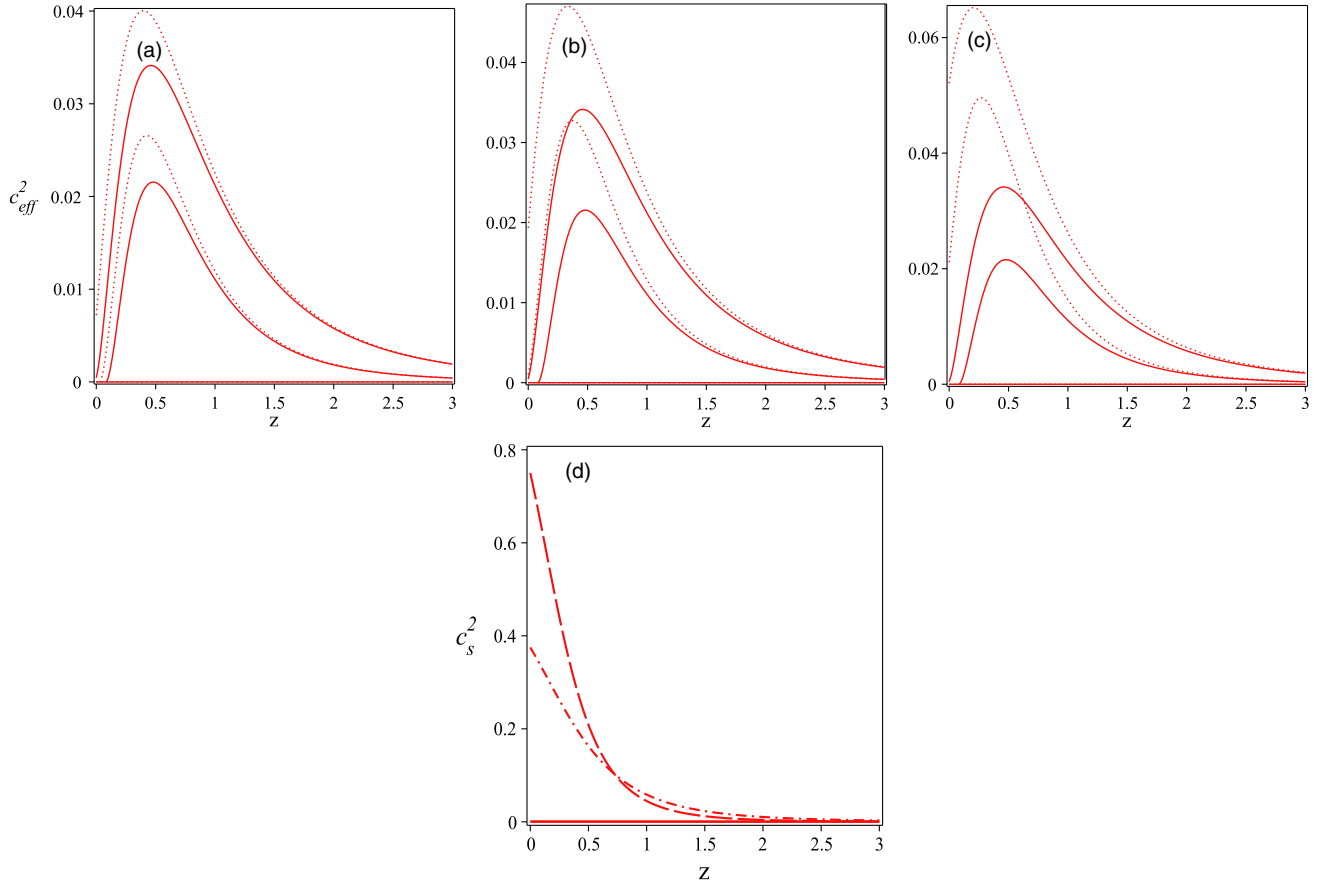
FIG. 3 (color online).   Evolution of c_{eff}^2 and c_s^2 with z for gCg universes. Panels (a)–(c): from left to right α goes from 1, 0.5, 0. The solid line (dotted line) represents c_{eff}^2 without (with) the effect of the additive term. The value of β is 0.01 in panel (a), 0.02 in panel (b), and 0.04 in panel (c). Panel (d): c_s^2 versus z for $\alpha = 1$ (dashed line), 0.5 (dot-dashed line), and 0 (solid line).

In Figs. 2(a)–2(c), we calculate w_c using Eq. (23) and the previously calculated values of δ_{gCg}, while in Fig. 2(d) we calculate w. Solid lines in Figs. 2(a)–2(d) represent w_c when $\sigma^2 - \omega^2$ is not taken into account for $\alpha = 1$ (top curve), 0.5 (middle curve), 0 (bottom curve). α has a strong effect on the results. A larger α produces a faster collapse and a w_c closer to zero, and moreover results in a later transition from DM- to DE-dominated stages of the gCg universes. The quoted result is obtained for a fixed value of $\bar{C}$ ($\bar{C} = 0.75$ in our case). If we increase the DE content of the system—corresponding to increasing the value of $\bar{C}$—the collapse will happen at later times or it will be prevented, with the consequence that w_c will no longer be close to zero. The dotted lines represent the same quantity when the additive term is taken into account. In Fig. 2(a) the value of β is 0.01, while in Figs. 2(b) and 2(c) it is 0.02 and 0.04, respectively. Since the effect of the additive term is to slow down the collapse, the effect on w_c is that of showing a more pronounced departure from zero. Figure 2(d) represents w given by Eq. (3), depending on $\bar{C}$, a, and α—and therefore independent from our additive parameter—and consequently identical to Fig. 2(b) of Ref. [42]. The solid line represents the case $\alpha = 0$, the dash-dotted line the case $\alpha = 0.5$, and the dashed line the case $\alpha = 1$.

In Figs. 3(a)–3(c) we plot c_{eff}^2 and in Fig. 3(d) $c_s^2 = -\alpha w$ for the same values of α and β, and with the lines having the same meaning as in previous figures. The plot shows the different behavior of c_{eff}^2 and c_s^2, implying a different behavior of the gCg component locally (c_{eff}^2) and in the background (c_s^2). Again, notice that our Fig. 3(d) for c_s^2 is the same as Fig. 3(b) of Ref. [42], since the sound speed is not dependent upon the additive parameter.

Finally, Fig. 4 plots the evolution of $h = H + \frac{\theta}{3a}$ with z. Solid lines again represent $h(z)$ without the additive term. In each plot, α has the values $\alpha = 0, 0.5, 1$, from right to left. Larger values of α give rise to a faster decrease in h. Since the turn-around redshift, z_{ta}, can be defined as the z at which $h = 0$, it is clear that a higher α implies a larger z_{ta} and an earlier collapse. Taking into account the additive term, z_{ta} becomes smaller with respect to the case in which the term is not present. The effect of the term $\sigma^2 - \omega^2$ is represented by the dotted lines: $\beta = 0.01, 0.02$, and 0.04 in Figs. 4(a)–4(c), respectively.

Previous works (e.g., Refs. [75,76]) have shown a problem in UDM models, namely oscillations or exponential blowup of the dark matter power spectrum not seen in observations—a problem which is evident on galactic
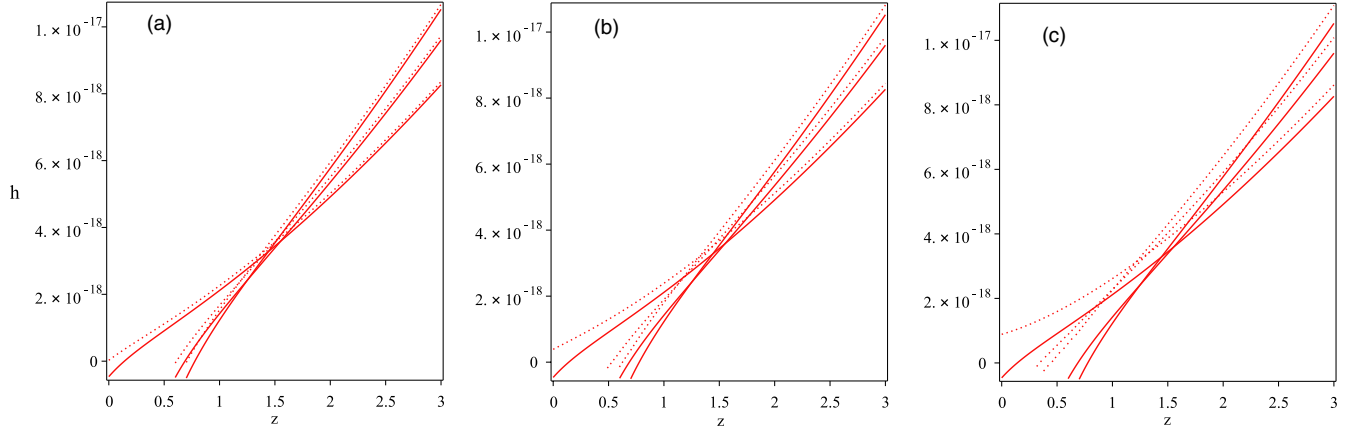
FIG. 4 (color online). Evolution of the expansion rate, h, with z. Solid lines represent the case $\sigma^2 - \omega^2 = 0$. From left to right, in each panel: $\alpha = 0, 0.5$, and 1. Dotted lines: $\sigma^2 - \omega^2 \neq 0$, with $\beta = 0.01$ [panel (a)], $\beta = 0.02$ [panel (b)], and $\beta = 0.04$ [panel (c)].

scales and only at recent times and that cannot be solved by taking baryons into account, as proposed by Ref. [77]. Both Refs. [75,78] have shown that gravitational effects of DM at late times can add fluctuations to baryons but that they are unable to erase the ones already present.

Our result concerning the effect of α on the growth of perturbations is partially in disagreement with the linear theory of perturbation in gCg universes (e.g., Refs. [75,76]), and in agreement with the findings of Ref. [42]. However, in our study—due to the additive term, which has its maximum effect on galactic scales—the growth of perturbations is slowed down. This somehow implies that the additive term presence works in the direction of reducing possible present oscillations as found by Refs. [75,76].

The previous results are perfectly framed in top-hat profiles for density and pressure. Since the profile is flat, it does not contain pressure gradients, and the growth of perturbations can be suppressed only by an accelerated expansion. Assuming a nonflat initial perturbation, it would be possible to improve the understanding of how α affects structure formation.

The study of the effect of pressure gradients by using alternative profiles-such as a Gaussian profile or a Navarro-Frenk-White profile—will be the object of a future study.

Similarly to Ref. [42], we also studied how changing the ICs changes the turn-around epoch. Small changes of the ICs can produce the same turn-around redshift in all models.

IV. CONCLUSIONS

In the present paper, we used the SCM to study how perturbations evolve in gCg universes, taking into account

the effect of shear and rotation. We used the same c_{eff}^2 as Ref. [42], and (in agreement with their work) we found that larger values of the parameter α speed up the collapse, but the additive term $\sigma^2 - \omega^2$ produces a slowing down of this acceleration, visible in the figures showing the evolution of δ_{b}, δ_{gCg} (Fig. 1), w_{c}, c_{eff}^2, and h. The comparison of w_{c} and c_{eff}^2 (the local, nonlinear parameters) with w and c_{s}^2 (the global, linear parameters) shows clear evidence of the difference in the linear and nonlinear dynamical behavior of the gCg.

Notwithstanding that the SCM is usually a faithful technique to study gravitational collapse and structure formation—with results comparable to those of simulations [79]—it would be worthwhile to check the results of this paper against smoothed-particle hydrodynamics simulations, which would allow one to take account of spatial pressure gradients. Moreover, more realistic profiles would improve our understanding of the local dynamics of gCg universes and how the background dynamics is influenced by local nonlinear inhomogeneities.

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