Time-like and space-like pion form factors within front-form dynamics

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Abstract. An approach for a unified description of the pion electromagnetic form factor in the space- and time-like regions, within a constituent quark model on the light front is briefly illustrated. Three main ingredients enter our approach: i) the on-shell quark-hadron vertex functions in the valence sector, ii) the dressed photon vertex where a photon decays in a quark-antiquark pair, and iii) the emission and absorption amplitudes of a pion by a quark.

The analysis of hadron em form factors in both the space- and time-like regions, within the light-front framework [1], opens a unique possibility to study hadron states in the valence and in the nonvalence sectors. Indeed, within the light-front approach the Fock expansion for mesons and baryons

$$
\begin{aligned}
|meson\rangle &= |q\bar{q}\rangle + |q\bar{q}q\bar{q}\rangle + |q\bar{q}\, g\rangle + \ldots.. \\
|baryon\rangle &= |qqq\rangle + |qqq\; q\bar{q}\rangle + |qqq\; g\rangle + \ldots..
\end{aligned} \tag{1}
$$

is particularly meaningful, because the mathematical and the physical vacuum coincide.

Our starting point is the Mandelstam formula for the matrix elements of the em current [2]. Following Ref. [2], the matrix elements for the pion in the time-like (TL) region read

$$
j^\mu = e\frac{2m^2}{\imath f_\pi^2} N_c \int \frac{d^4k}{(2\pi)^4} \bar{\Lambda}_\pi(k,P_\pi)\Lambda_{\bar{\pi}}(k-P_\pi,P_{\bar{\pi}}) Tr[S(k-P_\pi)\gamma^5 S(k-q)\Gamma^\mu S(k)\gamma^5] \tag{2}
$$

where $S(p) = [\not{p} - m + \imath\varepsilon]^{-1}$, with m the mass of the constituent quark, $\Gamma^\mu(k,q)$ is the quark-photon vertex, q^μ the virtual photon momentum, $\Lambda_\pi(k,P_\pi)$ the pion vertex function, $P^\mu_{\bar{\pi}}$ and P^μ_π are the pion momenta. $N_c = 3$ is the number of colors and the factor 2 comes from the isospin weight. For the space-like (SL) region, $P^\mu_{\bar{\pi}}$ has to be replaced by $-P^\mu_{\bar{\pi}}$ and $\bar{\pi}$ by π'. We project out the Mandelstam formula on the light front by a k^- integration and we assume that: i) the meson vertex functions do not diverge in the complex plane k^- for $|k^-| \to \infty$; ii) the contributions of their singularities are negligible. To emphasize the unified description of the em form factor in TL and SL regions, the simplifying assumption of a chiral pion ($m_\pi = 0$) is adopted.

To define our model we have to answer the following questions : i) how to describe the $q\bar{q}$-meson vertices? ii) how to model the dressed quark-photon vertex? iii) how to deal with the amplitude for the emission or absorption of a pion by a quark?

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To answer the first question we assume that, when both quarks are on their mass shell and in the interval $0 \le k^+ \le P_\pi^+$, the momentum-dependent component of the pion vertex function, $\Lambda_\pi(k, P_\pi)$, can be related to the "front-form pion wave function", ψ_π, obtained within the Hamiltonian front-form dynamics, through the following equation

$$(\not{k}_{on} + m)\gamma^5 \psi_\pi(k^+, \mathbf{k}_\perp; P_\pi^+, \mathbf{P}_{\pi\perp}) [(\not{k} - \not{P}_\pi)_{on} + m] =$$
$$(\not{k}_{on} + m)\gamma^5 \frac{m}{f_\pi} \frac{P_\pi^+}{[m_\pi^2 - M_0^2(k^+, \mathbf{k}_\perp; P_\pi^+, \mathbf{P}_{\pi\perp})]} [\Lambda_\pi(k, P_\pi)]_{[k^- = k_{on}^-]} [(\not{k} - \not{P}_\pi)_{on} + m] \quad (3)$$

where $k_{on}^- = (\mathbf{k}_\perp^2 + m^2)/k^+$ and M_0 is the front-form free mass (see Ref. [3]). A similar relation is also adopted for the vector mesons (VM). For the pion and the VM wave functions we use the eigenfunctions of the square mass operator proposed in Refs. [4, 5] within a relativistic constituent quark model which achieves a natural explanation of the "Iachello-Anisovitch law" [6, 7]. The VM eigenfunctions are normalized to the probability of the lowest $(q\bar{q})$ Fock state, roughly estimated to be $\sim 1/\sqrt{2n + 3/2}$ in a simple model [8] that reproduces the "Iachello-Anisovitch law" [6, 7].

As for the second question, following Ref. [8], a Vector Meson Dominance approximation is applied to the quark-photon vertex $\Gamma^\mu(k, q)$, when a $q\bar{q}$ pair is produced. In particular, the plus component of the quark-photon vertex reads as follows

$$\Gamma^+(k, q) = \sum_{n,\lambda} \left[\varepsilon_\lambda \cdot \widehat{V}_n(k, k-q)\right] \Lambda_n(k, P_n) \; \frac{\sqrt{2}[\varepsilon_\lambda^+]^* f_{Vn}}{(q^2 - M_n^2 + \imath M_n \Gamma_n(q^2))} \quad (4)$$

where f_{Vn} is the decay constant of the n-th vector meson into a virtual photon, $M_n(P_n)$ the mass (four-momentum) of the VM, $\Gamma_n(q^2) = \Gamma_n q^2/M_n^2$ (for $q^2 > 0$) the corresponding total decay width, ε_λ the VM polarization, and $[\varepsilon_\lambda \cdot \widehat{V}_n(k, k-q) \; \Lambda_n(k, q)$ the VM vertex function. For the VM Dirac structure, $\widehat{V}_n$, the on-shell expression suggested in Ref. [3] is used. The decay constant, f_{Vn}, is evaluated starting from a four dimensional representation in terms of the VM Bethe-Salpeter vertex and integrating over k^-. Only contributions from the isovector ρ-like vector mesons are included in the calculations.

The third question is answered by describing with a constant [9] the amplitude for the emission or absorption of a pion by a quark, i.e. the pion vertex function in the non-valence sector. The value of the constant is fixed by the pion charge normalization.

In the limit $m_\pi \to 0$ the pion form factor receives contributions only from processes where the photon decays in a $q\bar{q}$ pair. Then by Eq. (4) the matrix element j^+ can be written as a sum over the vector mesons and consequently the form factor becomes

$$F_\pi(q^2) = \sum_n \frac{f_{Vn}}{q^2 - M_n^2 + \imath M_n \Gamma_n(q^2)} \; g_{Vn}^+(q^2) \quad (5)$$

where $g_{Vn}^+(q^2)$, for $q^2 > 0$, is the form factor for the VM decay in a pair of pions.

Each VM contribution to the sum (5) is invariant under kinematical front-form boosts and can be evaluated in the rest frame of the corresponding resonance (with $q^+ = M_n$ and $\mathbf{q}_\perp = 0$).

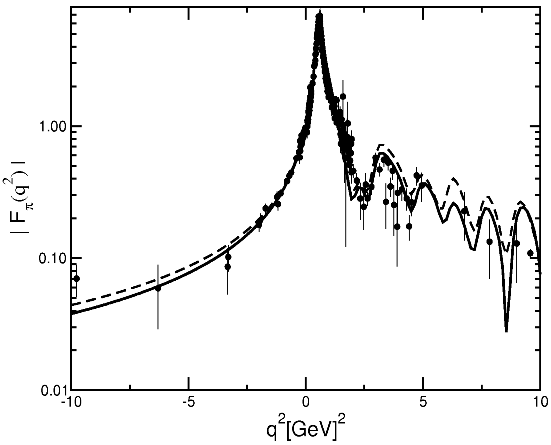

Fig.1. Pion electromagnetic form factor vs. the square momentum transfer q^2. Dashed and solid lines are the results with the asymptotic and the full pion wave function, respectively (after Ref. [8]).

It turns out that the same expression for $g^{+}_{Vn}(q^2)$ holds both in the TL and in the SL regions [8]. In our calculations the up-down quark mass is fixed at 0.265 GeV [5] and the oscillator strength at $\omega = 1.39$ GeV^2 [7]. For the first four vector mesons the known experimental masses and widths are used [10]. For the VM with $M_n >$ 2.150 GeV, the mass values corresponding to the model of Ref. [5] are used, while for the unknown widths we use a single value $\Gamma_n = 0.15$ GeV. To obtain stability of the results up to $q^2 =$ 10 (GeV$/c)^2$ twenty resonances are considered.

The calculated pion form factor is shown in Fig. 1 in a wide region of square momentum transfers, from -10 (GeV$/c)^2$ up to 10 (GeV$/c)^2$. The VM dominance ansatz for the (dressed photon)-($q\bar{q}$) vertex, within a CQ model consistent with the meson spectrum, is able to give a unified description of the pion form factor both in the SL and TL regions. The SL form factor is notably well described. It has to be stressed that the heights of the TL bumps directly depend on the calculated values of f_{Vn} and g^{+}_{Vn}.

These results encourage an investigation of the TL form factors of the nucleon by using an approach based on a simple ansatz for the nonvalence component of the nucleon state, following as a guideline the pion case.

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