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Objective Bayesian Analysis of Process Capability Indexes for Some Lifetime Models

Marcello Henrique de Almeida

Academic Supervisor: Prof. Dr. Fernando Antonio Moala

Post-Graduate Program in Applied and Computational Mathematics

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TÍTULO DA DISSERTAÇÃO: Objective Bayesian Analysis of Process Capability Indexes for Some Lifetime Models

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I dedicate this work to my mother Margarida
and my father Antonio.

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"Maybe I couldn't do the best, but I fought for the best to be done.
I'm not what I should be, but thank God, I am not what I was before".
Marthin Luther King

Resumo

A distribuição Gama tem sido aplicada em pesquisas em diversas áreas do conhecimento, devido a sua boa flexibilidade e adaptabilidade. A distribuição Weibull desempenha um papel importante no controle de confiabilidade e monitoramento da qualidade. Estes modelos têm sido amplamente utilizados para descrever o índice de capacidade de processo (PCI) quando os dados não seguem uma distribuição normal. Sob este cenário, os estudos atuais se concentram na estimativa dos parâmetros usando a inferência clássica. Neste trabalho, consideramos métodos Bayesianos para estimar o PCI denominado C_{pk} a partir de uma perspectiva objetiva, utilizando matching priori e priores de referência. A inferência proposta é ampliada para uma versão generalizada da distribuição Weibull que fornece um bom ajuste para dados mais complexos com comportamento de risco não-monotônico. As distribuições posteriores são construídas e são propostos estimadores Bayesianos com base na mediana para a distribuição Weibull e estimadores Bayesianos com base na média para a distribuição Gama. Neste caso, os métodos Markov Chain Monte Carlo são usados para obter as estimativas e, a partir de um extenso estudo de simulação, observamos que bons resultados são observados em termos de erros médios relativos e quadráticos. A abordagem proposta também é utilizada para construir intervalos de credibilidade adequados com baixo custo computacional e probabilidades de cobertura precisas. É apresentada uma aplicação de dados reais que confirma que nossa abordagem proposta tem um desempenho superior aos métodos atuais.

Palavras chave: *inferência bayesiana objetiva; matching priori; prioris de referência; distribuição Gama; distribuição Weibull; índice de capabilidade de processo* C_{pk}.

Abstract

The Gamma distribution has been applied in research in several areas of knowledge, due to its good flexibility and adaptability nature. The Weibull distribution plays an important role in reliability control and quality monitoring. These models have been widely used to describe the process capability index (PCI) when the data do not follow a normal distribution. Under this scenario, current studies focus on parameter estimation using classical inference. In this work, we consider Bayesian methods to estimate the PCI denominated C_{pk} from an objective perspective using matching prior and reference priors. The proposed inference is extended to a generalized version of the Weibull distribution that provides a good fit for more complex data with non-monotonic risk behavior. The posterior distributions are constructed and median based Bayesian estimators are proposed for the Weibull distribution and mean-based Bayesian estimators for the Gamma distribution. In this case, Markov Chain Monte Carlo methods are used to obtain the estimates and from an extensive simulation study, we observe that good results are observed in terms of mean relative and quadratic errors. The proposed approach is also used to construct adequate credibility intervals with low computational cost and accurate coverage probabilities. An application of real data is presented that confirms that our proposed approach outperforms current methods.

Keywords: *objective Bayesian inference; matching prior; reference priors; Gamma distribution; Weibull distribution; Process capability index* C_{pk}.

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List of Acronyms

MLE: Maximum Likelihood Estimation.

MRE: Mean Relative Error.

MSE: Mean Square Error.

MCMC: Markov Chain Monte Carlo.

ASQC: American Society for Quality Control.

ABNT: Brazilian Association of Technical Standards.

ISO: International Standardization Organization.

INMETRO: National Institute of Metrology Quality and Technology.

PCI: Process Capability Index.

PPM: Part per Million.

CDF: Cumulative Density Function.

GW: Generalized Weibull.

PDF: Probability Density Function.

LSL: Lower specification Limit.

USL: Upper Specification Limit.

SPC: Statistical Process Control.

IPEM: Weights and Measures Institute.

Contents

Chapter

1

Introduction

The concept of statistical quality control was first introduced in the mid-1920s by mathematician Walter Shewhart. From then on several organizations related to quality in general emerged, such as the American Society for Quality Control (ASQC), the Brazilian Association of Technical Standards (ABNT) and the International Standardization Organization (ISO), National Institute of Metrology Quality and Technology (INMETRO).

Later, principally with the advance of technology, industries began to fight more and more for space in the labor market. Thus, it is clear that in all industrial sectors it is essential to have quality in their production in order to be able to guarantee themselves in the face of such a disputed market.

Still in the production scope, in the confection of products in general can occur failures in their specification due to lack of monitoring. In this sense, there are two possibilities: the first of them is related to the dissatisfaction of the clients by the lack of quality of the product; on the other hand it can happen that the company is wasting its input during the production to try to avoid that the client is deceived.

From this perspective we have two tools: the control chart and the process capability indexes. These two mechanisms can contribute to the verification of the measurement system used by the company so that we have good results in terms of the products manufactured and whether the process is being able to generate items within the desired specifications.

Studies through probabilistic models is one of the main challenges currently highlighted in the field of statistical analysis. It is important to realize that with the advent and the new discoveries connected with the advance of new technologies has made possible the analysis through more robust models in order to be able to apply them in real life problems.

In the literature we can find several probability distributions that are generally applied to problems related to survival study, reliability, quality control, among others. Among them we can mention Gamma, Weibull, Exponential, Generalized Weibull (or Gamma-Weibull), among others.

In this work we use three probabilistic models to develop the study of interest. First, we will talk about the Gamma distribution, which is a model that has two parameters (α, β) where α represents the shape and β denotes the scale. This model is one of the best known in statistical analysis and commonly used in several application areas due to its great flexibility in adjusting data in areas such as environmental analysis, clinical trials among other real situations.

The Weibull distribution is another distribution widely used in studies related to practical applications because this model presents a monotonous risk function, that is, increasing or decreasing, but it can also be constant. It is worth remembering that this model has great importance because this distribution is the generalization of the exponential model that unlike the Exponential Distribution, the Weibull Distribution has an importance property of keeping memory, in other words, here is considered the time already used of an equipment in relation to a new unit and without use. See some applications of this Distribution in Teimouri et al. [56] and Ramos et al. [38].

The Generalized Weibull Distribution is also a model that has proven very useful in the field of analysis of survival data due to its various forms presented in its risk function such as: growing, decreasing, constant, bathtub shape and inverse bathtub. This model also has several particular cases of distributions that can be reached from its parameters. Some examples are: Gamma, Weibull, Exponential, Chi-square, Rayleigh. Thus, this model can be very effective in explaining data in different fields of study see Ahsanullah et al. [1], Raju and Srinivasan [37], Marani et al. [26] and Santos [44].

In the literature, it is easy to find results related to the process capability analysis assuming the data come from a Normal Distribution. In the opposite direction, it does not happen the same thing for data with Non-Normal Distribution, with that there are very few references that approach this subject. Thus, this work aims to treat the analysis of process capability through data from Non-Normal Distributions. For more details on the approach to non-normal data and see Gonçalez and Herner [14].

We showed some of the capability indexes that are available in the literature, but in the present work we have used the index called C_{pk} to obtain results about the capacity of a certain process to produce items with the least possible variation. C_{pk} has the objective of saying if a specific process is being viable in relation to loss, time, resources and costs and, mainly, if it is being able to offer good manufacturing and that it meets the needs and desires of the clientele.

In order to exemplify the present work, all the content presented is exposed as follows: In the first chapter, we present an introduction to the subject of interest where we briefly talk about statistical quality control, the emergence of industries and the need to produce with quality and respecting all the established standards, we also emphasize the importance of computer technology for the development of new research applied to real data. We also mention some important distributions that are very well known and also widely used in studies related to the most varied types of data of interest. Finally, in the introduction we also talk about the C_{pk} capability index that we use in the study under the perspective of data originated from Non-Normal Distribution. In the second chapter, we dedicate to the different capability indexes arranged in the literature under the normal and non-normal approach. We defined each index and explained in a way that stimulates the calculation of each one of them and the C_{pk} classification. Still in chapter 2, we present in a general way the maximum likelihood estimator, we show each of the distributions that we will use throughout the work and also their properties through the moments of each one and we also graphically illustrate their functions PDF, CDF, risk functions and survival functions. Finally, in the second chapter, we briefly review Bayesian inference. Chapter 3, composes the first paper we developed with the objective of estimating the C_{pk} index through the Gamma distribution. Chapter 4 and 5 is based on the second paper that we developed for estimating the C_{pk} index using the Weibull and the GW distributions, respectively. Finally, in chapter 6 illustrates our proposed approach in a real data set. Finally, in chapter 7 we conclude by summarizing the present study and also some future proposals.

Chapter

7

Final Considerations and Future Proposals

Process capability indexes usually assumes that the random process that generates the data follows a Gaussian model. However, in many situations the presence of asymmetric data is observe. In the proposed study, we considered the Weibull and Generalized Weibull distribution as asymmetric models to estimate the C_{pk} capability Index. The development of improved inferential procedures plays an important role in estimating PCI. Here, the parameters are assumed as random variables and are estimated from the Bayesian point of view. We proposed a Bayesian analysis through the gamma distribution together with an objective prior, called previous by matching prior that can return us a Bayesian estimator with good properties in terms of the mean relative errors, the mean square errors, and credibility intervals for the parameters and process capability index. For Weibull and WG we considered the objective reference priors to obtain posterior distributions with important properties, such as invariance under one-to-one transformation, consistency marginalization, and consistency sample properties. Hence, we provided an important review of the objective Bayesian methods and applied them successfully to estimate the PCI. As the posterior distributions do not have closed-form expressions, we considered Markov Chain Monte Carlo methods to sample from the posterior distributions, more specifically, the Metropolis-Hasting algorithms are derived for both models to estimate the C_{pk}.

A simulation study was presented to estimate the indexes assuming different values. For Gamma distribution we use posterior mean using the matching prior indicates better performance than the obtained with maximum likelihood estimator in terms of MREs and MSEs. For the Weibull and WG distributions we used Bayesian estimators based on a posterior medians that returned excellent results also in terms of MREs and MSEs. Additionally, the proposed posterior distributions achieved good results in terms of coverage probabilities obtained from the credibility intervals. The proposed approach allows us to estimate such intervals in a fast way with good precision, avoiding the need for resampling techniques that depend on maximization procedure methods and usually carried a high computational cost. A real dataset that describes the filling bags in a Brazilian company is present. We observed that the company has adopted a policy to include an additional gram of juice powder to avoid receive sanctions from the regulatory agency. From the tolerance limits, it can be noticed that the process is not adequate with respect to the production tolerances. After removing the damaged units and attesting that the samples were within the limits we computed the C_{pk} assuming that the data follows a Gaussian model as well as the proposed asymmetric models. The discrimination methods showed that the random process that generates the data is not symmetric and the Weibull and Gamma distributions were the best models to describe the data, the adjusted C_{pk} showed that the estimates values under objective Bayesian analysis were smaller than 1.00 which indicates that the process is still not adequate with respect to the production tolerances and improvements in the process should be considered to decrease financial losses.

There is a large number of possible extensions of this current work. For example, the presence of covariates that may affect the quality of the product can be considered from a Bayesian perspective. In this case, new priors needs to be defined to take into account the additional parameters in the model. The development and comparison with other generalizations of the Weibull distribution should further be considered.

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