

Notes on the two-brane model with variable tension

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Motivated by possible extensions of the braneworld models with two branes, we investigate some consequences of a variable brane tension, using the well established results on consistency conditions. By a slight modification of the usual stress-tensor used in order to derive the braneworld sum rules, we find some important constraints obeyed by time-dependent brane tensions. In particular, it is shown that the tensions of two Randall-Sundrum-like branes obeying, at the same time, an Eötvös law, aggravate the fine-tuning problem. Also, it is shown that if the hidden brane tension obeys an Eötvös law, then the visible brane has a mixed behavior allowing a bouncinglike period at early times while it is dominated by an Eötvös law now. To finalize, we discuss some qualitative characteristics, which may arise in the scope of dynamical brane tensions, as anisotropic background and branons production.

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I. INTRODUCTION

Over the past years, the amount of work dealing with the possibility of a universe with extra dimensions is increasing greatly. It is, in part, due to formal progress in string theory [1], but also (and perhaps mainly) due to the possibility of the hierarchy problem explanation [2]. Moreover, the scenarios investigated by Randall and Sundrum (RS) in [3,4] allow the existence of a large extra dimension in a nonfactorizable background geometry, where an exponential warp factor intervenes in the Higgs mechanism on the brane, making any hierarchy unnecessary. The basic setup of the RS model [3] is composed of two mirror domain walls (the 3-branes) placed at the extremity of an extra transverse dimension, an $S^1/\mathbb{Z}_2$ orbifold. Our universe is described by one of those branes (the so-called visible brane) and the metric is established due to a quite important (and necessary) constraint between the branes tensions: they must be equal and opposite. That last characteristic is the so-called RS model fine-tuning.

Recently, in the context of braneworld scenarios [4], the hypothesis of the Universe being better described by a variable brane tension [5] has been raised. In fact, keeping in mind the cosmological evolution of the Universe, a variable brane tension appears as a necessity. Such a possibility, however, should be explored with great care. To solve the Einstein equations for a variable tension brane in a complete scenario, with all the required constraints for a braneworld model, is far from trivial. For an illustrative example let us consider a RS-type model with the hypothesis of variable tension. Following the notation and conventions used in [3], one sees that the variation on the

tension is achieved by V_{vis} and V_{hid} , the vacuum energy of the visible and the hidden brane, nonconstants. Consistency is achieved if the term $\sigma(\phi)$ in the warp factor is replaced by $\sigma(x^\mu, \phi)$, where ϕ is the extra dimension and x^μ are the noncompact brane dimensions. Let us restrict the analysis to the case in which there is only time dependence on the warp factor, i.e., $\sigma(t, \phi)$ since, in the light of [5], it is the physics-motivated situation. In such a case, when the Einstein equations are computed, starting from a diagonal metric ansatz ($G_{\mu\nu}$), a nondiagonal component of the Einstein tensor appears in the form

$$\begin{aligned} \sqrt{-G} \frac{\partial^2 \sigma}{\partial t \partial \phi} = & -\frac{1}{12M^3} [\Lambda \sqrt{-G} G_{04} + V_{\text{vis}} \sqrt{-g^{\text{vis}}} g_{04}^{\text{vis}} \\ & \times \delta(\phi - \pi) + V_{\text{hid}} \sqrt{-g^{\text{hid}}} g_{04}^{\text{hid}} \delta(\phi)], \end{aligned} \quad (1)$$

and since the right-hand-side of the above equation is zero, one arrives at $\partial^2 \sigma / \partial t \partial \phi = 0$, which invalidates the premise. Besides, physical arguments point into the same direction: σ is present in the warp factor and, consequently, it enters in the rescaling of masses in the Higgs mechanism. Therefore, it cannot be time dependent.

The crucial point is that once the idea of a constant tension is not taken into account in the two-brane model, the line element may be no longer diagonal, since it may also lead to an anisotropic background. In such a case, the brane tension cannot be understood as the brane vacuum energy, since the brane is not a boost-invariant isotropic brane anymore [6]. It is possible to show, however, that starting from an off-diagonal five-dimensional metric, after a diagonalization by an anholonomic frame, the hierarchy problem can be solved [7].

We should emphasize, however, that the variable brane-world tension models arising from a generalization of [4]

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do not lead to an anisotropic background. In the case of an Eötvös brane tension, like the one studied in Ref. [5], a complete solution highly compatible to the observable symmetries is shown.

In the presence of a complete solution obtained from first principles, it is important to support the model by respecting general consistence conditions obtained from the Einstein equations, when applied to the braneworld scenario. The consistency conditions are even more essential in the absence of a complete solution. Such conditions were first developed in the context of five-dimensional braneworlds [8] and then extended to arbitrary dimensions [9]. In particular, those works corroborate the necessity of equal brane tensions—of opposite signs—in the RS model [3] in order to describe a nearly flat universe.

The main aim of this paper is to look at the consistency conditions of RS-like braneworlds in five dimensions, taking into account variable tensions in both mirror branes. It is possible to implement a variable brane tension into the general stress-tensor used to develop the braneworld sum rules. We are especially concerned about Eötvös branes, i.e., branes whose tensions are given by the phenomenological law¹ [10]

$$T_3 = \lambda(T_c - T), \quad (2)$$

where λ is a constant and T_c is the critical temperature above where any brane exists. The usual interpretation is given as follows: the tension T_3 is an intrinsic characteristic of the brane and it increases as the temperature continuously decreases with time. Therefore, as the Universe expands the temperature decreases and the brane becomes more rigid. The case encoded in Eq. (2) is the physical one analyzed in Ref. [5] for the one-brane model. Here we study some possible configurations for a time-dependent brane tension. In particular, we show that two mirror branes endowed with Eötvös tensions, at the same time, are *not allowed*, except if the branes tensions obey a dynamical fine-tuning, i.e., if the branes tensions are equal and opposite, time dependent. It is, in some sense, an aggravation of the fine-tuning problem, indicating that both branes should at least be part of a more general (unknown) global dynamics.

Going one step further, it is shown that if the hidden brane gets an Eötvös tension, then the visible brane tension is dominated by an exponential damping factor at early times, showing an Eötvös-like behavior now. The damping era, in which the brane tension decreases, is finished at a specific time, say $\bar{t}$, which denotes the inflection point of the tension variation. Keeping the usual interpretation of variable tensions, the present time (of increasing tension) can be associated with an expanding universe, while the damping era (before $\bar{t}$) may be understood as a bouncing-like behavior of the Universe.

We emphasize that all of this paper's content is based upon toy models since a complete scenario for a dynamical tension must be obtained from first principles. However, it is still interesting to study those specific cases. The impossibility of both branes presenting Eötvös tensions at the same time, and a bouncinglike behavior of the visible brane, are quite important characteristics which can serve, in some sense, as guidelines for the study of more elaborated variable braneworld tension models.

We remark that in the string theory framework, the possibility of variable tensions was analyzed in several contexts. For example, in [11] it was implemented as an integration constant while in [12] as a dynamical variable. Apart from that, some stabilization mechanisms as well as supersymmetric branes in singular spaces also used variable brane tensions, but in those cases the tension depends on a bulk scalar field in order to stabilize the distance between the branes [13], and also on the superpotential [14], respectively.

Here, we shall keep the scope to the analysis only about braneworld models. In fact, we are not concerned with the mechanism under which the tension becomes variable, instead we are looking for the possible dynamics it obeys. As a first approximation, we consider only time variable tensions. In this way, the Eötvös law (2) should be replaced by

$$T_3^{(i)} = \pm \lambda^{(i)} t + \beta^{(i)}, \quad (3)$$

where $\lambda^{(i)}$ is a positive constant and $\beta^{(i)}$ is a constant representing the lower value for the brane tension. The upper index (i) denotes a specific brane and runs in the range $i = \text{vis, hid}$, the visible and the hidden branes. In what follows, we absorb the constant $\beta^{(i)}$ into the definition of $T_3^{(i)}$, just for convenience, in such a way that Eq. (3) can be rewritten just as $T_3^{(i)} = \pm \lambda^{(i)} t$. Note that the standard interpretation still holds for the universe evolution: for a positive tension, for instance, as the universe expands—and cools—the tension increases. We just change the order parameter from temperature to time. In fact, it can be accomplished just in *very strict cases* in which the relation between time and temperature is linear. We assume that it is the case for an Eötvös tension. As we will see in the next Sec. II C, this assumption does not preclude the possibility of nonlinear effects in the tension.

This paper is organized in the following way: in the Sec. we derive the consistency conditions for the braneworld in five dimensions in the scope of variable tensions. After that, we study the possible time variations in three specific cases: both branes endowed with Eötvös tensions, both branes respecting the obtained sum rules independently, and the last case when we fix the hidden brane tension as an Eötvös one. We finalize by pointing out some important effects to be explored in the framework of variable brane tension models.

¹The subindex of T_3 denotes a 3-brane.

II. ALLOWED DYNAMIC TENSIONS

In this section we shall re-obtain, for bookkeeping purposes, the basic results about consistency conditions in five dimensions, in close analogy to the results in [8,9]. This preliminary setup allows the future analysis of the possible dynamics for the branes' tensions.

We analyze a D -dimensional bulk spacetime endowed with a nonfactorizable geometry, characterized by a metric given by

$$ds^2 = G_{AB}dX^A dX^B = W^2(r)g_{\alpha\beta}dx^\alpha dx^\beta + g_{ab}(r)dr^a dr^b, \quad (4)$$

where $W^2(r)$ is the warp factor, X^A denotes the coordinates of the full D -dimensional spacetime, x^α stands for the $(p+1)$ noncompact coordinates of the spacetime, and r^a labels the $(D-p-1)$ directions in the internal compact space. The D -dimensional Ricci tensor can be related to its lower dimensional partners by [8]

$$R_{\mu\nu} = \tilde{R}_{\mu\nu} - \frac{g_{\mu\nu}}{(p+1)W^{p-1}} \nabla^2 W^{p+1}, \quad (5)$$

$$R_{ab} = \tilde{R}_{ab} - \frac{p+1}{W} \nabla_a \nabla_b W, \quad (6)$$

where $\tilde{R}_{ab}$, ∇_a , and ∇^2 are, respectively, the Ricci tensor, the covariant derivative, and the Laplacian operator constructed by means of the internal space metric g_{ab} . $\tilde{R}_{\mu\nu}$ is the Ricci tensor derived from $g_{\mu\nu}$. Denoting the three curvature scalars by $R = G^{AB}R_{AB}$, $\tilde{R} = g^{\mu\nu}\tilde{R}_{\mu\nu}$, and $\tilde{R} = g^{ab}\tilde{R}_{ab}$ we have, from Eqs. (5) and (6),

$$\frac{1}{p+1} (W^{-2}\tilde{R} - R_\mu^\mu) = pW^{-2}\nabla W \cdot \nabla W + W^{-1}\nabla^2 W \quad (7)$$

and

$$\frac{1}{p+1} (\tilde{R} - R_a^a) = W^{-1}\nabla^2 W, \quad (8)$$

where $R_\mu^\mu \equiv W^{-2}g^{\mu\nu}R_{\mu\nu}$ and $R_a^a \equiv g^{ab}R_{ab}$. Note that with such a notation we have $R = R_\mu^\mu + R_a^a$. It can be verified that for an arbitrary constant ξ the following identity holds

$$\frac{\nabla \cdot (W^\xi \nabla W)}{W^{\xi+1}} = \xi W^{-2}\nabla W \cdot \nabla W + W^{-1}\nabla^2 W. \quad (9)$$

Combining the equation above with Eqs. (5) and (6) we have

$$\begin{aligned} \nabla \cdot (W^\xi \nabla W) &= \frac{W^{\xi+1}}{p(p+1)} [\xi(W^{-2}\tilde{R} - R_\mu^\mu) \\ &\quad + (p-\xi)(\tilde{R} - R_a^a)]. \end{aligned} \quad (10)$$

By using the D -dimensional Einstein equation

$$R_{AB} = 8\pi G_D \left(T_{AB} - \frac{1}{D-2} G_{AB} T \right), \quad (11)$$

where G_D is the gravitational constant in D dimensions, it is easy to write down the following equations

$$R_\mu^\mu = \frac{8\pi G_D}{D-2} (T_\mu^\mu (D-p-3) - T_m^m (p+1)), \quad (12)$$

$$R_m^m = \frac{8\pi G_D}{D-2} (T_m^m (p-1) - T_\mu^\mu (D-p-1)). \quad (13)$$

In the equations above we set $T_\mu^\mu = W^{-2}g_{\mu\nu}T^{\mu\nu}$, so that $T_M^M = T_\mu^\mu + T_m^m$. Now, it is possible to relate R_μ^μ and R_m^m in Eq. (10) in terms of the stress-tensor. Note also that the left-hand-side of the Eq. (10) vanishes upon a line integration over a closed path along the compact internal space. Hence, taking all that into account we have

$$\oint W^{\xi+1} \left(T_\mu^\mu [(p-2\xi)(D-p-1) + 2\xi] \right. \\ \left. + T_m^m p(2\xi - p + 1) + \frac{D-2}{8\pi G_D} [(p-\xi)\tilde{R} \right. \\ \left. + \xi\tilde{R}W^{-2}] \right) = 0. \quad (14)$$

This last equation provides a one parameter family of consistency conditions for warped braneworlds in arbitrary dimensions, just like in [9]. Each choice of ξ lead to a specific consistency condition. Let us hereon, particularize the analysis to the five-dimensional bulk case, since it is, indeed, the current case in variable tension one-brane models [5]. In the codimension one case, we have $D=5$, $p=3$, and $\tilde{R}=0$. Besides, we shall consider the specific case where $\xi=-1$ since it eliminates the overall warp factor, resulting in the most interesting case. With these assumptions, Eq. (14) takes a simple form given by

$$\oint (T_\mu^\mu - 4T_m^m) = \frac{\tilde{R}}{8\pi G_5} \oint W^{-2}. \quad (15)$$

In order to put the consistence conditions in terms of the brane tensions, let us establish the following stress-tensor ansatz

$$\begin{aligned} T_{MN} &= -\frac{\Lambda G_{MN}}{8\pi G_5} + \tau_{MN} - \sum_i \left[T_3^{(i)} + \kappa^{(i)} \frac{\partial T_3^{(i)}}{\partial t} \right] \\ &\quad \times P[G_{MN}]_3^{(i)} \Delta(r - r_i), \end{aligned} \quad (16)$$

where Λ is the bulk cosmological constant, τ_{MN} is the bulk matter fields stress-tensor, $T_3^{(i)}$ is the (time variable) tension of the i th 3-brane, $P[G_{MN}]_3^{(i)}$ is the pullback of the metric to the 3-brane, and $\Delta(r - r_i) \equiv \delta(r - r_i)/\sqrt{G_{rr}} = \delta(r - r_i)$ stands for the covariant delta function necessary to position the brane. The time variation of the i th-brane tension is, as usual, computed by the $\partial_t T_3^{(i)}$ term and the positive con-

stant $\kappa^{(i)}$, responsible to the magnitude of the variation, has units of $(\text{energy})^{-1}$, while the brane tension still has units of $[\text{energy}/(\text{length})^3]$. We emphasize that Eq. (16) is nothing but a slight modification of the ansatz presented in [9]. Imposing $\kappa^{(i)}$ to vanish or, equivalently, taking both the tensions constant, the same stress-tensor of Ref. [9] is obtained.²

We remark that the generalization implemented by Eq. (16) is just a first approximation, since it only takes linear contributions coming from the variation of the tension. We shall, however, keep our analysis in this straightforward case for two main reasons: first, as we will see, there are important results coming from the simplest case. Second, there are no physical reasons to encode higher derivatives as well as nonlinear terms in the time variation of the tension in the classical approach. The possibility of a spatial variation of the tension will be briefly discussed in the final section.

Before computing the necessary partial traces of the stress-tensor to complete the analysis via Eq. (15), let us assume for simplicity $\tau_{MN} = 0$, i.e., there is no contribution from bulk matter fields. Hence, it is easy to see from (16) that

$$T_\mu^\mu = \frac{-4\Lambda}{8\pi G_5} - 4 \sum_i [T_3^{(i)} + \kappa^{(i)} \partial_i T_3^{(i)}] \Delta(r - r_i) \quad (17)$$

and

$$T_m^m = \frac{-\Lambda}{8\pi G_5}. \quad (18)$$

Taking into account Eqs. (15), (17), and (18), we find

$$-\bar{R} \oint W^{-2} = 32\pi G_5 \sum_i [T_3^{(i)} + \kappa^{(i)} \partial_i T_3^{(i)}]. \quad (19)$$

Note that in the equation above, the bulk cosmological constant is factored out. Furthermore, Eq. (19) is valid only for tensions, which do not depend on the extra dimension. Now, in trying to describe our universe, one can affirm that $\bar{R} = 0$ with an accuracy of $10^{-120} M_{\text{Pl}}$, where M_{Pl} is the four-dimensional Planck mass. Then, the final result of the consistency analysis appears to be quite simple. In fact, from Eq. (19) one simply has

$$\sum_i [T_3^{(i)} + \kappa^{(i)} \partial_i T_3^{(i)}] = 0. \quad (20)$$

Note that in the context of constant tensions we recover the well-known RS fine-tuning between the brane tensions

$$T_3^{\text{vis}} = -T_3^{\text{hid}}, \quad (21)$$

as expected.

Hereon, some specific cases are analyzed in order to obtain physical insights about the variable tension possibility. We start by showing an aggravation of the fine-

tuning problem, appearing in the scope of time variable tensions.

A. Eötvös branes and the dynamical fine-tuning

The first case we consider is when both branes are endowed of tensions obeying the Eötvös law $T_3^{(i)} = \pm \lambda^{(i)} t$. To do so respecting the positivity of the T_3^{vis} , we impose³

$$T_3^{\text{vis}} = \lambda^{\text{vis}} t, \quad T_3^{\text{hid}} = -\lambda^{\text{hid}} t, \quad (22)$$

where, as remarked, $\lambda^{(i)} > 0$. Substituting Eqs. (22) into (20), one easily obtains the following constraints

$$\lambda^{\text{vis}} = \lambda^{\text{hid}} \quad (23)$$

and

$$\kappa^{\text{vis}} = \kappa^{\text{hid}}. \quad (24)$$

However, in the light of Eqs. (22), it means that the fine-tuning must be time dependent, i.e.,

$$T_3^{\text{vis}} + T_3^{\text{hid}} = 0 \Rightarrow T_3^{\text{vis}}(t) + T_3^{\text{hid}}(t) = 0. \quad (25)$$

Therefore, in order to satisfy the consistency conditions the time variable Eötvös brane tensions must obey a dynamical fine-tuning. It is indubitably a worsening of the previous (static tension) case. In particular, the relation (25) indicates that both branes need, at least, to respect some global (unknown) dynamics. It is hard to accept that the condition (25) is satisfied by chance. As a first consequence, we emphasize, in passing, a weak brane tension (as on the hidden brane at later times) complicates the modulo stabilization mechanism [15], indicating once more the necessity of including the backreaction on the metric to stabilize the distance between the branes [8].

In order to circumvent the cumbersome situation of a dynamical fine-tuning, several different approaches that lead to consistency conditions other than (20) can be used. Here we just enumerate some possibilities. The first possibility one can try is working in a model with more than one extra dimension. In this case $\bar{R}$ can be different from zero in (14) and the bulk cosmological constant is not factored out, both contributing to relax the constraint (20). In this case two possibilities emerge: higher codimension scenarios and hybrid compactification models (when there is codimension one with a p -brane, being $p > 3$). If the codimension is greater than one it is very difficult to extract gravitational information from the system. There are very interesting results in codimension two models [16,17], but it is far from being a closed issue. Indeed, for a higher codimension the problem is completely open. Another possibility, the hybrid compactification models, is poten-

²Compare with Eq. (14) of Ref. [9].

³Note that for our universe in a RS-like model the brane tension must be positive in order to recover a positive Newton's gravitational constant.

tially interesting, specially in the context of $\mathbb{R}^{10} \times S^1/\mathbb{Z}_2$ Horava-Witten compactifications [1]. But there are strong restrictions, coming from the experiment, concerning the size of the extra on-brane dimensions [18]. Such constraints can, in some cases, aggravate the hierarchy problem for static brane tension models. To sum up, more dimensions do not seem to be the best strategy to solve the dynamical fine-tuning problem.

In the five-dimensional general relativity scope, one can escape from the implications of Eqs. (20) and (22) by allowing the presence of (unknown) matter fields in the bulk. Obviously, if τ_{MN} is not zero in (16) the sum over the branes tensions (plus time variations) can be counterbalanced by the presence of the traces (τ_m^m , τ_μ^μ) in the consistency conditions. Of course, it can lead to another strong constraint, this time on the bulk matter, but the dynamical fine-tuning is not necessary anymore. The crucial problem arising from this approach rests, perhaps, on the fact that we do not have any information *a priori* about the behavior, and even the nature of those bulk fields.

In this vein, we shall point out another way to overcome the dynamical fine-tuning problem simply by lifting the necessity of two Eötvös branes tensions at the same time. In fact, in light of the previous discussion, two Eötvös branes appear to be problematic enough. So, the first different situation that might be thought of is when we impose that, just one of the two mirror branes obeys the Eötvös law. More precisely, what happens if we fix the behavior of the hidden brane as an Eötvös brane? Before looking at that point, let us briefly comment one more situation in the next section.

B. Independent branes tensions

Equation (20) means that the sum of the bracket terms must vanish. However, it is possible that the terms $T_3^{(i)}(t) + \kappa^{(i)} \partial_t T_3^{(i)}(t)$ vanish for each brane independently. This is not the most interesting case, nevertheless we shall make a brief comment on it, since it will be important for the next case.

Setting $T_3^{(i)}(t) + \kappa^{(i)} \partial_t T_3^{(i)}(t) = 0$ for all (i) one finds

$$T_3^{(i)}(t) \sim \exp(-t/\kappa^{(i)}). \quad (26)$$

So, in a scenario where the branes do not feel any influence from each other, the shape of the tension time variation must be like the one given in Eq. (26). Note that the exponential behavior of $T_3^{(i)}(t)$ is dominated by the constant $\kappa^{(i)}$, i.e., for a weak (strong) variation coupling, $\kappa^{(i)} \gg 1$ ($\ll 1$), the tensions time variation is also weak (strong). Now let us analyze the behavior of the visible brane tension when the hidden brane tension obeys an Eötvös law.

C. Bouncinglike behavior

A very interesting case is found when one of the brane's tension configuration is taken into account, with the con-

straint that the another brane tension must obey the Eötvös law. In order to implement it, let us impose $T_3^{\text{hid}} = -\lambda^{\text{hid}} t$.

Substituting the previous requirement into the consistency condition (20), we arrive at the following ordinary differential equation

$$\frac{dT_3^{\text{vis}}(t)}{dt} + \frac{T_3^{\text{vis}}(t)}{\kappa^{\text{vis}}} - \frac{\lambda^{\text{hid}}}{\kappa^{\text{vis}}} t - \frac{\kappa^{\text{hid}} \lambda^{\text{hid}}}{\kappa^{\text{vis}}} = 0, \quad (27)$$

whose solution is given by

$$T_3^{\text{vis}}(t) = \lambda^{\text{hid}} t + C e^{-t/\kappa^{\text{vis}}} + \lambda^{\text{hid}} (\kappa^{\text{hid}} - \kappa^{\text{vis}}), \quad (28)$$

where C is an arbitrary integration constant with (energy) $^{-2}$ units, just as $\lambda^{(i)} t$. Absorbing the last factor into the visible brane tension, Eq. (28) reads

$$T_3^{\text{vis}}(t) = -T_3^{\text{hid}}(t) + C e^{-t/\kappa^{\text{vis}}}. \quad (29)$$

The first characteristic to note is the mixed behavior of T_3^{vis} . It is governed by a negative exponential at early times in such a way that the tension was decreasing. Nevertheless after a critical time, say $\bar{t}$, the dominant term is given by the Eötvös law of the hidden brane. It is easy to compute the critical time $\bar{t}$, since it is the minimal point of the T_3^{vis} function. By imposing $\frac{dT_3^{\text{vis}}(t)}{dt} = 0$ it follows that

$$\bar{t} = \kappa^{\text{vis}} \ln \left(\frac{C}{\lambda^{\text{hid}} \kappa^{\text{vis}}} \right), \quad (30)$$

in such a way that the constant C should be positive. So, it is reasonable to interpret C as an independent contribution to $T^{(\text{vis})}$.

The qualitative behavior of $T_3^{\text{vis}}(t)$ can be better visualized in Fig. 1. The usual interpretation of this result is given as follows: for times when $t > \bar{t}$, the exponential term is suppressed and the dominant term of the visible brane tension is given by the Eötvös one. As time goes on, the brane becomes more rigid and the Universe expands. In particular, this dominant term is given by the hidden brane tension with opposite signal. For times when $t < \bar{t}$, however, the dominant factor is the exponential one. Nevertheless, as time flows, the visible brane tension decreases, allowing a bouncinglike period for the Universe described by such a brane. It is an interesting characteristic of this toy model. It emerges naturally from the consistency conditions in five dimensions by requiring an Eötvös hidden brane tension. We should emphasize that this bouncinglike period can be eliminated by imposing the additional fine-tuning on the couplings

$$C = \lambda^{\text{hid}} \kappa^{\text{vis}}, \quad (31)$$

as one can see from Eq. (30).

The functional form of the visible brane tension can be used then, at least qualitatively, as a tool to understand the general behavior of the Universe it describes. Returning again to the content of the Fig. 1, one sees that at early times ($0 < t < \bar{t}$) the visible brane has, predominantly, a independent behavior (in the sense of the previous sub-

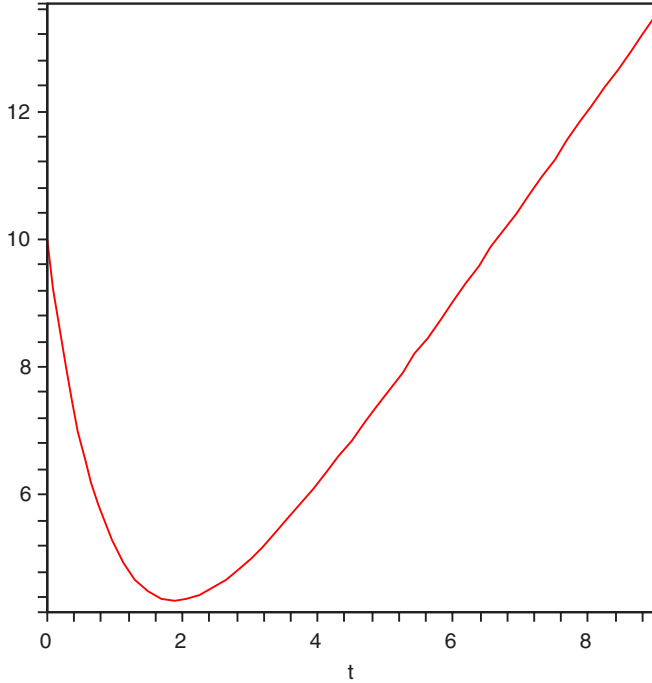


FIG. 1 (color online). Illustrative graphic of $T_3^{\text{vis}}(t)$ for numerical values $\lambda^{\text{hid}} = 1.5$, $\kappa^{\text{vis}} = 1$, and $C = 10$. Note the bouncing-like behavior before the minimal point ($\bar{t} \approx 1.897$). A fine-tuning between λ^{hid} , κ^{vis} , and C ($C = \lambda^{\text{hid}} \kappa^{\text{vis}}$) eliminates the bouncinglike period.

section) from the hidden brane. It contracts until $\bar{t}$, and after that the effects coming from the presence of the hidden brane begin to dominate.

The complete dynamics involved in such a process is far beyond the scope of this paper. The point, however, is that some interesting possibilities arise even studying simple models like that, based upon the formal context of the consistency conditions.

III. CONCLUDING REMARKS AND OUTLOOK

By applying the consistency conditions to five-dimensional variable tension branes in a RS-like setup, we investigate the behavior of some time variable tension specific scenarios. In particular, we demonstrate that the requirement of two Eötvös branes seems to aggravate the fine-tuning problem, indicating that both branes should obey a more general unknown dynamic.

An interesting scenario arises, however, when we require that just one of the branes respects the Eötvös law. For instance, if the hidden brane satisfies this last requirement, the visible brane tension shows up as a mixed behavior indicating a bouncinglike period at early times. It occurs when the visible brane seems not to “feel” the hidden brane, after which an expansion period, encoded in a mainly Eötvös tension term, comes to dominate. We remark that the contracting period can be eliminated by an additional fine-tuning in the couplings of the model.

This work deals only with the possibility of time variable brane tensions, in particular, when time variations related by the Eötvös law, since it is a physically motivated one [5]. We shall, however, make some brief comments about the possibility of a spatial tension variation. Perhaps a more complete scenario can arise from the implementation of a spatial variation in the tension in addition to the time variation. In the context of consistency conditions, it can be accomplished by another extension of the stress-tensor (16). We expect a class of nontrivial solutions, arising from such an extension, since the spacial variations must be suppressed by, for instance, a time damping factor in order to reproduce the observable isotropic universe at large scales. However, a definitive word requires a more careful analysis.

After these brief notes concerning the viability of variable tensions braneworld models, we shall use part of this final section for pointing out some possible future research lines, in order to bring some reasons why the analysis of such scenarios are worthwhile. First, it has to be taken very seriously how to match an essentially anisotropic brane coming from some variable tension models with a large scale isotropic universe, as with our experiment. As shown in [5], it is not the case of Eötvös branes in the one-brane setup. However, new cosmological signatures are also expected in the scope of those (not-Eötvös) variable tension branes. The anisotropic background generated by such branes in the two-brane models breaks some standard model fields symmetries as well as can lead to subtle violation of the equivalence principle. Therefore, it constitutes a good laboratory to the study of fundamental principles limits.

Apart from that, a variable tension braneworld model can also incorporate, at least in some regime, the production of branons [19,20]. Starting from the fact that any brane must fluctuate, because a completely rigid object cannot exist in the scope of a relativistic theory, one concludes that a new (scalar) field has to be included to represent the brane position in the bulk. Such a field gives rise to Goldstone bosons (the branons), arising from the spontaneous symmetry breaking of the full bulk diffeomorphism due to the presence of the brane. It seems that when the brane is flexible enough (with a tension scale much smaller than the fundamental gravity scale), branons are the unique particles to be expected. We remark that this kind of analysis was also developed in the context of orbifold compactifications [21]. The context analyzed in the previous section does not rule out a set of parameters for branons production on the visible brane, for instance, in the early universe.

We conclude by saying that variable tension braneworlds can also impose an imprint in effective projected four-dimensional field theory, since the Kaluza-Klein mass spectrum of higher dimensional theories is indirectly affected by the brane tension. Therefore, it can, potentially,

lead to new phenomena in particle physics. We shall finish by emphasizing the usefulness of the consistency conditions in the establishment of this type of braneworld models, in the sense of avoiding ill-defined scenarios. In this vein, the consistency conditions are also useful as starting point of further investigations in variable tension models, in order to study links with experimental signatures coming from new phenomenology obtained by studying mathematically consistent models.

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