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New accelerating cosmology without dark energy: the particle creation approach and the reduced relativistic gas

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Abstract The standard procedure to explain the accelerated expansion of the Universe is to assume the existence of an exotic component with negative pressure, generically called dark energy. Here, we propose a new accelerating flat cosmology without dark energy, driven by the negative creation pressure of a reduced relativistic gas (RRG). When the hybrid dark matter of the RRG is identified with cold dark matter, it describes the so-called CCDM cosmology whose dynamics is equivalent to the standard ΛCDM model at both the background and perturbative levels (linear and nonlinear). This effect is quantified by the creation parameter α. However, when the pressure from the RRG slightly changes the dynamics of the universe, as measured by a parameter b, the model departs slightly from the standard ΛCDM cosmology. Therefore, this two-parametric model (α, b) describes a new scenario whose dynamics is different but close to the late-time scenarios predicted by CCDM and ΛCDM models. The free parameters of the RRG model with creation are constrained based on SNe Ia data (Pantheon+SH0ES) and also using $H(z)$ from cosmic clocks. In principle, this mild distinction in comparison with both CCDM or ΛCDM may help alleviate some cosmological problems plaguing the current standard cosmology.

1 Introduction

It is widely believed that present day cosmology is dominated by dark energy (DE), an exotic fluid possessing negative pressure, introduced to explain the current accelerating stage of the universe, and also to solve the conflict between the low value of the mass density parameter (Cold Dark Matter + Baryons) and the high degree of flatness ($\Omega_{total} \sim 1$), as predicted by the inflationary paradigm. The resulting model is supported by a plethora of observational results, including, among others, high redshift supernovae Ia [1–4], estimates of the total age of the Universe [5–10], galaxy clusters evolution [11–16], baryon acoustic oscillations [17–20], cosmic background radiation [21,22], and cosmic chronometers [23–25].

Later, dozens of different dark energy candidates have been proposed in the literature [26–31]. However, until now, the simplest and most theoretically compelling DE candidate is the cosmological constant (Λ) or, equivalently, the vacuum energy density ($\rho_v = \Lambda/8\pi G$). At the level of Einstein Field Equations (EFE), it behaves like a rigid substance (constant density) and negative pressure, obeying the equation of state (EoS), $p_v = -\rho_v$. The emerging cosmology is driven at late times by three conserved and noninteracting dominant components ($\Omega_B \sim 5\%, \Omega_{CDM} \sim 25\%, \Omega_{DE} \sim 70\%$), thereby defining the standard ΛCDM cosmology, also named cosmic concordance model. In addition, when combined with the very early inflationary scenario, it is usually considered to be pretty simple (spatially flat geometry), realistic, appealing, and predictive, provided that it is fine-tuned to fit the data.

Although explaining most of the current astronomical observations, there are some old unsolved theoretical "puzzles" and also a couple of recent observational difficulties plaguing the ΛCDM model. In theoretical grounds, one may quote: (i) The old cosmological constant problem (CCP), and (ii) the cosmic coincidence "mystery" (CCM). Basically, the Λ-problem is related with the extreme smallness of its present observed value. Actually, whether dark energy is represented

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by the vacuum state, the CCP constitutes one of the greatest challenges for our current understanding of fundamental Physics [32–34]. The CCM arises because the energy density of the rigid vacuum constant was astonishingly small in the radiation phase, but now it is finely tuned with the variable decreasing density of the dark matter component, or equivalently, $\Omega_\Lambda \sim \Omega_M$ at present [35,36]. Furthermore, at least two observational difficulties has been recently added: (iii) the Hubble tension, and (iv) the S_8 tension, thereby showing persistent discrepancies of the ΛCDM predictions with the astronomical observations.

The H_0 tension is an inconsistency between measurements of the Hubble constant (H_0), using Cepheid and calibrated supernovae of the SH0ES collaboration [37] + strong lensing time delays [38] and also the value $H_0 = 74$ km/s/Mpc from data at low and intermediate redshifts [39,40], while the Planck CMB power spectrum at high redshifts prefer values close to 68 km/s/Mpc. These results imply a statistical discrepancy around 5.4σ. The ongoing tension S_8 on the plane (Ω_M, σ_8), where σ_8 is the current mass fluctuation on a scale of 8Mpc, arises when confronting ΛCDM + Planck estimates with cosmic shear experiments [41].

In this way, although ΛCDM being an interesting cosmic setting, until now it seems like an incomplete description of the Universe. In certain sense, the cosmic concordance model is a comprehensible collection of ingredients brought together in order to explain the complete cosmic evolution of the Universe over all time and length scales. Such disparate ingredients are needed in order to smoothly connect the very early and late-time accelerating regimes [42]. Since the true nature of DE remains elusive, and, even the cosmic concordance model is not free of problems, new possibilities are being carefully scrutinized, mainly the ones slightly departing from ΛCDM cosmology. Some examples are the research line started long ago by decaying vacuum models [43–49], just the first kind of models assuming interactions in the dark sector [50–52], cosmologies based on modifications of Einstein's gravity [53,54], and also models reducing the dark sector and whose dominant component is dark matter (see below).

Actually, part of the ΛCDM problems inspired some authors to explore the possibility of reducing the dark sector by taking $\Omega_{DE} \equiv 0$. The basic idea is that different from DE, the DM component is needed in practically all relevant scales (from galaxies to cosmology). Such an approach quite investigated recently, is based on the assumption that the accelerating fuel is related to a negative pressure provided by the matter creation process in the evolving Universe [55–59]. This reduction in the dark sector is quite interesting because it replaces the idea of an unknown component by a cosmic mechanism of microscopic origin (matter-creation), and as such, bringing new possibilities to solve the problems plaguing the standard model. In particular, it has been shown that the so-called creation cold dark matter cosmology (CCDM), sometimes referred to as LJO model, emulates the ΛCDM cosmology with just one free parameter α defining the creation rate [60]. Interestingly, the equivalence is not restricted to the background solution, since it also happens at the perturbative linear and non-linear levels [61–63]. Even so, apart the CCP and CCM puzzles which are absent in such scenario, the CCDM model may share the same observational problems of the ΛCDM model. However, is has been argued that such model (or some variant of it) may solve in a natural way, the theoretical and observational problems of ΛCDM [64–67].

In this connection, we recall that some attention has also been paid to a new warm dark matter (WDM) component, the so-called reduced relativistic gas (RRG). Its possible relevance to cosmology has also been investigated. In particular, it has been shown that RRG models provide relativistic corrections to the dynamics of the universe, and this kind of warm dark matter (WDM) particles allows large-scale structure (LSS) to be formed as we observe today [68–70]. Nevertheless, since the RRG at late time behaves exactly like a cold dark-matter component, the late time accelerating stage cannot be driven by the RRG model. So, as far as we know, a dark energy component has usually been adopted in the literature of RRG cosmology (see also Sect. 3).

In this paper, we propose a model in which the fuel accelerating the universe is generated by the gravitationally induced particle creation of the RRG component. More precisely, it is assumed that WDM particles spring up into spacetime at the same rate as proposed in the CCDM scenario, while the EoS of this gas is described within the so-called RRG approximation. This approach not only describes an accelerated expansion phase at late times of the universe without dark energy, but also accounts for relativistic contributions from the RRG gas particles in the early dynamics of the universe. In this way, the CCDM model which mimicks the ΛCDM models is slightly modified. These relativistic contributions can be quantified by the warmness parameter b, which is the ratio between the relativistic and non-relativistic energy densities of the massive particles. In order to constrain the values of the pair of free parameters (α, b) we analyse apparent magnitudes from type Ia supernovae (SNe Ia) combined with Cepheid distances from the Pantheon+SH0ES data set and $H(z)$ measurements from cosmological clocks.

The paper is organized as follows. In Sect. 2, the basics of macroscopic particle production in cosmology and the associated dynamics are briefly discussed. Section 3 introduces the equation of state approximation proposed long ago for the reduced relativistic gas (RRG). Section 4 discusses the model in which WDM particles are gravitationally created at the production rate proposed by the CCDM model. In Sect. 5, we present our statistical analyses and constraints to the free parameters of the model. The main findings of the paper are summarized in Sect. 6. Finally, in Appendix A, all details describing the transition redshift are presented.

2 Cosmic dynamics and thermodynamics with particle production

In this section, we briefly review the cosmic dynamics and thermodynamics for a single fluid endowed with 'adiabatic' particle production, focusing especially on those aspects relevant to the RRG component that will be discussed next section.

To begin with, let us consider the space-time described by a flat Friedmann–Lemaître–Robertson–Walker (FLRW) geometry:

$$ds^2 = dt^2 - a^2(t)\left[dx^2 + dy^2 + dz^2\right], \tag{1}$$

where $a(t)$ is the scale factor.

For now we will ignore the baryonic contribution. So, the cosmic dynamics is driven by the RRG gas endowed with the macroscopic description of particle production put forward by Prigogine et al. [71] and extended through a manifestly covariant approach by Calvão et al. [72]. Later, it was shown that the matter creation process is completely different from the bulk viscosity [73], an effective mechanism proposed long ago by Zeldovich [74] and also adopted by several authors to describe the matter creation process [75,76].

The matter-creation mechanism is assumed here to be the unique source of acceleration acting in the current stage of the universe. Its creation pressure (p_c) provides the macroscopic phenomenological description of the quantum back reaction effect on the geometry associated with the gravitational creation process induced by the expanding universe. This basic creation effect has a quantum origin, as many authors have discussed in the literature [77,78].

In this case, the Einstein Field Equations (**EFE**) for a single fluid endowed with matter-creation can be written as (for a multifluid component with matter creation see [63–65])

$$8\pi G\rho = 3H^2, \tag{2}$$

$$8\pi G(p + p_c) = -2\dot{H} - 3H^2, \tag{3}$$

where an overdot means time comoving derivative, $H = \dot{a}/a$ is the Hubble parameter, ρ, p and p_c are the energy density, the equilibrium pressure, and creation pressure, respectively.

The thermodynamic states of a relativistic fluid are defined by the energy conservation law (ECL), which is also contained in the **EFE** ($u_\mu T^{\mu\nu}_{;\nu} = 0$), and the conservation of the particle and entropy fluxes $N^\mu_{;\mu} = 0$ and $S^\mu_{;\mu} = 0$. However, in the presence of gravitationally induced particle creation, the ECL contained in the **EFE**, must be complemented by the balance equations for N^μ and S^μ, respectively. In this case, one may write:

$$u_\mu T^{\mu\nu}_{;\nu} = 0 \iff \dot{\rho} + 3H(\rho + p + p_c) = 0, \tag{4}$$

$$N^\mu_{;\mu} = n\Gamma \iff \dot{n} + 3nH = n\Gamma \iff \frac{\dot{N}}{N} = \Gamma, \tag{5}$$

$$S^\mu_{;\mu} = s\Gamma \iff \dot{s} + 3sH = s\Gamma \iff \frac{\dot{S}}{S} = \Gamma, \tag{6}$$

where n and s are the particle concentration and entropy density and Γ is the particle creation rate. The quantities $N = na^3$ and $S = sa^3$, are the total number of particles and entropy in a comoving volume, respectively. The concept of 'adiabatic' creation is very simple to understand. From Eqs. (5) and (6) we see that

$$\frac{\dot{S}}{S} = \frac{\dot{N}}{N} = \Gamma \geq 0, \tag{7}$$

where the inequality is a consequence of the second law of thermodynamics. The positiviness of Γ means that the space-time may only create matter (entropy). The natural solution is $S = k_B N$, where k_B is the boltzmann constant. In addition, since the entropy per particle is $\sigma = S/N$, the first equality above means that

$$\dot{\sigma} = \sigma\left(\frac{\dot{S}}{S} - \frac{\dot{N}}{N}\right) = 0. \tag{8}$$

Hence, 'adiabatic' particle creation means the total entropy S and number of particles N increases, but the specific entropy remains constant ($\dot{\sigma} = 0$).

It is easy to see that thermodynamics also determines the expression of the creation pressure. The thermodynamic quantities are related by the local Gibbs law: $nTd\sigma = d\rho - (\rho + p)\frac{dn}{n}$, where T is the temperature. So, by using that $\dot{\sigma} = 0$ and combining that result with (4) and (5), it is readily seen that [57–67]

$$p_c = -(\rho + p)\frac{\Gamma}{3H}. \tag{9}$$

In the 'adiabatic condition', Γ is positive definite. Therefore, the creation pressure may be always negative provided that $\rho + p$ is positive. This happens, for instance, by assuming a normal equation of state (EoS), that is, without any kind dark energy

$$p = \omega\rho, \quad \omega \geq 0. \tag{10}$$

Now, by combining Eq. (4) with (9) and (10), one obtains the differential equation governing the evolution of the energy density:

$$\dot{\rho} + 3H\rho(1 + \omega)\left(1 - \frac{\Gamma}{3H}\right) = 0. \tag{11}$$

In the absence of particle creation ($\Gamma = 0$), equation (11) reduces to the flat FLRW equation. For $\Gamma = 3H$, the above equation yields $\dot{\rho} = 0$, that is, ρ constant, regardless of the value of ω. In other words, 'adiabatic' particle production allows de Sitter space-times whose matter-energy content is supported by a pressureless fluid or even radiation [71,72]. Next section, we shall consider the possibility that the gas of created particles is a reduced relativistic gas (RRG).

3 Reduced relativistic gas (RRG)

Contributions from WDM particles to cosmology, along with other implications to astronomy, have increased the interest in the relativistic behavior of dark matter particles [79–81]. The appropriate EoS for an ideal gas of relativistic massive particles was derived long ago [82–84]

$$\rho = \frac{K_3(\rho_m/P)}{K_2(\rho_m/P)}\rho_m - P, \tag{12}$$

where $\rho_m = nmc^2$ is the rest energy density of a gas with particle number density $n = N/V$, while K_2 and K_3 are modified Bessel functions of the second kind.

The EoS (12) leads to a complicated differential equation governing the evolution of the cosmos. On the other hand, an alternative description of a relativistic gas was proposed by Sakharov in which a phenomenological equation of state describes a fluid that transits from an ultra-relativistic to a pressureless matter behavior as the universe expands [85]. Later, an approximation to the relativistic description of the ideal gas was obtained explicitly under the hypothesis that identical particles of the gas share equal momentum magnitudes [86].

In this new approach, the EoS describing the so-called Reduced Relativistic Gas (RRG) takes the following form:

$$P = \frac{\rho}{3}\left[1 - \frac{\rho_{mo}^2}{\rho^2}\left(\frac{n}{n_o}\right)^2\right], \tag{13}$$

where p and ρ are, respectively, the pressure and energy density of the gas and $\rho_{mo} = n_o m$ is the rest energy density of massive particles $\rho_m = nm$ evaluated at the present time ($t = t_o$).

As it appears, the above EoS is much simpler than Eq. (12) and allows analytical solutions to be obtained for models in which the relativistic contributions of massive particles are significant in the dynamics of the universe. Moreover, it was shown [86,87] that the RRG equation of state is a very efficient approximation to the EoS (12). Note also that it depends explicitly on n so that a third differential equation is required to solve the Einstein Equations (2) and (3). Accordingly, the evolution of the particle concentration as given by the balance equation (5) must be used. Conversely, when the number of particles in a comoving volume ($N = na^3$) is conserved, $n \propto a^{-3}$ and $\Gamma = 0$ so that from (9) the creation pressure is also nullified ($p_c = 0$). Thus, the cosmic equations (2) and (3) can be easily solved with the help of EoS (13). In this case EoS (13) can also be used to rewrite (11) as:

$$\frac{d\rho}{da} = -3\frac{\rho}{a}\left[\frac{4}{3} - \frac{1}{3}\frac{\rho_{mo}^2}{\rho^2}\left(\frac{a_o}{a}\right)^6\right], \tag{14}$$

whose solution is given by

$$\rho(a) = \sqrt{\rho_{mo}^2\left(\frac{a_o}{a}\right)^6 + \rho_{ro}^2\left(\frac{a_o}{a}\right)^8}, \tag{15}$$

where ρ_{ro} is an integration constant whose interpretation is a simple one: when $a \ll a_o$, the second term dominates over the first and the energy density behaves as $\rho \propto a^{-4}$, which has the same evolution as radiation. Eventually, as $a(t)$ increases, the second term becomes negligible compared to the first and ρ becomes $\rho \propto a^{-3}$, which is consistent with the behavior of pressureless matter at late times. It is also convenient to write ρ as

$$\rho = \rho_{mo}\left(\frac{a_o}{a}\right)^3\sqrt{1 + b^2\left(\frac{a_0}{a}\right)^2}, \tag{16}$$

where $b = \rho_{ro}/\rho_{mo}$ is known as the warmness parameter. This parameter is important in determining the relevance of relativistic contributions to energy density as the universe expands.

Several models have considered relativistic contributions from the RRG approximation to cosmology. Solutions for models with pure WDM, WDM with radiation, WDM + Λ and others multifluids have been obtained in the literature both in background and perturbative levels of cosmology assuming the EoS (13) [69,70,88–90]. Other approaches such as running vacuum cosmologies, alternative space-time geometry and scalar fields have also been studied in the context of RRG [91,92]. In the next Section we will investigate a scenario where the particles of the RRG gas are allowed to be created from the mechanism of gravitationally induced particle creation. As remarked in the Introduction, the derived accelerating cosmology has no dark energy ($\Omega_{DE} = 0$).

4 Modified CCDM: RRG model with particle creation

Let us now consider a model endowed with the "adiabatic" mechanism of warm particle creation. As discussed in the Introduction, the particle production rate Γ is fundamental in determining the cosmological parameters in models that $\dot{N} \neq 0$. Rigorously, Γ should be determined from quantum field theory (QFT) of non-equilibrium in curved spacetimes. However, this theory has yet to be formulated. On the other hand, the particle production rate can be obtained phenomenologically. Similarly to the creation rate proposed in [60], we investigate the case where Γ is given by

$$\Gamma = 3\alpha \left(\frac{\rho_o}{\rho_m}\right) H, \tag{17}$$

where α is a dimensionless constant, ρ_o is the critical energy density evaluated today, 3 is only a mathematical convenience and ρ_m is the rest energy density $\rho_m = nmc^2$ of dark matter and baryon particles $\rho_m = \rho_{dm} + \rho_b$. As one may recall from the previous section, the particle concentration n can be solved in terms of the scale factor from the balance equation (4) and the production rate (17) yields:

$$\frac{n}{n_o} = \frac{\rho_m}{\rho_{mo}} = \left(1 - \alpha\frac{\rho_o}{\rho_{mo}}\right)\left(\frac{a_o}{a}\right)^3 + \alpha\frac{\rho_o}{\rho_{mo}}. \tag{18}$$

It is clear from this equation that ρ_m can be written as

$$\rho_m = (\rho_{mo} - \alpha\rho_o)(1+z)^3 + \alpha\rho_o, \tag{19}$$

which is exactly the solution for the matter energy density of the CCDM model [60].

If the particle creation rate is neglected, that is, $\alpha = 0$, the standard case $n \propto \rho_m \propto a^{-3}$ is recovered from the above equation, as expected. From the particle production rate (17) and the particle concentration (18) one can write the ECL for the RRG gas modified by the mechanism of particle creation

$$\frac{d\rho}{da} = -3\frac{\rho}{a}\left(1 - \alpha\frac{\rho_o}{\rho_m}\right)\left[\frac{4}{3} - \frac{1}{3}\frac{\rho_m^2}{\rho^2}\right], \tag{20}$$

where ρ, in this case, is the energy density of the reduced relativistic gas of massive particles. Note that Eq. (14) is a particular case of the ECL (20) when $\alpha = 0$.

The differential equation in (20) can be extremely simplified if the scale factor and its differential are written in terms of ρ_m and other constants. Note that differentiating equation (19) with respect to a yields

$$\frac{da}{a} = -\frac{1}{3}\left(\frac{a}{a_o}\right)^3 \frac{d\rho_m}{\rho_{mo} - \alpha\rho_o}, \tag{21}$$

and, also from Eq. (19), note that

$$\left(\frac{a}{a_o}\right)^3 = \frac{\rho_{mo} - \alpha\rho_o}{\rho_m - \alpha\rho_o}. \tag{22}$$

Now, by inserting Eqs. (21) and (22) into (20) the ECL becomes simply

$$\frac{d\rho}{d\rho_m} = \frac{4}{3}\frac{\rho}{\rho_m} - \frac{1}{3}\frac{\rho_m}{\rho}. \tag{23}$$

This first-order differential equation is easily solved. Introducing a new variable, $u \equiv \rho/\rho_m$, the solution reads:

$$u = \sqrt{1 + (C\rho_m)^{2/3}}, \tag{24}$$

where C is a constant of integration. As $\rho = \rho_m u$, it follows that

$$\rho = \sqrt{\rho_m^2 + \left(\rho_o^2 - \rho_{mo}^2\right)\left(\frac{\rho_m}{\rho_{mo}}\right)^{8/3}}, \tag{25}$$

in which $C = (\rho_o^2 - \rho_{mo}^2)/\rho_m^{8/3}$ can be obtained as an initial condition for ρ at $a = a_o$ in (24). Finally, by defining $\rho_{ro} = \sqrt{\rho_o^2 - \rho_{mo}^2}$ as the relativistic contribution of the massive particles to the energy density

$$\rho = \sqrt{\rho_{mo}^2\left(\frac{\rho_m}{\rho_{mo}}\right)^2 + \rho_{ro}^2\left(\frac{\rho_m}{\rho_{mo}}\right)^{8/3}} = \rho_m\sqrt{1 + b^2\left(\frac{\rho_m}{\rho_{mo}}\right)^{2/3}}, \tag{26}$$

where $b = \rho_{ro}/\rho_{mo}$ is the warmness parameter. In terms of the redshift, ρ can be rewritten as

$$\rho = \left[(\rho_{mo} - \alpha\rho_o)(1+z)^3 + \alpha\rho_o\right] \times\sqrt{1 + b^2\left[\left(1 - \frac{\alpha\rho_o}{\rho_{mo}}\right)(1+z)^3 + \frac{\alpha\rho_o}{\rho_{mo}}\right]^{2/3}}. \tag{27}$$

Note that in the absence of particle production ($\alpha = 0$), the equation above recovers the reduced relativistic gas from (15). In addition, for a gas with negligible warmness parameter ($b = 0$), that is, a gas with non-relativistic particles, the solution (27) reduces to Eq. (19), as expected.

Let us now determine the contributions of the warm particle creation to the dynamics of the universe. Considering that the only component of the universe being the gas described above, the energy density (27) coincides with the critical energy density in (2). Thus, the Hubble parameter for this model can be obtained by substituting Eq. (27) into (2) and, considering $\Omega_m = \rho_{mo}/\rho_o$, it reads

$$H^2 = H_o^2\left[(\Omega_m - \alpha)(1+z)^3 + \alpha\right] \times\sqrt{1 + \frac{b^2}{\Omega_m^{2/3}}\left[(\Omega_m - \alpha)(1+z)^3 + \alpha\right]^{2/3}}. \tag{28}$$

In models which the acceleration of the universe is due to the mechanism of particle creation, the late phase of the universe is dominated by matter, $\Omega_m = 1$. However, galaxy cluster observations and other probes suggest that $\Omega_m \approx 0.3$ [11–13,93–95]. The explanation for the low Ω_m constraints in models with particle creation is that the effective matter energy density, which scales as $(1+z)^3$ and is responsible for the structure formation, is $\Omega_{meff} = \Omega_m - \alpha$. In this point of view, the matter that did not agglomerate is homogeneously distributed across the universe [60]. This is similar to the model proposed in this paper (see equation (28)), except it is modified by relativistic contributions.

Additionally, by evaluating $H(z)$ at $z = 0$ in Eq. (28), we see that $\Omega_m = 1/\sqrt{1+b^2}$. Thus, the Hubble parameter can be written only in terms of H_0, α and b as

$$H^2 = H_o^2 \left[\left(\frac{1}{\sqrt{1+b^2}} - \alpha\right)(1+z)^3 + \alpha\right] \times \sqrt{1 + b^2 \left(1+b^2\right)^{1/3} \left[\left(\frac{1}{\sqrt{1+b^2}} - \alpha\right)(1+z)^3 + \alpha\right]^{2/3}}. \tag{29}$$

The effective matter energy density is now a function of α and b, and can be written as $\Omega_{meff} = (1/\sqrt{1+b^2}) - \alpha$. Note from the equation above that as $z \to -1$ ($a \to \infty$), the expansion rate $H_f = H(a \to \infty)$ becomes approximately constant

$$H_f \approx H_o\sqrt{\alpha}\left[1 + b^2\left(1+b^2\right)^{1/3}\alpha^{2/3}\right]^{1/4}, \tag{30}$$

which implies an exponential expansion in the very late universe since $\dot{a}/a = H_f$ yields ($a \propto e^{H_f t}$). This dynamic behavior at $a \to \infty$ is consistent with both the CCDM ($H_f = H_o\sqrt{\alpha}$) and ΛCDM ($H_f = H_o\sqrt{\Omega_\Lambda}$) models, except for different values of H_f.

Several authors have discussed the degeneracy of the dynamics of the universe between the ΛCDM model and models with non-relativistic particle creation, for Γ in the form (17), if $\alpha = \Omega_\Lambda$. On the other hand, the dynamics of the universe in a model with RRG is clearly different from the one in the ΛCDM model. Nonetheless, these relativistic corrections might solve or alleviate some problems of the standard model. In the next section, we confront the model against late-time cosmological observational data in order to constrain its free parameters.

5 Analysis and results

In order to constrain the free parameters of the RRG model with particle creation, we make use of the following observational data: apparent magnitudes from SNe Ia combined with Cepheid distances, as given by the Pantheon+SH0ES compilation [4] and $H(z)$ from cosmic chronometers [24,25].

5.1 Data and methodology

The Pantheon+SH0ES compilation [4] comprises currently one of the largest SNe Ia compilations, combining Pantheon+ SNe Ia with Cepheid distances from SH0ES. The Pantheon+ consists of 1701 light curves of 1550 distinct SNe Ia, spanning a redshift interval $0.001 < z < 2.26$. By using this compilation, it is possible to constrain the free parameters of the model including H_0, which is obtained from the SH0ES calibration.

Besides being a large compilation yielding powerful constraints, the Pantheon+SH0ES also is independent of the cosmological model choice, depending only on the SNe Ia and Cepheid Astrophysics, being then suitable for constraining the RRG cosmological model.

Another dataset which is independent of the cosmological model choice and dependent only on astrophysical assumptions, is the $H(z)$ data from cosmic chronometers (CC). $H(z)$ data obtained this way depends only on chemical evolution models, from which ages can be obtained, so $H(z)$ can be obtained through the relation $H(z) = -\frac{1}{(1+z)dt/dz}$ [23].

We use the 32 CC $H(z)$ data compiled from Moresco et al. [24,25]. This compilation includes the first treatment of the CC systematic errors, which increased the errors when compared with previous compilations, but yields more robust constraints.

Following the Bayes' Theorem, in both datasets, we aim to probe the posterior p of the parameters $\theta = (M, H_0, \alpha, b)$ given the data, D:

$$p(\theta|D) \propto \pi(\theta)\mathcal{L}(\theta|D), \tag{31}$$

where π is the prior, which we choose to be flat for all parameters and $\mathcal{L}$ is the likelihood, given by

$$\mathcal{L} = \mathcal{L}_{SN}\mathcal{L}_H, \tag{32}$$

where $\mathcal{L}_{SN}$ is the likelihood of the Pantheon+SH0ES compilation and $\mathcal{L}_H$ is the CC likelihood. Both likelihoods can be written as:

$$\mathcal{L}_D \propto e^{-\chi_D^2/2}, \tag{33}$$

where $D = (SN, H)$ and

$$\chi_D^2 = \sum_{i,j}(y(z_i,\theta) - y_{obs,i})C_{ij}^{-1}(y(z_j,\theta) - y_{obs,j}), \tag{34}$$

where C_{ij} is the covariance matrix and the data y_i is the apparent magnitude m_i for SNe Ia and H_i for CC $H(z)$ data.

In order to probe the posteriors, we used the python package emcee [96], which generates Monte Carlo-Markov Chains (MCMC) by sampling the posteriors. This package

Table 1 Combined constraints on the free parameters at 68% and 95% confidence levels (C.L.) from Pantheon+SH0ES+$H(z)$

Parameter	68% C.L	95% C.L
M	$-19.262^{+0.028}_{-0.028}$	$-19.262^{+0.056}_{-0.056}$
H_0	$73.1^{+0.97}_{-0.97}$	$73.1^{+2.0}_{-1.9}$
α	$0.663^{+0.029}_{-0.012}$	$0.663^{+0.052}_{-0.068}$
b	< 0.266	< 0.691

is based on the Affine-Invariant Ensemble Sampler [97]. To plot the results, we used the package getdist [98], which uses the method of Kernel Density Estimation (KDE) in order to smooth the MCMC histograms.

5.2 Results

The constraints on the free parameters from all the datasets can be seen on Fig. 1.

As can be seen on this Figure, the Pantheon+SH0ES dataset yields the strongest constraints. However, $H(z)$ data was essential in order to reduce the degeneracy between α and b parameters. In fact, the degeneracy between α and b was such that b was superiorly limited only by the **flat** prior $b \in [0, 1]$ both for Pantheon+SH0ES and $H(z)$ data. For the combination, however, it was superiorly limited at $b \sim 1$ at 95% c.l.

In Fig. 2, we see the combined results.

As can be seen on this Figure, all parameters are well constrained by the joint analysis, except for b. Due to the degeneracy between α and b, the b parameter is weakly constrained, although this degeneracy is reduced when combining with $H(z)$ data, as explained above. There is, also, a strong correlation between H_0 and M, the SNe Ia absolute magnitude, induced by Pantheon+SH0ES data. Nevertheless, this correlation also appears in analyses involving the standard cosmological model. The results for the free parameters at 1 and 2σ c.l. can be seen at Table 1.

As can be seen from this Table, the b parameter is well constrained at 1σ c.l., if one considers that we have used only background data. It may indicate that the combination with clustering data or even other background data can provide strong constraints for this parameter.

6 Final comments and conclusion

In this paper, we have discussed based on Einstein's gravity theory, a new accelerating cosmology without vacuum, quintessence or extra dimensions. The late time accelerating phase is powered by the negative pressure (back reaction) due to the particle creation mechanism of massive, warm dark matter particles forming the reduced relativistic gas.

In comparison with the standard cosmology (ΛCDM), the first advantage of our model is that the unknown dark sector (DE + DM) is reduced to only dark matter. In fact, the absence of vacuum energy density ($\Omega_\Lambda = 0$) implies that the old cosmological constant problem and also the coincidence mystery are absent in this framework. Secondly, it is known that the ΛCDM model is degenerated with the original creation cold dark matter (CCDM) cosmology [60], not only for background results, but also in the linear and nonlinear perturbative levels. It was shown that the inclusion of the RRG warm dark matter broke the degeneracy between ΛCDM and CCDM cosmology, and, as such, a quasi-ΛCDM dynamics is obtained. In principle, this quasi-CCDM evolution cosmology may help to solve the H_0 and $S8$ tensions.

On the other hand, the idea of relativistic dark matter particles as a significant component of the universe has problems nonetheless. It is widely believed that relativistic dark matter prevents large scale structure to be formed due to the free streaming process. However, as discussed in the introduction, some papers show that warm dark matter (WDM) can be consistent with structure formation while providing relativistic corrections to cosmology based on the RRG model. In this context, we were able to obtain an analytical model that recovers the pure RRG model when the number of particles is conserved ($\alpha = 0$) and the CCDM cosmology when only non-relativistic particles are considered ($b = 0$). In other words, this new model $RRG + \alpha$ is a generalization of the CCDM(LJO) cosmology in the context of massive relativistic particles.

It is also important to stress that the energy density of matter in the late phase of the universe is $\Omega_m \sim 1$ for models without dark energy. However, observations indicate that $\Omega_m \sim 0.3$. Nevertheless, the fraction of matter that clusters in galaxies is represented, in models with particle creation, by the effective energy density $\Omega_{meff} = \Omega_m - \alpha$ which is consistent with $\Omega_{meff} \approx 0.3$ as shown in CCDM model [60]. Additionally, the remaining 0.7 fraction of matter is said to be homogeneously distributed across the universe by the mechanism of particle creation in these cases.

In order to constrain the free parameters of the model, we have used background data as SNe Ia + Cepheids from Pantheon+SH0ES and $H(z)$ from Cosmic Chronometers. While the b parameter is weakly constrained by this analysis, it enables distinctions from the standard ΛCDM model, differently of what happened with CCDM, where no distinction could be made, even at higher orders of density perturbations. Figures 1 and 2 show that this combined data attribute the maximum probability for b (for b up to 1) at $b \ll 1$, approximating the proposed model to CCDM and ΛCDM. This result is consistent with other background tests in models with conserved number of particles of RRG, as can be

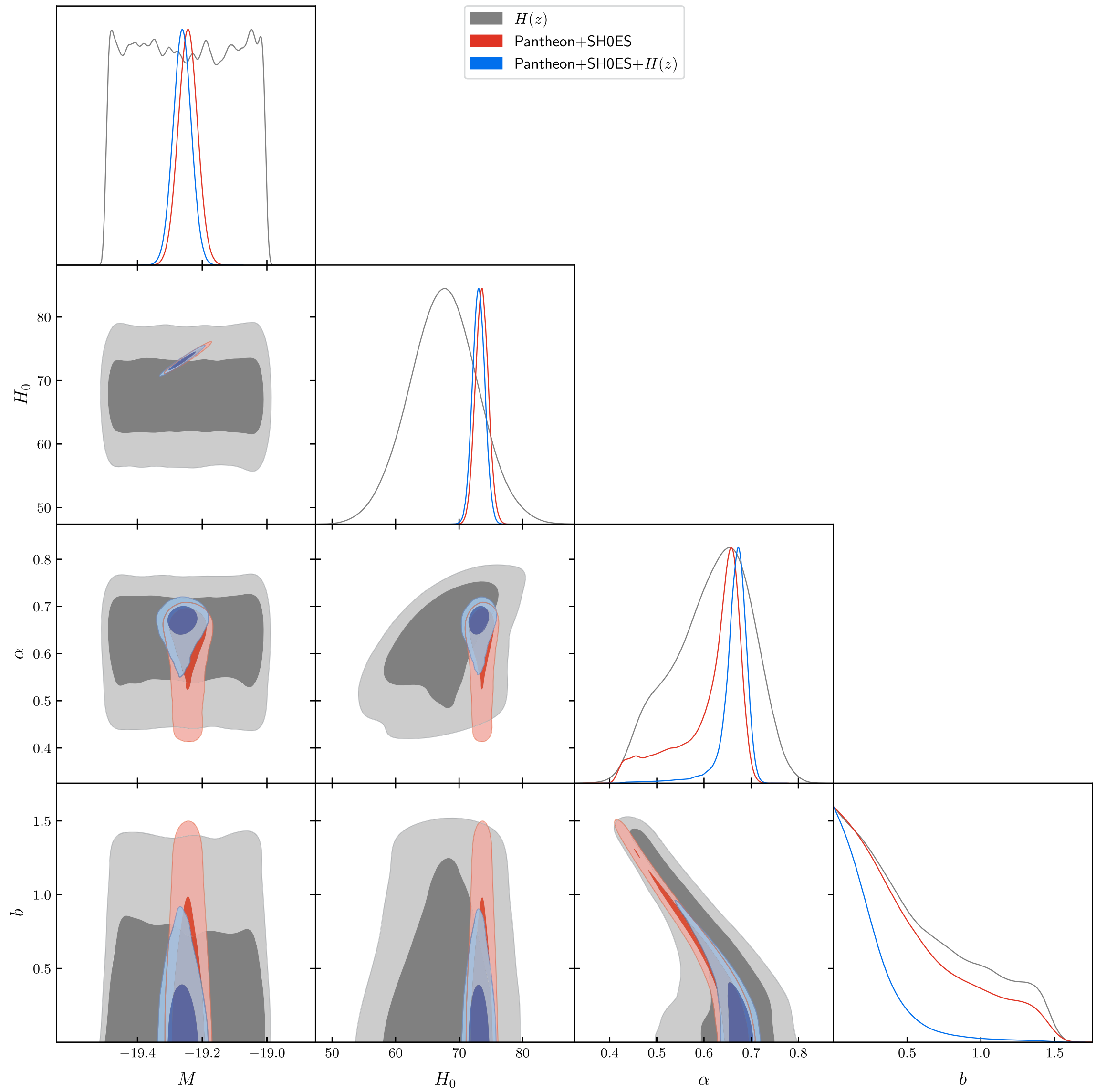

Fig. 1 Constraints on M, H_0, α, b, using the SNe Ia Pantheon+SH0ES data set, along with $H(z)$ from cosmic clocks

seen in [88,89]. On the other hand, these observations suggest that the transition from a decelerated to an accelerated regime of the universe happens slightly later in the proposed model at $z_t \approx 0.60$ (see Appendix), whereas in CCDM (LJO) model, $z_t \approx 0.71$. This result is expected, since the relativistic contributions increases the positive pressure from the gas, thereby intensifying the deceleration regime at high redshifts. Although b also increases the negative pressure of particle creation, these contributions are less significant since they are stronger at high values of z, i.e. when p_c is minimal.

These results endorse the claim that this new model has mild differences in the cosmic dynamics, as compared to CCDM and ΛCDM. Therefore, these differences might clarify some of the problems plaguing the standard model. From the statistical analysis in Sect. 5, we can also verify the estimate of the effective energy density of matter according to the constraints on α and b. For instance, consider the upper limit at 1σ c.l. constraint on b at $b = 0.266$ along with the best fit for α at $\alpha = 0.663$ (see Table 1) which yield $\Omega_{meff} \approx 0.30$ as expected. Nevertheless, more data are required in order to better constrain the parameters within the $RRG + \alpha$ context. In fact, since cosmology is in the precision era, this model must also be confronted with observations in the perturbative level of cosmology. Based on some assumptions, the warmness parameter have already been constrained by analysing density perturbations, however, the treatment was applied for models with a conserved number of RRG particles. A perturbative extension of such analyses to the context of particle creation will be addressed in a forthcoming paper.

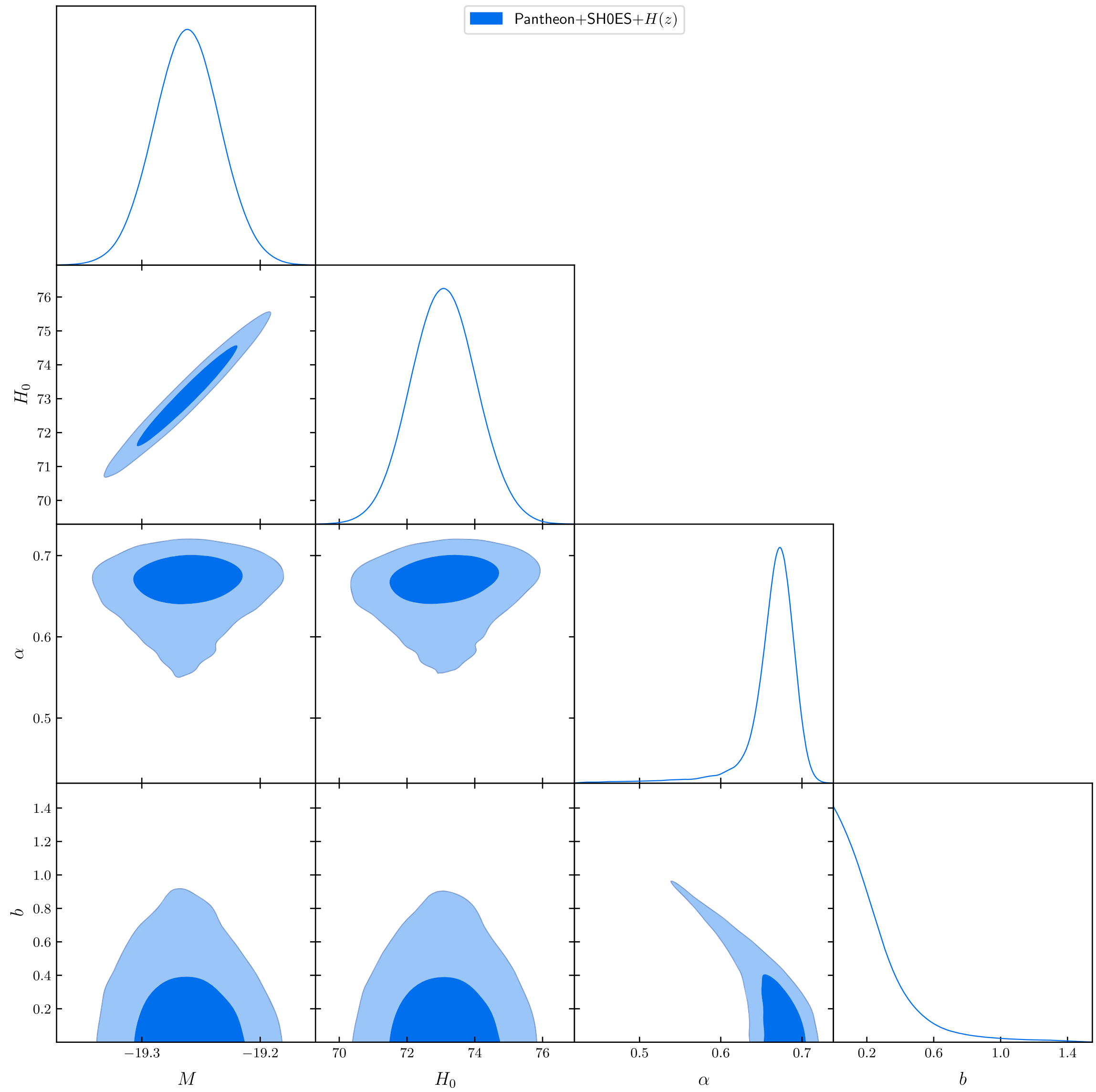

Fig. 2 Combined constraints on α, b, H_0 and M, using the SNe Ia Pantheon+SH0ES+$H(z)$ from cosmic clocks

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Data Availability Statement This manuscript has associated data in a data repository. [Author's comment: Pantheon+SH0ES data is available at GitHub: https://github.com/PantheonPlusSH0ES/DataRelease H(z) Cosmic Chronometers data is available at Gitlab: https://gitlab.com/mmoresco/CCcovariance and at Ref.[25], Table 1.]

Code Availability Statement Code/software will be made available on reasonable request. [Author's comment: The code/software generated during and/or analysed during the current study is available from the corresponding author on reasonable request.]

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Appendix A: The deceleration parameter $q(z)$ and the transition redshift z_t

Two useful dynamical physical parameters in cosmology are the deceleration parameter $q(z)$ and the value of the transition redshift, z_t. These parameters are very useful to discriminate cosmological models [99–103]. In particular, ΛCDM and CCDM models predict the same values for both quantities. Since now we have two free parameter (α, b), we show in this appendix that their values are slightly modified. The value of q is defined by:

$$q = -\frac{a\ddot{a}}{\dot{a}^2} = -\frac{\dot{H}}{H^2} - 1. \tag{A1}$$

From the second Friedmann equation (3), the Hubble parameter (29), and the expression above, the deceleration parameter can be obtained for the model proposed in this paper, and written in terms of the redshift, as

$$q\,(z) = -1 + 2\left(1 - \frac{\alpha}{\left(\frac{1}{\sqrt{1+b^2}} - \alpha\right)(1+z)^3 + \alpha}\right) \times \left(1 - \frac{1}{4}\,\frac{1}{1 + b^2\left(1+b^2\right)^{1/3}\left[\left(\frac{1}{\sqrt{1+b^2}} - \alpha\right)(1+z)^3 + \alpha\right]^{2/3}}\right). \tag{A2}$$

Note that, for $b = \alpha = 0$, the deceleration parameter above yields $q = 0.5$, which corresponds to a model with pure pressureless matter, as expected. On the other hand, for $\alpha \neq 0$ and $b \neq 0$ the deceleration parameter depends on z, and as $z \to \infty$ we notice that $q \to 1$ which corresponds to the ultra-relativistic case. Finally, when $z \to -1$, i.e. $a \to \infty$, Eq. (A2) approaches $q \approx -1$, which is the q for vacuum. The dynamics of the universe for this model, regarding the deceleration parameter, can be visualized in Fig. 3.

As in the ΛCDM and CCDM cosmologies, the model described above also transits from a decelerated to an accelerated expansion phase. This exchange occurs when $\ddot{a} = 0$, or equivalently $q(z) = 0$, at the transition redshift z_t. Hence, the transition redshift obtained by evaluating $q(z) = 0$ in Eq. (A2), is

$$z_t = \left(\frac{2\alpha}{\frac{1}{\sqrt{1+b^2}} - \alpha}\right)^{1/3} \times \left\{1 + \frac{b^2\left(1+b^2\right)^{1/3}\left[\left(\frac{1}{\sqrt{1+b^2}} - \alpha\right)(1+z_t)^3 + \alpha\right]^{2/3}}{1 + b^2\left(1+b^2\right)^{1/3}\left[\left(\frac{1}{\sqrt{1+b^2}} - \alpha\right)(1+z_t)^3 + \alpha\right]^{2/3}}\right\}^{-1/3} - 1. \tag{A3}$$

Notably, for $b = 0$, the equation above recovers z_t from the CCDM(LJO) model $z_t = [2\alpha/(1-\alpha)]^{1/3} - 1$ [60].

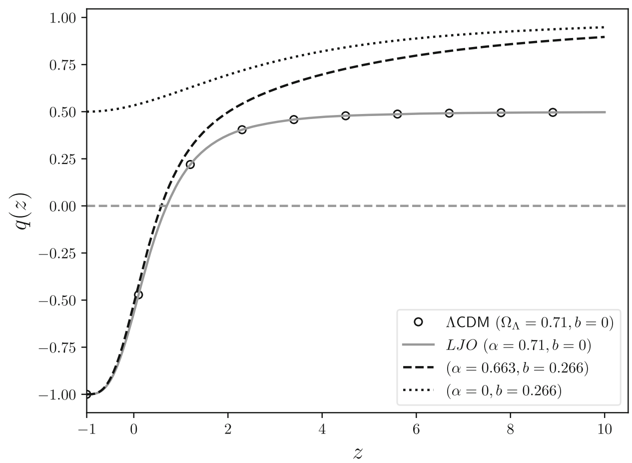

Fig. 3 The deceleration parameter q with respect to the redshift for different cosmological models. The solid grey line represents the CCDM (LJO) model, which expectedly coincides with the ΛCDM model represented by the black circles. The dashed line corresponds to the $q(z)$ of the RRG with particle creation for the values $\alpha = 0.663$ and $b = 0.266$ (see Table 1). Finally, the dotted line indicates $q(z)$ of a model with RRG and absent of particle creation

However, for $b \neq 0$, the transition redshift is modified by relativistic corrections. The value of z_t is not easily obtained from the equation above due to higher order terms of z_t in this expression. An approximation to the Eq. (A3) can be obtained using a recurrence procedure, where at the zeroth-order, $z_t^{(0)} = z_t(b = 0)$, and replacing this at the rhs of the equation above, we obtain the first order approximation:

$$z_t^{(1)} = \left(\frac{2\alpha}{\frac{1}{\sqrt{1+b^2}} - \alpha}\right)^{1/3} \times \left\{1 + \frac{b^2\left(1+b^2\right)^{1/3}\left[\left(\frac{1}{\sqrt{1+b^2}} - \alpha\right)\left(\frac{2\alpha}{1-\alpha}\right) + \alpha\right]^{2/3}}{1 + b^2\left(1+b^2\right)^{1/3}\left[\left(\frac{1}{\sqrt{1+b^2}} - \alpha\right)\left(\frac{2\alpha}{1-\alpha}\right) + \alpha\right]^{2/3}}\right\}^{-1/3} - 1. \tag{A4}$$

The approximation above, for the values $\alpha = 0.663$ and $b = 0.266$ (see Table 1), yields $z_t^{(1)} \approx 0.58$.

A more realistic estimation of z_t can be obtained by calculating the likelihood of z_t (see Fig. 4) with the constraints on α and b previously sampled from the combined Pantheon+SH0ES+$H(z)$ data, discussed in the previous section, and the theoretical prediction of z_t in Eq. (A3). These results yield $z_t = 0.602^{+0.041}_{-0.041}$ for a 68% C.L. and $z_t = 0.602^{+0.085}_{-0.079}$ for a 95% C.L.. As expected, this transition redshift is slightly smaller than $z_t(LJO) \approx 0.71$ in the CCDM(LJO) model [60], since the relativistic contribution of the gas increases the positive pressure (13). In other words, the pressure, which is supposed to be zero for massive particles, receives a positive contribution from the relativistic particles that slows the expansion of the universe, compared to a standard matter-dominated era, and retards the transition

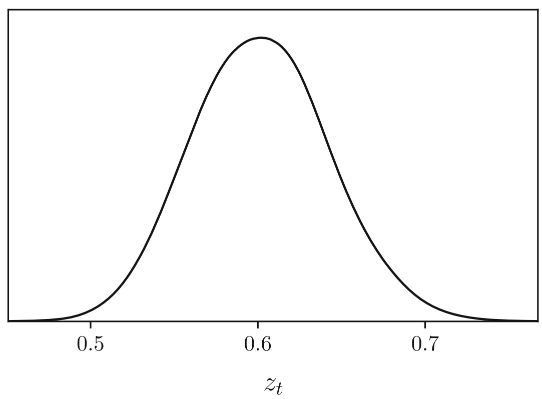

Fig. 4 The likelihood of the transition redshift (z_t) for the RRG model in the context of particle creation given by the combined observational data Pantheon+SH0ES [4] with $H(z)$ data from cosmic clocks [24,25], which constrains z_t to $z_t = 0.602^{+0.041+0.085}_{-0.041-0.079}$

for an accelerated regime. It is interesting to notice that b also contributes to the negative creation pressure (see 9 and 13), although this contribution is less relevant due to the fact that Γ is only significant when ρ_m becomes small (see Eq. 17).

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