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How may confinement affect technicolour?

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Abstract. Confinement has been introduced into the quark gap equation, as proposed by Cornwall, as a possible solution to the problem of chiral symmetry breaking in quantum chromodynamics with dynamically massive gluons. We argue that the same mechanism can be applied for technicolour with dynamically massive technigluons. Within this approach both theories develop a hard self-energy dynamic, resulting from an effective four-fermion interaction, which does not lead to the known technicolour phenomenological problems. We outline a quite general type of technicolour model within this proposal that may naturally explain the masses of different fermion generations.

Contents

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1. Introduction

The nature of electroweak symmetry breaking is one of the most important problems in particle physics. There are many questions that may be answered in the near future by the Large Hadron Collider experiments, such as the following: is the Higgs boson, if it exists at all, elementary or composite, and what are the symmetries behind the Higgs mechanism? The possibility that the Higgs boson is a composite state instead of an elementary one is more akin to the phenomenon of spontaneous symmetry breaking that originated from the Ginzburg–Landau Lagrangian, which can be derived from the microscopic Bardeen–Cooper–Schrieffer (BCS) theory of superconductivity describing the electron–hole interaction (or the composite state in our case). This dynamical origin of spontaneous symmetry breaking has been discussed with the use of many models, the technicolour (TC) one being the most popular [1].

Ordinary fermion masses ($m_{\rm f}$) result from the interaction of these fermions with technifermions through the exchange of a extended technicolour (ETC) boson and depend crucially on the technifermion self-energy. In early models this self-energy was considered to be given by the standard operator product expansion result [2]: $\Sigma_{\rm TC}(p^2) \propto \langle \bar{T}_{\rm f} T_{\rm f}\rangle / p^2$, where $\langle \bar{T}_{\rm f} T_{\rm f}\rangle$ is the TC condensate and of the order of a few hundred GeV. With this self-energy the fermion masses are given by $m_{\rm f} \approx \langle \bar{T}_{\rm f} T_{\rm f}\rangle / M^2_{\rm etc}$. In order to obtain the fermion masses of the second and third generations, the ETC gauge boson masses had to be very light. Since these bosons connect different fermionic generations and must be light, they may produce flavour changing neutral currents (FCNC) incompatible with the experimental data. A possible way out of this dilemma was proposed by Holdom [3], remembering that the self-energy behaves as

$$\Sigma_{\rm TC}(p^2) \approx \frac{\langle \bar{T}_{\rm f} T_{\rm f}\rangle_\mu}{p^2} \left(\frac{-p^2}{\mu^2}\right)^{\gamma_m/2}, \tag{1}$$

where μ is the characteristic TC scale and γ_m the anomalous dimension associated with the condensate operator.

If $\gamma_m \geqslant 1$, the fermion masses have a smaller dependence on the ETC gauge boson masses, which can be larger when resolving the FCNC problem. Models proposing such large anomalous dimensions were reviewed in [4]. Therefore, theories with large anomalous dimensions (γ_m) are quite desirable for TC phenomenology [1]. Lattice simulations were used to study many models that could have a large γ_m; regrettably, some of these models do have an appreciable anomalous dimension but are not large enough to solve the phenomenological problems of TC theories [5], indicating how difficult it is to build a realistic TC model.

It is quite possible that the TC problems are related to the poorly known self-energy expression, or the way chiral symmetry breaking (CSB) is realized in non-Abelian gauge theories. Actually, the only known laboratory to test the CSB mechanism is QCD, and even in this case, considering several recent results about dynamical mass generation in QCD that we shall discuss throughout the paper, imply that the dynamical quark mass generation mechanism is not fully understood. We will argue that a recent proposal to understand the CSB mechanism in QCD [6, 7] may shed light on the same mechanism in TC models and possibly lead to viable theories for dynamical symmetry breaking of the standard model.

The effect of confinement in TC, as well as in QCD, may be so strong that an effective four-fermion interaction (like the famous Nambu–Jona–Lasinio gauged model) can be generated. The theory develops quite a hard self-energy implying that FCNCs can be avoided by decoupling the techni-gauge fields, while different family fermion masses are generated via CSB of the TC and QCD theories.

In section 2 of this work we will recall that QCD possess the property of dynamical generation of gluon masses. This property has been verified through lattice simulations as well as through Schwinger–Dyson calculations. We argue that the same mechanism happens in TC theories (i.e. generation of dynamically massive technigluons), as long as the TC model is not conformal. If this is the case, the same CSB problem appearing in QCD will appear in TC: QCD (TC) with dynamically massive (techni)gluons does not have strength enough to generate the expected (techni)quark masses, or (techni)quark condensates. A possible solution to this problem, as proposed by Cornwall [6, 7] and followed by the authors [8], is that confinement is responsible for the CSB in QCD, and we assume this to also be true for TC. In section 3 we discuss how we can model confinement in non-Abelian theories in order to obtain the right amount of CSB. We argue that in QCD, as well in TC, the confinement effect is so strong that it can generate an effective four-fermion interaction (or an effective Nambu–Jona–Lasinio gauged model). This results in quite hard self-energies for quarks as well as techniquarks. In section 4 we discuss how QCD and TC theories with hard self-energies, or self-energies that have been named in the past as irregular solutions for $\Sigma(p^2)$, may lead to models where both theories contribute to the ordinary fermion masses, and do not lead to FCNC problems. Section 5 contains our conclusions.

2. Technicolour with dynamically massive technigluons

Many years ago Cornwall proposed that a dynamical gluon mass could be generated in QCD [9]. Only recently was this possibility confirmed by lattice simulations [10] and checked rigorously through Schwinger–Dyson equations (SDE) [11]. It seems that this is a general property of non-Abelian gauge theories [12]. There is no reason to believe that the same mechanism does not happen in TC theories. The only possibility for technigluons not acquiring a dynamical mass that we can think of is the case of a conformal or non-asymptotically free TC model, where the effect of technifermion loops in the SDE cancel the gauge loop effects responsible for the dynamical technigluon mass.

We assume a TC theory based, for instance, on a $SU(N)$ gauge group, with a fermion content such that the theory is asymptotically free and is not almost conformal (or not near a perturbative fixed point). We also assume that in this theory the technigluons will acquire a dynamical mass, and CSB breaking can be studied in the same way it is studied for QCD, through SDE for technifermions. The technifermion self-energy will be given by

$$\begin{aligned}\Sigma_{\rm TC} &\equiv M(p^2)\\ &= \frac{C_2}{(2\pi)^4}\int \mathrm{d}^4k \frac{\bar{g}^2_{\rm tc}(p-k)3M(k^2)}{[(p-k)^2+m^2_{\rm tg}(p-k)][k^2+M^2(k^2)]},\end{aligned} \tag{2}$$

where we consider the Landau gauge, C_2 is the Casimir operator for the fermionic representation, $m_{\rm tg}(k^2)$ is the dynamical technigluon mass and $\bar{g}_{\rm tc}$ the effective TC coupling constant. Firstly, we must say that, as far as we know, the CSB mechanism in TC models has not been studied up to now in the presence of dynamical technigluon mass generation. Secondly, to understand what may happen in a TC theory we will recall some QCD results.

When dynamical gauge boson masses are generated in any asymptotically free non-Abelian gauge theory we also expect that the coupling constant develops a non-perturbative infrared fixed point [13]. In QCD it was predicted many years ago that the coupling constant would behave as [9]

$$\bar{g}^2_{\rm QCD}(k^2) = \frac{1}{b\ln[(k^2+4m_{\rm g}^2)/\Lambda^2_{\rm QCD}]}\,, \tag{3}$$

where b is the first β function coefficient, and $m_{\rm g} \equiv m_{\rm g}(k^2=0) \approx 2\Lambda_{\rm QCD} \approx 500$–$600\,{\rm MeV}$ (the phenomenologically preferred infrared value of the gluon mass [14]). This charge's value at the infrared fixed point ($\alpha_{\rm s}(0) \equiv \bar{g}^2(0)/4\pi$) is of order 0.5. This number may be considered surprisingly small but there are several phenomenological calculations indicating that this value should not be larger than 1; see, for instance, a compilation of infrared values of the QCD coupling constant shown in [15]. Now, the gluon propagator in the fermionic SDE kernel, no longer behaves as $1/k^2$ but as $1/(k^2+m_{\rm g}^2)$ in the infrared, what diminishes is the strength of the interaction, and we also add to this fact the damping caused by the small value of the infrared coupling constant ($\bar{g}^2(0)$). The consequence is that we do not generate dynamical quark masses ($M(k^2)$) (or quark condensates) compatible with the experimental data in QCD for quarks in the fundamental representation [16]! QCD could only generate CSB if quarks were in higher dimensional representations, i.e. with higher values for the Casimir operator in order to compensate for the infrared damping discussed above [17].

TC theories will also present dynamical technigluon mass generation, and for the same reasons that we discussed in the QCD case, i.e. a small infrared TC coupling constant and the damping caused by the $1/m_{\rm tg}^2$ infrared value of the technigluon propagator, we do not expect that they will develop enough CSB to form the TC condensates. In this work we will follow the idea of [6, 7] that confinement is necessary and sufficient to promote CSB and develop the expected (techni)quark condensates. Actually, our next section will start discussing the evidence for a relation between CSB and confinement.

3. Chiral symmetry breaking as a consequence of confinement

The majority of studies about CSB in gauge theories, no matter if in QCD or TC, relied on the one-gauge boson exchange. If we deal with dynamically massive gauge bosons, as discussed in the previous section, CSB will not be achieved at least if we have fermions in the fundamental representation. We will then consider the case where confinement is necessary for CSB, and in order to emphasize this possibility we will review some QCD aspects that point in this direction. These arguments are going to be used in order to justify the fact that confinement is also necessary for TC CSB.

In [9] the following scenario was proposed for QCD: (a) gluons acquire a dynamical mass, (b) the theory with dynamically massive gluons generate vortices, and (c) these center vortices generate confinement. Lattice simulations are showing evidence for a relation between CSB and confinement, where center vortices play a fundamental role. In the $SU(2)$ case the artificial center vortices' removal also implies a recovery of the chiral symmetry [18–20]! We also present another lattice result indicating the importance of the deep infrared region for CSB in QCD [21]. In this simulation the quark condensate $\langle\bar{q}q\rangle$ is drastically reduced ($\approx$40%) by removing very low momentum gluons. This last result is consistent with the CSB mechanism obtained in the confinement model of [6], as shown in [8], where most of the CSB is due to gluons with a momentum smaller than a few hundred MeV. Finally, continuum arguments also claim that confinement is necessary and sufficient for CSB [22].

There is also another QCD confinement fact indicating that we need something else other than the one dynamically massive gauge boson exchange to explain the strong force. It has been known for a long time that the following static potential leads to a quite successful quarkonium phenomenology

$$V_{\mathrm{F}}(r) = K_{\mathrm{F}} r - \frac{4}{3}\frac{\alpha_{\mathrm{s}}}{r}, \tag{4}$$

where the first (confining) term is linear with the distance and proportional to the string tension K_{F}. The second term, that is of order α_{s}, the strong coupling constant describes the one gluon exchange contribution. The classical potential between static quark charges is related to the Fourier transform of the time–time component of the full gluon propagator in the following way:

$$V(\mathbf{r}) = -\frac{2C_2}{\pi}\int \mathrm{d}^3\mathbf{q}\alpha_{\mathrm{s}}(\mathbf{q}^2)\Delta_{00}(\mathbf{q})\exp^{\imath\mathbf{q}\cdot\mathbf{r}}, \tag{5}$$

where the bold terms, **q** and **r**, are three vectors. As noted in [23], *the linear confining term of the potential* ($K_{\mathrm{F}}r$) *cannot be obtained from the gluon propagator determined in the lattice or from the gluonic SDE*, i.e. we could roughly say that the dynamically massive gluon propagator also does not lead to quark confinement as it may not lead to CSB for fermions in the fundamental representation. The existence of a linear confining potential felt by quarks is supported by lattice simulations [24], and is a strong justification for a confining effective propagator. This linear confining part of the potential must also show a cutoff at some distance. For $n_{\mathrm{f}} = 2$ quarks in the fundamental representation, lattice QCD data seems to indicate that the string breaks at the following critical distance [25]:

$$r_{\mathrm{c}} \approx 1.25\,\mathrm{fm}, \tag{6}$$

which corresponds to a critical mass (or momentum), compatible with the m value necessary to generate the expected amount of CSB in the gap equation. This distance may change with the fermionic representation (because the string tension changes with the fermionic representation [24]), but there shall always be a critical value associated with the string breaking or to the force screening.

All the above facts were collected in order to show that a theory with dynamically massive gauge bosons, as expected for any asymptotically free non-Abelian gauge theory, may not have enough strength to generate CSB with fermions in the fundamental representation. Of course, for large fermionic representations, with a large value for the Casimir operator (C_2 in equation (2)) this may not be true [17]. Confinement and CSB seem to be intimately connected. The Fourier transform (equation (5)) of a dynamically massive gauge boson propagator does not lead to a confining potential ($\propto K_{\mathrm{F}}r$), although it can explain its short distance behaviour ($\propto \alpha_{\mathrm{s}}/r$) [23]. In some way confinement must also be limited to some scale as described by equation (6). Therefore, to model CSB in QCD or TC, as we intend to do in a Schwinger–Dyson equation approach, we must introduce confinement explicitly and also consider the one-gauge dynamically massive boson exchange. The propagators that we shall use in the fermionic Schwinger–Dyson equation, when plugged into equation (5), have to reproduce at some extent the behaviour of equation (4) and the confining contribution has to reflect the limit shown in equation (6). These ideas, introduced in [6] and applied phenomenologically in [8] in the QCD case, are going to be extended to TC theories in this work.

Cornwall introduced a confinement effect explicitly into the gap equation through the following *effective propagator, which is not at all related to the propagation of a standard quantum field* [6]:

$$D^{\mu\nu}_{\text{eff}}(k) \equiv \delta^{\mu\nu} D_{\text{eff}}(k), \quad D_{\text{eff}}(k) = \frac{8\pi K_{\text{F}}}{(k^2+m^2)^2}, \tag{7}$$

where K_{F} is the string tension. In the $m \to 0$ limit we would obtain the standard effective propagator $8\pi K_{\text{F}}\delta^{\mu\nu}/k^4$ that yields approximately an area law for the Wilson loop. This propagator has an Abelian gauge invariance that appears in the quark action obtained by integrating over quark world lines implying an area-law action [6]. We must necessarily have a finite $m \neq 0$ value due to entropic reasons as demonstrated in [6], and its value is related to the dynamical quark mass ($m \approx M(0)$), as required by gauge invariance, originating a negative term $-K_{\text{F}}/m$ in the static potential in order to generate the Goldstone bosons associated with CSB.

We can now turn to TC and write down what we may expect for the gap equation. As happens in the QCD case, the technifermion SDE can be modelled by the sum of a part containing the confining effective propagator plus another contribution with a massive one-techni-gluon exchange [6, 8], which, in the Abelian techni-gluon approximation, is given by

$$M(p^2) = \frac{1}{(2\pi)^4}\int \mathrm{d}^4k\, D^{\text{tc}}_{\text{eff}}(p-k)\frac{4M(k^2)}{k^2+M^2(k^2)} + \frac{C_2}{(2\pi)^4}\int \mathrm{d}^4k \frac{\bar{g}^2_{\text{tc}}(p-k)3M(k^2)}{[(p-k)^2+m^2_{\text{tg}}(p-k)][k^2+M^2(k^2)]}, \tag{8}$$

where $M(p^2) = M_{\text{c}}(p^2) + M_{1\text{tg}}(p^2)$ is the dynamical techni-quark mass generated by the effective confining and the dressed techni-gluon contributions. This last equation is the basic one that we shall explore in this work. Note that the effective propagator in the first integral of equation (8) leads to a confining potential ($\propto K_{\text{tc}}r$) and the massive techni-gluon exchange to the short distance contribution ($\propto \alpha_{\text{tc}}/r$) of the static TC potential. We have just replaced the QCD quantities (K_{F}, $\bar{g}^2_{\text{QCD}}$ and m_{g}) by the equivalent TC quantities (K_{tc}, $\bar{g}^2_{\text{tc}}$ and m_{tg}). In the following we also assume that the string tension in the confining propagator has also got to be changed according to the fermionic representation [8], but much of our discussion will be related to fermions in the fundamental representation.

If the TC theory contains fermions in the fundamental representation, it can be shown that just the first integral on the right hand side of equation (8), i.e. the gap equation without the massive technigluon exchange, is enough to generate the desirable amount of CSB (with appropriate values K_{tc} and $m \approx M(0)$). The asymptotic behaviour of the self-energy in this case is

$$M(p^2)|_{p^2\to\infty} \propto 1/p^4\,,$$

which is a very soft behaviour. The one-technigluon exchange enters only to modify the asymptotic behaviour of the gap equation, as happens in the QCD case [8].

The full gap equation can be transformed into a differential equation and it is possible to verify that the solution is a linear combination of two independent solutions of the form $f(x) = b_1 f_{\text{reg}}(x) + b_2 f_{\text{irreg}}(x)$, where b_1 and b_2 are determined by the boundary conditions. The asymptotic behaviour is dominated by the one-technigluon exchange contribution, whereas *the effects of the confining propagator enter only through the boundary conditions* [8]. In [8] we verified that the irregular solution dominates when a cutoff $\Lambda \approx m$ is introduced. In a $SU(N)$ TC theory this ultraviolet behaviour would be equal to [8]

$$M(p^2)|_{p^2\to\infty} \propto M(\ln p^2/M^2)^{-d} , \tag{9}$$

where $d = 9C_2/(11N - 2n_f)$ for n_f flavours. This solution minimizes the vacuum energy and has a vacuum expectation value proportional to $1/g^2$ [26]. All the above comments are included to recall just how the boundary conditions may affect the behaviour of the self-energy. We shall not consider $\Lambda \approx m$ in the sequence, but we will argue that the integrals in equation (8) should be performed in different momentum regions.

We now suppose that the confining propagator is limited to a specific momentum interval. The confining propagator that we are discussing here is not the one of a fundamental field, therefore we argue that it must specify a certain region where confinement should exist. If the string breaking happens at a certain critical distance (r_c), and if the phenomenological classical potential between static quark charges is given by the Fourier transform of the time–time component of this confining propagator, the confining propagator will not reflect this breaking unless we cut the momentum up to a maximum value where the confinement region exists, or we can understand the momentum flowing in the confining propagator as the energy that may flow between confined quarks. If this hypothesis is correct, it is natural to have the following four-fermion approximation [8]:

$$M(p^2) \approx M_{4f}(p^2) = \frac{2}{\pi^3}\frac{K_R}{m_{tc}^4}\int d^4k\, \frac{M_{4f}(k^2)\theta(m_{tc}^2 - k^2)}{k^2 + M_{4f}^2(k^2)} + \frac{C_2}{(2\pi)^4} \times \int^{\Lambda} d^4k \frac{\bar{g}_{tc}^2(p-k)3M_{4f}(k^2)}{[(p-k)^2 + m_{tg}^2(p-k)][k^2 + M_{4f}^2(k^2)]}. \tag{10}$$

In [8] we verified that *the critical behaviour of equation* (10) *and the one of equation* (8) *are basically the same in what concerns the critical values of the 'constants'* $K_{F,tc}$ *and* m, with the massive one-gauge boson exchange barely affecting the symmetry breaking. The value of the chiral parameters, like the dynamical fermion mass and condensates, are not so much different, implying that the approximation is quite reasonable. This is a consequence of the very strong confining force and the fact that most of the symmetry breaking is dominated by the physics at very low momenta.

The solution of equation (10) has a slow decrease with the momentum [8] and is typical of the gauged Nambu–Jona–Lasinio (NJL) type of model [27]. The dressed one-gluon exchange has not enough strength to generate such a type of four-fermion interaction [8], which occurs only due to the large ratio between the string tension and the factor m in the confining potential. Actually, we have simple reasoning to explain why the self-energy solution is the one corresponding to what is called irregular behaviour (or an NJL type of solution). Equation (8) is a particular case of the following equation:

$$M(p^2) \approx \beta \int^{m^2} d^2k k^2\, G(p,k)\frac{M(k^2)}{k^2 + M^2(k^2)} + \alpha \int^{\Lambda^2} d^4k\, \frac{\bar{g}^2(p-k)M(k^2)}{[(p-k)^2][k^2 + M^2(k^2)]}, \tag{11}$$

where Λ is an ultraviolet cutoff, $G(p,k)$ is an integrable function in the interval $[0, m^2]$, where the interval is understood for p and k, and we have arbitrarily chosen m as the momentum limit to where confinement is propagated. $M(k^2)$ is a well behaved function in the infrared with $M(0) \approx m$. We can verify that the ultraviolet boundary condition behaviour ($p^2 \to \Lambda^2$) of equation (11) is given by

$$M(\Lambda^2) \propto \beta \int^{m^2} \mathrm{d}^2 k k^2 \frac{M(k^2)}{k^2 + M^2(k^2)}, \tag{12}$$

which is a constant and not different from a bare mass in the gap equation, leading to a hard behaviour for the dynamical mass. Another argument in favour of limiting the confining propagator to a certain momentum region can also be abstracted from [28] and references therein, although we do not need necessarily to interpret the quark condensate that is generated in our case as an 'inside hadron' condensate.

It is known that the introduction of a four-fermion interaction is responsible for harder self-energy solutions in non-Abelian gauge theories [27]. In our case this four-fermion interaction is natural because of the enormous strength of the effective confining propagator. We can provide more arguments for the four-fermion approximation, which also support the introduction of an effective confining propagator: four-fermion interactions have been known, for a long time, to describe the low energy strong interaction behaviour, and it would be quite difficult to imagine that only a massive (techni)gluon propagator could lead to an effective four-fermion interaction, because the actual interaction strength for the perturbative gap equation is measured by the product 'coupling$\otimes$propagator', and we know from equation (3) that the one-(techni)gluon exchange has not enough strength to generate such effective coupling. On the other hand the confining effective propagator, with the usual values for the string tension, is strong enough to generate the effective gap equation (10)! Apart from the (techni)gluon mass effect appearing in the one-(techni)gluon contribution, equation (10) has been extensively studied in [27], and it does lead to a self-energy solution that decreases slowly with the momentum, although the origin and the *cutoffs* are totally different. *This can also be verified comparing the four-fermion coupling constant* (λ) *of* [27] *with our 'effective coupling'* K_R/m_{tc}^2, related to the representation R of the TC group. The fermion condensate in a given representation R obtained from equation (10), as shown in [8], has the same form found by Takeuchi (equation (6) of [27])

$$\langle \bar{q} q \rangle_R (m_{\mathrm{tc}}^2) \approx -\frac{N_R}{8\pi} \frac{m_{\mathrm{tc}}^4}{K_R} M_R(m_{\mathrm{tc}}^2), \tag{13}$$

corresponding to a broken-symmetry phase characterized by $K_R/m_{\mathrm{tc}}^2 > 1$ (or $\lambda > 1$ in figure 1 of the first paper in [27]), leading naturally to large anomalous dimensions produced by the confining propagator and very hard dynamics for the self-energy. Therefore, there is no reason to expect different behaviour in a TC model, or any asymptotically free non-Abelian theory, as long as the theory is in the confining phase [8].

Summarizing our discussion we can say that the explicit introduction of confinement into the gap equation [6] gives a possible solution for the problem of CSB when the gauge bosons have a dynamically generated mass. It is necessary to generate the linear potential as well as to promote the symmetry breaking associated with the deep infrared region, which are facts observed in lattice simulations [18–21]. The introduction of the scale $m \approx M(0)$ into the confining propagator is necessary for entropic reasons, otherwise it would be extremely difficult to generate the Goldstone bosons associated with CSB [6]. Within the approximations discussed here, these results are valid for any non-Abelian gauge theory in the confining phase.

4. Technicolour models with dynamically massive technigluons and confinement effects

It is possible to outline a class of TC models that can be built based on the irregular solution for the self-energy, with *the main advantage is that QCD and TC have the same type of self-energy and participate equally in the mass generation mechanism for the known fermions* [29]. Considering that QCD and TC have the so called 'irregular' self-energy [8, 27], which will be parameterized as [29, 30]

$$\Sigma(p^2) \sim \mu[1 + bg^2 \ln(p^2/\mu^2)]^{-\gamma}, \tag{14}$$

where μ is the characteristic scale of mass generation (QCD or TC), $\gamma = 3c/16\pi^2 b$ and $c = \frac{1}{2}[C_2(R_1) + C_2(R_2) - C_2(R_3)]$. $C_2(R_i)$ are the Casimir operators for fermions in the representations R_1 and R_2 that condense in the representation R_3, when we compute the ordinary fermion mass (m_f) we obtain [29]

$$m_\mathrm{f} \approx g^2_{\mathrm{etc}}\mu_{\mathrm{TC(QCD)}} \left[1 + b_{\mathrm{TC(QCD)}} g^2_{\mathrm{TC(QCD)}} \ln\left(M^2_{\mathrm{etc}}/\mu^2_{\mathrm{TC(QCD)}}\right)\right]^{-\gamma}. \tag{15}$$

In the above equation $\mu_{\mathrm{TC(QCD)}}$ is the characteristic TC(QCD) CSB scale, g^2_{etc} is the ETC coupling constant, $b_{\mathrm{TC(QCD)}}$ is the first β function coefficient, $g^2_{\mathrm{TC(QCD)}}$ is the TC(QCD) coupling constant, M_{etc} the ETC boson mass, and we also neglected some constants. Three points are very important: (a) the fermion masses depend quite weakly on the ETC boson mass, which may have very large values not leading to FCNC problems, (b) small fermion masses are generated when the CSB is due to QCD (this is quite different from usual models where it is assumed that QCD has a very soft solution for the self-energy), (c) the largest mass that we can generate, if μ_{TC} is of the order of a few hundred GeV, is roughly of the order of $g^2_{\mathrm{etc}}\mu_{\mathrm{TC}}$ and not too much different from the top quark mass [30].

According to the previous paragraph we can say that we may generate two different mass values for the ordinary fermions:

$$m_\mathrm{f}^{\mathrm{light}} \approx g^2_{\mathrm{etc}}\mu_{\mathrm{QCD}}, \qquad m_\mathrm{f}^{\mathrm{heavy}} \approx g^2_{\mathrm{etc}}\mu_{\mathrm{TC}}, \tag{16}$$

where we neglected the (small) contribution of the term between brackets in equation (15). If we compute the condensates we also can verify that we have QCD and TC condensates with scales separated by an $\mathcal{O}(10^3)$. The light masses are of the order of first generation fermion masses, while the heavier are of the order of third generation masses [29]. But how is it possible to prevent light fermions acquiring heavy masses? This can be solved with the help of a family, or horizontal, symmetry.

In the sequence we sketch a scheme quite similar to the one proposed by Berezhiani and Gelmini *et al* in [31] where their vacuum expectation values (vevs) of fundamental scalars are substituted by QCD and TC condensates [29]. Let us suppose that we have a horizontal symmetry based on the $SU(3)_\mathrm{H}$ group and the TC theory has technifermions in the fundamental representation of $SU(4)_{\mathrm{TC}}$. The technifermions form a quartet under $SU(4)_{\mathrm{TC}}$ and the quarks are triplets of QCD. The TC and colour condensates will be formed at the scales μ_{TC} and μ_{QCD} in the most attractive channel (mac) of the products $\bar{\mathbf{4}} \otimes \mathbf{4}$ and $\bar{\mathbf{3}} \otimes \mathbf{3}$ of each strongly interacting theory. We assign horizontal quantum numbers to technifermions and quarks such that these same products can be decomposed in the following representations of $SU(3)_\mathrm{H}$: $\bar{\mathbf{6}}$ in the case of the TC condensate, and $\mathbf{3}$ in the case of the QCD condensate. For this it is enough that the standard left-handed (right-handed) fermions transform as triplets (antitriplets) under $SU(3)_\mathrm{H}$, assuming that the TC and QCD condensates are formed in the $\bar{\mathbf{6}}$ and in the $\mathbf{3}$ of the $SU(3)_\mathrm{H}$ group. This is consistent with the mac hypothesis, although a complete analysis of this problem is out of the scope of this work. The above choice for the condensation channels is crucial for our model, because the TC condensate in the representation $\bar{\mathbf{6}}$ (of $SU(3)_{\rm H}$) will interact only with the third fermionic generation while the $\mathbf{3}$ (the QCD condensate) will interact only with the first generation. In this way we can generate coefficients C and A respectively of a Fritzsch type matrix [32], because when we add these condensates (vevs) and write them as a 3×3 matrix we will end up (*at leading order*) with

$$M_{\rm f} = \begin{pmatrix} 0 & A & 0 \\ A^* & 0 & 0 \\ 0 & 0 & C \end{pmatrix}. \tag{17}$$

The points that must still be discussed are how we generate the intermediate masses and why the contribution of the term between the brackets in equation (15) is indeed small and can be neglected.

In the scenario that we shall consider, the ETC group can connect all fermions and contain the TC and QCD interactions. Actually the ETC role can be played by a grand unified theory (GUT), which has exactly these characteristics. This is possible because the fermion mass barely depends on the ETC or GUT gauge boson masses, as can be verified from equation (15). These ETC or GUT bosons can intermediate neutral flavour changing interactions, however, they can be very heavy in order to be consistent with all experimental constraints on FCNC interactions [29]. We can build a TC model based on a GUT such that

$$G_{\rm gut} \supset G_{\rm SM} \otimes SU(N)_{\rm TC} \otimes G_{\rm H}\,, \tag{18}$$

where $G_{\rm SM}$ is the Standard Model group, $SU(N)_{\rm TC}$ is the TC group and $G_{\rm H}$ corresponds to a horizontal symmetry, which is not necessarily a local one, but with a characteristic scale of the order of the GUT scale, and, for simplicity, couplings are assumed to be of the same order (i.e. $g^2_{\rm H} \approx g^2_{\rm gut}$). TC should condensate at the TeV scale. All groups are embedded into the GUT, therefore we may have all kinds of neutral flavour changing interactions but at the GUT scale, since this theory will play the role of the ETC theory [1].

In the example that we discussed before, where $G_{\rm H} \equiv SU(3)_{\rm H}$ with technifermions condensing in the $\bar{\mathbf{6}}$ and quarks condensing in the $\mathbf{3}$ representations of the horizontal group, we can obtain the following mass matrix [29]:

$$M_{\rm f} = \begin{pmatrix} 0 & A & 0 \\ A^* & 0 & B \\ 0 & B^* & C \end{pmatrix}, \tag{19}$$

where $A \propto g^2_{\rm gut}\mu_{\rm QCD} \approx \mathcal{O}(\text{MeV})$ and $C \propto g^2_{\rm gut}\mu_{\rm TC} \approx \mathcal{O}(\text{GeV})$. The B term has an intermediate value naturally generated by the effective potential of the *composite* $\bar{\mathbf{6}}$ and $\mathbf{3}$ Higgs system, as shown in [29]. Note that we can only obtain a mass as heavy as the top quark one in TC models with the use of equation (14) [30].

To show that a mass matrix like the one of equation (19) is a feasible one, we can use the technique of effective potential for composite operators as discussed in [33], and verify that QCD and TC lead to a two composite Higgs boson system, indicated respectively by η and ϕ, with, due to the horizontal symmetry, the following vevs:

$$\langle\eta\rangle \approx \begin{pmatrix} 0 \\ 0 \\ v_\eta \end{pmatrix}, \quad \langle\phi\rangle \approx \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & v_\phi \end{pmatrix}, \tag{20}$$

where the first vev will be of the order of 250 MeV and the second one of order 250 GeV. The intermediate term in equation (19) will be originated by mixed terms in the effective potential of our composite system [29]. These terms will come out naturally from one-loop standard model interactions connecting η and ϕ (or quarks and techniquarks scalar composites), being of the following type:

$$V_2(\eta, \phi) = \epsilon \eta^\dagger \eta \phi^\dagger \phi + \delta \eta^\dagger \phi \eta \phi^\dagger + \cdots . \tag{21}$$

The details of how this effective potential contribution originates in such types of models were worked out in [29].

Let us summarize why we consider this is quite a general type of model. Firstly, due to the fact that confinement is responsible for a dynamical mass typical of a NJL gauged model, or of the irregular type, we end up with two scales of ordinary fermion masses: QCD and TC. Secondly, due to the form of the self-energy, the fermion masses barely depend on the ETC gauge boson masses, and these can be quite heavy and do not generate FCNC problems. Finally, to generate a reasonable fermion mass matrix we only need a horizontal or family symmetry. There are possibly many theories based on different groups that may fit into this scheme, and we do not need to appeal to technifermions that belong to higher TC representations.

5. Conclusions

We initiated our work by calling attention to the fact that in asymptotically free non-Abelian gauge theories the gauge bosons may acquire naturally dynamical masses. This fact has already been checked for QCD through lattice simulations and SDE. We expect that the same phenomenon occurs in TC theories if the theory does not have too many fermions in order to spoil the gauge boson mass generation mechanism. CSB in the TC model with dynamically generated technigluon masses was discussed here, as far as we know, for the first time. We argue, based on the QCD results, that with dynamically massive technigluons it may be quite difficult to promote CSB in TC theories, particularly if technifermions are in the fundamental representation of the TC group.

Based on lattice results and continuum arguments, we followed Cornwall's idea that confinement is necessary and sufficient for CSB [6, 7]. Confinement has to be introduced explicitly into the fermionic Schwinger–Dyson equation. This is performed with the introduction of an effective confining propagator, in a way quite similar to the proposition of the phenomenological static potential of equation (4), which is quite successful in describing the quarkonium spectra. The new gap equation, in the QCD case, was extensively discussed in [6–8], and we just rewrite it in the TC case.

The TC gap equation containing a confining propagator and dynamically massive technigluons is discussed following the steps already pointed out in [8]. The main point is the introduction of an infrared cutoff in the confining part of the gap equation. The result is that the confining propagator is responsible for generating a contribution typical of a bare mass in the fermionic Schwinger–Dyson equation, which leads to a very 'hard' self-energy, or a self-energy of the irregular type. One important fact is that the full gap equation is well approximated by a four-fermion interaction, and its critical behaviour is not different from the one of the full equation, as shown in [8]. Moreover, the numerical values for the chiral parameters obtained with the four-fermion approximation do not differ from the ones of the full equation [8]. In TC we should also expect the same four-fermion interaction as happens in QCD; they appear here in the same way that they appear in walking TC theories [27], although their origin in our case is totally different and based on the confinement effect.

As a consequence of confinement and dynamical gauge boson mass generation, leading to a very particular fermionic self-energy, we see that the CSB of both theories, QCD and TC, participate in the generation of ordinary fermion masses. These fermion masses barely depend on the ETC mass scale. This allow us to build a quite general type of TC model, where the ETC interaction can be naturally substituted by a GUT interaction, at the cost of introducing a horizontal or family symmetry to prevent light fermions acquiring masses directly from the TC condensates. The new gauge boson interactions (horizontal or GUT) appear at a very high energy scale and we do not expect FCNC at undesirable levels in this type of model.

The fact that most of the first fermionic family masses originated from the QCD CSB, and the third fermionic family masses comes from the TC chiral breaking, is a novelty. This is a direct consequence of the possibility that confinement, in non-Abelian gauge theories with fermions in the fundamental representation, induces an effective four-fermion interaction simulating a bare mass, where the self-energy decreases very slowly with the momentum and may be a solution for the phenomenological TC problems.

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