

Fermion helicity flip in weak gravitational fields

R. Aldrovandi, G. E. A. Matsas, S. F. Novaes, and D. Spehler*

*Instituto de Física Teórica, Universidade Estadual Paulista,
Rua Pamplona 145, CEP 01405-900 São Paulo, Brazil*

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The helicity flip of a spin- $\frac{1}{2}$ Dirac particle interacting gravitationally with a scalar field is analyzed in the context of linearized quantum gravity. It is shown that massive fermions may have their helicity flipped by gravity, in opposition to massless fermions which preserve their helicity.

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The behavior of a spinning particle in a gravitational field is a long standing problem [1,2], which has recently shown a revival of interest. In particular, whether or not gravity flips the helicity of a spin- $\frac{1}{2}$ Dirac particle has been examined lately in the semiclassical context [3,4]. Although a final answer will have to wait for a complete theory of quantum gravity, we believe the linearized quantum approach can shed new light on the subject. Our aim in this paper is to examine the problem of fermion helicity flip in this context. Namely, we calculate at the tree level the helicity flip probability for a spin- $\frac{1}{2}$ Dirac particle interacting via a graviton exchange with a massive scalar particle in Minkowski space. We conclude that only massive fermions can have their helicity flipped. The main virtue of our treatment is the fact that all concepts employed are simple, and widely used in field theory.

The coupling of matter fields with gravity in the context of the linearized theory is obtained using weak field

approximation. In this vein, the spacetime metric may be written as $g_{\mu\nu} = \eta_{\mu\nu} + \kappa h_{\mu\nu}$, where $\eta_{\mu\nu}$ is the Minkowski metric, $\kappa \equiv \sqrt{32\pi G} = 8.211 \times 10^{-19} \text{ GeV}^{-1}$ plays the role of a small coupling constant, and the perturbation $h_{\mu\nu}$ represents the graviton field which is supposed to be quantized in Minkowski space using Cartesian coordinates. Hence, it is the metric tensor $\eta_{\mu\nu}$ which will provide the canonical isomorphism between the vector space and the corresponding dual space.

Next, by *minimally* coupling gravity to the scalar field ϕ [5], and to the fermionic field ψ [6], we obtain the following interaction actions to first order in κ :

$$S_{\phi\phi h} = \int d^4x \frac{\kappa}{4} [h (\partial^\alpha \phi \partial_\alpha \phi - M^2 \phi^2) - 2h^{\mu\nu} \partial_\mu \phi \partial_\nu \phi] \quad (1)$$

and

$$S_{\psi\psi h} = \int d^4x \frac{\kappa}{2} \left\{ \left[-\frac{i}{4} \bar{\psi} (\gamma^\nu \partial_\mu + \gamma_\mu \partial^\nu) \psi + \frac{i}{4} (\partial_\mu \bar{\psi} \gamma^\nu + \partial^\nu \bar{\psi} \gamma_\mu) \psi \right] h_\nu{}^\mu + \left(\frac{i}{2} \bar{\psi} \gamma^\nu \partial_\nu \psi - \frac{i}{2} \partial_\nu \bar{\psi} \gamma^\nu \psi - m \bar{\psi} \psi \right) h \right\}, \quad (2)$$

where M and m are the scalar and fermion masses, respectively, and $h \equiv h_\mu{}^\mu$. The vertices associated with the couplings $\phi(k_1) - \phi(k_2) - h^{\mu\nu}$ and $\bar{\psi}(k_1) - \psi(k_2) - h^{\mu\nu}$ derived from the actions (1) and (2) are

$$V_{\phi\phi h}^{\mu\nu}(k_1, k_2) = i \frac{\kappa}{2} [k_1^\mu k_2^\nu + k_1^\nu k_2^\mu - \eta^{\mu\nu} (k_1 \cdot k_2 + m^2)] \quad (3)$$

and

$$V_{\psi\psi h}^{\mu\nu}(k_1, k_2) = i \frac{\kappa}{4} \left\{ \frac{1}{2} [\gamma^\mu (k_1^\nu + k_2^\nu) + \gamma^\nu (k_1^\mu + k_2^\mu)] + \eta^{\mu\nu} [(k_1 + k_2) - 2m] \right\}, \quad (4)$$

respectively. In Eq. (3) all the momenta are considered incoming, whereas in Eq. (4) they follow the fermionic arrow.

*Also at Université Louis Pasteur, Institut Universitaire de Technologie 3 rue St. Paul, 67300 Strasbourg, France.

We are now able to evaluate the helicity flip rate of a fermion (F) interacting with a scalar (S) via a graviton exchange, i.e., $F(p) + S(q) \rightarrow F(k) + S(l)$ [see Fig. (1)]. In particular, the squared invariant amplitude for an initial

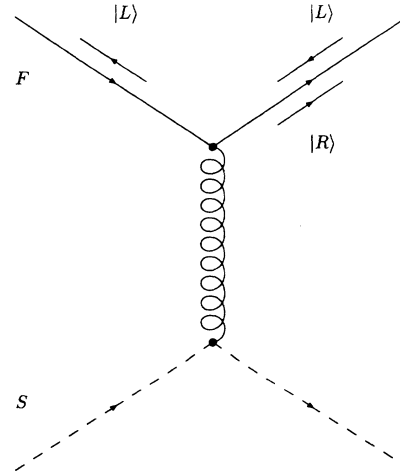


FIG. 1. Feynman diagrams for the process $F(p) + S(q) \rightarrow F(k) + S(l)$.

left-handed fermion $|L\rangle$ suffering a transition to a right-handed one $|R\rangle$ due to a graviton exchange with a scalar field is

$$N_{|L\rangle\rightarrow|R\rangle} = |\bar{u}(k, \mathcal{S}_R) V_{\psi\psi h}^{\mu\nu}(p, k) u(p, \mathcal{S}_L) \times D_{\mu\nu, \alpha\beta}(p-k) V_{\phi\phi h}^{\alpha\beta}(q, -l)|^2, \quad (5)$$

where $D_{\mu\nu, \alpha\beta}$ is the usual graviton propagator

$$D_{\mu\nu, \alpha\beta}(p-k) = \frac{i}{2(p-k)^2} (\eta_{\mu\alpha}\eta_{\nu\beta} + \eta_{\mu\beta}\eta_{\nu\alpha} - \eta_{\mu\nu}\eta_{\alpha\beta}). \quad (6)$$

The fermion spinors satisfy

$$u(p, \mathcal{S}_L) = \frac{(1 + \gamma_5 \gamma_\mu \mathcal{S}_L^\mu)}{2} u(p, \mathcal{S}_L)$$

and

$$u(k, \mathcal{S}_R) = \frac{(1 + \gamma_5 \gamma_\mu \mathcal{S}_R^\mu)}{2} u(k, \mathcal{S}_R)$$

as usual, where we have introduced the polarization four-vectors

$$\begin{aligned} \mathcal{S}_L^\mu &= -\frac{p^\mu}{m\beta_i} + \frac{\sqrt{1-\beta_i^2}}{\beta_i} \eta^{\mu 0}, \\ \mathcal{S}_R^\mu &= \frac{k^\mu}{m\beta_f} - \frac{\sqrt{1-\beta_f^2}}{\beta_f} \eta^{\mu 0}, \end{aligned} \quad (7)$$

with $\beta_{i(f)}$ being the initial (final) fermion velocities. Thus, letting (3), (4), and (6) in (5) we obtain

$$\begin{aligned} N_{|L\rangle\rightarrow|R\rangle} &= \frac{\kappa^4}{512t^2} \left\{ [1 - (\mathcal{S}_L \cdot \mathcal{S}_R)] \{ (s-u)^2 [(s-u)^2 - t(t-4M^2)] - 16m^2M^2 [M^2(t-4m^2) + (s-u)^2] \} \right. \\ &\quad - 4[(A \cdot \mathcal{S}_R)(k \cdot \mathcal{S}_L) + (A \cdot \mathcal{S}_L)(p \cdot \mathcal{S}_R)] [(s-u)^2 + 2t(t+3M^2) - 8m^2M^2] \\ &\quad + 2(p \cdot \mathcal{S}_R)(k \cdot \mathcal{S}_L) [4(t+3M^2)^2(t-4m^2) + (s-u)^2(5t+8M^2) + 16m^2(t+2M^2)^2] \\ &\quad \left. + 8(A \cdot \mathcal{S}_L)(A \cdot \mathcal{S}_R)t \right\}, \end{aligned} \quad (8)$$

where $s = (p+q)^2$, $t = (p-k)^2$, and $u = (p-l)^2$ are the Mandelstam variables, and we have introduced the four-vector

$$A^\mu \equiv (t+3M^2)(k^\mu + p^\mu) - \frac{1}{2}(s-u)(q^\mu + l^\mu). \quad (9)$$

In a similar way we can evaluate the squared invariant amplitude for an initially left-handed fermion to remain with the same helicity after the interaction.

The polarization of the scattered fermion can be measured by

$$P = 1 - \frac{2N_{|L\rangle\rightarrow|R\rangle}}{N_{|L\rangle\rightarrow|L\rangle} + N_{|L\rangle\rightarrow|R\rangle}}, \quad (10)$$

where

$$N_{|L\rangle\rightarrow|L\rangle} + N_{|L\rangle\rightarrow|R\rangle} = \frac{\kappa^4}{256t^2} \{ (s-u)^2 [(s-u)^2 - t(t-4M^2)] - 16m^2M^2 [M^2(t-4m^2) + (s-u)^2] \} \quad (11)$$

depends only on the Mandelstam variables and the relevant masses. Notice that $P = 1$ indicates no depolarization (flip) of the initial fermions, whereas $P = -1$ represents that all the left-handed initial fermions were flipped.

In order to evaluate P , we can choose, for instance, the laboratory frame where the scalar particle is initially at rest $[q^\mu = (M, \vec{0})]$. In this frame, $s_{\text{lab}} = m^2 + M^2 + 2ME$, and $u_{\text{lab}} = 2(m^2 + M^2) - s_{\text{lab}} - t_{\text{lab}}$ with

$$t_{\text{lab}} = -2M(E^2 - m^2) \left[\frac{M + E \sin^2 \theta - \sqrt{M^2 - m^2 \sin^2 \theta} \cos \theta}{(E+M)^2 - (E^2 - m^2) \cos^2 \theta} \right].$$

Here E is the initial fermion energy, and θ the fermion scattering angle in this frame. The final expression for P is rather lengthy, which is omitted here. In Fig. 2 we show the behavior of P as a function of the fermion mass. We have fixed the initial fermion energy and the scalar particle mass, and shown the curves for different values of the scattering angles. Notice that the larger

is the scattering angle, the larger is the helicity flip. It is clear from this figure that $P \rightarrow 1$ when $m \rightarrow 0$; i.e., there is no helicity flip for massless fermions. In Fig. 3 we present P as function of the scalar particle mass at a fixed scattering angle ($\theta = 30^\circ$), and for various values of the fermion mass.

A simple expression for P can be obtained by taking

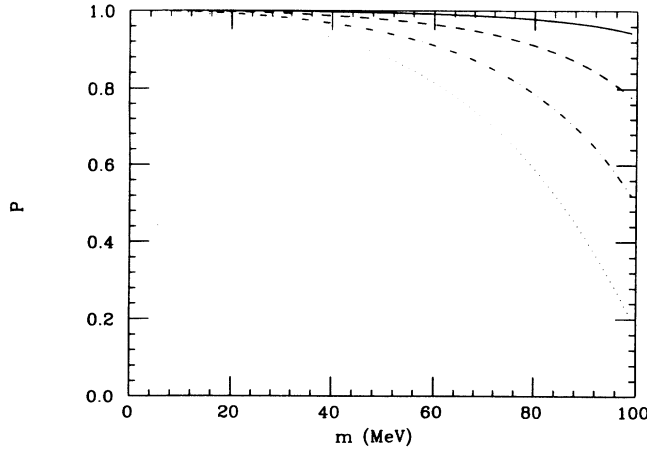


FIG. 2. P as a function of the fermion mass (m) for $E = 100$ MeV, $M = 1000$ MeV, and $\theta = 20^\circ$ (solid line), 40° (dashed line), 60° (dot-dashed line), and 80° (dotted line).

the limit of very large scalar masses, (i.e., $M \gg E > m$), which corresponds to small squared transferred momentum approximation ($t \sim 0$). In this regime, we have $S_R^\mu = -S_L^\mu \rightarrow p^\mu/m$, which leads to

$$P \simeq 1 - \left(\frac{m^2}{E^2} \right), \quad (12)$$

and makes clear that the helicity flip probability is proportional to the square of the fermion mass. In this approximation, taking for instance a fermion with mass $\simeq 10$ eV, which is about the upper bound for the electron-neutrino mass, with an energy in the range $0.1 - 10$ MeV, we obtain a helicity flip probability of $\sim 10^{-8}$ to 10^{-12} .

Summarizing, we have shown that, in the context of linearized quantum gravity, massive spin- $\frac{1}{2}$ Dirac particles have their helicity flipped due to gravitational interaction. Notwithstanding, no helicity flip is expected for

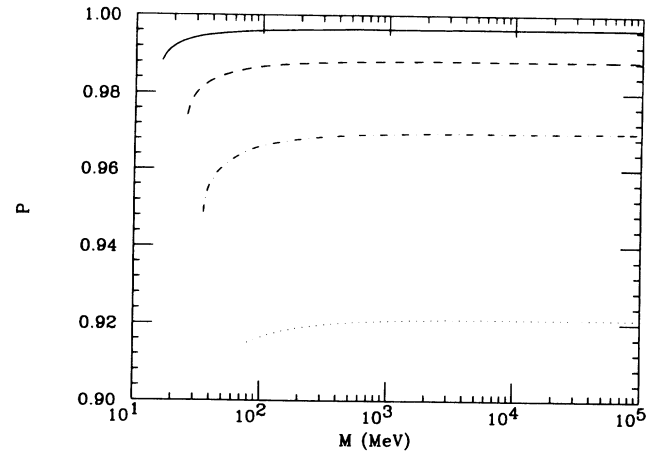


FIG. 3. P as a function of the scalar mass (M) for $E = 100$ MeV, $\theta = 30^\circ$, and $m = 30$ MeV (solid line), 50 MeV (dashed line), 70 MeV (dot-dashed line), and 90 MeV (dotted line).

massless fermions. Our approach differs from the previous semiclassical ones [3,4] in various aspects. The gravitational interaction is described here by a massless spin-2 quantum field $h_{\mu\nu}$, and not by the spacetime curvature as in the semiclassical approach. Once we have assumed the reasonable actions (1) and (2) obtained by minimally coupling the matter fields to gravity, our results follow straightforwardly. The fermion helicity flip appears as a dynamical effect, coming from the local coupling of spin to gravity.

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