Asymptotic Scenarios for the Proton's Central Opacity: An Empirical Study

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3 authors, including:

Daniel Almeida Fagundes
University of Campinas

Paulo Victor Recchia Gomes Silva
Universidade Federal de Pelotas

Asymptotic Scenarios for the Proton's Central Opacity: An Empirical Study

D.A. Fagundes[1], M.J. Menon[2], and P.V.R.G. Silva[2]

[1]*Instituto de Física Teórica, Universidade Estadual Paulista , 01140-070 - São Paulo, SP, Brazil*
[2]*Instituto de Física Gleb Wataghin, Universidade Estadual de Campinas , 13083-859 Campinas, SP, Brazil*

Abstract

We present a model-independent analysis of the experimental data on the ratio X between the elastic and total cross-sections from pp and $\bar{p}p$ scattering in the c.m. energy interval 5 GeV - 8 TeV. Using a novel empirical parametrization for that ratio as a function of the energy and based on theoretical and empirical arguments, we investigate three distinct asymptotic scenarios: either the black-disk (BD) limit or scenarios above and below that limit. Our analysis favors a scenario below the BD, with asymptotic ratio $X = 0.36 \pm 0.08$.

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1 Introduction

The dependence of the ratio between the elastic and total hadronic cross-sections as a function of the c.m. energy,

$$X(s) = \frac{\sigma_{el}}{\sigma_{tot}}(s), \tag{1}$$

constitutes an important quantity in the investigation of elastic and soft diffractive processes. Besides giving information on the hadron's central opacity (profile function at $b = 0$) and on the ratio of the inelastic to total cross-sections, it is also connected with the ratio between the total cross section and the elastic slope parameter through the approximated relation $X = \sigma_{tot}/16\pi B_{el}$.

Presently, in the lack of a theoretical framework able to describe the elastic scattering states from the first principles of QCD, one possible way to look for new phenomenological insights and/or inputs is the empirical approach. In this context, Fagundes and Menon have recently developed a model-independent analysis of the experimental data on the ratio X from pp scattering in the energy interval 10 GeV - 7 TeV [1]. The empirical parametrization is given by $X(s) = Af(s)$, with $f(s) = \tanh\{a + b\ln(s/s_0) + c\ln^2(s/s_0)\}$, where $s_0 = 1$ GeV^2, a, b, c are dimensionless free fit parameters and A the asymptotic limit. In order to estimate the uncertainties in extrapolations to higher energies, two asymptotic limits have been considered: either $A = 1/2$ (black-disk limit) or $A = 1$ (maximum unitarity). Beyond consistent data reductions of the experimental information on $X(s)$, the approximate relation has allowed extrapolations of the uncertainty regions in the ratio σ_{tot}/B_{el} that may be useful in the determination of the proton-proton total cross-section from proton-air production cross-section in cosmic-ray experiments [1].

In this communication, this empirical analysis of the X data is updated and developed in several aspects. The experimental data from $\bar{p}p$ scattering, all the pp TOTEM data at 7 TeV (four points) and 8 TeV (one point) are included in the dataset and the energy cutoff is down to 5 GeV. The description of the change of curvature in $X(s)$ demands a novel empirical ansatz for $f(s)$ and as explained in what follows, we investigate all the three possible asymptotic scenarios: either the black-disk limit or scenarios above or below that limit. Our main conclusions are: a) the black-disk does not represent a definitive solution; b) the data reductions, using the novel parametrization, favor a scenario below the black-disk, with asymptotic ratio $A = 0.36 \pm 0.08$.

After discussing the arguments for investigating the three scenarios, we introduce the new parametrization, discuss the fit procedures and results and then present a summary and our conclusions.

2 Asymptotic Scenarios

The *Black-Disk* limit represents a standard phenomenological expectation, typical, for example, of eikonal models. We have the arguments that follows for investigating scenarios either below or above that limit.

Below the Black Disk. We have recently developed an amplitude analysis on the quantities σ_{tot}, ρ parameter and σ_{el}, including the TOTEM Collaboration results at 7 and 8 TeV [2–4]. For our purposes, we recall that the parametrization for the total cross section is expressed by $\sigma_{tot}(s)$ = Regge terms $+ \alpha + \beta \ln^{\gamma}(s/s_h)$ and fits to σ_{tot} and ρ data from pp and $\bar{p}p$ scattering above 5 GeV, led to statistically consistent solutions either for $\gamma = 2$ (fixed) or $\gamma > 2$ (free fit parameter). In both cases, extension of the parametrization to σ_{el} data (same γ value) allowed to extract the ratio $X(s)$ and its asymptotic value A. In all cases investigated, we have obtained $A < 1/2$ within the uncertainties and lowest central value around 0.3 (see a summary of the results in [4], Figure 10). Moreover, we recall that in the publications by the TOTEM Collaboration, the authors quote the COMPETE Collaboration prediction for $\sigma_{tot}(s)$ [5], presenting also their own fit to the $\sigma_{el}(s)$ data [6]. As shown in [4], from these two results and using the central values of the parameters, one obtains $X(s) \to A = 0.436$ as $s \to \infty$, suggesting, therefore a scenario below the black disk (see also this point in [4], Figure 10).

Above the Black Disk. As discussed in [1], besides the obvious maximum bound allowed by Unitarity, namely $A = 1$, the U-matrix unitarization scheme by Troshin and Tyurin predicts an asymptotic limit beyond the black disk, $1/2 < A \leq 1$ [7]. Here we also recall that in a formal context, two well known bounds for the total and inelastic cross-sections read [8]:

$$\sigma_{tot}(s) < \frac{\pi}{m_\pi^2} \ln^2(s/s_0), \qquad \sigma_{inel}(s) < \frac{\pi}{4m_\pi^2} \ln^2(s/s_0).$$

Therefore, in case of simultaneous saturation of both bounds as $s \to \infty$, it is possible that $\sigma_{inel}/\sigma_{tot} \to 1/4$, which from unitarity, implies in $X(s) \to A = 3/4 = 0.75$.

3 Parametrization, Fit Procedures and Results

Our dataset comprises all the experimental data on the ratio X from pp and $\bar{p}p$ scattering in the energy interval from 5 GeV up to 8 TeV (41 points, 28 from pp and 13 from $\bar{p}p$) [9]. With this enlarged set (as compared with that in [1]), preliminary tests led us to change the parametrization used in [1] by the following suitable empirical ansatz

$$f(s) = \tanh\{\alpha + \beta\sqrt{\ln(s/s_0)} + \gamma \ln(s/s_0)\}, \tag{2}$$

where $s_0 = 25$ GeV^2 (the energy cutoff), α, β and γ are free fit parameters and for A representing the asymptotic limit, the ratio is given by

$$X(s) = Af(s). \tag{3}$$

The data reductions have been performed with the objects of the class TMinuit of ROOT Framework, with confidence level fixed at 68 %. For tests on the goodness of fit we shall consider the reduced Chi squared, χ^2/ν, and the corresponding integrated probability, $P(\chi^2, \nu)$. Since the parametrization is nonlinear in three parameters, different initial values must be tested in order to check the stability of the result. We have considered two variants in the fit procedures: either A fixed, so as to *impose* an asymptotic limit, or A as a free fit parameter, in order to *select* a possible asymptotic scenario.

Variant 1 - A Fixed. We have developed 5 tests with the three scenarios: (1) below the black-disk, either $A = 0.3$ (lowest value we have obtained in [2–4]) or $A = 0.436$ (the result from the TOTEM and

COMPETE parameterizations); (2) the black disk, $A = 0.5$; (3) above the black-disk, either $A = 0.75$ (possible "formal" result) or $A = 1$ (maximum unitarity). The statistical information on the fit results are given in Table 1 and the comparison with the experimental data in Figure 1. As illustration it is also shown the estimation of the ratio X from the Pierre Auger Collaboration results for σ_{tot} and σ_{inel} at 57 TeV (not included in the dataset). We conclude that all results present consistent and equivalent descriptions of the experimental data analyzed. In other words, the fit results with our empirical parametrization can not discriminate or select an asymptotic scenario.

Table 1: Statistical information on the fit results with Variant 1, $\nu = 38$ DOF.

A (fixed):	0.3	0.436	0.5	0.75	1.0
χ^2/ν:	0.789	0.774	0.778	0.787	0.790
$P(\chi^2, \nu)$:	0.812	0.840	0.843	0.823	0.818

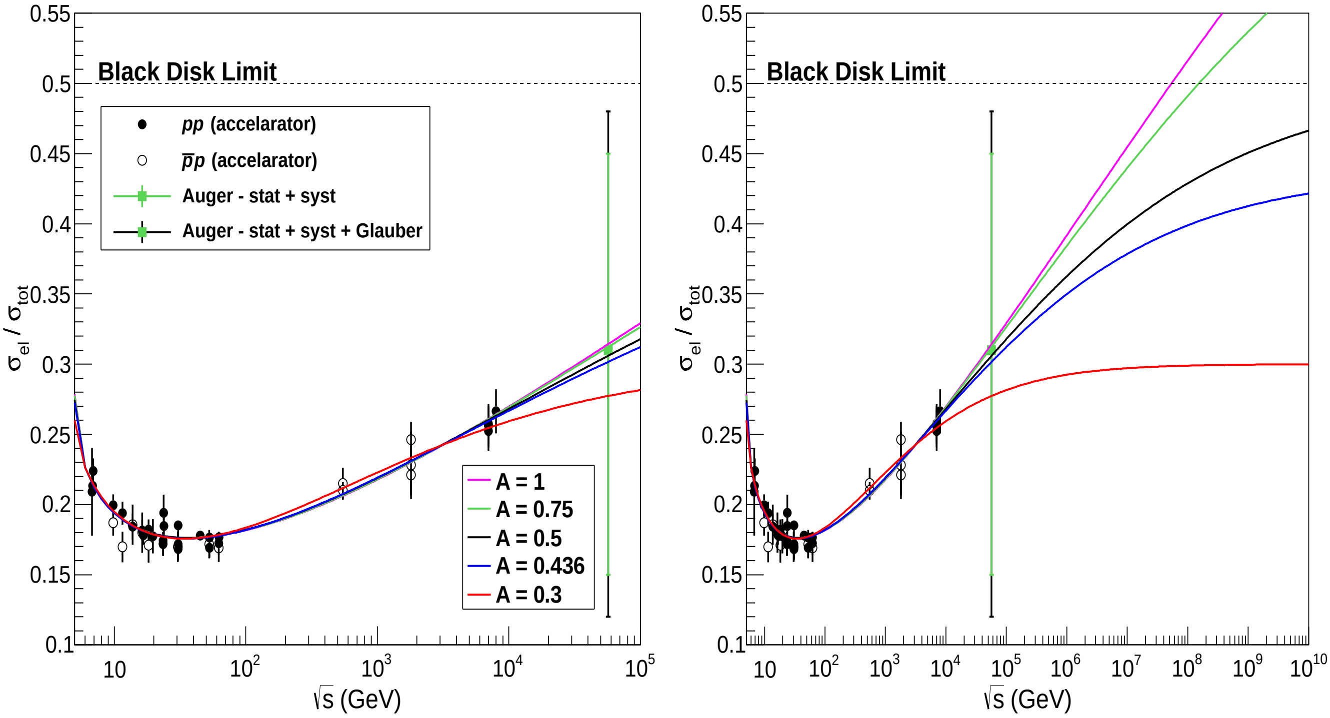

Figure 1: Fit results for the ratio $X(s)$ with Variant 1 (A fixed) and experimental data: up to $\sqrt{s} = 100$ TeV (left) and extrapolation to higher energies (right).

Variant 2 - A as a Free Parameter. Using as initial values of the parameters the final values obtained in Variant 1 and the corresponding values of A, the data reductions with four free parameters lead to the selected asymptotic scenario, defined by the final value of A. In the 5 cases investigated the data reductions converged to an unique solution within the uncertainties in the free parameters, with statistical results $\chi^2/\nu = 0.791$ and $P(\chi^2, \nu) = 0.814$, for $\nu = 37$ DOF and the following values of the free parameter:

$$A = 0.361 \pm 0.078, \quad \alpha = 0.96 \pm 0.32, \quad \beta = -0.43 \pm 0.19, \quad \gamma = 0.109 \pm 0.048.$$

The fit result with the uncertainty region, evaluated through analytical error propagation from the free parameters (one standard deviation), is displayed in Figure 2. For comparison, we have also included the result and corresponding uncertainty region for the case $A = 0.5$ fixed (black disk) and the central values for the cases $A = 0.436$ and $A = 0.3$ (same as Figure 1). We conclude that, asymptotically and within the uncertainties, our solution is not compatible with the black disk limit and the central values for the cases $A = 0.75$ and $A = 1.0$ neither (namely scenarios above the black disk). The central values in the cases of $A = 0.3$ and $A = 0.436$ lie within our uncertainty region. Therefore, our unique solution favors a scenario below the black-disk limit and we can infer, also from unitarity:

$$\frac{\sigma_{el}}{\sigma_{tot}} \to 0.36 \pm 0.08, \qquad \frac{\sigma_{inel}}{\sigma_{tot}} \to 0.64 \pm 0.08 \quad \text{as} \quad s \to \infty.$$

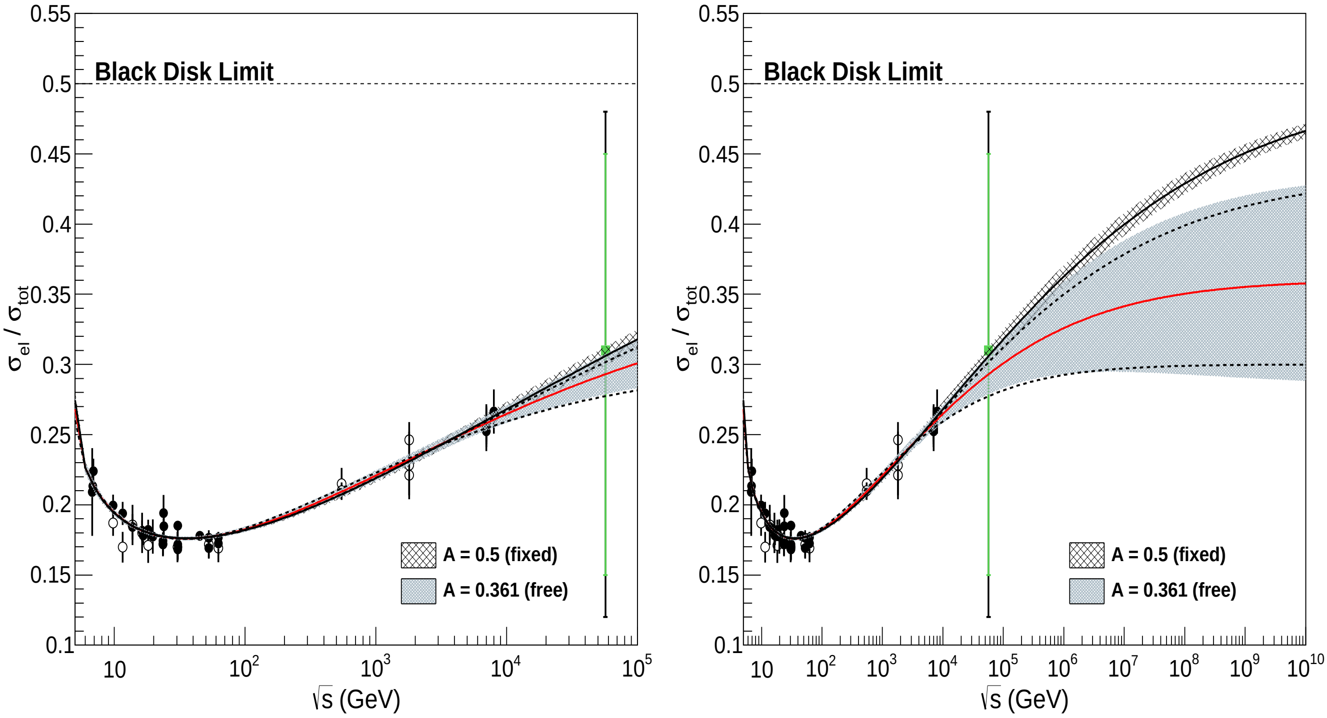

Figure 2: Fit results for the ratio $X(s)$ with Variant 2 (A as a free fit parameter) and experimental data: up to $\sqrt{s} = 100$ TeV (left) and extrapolation to higher energies (right).

4 Summary and Conclusions

We have introduced a novel suitable analytical parametrization for the ratio X and developed two variants as fit procedures to our dataset (pp and $\bar{p}p$ data at 5 GeV $\leq \sqrt{s} \leq 8$ TeV). In Variant 1, we impose different asymptotic limits by fixing A at 0.3, 0.436, 0.5, 0.75 and 1.0. All the results are consistent with the experimental data (Table 1 and Figure 1). Although the results do not discriminate an asymptotic scenario we can conclude that the black disk limit does not represent a definitive or unique solution. In Variant 2, with A as a free parameter, we have obtained an unique convergent solution, indicating a scenario below the black-disk: $A = 0.36 \pm 0.08$. Within the uncertainty, this asymptotic value is in agreement with the results obtained by Fagundes, Menon and Silva [2–4], the prediction from the parameterizations by the COMPETE and TOTEM Collaborations [5,6] and also with recent phenomenological analysis by Kohara, Ferreira and Kodama which indicates A approximately 1/3 [10].

As we have discussed [2–4], a scenario below the black disk is not in disagreement with the Pumplin bound, namely

$$\frac{\sigma_{el}}{\sigma_{tot}} + \frac{\sigma_{diff}}{\sigma_{tot}} \leq 1/2,$$

where σ_{diff} stands for the soft diffractive processes (single and double dissociation). Therefore, in case of saturation of the Pumplin bound, it is possible to infer $\sigma_{diff}/\sigma_{tot} \rightarrow 0.14 \pm 0.08$ as $s \rightarrow \infty$.

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