PHYSICAL REVIEW D VOLUME 43, NUMBER 10 15 MAY 1991

Cornwall-Norton model in the strong-coupling regime

A. A. Natale
Instituto de Física Teórica, Universidade Estadual Paulista, Rua Pamplona 145, 01405 São Paulo, São Paulo, Brazil
(Received 18 December 1990)

The Cornwall-Norton model is studied in the strong-coupling regime. It is shown that the fermionic self-energy at large momenta behaves as $\Sigma(p)\sim(m^2/p)\ln(p/m)$. We verify that in the strong-coupling phase the dynamically generated masses of gauge and scalar bosons are of the same order, and the essential features of the model remain intact.

The Cornwall-Norton model[1] is one of the simplest models of dynamical gauge-boson-mass generation in four-dimensional gauge theories. In this model the Schwinger-Dyson equation for the fermionic self-energy is quite similar to the one of quantum electrodynamics (QED), and assuming that this equation has a nontrivial solution such as the one proposed for QED by Johnson, Baker, and Willey[2] (JBW), it was shown that the gauge boson acquires a dynamical mass. *A posteriori*, with the use of an effective potential for composite operators,[3] it was found that the mass generation occurs only when the coupling constant has a moderately small critical value. The model also contains a composite scalar boson,[4] which plays the role of the standard-model Higgs boson and whose mass is numerically small when compared to the gauge-boson mass.

Nowadays it is believed that the JBW solution[2] of mass generation in QED is not realized in nature. Maskawa and Nakagima[5] have shown that QED admits a nontrivial solution for the gap equation only when the coupling constant α is larger than $\alpha_c\equiv\pi/3$. Further studies[6] confirmed this result, which also imply that even in the presence of a bare mass the nontrivial solution for $\alpha<\alpha_c$ disappears when we go to the chiral limit.[5,6]

If the chiral-symmetry-breaking solution of QED at weak coupling does not exist, we could imagine that many of the results of Refs. 1, 3, and 4 can be substantially modified, because part of them were obtained assuming a weak-coupling regime. However, we believe that the nice characteristics of the model should remain intact, as long as they only depend on the existence of a nontrivial fermionic self-energy solution. Recalling again the similarity between the gap equation of QED and the Cornwall-Norton one, we come to the conclusion that also the Cornwall-Norton model must be realized in the strong-coupling regime.

It has been claimed that QED possesses a nontrivial ultraviolet fixed point at $\alpha_c=\pi/3$,[7] and this hypothesis is corroborated by numerical simulations.[8] One might wonder if the same fixed point does not appear in the Cornwall-Norton model. If this is the case, the model is one example of gauge-boson-mass generation in the presence of a fixed point, and its study may also be interesting to learn aspects of dynamical mass generation with U(1) technicolor theories.[9]

In this work we study the Cornwall-Norton model in the strong-coupling regime. We discuss the solution of the gap equation, compute the effective potential of composite operators at stationary points, and calculate the gauge- and scalar-boson masses.

The Schwinger-Dyson equation for the fermionic propagator of the model is[1]

$$S^{-1}(p)=\not p+m_0+ig_A^2\int\frac{d^4k}{(2\pi)^4}G^A_{\mu\nu}(k)\gamma^\mu S(p-k)\Gamma^\nu_A(p-k,p)+ig_B^2\int\frac{d^4k}{(2\pi)^4}G^B_{\mu\nu}(k)\gamma^\mu\tau_2S(p-k)\Gamma^\nu_B(p-k,p)\ ,\quad(1)$$

where $G^{A(B)}$, S indicate, respectively, the complete gauge bosons and fermion propagators, and Γ are the vertex functions. In terms of the self-energy $\Sigma(p)$ we can write

$$S^{-1}(p)-\not p-m_0=\Sigma(p)\equiv\Sigma_S(p)+\tau_3\Sigma_A(p)\ ,\quad(2)$$

where we divided $\Sigma(p)$ in the chirally symmetric (Σ_S) and asymmetric (Σ_A) parts. Using Eqs. (1) and (2) in the lowest order of Γ, and working in the Landau gauge we obtain

$$\Sigma_A(p)=i(g_B^2-g_A^2)\int\frac{d^4k}{(2\pi)^4}\frac{g_{\mu\nu}-k_\mu k_\nu/k^2}{k^2}\gamma^\mu\frac{\Sigma_A(p-k)}{[\not p-\not k+m_0+\Sigma_S(p-k)]^2-\Sigma_A^2(p-k)}\gamma^\nu\ .\quad(3)$$

Changing $(g_A^2-g_B^2)$ in Eq. (3) by e^2, where e is the electromagnetic coupling constant, we arrive at the QED gap equation. Therefore, following Ref. 2, it was assumed that Eq. (3) has the solution

$$\Sigma_A(p^2) \underset{p^2\to\infty}{\sim} \delta m \left[\frac{p^2}{m^2}\right]^{-\epsilon}, \tag{4}$$

where δm is the dynamical mass, and

$$\epsilon=\frac{3}{16\pi^2}(g_A^2-g_B^2)+O(g_A^2,g_B^2) \tag{5}$$

subjected to the condition $0<g_A^2-g_B^2<4\pi^2/3$, which delimits the weak-coupling region.

Analyzing the B-gauge-boson self-energy

$$\Pi^B_{\mu\nu}\cong -ig_B^2\int\frac{d^4p}{(2\pi)^4}\mathrm{Tr}[\Gamma^B_\mu(p-k,p)S(p)\gamma_\nu\tau_2S(p-k)]\ , \tag{6}$$

we can verify that at zero momentum the B boson acquires a mass given as a function of $\Sigma_A(p)$ by[1]

$$\Pi^B(0)\cong 16\pi^2g_B^2\int\frac{dp^2}{(2\pi)^4}\frac{p^2\Sigma_A^2(p^2)}{(p^2+m^2)^2}\ , \tag{7}$$

where the momentum is in the Euclidean space and m is the total mass (i.e., bare plus dynamical). The introduction of Eq. (4) into Eq. (7) entails

$$\Pi^B(0)\equiv M_B^2=\frac{g_B^2}{2\pi^2\epsilon}(\delta m)^2+O(g_A^2,g_B^2)\ . \tag{8}$$

The existence of a massive scalar boson was inferred in an ingenious way.[4] The kinetic term of the effective action for composite operators was computed, leading to an effective Ginzburg-Landau Lagrangian density of the form[4]

$$\Omega=\int d^4x\left[\tfrac{1}{2}(\partial_\mu\phi)^2+\frac{A}{4}\phi^4-\frac{B}{6}\phi^6\right], \tag{9}$$

where A and B are functions of ϵ, δm, and m, and whose minimization imply in one composite scalar-boson mass given by[4]

$$M_\phi^2=\frac{4\epsilon}{5}\frac{(\delta m)^4}{m^2}\ . \tag{10}$$

With the critical value found for ϵ in Refs. 3 and 4 it was determined one numerical hierarchy between M_ϕ and M_B, where the gauge boson is at least 1 order of magnitude heavier than the scalar. The procedure to arrive at Eq. (9) is explained in Ref. 4 (see also Ref. 10), and as emphasized by Cornwall and Shellard,[4] it is not expected to be valid at strong coupling. Notice that the hierarchy between Eqs. (8) and (10) is surely one of the results that can be modified in the strong-coupling phase.

Because of the formal identity between the Cornwall-Norton gap equation with the QED one when we change $(g_A^2-g_B^2)/4\pi$ by α ($\equiv e^2/4\pi$) in Eq. (3), we may translate all the results of Refs. 5 and 6, concerning the QED gap equation, to our problem. These results can be stated as follows. (a) In a theory chirally symmetric ($m_0=0$), (i) when $\alpha<\alpha_c\equiv\pi/3$ there is only the trivial solution ($\Sigma=0$), and (ii) when $\alpha>\alpha_c$, in addition to the trivial solution, $\Sigma(p)$ has an infinite number of oscillating solutions. (b) In a theory with a bare mass ($m_0\neq 0$), (i) when $\alpha<\alpha_c$ there is only one nontrivial solution, but it turns out to be trivial in the chiral limit ($m_0\to 0$), and (ii) when $\alpha>\alpha_c$ there is a finite number of nontrivial solutions. For simplicity, and as we want to study a real theory of dynamical mass generation, we will set $m_0=0$ in Eq. (1), consequently the dynamical mass, denoted by δm, will be equal to the total mass m and $\Sigma(p)$ will be nontrivial for $\alpha>\alpha_c\equiv\pi/3$.

Equation (3) can be transformed into a differential equation whose solution is[7] [hereafter we drop the symbol A from $\Sigma_A(p)$]

$$\Sigma(p^2)=Cm\ {}_2F_1\left[\tfrac{1}{2}+\gamma',\tfrac{1}{2}-\gamma',2;\frac{-p^2}{2}\right], \tag{11}$$

where C is a constant determined by the boundary conditions and

$$\gamma'\equiv i\gamma=\frac{1}{2}\left[1-\frac{3}{\pi}\left[\frac{g_A^2-g_B^2}{4\pi}\right]\right]^{1/2}. \tag{12}$$

At large momenta Eq. (11) can be expanded as

$$\Sigma(p)\underset{p^2\gg m^2}{\sim} m\left[\frac{p^2}{m^2}\right]^{-1/2}\left[\frac{\coth\pi\gamma}{\pi\gamma(\gamma^2+\frac{1}{4})}\right]^{1/2}\times\sin\left[2\gamma\ln\left[\frac{p}{m}\right]+\beta\right] \tag{13}$$

where

$$\beta=\arg\left[\frac{\Gamma(1+2\gamma')}{\Gamma^2(\frac{1}{2}+\gamma')}\right]-\arctan 2\gamma\ . \tag{14}$$

At $\gamma=0$ (or $g_A^2-g_B^2=4\pi^2/3$) Eq. (13) assumes the simpler form

$$\Sigma(p)\underset{p^2\gg m^2}{\sim}\frac{m^2}{p}\ln\left[\frac{p}{m}\right]. \tag{15}$$

With the help of Eq. (13) we can determine the gauge-boson mass [Eq. (7)], and, following Ref. 3, we can compute the effective potential for composite operators (Ω). The idea of computing Ω is to determine for which value of γ (or, $g_A^2-g_B^2$) occurs the symmetry breaking; i.e., the minimum of energy will happen for a specific value of the coupling constant. However, an easier way to extract information about the minimum of energy is to compute $\langle\Omega\rangle$, i.e., the values of Ω at stationary points:

$$\langle\Omega\rangle=2i\int\frac{d^4p}{(2\pi)^4}\{\ln[1-\Sigma^2(p)/p^2]+\Sigma^2(p)/[p^2-\Sigma^2(p)]\}\ . \tag{16}$$

The computation of Eq. (16) is enough to determine the critical value of $(g_A^2-g_B^2)$ as will be shown in the following. The value of $\langle\Omega\rangle$ is much less sensible than Ω to any possible deviation of a linearized expression of $\Sigma(p)$ from the actual solution of the nonlinear gap equation. Notice that all the information about the gauge bosons which appears in Ω was swept away in Eq. (16); it enters only in the dependence of $\Sigma(p)$ on $(g_A^2-g_B^2)$.

To determine $\langle\Omega\rangle$ as a function of γ we need only the ultraviolet part of $\Sigma(p)$ given by Eq. (13). The infrared part, at least in the leading approximation, does not depend on γ.[6] Considering that $\Sigma(p)$ naturally damps the integrals in Eq. (16), we can expand the ultraviolet part of $\langle\Omega\rangle$ in powers of $\Sigma(p)/p$, and introducing the variable $x=p^2/m^2$ we obtain the leading term of $\langle\Omega\rangle$:

$$\frac{8\pi^2}{m^4}\langle\Omega\rangle\simeq-\frac{1}{2}\int_1^\infty dx\frac{\bar{\Sigma}^4}{x}+O\left[\frac{\bar{\Sigma}^6}{x^2}\right], \tag{17}$$

where $\bar{\Sigma}=\Sigma/m$, and only the region $p^2>m^2$ was considered. This calculation has already been done for QED, where the substitution of Eq. (13) into Eq. (17) yields[11]

$$\frac{8\pi^2}{m^4}\langle\Omega\rangle\simeq\frac{-1}{32}\left[\frac{\coth(\pi\gamma)}{\pi\gamma(\gamma^2+\frac{1}{4})}\right]^2\left[3-\frac{(\gamma^2+1)(2\gamma\sin4\beta-\cos4\beta)+4(4\gamma^2+1)(\cos2\beta-\gamma\sin2\beta)}{(4\gamma^4+5\gamma^2+1)}\right]. \tag{18}$$

For $(g_A^2-g_B^2)/4\pi<\pi/3$ only the trivial solution $[\Sigma(p)=0]$ exists,[5,6] and for $(g_A^2-g_B^2)/4\pi\geq\pi/3$ we verify that the minimum of energy [i.e., the minimum of Eq. (18)] occurs at $(g_A^2-g_B^2)/4\pi=\pi/3$ (or $\gamma=0$). We conclude that this point corresponds to an absolute minimum.[11] Notwithstanding, following Wilson's work on the renormalization-group equations[12] we know that a fixed point is a stationary point of a potential; therefore, we identify the value of $(g_A^2-g_B^2)/4\pi=\pi/3$ as a fixed point of the theory.

The dynamical symmetry breaking in the Cornwall-Norton model, as discussed above, is realized at strong coupling when $(g_A^2-g_B^2)/4\pi=\pi/3$; therefore, the fermionic self-energy at large momenta has the universal behavior given by Eq. (15), and this is the expression of $\Sigma(p)$ to be used in the calculation of gauge- and scalar-boson masses.

The masses of gauge and scalar bosons and the hierarchy found between them in the weak-coupling regime,[4] as will be shown in the following, are modified at strong coupling. The calculation of the gauge-boson mass is straightforward; from Eqs. (7) and (15) we find

$$M_B^2\approx\frac{0.41g_B^2}{\pi^2}m^2, \tag{19}$$

where the integral in Eq. (7), for simplicity, was cut in the infrared at $p=m$ and evaluated numerically. The main difference with respect to the weak-coupling result [Eq. (8)] is a factor $1/\epsilon$.

In the case of the scalar-boson mass a method devised by Elias and Scadron[13] to compute the σ-meson mass of quantum chromodynamics may be helpful. The scalar mass appears when we compute the fermion-tadpole diagram of Fig. 1, whose result is

$$m=-g_{\phi\bar{\psi}\psi}^2\frac{\langle\bar{\psi}\psi\rangle_R}{M_\phi^2}, \tag{20}$$

where $g_{\phi\bar{\psi}\psi}$ is the composite scalar boson coupling to the fermions, and $\langle\bar{\psi}\psi\rangle_R$ is the renormalized fermion condensate. The calculation of the condensate is identical to the case of QED, which was determined by Bardeen, Leung, and Love,[14] and is given by

$$\langle\bar{\psi}\psi\rangle_R\simeq-\frac{1.09}{2\pi^2}m^3 \tag{21}$$

when evaluated at the critical coupling constant.

When the chiral symmetry is broken the theory also forms Goldstone bosons, and the coupling of these bosons to the fermions ($g_{\pi\bar{\psi}\psi}$) as well as the scalar coupling have their strength given by the Goldberger-Treiman relation

$$g_{\phi\bar{\psi}\psi}=g_{\pi\bar{\psi}\psi}=\frac{m}{f_\pi}, \tag{22}$$

where f_π is the Goldstone-boson decay constant obtained through the Pagels-Stokar formula[15]

$$f_\pi^2=\frac{m^2}{4\pi^2}\int_1^\infty dx\,x\bar{\Sigma}(x)\frac{\bar{\Sigma}(x)-\frac{1}{2}x\bar{\Sigma}'(x)}{x+\bar{\Sigma}^2(x)^2}, \tag{23}$$

where $x=p^2/m^2$ and, for simplicity and consistency with our previous calculations, we introduced a cutoff at $p^2=m^2$. Using $\Sigma(p)$ given by Eq. (15) we obtain $f_\pi^2\simeq0.49m^2/4\pi^2$, and combining the above expressions we arrived at the scalar-boson mass $M_\phi\simeq0.7m$. Comparing this result with Eq. (10) we notice again a difference of a factor ϵ.

The ratio between the gauge- and scalar-boson mass is

$$\frac{M_B^2}{M_\phi^2}\approx\frac{0.26g_B^2}{\pi}. \tag{24}$$

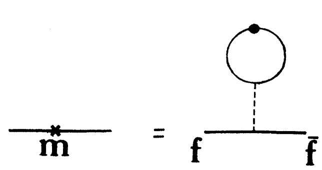

FIG. 1. Fermion mass gap equation in terms of the scalar-boson tadpole graph.

Apart from the condition $(g_A^2-g_B^2)/4\pi\approx\pi/3$, g_B^2 is not constrained, and if $g_B^2/4\pi\sim 1$ we do not have any hierarchy between gauge and scalar masses, as can be seen from Eq. (24). However, if g_B^2 is small we have the opposite of what was obtained at weak coupling, i.e., $M_B^2 \ll M_\phi^2$.

The existence of a scalar boson lighter than the dynamical mass, as obtained in Ref. 4 at weak coupling, is an interesting feature because in a realistic theory this scalar would play the role of the Higgs boson, and we could think of a model where the Higgs-boson mass would lie below the characteristic mass scale of the standard model. Unfortunately, as we verified here, if the theory is realized at strong coupling, the Higgs-boson mass will probably be of the order of the dynamical mass.

In conclusion, in this paper we have computed the effective potential for composite operators at stationary points and shown that the minimum of energy occurs for $(g_A^2-g_B^2)/4\pi\approx\pi/3$. Therefore, the asymptotic fermionic self-energy behaves as $\Sigma(p)\sim(m^2/p)\ln(p/m)$. We computed the gauge- and scalar-boson masses, and verified that they are of the same order at strong coupling. Although the masses are modified, the many nice properties of the model remain intact. It would be interesting to investigate if these results are changed by the addition of the chirally invariant four-fermion operator (with coupling constant G) to the Lagrangian of the model.[16] Naively, even if the fixed point is moved to some point in the plane (α, G) (Ref. 17) the symmetry breaking still happens at strong coupling (α or G), and we should not expect any gross deviation in our results.

This work was partially supported by Conselho Nacional de Pesquisa.

[1]J. M. Cornwall and R. E. Norton, Phys. Rev. D **8**, 3338 (1973).

[2]K. Johnson, M. Baker, and R. Willey, Phys. Rev. **136**, B1111 (1964); Th. A. J. Maris, V. E. H. Herscovitz, and G. Jacob, Phys. Rev. Lett. **12**, 313 (1964).

[3]J. M. Cornwall, R. Jackiw, and E. Tomboulis, Phys. Rev. D **10**, 2428 (1974).

[4]J. M. Cornwall and R. C. Shellard, Phys. Rev. D **18**, 1216 (1978).

[5]T. Maskawa and H. Nakagima, Prog. Theor. Phys. **52**, 1326 (1974); **54**, 860 (1975).

[6]R. Fukuda and T. Kugo, Nucl. Phys. **B117**, 250 (1976).

[7]V. A. Miransky, Nuovo Cimento A **90**, 149 (1985); Zh. Eksp. Teor. Fiz. **88**, 1514 (1985) [Sov. Phys. JETP **61**, 905 (1985)].

[8]J. B. Kogut, E. Dagotto, and A. Kocic, Phys. Rev. Lett. **61**, 2416 (1988); Nucl. Phys. **B317**, 273 (1989); **B317**, 271 (1989).

[9]B. Holdom, Phys. Rev. D **24**, 1441 (1981); M. Bando, T. Morozumi, H. So, and K. Yamawaki, Phys. Rev. Lett. **59**, 389 (1987).

[10]A. A. Natale, Nucl. Phys. **B226**, 365 (1983).

[11]A. A. Natale, Phys. Lett. B **250**, 139 (1990).

[12]K. G. Wilson, Phys. Rev. B **4**, 3174 (1971); K. G. Wilson and J. Kogut, Phys. Rep. **12C**, 75 (1974).

[13]V. Elias and M. D. Scadron, Phys. Rev. Lett. **53**, 1129 (1984).

[14]W. A. Bardeen, C. N. Leung, and S. T. Love, Nucl. Phys. **B323**, 493 (1989).

[15]H. Pagels and S. Stokar, Phys. Rev. D **20**, 2947 (1979).

[16]W. A. Bardeen, C. N. Leung, and S. T. Love, Phys. Rev. Lett. **56**, 1230 (1986); Nucl. Phys. **B273**, 649 (1986).

[17]A. Kocic, S. Hands, J. B. Kogut, and E. Dagotto, Nucl. Phys. **B347**, 217 (1990); E. Dagotto, University of California at Santa Barbara Report No. ITP-Santa Barbara-90-149, 1990 (unpublished).