Covariance for electromagnetic current for two bosons at higher order in the light front

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Abstract. We show that the electromagnetic current for a system composed by two charged bosons has a structure of many bodies even in the impulse approximation, when described in the time x^+. In terms of the two body component for the bound state, the current contains two body operators. We discuss the process of pair creation by the photon and interpret it as a zero mode contribution to the current and its consequences for the components of currents in the light front.

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INTRODUCTION

In recent years two relativistic approaches have united the description of the bound state and scattering states with a consistent description of the electromagnetic current operator, necessary to investigate the bosonic system [1]. The first one is based in the Bethe-Salpeter formalism [2]. This formalism is four dimensional and explicitly covariant under Lorentz transoformations. The technique of calculations is based on Feynman diagrams and their associated rules. The second one is based on the light-front dynamics [3].

A description of few body systems in the light front allows an optimization in the Fock space representation. In the instantaneous form description, any Fock space truncation is stable under certain Lorentz transformations (kinematic transformations), which does not occur in the light front [4].

DESCRIPTION OF THE METHOD

In this work we build the electromagnetic current operator for a system composed of two bosons up to order g^2 in the light front in the *ladder* approximation. The technique employed to deduce these operators is the definition of global propagators in the light front when an electromagnetic background field acts on one of the particles. In this manner we can separate the diagrams that already are contained in the expansion of the Bethe-Salpeter equation when we consider the one body electromagnetic current from the corresponding two body diagrams, irreducible to one body in the light front. Although we are in fact calculating corrections to the propagator of two bosons in the

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background electromagnetic field, we stretch the language using terms such as "current operator" and "current" to designate such corrections.

Using this technique we obtain the electromagnetic current for a bound system with constant vertex in the Breit reference frame in the limit of photon's momentum transfer going to zero $q^+ = q^0 + q^3 \to 0$. In this limit the pair creation term survives for the current component J^- and is responsible for current covariance in this model for order g^2.

Particles propagating forwardly in time of the null plane have positive k^+. Integration in the null plane energy is due to projecting the propagation of the physical system in the time x^+, which imposes that in the Bethe-Salpeter equation and in the pertubative expansion of the propagator for two particle system between two time instants x^+, it will only appear non vanishing contributions for $k^+ > 0$.

Contruction of the propagator

In a recent article [1] it has been considered, in the zeroth order of perturbative coupling, the calculation of the electromagnetic current in the light front coordinates for scalar bosons in the electromagnetic background field. The calculation is considered only in the region $0 < k_2^+ < k_i^+ < k_4^+ < k_f^+$ and its combinations. The same result is found in the article by Marinho, Frederico and Sauer [5], using a different technique.

The Lagrangean density for interacting scalar and electromagnetic fields is given by

$$\begin{aligned} \pounds &= D_\mu \phi D^\mu \phi^* - m^2 \phi^* \phi \\ &= \partial_\mu \phi \partial^\mu \phi^* - m^2 \phi^* \phi + ieA^\mu \left(\phi \partial_\mu \phi^* - \phi^* \partial_\mu \phi \right) + e^2 A^\mu A_\mu \phi \phi^* . \end{aligned} \tag{1}$$

In the calculation of the propagator for a particle in a background field we use the interaction Lagrangean of a scalar field and electromagnetic field. The Lagrangean (1) shows immediately that there are two types of vertices. The first term corresponds to a vertex containing a photon and two scalar particles. The second vertex contains two photons and two scalar particles.

The propagator in the light front with background field in the second order in the electromagnetic coupling constant is:

$$\begin{aligned} S(x^+) &= \int d\bar{x}_1^+ d\bar{x}_2^+ d\bar{x}_3^+ dq^- F(q^-) e^{-\frac{i}{2} q^- \bar{x}_3^+} S_1(\bar{x}_1^+) S_2(\bar{x}_2^+) \times \\ &\quad \times S_4(x^+ - \bar{x}_2^+) S_\sigma(\bar{x}_2^+ - \bar{x}_1^+) \left[S_3 \frac{\partial S_5}{\partial \bar{x}_3^+} - \frac{\partial S_3}{\partial \bar{x}_3^+} S_5 \right], \end{aligned} \tag{2}$$

where $F(q^-)$ is the form factor and

$$S_i(x^+) = \frac{i}{2(2\pi)} \int \frac{dk_i^-}{k_i^+} \frac{e^{-\frac{i}{2} k_i^- \bar{x}_i^+}}{(k_i^- - \frac{k_{1\perp}^2 + m^2}{k_i^+} + \frac{i\varepsilon}{k_i^+})}, \tag{3}$$

$$\frac{\partial S_{[3,5]}}{\partial \bar{x}_3^+} = \frac{1}{2^2(2\pi)} \int \frac{dk^-_{[3,5]}}{k^+_{[3,5]}} \frac{k_3^- e^{-\frac{i}{2}k_3^-[(\bar{x}_3^+ - \bar{x}_1^+),\,(\bar{x}^+ - \bar{x}_3^+)]}}{(k^-_{[3,5]} - \frac{k^2_{[3,5]\perp}+m^2}{k^+_{[3,5]}} + \frac{i\varepsilon}{k^+_{[3,5]}})}\,, \tag{4}$$

with $i = 1,2,3,4,5,\sigma$ labeling the bosons respectively.

Substitution of relations in equation (4) into equation (2) and integration over $d\bar{x}_1^+ d\bar{x}_2^+ d\bar{x}_3^+$ and posterior Fourier transformation via $\widetilde{S}(k_f^-) = \int dx^+ e^{\frac{i}{2}k_f^- x^+} S(x^+)$, yields momentum conservation delta functions among k_1^+, k_2^+, k_3^+, k_4^+, k_5^+ and k_σ^+, which therefore entails the following relations:

$$\begin{array}{lllll} k_i = \text{initial momentum} & k_f = k_i + q & k_\sigma = k_4 - k_2 & k_\sigma = k_1 - k_3 & q = k_5 - k_3 \\ k_f = \text{final momentum} & k_3 = k_i - k_4 & k_i = k_1 + k_2 & k_f = k_4 + k_5 & \end{array} \tag{5}$$

The final propagator then can be written in terms of two integration momenta only, and in this case we choose "spectator" particles in relation to the current the bosons 2 and 4 such that

$$\begin{aligned} \widetilde{S}(k_f^-) &= \frac{ig^2}{2^3(2\pi)^2} \int \frac{dq^- F(q^-) dk_2^- dk_4^- \left(k_f^- + k_i^- - 2k_4^-\right)}{(k_i - k_2)^+ k_2^+ (k_i - k_4)^+ k_4^+ \left(k_f - k_4\right)^+ (k_4 - k_2)^+} \times \\ &\times \frac{\left[k_4^- - k_i^- + (k_i - k_4)_{on} - \frac{i\varepsilon}{(k_i - k_4)^+}\right]^{-1} \left[k_4^- - k_{4_{on}} + \frac{i\varepsilon}{k_4^+}\right]^{-1}}{\left[k_2^- - k_i^- + (k_i - k_2)_{on} - \frac{i\varepsilon}{(k_i - k_2)^+}\right] \left[k_2^- - k_{2_{on}} + \frac{i\varepsilon}{k_2^+}\right]} \times \\ &\times \frac{\left[k_4^- - k_f^- + (k_f - k_4)_{on} - \frac{i\varepsilon}{\left(k_f - k_4\right)^+}\right]^{-1}}{\left[k_2^- - k_4^- + (k_4 - k_2)_{on} - \frac{i\varepsilon}{(k_4 - k_2)^+}\right]}. \end{aligned} \tag{6}$$

The propagator in the momentum representation is given by (6), in terms of k_i^-, k_f^-, k_4^- and k_2^-. To proceed with the calculation, we need to choose the integration regions that relate these momenta. There are altogether 20 such regions, some of which yield vanishing results and some which give Z-type diagram contributions.

THE ELECTROMAGNETIC CURRENT

In order to calculate the electromagnetic current generated by the diverse configurations, we need to consider the matrix elements $J^{-,+,\perp} = \langle \Gamma | \widehat{O}^{-,+,\perp} | \Gamma \rangle$, where Γ is the constant vertex and $\widehat{\mathscr{O}}^{-,+,\perp}$ are the current operators which can be obtained directly from the sum of final results from each region. As an example, for region 3, we have

$$\widehat{\mathscr{O}}_3^- = -\frac{ig^2}{8} \frac{\theta\left(k_f^+ - k_4^+\right) \theta\left(k_4^+ - k_i^+\right) \theta\left(k_i^+ - k_2^+\right) \theta\left(k_2^+\right) \left[k_f^- - k_i^- - 2(k_f - k_4)_{on}\right]}{(k_i - k_2)^+ k_2^+ (k_i - k_4)^+ k_4^+ \left(k_f - k_4\right)^+ (k_4 - k_2)^+} \times$$

$$\times \frac{\left[k_f^- - k_i^- + (k_i - k_4)_{on} - (k_f - k_4)_{on}\right]^{-1} \left[k_f^- - k_{4_{on}} - (k_f - k_4)_{on}\right]^{-1}}{\left[k_i^- - (k_i - k_2)_{on} - k_{2_{on}}\right] \left[k_f^- - k_{2_{on}} - (k_f - k_4)_{on} - (k_4 - k_2)_{on}\right]}. \tag{7}$$

Introducing the unity resolution in $J^{-,+,\perp} = \langle \Gamma \,| \widehat{O}^{-,+,\perp} |\, \Gamma \rangle$,

$$\left\langle \Gamma \left| \widehat{O}^{-,+,\perp} \right| \Gamma \right\rangle = \Gamma^2 \int dk_2^+ d^2k_{2_\perp} dk_4^+ d^2k_{4_\perp} \mathscr{O}^{-,+,\perp}, \tag{8}$$

where we integrate in all momenta and so as in the case of propagator, we use the deltas to solve all integrals, except dk_2 and dk_4. The next step is to make two changes of variables that facilitate the integration, $x = \left(k_i^+ - k_2^+\right)\left(q^+\right)^{-1}$ and $y = \left(k_f^+ - k_4^+\right)\left(q^+\right)^{-1}$. Using the results in a general form, we count the terms in q^+ to a quick analysis of the result in the Breit's frame, where $q^+ \to 0$.

We then perceive that multiplying all these factors that it will always remain at least a $\left(q^+\right)^1$, and in the limit $q^+ \to 0$, the current for all regions vanishes. The conclusion is that the introduction of a virtual boson in comparison to the configuration worked out in the former article [1], does not alter the current, since there is this vanishing off of the terms in the Breit's reference frame.

CONCLUSION

In this work we performed the calculations for corrections to the propagator in a background field up to second order in the coupling. We obtained more diagrams than it was considered in a recent article [5], just those in which antiparticles appear. The "Z-graph" appears naturally in our approach. Yet in the Breit's reference frame these diagrams do not contribute to the current in order g^2.

For orders in g^n, perhaps it may be possible to devise a recipe on how to introduce correctly the orders in q^+ so that the results in some regions survive, as in [1] in the Breit's frame.

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